Nonlinearities in the Regional Phillips Curve with Labor Market Tightness

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 - Δ economic activity $\Rightarrow \Delta$ labor costs $\Rightarrow \Delta$ prices

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 - ls relation b/w economic activity and π different in tight vs slack labor markets?
- ls the Phillips curve nonlinear in labor market tightness (θ) ?
 - $\theta = \text{job vacancies}/\text{unemployed workers}$
 - Limited episodes of tight labor markets in time-series data
 - Turn to regional panel data

Regional Tightness and Inflation

- Evidence of nonlinear relationship between heta and π in raw data



Notes. Scatter plot of logarithm of labor market tightness and 12-month core inflation across 21 US Metropolitan Statistical Areas (MSAs) from December 2000 to July 2024.

This Paper

- 1. Theoretical framework
 - Multi-sector, 2-region New Keynesian model with
 - Search-and-matching frictions in the labor markets
 - Wage rigidities
 - Derive nonlinear regional Phillips Curve (NRPC) with θ

This Paper

- 1. Theoretical framework
 - Multi-sector, 2-region New Keynesian model with
 - Search-and-matching frictions in the labor markets
 - Wage rigidities
 - Derive nonlinear regional Phillips Curve (NRPC) with θ
- 2. Empirical exercise
 - Impute MSA-level job vacancies from state-level data
 - Construct MSA-level labor market tightness
 - Estimate NRPC with panel variation + IV approach
 - IV: shift-share instrument exploiting sectoral labor demand shocks

Contributions to the Literature

 Theoretical microfoundation of nonlinear Phillips curve with wage rigidities: Benigno and Antonio Ricci (2011), Daly and Hobijn (2014), Schmitt-Grohé and Uribe (2022), Benigno and Eggertsson (2023)

Contribution: first to microfound nonlinear regional Phillips curve

 Identification of Phillips curve with regional panel variation: Mavroeidis, Plagborg-Møller, and Stock (2014), Fitzgerald and Nicolini (2014), McLeay and Tenreyro (2020), Hazell, Herreño, Nakamura, and Steinsson (2022), Cerrato and Gitti (2022), Autor, Dube, and McGrew (2023), Smith, Timmermann, and Wright (2023)

Contribution: estimate regional Phillips curve with θ & nonlinearity

 Estimation of nonlinearities in regional Phillips curve: Kiley (2015), Murphy (2017), Babb and Detmeister (2017), Leduc and Wilson (2017), Hooper, Mishkin, and Sufi (2020)

Contribution: use θ + estimate NRPC with IV

Outline

Introduction

Theoretical Framework

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Key Ingredients of the Model

- Two regions in a monetary union: i, j
- Households
 - GHH preferences over consumption and members' labor force participation
 - Unemployed members search for job and get hired according to matching function
 - Labor perfectly mobile across sectors, immobile across regions
- Firms
 - Commodity sector: traded on international market at exogenous price
 - Intermediate-input sector: perfect competition, tradable
 - Final-goods sector: monopolistic competition, non-tradable
- Employment agencies
 - Post vacancies of intermediate-input and final-goods firms at a cost
 - Match unemployed workers to vacancies charging a fee
- Monetary authority sets common interest rate according to Taylor rule

Wage-Setting Mechanism

Phillips (1958): nonlinear relation between θ and nominal wage growth

Wage-setting mechanism

$$W_{it} = \max\{W_{it}^{flex}, W_{it}^{norm}\}$$
(1)

W^{flex}: flexible nominal wage pinned down through model of employment agencies
 W^{norm}_{it} ≡ (W
i)^λ(W^{flex}{it})^{1-λ}: prevailing nominal wage, λ = degree of wage rigidity

Asymmetric response of real wages to state of labor market

$$w_{it} = \begin{cases} w_{it}^{flex} & \theta_{it} > \theta_{it}^{*} \\ (\bar{w}_{i})^{\lambda} (w_{it}^{flex})^{1-\lambda} & \theta_{it} \le \theta_{it}^{*} \end{cases}$$
(2)

•
$$\theta_{it}^* = \left(\frac{1}{m_{it}}\frac{\gamma_{it}^c}{\gamma_{it}^b}\bar{w}_i\right)^{\frac{1}{\eta}}$$
: region-specific and varies along time

Nonlinear Regional Phillips Curve

Log-linearizing the model around zero inflation steady state:

$$\pi_{it} = \begin{cases} \beta E_t \pi_{it+1} + \kappa_{\theta}^{tight} \hat{\theta}_{it} + \kappa_{p} \hat{\rho}_{it}^{x} + \kappa_{\nu}^{tight} \hat{\nu}_{it} + \kappa_{a} \hat{a}_{it}^{y} & \hat{\theta}_{it} > \hat{\theta}_{it}^{*} \\ \beta E_t \pi_{it+1} + \kappa_{\theta} \hat{\theta}_{it} + \kappa_{p} \hat{\rho}_{it}^{x} + \kappa_{\nu} \hat{\nu}_{it} + \kappa_{a} \hat{a}_{it}^{y} & \hat{\theta}_{it} \le \hat{\theta}_{it}^{*} \end{cases}$$
(3)

