# Strike while the Iron is Hot:

# Optimal Monetary Policy with a Nonlinear Phillips Curve

Peter Karadi[1,4] Anton Nakov[1,4] Galo Nuño[2,4] Ernesto Pasten[3] Dominik Thaler[1]

[1]ECB · [2]Bank of Spain · [3]Central Bank of Chile · [4]CEPR

Inflation: Drivers and Dynamics Conference, October 24-25, 2024

# Motivation

- The standard framework for monetary policy analysis is (log) linear-quadratic and assumes constant frequency of price changes (Galí, 2008; Woodford, 2003).
- In contrast, the recent inflation surge featured
  - A significant increase in inflation and frequency of price changes. US
  - Steeper Phillips curve (Benigno and Eggertsson, 2023; Cerrato and Gitti, 2023). US

**How different is optimal monetary policy in a non-linear framework where frequency endogenously varies?**

# What we do

- Take off-the-shelf menu cost models: Golosov and Lucas (2007) and, for robustness, "Calvoplus" (Nakamura and Steinsson, 2008).
  - For exposition, focus on Golosov and Lucas (2007).
  - U.S. calibration matching frequency and size of price changes.
- Positive analysis: Document non-linearities under a Taylor rule.
- Solve the non-linear Ramsey problem over the sequence space under perfect foresight.
  - New algorithm.
  - Characterize optimal policy in the long run, in the short run and in response to shocks.

# What we find

- Positive analysis:
  - The Phillips curve is non-linear: it gets steeper as frequency increases.
- Normative analysis:
  - When cost-push shocks are small, business as usual.
  - When cost-push shocks are large, more *hawkish* policy: "strike while the iron is hot."
  - Divine coincidence holds for efficient shocks, either small or large.
  - Optimal long-run inflation is slightly positive.
  - The time-inconsistency problem is there, but weakened relative to standard framework.

# Literature

- Non-linear PC (Benigno and Eggertsson, 2023; Cerrato and Gitti, 2023; Blanco et al., 2024a)
  - Microfounded in menu cost models (Golosov and Lucas, 2007; Auclert et al., 2022)
  - Important for large cost-push shocks (as in Karadi and Reiff, 2019; Alvarez and Neumeyer, 2020; Costain et al., 2022; Alexandrov, 2020; Blanco et al., 2024b; Cavallo et al., 2023)
- Optimal policy in a menu cost economy
  - Focus on target (Burstein and Hellwig, 2008; Nakamura and Steinsson, 2018; Blanco, 2021)
  - Small shocks, large shocks, optimal non-linear target (comp. Galí, 2008; Woodford, 2003)
  - Focus on aggregate and volatility shocks (differently from Caratelli and Halperin, 2023, who focus on small *sectoral* shocks)

# Overview of (our version of) the Golosov-Lucas model

= Textbook, Discrete-time New-Keynesian model with Calvo pricing (e.g. Galí, 2008)

– Calvo fairy [Calvoplus also includes this component]

+ fixed costs of price adjustments $\eta$

+ stochastic, idiosyncratic product quality $A_t(i)$

= Heterogeneous-firm NK DSGE model.

# Sketch of the model

- Households consume a Dixit and Stiglitz (1977) basket of goods, work and save.
- Per-period utility of consumption is log and disutility of labor is linear.
- Idiosyncratic quality $A_t(i)$ implies that

$$C_t = \left\{ \int [A_t(i) C_t(i)]^{\frac{\epsilon - 1}{\epsilon}} di \right\}^{\frac{\epsilon}{\epsilon - 1}}.$$

- Monopolistic producers with $Y_t(i) = A_t \frac{N_t(i)}{A_t(i)}$, $A_t$ is aggregate productivity.
- Firms face a fixed cost in labor units $\eta$ to update prices and an employment subsidy $\tau_t$.

# Pricing decision

- Define $p_t(i) \equiv \log\left(P_t(i)/(A_t(i)P_t)\right)$ be the quality-adjusted log real price.
- Define $\lambda_t(p)$ be the price-adjustment probability. Value function is

$$
\begin{aligned}
V_t(p) &= \Pi(p, w_t, A_t, A_t(i), \tau_t) \\
&+ \mathbb{E}_t\left[(1-\lambda_{t+1}(p-\sigma_{t+1}\varepsilon_{t+1}-\pi_{t+1}))\,\Lambda_{t,t+1}V_{t+1}(p-\sigma_{t+1}\varepsilon_{t+1}-\pi_{t+1})\right] \\
&+ \mathbb{E}_t\left[\lambda_{t+1}(p-\sigma_{t+1}\varepsilon_{t+1}-\pi_{t+1})\,\Lambda_{t,t+1}\left(\max_{p'} V_{t+1}(p') - \eta w_{t+1}\right)\right].
\end{aligned}
$$

