The Inflation Accelerator

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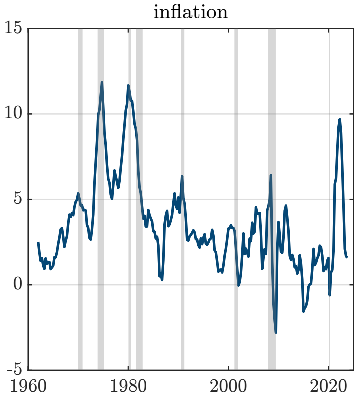
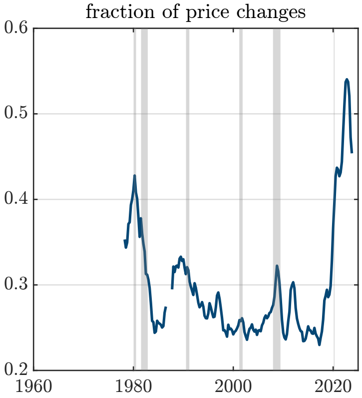
[1] The views expressed herein are those of the authors and not necessarily those of the Board of Governors of the Federal Reserve System, the Federal Reserve Bank of Atlanta or the Federal Reserve System.

Motivation

- Slope of Phillips curve a key ingredient in monetary policy analysis
- In sticky price models pinned down by fraction of price changes, n
- Data: fraction of price changes increases with inflation
  - Gagnon (2009), Alvarez et al. (2018), Blanco et al. (2024)

Evidence from the U.S.





- Source: Nakamura et al. (2018), Montag and Villar (2023). Fraction quarterly.
- Inflation computed using CPI without shelter (year-to-year changes).

▸ extensive margin decomposition

Motivation

- Slope of Phillips curve a key ingredient in monetary policy analysis
- In sticky price models, key determinant: fraction of price changes, n
- Data: fraction of price changes increases with inflation
  - Gagnon (2009), Alvarez et al. (2018), Blanco et al. (2024)
- How does slope fluctuate in U.S. time series?
  - answer using model that reproduces this evidence

Existing Models

- Time-dependent models
  - widely used due to their tractability
  - constant fraction of price changes

- State-dependent (menu cost) models
  - less tractable: state of the economy includes distribution of prices
  - calibration consistent with micro price data: fraction nearly constant

- We develop tractable alternative with endogenously varying fraction
  - multi-product firms choose *how many*, but not *which*, prices to change
  - exact aggregation: reduces to one-equation extension of Calvo

Our Findings

- Our model predicts highly non-linear Phillips curve
  - slope fluctuates from 0.02 in 1990s to 0.12 in 1970s and 1980s
- Mostly due to a feedback loop between fraction and inflation
  - *inflation accelerator*
  - inflation more sensitive to changes in fraction when inflation is high
- Absent feedback loop slope increases to only 0.04 in 1970s and 1980s

Model

Consumers

- Life-time utility

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left(\log c_t - h_t\right)$$

- Budget constraint

$$P_t c_t + \frac{1}{1+i_t} B_{t+1} = W_t h_t + D_t + B_t$$

- Monetary policy targets nominal spending $M_t \equiv P_t c_t$

$$\log M_{t+1}/M_t = \mu + \varepsilon_{t+1},\ \varepsilon_{t+1} \sim N\left(0, \sigma^2\right)$$

- Log-linear preferences imply $W_t = P_t C_t = M_t$

Technology

- Multi-product firms i sell continuum of goods k each
  - final good sector competitive:

$$c_t = y_t = \left(\int_0^1 \int_0^1 (y_{ikt})^{\frac{\theta-1}{\theta}} \, \mathrm{d}k\mathrm{d}i \right)^{\frac{\theta}{\theta-1}}$$

  - demand for individual variety:

$$y_{ikt} = \left(\frac{P_{ikt}}{P_t} \right)^{-\theta} y_t, \qquad P_t = \left(\int_0^1 \int_0^1 (P_{ikt})^{1-\theta} \, \mathrm{d}k\mathrm{d}i \right)^{\frac{1}{1-\theta}}$$

  - each produced with DRS technology $y_{ikt} = (l_{ikt})^{\eta}$

Firm Problem

- Real discounted flow profits of firm i

$$\frac{1}{P_t c_t}\int_0^1 \left(P_{ikt}y_{ikt} - \tau W_t l_{ikt}\right)\mathrm{d}k = \left(\frac{P_{it}}{P_t}\right)^{1-\theta} - \tau\left(\frac{X_{it}}{P_t}\right)^{-\frac{\theta}{\eta}} y_t^{\frac{1}{\eta}}$$

  - flow profits depend on two moments of its price distribution

$$P_{it} = \left(\int_0^1 \left(P_{ikt}\right)^{1-\theta}\mathrm{d}k\right)^{\frac{1}{1-\theta}} \quad \text{and} \quad X_{it} = \left(\int_0^1 \left(P_{ikt}\right)^{-\frac{\theta}{\eta}}\mathrm{d}k\right)^{-\frac{\eta}{\theta}}$$

- Firm chooses fraction of price changes n_{it}, cost $\frac{\xi}{2}\left(n_{it}-\bar{n}\right)^2$ if $n_{it} > \bar{n}$
  - but not which, so history encoded in two state variables, P_{it-1} and X_{it-1}
  - e.g. $P_{it} = \left(n_{it}\left(P_{it}^{*}\right)^{1-\theta} + \left(1-n_{it}\right)\left(P_{it-1}\right)^{1-\theta}\right)^{\frac{1}{1-\theta}}$

Symmetric Equilibrium

- Let $p_t^* = P_t^*/P_t$, $x_t = X_t/P_t$, $\pi_t = P_t/P_{t-1}$

- Optimal reset price similar to Calvo, except n_t varies

$$\left(p_t^*\right)^{1+\theta\left(\frac{1}{\theta}-1\right)} = \frac{1}{\eta} \frac{\left.\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left(y_{t+s}\right)^{\frac{1}{\eta}} \prod_{j=1}^{s} \left(1-n_{t+j}\right)\left(\pi_{t+j}\right)^{\frac{\theta}{\eta}} \right\} b_{2t}}{\left.\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \prod_{j=1}^{s} \left(1-n_{t+j}\right)\left(\pi_{t+j}\right)^{\theta-1} \right\} b_{1t}}$$

- Fraction of price changes

$$\xi\left(n_t-\bar{n}\right) = \underbrace{b_{1t}\left(\left(p_t^*\right)^{1-\theta}-\left(\pi_t\right)^{\theta-1}\right)}_{\text{change price index}} - \underbrace{\tau b_{2t}\left(\left(p_t^*\right)^{-\frac{\theta}{\eta}}-\left(x_{t-1}\right)^{-\frac{\theta}{\eta}}\left(\pi_t\right)^{\frac{\theta}{\eta}}\right)}_{\text{reduce misallocation}}$$

Symmetric Equilibrium

- Inflation pinned down by the definition of price index

$$1 = n_t \left(p_t^*\right)^{1-\theta} + (1 - n_t)\left(\pi_t\right)^{\theta-1}$$

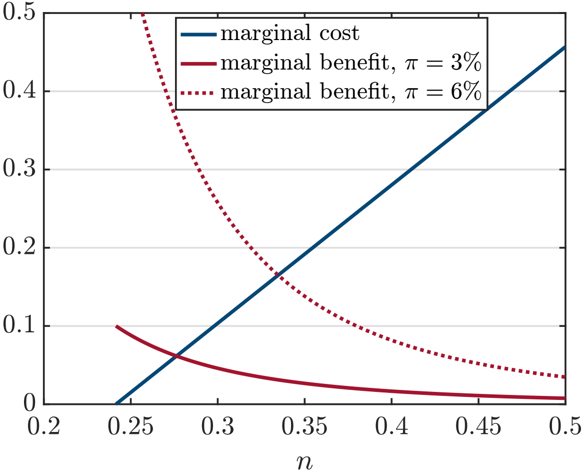
- Losses from misallocation

$$\left(x_t\right)^{-\frac{\theta}{\eta}} = n_t \left(p_t^*\right)^{-\frac{\theta}{\eta}} + (1 - n_t)\left(x_{t-1}\right)^{-\frac{\theta}{\eta}}\left(\pi_t\right)^{\frac{\theta}{\eta}}$$

