Online Appendix to
"Federal Funds Rates Based on Seven Simple Monetary Policy Rules"
Federal Reserve Bank of Cleveland *Economic Commentary*


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This online appendix accompanies and provides additional background materials for the Federal Reserve Bank of Cleveland *Economic Commentary* "Federal Funds Rates Based on Seven Simple Monetary Policy Rules."


## I. Additional Background on General Simple Monetary Policy Rules


We start with a generalized form for a simple monetary policy rule:

$$i_t = \rho i_{t-1} + (1-\rho)[r^* + \pi_t + \alpha(\text{inflation gap}) + \beta(\text{activity gap})].$$

In this rule, $i_t$ is the central bank's policy rate at time $t$; $r^*$ is the equilibrium real interest rate, which within the above rule can be thought of as the long-run funds rate adjusted for inflation; the inflation gap is the difference between inflation $\pi_t$ and target inflation $\pi^*$; and the activity gap is either the output gap—the percentage difference between actual output $Y_t$ and potential output $Y_t^*$—or the negative of the unemployment gap—the percentage point difference between the unemployment rate $u_t$ and the natural rate of unemployment $u_t^*$.

We cover inertia ($\rho$) in the policy rule in the next section and walk through the other elements of this general rule in this section.


## The Equilibrium Real Interest Rate, $r^*$


Several variables in the general policy rule refer to the level of a macroeconomic variable that would prevail in a hypothetical equilibrium state. Because such an equilibrium is unobservable, and because of data limitations—in terms of timing, quality, or both—considerable uncertainty surrounds any estimate of an equilibrium variable.

Carlstrom and Stehulak (2015) define the equilibrium real interest rate, or the long-run natural rate of interest, as the "real (inflation-adjusted) safe interest rate that the economy will converge to over time." But in more general terms, $r*$ is defined as the real interest rate consistent with a zero output gap—i.e., a state in which the economy is operating at its full potential, discussed below—and stable inflation. Monetary policy is contractionary if $r_t > r*$, expansionary if $r_t < r*$, and neutral if $r_t = r*$ (see Laubauch and Williams 2003 for more discussion), where $r_t$ denotes the current real interest rate. Carlstrom and Stehulak (2015) caution, "It is important to remember that this [equilibrium] interest rate [$r*$] is not a policy choice but instead is governed by factors such as world savings and interest rates, productivity growth, and demographics." In other words, the equilibrium real interest rate is determined by deep forces within the economy, and is not under the control of the monetary authority. While monetary policy can shift the current real interest rate away from this equilibrium rate, it cannot do this indefinitely.

As Canzoneri et al. (2015) note, it would be quite helpful for the central bank to know this rate. The original "Taylor rule" (Taylor 1993) assumed that $r*$ was both known and fixed at 2 percent. But its value at any given time is highly uncertain and there are a number of competing methods for estimating its value.[1] There is increasing evidence that this rate is not constant over time, so it should have a $t$ subscript, and that the current long-run normal level for the real interest rate is notably lower than 2 percent (see, e.g., Carlstrom and Stehulak 2015 and Williams 2015).[2]

Arguably, the most well-known method for estimating $r_t*$ is that of Laubach and Williams (2003). The original paper estimated $r_t*$ over the period 1961 to 2002, where the $t$ subscript captures potential time-variation, but estimates are now updated quarterly and made publicly available on the website of the Federal Reserve Bank of San Francisco. Unfortunately, $r_t*$ estimates "are very imprecise and are subject to considerable real-time mismeasurement" (Laubach and Williams 2003), leading the authors to advocate for monetary policy tools and approaches that are robust to this uncertainty (Laubach and Williams 2015).

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[1] See Laubach and Williams (2003), Clark and Kozicki (2005), Justiniano and Primiceri (2010), Barsky et al. (2014), Kiley (2015), and Hamilton et al. (2015) for various methods and further discussion.
[2] See Carvalho et al. (2016) for a discussion of the influence of demographic changes on the equilibrium real interest rate.