- The price adjustment probability is characterized by a (s,S) rule:

$$
\lambda_t(p) = I[\max_{p'} V_t(p') - \eta w_t > V_t(p)].
$$

# Monetary Policy and shocks processes

- For positive analysis only, monetary policy follows a Taylor rule:

$$\log(R_t) = \rho_r \log(R_{t-1}) + (1-\rho_r)\left[\phi_\pi(\pi_t - \pi^*) + \phi_y(y_t - y_t^e)\right] + \varepsilon_{r,t} \quad \varepsilon_{r,t} \sim N(0,\sigma_r^2)$$

- Shocks on quality $A_t(i)$, TFP $A_t$, employment subsidy $\tau_t$, and volatility $\sigma_t$:

$$\log(A_t(i)) = log(A_{t-1}(i)) + \varepsilon_t(i), \quad \varepsilon_t(i) \sim N(0,\sigma_t^2),$$

$$\log(A_t) = \rho_A \log(A_{t-1}) + \varepsilon_{A,t} \quad \varepsilon_{A,t} \sim N(0,\sigma_A^2),$$

$$\tau_t - \tau = \rho_\tau(\tau_{t-1} - \tau) + \varepsilon_{\tau,t} \quad \varepsilon_{\tau,t} \sim N(0,\sigma_\tau^2),$$

$$\log(\sigma_t/\sigma) = \rho_\sigma \log(\sigma_{t-1}/\sigma) + \varepsilon_{\sigma,t} \quad \varepsilon_{\sigma,t} \sim N(0,\sigma_\sigma^2)$$

# Aggregation and market clearing

- Aggregate price index

$$1 = \int e^{p(1-\epsilon)} g_t(p) dp,$$

- Labor market equilibrium

$$N_t = \frac{C_t}{A_t} \underbrace{\int e^{p(-\epsilon)} g_t(p) dp}_{\text{dispersion}} + \eta \underbrace{\int \lambda_t(p - \sigma_t \varepsilon - \pi_t) g_{t-1}(p) dp}_{\text{frequency}},$$

where $g_t(p)$ is endogenous object.

# The model in one slide

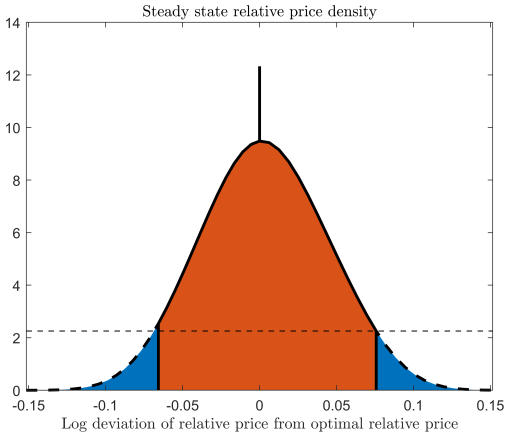
$$\max_{\{g_t^c(\cdot), g_t^0, V_t(\cdot), C_t, w_t, p_t^*, s_t, S_t, \pi_t^*\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\gamma}}{1-\gamma} - \upsilon \frac{C_t}{A_t} \left( \int e^{(x+p_t^*)(-\epsilon_t)} g_t^c(p)\, dx + g_t^0 e^{(p_t^*)(-\epsilon)} \right) - \upsilon \eta g_t^0 \right)$$

subject to

$$\begin{aligned}
1 &= \int e^{(x+p_t^*)(1-\epsilon)} g_t^c(x)\, dx + g_t^0 e^{(p_t^*)(1-\epsilon)}, \\
0 &= \Pi_t'(x) + \frac{1}{\sigma} \Lambda_{t,t+1} \int_{s_{t+1}}^{S_{t+1}} V_{t+1}(x') \frac{\partial \phi\left(\frac{x-x'-\pi_t^*}{\sigma}\right)}{\partial x} dx' \\
&+ \Lambda_{t,t+1} \left( \phi\left(\frac{S_{t+1}-\pi_t^*}{\sigma}\right) - \phi\left(\frac{s_{t+1}-\pi_t^*}{\sigma}\right) \right) \left(V_{t+1}(0) - \eta w_{t+1}\right), \\
V_t(s_t) &= V_t(0) - \eta w_t, \\
V_t(S_t) &= V_t(0) - \eta w_t, \\
w_t &= \upsilon C_t^{\gamma}, \\
V_t(x) &= \Pi(x, p_t^*, w_t, A_t) + \Lambda_{t,t+1} \frac{1}{\sigma} \int_{s_t}^{S_t} \left[ V_{t+1}(x') \phi\left(\frac{(x-x') - \pi_{t+1}^*}{\sigma}\right) \right] dx' \\
&+ \Lambda_{t,t+1} \left( 1 - \frac{1}{\sigma} \int_{s_t}^{S_t} \left[ \phi\left(\frac{(x-x') - \pi_{t+1}^*}{\sigma}\right) \right] dx' \right) \left[ \left(V_{t+1}(0) - \eta w_{t+1}\right) \right], \\
g_t^c(x) &= \frac{1}{\sigma} \int_{s_{t-1}}^{S_{t-1}} g_{t-1}^c(x_{-1}) \phi\left(\frac{x_{-1} - x - \pi_t^*}{\sigma}\right) dx_{-1} + g_{t-1}^0 \phi\left(\frac{-x - \pi_t^*}{\sigma}\right), \\
g_t^0 &= 1 - \int_{s_t}^{S_t} g_t^c(x) dx.
\end{aligned}$$

# Model: Intuitive summary

- After observing shocks, firm $i$ chooses whether to adjust prices and by how much ($p_t^*(i)$).
- Due to idiosyn. shock, endogenous price (gap) distribution, $x_t(i) \equiv p_t(i) - p_t^*(i)$.
- Distortions: price dispersion and menu cost.
- Market power creates another distortion.
- Monetary policy must deal with them all.

