Keeping Control Over Boundedly Rational Expectations

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* The views expressed do not necessarily represent those of the Banque de France or the Eurosystem

How to Avoid Losing Control over Inflation Expectations?

- **Canonical Response: Respect the Taylor principle** $i_t = \phi \pi_t, \phi > 1$
  - Failure to do so as root of the stagflation of the 1970s (Clarida Gali Gertler 2000)
  - NK models under RE: If not, self-fulfilling inflation (Sargent Wallace 1973)
- **But no longer necessary in NK models purged of the FG puzzle**
  - Most solutions to the FG puzzle involve extra discounting of the future
  - Once enough discounting to solve FG puzzle, determinacy under a peg
- **So stop worrying and love passive monetary policy?**
  - Need for active MP was just pathology of baseline models, like FG puzzle?
  - But what about the 1970 then? What about Turkey today?

Reconsider How to Keep Control over Expectations away from RE

- **Consider large set of boundedly rational expect. based on Woodford (2019)**
  1. Finite Planning Horizons (solution to the FG puzzle)
  2. Long-run learning (to argue against neo-Fisherian predictions)

$\rightarrow$ Framework can replicate dynamics of expect. in survey (Gust et al. 2024)

RESULTS

1. **Using Taylor rules: Active MP prevents inflation spirals**
   - Cumulative process (Wicksell 1898, Friedman 1968, Adaptive Learning)
   - Whenever future discounted enough to avoid FG puzzle
2. **Characterize active MP beyond Taylor rules**
   - Characterized as $E_t(\sum_{n=0}^{\infty} \gamma(n) i_{t+n})$ high enough, for some $\gamma(n)$
   - Weight $\gamma(n)$ not monotonic: first increases, then decreases
3. **Hike now or play on expectations of future high rates? Optimal policy**
   - If rely on higher rates tomorrow, only a larger recession tomorrow?
   - Delay hikes more and rely more on future high rates, the larger the weight on output

Literature

- **FPH, FPH-learning**: Woodford (2019), Woodford Xie (2020), Xie (2020), Gust Herbst Lopes-Salido (2020), Gust Herbst Lopes-Salido (2023), Dupraz et al. (2021), Na Xie (2022)
- **Other bounded rationality solutions (and others) to the FG puzzle**: Gabaix (2020), Angeletos Lian (2018), Farhi Werning (2019), (Bilbiie 2020, Werning 2015, Acharya Dogra 2020)
- **Models of imprecise memory & perpetual learning**: Afrouzi et al. (2023), Azeredo da Silveira Sung Woodford (2020), Sung (2022), Nagel Xu (2022)
- **Models of long-term drifts**: Kosicki Tinsley (2001), Gurkaynak Sack Swanson (2005), Cogley Primiceri Sargent (2005), Cogley Sbordone (2008), Crump et al. (2023)
- **Models of Bounded Rationality in macro more broadly**: Angeletos Huo Sastry (2020), Bordalo Gennaioli Shleifer (2018), Bordalo et al. (2019, 2020), L'Huillier Singh Yoo (2023), Beaudry Carter Lahiri (2023), Hadjini (2023)
- **Cumulative process under adaptive expectations**: Wicksell (1898), Friedman (1968), Howitt (1992)
- **Least-Square Learning**: Bullard Mitra (2002), Evans Honkapohja (2001), Preston (2005), Orphanides Williams (2005), Milani (2007), Molnar Santoro (2014), Eusepi Preston (2018), Gaspar Smets Vestin (2010), Carvalho et al. (2021), Gati (2022)

Woodford's Boundedly Rational NK Model & The Taylor Principle

The Rational Expectations Baseline NK Model

- Canonical 3-equation NK model under rational expectations

$$
\begin{aligned}
y_t &= -\sigma(i_t - E_t(\pi_{t+1})) + E_t(y_{t+1}) + \nu_t^y \\
\pi_t &= \kappa(y_t - y_t^e) + \beta E_t(\pi_{t+1}) + \nu_t^p \\
i_t &= \phi_\pi \pi_t + \phi_y y_t
\end{aligned}
$$