To capture uncertainty surrounding $r_t^*$, we considered two alternative estimates. For most policy rules, we took $r_t^*$ to be constant going forward and equal to the most recently available median long-run projection of the federal funds rate as reported by members of the FOMC in the Summary of Economic Projections minus two percent (the long-run objective of the FOMC for annual PCE inflation). While we use the median, it is worth noting that FOMC participants also often disagree about the value of $r_t^*$, which further underscores the general level of uncertainty surrounding its estimation.[3] In one policy rule variant, we use the most recently available Laubach-Williams estimate of $r_t^*$, which we assume will be constant going forward.[4]

## Inflation Measurement and the Choice of a Price Index

The policy rules we consider require as an input a measure of the rate of inflation. This raises the question of what inflation measure to use. The earliest policy rules were specified in terms of the GDP deflator.[5] The inflation measure typically used in financial markets is the consumer price index (CPI) produced by the Bureau of Labor Statistics. Federal Reserve policymakers usually look at a wide range of inflation measures.[6] However, the FOMC's long-run objective, commonly called target inflation, is for 2 percent inflation as measured by the annual change in the price index for personal consumption expenditures (PCE), and so we focus on PCE-based inflation rates.[7]

A crucial issue in inflation measurement is noise: in the short run, inflation is often subject to large transitory influences, coming from price shocks to particular goods and services that are unrelated to underlying inflation trends. Even when these shocks are fairly transitory, they can be large enough to shift the price index for one or more quarters, distorting or masking the underlying trend in inflation.

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[3] For example, in the March 2016 SEP, estimates of $r^*$ among FOMC members ranged from 3 to 4 percent. See Carlstrom and Stehulak (2015) for more details. In the June SEP, estimates of $r^*$ ranged from 2.75 to 3.75 percent.

[4] We use the one-sided Laubach-Williams $r^*$ estimate, which "corresponds more closely to 'real-time' estimates, in that only current and past observations are used" to estimate the rate (Laubach and Williams 2003).

[5] Prior to mid-1988, inflation as measured by the GNP deflator was prominent in Federal Reserve monetary policy deliberations (Mehra and Sawhni 2010). Taylor (1993) and Taylor (1999) both explicitly considered the GDP deflator as the relevant indicator of inflation.

[6] See the Frequently Asked Questions at the website of the Board of Governors of the Federal Reserve System.

[7] See the FOMC's Statement on Longer-Run Goals and Monetary Policy Strategy.

Some policymakers have argued that the central bank should not respond to short-term or transitory fluctuations in inflation; see, e.g., Mishkin (2007) or Bernanke (2010). This line of reasoning suggests that policy rules that use headline inflation may be unrealistic, in that policymakers likely choose not to respond to price swings that they view as temporary. Instead, policymakers would prefer to respond to the true underlying inflation trend; Ashley et al. (2014) present evidence that policymakers have, in fact, responded only to trend movements in inflation.

If policymakers wish to respond only to trend movements in inflation, they must identify and remove the noise. One approach to this filtering problem uses so-called "core inflation"—which is more accurately described as inflation in a price index excluding food and energy prices—as the measure of trend, on the basis that food and energy price movements are volatile at high frequencies and may mask trend inflation. Many analysts and policymakers regularly refer to and make use of these core inflation rates, and we follow this convention in some cases.[8] However, excluding food and energy prices has myriad limitations—for example, sharp transitory shocks can affect other inflation components as well—and a number of approaches can produce trend inflation estimates that are both unbiased and robust to shocks in any underlying component of the price index.[9] Exploring these alternative inflation measures inside of simple policy rules is beyond the scope of our analysis.

In forward-looking policy rules—which are rules in which the quarter $t$ federal funds rate responds to the *expected* or *forecasted* inflation rate in future quarters—the difference between core and headline inflation is less relevant. The reason is that, generally speaking, forecasts of both PCE and core PCE inflation tend to converge to about the same value as the forecast horizon increases (e.g., Mishkin 2007, Liu and Weidner 2011), consistent with an assumption that the effects of food and energy price shocks dissipate with time.[10]

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[8] For cases in which the inflation gap is specified using core PCE inflation, we assume that the inflation target remains at 2 percent.

[9] For further criticism of the use of "core" inflation measures, and advocacy of the use of headline measures, see Bullard (2011). One time-tested alternative to core inflation is the median CPI, produced by the Federal Reserve Bank of Cleveland (see Bryan and Cecchetti 1994). Other notable approaches include the Federal Reserve Bank of Dallas's trimmed mean PCE (Dolmas 2005) and the Federal Reserve Bank of Cleveland's trimmed mean CPI.