- Model reduces to one-equation extension of Calvo
  - as $\xi \to \infty$, $n_t = \bar{n}$ so our model nests Calvo

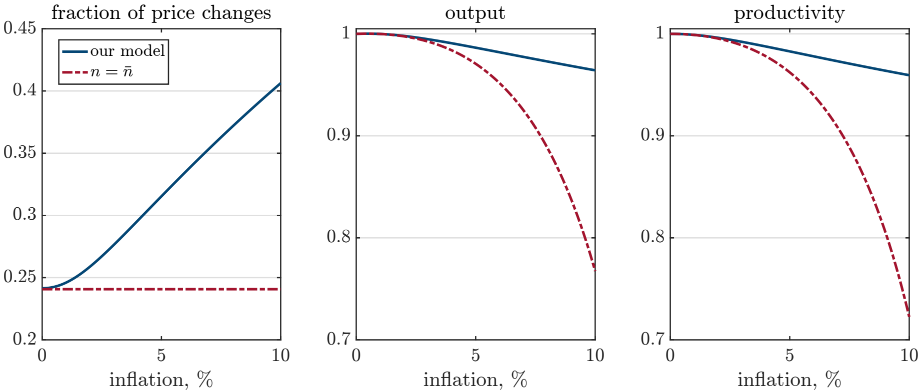
- Unlike Calvo, important non-linearities so solve using global methods
  - third-order perturbation reasonably accurate

Steady-State Fraction of Price Changes



Fraction of price changes increases with inflation

Steady-State Output and Productivity



- Inflation less distortionary in our model

  equations

  – because more frequent price changes, as in menu cost models

Slope of the Phillips Curve

Parameterization

- Assigned parameters
  - period 1 quarter, $\beta = 0.99$, $\theta = 6$, $\eta = 2/3$

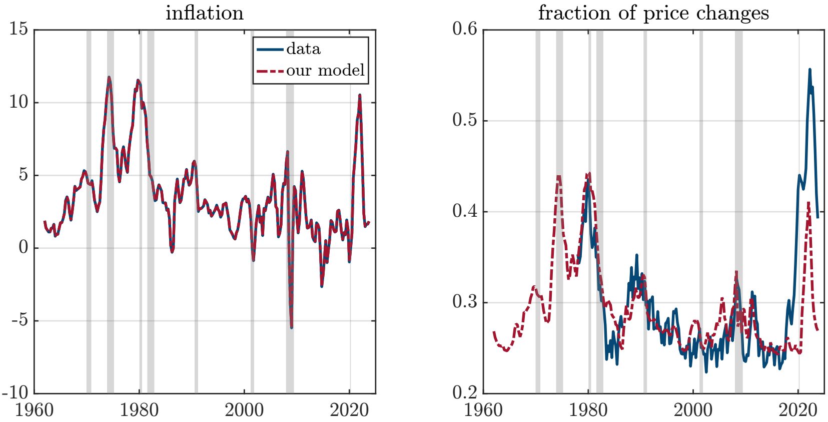
- Calibrated parameters
  - mean and standard deviation of nominal spending growth μ and σ
  - fraction of free price changes $\bar{n}$, price adjustment cost ξ

- Calibration targets

	Data	Model
mean inflation	0.035	0.035
s.d. inflation	0.027	0.027
mean fraction	0.297	0.297
slope of n_t on $\|\pi_t\|$	0.016	0.016

Fraction of Price Changes

- Use non-linear solution to recover shocks that reproduce U.S. inflation



- Reproduces fraction well, except post-Covid
  - many price decreases due to sectoral shocks

extensive margin model

Towards the Slope of the Phillips Curve

- First order perturbation around equilibrium point at each date t
  - hats denote deviations from equilibrium at that date

- Aggregate price index:

$$\hat{\pi}_t = \underbrace{\frac{1}{(1-n_t)\,\pi_t^{\theta-1}}\frac{\pi_t^{\theta-1}-1}{\theta-1}}_{\mathcal{M}_t}\hat{n}_t + \underbrace{\frac{1-(1-n_t)\,\pi_t^{\theta-1}}{(1-n_t)\,\pi_t^{\theta-1}}}_{\mathcal{N}_t}\hat{p}_t^*$$

- Elasticity $\mathcal{N}_t$ to reset price: identical to Calvo
  - increases with n_t, decreases with π_t (lower weight on new prices)

- Elasticity $\mathcal{M}_t$ to frequency: zero if $\pi_t = 1$, increases with inflation

Intuition

- Why is inflation more sensitive to changes in n_t when inflation is high?

$$\mathcal{M}_t = \frac{1}{(1-n_t)\,\pi_t^{\theta-1}} \frac{\pi_t^{\theta-1}-1}{\theta-1}$$

- Inflation ≈ average price change × fraction of price changes
  - $\pi_t = 1$: average price change $= 0$
    - so fraction inconsequential
  - π_t is high: average price change is large
    - so Δn_t increases inflation considerably

Inflation Accelerator

- Recall aggregate price index

$$\hat{\pi}_t = \mathcal{M}_t \hat{n}_t + \mathcal{N}_t \hat{p}_t^*$$

  – elasticity $\mathcal{M}_t$ increases with inflation, zero if $\pi_t = 1$

- Optimal fraction of price changes

$$\hat{n}_t = \mathcal{A}_t \hat{\pi}_t + \mathcal{B}_t \hat{p}_t^* - \mathcal{C}_t \hat{x}_{t-1} + \frac{n_t - \bar{n}}{n_t} \hat{b}_{1t}$$

  – elasticities $\mathcal{A}_t$ and $\mathcal{B}_t$ also increase with π_t

- Feedback loop amplifies inflation response to changes in reset price

$$\hat{\pi}_t = \frac{\mathcal{M}_t \mathcal{B}_t + \mathcal{N}_t}{1 - \mathcal{M}_t \mathcal{A}_t} \hat{p}_t^* - \frac{\mathcal{M}_t \mathcal{C}_t}{1 - \mathcal{M}_t \mathcal{A}_t} \hat{x}_{t-1} + \frac{\mathcal{M}_t}{1 - \mathcal{M}_t \mathcal{A}_t} \frac{n_t - \bar{n}}{n_t} \hat{b}_{1t}$$