- In 2-by-2 matrix form, $Y_t = (y_t, \pi_t)'$, $\nu_t = (\nu_t^y, \nu_t^p - \kappa y_t^e)$

$$
Y_t = AE_t(Y_{t+1}) + b\nu_t
$$

- Unique bounded solution iff both eigenvalues $\lambda_i^*(\phi)$ of A^{-1} outside unit circle, iff

$$
\phi_\pi + \frac{1-\beta}{\kappa}\phi_y > 1.
$$

Woodford (2019)'s Boundedly Rational NK Model: Finite Planning Horizons

1. **Finite Planning Horizons**

   - Agents perceive shocks and reason through GE only until h periods ahead
   - Beyond h, assume all variables beyond their control back to steady-state

$$Y_t^h = E_t \sum_{j=0}^{h} A^j b\nu_{t+j}$$

   - Assuming geometric distribution of planning horizons, aggregate economy satisfies

$$Y_t = \rho A E_t(Y_{t+1}) + b\nu_t$$

   - Solves FGP if ρ low enough, $\rho < \rho^*$ ▸ FGP

Proposition

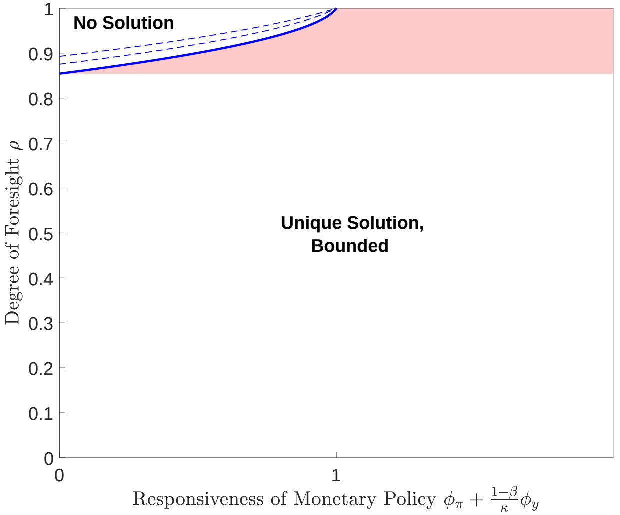
Let $\lambda_2^*(\phi)$ be the smaller root of A^{-1}.

1. *If $\rho < |\lambda_2^*(\phi)|$, the economy has a unique equilibrium, which is bounded. It is*

$$Y_t = E_t \sum_{j=0}^{\infty} (\rho A)^j b\nu_{t+j}$$

2. *If $\rho > |\lambda_2^*(\phi)|$, the economy has no solution in general.*

Stability Condition with FPH and Long-Term Learning



Note: The pink area is the region where the model is subject to the FG puzzle

Woodford (2019)'s Boundedly Rational NK Model: Long-Term Learning

1. **Finite Planning Horizons**

   - Agents perceive shocks and reason through GE only until h periods ahead
   - Beyond h, assume all variables beyond their control back to steady-state

$$Y_t^h = E_t \sum_{j=0}^{h} A^j b \nu_{t+j}$$

2. **Long-Term Learning**

   - FPH can be combined with a second, distinct assumption: long-term learning
   - Beyond h, terminal value functions now updated with past realizations of π and y
   - Long-run expectations $Y_{t-1}^* = (y_{t-1}^*, \pi_{t-1}^*)$ satisfy

$$y_t^\star = \mu y_{t-1}^\star + (1-\mu) y_t,$$
$$\pi_t^\star = \mu \pi_{t-1}^\star + (1-\mu) \pi_t,$$

   - The parameter $\mu \in [0,1]$ captures the learning gain on long-term expectations
   - We show that reduces to