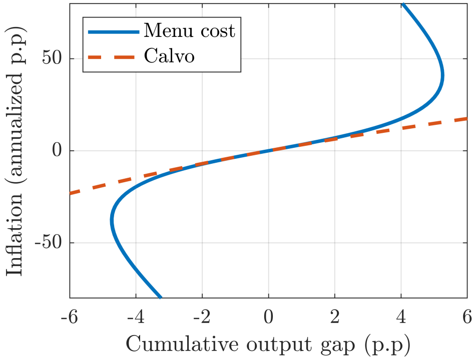


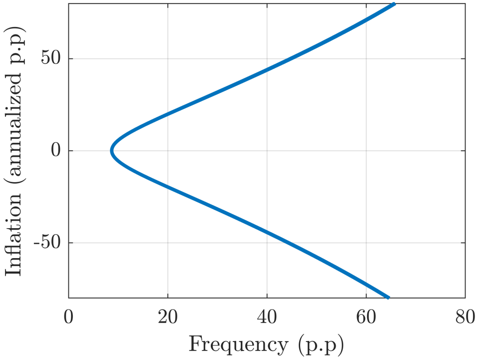
# Calibration

| | | | |
|---|---|---|---|
| **Households** | | | |
| $\beta$ | $0.96^{1/12}$ | Discount rate | Golosov and Lucas (2007) |
| $\epsilon$ | 7 | Elasticity of substitution | Golosov and Lucas (2007) |
| $\gamma$ | 1 | Risk aversion parameter | Midrigan (2011) |
| $\upsilon$ | 1 | Utility weight on labor | Set so that $w = C$ |
| **Price setting targets** | | | |
| Frequency | 8.7% | Frequency of price changes | Nakamura and Steinsson (2008) |
| Size | 8.5% | Absolute size of price changes | Nakamura and Steinsson (2008) |
| **Monetary policy** | | | |
| $\phi_\pi$ | 1.5 | Inflation coefficient in Taylor rule | Taylor (1993) |
| $\phi_y$ | 0.125 | Output coefficient in Taylor rule | Taylor (1993) |
| **Shocks** | | | |
| $\rho_A$ | $0.95^{1/3}$ | Persistence of the TFP shock | Smets and Wouters (2007) |
| $\rho_\tau$ | $0.25^{1/3}$ | Persistence of the cost-push shock | |

# Main positive result: Non-linear Phillips curve

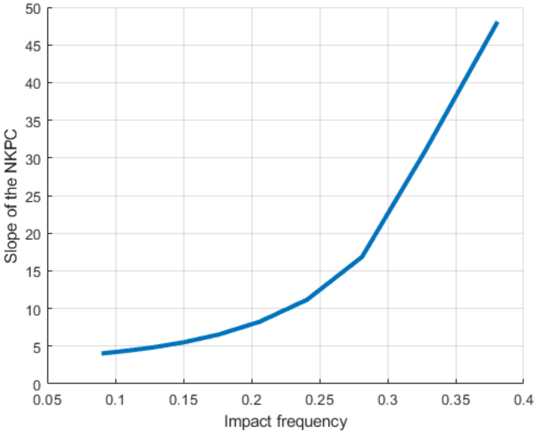
Small shocks: like *adjusted* Calvo; large shocks: non-linear, even bending backwards.



# Corollary: State-dependent monetary policy

- P.C. slope determines the sacrifice ratio: the relative impact on inflation versus output gap of a marginal monetary policy tightening.
- Key: state-dependent monetary policy effects.



# Normative analysis: Computation

- Challenges
  - Price distribution $g_t(p_t)$ and value function $V_t(p_t)$ are infinite-dimensional objects
  - We need sufficient accuracy for optimal policy assessment
- New algorithm, in discrete time
  - Approximate distribution and value functions by piece-wise linear functions on grid.
  - Endogenous grid points: (S,s) bands and the optimal reset price.
  - Evaluate integrals analytically.
  - Solve non-linearly in the sequence space using Dynare's perfect foresight Ramsey solver.