Slope of the Phillips Curve

- Let $\hat{mc}_t = \frac{1}{\eta}\hat{y}_t$ aggregate real marginal cost

$$\hat{\pi}_t = \mathcal{K}_t \hat{mc}_t + \dots$$

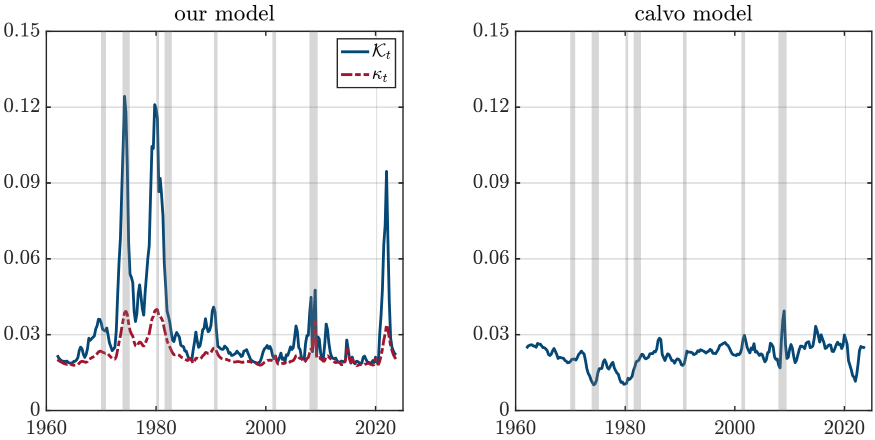
- Slope of the Phillips curve

$$\mathcal{K}_t = \underbrace{\frac{1}{1+\theta\left(\frac{1}{\eta}-1\right)}}_{\text{complementarities}} \times \underbrace{\frac{y_t^{\frac{1}{\eta}}}{b_{2t}}}_{\text{horizon}} \times \underbrace{\frac{\mathcal{M}_t\mathcal{B}_t+\mathcal{N}_t}{1-\mathcal{M}_t\mathcal{A}_t}}_{\text{reset price}}$$

- Absent endogenous frequency response ($\mathcal{A}_t = \mathcal{B}_t = 0$)

$$\kappa_t = \frac{1}{1+\theta\left(\frac{1}{\eta}-1\right)} \times \frac{y_t^{\frac{1}{\eta}}}{b_{2t}} \times \underbrace{\frac{1-(1-n_t)\,\pi_t^{\theta-1}}{(1-n_t)\,\pi_t^{\theta-1}}}_{\mathcal{N}_t}$$

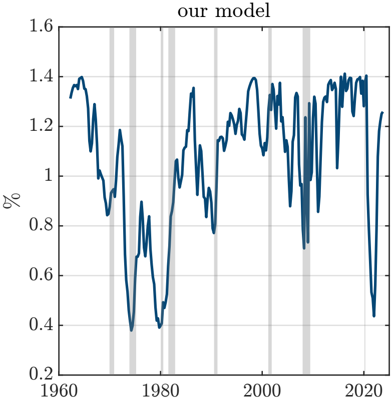
Time-Varying Slope of the Phillips Curve

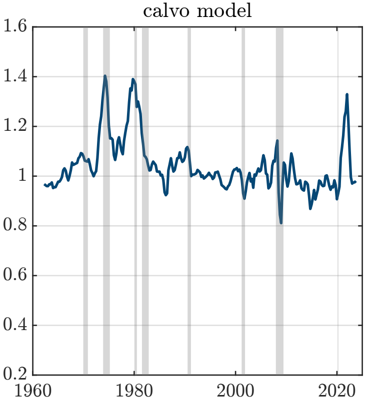


Ranges from 0.02 to 0.12, mostly due to inflation accelerator

Sacrifice Ratio

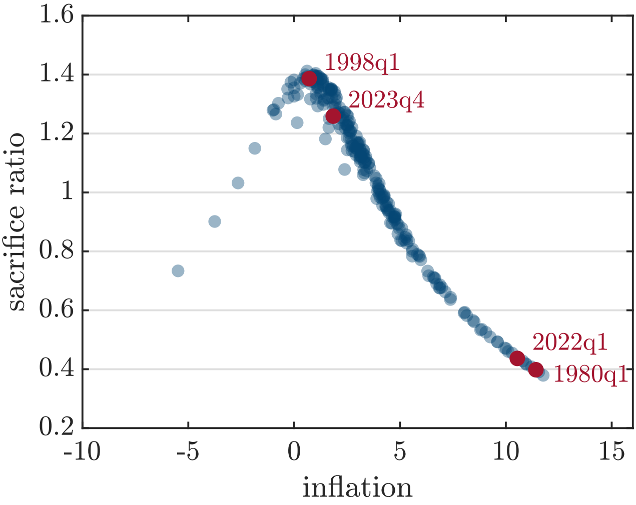
- Calculate decline in annual output needed to reduce π by 1% over a year





Ranges from 0.4% to 1.4%, opposite of Calvo

Inflation and the Sacrifice Ratio



Robustness

Eliminate Strategic Complementarities

- Set $\eta = 1$, recalibrate model

Targeted Moments

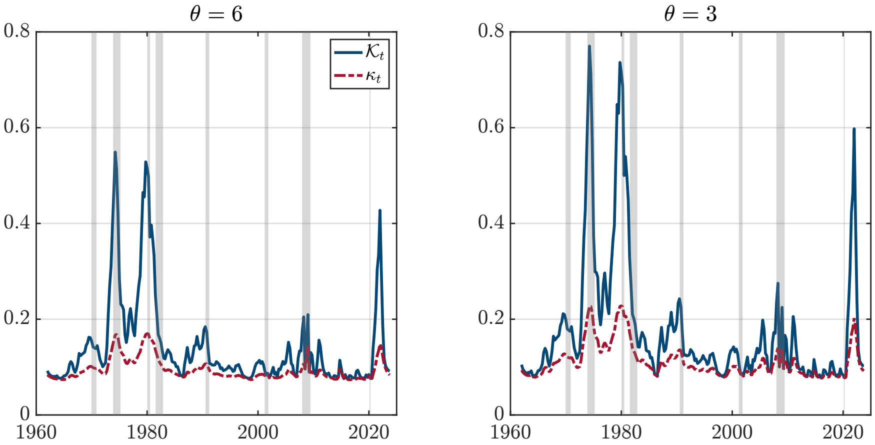
	Data	$\theta = 6$	$\theta = 3$
mean inflation	3.517	3.517	3.517
s.d. inflation	2.739	2.739	2.739
mean fraction	0.297	0.297	0.297
slope of n_t on $\lvert\pi_t\rvert$	0.016	0.016	0.016

Calibrated Parameters

		$\theta = 6$	$\theta = 3$
μ	mean spending growth rate	0.035	0.035
σ	s.d. monetary shocks	0.019	0.018
$\bar{n}$	fraction free price changes	0.232	0.227
ξ	adjustment cost	0.365	0.109

- Smaller price adjustment costs because less curvature in profit function

Slope of the Phillips Curve



Larger absent complementarities, but fluctuates as much

Taylor Rule

- Replace nominal spending target with Taylor rule

$$\frac{1+i_t}{1+i} = \left(\frac{1+i_{t-1}}{1+i}\right)^{\phi_i} \left(\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi} \left(\frac{y_t}{y_{t-1}}\right)^{\phi_y}\right)^{1-\phi_i} u_t$$

- Two versions
  - u_t shocks iid
  - serially correlated with persistence ρ to match autocorrelation inflation
- Use Justiniano and Primiceri (2008) estimates
  - $\phi_i = 0.65$, $\phi_\pi = 2.35$, $\phi_y = 0.51$

Calibration of Economy with a Taylor Rule

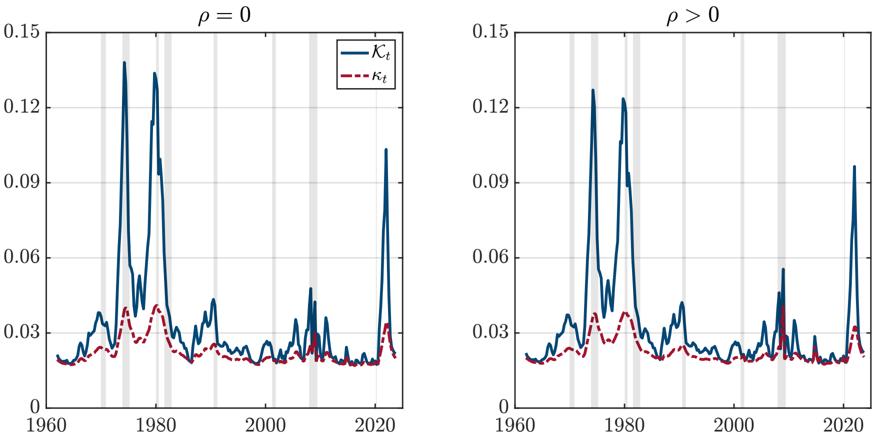
Targeted Moments

	Data	$\rho = 0$	$\rho > 0$
mean inflation	3.517	3.517	3.517
s.d. inflation	2.739	2.739	2.739
mean fraction	0.297	0.297	0.297
slope of n_t on $\lvert\pi_t\rvert$	0.016	0.016	0.016
autocorr. inflation	0.942	*0.913*	0.942

