Monetary Policy & Anchored Expectations
An Endogenous Gain Learning Model

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¹The views expressed are solely the views of the author and do not necessarily reflect the views of the European Central Bank or the Eurosystem.
Anchoring

“Essential to anchor inflation expectations at some low level.”

“We don’t see a de-anchoring.”

“Failure of the Fed to stably achieve its 2 percent target could de-anchor inflation expectations.”

“Long-run inflation expectations [...] are not perfectly anchored in real economies; moreover, the extent to which they are anchored can change.”
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   • anchors expectations to inflation target
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- anchors expectations to inflation target
- responds aggressively to movements in long-run expectations
Related literature

• **Optimal monetary policy in the New Keynesian model**

• **Adaptive learning**

• **Anchoring and the Phillips curve**
MODEL OF ANCHORING EXPECTATIONS

QUANTIFICATION OF ANCHORING

OPTIMAL MONETARY POLICY
Households: standard up to $\hat{E}$

Maximize lifetime expected utility

$$\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[ U(C_T^i) - \int_0^1 v(h_T^i(j))dj \right]$$

Budget constraint

$$B_t^i \leq (1 + i_{t-1})B_{t-1}^i + \int_0^1 w_t(j)h_t^i(j)dj + \Pi_t^i(j)dj - T_t - P_tC_t^i$$
Firms: standard up to $\hat{E}$

Maximize present value of profits

$$\hat{E}_t^j \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \left[ \Pi_t^j(p_t(j)) \right]$$

subject to demand

$$y_t(j) = Y_t \left( \frac{p_t(j)}{P_t} \right)^{-\theta}$$
Aggregate relationships

- New Keynesian core: standard IS and Phillips curves

\[
x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} ((1 - \beta)x_{T+1} - \sigma(\beta i_T - \pi_{T+1}) + \sigma r_T^n)
\]

\[
\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} (\kappa \alpha \beta x_{T+1} + (1 - \alpha) \beta \pi_{T+1} + u_T)
\]

Observables: \((\pi, x, i)\)

Exogenous states: \((r^n, u)\)

Laura Gáti (ECB)
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Observables: \((\pi, x, i)\) inflation, output gap, interest rate
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Observables: \((\pi, x, i)\) inflation, output gap, interest rate
Exogenous states: \((r^n, u)\) natural rate and cost-push shock
Uncertainty on mean inflation

- Need a model of fluctuating long-run inflation expectations
- Main info assumption: $\hat{E}_i = \hat{E}_j = \hat{E}$ captures
- Firms and households do not know mean inflation
- They learn it from observed data
- Forecasts of inflation tomorrow centered around long-run expectation:
  $$\hat{E}_t \pi_{t+1} = \bar{\pi}_t + E_t \pi_{t+1}$$
- $E$: rational (model-consistent) expectations
- Short-run surprises informative about long-run inflation expectations
Uncertainty on mean inflation

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Uncertainty on mean inflation

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→ Main info assumption: \( \hat{E}^i = \hat{E}^j = \hat{E} \) captures

\[
\bar{\pi}_t + \pi_{t+1} = \hat{E}_t \pi_{t+1}
\]

E: rational (model-consistent) expectations

\( \bar{\pi} \): short-run surprises informative about long-run inflation expectations
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→ short-run surprises informative about long-run inflation expectations $\bar{\pi}_t$
Learning mean inflation from data

Yesterday’s one-period ahead inflation forecast:

\[ \hat{E}_{t-1} \pi_t = \bar{\pi}_{t-1} + E_{t-1} \pi_t \]
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\[ f_{t|t-1} = \pi_t - \hat{E}_{t-1} \pi_t \]
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→ **Update** for long-run inflation expectations:

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t f_{t|t-1}$$
Learning mean inflation from data

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→ Update for long-run inflation expectations:

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\(k_t \in (0, 1)\) learning gain as sensitivity to surprises
Alternatives for the gain

1. **Decreasing** gain:

\[
\bar{\pi}_t = \bar{\pi}_{t-1} + \frac{1}{t} f_{t|t-1}
\]

2. **Constant** gain:

\[
\bar{\pi}_t = \bar{\pi}_{t-1} + k f_{t|t-1}
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3. Endogenous gain:

\[ \bar{\pi}_t = \bar{\pi}_{t-1} + g(f_{t|t-1}) f_{t|t-1} \]
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Carvalho et al (2022): endogenous gain as a **metric for unanchoring**