[10] Liu and Weidner (2011) argue that the convergence of the inflation forecasts is because inflation expectations are well anchored and shocks to food and energy prices have a limited effect on core prices. Note that unbiasedness of "core" inflation measures requires an absence of relative price trends in either food or energy; see Bullard (2011) for further discussion.

The activity gap measures the extent to which economic activity exceeds, or falls short of, its "normal" or equilibrium level. An "overheating" economy is normally characterized by output above potential output $Y_t > Y_t^*$ and unemployment below the natural rate of unemployment $u_t < u_t^*$. Overheating, via the Phillips curve, puts upward pressure on inflation. To resist this inflationary pressure, the general policy rule above would posit that the central bank should raise interest rates to slow the economy. The opposite is true in a "slack" economy.

We measure the output gap as the percentage difference between actual output and potential output, where potential output is defined by the Congressional Budget Office (CBO) as the "maximum sustainable amount of real (inflation-adjusted) output that the economy can produce." Estimating potential output $Y_t^*$ is an especially challenging task, because this variable is inherently tied to medium- and longer-term economic growth prospects. A decline in trend productivity growth, for example, reduces growth prospects, and would accordingly reduce estimates of $Y_t^*$ going forward.

While the CBO produces an estimate of—and a forecast for—the output gap, the challenges associated with estimating $Y_t^*$ prompt many analysts to use the unemployment gap, $u_t - u_t^*$, as a proxy for the output gap in simple policy rules. Researchers often refer to $u_t^*$ as the natural rate of unemployment, but there are a number of alternative and not-exactly-equivalent concepts for this variable; see Tasci and Verbrugge (2014). One popular concept for $u_t^*$ is the "nonaccelerating inflation rate of unemployment" (NAIRU), which is the level of unemployment thought to be "consistent with an unchanging inflation rate" (Stiglitz 1997).[11] Another concept for $u_t^*$ is the level of unemployment consistent with projections of current flows in the labor market (Tasci 2013). A third concept for $u_t^*$ is simply its long-run forecast. Estimates of $u_t^*$ can differ across concepts.

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[11] This concept, in turn, rests upon a Phillips curve linking the unemployment gap to the level or the first difference in inflation. Tallman and Zaman (2015) argue that the Phillips curve relationship is mainly evident in services prices.

To link the unemployment gap and the output gap, we use the gap version of Okun's law, which posits an empirical relationship of the form:

$$u_t - u_t^* = b(100[Y_t / Y_t^* - 1]) + e_t$$

with estimates suggesting Okun's coefficient, $b$, is negative, consistent with the notions of an "overheating" economy having output above potential and an unemployment rate below the natural rate, and a "slack" economy having output below potential and an unemployment rate above the natural rate, and $e_t$ is a regression residual. Accounting for time-variation in the difference version of Okun's law tends to improve forecasting performance (Knotek 2007), and we apply this intuition to the gap version as well. As in Knotek (2007), we use a 13-year rolling window to estimate $b$, based on CBO's estimates for the unemployment gap and output gap.[12] We then substitute $(1/b)(u_t - u_t^*)$ for the output gap when working with SPF and FRBC BVAR forecasts, where $1/b$ is the inverse Okun's coefficient.

## II. Additional Background on Seven Simple Policy Rules

In two of the simple rules we consider, we set the amount of inertia, or interest rate smoothing, to 0.8, and the last rule we consider places a relatively low weight on the output gap. We discuss the basis for these choices here.

### Policy Inertia

Empirical estimates of central bank policy functions generally suggest substantial inertia, or partial adjustment of the actual funds rate toward its target.[13] There is a wide variety of empirical evidence consistent with such inertial response by central banks around the world (e.g.,

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[12] The CBO derives its estimate of potential output in part from an Okun's law relationship, a practice criticized by Tetlow (2009). For comparison, we also investigated two alternative approaches to our baseline. The first alternative used the entire time series history of CBO output gaps and unemployment gaps to estimate Okun's coefficient. The second alternative constructed an output gap using a one-sided rolling-window implementation of the univariate Hodrick-Prescott (HP) filter, with smoothing parameter 1600 and three years of padded output forecasts added to each rolling window. (A one-sided implementation is necessary; see Ashley and Verbrugge 2009.) We then combined that output gap with the implied unemployment gap from the SPF and estimated Okun's coefficient over a 13-year window. The inverse Okun's coefficients are −1.80 in our baseline, −1.92 using the entire CBO time series history, and −1.41 using the HP output gap and the SPF unemployment gap.