Calibrated Parameters

		$\rho = 0$	$\rho > 0$
$\log \pi$	inflation target	0.040	0.037
σ	s.d. monetary shocks $\times 100$	2.626	0.551
ρ	persistence money shocks	–	0.685
$\bar{n}$	fraction free price changes	0.241	0.241
ξ	adjustment cost	1.671	1.688

Slope of the Phillips Curve



Our results robust to assuming a Taylor rule

Conclusion

- Data: fraction of price changes increases with inflation
- Developed tractable model consistent with this evidence
  - firms choose how many, but not which prices to change
  - reduces to one-equation extension of Calvo
- Implies slope of Phillips curve increases considerably with inflation
  - partly because more frequent price changes
  - primarily due to endogenous frequency response – *inflation accelerator*

Losses from Misallocation

$$(X_{it+s})^{-\frac{\theta}{\eta}} = n_{it+s}\left(P^*_{it+s}\right)^{-\frac{\theta}{\eta}} + (1-n_{it+s})\, n_{it+s-1}\left(P^*_{it+s-1}\right)^{-\frac{\theta}{\eta}} + \cdots$$

$$+ \prod_{j=1}^{s}(1-n_{it+j})\, n_{it}\left(P^*_{it}\right)^{-\frac{\theta}{\eta}} + \prod_{j=1}^{s}(1-n_{it+j})(1-n_{it})\left(X_{it-1}\right)^{-\frac{\theta}{\eta}}$$

back

Steady-State Output and Productivity

$$y^{\frac{1}{\eta}} = \eta \frac{1-\beta(1-n)\pi^{\frac{\theta}{\eta}}}{1-\beta(1-n)\pi^{\theta-1}} \left(\frac{n}{1-(1-n)\pi^{\theta-1}}\right)^{\frac{1+\theta\left(\frac{1}{\eta}-1\right)}{\theta-1}}$$

$$x^{\theta} = \left(\frac{1-(1-n)\pi^{\frac{\theta}{\eta}}}{n}\right)^{\eta} \left(\frac{1-(1-n)\pi^{\theta-1}}{n}\right)^{-\frac{\theta}{\theta-1}}$$

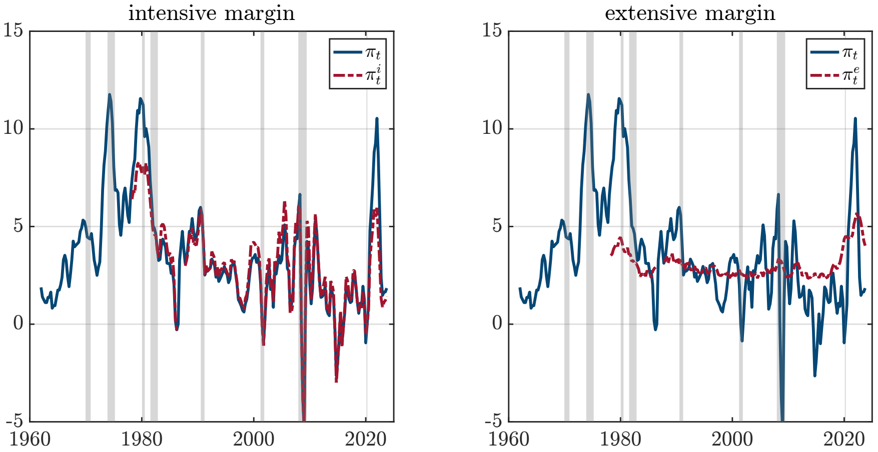
back

Role of Extensive Margin

- Decompose $\pi_t = \Delta_t n_t$ into two components
  - Δ_t : average price change conditional on adjustment
  - n_t : fraction of price changes

- Isolate role of each using Klenow and Kryvtsov (2008) decomposition
  - intensive margin: $\pi_t^i = \Delta_t \bar{n}$
    - $\bar{n}$: mean fraction of price changes
  - extensive margin: $\pi_t^e = \bar{\Delta} n_t$
    - $\bar{\Delta}$: mean average price change

Role of Extensive Margin: Data



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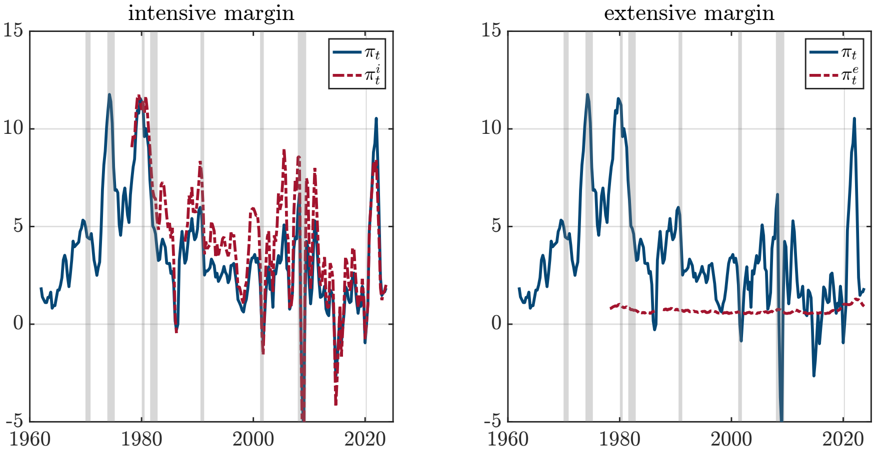
Montag and Villar (2024)

- Argue that extensive margin plays no role post Covid
- Same decomposition but set $\bar{n}$ and $\bar{\Delta}$ equal to January 2020 values
  - due to seasonality, unusually large n and low Δ
- Illustrate fixing $\bar{n}$ and $\bar{\Delta}$ at January 2020 values

Role of Extensive Margin using January 2020



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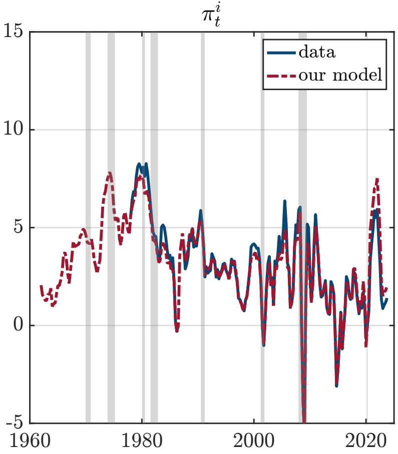
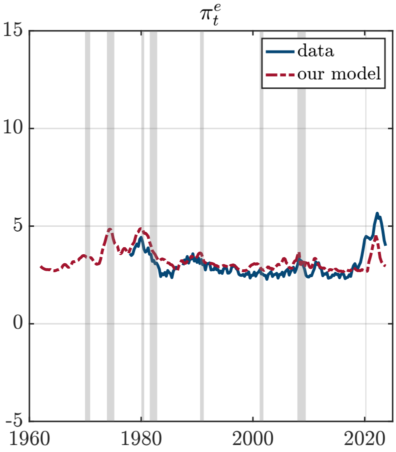
Role of Extensive Margin: Our Model