- \blacktriangleright π_{it} : final-goods price inflation in region *i* and period *t*
- $E_t \pi_{it+1}$: short-run inflation expectations
- $\hat{\theta}_{it}$: labor market tightness
- \hat{p}_{it}^{x} : incidence of intermediate-input price shock on regional price level
- \triangleright $\hat{\nu}_{it}$: search-and-matching shock
- \hat{a}_{it}^{y} : final-goods sector productivity shock

$$\Rightarrow \kappa_{\theta} = \kappa_{\theta}^{tight} (1 - \lambda) < \kappa_{\theta}^{tight}$$

Solve NRPC Forward

Assume

Labor market is slack in t, expected to remain slack forever

- Labor market is tight in t, expected to remain tight until t + T 1
 - Labor market expected to turn slack in t + T and remain slack forever

NRPC becomes

$$\pi_{it} = \begin{cases} E_t \pi_{t+\infty} + \xi E_t \hat{\theta}_{it+\infty} + \psi_{\theta}^{tight} \tilde{\theta}_{it} + \psi_{\rho} \hat{\rho}_{it}^{\mathsf{x}} + \varepsilon_{it}^{tight} & \tilde{\theta}_{it} > \tilde{\theta}_{it}^{*} \\ E_t \pi_{t+\infty} + \psi_{\theta}^{slack} \tilde{\theta}_{it} + \psi_{\rho} \hat{\rho}_{it}^{\mathsf{x}} + \varepsilon_{it} & \tilde{\theta}_{it} \le \tilde{\theta}_{it}^{*} \end{cases}$$
(4)

- $E_t \pi_{t+\infty}$: long-run inflation expectations depending on monetary policy regime
- $E_t \hat{\theta}_{it+\infty}$: permanent component of θ variation
- $\tilde{\theta}_{it} = \hat{\theta}_{it} E_t \hat{\theta}_{it+\infty}$: transitory component of θ variation
- ε_{it} , ε_{it}^{tight} : PDV of search-and-matching & final-goods sector productivity shocks

$$\Rightarrow \psi^{\textit{slack}}_{\theta} = \psi^1_{\theta}$$
 , $\psi^{\textit{tight}}_{\theta} = \psi^1_{\theta} + \psi^2_{\theta}$

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Measuring Job Vacancies at the MSA Level

- Challenge
 - Consumer Price Index (CPI) data available at MSA level $\Rightarrow \pi$
 - ▶ Job vacancy data available at state level $\Rightarrow \theta$
- Newly imputed data series of job vacancies at MSA level
 - Imputed from state-level data (JOLTS-Bureau of Labor Statistics)
 - Using yearly employment weights from Current Population Survey (CPS)
- Advantages:
 - 1. Job postings from a representative sample of firms
 - 2. Available since December 2000
- Distribution of vacancies in MSAs different from states
 - ► IV takes care of measurement errors

ata distributions π and heta aggregate heta cross-sectional variation

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From the Model to the Data

Empirical specification:

 $\pi_{it} = \boldsymbol{c} + \alpha_i + \gamma_{t_q} + \delta_{it_q} + \psi_{\theta}^1 \ln(\theta_{it}) + \psi_{\theta}^2 \ln(\theta_{it}) \times \boldsymbol{I}_{\{\theta_{it} > 1\}} + \beta \boldsymbol{I}_{\{\theta_{it} > 1\}} + \psi_{\rho} \boldsymbol{p}_{it}^{\mathsf{x}} + \varepsilon_{it} \quad (5)$

 \blacktriangleright π_{it} : 12-month core inflation in MSA *i* and year-month *t*

- **c**, α_i , γ_t , δ_{it_a} : constant, MSA FE, year-quarter FE, and their interaction
- $\ln(\theta_{it})$: log of labor market tightness
- I_{{θit>1}}: dummy that takes value of 1 when θ_{it} > 1
- *p*^x_{it}: ratio of national manufacturing PPI and MSA-level CPI

$$\Rightarrow \psi_{\theta}^{\textit{slack}} = \psi_{\theta}^{1}, \, \psi_{\theta}^{\textit{tight}} = \psi_{\theta}^{1} + \psi_{\theta}^{2}$$

Identification of NRPC Slope

Empirical specification:

 $\pi_{it} = \boldsymbol{c} + \alpha_i + \gamma_{t_q} + \delta_{it_q} + \psi_{\theta}^1 \ln(\theta_{it}) + \psi_{\theta}^2 \ln(\theta_{it}) \times \boldsymbol{I}_{\{\theta_{it} > 1\}} + \beta \boldsymbol{I}_{\{\theta_{it} > 1\}} + \psi_{\rho} \boldsymbol{p}_{it}^{\mathsf{x}} + \varepsilon_{it} \quad (5)$

- Simultaneity bias between demand and supply
 - Demand shocks $\Rightarrow \theta \uparrow$, $\pi \uparrow$
 - Supply shocks $\Rightarrow \theta \downarrow$, $\pi \uparrow$
 - Supply shocks induce downward bias in slope of Phillips curve

Identification of NRPC Slope

Empirical specification:

 $\pi_{it} = c + \alpha_i + \gamma_{t_q} + \delta_{it_q} + \psi_{\theta}^1 + \ln(\theta_{it}) + \psi_{\theta}^2 \ln(\theta_{it}) \times I_{\{\theta_{it} > 1\}} + \beta I_{\{\theta_{it} > 1\}} + \psi_p p_{it}^{\mathsf{x}} + \varepsilon_{it}$ (5)

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- Empirical strategy tackles simultaneity bias
 - Aggregate level \Rightarrow fixed effects capture aggregate demand and supply shocks

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- Empirical strategy tackles simultaneity bias
 - Aggregate level \Rightarrow fixed effects capture aggregate demand and supply shocks
 - Regional level \Rightarrow IV addresses unobservable regional supply shocks in ε_{it}
 - Search-and-matching and final-goods sector productivity shocks

Empirical specification:

 $\pi_{it} = \boldsymbol{c} + \alpha_i + \gamma_{t_q} + \delta_{it_q} + \psi_{\theta}^1 \ln(\theta_{it}) + \psi_{\theta}^2 \ln(\theta_{it}) \times \boldsymbol{I}_{\{\theta_{it} > 1\}} + \beta \boldsymbol{I}_{\{\theta_{it} > 1\}} + \psi_{\rho} \boldsymbol{p}_{it}^{\mathsf{x}} + \boldsymbol{\varepsilon}_{it} \quad (5)$

• ε_{it} contains unobservable regional supply shocks \Rightarrow IV

SSIV: intermediate-input sectors' labor demand shocks

$$z_{it}^{\times} = \sum_{k=1}^{N} e_{ki} imes g_{kt},$$

g_{kt}: 3-year growth national employment of industry k at time t

- e_{ki}: average employment share of industry k in MSA i
- \blacktriangleright k = tradable intermediate-input 2-digit Census industries
- Exogeneity stems from g_{kt} rather than e_{ki} [Borusyak et al. 2022]

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SSIV: intermediate-input sectors' labor demand shocks

- Mechanism: $z_{it}^{\times} \uparrow \Rightarrow \theta_{it} \uparrow \Rightarrow w_{it} \uparrow \Rightarrow \pi_{it} \uparrow$
 - Following a positive labor demand shock in national manufacturing sector
 - Final-goods firms in Detroit experience larger cost increases

Empirical specification:

 $\pi_{it} = \boldsymbol{c} + \alpha_i + \gamma_{t_q} + \delta_{it_q} + \psi_{\theta}^1 \ln(\theta_{it}) + \psi_{\theta}^2 \ln(\theta_{it}) \times \boldsymbol{I}_{\{\theta_{it} > 1\}} + \beta \boldsymbol{I}_{\{\theta_{it} > 1\}} + \psi_{\rho} \boldsymbol{p}_{it}^{\mathsf{x}} + \boldsymbol{\varepsilon}_{it} \quad (5)$

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• Mechanism:
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Identifying assumption: on average, cost increase no larger in Detroit than NYC

Empirical specification:

 $\pi_{it} = \boldsymbol{c} + \alpha_i + \gamma_{t_q} + \delta_{it_q} + \psi_{\theta}^1 \ln(\theta_{it}) + \psi_{\theta}^2 \ln(\theta_{it}) \times \boldsymbol{I}_{\{\theta_{it} > 1\}} + \beta \boldsymbol{I}_{\{\theta_{it} > 1\}} + \psi_{\rho} \boldsymbol{p}_{it}^{\mathsf{x}} + \varepsilon_{it} \quad (5)$

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SSIV: intermediate-input sectors' labor demand shocks

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Threat to exclusion restriction

►
$$z_{it}^{x}$$
 $\uparrow \Rightarrow p_{t}^{x} \downarrow \Rightarrow \pi_{it} \downarrow \rightarrow \text{controlled for by } p_{it}^{x}$

• Instrument
$$p_{it}^{x} = \frac{p_{t}^{x}}{p_{it}}$$
 with $\frac{p_{t}^{x}}{p_{it-24}}$

Empirical specification:

 $\pi_{it} = \boldsymbol{c} + \alpha_i + \gamma_{t_q} + \delta_{it_q} + \psi_{\theta}^1 \ln(\theta_{it}) + \psi_{\theta}^2 \ln(\theta_{it}) \times \boldsymbol{I}_{\{\theta_{it} > 1\}} + \beta \boldsymbol{I}_{\{\theta_{it} > 1\}} + \psi_p \boldsymbol{p}_{it}^{\mathsf{x}} + \varepsilon_{it} \quad (5)$

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SSIV: intermediate-input sectors' labor demand shocks

• Mechanism:
$$z_{it}^{\times} \uparrow \Rightarrow \theta_{it} \uparrow \Rightarrow w_{it} \uparrow \Rightarrow \pi_{it} \uparrow$$

- Identifying assumption: on average, cost increase no larger in Detroit than NYC
- ▶ Threat to exclusion restriction \Rightarrow controlled for by p_{it}^{x}