$$Y_t^h = E_t \sum_{j=0}^{h} A^j b \nu_{t+j} + A^{h+1} Y_{t-1}^*$$

Stability Condition with FPH and Long-Term Learning

Proposition

1. *If* $\rho < |\lambda_2^*(\phi)|$, *the economy has a unique equilibrium*

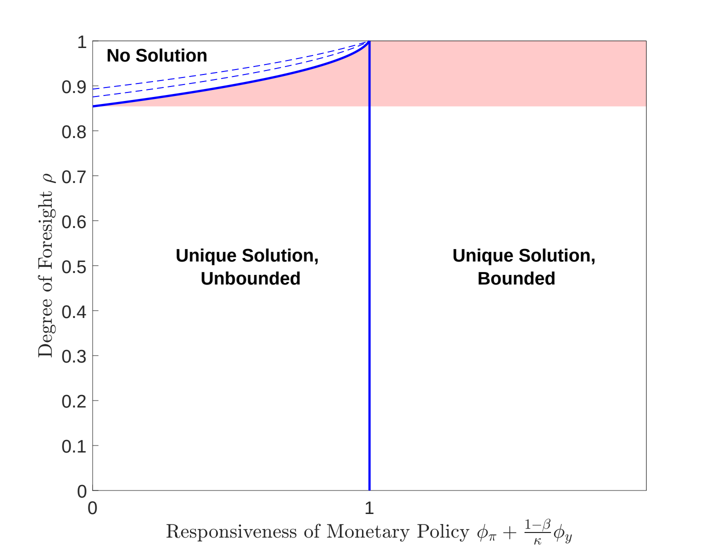
$$Y_t = E_t \sum_{j=0}^{\infty} (\rho A)^j b\nu_{t+j} + (I - \rho A)^{-1}(1-\rho)AY_{t-1}^*.$$

   1.1 *If the Taylor principle is satisfied, then the unique equilibrium is bounded.*
   1.2 *If the Taylor principle is not satisfied, then the unique equilibrium is unbounded.*

2. *If* $\rho > |\lambda_2^*(\phi)|$, *the economy has no solution in general.*

- When TP not satisfied, only 1 root inside unit circle for 2 state variables ▸ Roots

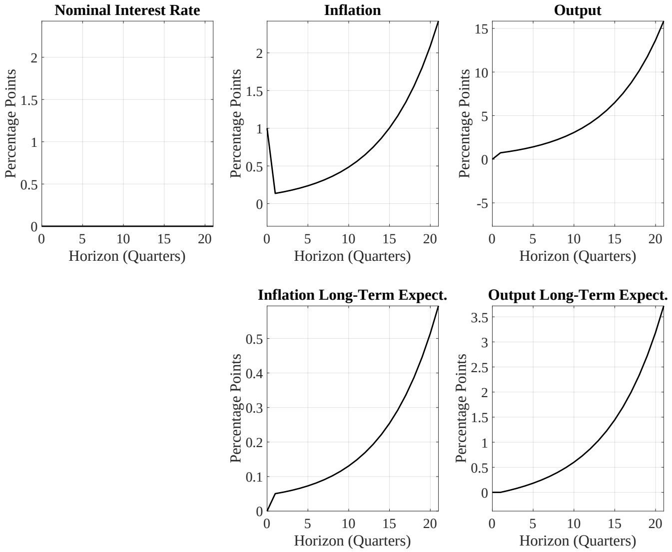
Stability Condition with FPH and Long-Term Learning



Note: The pink area is the region where the model is subject to the FG puzzle

Illustration: Dynamics Following a Transitory Cost-Push Shock

Example for fully passive monetary policy: $\phi_\pi = \phi_y = 0$



Beyond Taylor Rules

Beyond Taylor Rules

- Arbitrary exogenous interest rate path: $\phi_\pi = \phi_y = 0$ & arbitrary intercept i_t

	Rational Expectations	FPH-learning, $\rho < \rho^*$
Active Taylor Rule	Unique Bounded Inflation Path	Unique Stable Inflation Path
Passive Taylor Rule	Self-Fulfilling Inflation	Unique Inflation Spiral
Exo. Interest-Rate Path	Self-Fulfilling Inflation	Depends on Interest Rate Path

- For $\rho < \rho^*$: **Bounded Inflation or Inflation Spiral, depending on Rate Path**
- Characterize all rate paths where π comes back to target (i.e. $\lim_{t\to\infty} \pi_t = 0$)
- Captures which paths have active enough MP without restricting to Taylor rule