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Model

Model Overview

- Continuum of multi-product firms
  - each sells continuum of goods
  - decreasing returns labor-only technology
  - cost of changing prices
- Monetary policy targets nominal spending
  - only source of aggregate uncertainty
- Golosov-Lucas log-linear assumptions on preferences

Consumers

- Life-time utility

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left(\log c_t - h_t\right)$$

- Budget constraint

$$P_t c_t + \frac{1}{1+i_t} B_{t+1} = W_t h_t + D_t + B_t$$

- Monetary policy targets nominal spending $M_t \equiv P_t c_t$

$$\log M_{t+1}/M_t = \mu + \varepsilon_{t+1},\ \varepsilon_{t+1} \sim N\left(0, \sigma^2\right)$$

- Log-linear preferences imply $W_t = M_t$

Final Goods Producer

- Final good used for consumption, produced using CES aggregator

$$c_t = y_t = \left(\int_0^1 \int_0^1 (y_{ikt})^{\frac{\theta-1}{\theta}} \, \mathrm{d}k \mathrm{d}i \right)^{\frac{\theta}{\theta-1}}$$

  - y_{ikt} output of good k produced by firm i, sold at price P_{ikt}

- Demand for individual product

$$y_{ikt} = \left(\frac{P_{ikt}}{P_t} \right)^{-\theta} y_t, \qquad \text{where} \qquad P_t = \left(\int_0^1 \int_0^1 (P_{ikt})^{1-\theta} \, \mathrm{d}k \mathrm{d}i \right)^{\frac{1}{1-\theta}}$$

Intermediate Goods Producers

- Individual goods produced with decreasing returns technology

$$y_{ikt} = (l_{ikt})^{\eta}$$

  – $\eta \leq 1$: micro-strategic complementarities in price setting

- Nominal flow profits of firm i from producing product k

$$P_{ikt} y_{ikt} - \tau W_t l_{ikt}$$

  – subsidy to eliminate markup distortion $\tau = 1 - 1/\theta$

- Real flow profits of firm i

$$\int_0^1 \left(\left(\frac{P_{ikt}}{P_t} \right)^{1-\theta} y_t - \tau \frac{W_t}{P_t} \left(\frac{P_{ikt}}{P_t} \right)^{-\frac{\theta}{\eta}} y_t^{\frac{1}{\eta}} \right) \mathrm{d}k$$

Price Adjustment Costs

- Firm chooses fraction of prices to change $n_{it} \in [0, 1]$
  - but not which prices to change (similar to Greenwald 2018)

- Price adjustment cost, denominated in units of labor

$$\frac{\xi}{2}\left(n_{it} - \bar{n}\right)^2, \quad \text{if } n_{it} > \bar{n}$$

  - when $\xi \to \infty$, model collapses to Calvo with constant frequency $\bar{n}$

- If adjust $P_{ikt} = P_{it}^*$, otherwise $P_{ikt} = P_{ikt-1}$

Within-Firm Misallocation

- Firm-level output y_{it} and labor l_{it}

$$y_{it} = \left(\int_0^1 (y_{ikt})^{\frac{\theta-1}{\theta}} \, \mathrm{d}k \right)^{\frac{\theta}{\theta-1}} \qquad \text{and} \qquad l_{it} = \int_0^1 l_{ikt} \mathrm{d}k$$

- Firm production function

$$y_{it} = \left(\frac{X_{it}}{P_{it}} \right)^{\theta} (l_{it})^{\eta}$$

- Depends on firm price index P_{it} and losses from misallocation X_{it}

$$P_{it} = \left(\int_0^1 (P_{ikt})^{1-\theta} \, \mathrm{d}k \right)^{\frac{1}{1-\theta}} \qquad \text{and} \qquad X_{it} = \left(\int_0^1 (P_{ikt})^{-\frac{\theta}{\eta}} \, \mathrm{d}k \right)^{-\frac{\eta}{\theta}}$$

- Absent price dispersion $X_{it}/P_{it} = 1$, otherwise $X_{it}/P_{it} < 1$

Firm Problem

- Choose reset price P_{it}^* and fraction of prices to change n_{it} to maximize

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left[\underbrace{\left(\frac{P_{it+s}}{P_{t+s}}\right)^{1-\theta}}_{\text{sales}} - \underbrace{\tau \left(\frac{X_{it+s}}{P_{t+s}}\right)^{-\frac{\theta}{\eta}} y_{t+s}^{\frac{1}{\eta}}}_{\text{labor costs}} - \underbrace{\frac{\xi}{2}\left(n_{it+s} - \bar{n}\right)^2}_{\text{repricing costs}} \right]$$

- Choices at t affect firm price index and misallocation at all future dates

$$\left(P_{it+s}\right)^{1-\theta} = n_{it+s}\left(P_{it+s}^*\right)^{1-\theta} + \left(1 - n_{it+s}\right) n_{it+s-1}\left(P_{it+s-1}^*\right)^{1-\theta} + \cdots$$

$$+ \prod_{j=1}^{s}\left(1 - n_{it+j}\right) n_{it}\left(P_{it}^*\right)^{1-\theta} + \prod_{j=1}^{s}\left(1 - n_{it+j}\right)\left(1 - n_{it}\right)\left(P_{it-1}\right)^{1-\theta}$$

misallocation profit

- History encoded in two state variables: P_{it-1} and X_{it-1}
  - exact aggregation because adjustment hazard does not depend on P_{ikt-1}

Optimal Reset Price

- Optimal reset price

intuition

$$\frac{P_{it}^*}{P_t} = \left(\frac{1}{\eta}\frac{b_{2it}}{b_{1it}}\right)^{\frac{1}{1+\theta\left(\frac{1}{\eta}-1\right)}}$$

- Depends on present value of revenue and marginal costs
  - weighted by the probability that a price is still in effect at a future date

$$b_{1it} = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \prod_{j=1}^{s} (1 - n_{it+j}) \left(\frac{P_{t+s}}{P_t}\right)^{\theta-1}$$

$$b_{2it} = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \prod_{j=1}^{s} (1 - n_{it+j}) \left(\frac{P_{t+s}}{P_t}\right)^{\frac{\theta}{\eta}} (y_{t+s})^{\frac{1}{\eta}}$$

- Similar to Calvo, except n_{it} time-varying

Optimal Fraction of Price Changes

- Equate marginal cost to marginal benefit

$$\xi\left(n_{it}-\bar{n}\right)=b_{1it}\left(\left(\frac{P_{it}^{*}}{P_{t}}\right)^{1-\theta}-\left(\frac{P_{it-1}}{P_{t}}\right)^{1-\theta}\right)-\tau b_{2it}\left(\left(\frac{P_{it}^{*}}{P_{t}}\right)^{-\frac{\theta}{\eta}}-\left(\frac{X_{it-1}}{P_{t}}\right)^{-\frac{\theta}{\eta}}\right)$$

- Marginal benefit
  - changes firm price index
  - and reduces misallocation
  - weighted by the same terms b_{1it} and b_{2it} that determine P_{it}^{*}

Symmetric Equilibrium

- Since firms are identical, in equilibrium $P_{it}^* = P_t^*$, $n_{it} = n_t$, …

- Going forward: $p_t = P_t/M_t$, $p_t^* = P_t^*/M_t$, $x_t = X_t/P_t$ and $\pi_t = P_t/P_{t-1}$

- Equilibrium conditions

  – reset price: $\frac{p_t^*}{p_t} = \left(\frac{1}{\eta}\frac{b_{2t}}{b_{1t}}\right)^{\frac{1}{1+\theta\left(\frac{1}{\eta}-1\right)}}$

  – fraction of price changes:
  $$\xi\left(n_t - \bar{n}\right) = b_{1t}\left(\left(\frac{p_t^*}{p_t}\right)^{1-\theta} - \left(\frac{1}{\pi_t}\right)^{1-\theta}\right) - \tau b_{2t}\left(\left(\frac{p_t^*}{p_t}\right)^{-\frac{\theta}{\eta}} - \left(\frac{x_{t-1}}{\pi_t}\right)^{-\frac{\theta}{\eta}}\right)$$