► Instrument
$$\ln(\theta_{it}) \times I_{\{\theta_{it} > 1\}}$$
 with $z_{it}^{\times} \times I_{\{\theta_{it} > 1\}}$

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Estimate of NRPC

• π deviations = π - estimated controls and FEs

• Regional Phillips curve steepens when $\theta_{it} > 1$



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Robustness Checks



Results robust to use of different weights to impute vacancies at MSA level different weight

NRPC with θ robust to other nonlinear functional forms

NRPC with θ robust to inclusion of proxy for final-goods sector productivity shock final-goods sector productivity

Conclusion

- Key result: NRPC with θ
 - Derivation of NRPC through a NK model
 - Newly imputed measure of vacancies at MSA level
 - Estimation of NRPC combining panel variation and IV
 - When θ exceeds 1, NRPC significantly steepens
- Policy implications:
 - Ignoring nonlinearity of Phillips curve \Rightarrow unexpected surge in π
 - Nonlinearity allows to bring down π at lower economic cost in tight labor markets
- Path for future research
 - Aggregation from regional to national Phillips curve

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Time Series of Aggregate θ & π



Notes. Time series of labor market tightness and inflation from Benigno and Eggertsson (2023).

Regional Labor Market Tightness and Inflation



Notes. Scatter plot of labor market tightness and 12-month core inflation across 21 US Metropolitan Statistical Areas (MSAs) from December 2000 to July 2024.

Dack

Aggregate Labor Market Tightness and Inflation



Notes. Scatter plot of aggregate logarithm of labor market tightness and 12-month core inflation from December 2000 to July 2024.

Aggregate Labor Market Tightness and Inflation



Notes. Scatter plot of aggregate labor market tightness and 12-month core inflation from December 2000 to July 2024.

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Households' Preferences

Continuum of measure ζ of households: h

GHH utility [Greenwood, Hercowitz, and Huffman 1988]

$$u(C_{it}^{h}, F_{it}^{h}, \chi_{it}) = \frac{1}{1 - \sigma} \left(C_{it}^{h} - \chi_{it} \int_{0}^{F_{it}^{h}} f^{\omega} df \right)^{1 - \sigma}, \ \sigma > 0$$
(6)

• $C_{it}^{h} = \left[\int_{0}^{1} C_{it}^{h}(z)^{\frac{\epsilon-1}{\epsilon}} dz\right]^{\frac{\epsilon}{\epsilon-1}}, \epsilon > 1$: consumption good aggregator

• CES preferences over final-goods varieties $C_{it}^h(z)$

 \blacktriangleright F_{it}^h : total number of members who participate in labor market

e

- f^{ω} : fixed disutility from working of member f
- Household members ordered by their disutility from working

$$\int_{0}^{F_{it}^{h}} f^{\omega} df = \frac{\left(F_{it}^{h}\right)^{1+\omega}}{1+\omega}$$
(7)



At beginning of period t

- F_{it}^h : labor force supplied by household h
- (1 s) F_{it}^h : employed
- sF_{it}^h : unemployed and search for a job



Agencies match unemployed workers to firms' vacancies

Through regional matching function:

$$M_{it} = m_{it} U_{it}^{\eta} V_{it}^{1-\eta}, \ \eta \in [0,1]$$
(8)

- *M_{it}*: unemployed workers matched to vacancies
- *m_{it}*: matching efficiency
- U_{it}: total unemployment in region i
- V_{it}: total vacancies posted by agencies in region i
- V_{it} and U_{it} taken as given by households



Probability of job seeker finding a job

$$f(\theta_{it}) = \frac{M_{it}}{sF_{it}} = u_{it}\frac{m_{it}\theta_{it}^{1-\eta}}{s}$$
(9)

- $\theta_{it} \equiv V_{it}/U_{it}$: labor market tightness
- $u_{it} \equiv U_{it}/F_{it}$: unemployment rate
- Taken as given by households

Successful employment matches for household h

$$H_{it}^{h} = M_{it}^{h} = f(\theta_{it})sF_{it}^{h} = u_{it}m_{it}\theta_{it}^{1-\eta}F_{it}^{h},$$
(10)



At the end of period t

- Labor force supplied by household h: $F_{it}^h = N_{it}^h + U_{it}^h$
 - *N^h_{it}*: employed household members
 - U_{it}^h : unemployed household members after search and matching

Number of employed household members

$$N_{it}^{h} = (1 - s)F_{it}^{h} + H_{it}^{h}$$

= $(1 - s + m_{it}u_{it}\theta_{it}^{1-\eta})F_{it}^{h}.$ (11)



• H_{it}^{h} pay to employment agencies fraction γ_{it}^{b} of their income

Households' Problem

$$\max_{\{C_{it}^{h},F_{it}^{h},B_{it}^{h}\}} E_{0} \sum_{t=0}^{\infty} \beta^{t} \frac{1}{1-\sigma} \left(C_{it}^{h} - \chi_{it} \frac{\left(F_{it}^{h}\right)^{1+\omega}}{1+\omega}\right)^{1-\sigma}$$

subject to

$$C_{it}^{h}P_{it} + B_{it}^{h} \leq (1 + i_{t-1})B_{it-1}^{h} + \left[(1 - s) + (1 - \gamma_{it}^{b})m_{it}u_{it}\theta_{it}^{1 - \eta}\right]F_{it}^{h}W_{it}$$
$$+ \int_{0}^{1}\Pi_{it}^{F}(z) dz + \int_{0}^{1}\Pi_{it}^{E}(I) dI$$