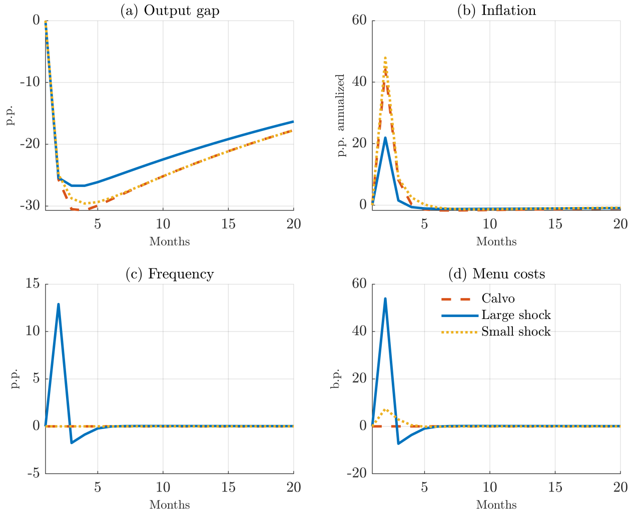
# Normative result 1: Optimal response to cost-push shocks is non-linear

- In the textbook, LQ framework, optimal policy is a price-level targeting rule

$$\hat{p}_t = -\frac{1}{\epsilon}\tilde{y}_t^e$$

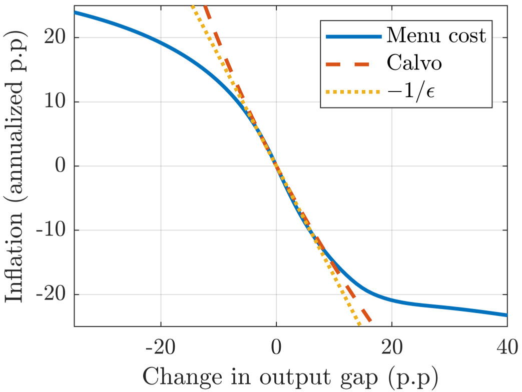
- For small cost-push shocks, optimal policy in the menu cost model is about the same.
- For large cost-push shock, strike while the iron is hot!

# Strike while the iron is hot!



# Strike while the iron is hot: Optimal non-linear target rule

- S-shaped optimal targeting rule.
- Monetary policy is more *hawkish* as shocks get larger.
- Non-linearity kicks in for $\pi_t > 10\%$



# Why is the optimal targeting rule S-shaped?

- Nonlinear loss function has three components:

$$-\underbrace{\left(Y_t^{gap} - e^{-Y_t^{gap}}\right)}_{\text{output gap}} - \underbrace{\left(e^{-Y_t^{gap}}(\zeta_t^{\mu-\overline{\mu}} - 1)\right)}_{\text{dispersion} \overset{\text{in GL}}{\approx} 0} - \underbrace{\eta g_t^0}_{\text{menu cost} \overset{\text{in Calvo}}{=} 0}$$

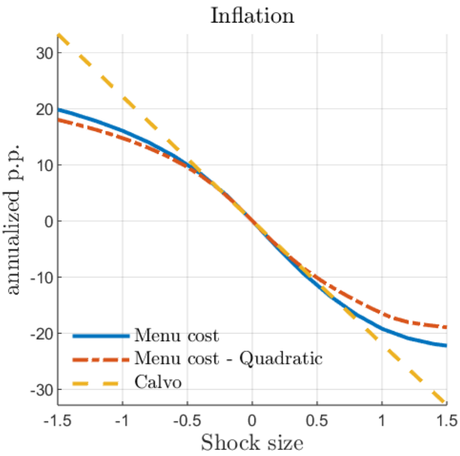
- 2nd-order approximation in Calvo

$$-\left(Y_t^{gap}\right)^2 - \alpha\pi_t^2$$

- For the baseline U.S. calibration, the standard 2nd-order approximation is just fine.

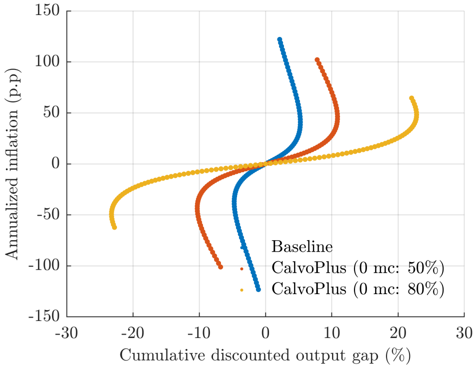
# Why is the optimal targeting rule S-shaped?

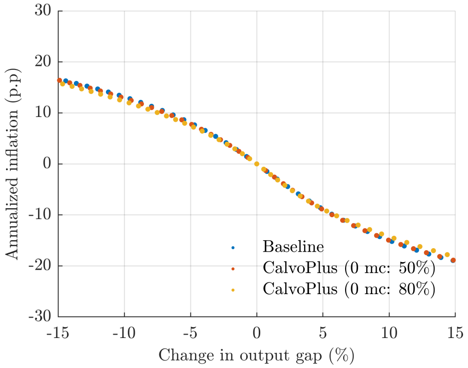
- A quadratic loss function yields very similar optimal targeting rule.
- The non-linearity comes from the PC.



# Normative result 1.1: Calvoplus

Calvo plus: very different Phillips curve slope, almost the same optimal monetary policy.