  – price index: $1 = n_t\left(\frac{p_t^*}{p_t}\right)^{1-\theta} + \left(1-n_t\right)\pi_t^{\theta-1}$

  – losses from misallocation: $x_t^{-\frac{\theta}{\eta}} = n_t\left(\frac{p_t^*}{p_t}\right)^{-\frac{\theta}{\eta}} + \left(1-n_t\right)x_{t-1}^{-\frac{\theta}{\eta}}\pi_t^{\frac{\theta}{\eta}}$

Computation

- Model collapses to one-equation extension of Calvo
  - the additional equation determines the fraction of price changes
  - as $\xi \to \infty$, $n_t = \bar{n}$ so our model nests Calvo
- Two state variables
  - previous period price: $s_t = P_{t-1}/M_t = p_{t-1}/\exp(\mu + \varepsilon_t)$
  - previous period misallocation: x_{t-1}
- Do not need to keep track of joint distribution of these variables
  - because firms are ex-post identical
- Solve the model globally, but third-order perturbation reasonably accurate

Parameterization

Calibration Strategy

- Assigned parameters
  - period 1 quarter so $\beta = 0.99$
  - demand elasticity $\theta = 6$ and span of control $\eta = 2/3$

- Calibrated parameters
  - mean and standard deviation of nominal spending growth μ and σ
  - fraction of free price changes $\bar{n}$ and price adjustment cost ξ

- Calibration targets
  - mean and standard deviation of inflation
  - mean fraction of price changes
  - slope of fraction of price changes on absolute value of inflation

Calibrated Parameters

Targeted Moments

	Data	Our model	Calvo
mean inflation	3.517	3.517	3.517
s.d. inflation	2.739	2.739	2.739
mean fraction	0.297	0.297	0.297
slope of n_t on $\|\pi_t\|$	0.016	0.016	–

Calibrated Parameters

		Our model	Calvo
μ	mean spending growth rate	0.035	0.035
σ	s.d. monetary shocks	0.022	0.022
$\bar{n}$	fraction free price changes	0.241	0.297
ξ	adjustment cost	1.767	–

- Price adjustment costs account for 0.65% of all labor costs

Steady State Analysis

Overview

- Show how steady-state outcomes vary with trend inflation
- Responses to monetary shocks
- Derive Phillips curve

Fraction of Price Changes

- Let $\pi = \exp(\mu)$ denote level of trend inflation
  - variable without t subscript is value in non-stochastic steady state

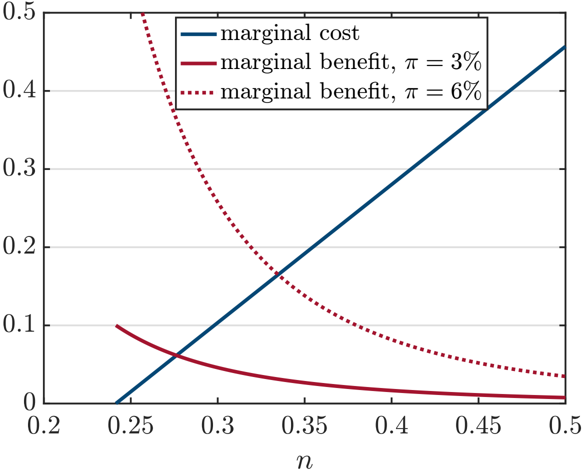
- Steady state fraction of price changes n

$$\xi(n - \bar{n}) = \frac{1}{1 - \beta(1-n)\pi^{\theta-1}} \frac{1}{n} \left(1 - \pi^{\theta-1} - \tau\eta \frac{1 - (1-n)\pi^{\theta-1}}{1 - (1-n)\pi^{\frac{\theta}{\eta}}} \left(1 - \pi^{\frac{\theta}{\eta}}\right) \right)$$

- Marginal cost linearly increasing in n

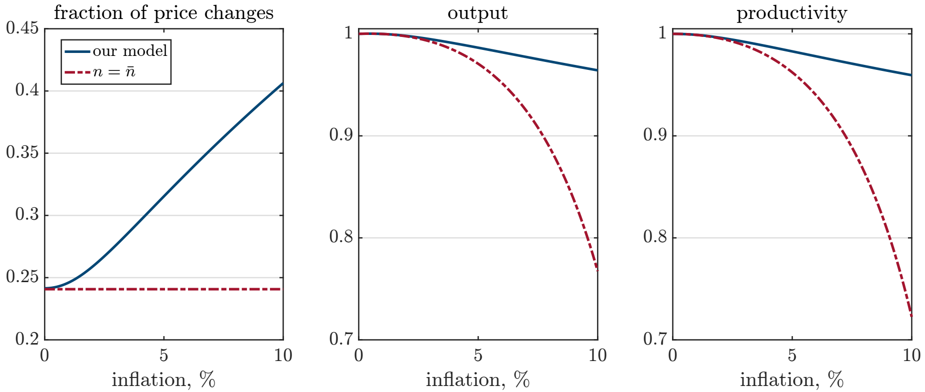
- Marginal benefit
  - absent trend inflation (i.e. $\pi = 1$), marginal benefit is zero and $n = \bar{n}$
  - when $\pi > 1$, decreasing in n

Fraction of Price Changes



Fraction of price changes increases with inflation

Output and Productivity



- Inflation less distortionary in our model

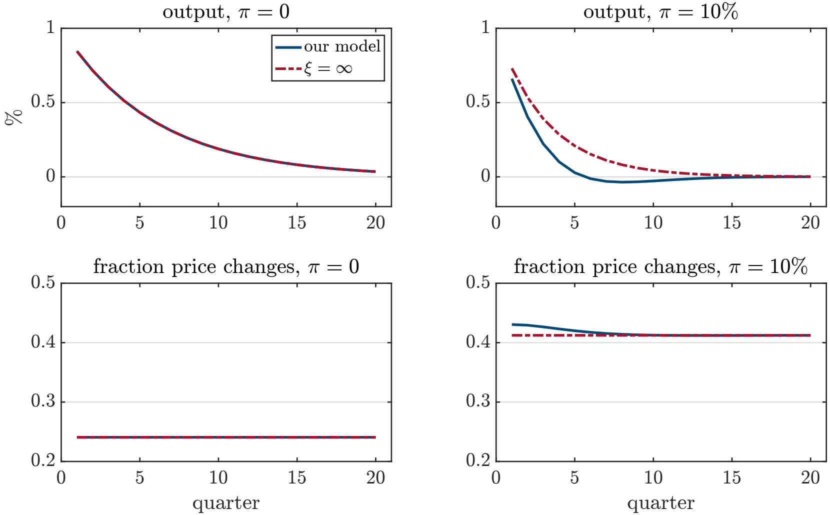
equations

  – because more frequent price changes, as in menu cost models

Real Effects of Monetary Shocks

- Response to 1% monetary shock
  - in economies with 0 and 10% trend inflation
  - compare to economy with steady-state frequency as our model, but $\xi = \infty$
- Focus on output response
  - $M_t = P_t y_t$, so output response depends on how sticky prices are

Response to 1% Monetary Shock



Understanding the Result

- Small jump in frequency has large effect on price level

- To see why, log-linearize expression for aggregate price index

$$\hat{\pi}_t = \underbrace{\frac{1}{(1-n)\,\pi^{\theta-1}} \frac{\pi^{\theta-1}-1}{\theta-1}}_{\mathcal{M}} \hat{n}_t + \underbrace{\frac{1-(1-n)\,\pi^{\theta-1}}{(1-n)\,\pi^{\theta-1}}}_{\mathcal{N}} (\hat{p}_t^* - \hat{p}_t)$$

- Elasticity $\mathcal{N}$ to reset price changes: identical to Calvo
  - decreases with inflation (lower weight on new prices)

- Elasticity $\mathcal{M}$ to frequency: zero if $\pi = 1$, increases with inflation
  - so price level more responsive to changes in n at high inflation

Intuition

- Why is price level more responsive to changes in n at high inflation?