[13] Indeed, there is evidence of "double inertia"; see Carlstrom and Fuerst (2014).

Goodhart 1999, Clarida et al. 1998, Coibion and Gorodnichenko 2012), although some research raises the possibility that the inertia is coming from the target rule variables (e.g., Rudebusch 2006). In their study of the U.S. Great Inflation episode in the 1970s, Humpage and Mukherjee (2015) find that inertia was deliberate.

There may be significant advantages to including inertia in a monetary policy rule. As discussed in Woodford (2003b) and Levin et al. (2003), inertial policy rules could allow small changes in the policy rate to have a relatively large impact on economic activity by affecting the expected path of future short-term rates (see also Goodfriend 1991) and appear to be more robust to uncertainty about the structure of the economy. In some models, inertia can considerably improve economic performance in terms of reducing output and inflation variability (see, e.g., Taylor and Williams 2011). A possible interpretation of policy inertia is that it reflects a central bank's attempt to signal commitment to a rule and thus avoid potential "inflation bias" associated with discretionary policy (Svensson and Woodford 2003). Further, in some models, inertia helps the public learn or discern how the central bank is behaving, or allows for existence of an equilibrium (see, e.g., Bullard and Mitra 2007, Duffy and Xiao 2008, Gasteiger 2014). Inertia is also consistent with the insight of Brainard (1967), who demonstrates that, under some conditions, parameter uncertainty suggests more caution in policy.[14]

While there are a variety of reasons for including inertia in policy rules, parameterizations vary widely. In empirical specifications that include one lag of the federal funds rate, many estimates are near 0.8 (see, e.g., Clarida et al. 2000, Kozicki 1999, Amato and Laubach 1999, Orphanides 2001, Rudebusch 2006) although in some specifications this inertia is as small as 0.6 or less (e.g., Orphanides 2001, Mehra 2002, Rabanal 2004, Molodtsova et al. 2008, Coibion and Gorodnichenko 2011) and in other specifications it can be as large as 0.9 or more (e.g., Amato and Laubach 1999, McCulloch 2007, Coibion and Gorodnichenko 2011, 2012, Belongia and Ireland 2015b).[15] We follow Canzoneri et al. (2015) and set $\rho$=0.8, a coefficient that these authors state is "typical of estimates that are found in the empirical literature" (p. 385).[16]

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[14] It is not always the case that uncertainty favors more inertia and weaker responses; see, e.g., Giannoni (2007).
[15] The data appear more consistent with two or more lags; see, e.g., McCulloch (2007), Qin and Enders (2008), Fernandez et al. (2008), or Ashley et al. (2014).
[16] Rudebusch (2006) also states: "Based on historical data, estimates of $\rho$ are often in the range of 0.8" (p. 92). Following Rudebusch (2006), Clark (2012) uses an inertia parameter of 0.8 in imposing a Taylor-rule prior in a BVAR.

## The Weight on the Output Gap

While simple policy rules generally put a weight of at least one on the inflation rate in order to satisfy the Taylor principle (see, e.g., Woodford 2003a), a similar principle does not exist for the weight on the output gap. We consider a weight of 0.5 as in Taylor (1993) as well as 1.0 as in Taylor (1999), but to span the range of plausible parameterizations in the literature, we also consider a specification with a small weight on the output gap: namely, the parameters reported in Clarida et al. (2000) for a forward-looking rule with inertia estimated on U.S. data over the period 1982-1996. This rule estimates a weight of 0.14 on the one-step-ahead forecast for the output gap.

It is worth noting that such low weights are not uncommon: coefficients of similar or smaller magnitude appear in the rules of Rotemberg and Woodford (1997), Cogley et al. (2011), and Arias et al. (2015). The rules in Schmitt-Grohe and Uribe (2007) also place little weight on output gaps or changes in output.

A general finding in the literature (see, e.g., Smets 1998) is that uncertainty about the size of the output gap generally implies that the response to the output gap should be attenuated. The intuition is that errors in estimating the size of the output gap add unwanted noise to the setting of policy, and this source of noise can be reduced if the coefficient on the output gap is reduced. Orphanides et al. (2000) map out the optimal coefficient as a function of the degree of measurement uncertainty in the output gap; in their study, the optimal coefficient ranges from 0 to 1 (see also Swanson 2004).[17] McCallum and Nelson (2004) state that their "results are consistent with the suggestion of McCallum (1999a) and Orphanides (2003) that it is dangerous to respond strongly to measures of the output gap."