# Normative result 2: “Divine coincidence” holds

- In the standard NK model with Calvo pricing: divine coincidence holds after shocks affecting the efficient allocation: TFP ($A_t$) [also true for a discount rate shock].
- Optimal policy stabilizes inflation and closes the output gap.
- Same result holds in menu-cost models, regardless shocks are small or large.

# Normative result 3: Optimal long-run inflation rate

- The steady-state Ramsey inflation rate is slightly above zero: $\pi^* = 0.3\%$
- Why not zero?
  - Asymmetric profit function: negative price gaps more harmful $=>$ Asymmetric (S,s) bands.
  - At zero inflation, more mass around the lower than higher threshold.
  - Slightly positive inflation raises $p^*$ and pushes the mass of firms upwards.
  - $=>$ Lower frequency $=>$ less waste of resources paying for the menu cost.

# Normative result 4: Time inconsistency is weakened by endogenous frequency

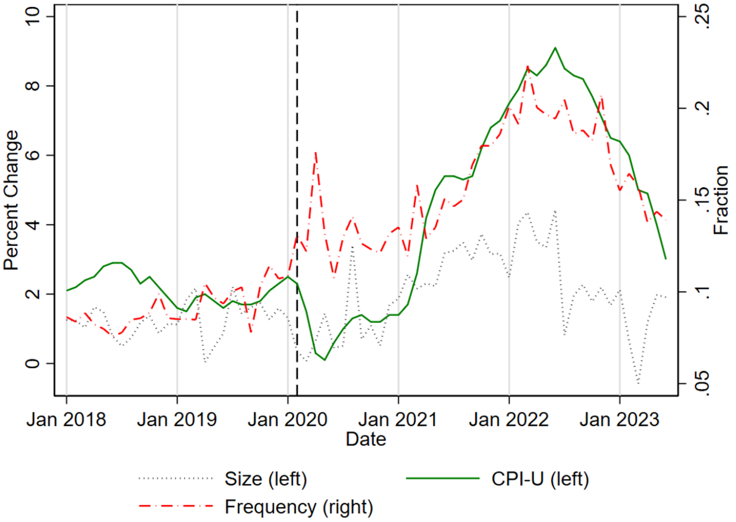
- Well-known time inconsistency when steady-state output is inefficiently low ($\tau = 0$)
  - A re-optimizing central bank should generate a surprise inflation and a temporary boom.
- Smaller time inconsistency in menu costs than in Calvo.
  - Inflation surprise raises frequency $=>$ output response is smaller.

# Conclusion

We study optimal policy in a menu cost model delivering a non-linear Phillips curve.

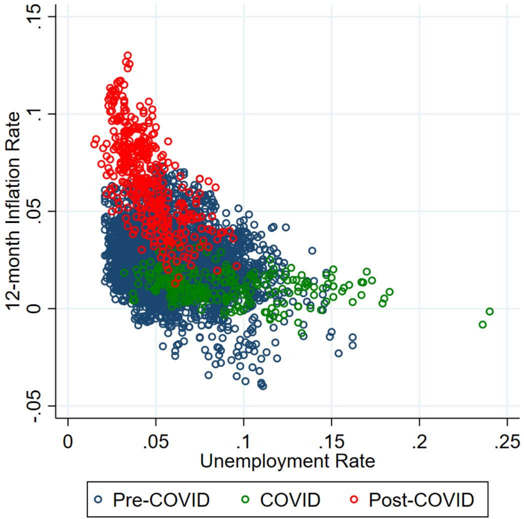
- Optimal response to small cost shocks similar to Calvo (1983).
- Lean against frequency for large cost-push shocks: strike while the iron is hot!
- Divine coincidence holds for efficient shocks, either small or large.
- Optimal long-run inflation is near zero.
- Time-inconsistency is there although weakened.