- Inflation $\approx$ average price change $\times$ fraction of price changes
  - $\pi = 0$: average price change $= 0$ so fraction inconsequential
  - $\pi = 10\%$: average price change is large
    - so Δn increases price level considerably
    - mechanism in Caplin and Spulber (1986) menu cost model

- Prices even more responsive to large shocks

large shock

  - strong non-linearities, as in menu cost model

Inflation Accelerator

- Expression for price index: higher frequency increases inflation

$$\hat{\pi}_t = \mathcal{M}\hat{n}_t + \mathcal{N}\left(\hat{p}_t^* - \hat{p}_t\right)$$

  - elasticity $\mathcal{M}$ increases with inflation, zero if $\pi = 1$

- Optimal frequency increases with inflation

$$\hat{n}_t = \mathcal{A}\hat{\pi}_t + \mathcal{B}\left(\hat{p}_t^* - \hat{p}_t\right) - \mathcal{C}\hat{x}_{t-1} + \frac{n - \bar{n}}{n}\hat{b}_{1t}$$

  - elasticities $\mathcal{A}$ and $\mathcal{B}$ increase with inflation, zero if $\pi = 1$

equations

- Feedback loop amplifies inflation response to changes in reset price

$$\hat{\pi}_t = \frac{\mathcal{M}\mathcal{B} + \mathcal{N}}{1 - \mathcal{M}\mathcal{A}}\left(\hat{p}_t^* - \hat{p}_t\right) - \frac{\mathcal{M}\mathcal{C}}{1 - \mathcal{M}\mathcal{A}}\hat{x}_{t-1} + \frac{\mathcal{M}}{1 - \mathcal{M}\mathcal{A}}\frac{n - \bar{n}}{n}\hat{b}_{1t}$$

Phillips Curve

- Let $\widehat{mc}_t = \frac{1}{\eta}\hat{y}_t$, can derive Phillips curve: $\hat{\pi}_t = \mathcal{K}\widehat{mc}_t + \dots$ Phillips Curve

- Slope of the Phillips curve

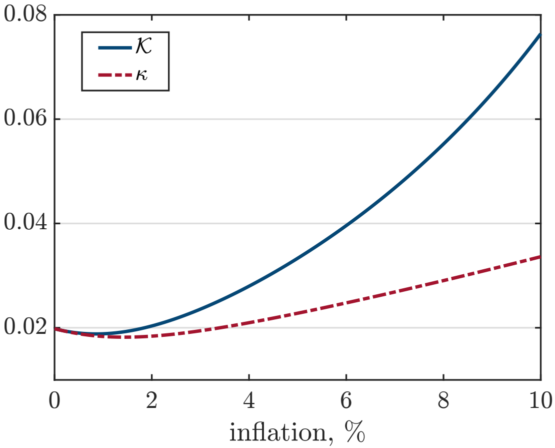
$$\mathcal{K} = \underbrace{\frac{1}{1+\theta\left(\frac{1}{\eta}-1\right)}}_{\text{complementarities}} \times \underbrace{\left(1-\beta\left(1-n\right)\pi^{\frac{\theta}{\eta}}\right)}_{\text{horizon effect}} \times \underbrace{\frac{\mathcal{MB}+\mathcal{N}}{1-\mathcal{MA}}}_{\text{reset price}}$$

- If $\xi = \infty$, reduces to slope in Calvo

$$\kappa = \frac{1}{1+\theta\left(\frac{1}{\eta}-1\right)} \times \left(1-\beta\left(1-n\right)\pi^{\frac{\theta}{\eta}}\right) \times \underbrace{\frac{1-\left(1-n\right)\pi^{\theta-1}}{\left(1-n\right)\pi^{\theta-1}}}_{\mathcal{N}}$$

- Difference between $\mathcal{K}$ and κ captures inflation accelerator

Slope of the Phillips Curve



Much steeper at high inflation, mostly due to inflation accelerator

Phillips Curve in the Time-Series

Approach

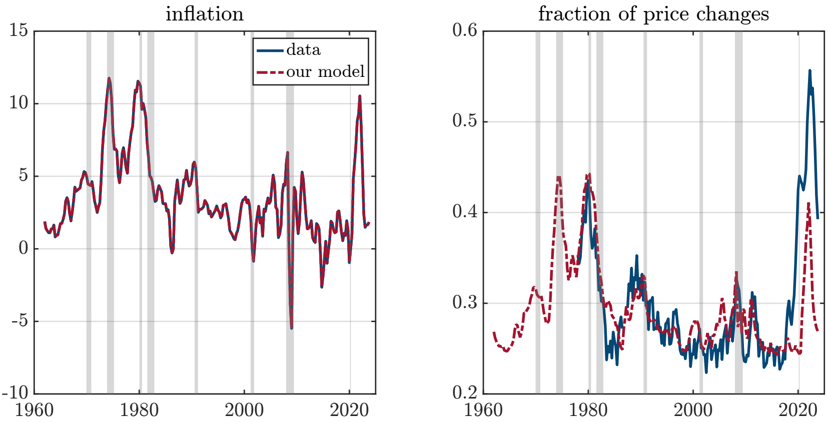
- Use non-linear solution to back out shocks that match U.S. inflation series

$$\pi_t = \pi\left(\frac{p_{t-1}}{\exp\left(\mu + \varepsilon_t\right)}, x_{t-1}\right)$$

  – initialize 1962 in stochastic steady state

- Derive Phillips curve by perturbing equilibrium conditions at each date

Fraction of Price Changes



Reproduces fraction well, except post-Covid

output gap extensive margin model

Perturbation of Equilibrium at Each Date

- First-order perturbation around equilibrium point at each date t
  - nominal spending growth rate at t is $\mu_t = \mu + \varepsilon_t$
  - consider additional shock $\tilde{\varepsilon}_t$, so that $\tilde{\mu}_t = \mu_t + \tilde{\varepsilon}_t$
  - let $\hat{\pi}_t = \log \tilde{\pi}_t - \log \pi_t$ be log-deviation from original equilibrium point

- Log-linearize aggregate price index

$$\hat{\pi}_t = \underbrace{\frac{1}{(1-n_t)\,\pi_t^{\theta-1}} \frac{\pi_t^{\theta-1}-1}{\theta-1}}_{\mathcal{M}_t} \hat{n}_t + \underbrace{\frac{1-(1-n_t)\,\pi_t^{\theta-1}}{(1-n_t)\,\pi_t^{\theta-1}}}_{\mathcal{N}_t} \left(\hat{p}_t^* - \hat{p}_t\right).$$

- Log-linearize expression for optimal fraction of price changes

$$\hat{n}_t = \mathcal{A}_t \hat{\pi}_t + \mathcal{B}_t \left(\hat{p}_t^* - \hat{p}_t\right) - \mathcal{C}_t \hat{x}_{t-1} + \frac{n_t - \bar{n}}{n_t} \hat{b}_{1t}$$