- ▶ P_{it} : price index associated with the consumption basket C_{it}^h
- ▶ B_{it}^h : risk-free nominal bond paying i_t in period t+1
- ► W_{it}: nominal wage rate
- ▶ $\Pi_{it}^{F}(z)$, $\Pi_{it}^{E}(I)$: profits of final-goods firm z and agency I

Households' Optimality Conditions

Optimal labor force participation:

$$F_{it} = \left[\frac{(1-s) + (1-\gamma_{it}^b)m_{it}u_{it}\theta_{it}^{1-\eta}}{\chi_{it}}w_{it}\right]^{\frac{1}{\omega}}$$
(12)

Demand for final-goods variety z:

$$C_{it}(z) = C_{it} \left(\frac{P_{it}(z)}{P_{it}}\right)^{-\epsilon}$$
(13)

Price index:

$$P_{it} = \left[\int_0^1 P_{it}(z)^{1-\epsilon} dz\right]^{\frac{1}{1-\epsilon}}$$
(14)

Commodity and Intermediate-Input Sectors

Commodity sector

- Global supply
- Production function: $P_t^o = c_t^o O_t$
 - c^o_t: exogenous marginal costs of production

Intermediate-input sector

- Representative firm producing one tradable homogeneous good: X_{it}
- Perfect competition: P_t^{\times}
- Production function: $X_{it} = A_{it}^{\times} N_{it}^{\times \rho} O_{it}^{1-\rho}$

Representative intermediate-input firm maximizes

$$P_t^{\mathsf{x}} A_{it}^{\mathsf{x}} N_{it}^{\mathsf{x}\rho} O_{it}^{1-\rho} - W_{it} N_{it}^{\mathsf{x}} - P_t^{\mathsf{o}} O_{it}$$

From Optimization of Intermediate-Input Firm's Problem

Intermediate-input firm's optimization implies:

Demand for labor

$$W_{it}N_{it}^{x} = \rho M C_{it}^{x} X_{it}$$

Demand for commodity

$$P_t^o O_{it} = (1 - \rho) M C_{it}^x X_{it}$$
where $M C_{it}^x = \frac{1}{A_{it}^x} \left(\frac{W_{it}}{\rho}\right)^{\rho} \left(\frac{P_t^o}{1 - \rho}\right)^{1 - \rho}$

Final-Goods Sector

- Continuum of firms (z) producing varieties of nontradable final goods: $Y_{it}(z)$
- Monopolistic competition: $P_{it}(z)$
- \blacktriangleright Calvo-style price frictions captured by parameter α
- Production function: $Y_{it}(z) = A_{it}^y N_{it}^y(z)^{\phi} X_{it}(z)^{1-\phi}$

Final-goods firm z maximizes

$$E_t \sum_{k=0}^{\infty} M_{it,t+k} [P_{it+k}(z) Y_{it+k}(z) - W_{it+k} N_{it+k}^{y}(z) - P_{t+k}^{x} X_{it+k}]$$

subject to

$$Y_{it}(z) = A^y_{it} N^y_{it}(z)^{\phi} X_{it}(z)^{1-\phi}$$

$$Y_{it}(z) = Y_{it} \left(\frac{P_{it}(z)}{P_{it}}\right)^{-\epsilon}$$

From Optimization of Final-goods Firms' Problem

Final-good firms' optimization implies:

Optimal pricing condition

$$\sum_{k=0}^{\infty} a^{k} E_{t} \left[M_{it,t+k} Y_{it+k}(z) \left(\frac{P_{it}(z)}{P_{it-1}} - \frac{\theta}{\theta - 1} RMC_{it+k} \frac{P_{it+k}}{P_{it-1}} \right) \right]$$

where $RMC_{it}^{y} = \frac{MC_{it}^{y}}{P_{it}} = \frac{1}{P_{it}} \frac{1}{A_{it}^{y}} \left(\frac{W_{it}}{\phi} \right)^{\phi} \left(\frac{P_{t}^{x}}{1 - \phi} \right)^{1 - \phi}$

Demand for labor

$$W_{it}N_{it}(z) = \phi M C_{it}^{y} Y_{it}$$

Demand for intermediate inputs

$$P_t^{\mathsf{x}} X_{it}(z) = (1 - \phi) M C_{it}^{\mathsf{y}} Y_{it}$$

Employment Agencies

- Continuum of measure 1 of employment agencies: I
- Carry out search-and-matching process in the labor market
 - Post vacancies of intermediate-input and final-goods firms
 - Real cost of posting a vacancy: γ^c_{it}
 - Match unemployed workers to vacancies through matching function