Illustration: Dynamics Following a Transitory Cost-Push Shock

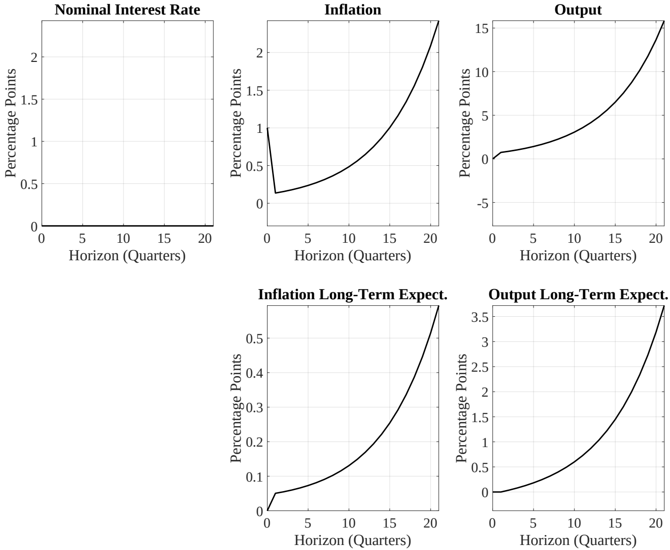


Illustration: Dynamics Following a Transitory Cost-Push Shock

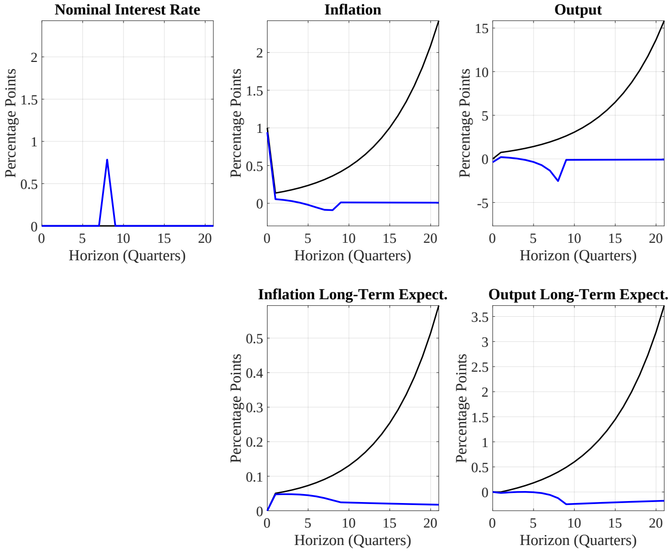
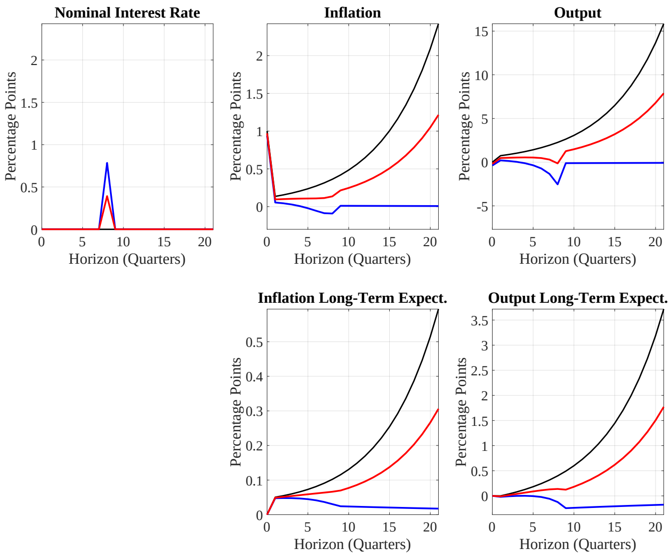