Finally, in some specifications, e.g. Schmitt-Grohe and Uribe (2007) and Ascari and Ropele (2009), a small coefficient on the output gap is necessary to prevent destabilization of the economy. Ascari and Ropele (2007) also find that for determinacy the existence of positive trend inflation requires a central bank to become more "conservative," i.e. to focus more

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[17] Swanson (2004) and Meyer et al. (2001) find that a nonlinear policy rule may be preferable to a linear rule in the presence of uncertainty about the gap. When the gap term is small, policymakers should significantly attenuate their response to it, reflecting the uncertainty they face about the magnitude and even the sign of the gap. But when gap estimates are large, policymakers can be more confident that significant slack is present, warranting a more aggressive response.

attention on inflation fluctuations than on output fluctuations—although determinacy depends upon the microfoundations of the price stickiness (Ascari and Rossi 2012). In keeping with this, Ascari and Florio (2012) find that higher inflation targets require more assertive inflation responses and less assertive output responses in order to adequately anchor inflation expectations and ensure stability. The existence of habits also suggests, in simple rules, a reduced weight on output fluctuations and a greater positive response to inflation (e.g., Leith et al. 2012).

Of course, not all models find it optimal to downweight the output gap. For instance, Tetlow (2015) finds that the optimal coefficient on the gap term depends upon the specific data generating process; in the 2007 version of the FRB/US model, for example, the optimal coefficients end up near to those in the Taylor (1999) rule. Giannoni (2007) finds that uncertainty can, in some circumstances, suggest *stronger* responses to the estimated output gap. But the conflicting findings underscore the general point that uncertainty about the nature of the economy greatly complicates the attempt to determine an "optimal" rule, to the benefit of simple monetary policy rules.

## III. Additional Background on the Data and Forecasts

This section provides additional details on the forecasts we use.

### SPF and CBO

The SPF survey is released around the middle of the middle (second) month of each quarter. For the unemployment rate, PCE inflation, and core PCE inflation, the SPF reports 6 quarterly readings: one for each of the previous quarter ($t-1$), the current quarter ($t$), and the subsequent four quarters ($t+1$ through $t+4$). PCE inflation and core PCE inflation are reported as quarterly seasonally adjusted annualized rates. To compute year-over-year inflation rates, we combine the most recently available PCE and core PCE price index data through quarter $t-2$ with forecasts for the PCE and core PCE price levels implied by the SPF median forecasts for quarters $t-1$ through $t+4$, and we then compute year-over-year (i.e., four-quarter) inflation rates.

The CBO provides forecasts typically twice per year for a large number of variables, including the PCE price index, the core PCE price index, the output gap, the unemployment rate,

and the natural rate of unemployment. We use the CBO's forecasts as-is; i.e., we make no corrections when the available data in quarter $t-1$ did not match the CBO's forecast for quarter $t-1$.

FRBC BVAR Forecasting Model

To augment the publicly available forecasts from SPF and CBO, we provide the forecasts coming from the small statistical Bayesian vector autoregression model used in previous Federal Reserve Bank of Cleveland *Economic Commentaries* by Knotek and Zaman (2013) and Knotek et al. (2015).[18] Because the Cleveland Fed staff consult a variety of forecasting models, this forecast does not necessarily represent the official forecast of Cleveland Fed staff or the president of the Cleveland Fed.

The model includes seven variables: real GDP, real PCE, the unemployment rate, unit labor cost growth, PCE inflation, core PCE inflation, and the federal funds rate. Real GDP and real PCE enter the model in natural log levels. Unit labor cost growth is defined as growth in the employment cost index for private workers less growth in nonfarm business sector labor productivity. PCE inflation and core PCE inflation are modeled as deviations from a slow-moving inflation trend. The trend for core PCE inflation is the long-term inflation expectations series from the Federal Reserve Board of Governor's FRB/US econometric model, denoted PTR; from 2007 onward, the trend is the 10-year expected PCE inflation rate from the Survey of Professional Forecasters. For PCE inflation, we use core PCE inflation for the trend. In autoregressive models, specifying inflation as a deviation from trend has been found to improve forecast accuracy (see, e.g., Kozicki and Tinsley 2001, Clark 2011, Zaman 2013, Faust and Wright 2013). We estimate this BVAR model using four lags and quarterly time series data starting in 1959Q1.