• Charge fee proportional to real wage of workers screened: γ_{it}^{b}

Number of matches per vacancy posted:

$$q(\theta_{it}) = \frac{M_{it}}{V_{it}} = \frac{m_{it} U_{it}^{\eta} V_{it}^{1-\eta}}{V_{it}} = m_{it} \theta_{it}^{-\eta}$$
(15)

• $q(\theta_{it})$ and real wage w_{it} taken as given by agency l

Number of matches generated by agency *I*: $q(\theta_{it})V_{it}^{I} = m_{it}\theta_{it}^{-\eta}V_{it}^{I}$

Employment Agencies' Problem

$$\max_{V_{it}^{l}} \gamma_{it}^{b} w_{it} m_{it} \theta_{it}^{-\eta} V_{it}^{l} - \gamma_{it}^{c} V_{it}^{l}$$



Flexible real wage rate

$$w_{it}^{flex} = \frac{1}{m_{it}} \frac{\gamma_{it}^c}{\gamma_{it}^b} \theta_{it}^{\ \eta} \tag{17}$$

Monetary Authority

Common monetary policy following the Taylor rule

$$r^{n} = \phi_{\pi}(\pi_{t} - \bar{\pi}_{t}) - \phi_{\theta}(\hat{\theta}_{t} - \bar{\theta}_{t}) + \varepsilon_{rt}$$

- $\pi_t = \zeta \pi_{it} + (1 \zeta) \pi_{jt}$: economy-wide inflation
- $\hat{\theta}_t = -(\zeta \hat{\theta}_{it} + (1 \zeta) \hat{\theta}_{jt})$: deviation of aggregate θ from steady state
- $\bar{\pi}_t$: time-varying inflation target
- $\bar{\theta}_t = \frac{(1-\beta)\bar{\pi}_t}{\kappa}$: θ consistent with long-run π target
- ϕ_{π} and ϕ_{θ} : ensure a unique locally bounded equilibrium
- \triangleright ε_{rt} : transitory monetary shock

Derivation 1/3

Labor market is slack at time t

$$\pi_{it}^{slack} = \beta E_t \pi_{it+1}^{slack} + \kappa_\theta \hat{\theta}_{it} + \kappa_p \hat{p}_{it}^{\mathsf{x}} + \kappa_a \hat{a}_{it}^{\mathsf{y}} + \kappa_\nu \hat{\nu}_{it}$$
(18)

Solve forward

$$\pi_{it}^{slack} = E_t \sum_{k=0}^{\infty} \beta^k \kappa_{\theta} \hat{\theta}_{it+k} + E_t \sum_{k=0}^{\infty} \beta^k \left[\kappa_p \hat{\rho}_{it+k}^{\mathsf{X}} + \kappa_a \hat{a}_{it+k}^{\mathsf{Y}} + \kappa_\nu \hat{\nu}_{it+k} \right]$$
(19)

• Assume
$$\tilde{\theta}_{it} = \hat{\theta}_{it} + E_t \hat{\theta}_{it+\infty}$$

$$\pi_{it}^{slack} = E_t \sum_{k=0}^{\infty} \beta^k \kappa_{\theta} \tilde{\theta}_{it+k} + \frac{\kappa_{\theta}}{1-\beta} E_t \hat{\theta}_{it+\infty} + E_t \sum_{k=0}^{\infty} \beta^k \left[\kappa_p \hat{\rho}_{it+k}^x + \kappa_{\vartheta} \hat{a}_{it+k}^y + \kappa_{\nu} \hat{\nu}_{it+k} \right]$$
(20)

Derivation 2/3

$$As E_t \pi_{it+\infty} = E_t \pi_{jt+\infty} = E_t \pi_{t+\infty} = \frac{\kappa_{\theta}}{1-\beta} E_t \theta_{it+\infty}$$

$$\pi_{it}^{slack} = E_t \pi_{t+\infty} + E_t \sum_{k=0}^{\infty} \beta^k \kappa_{\theta} \tilde{\theta}_{it+k}$$

$$+ E_t \sum_{k=0}^{\infty} \beta^k \left[\kappa_p \hat{\rho}_{it+k}^x + \kappa_a \hat{a}_{it+k}^y + \kappa_\nu \hat{\nu}_{it+k} \right]$$

$$(21)$$

Assume
$$\tilde{\theta}_{it}$$
, \hat{p}_{it}^{x} , and $\hat{a}_{it}^{y} \sim AR(1) \text{ w} / \rho_{\theta}$, ρ_{p} , and ρ_{a}
$$\pi_{it}^{slack} = E_{t}\pi_{t+\infty} + \psi_{\theta}^{1}\tilde{\theta}_{it} + \psi_{p}\hat{p}_{it}^{x} + \psi_{a}\hat{a}_{it}^{y} + \varepsilon_{it}$$
(22)

$$\psi_{\theta}^{1} = \frac{\kappa_{\theta}}{(1-\beta\rho_{\theta})}$$

$$\psi_{\rho} = \frac{\kappa_{\rho}}{(1-\beta\rho_{\rho})}, \ \psi_{a} = \frac{\kappa_{a}}{(1-\beta\rho_{a})}$$

$$\varepsilon_{it} = E_{t} \sum_{k=0}^{\infty} \beta^{k} \kappa_{\nu} \hat{\nu}_{it+k}$$