# CPI and frequency of price changes in the US, Montag and Villar (2023)




Back

# Phillips correlation across US cities, Cerrato and Gitti (2023)




Back

# Modified Phillips correlation time, Benigno and Eggertsson (2023)

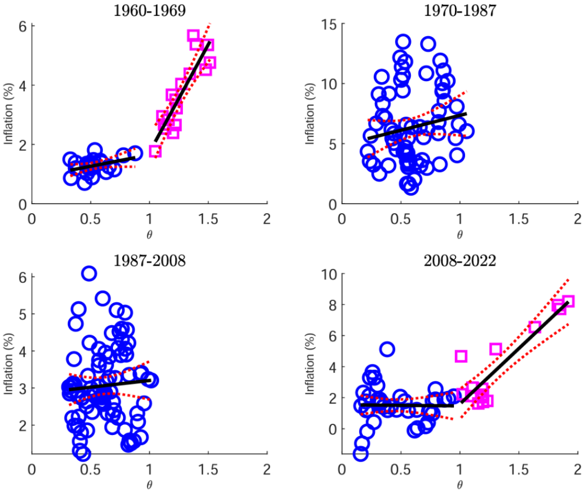
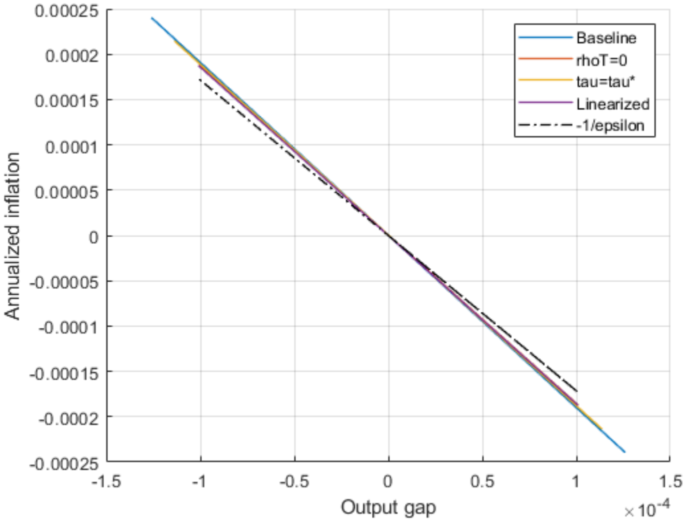



Figure 4: Inflation: CPI inflation rate at annual rates. $\theta$: vacancy-to-unemployed ratio.

Back

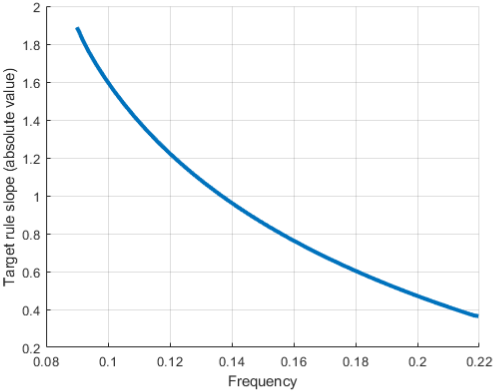
# Slope of the target rule for small shocks




Back

# State-dependent inflation-output tradeoff

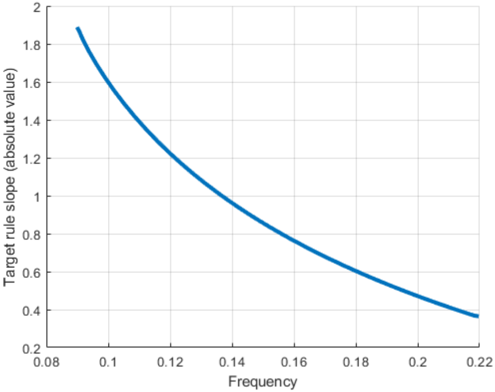
- Inflation-output tradeoff varies with frequency




Back

# State-dependent inflation-output tradeoff

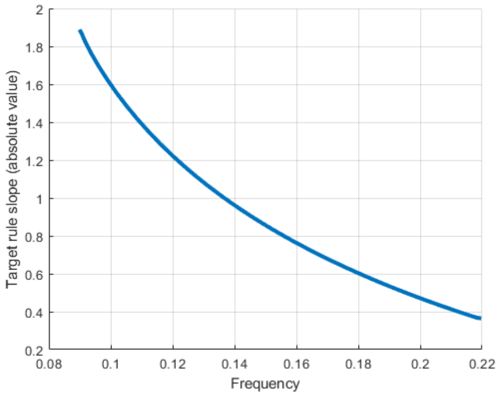
- Inflation-output tradeoff varies with frequency
- After large shocks, the planner stabilizes inflation relative to the output gap on the margin more Analogy with Calvo, 1983




Back

# State-dependent inflation-output tradeoff

- Inflation-output tradeoff varies with frequency
- After large shocks, the planner stabilizes inflation relative to the output gap on the margin more Analogy with Calvo, 1983
- Reduction in sacrifice ratio dominates decline in relative welfare weight of inflation




Back

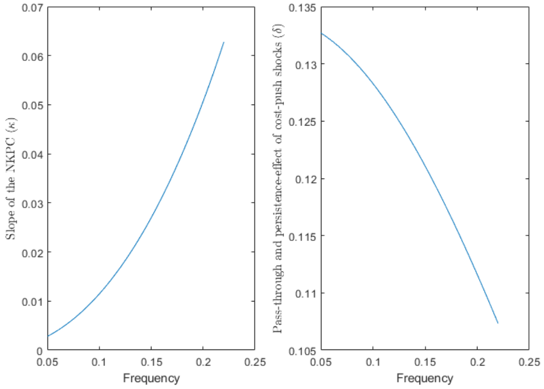
# Frequency and optimal policy in Calvo (1983)

- Optimal response to an iid cost-push shock ($u_t$)

$$\hat{p}_t = \delta \hat{p}_{t-1} + \delta u_t$$

$$x_t = \delta x_{t-1} + \delta \epsilon u_t,$$

where $\hat{p}_t \equiv p_t - p_{-1}$ is the change in the price level and $x_t$ is the output gap