Research also suggests that imposing near-term conditions can improve the medium-term forecast accuracy of quarterly models (see, e.g., Faust and Wright 2013, Del Negro and Schorfheide 2013). As such, we report forecasts that are conditional on nowcasts from the following sources. For real GDP, we use a nowcast for current quarter growth based on a

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[18] Knotek and Zaman (2013) and Knotek et al. (2015) imposed a Taylor-rule prior in the estimation of the BVAR, following Clark (2012). Given that we do not wish to impose a particular Taylor rule in the estimation of the model, we turn off the Taylor-rule prior for the purpose of generating this forecast.

Bayesian mixed frequency model with stochastic volatility as in Carriero et al. (2015). For the unemployment rate, we use the nowcast for the current quarter as reported in the most recently released Survey of Professional Forecasters. For PCE inflation and core PCE inflation, we use nowcasts from the [Cleveland Fed's inflation nowcasting website](#), which is based on the work of Knotek and Zaman (2015). Because Knotek and Garciga (2015) show that this inflation nowcasting model produces both more accurate nowcasts and more accurate one-quarter-ahead forecasts than a number of competitors, we use the model to nowcast PCE inflation and core PCE inflation for both the current quarter and the next quarter.

## IV. Other Simple Rules and Monetary Policy Options

We focus attention in the *Economic Commentary* on simple monetary policy rules that rely only on variables that are commonly forecasted, which allows us to produce federal funds rates based on the seven simple policy rules going forward. This narrow focus omits large parts of the literature on simple policy rules, as well as other options for monetary policy.

Most of the rules we consider take a form similar to that proposed by Taylor (1993), with the exception of the first-difference rule. An alternative class of simple rules that we do not consider targets the price level rather than the inflation rate; see, e.g., Eggertsson and Woodford (2003).[19] Such a rule may be particularly attractive when the economy is at the zero lower bound (ZLB): compared with a typical inflation-targeting policy rule, a price level rule promises more future monetary accommodation and higher inflation during the ZLB episode, which in turn more effectively stimulates economic activity and inflation. Reifschneider and Williams (2000) and Williams (2006, 2009) find that such price-level targeting rules are effective at reducing the costs of the ZLB, as long as the public understands the policy rule. The analysis of Giannoni (2014) suggests that they have some other nice properties; Walsh (2009) discusses pragmatics.[20]

In light of recent experiences with bubbles in stock markets in the 1990s and the housing market in the 2000s, some research has sought to include financial market indicators as additional variables in simple policy rules in an effort to bring policy closer to its theoretical

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[19] Price level targeting has a long history. Knut Wicksell suggested that Sweden should stabilize the price level in 1898, and Sweden actually did experiment with price level targeting a few decades later (see Berg and Jonung 1999). Hatcher and Minford (2013) survey the price-level targeting literature.

[20] Nominal income targeting rules offer another approach; see, e.g., Rudebusch (2002) and Woodford (2012). Belongia and Ireland (2015a) argue that the implementation of these rules may not be straightforward.

optimum. Some examples: Cecchetti et al. (2002) suggest the inclusion of a "stock bubble" term; Christiano et al. (2010) suggest the inclusion of a credit growth term; Curdia and Woodford (2010) suggest the inclusion of a credit spread term; and Leduc and Natal (2015) suggest the inclusion of the external finance premium. However, some research suggests that the benefit of adding additional variables is negligible (see, e.g., Taylor and Williams 2011), while other research suggests it can even be harmful (see, e.g., Carlstrom and Fuerst 2007).

The incidence, persistence, and potential future likelihood of the ZLB in many advanced economies has stimulated a great deal of research activity, given the limitations that the ZLB places on monetary policy to stabilize employment and inflation (see, e.g., Coenen et al. 2004, Williams 2009, Chung et al. 2012). In the vicinity of the ZLB, following an optimal monetary policy strategy, under either discretion or commitment, may notably differ from the policy pahts based on some simple monetary policy rules (see, e.g., Benhabib et al. 2001, Eggertsson and Woodford 2003, Adam and Billi 2006, 2007, Nakov 2008). Reifschneider and Williams (2000) propose a modification of a simple policy rule that accounts for episodes in which the policy rate had previously been constrained by the ZLB.

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