Derivation 3/3

- Labor market is tight at time t
 - Expected to remain tight until t + T 1
 - $\blacktriangleright E_{t+T-1}\pi_{t+T}^{slack}$
 - Solve backward

$$\pi_{it}^{tight} = \beta^T E_t \pi_{it+T}^{slack} + E_t \sum_{k=0}^{T-1} \beta^k \left(\kappa_{\theta}^{tight} \hat{\theta}_{it+k} + \kappa_p \hat{p}_{it+k}^{\mathsf{x}} + \kappa_a \hat{a}_{it+k}^{\mathsf{y}} + \kappa_{\nu}^{tight} \hat{\nu}_{it+k} \right)$$

• Substitute $E_t \pi_{it+T}^{slack}$ and apply other steps

$$\pi_{it}^{tight} = E_t \pi_{t+\infty} + \xi E_t \hat{\theta}_{it+\infty} + (\psi_{\theta}^1 + \psi_{\theta}^2) \tilde{\theta}_{it} + \psi_{\rho} \hat{p}_{it}^{\mathsf{x}} + \psi_{\mathfrak{a}} \hat{a}_{it}^{\mathsf{y}} + \varepsilon_{it}^{tight}$$

•
$$\xi = \frac{1-\beta^{T}}{1-\beta}\lambda\kappa_{\theta}^{tight}$$

• $\psi_{\theta}^{2} = \frac{1-(\beta\rho_{\theta})^{T}}{1-\beta\rho_{\theta}}\lambda\kappa_{\theta}^{tight}$

Additional Data

- Core CPI: Bureau of Labor Statistics (BLS)
 - 21 US cities, monthly or bi-monthly frequency
- Unemployment: LAUS-BLS
 - MSA level, monthly frequency
- Industry employment shares: Census, American Community Survey (ACS)
 - MSA level, 2000 Census; 2006-2022 ACS
- Industry employment: CPS
 - National level, monthly frequency
- Manufacturing PPI index: BLS
 - Mational level, monthly frequency
- Balanced panel of city-year-month observations, Dec00-Jul24

Descriptive Statistics: MSA-level π and θ



Descriptive Statistics: MSA-level and National θ



Variation in θ and π



Main Table

Table 1: Estimates of slope of NRPC, Dec00-Jul24

(1)	(2)	(3)
ÌŃ	ÒĹŚ	ÒĹŚ
0.16	0.84***	0.20**
(0.32)	(0.10)	(0.09)
1.79***	3.28***	0.54***
(0.63)	(0.51)	(0.18)
-0.15**	-0.32*	0.00
(0.06)	(0.17)	(0.04)
6.09***	2.75***	-0.13
(1.22)	(1.03)	(0.98)
4334	4456	4418
\checkmark	\checkmark	\checkmark
\checkmark		\checkmark
\checkmark		\checkmark
54.54		
43 23		
	$(1) \\ IV \\ \hline 0.16 \\ (0.32) \\ 1.79^{***} \\ (0.63) \\ -0.15^{**} \\ (0.06) \\ 6.09^{***} \\ (1.22) \\ \hline 4334 \\ \checkmark \\ \checkmark \\ 54.54 \\ 43.23 \\ \hline $	$\begin{array}{cccc} (1) & (2) \\ \hline V & OLS \\ \hline 0.16 & 0.84^{***} \\ (0.32) & (0.10) \\ 1.79^{***} & 3.28^{***} \\ (0.63) & (0.51) \\ -0.15^{**} & -0.32^{*} \\ (0.06) & (0.17) \\ 6.09^{***} & 2.75^{***} \\ (1.22) & (1.03) \\ \hline 4334 & 4456 \\ \checkmark & \checkmark \\ \checkmark \\ 54.54 \\ 43.23 \\ \end{array}$

First Stage

Table 2: First Stage Coefficients

	(1) $\ln(\theta_{it})$	$(2) \\ \ln(\theta_{it}) \times I_{\{\theta_{it} > 1\}}$	$(3) \\ \ln(p_{it}^{\times})$
Z_{it}^{X}	9.40***	1.42***	-0.11***
$z_{it}^{x} imes I_{\{\theta_{i}^{x} > 1\}}$	(1.31) 3.00**	(0.29) 8.95***	(0.03) -0.13***
$I_{\{\theta_{it}>1\}}$	(1.43) 0.19***	(1.16) 0.08***	(0.05) -0.00
$\ln(p_{it-24}^{x})$	(0.01) 1.30***	(0.01) 0.58*** (0.11)	(0.00) 0.68***
	(0.19)	(0.11)	(0.01)
MSA FE	4334 √	4334 √	4334 √
Year-Quarter FE	\checkmark	\checkmark	\checkmark
$MSA \times Year\text{-}Quarter \; FE$	\checkmark	\checkmark	\checkmark
F-stat	54.53	43.22	1314.25