Back

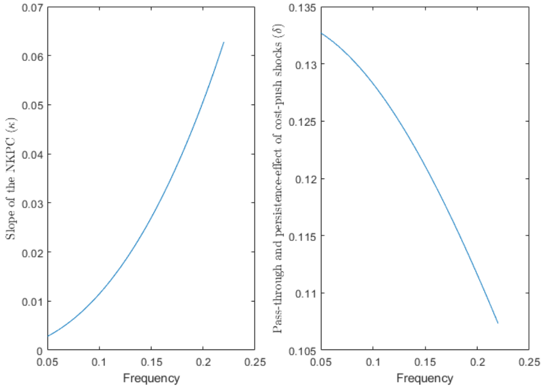
# Frequency and optimal policy in Calvo (1983)

- Optimal response to an iid cost-push shock ($u_t$)

$$\hat{p}_t = \delta \hat{p}_{t-1} + \delta u_t$$
$$x_t = \delta x_{t-1} + \delta \epsilon u_t,$$

where $\hat{p}_t \equiv p_t - p_{-1}$ is the change in the price level and $x_t$ is the output gap

- Parameter $\delta$ decreasing in frequency




Back

# Frequency and optimal policy in Calvo (1983)

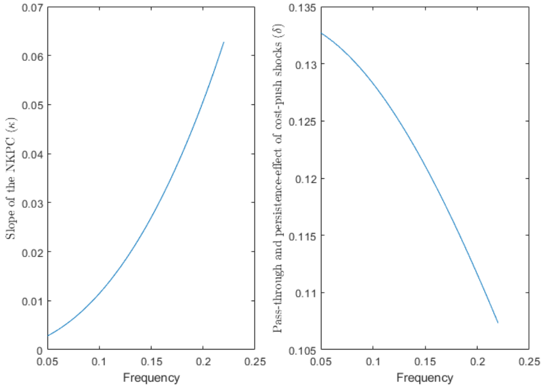
- Optimal response to an iid cost-push shock ($u_t$)

$$\hat{p}_t = \delta \hat{p}_{t-1} + \delta u_t$$

$$x_t = \delta x_{t-1} + \delta \epsilon u_t,$$

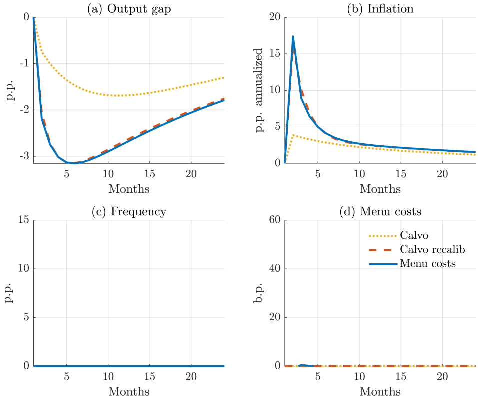
where $\hat{p}_t \equiv p_t - p_{-1}$ is the change in the price level and $x_t$ is the output gap

- Parameter $\delta$ decreasing in frequency
- Reduction in sacrifice ratio dominates

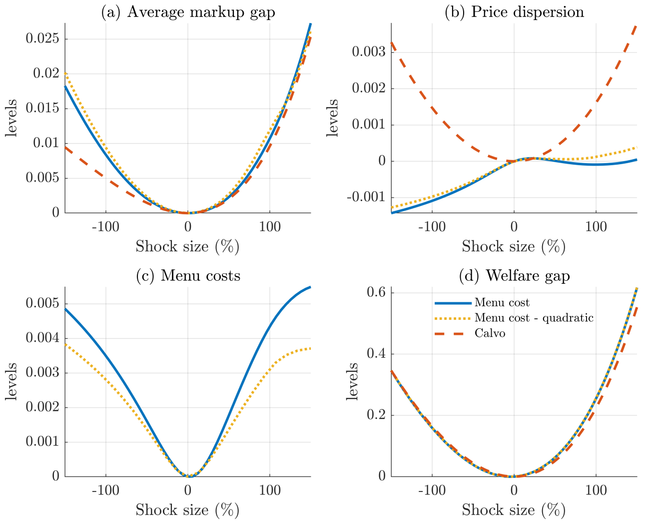



Back

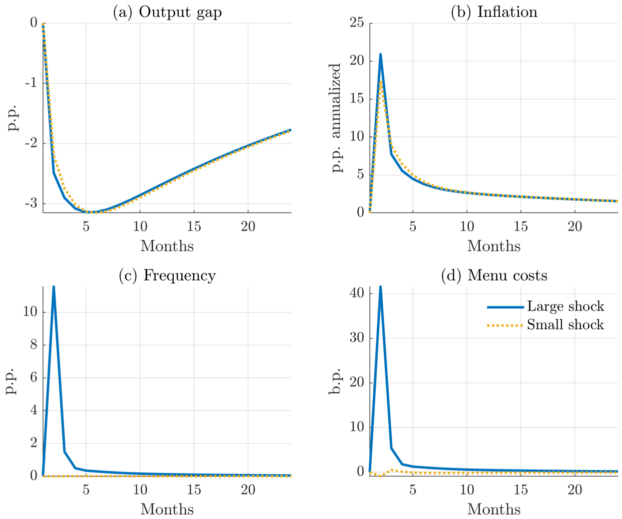
# Response to a cost-push shock under a TR (Calvo vs. Golosov-Lucas)