Slope of the Phillips Curve

- Slope of the Phillips curve

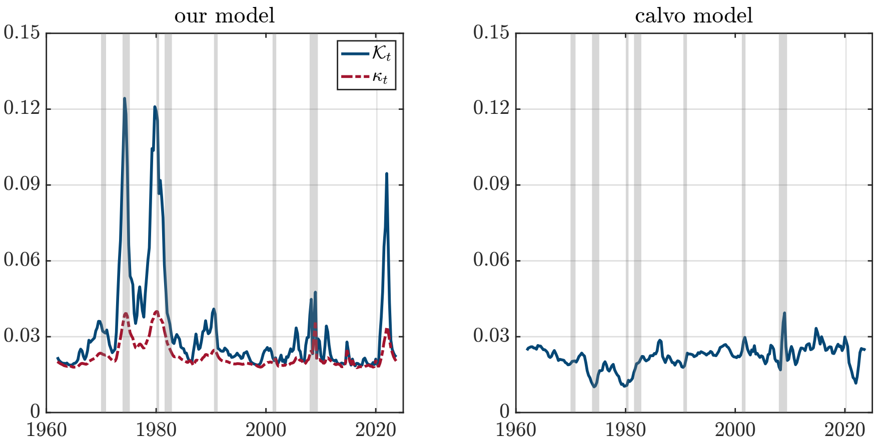
$$\mathcal{K}_t = \frac{1}{1+\theta\left(\frac{1}{\eta}-1\right)} \quad \times \quad \frac{y_t^{\frac{1}{\eta}}}{b_{2t}} \quad \times \quad \frac{\mathcal{M}_t\mathcal{B}_t+\mathcal{N}_t}{1-\mathcal{M}_t\mathcal{A}_t}$$

- Absent endogenous frequency response

$$\kappa_t = \frac{1}{1+\theta\left(\frac{1}{\eta}-1\right)} \quad \times \quad \frac{y_t^{\frac{1}{\eta}}}{b_{2t}} \quad \times \quad \underbrace{\frac{1-(1-n_t)\,\pi_t^{\theta-1}}{(1-n_t)\,\pi_t^{\theta-1}}}_{\mathcal{N}_t}$$

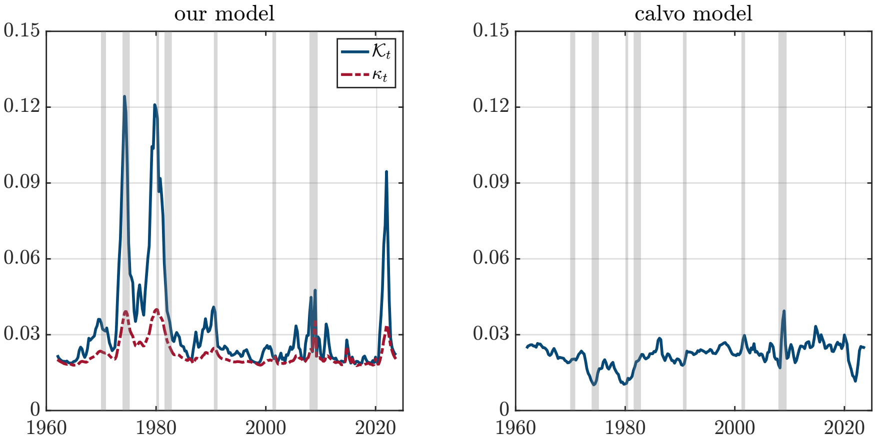
- The difference $\mathcal{K}_t - \kappa_t$ captures the inflation accelerator

Time-Varying Slope of the Phillips Curve



Ranges from 0.02 to 0.12, mostly due to inflation accelerator

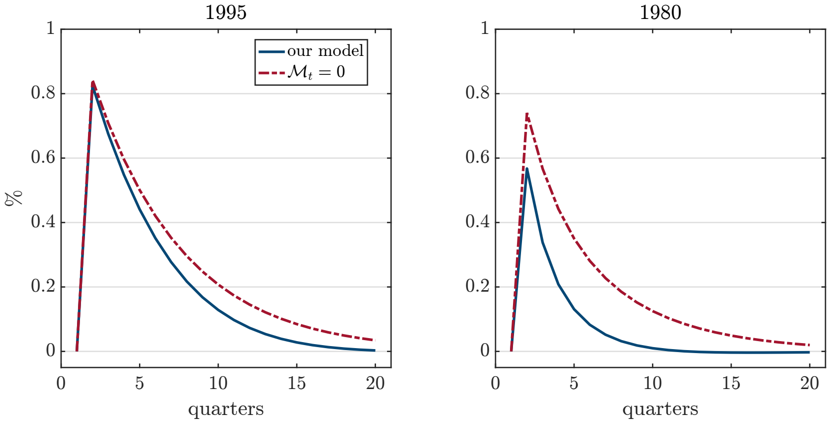
Time-Varying Slope of the Phillips Curve



In Calvo model slope falls in periods of high inflation

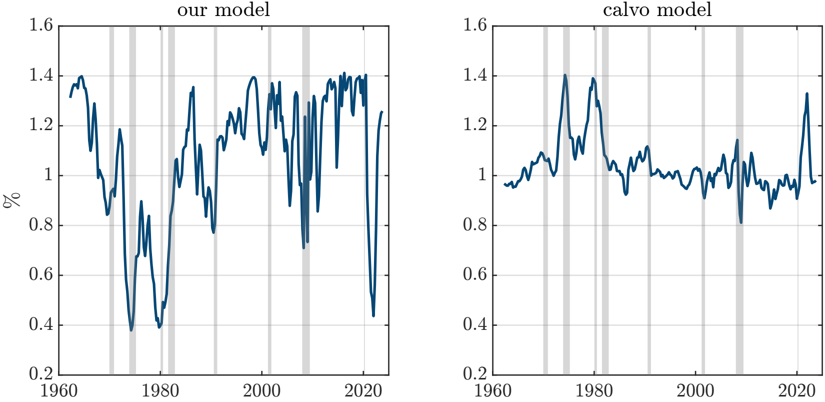
Implication 1: Time-Varying Responses to Shocks

- Consider response to 1% shock in 1995 (low π_t) and 1980 (high π_t)
- Build intuition by computing log-linear approximation details
  - repeat setting $\mathcal{M}_t = 0$ to isolate inflation accelerator



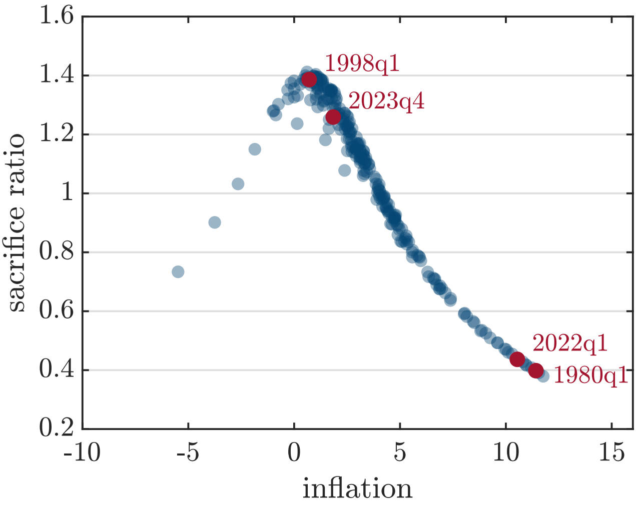
Implication 2: Sacrifice Ratio

- Time-varying slope: reducing inflation less costly when inflation is high
- Calculate average drop in output needed to reduce π by 1pp over a year



Ranges from 0.4% (high inflation) to 1.4% (low inflation), opposite of Calvo

Inflation and the Sacrifice Ratio



Robustness

Two Robustness Exercises

- Eliminate strategic complementarities
  - set $\eta = 1$ and recalibrate model with $\theta = 6$ and $\theta = 3$ ▸ calibration
  - slope of Phillips curve is larger, but fluctuates as in baseline ▸ Phillips curve
- Taylor Rule monetary policy
  - replace nominal spending target with Taylor rule ▸ calibration
  - slope of Phillips curve as in baseline ▸ Phillips curve