Implications for Monetary Policy 1/2

• Linear PC \Rightarrow underestimation of π in tight labor market



Implications for Monetary Policy 1/2

• Linear PC \Rightarrow underestimation of π in tight labor market



Implications for Monetary Policy 1/2

• Linear PC \Rightarrow underestimation of π in tight labor market



Implications for Monetary Policy 2/2

▶ Nonlinear PC \Rightarrow bring down π at lower economic cost in tight labor market



Implications for Monetary Policy 2/2

▶ Nonlinear PC \Rightarrow bring down π at lower economic cost in tight labor market



Implications for Monetary Policy 2/2

▶ Nonlinear PC \Rightarrow bring down π at lower economic cost in tight labor market



Other Kinks

NRPC robust to other definition of kinks

	(1)	(2) k _ abest-fit	(3) k = ā
	K — 1	$\kappa = v_i$	$\kappa = v_i$
$\ln(\theta_{it})$	0.16	0.20	0.17
	(0.32)	(0.43)	(0.36)
$\ln(\theta_{it}) \times I_{\{\theta_{it} > k\}}$	1.79***	1.89*	1.18**
	(0.63)	(1.09)	(0.55)
$I_{\{\theta_{it}>k\}}$	-0.15**	-0.27*	0.24
	(0.06)	(0.16)	(0.16)
$\ln(p_{it}^{x})$	6.09***	5.89***	6.10***
	(1.22)	(1.23)	(1.21)
Observations	4334	4334	4334
MSA FE	\checkmark	\checkmark	\checkmark
Year-Quarter FE	\checkmark	\checkmark	\checkmark
MSA x Year-Quarter FE	\checkmark	\checkmark	\checkmark
F-stat θ	54.54	46.11	47.06
F-stat $\theta imes I$	43.23	15.91	41.98

Table 3: IV Estimates of slope of NRPC, Dec00-Jul24

Population Weights for Vacancy Imputation

NRPC robust to use of population weights with different kinks

	(1)	(2)	(3)
	k = 1	$k = \theta_i^{\text{best-fit}}$	$k = \bar{\theta}_i$
$\ln(\theta_{it})$	0.32	0.24	0.19
	(0.34)	(0.40)	(0.34)
$\ln(\theta_{it}) \times I_{\{\theta_{it} > k\}}$	1.39**	1.72*	1.17**
	(0.69)	(1.01)	(0.55)
$I_{\{\theta_{it}>k\}}$	-0.08	-0.24	0.28
()	(0.06)	(0.16)	(0.18)
$\ln(p_{it}^{\times})$	6.12***	6.01***	6.06***
	(1.20)	(1.20)	(1.22)
Observations	4334	4334	4334
MSA FE	\checkmark	\checkmark	\checkmark
Year-Quarter FE	\checkmark	\checkmark	\checkmark
$MSA \times Year\operatorname{-Quarter} FE$	\checkmark	\checkmark	\checkmark
F-stat θ	55.44	46.00	47.54
F-stat $ heta imes I$	39.50	19.00	41.75

Table 4: IV Estimates of slope of NRPC, Dec00-Jul24

Other Nonlinear Functional Forms

NRPC robust to other nonlinear functional forms

	(1) Piecewise-log	(2) Log	(3) Quadratic-log	(4) Piecewise-lin	(5) Quadratic-lin	(6) Inverse
$\ln(\theta_{it})$	0.16	0.78***	1.05***			
$\ln(\theta_{it}) \times \mathit{I}_{\theta_{it} > 1}$	(0.32) 1.79*** (0.63)	(0.22)	(0.28)			
$\ln(\theta_{it})^2$	()		0.29** (0.14)			
θ_{it}			()	0.48	0.81	
$ heta_{it} imes I_{ heta_{it} > 1}$				0.59	(1.57)	
θ_{it}^2				(0.34)	0.01	
$\frac{1}{\theta_{it}}$					(0.41)	-0.34*** (0.11)
Observations	4334	4334	4334	4334	4334	4334
MSA FE	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
Year-Quarter FE	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
$MSA \times Year-Quarter FE$	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
F-stat θ	56.99	57.95	41.25	56.99	43.97	38.73
F-stat $\theta \times I/\theta^2$	45.17		14.59	35.57	26.86	

Table 5: IV Estimates of slope of NRPC, Dec00-Jul24

Add Proxy for Final-Goods Sector Productivity

NRPC robust to inclusion of proxy for final-goods sector productivity

	(1)	(2)
	Benchmark	Final-goods Productivity
$\ln(\theta_{it})$	0.16	0.14
	(0.32)	(0.33)
$\ln(heta_{it}) imes I_{ heta_{it} > 1}$	1.79***	1.68***
-	(0.63)	(0.62)
$I_{\theta_{it}>1}$	-0.15**	-0.13*
	(0.06)	(0.07)
$\ln(p_{it}^{x})$	6.09***	6.08***
	(1.22)	(1.20)
z_{it}^{y}		1.41
		(1.65)
Observations	4334	4334
MSA FE	\checkmark	\checkmark
Year-Quarter FE	\checkmark	\checkmark
MSA x Year-Quarter FE	\checkmark	\checkmark
F-stat θ	56.99	54.35
F-stat $\theta \times I$	45.17	36.88

Table 6: IV Estimates of slope of NRPC, Dec00-Jul24