# Welfare decomposition



# Response to a cost-push shock (large vs. small shock in Golosov-Lucas)



# References I

Alexandrov, Andrey (2020) "The Effects of Trend Inflation on Aggregate Dynamics and Monetary Stabilization," CRC TR 224 Discussion Paper Series, University of Bonn and University of Mannheim, Germany.

Alvarez, Fernando and Pablo Andres Neumeyer (2020) "The Passthrough of Large Cost Shocks in an Inflationary Economy," in Gonzalo Castex, Jordi Galí, and Diego Saravia eds. *Changing Inflation Dynamics,Evolving Monetary Policy*, Vol. 27 of Central Banking, Analysis, and Economic Policies Book Series: Central Bank of Chile, Chap. 2, pp. 007–048.

Auclert, Adrien, Rodolfo D Rigato, Matthew Rognlie, and Ludwig Straub (2022) "New Pricing Models, Same Old Phillips Curves?" Technical report, National Bureau of Economic Research.

# References II

Benigno, Pierpaolo and Gauti Eggertsson (2023) "It's Baaack: The Surge in Inflation in the 2020s and the Return of the Non-Linear Phillips Curve," NBER Working Papers 31197, National Bureau of Economic Research, Inc.

Blanco, Andrés (2021) "Optimal Inflation Target in an Economy with Menu Costs and a Zero Lower Bound," *American Economic Journal: Macroeconomics*, Vol. 13, pp. 108–141.

Blanco, Andrés, Corina Boar, Callum J Jones, and Virgiliu Midrigan (2024a) "Nonlinear Inflation Dynamics in Menu Cost Economies," NBER Working Papers 32094, National Bureau of Economic Research.

Blanco, Andrés, Corina Boar, Callum Jones, and Virgiliu Midrigan (2024b) "Nonlinear Inflation Dynamics in Menu Cost Economies,"Technical report, unpublished manuscript.

Burstein, Ariel and Christian Hellwig (2008) "Welfare Costs of Inflation in a Menu Cost Model," *American Economic Review*, Vol. 98, pp. 438–43.

# References III

Calvo, Guillermo A. (1983) "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics*, Vol. 12, pp. 383 – 398.

Caratelli, Daniele and Basil Halperin (2023) "Optimal monetary policy under menu costs," unpublished manuscript.

Cavallo, Alberto, Francesco Lippi, and Ken Miyahara (2023) "Large Shocks Travel Fast," NBER Working Papers 31659, National Bureau of Economic Research, Inc.

Cerrato, Andrea and Giulia Gitti (2023) "Inflation Since COVID: Demand or Supply," unpublished manuscript.

Costain, James, Anton Nakov, and Borja Petit (2022) "Flattening of the Phillips Curve with State-Dependent Prices and Wages," *The Economic Journal*, Vol. 132, pp. 546–581.

Dixit, Avinash K and Joseph E Stiglitz (1977) "Monopolistic Competition and Optimum Product Diversity," *American Economic Review*, Vol. 67, pp. 297–308.

# References IV

Galí, Jordi (2008) *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework*: Princeton University Press.

Golosov, Mikhail and Robert E. Lucas (2007) "Menu Costs and Phillips Curves," *Journal of Political Economy*, Vol. 115, pp. 171–199.

Karadi, Peter and Adam Reiff (2019) "Menu Costs, Aggregate Fluctuations, and Large Shocks," *American Economic Journal: Macroeconomics*, Vol. 11, pp. 111–146.

Midrigan, Virgiliu (2011) "Menu Costs, Multiproduct Firms, and Aggregate Fluctuations," *Econometrica*, Vol. 79, pp. 1139–1180.

Montag, Hugh and Daniel Villar (2023) "Price-Setting During the Covid Era," *FEDS Notes*.

Nakamura, Emi and Jón Steinsson (2008) "Five Facts about Prices: A Reevaluation of Menu Cost Models," *The Quarterly Journal of Economics*, Vol. 123, pp. 1415–1464.

# References V

——— (2018) "High-Frequency Identification of Monetary Non-Neutrality: The Information Effect," *The Quarterly Journal of Economics*, Vol. 133, pp. 1283–1330.

Smets, Frank and Rafael Wouters (2007) "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *The American Economic Review*, Vol. 97, pp. 586–606.

Taylor, John B (1993) "Discretion versus Policy Rules in Practice," in *Carnegie-Rochester Conference Series on Public Policy*, Vol. 39, pp. 195–214, Elsevier.

Woodford, Michael (2003) *Interest and Prices: Foundations of a Theory of Monetary Policy*: Princeton University Press.