- Low gain: anchored regime
- High gain: unanchored regime
Smoothly varying degrees of unanchoring

\[ k_t = g(f_t|t-1), \quad g'' > 0 \]
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\[ \rightarrow g(\cdot) \text{ smooth and continuous for optimal policy problem, convex} \]
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Convexity:

- Large surprises unanchor more than smaller ones
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- Large surprises unanchor more than smaller ones
- Pay more attention to inflation when it *really* surprises you
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Convexity:

- Large surprises unanchor more than smaller ones
- Pay more attention to inflation when it \textit{really} surprises you
  (rational inattention, expectations data, experimental studies)
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Optimal monetary policy: Molnár & Santoro (2014)

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Marcet & Nicolini (2003), Carvalho et al (2022)
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Marcet & Nicolini (2003), Carvalho et al (2022)
Optimal monetary policy: -
Model of anchoring expectations

Quantification of anchoring

Optimal monetary policy
Estimating form of gain function

- Calibrate parameters of New Keynesian core to literature

\[ \pi_t = \pi_{t-1} + g(f_{t|t-1}) \]

Moments: autocovariances of inflation, output gap, federal funds rate and 1-year ahead Survey of Professional Forecasters (SPF) inflation expectations at lags 0, ..., 4
Estimating form of gain function

• Calibrate parameters of New Keynesian core to literature

• Estimate flexible form of expectations process via simulated method of moments

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Calibration - parameters from the literature

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.98</td>
<td>stochastic discount factor</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>1</td>
<td>intertemporal elasticity of substitution</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.5</td>
<td>Calvo probability of not adjusting prices</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.0842</td>
<td>slope of the Phillips curve</td>
</tr>
<tr>
<td>$\psi_\pi$</td>
<td>1.5</td>
<td>coefficient of inflation in Taylor rule</td>
</tr>
<tr>
<td>$\psi_x$</td>
<td>0.3</td>
<td>coefficient of the output gap in Taylor rule</td>
</tr>
<tr>
<td>$\sigma_r$</td>
<td>0.01</td>
<td>standard deviation, natural rate shock</td>
</tr>
<tr>
<td>$\sigma_i$</td>
<td>0.01</td>
<td>standard deviation, monetary policy shock</td>
</tr>
<tr>
<td>$\sigma_u$</td>
<td>0.5</td>
<td>standard deviation, cost-push shock</td>
</tr>
<tr>
<td>$\bar{g}$</td>
<td>0.145</td>
<td>initial value of the gain</td>
</tr>
</tbody>
</table>

Carvalho et al (2022)
Estimated expectations process

\[ \bar{\pi}_t - \bar{\pi}_{t-1} = \hat{g}(f_{t|t-1}) f_{t|t-1} \]

Estimated change in long-run inflation expectations for various forecast errors
MODEL OF ANCHORING EXPECTATIONS

QUANTIFICATION OF ANCHORING

OPTIMAL MONETARY POLICY
Ramsey problem

$$\min_{\{y_t, \pi_t, k_t\}_{t=t_0}} \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} (\pi_t^2 + \lambda_x x_t^2)$$

s.t. model equations

s.t. evolution of expectations

• \(\mathbb{E}\) is the central bank’s (CB) expectation

• Assumption: CB observes private expectations and knows the model
Optimal policy - responding to unanchoring

\[ i(\bar{\pi}, \text{all other states at their means}) \]

Stabilizing \( \bar{\pi} \)
Optimal policy - responding to unanchoring

\[ i(\bar{\pi}, \text{all other states at their means}) \]

\[ \uparrow \bar{\pi} \text{ by 5 bp} \implies \uparrow i \text{ by 250 bp} \]
Optimal policy - responding to unanchoring

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\[ \uparrow \bar{\pi} \text{ by 5 bp} \implies \uparrow i \text{ by 250 bp} \]

Stabilizing \( \bar{\pi} \)

Mode: 0.3 bp movement in \( \bar{\pi} \)
Unanchoring amplifies shocks

Impulse responses after a cost-push shock *when policy follows a Taylor rule*
Conclusion
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First theory of monetary policy for potentially unanchored expectations
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Model-based notion of unanchoring
  • Sensitivity of long-run expectations to short-run fluctuations
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First theory of monetary policy for potentially unanchored expectations

Model-based notion of unanchoring
  • Sensitivity of long-run expectations to short-run fluctuations

Optimal monetary policy
  • Anchors expectations by responding aggressively to long-run expectations