Illustration: Dynamics Following a Transitory Cost-Push Shock



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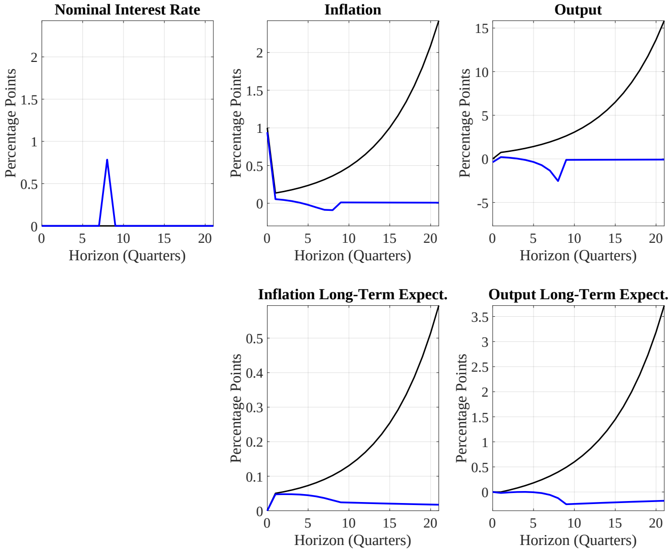
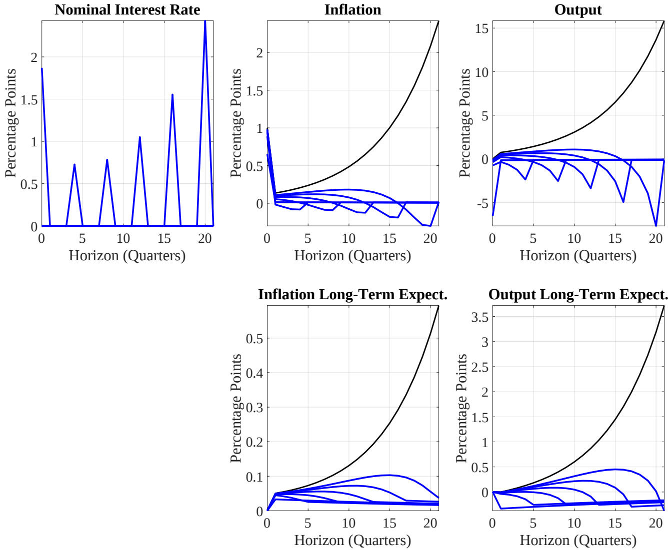


Illustration: Dynamics Following a Transitory Cost-Push Shock



Active Interest Rate Paths

- Inflation spiral arises from root $\lambda_2^b(0) > 1$: absent MP, makes inflation snowball
- Let $z_{2,t}^b = e_2' Y_{t-1}^b$, for e_2' the left eigenvector of root $\lambda_2^b(0)$
- Economy converges back to steady-state iff z_2^b does; can show:

$$E_t(z_{2,t+k}^b) = \lambda_2^b(0)^{k+1} \left(z_{2,t-1}^b + \left(1 - \frac{\mu}{\lambda_2^b(0)}\right) \left(\sum_{n=0}^{\infty} \gamma(n) E_t(v_{2,t+n} - c_2^i i_{t+n}(\nu)) \right) \right) + o(1)$$

Proposition

The economy returns to steady-state in the long run iff

$$\underbrace{z_{2,t-1}^b}_{\text{Already deanchored}} + \underbrace{\left(1 - \frac{\mu}{\lambda_2^b(0)}\right) E_t \left(\sum_{n=0}^{\infty} \gamma(n) v_{2,t+n} \right)}_{\text{Inflationary effect of shocks to come}} = \underbrace{\left(1 - \frac{\mu}{\lambda_2^b(0)}\right) c_2^i E_t \left(\sum_{n=0}^{\infty} \gamma(n) i_{t+n}(\nu) \right)}_{\text{Countering effect of monetary policy}}$$

The Relative Anchoring Effect of Interest Rates of Different Horizons

$$\underline{\text{Condition}}: \quad \sum_{n=0}^{\infty} \gamma(n) E_t(i_{t+n}(\nu)) \text{ large enough}$$

Definition

The relative anchoring effect $\gamma(n)$ of the rate i_{t+n} at horizon n is

$$\gamma(n) = \frac{\left(\frac{1}{\lambda_2^f(0)}\right)^{n+1} - \left(\frac{1}{\lambda_2^b(0)}\right)^{n+1}}{\frac{1}{\lambda_2^f(0)} - \frac{1}{\lambda_2^b(0)}}$$

- Single-peaked function of horizon n, peaking at an intermediary horizon
  $\rightarrow$ Strikes a balance between 2 opposing forces ▸ Twitter Feud

1. Current short-term rates matter little per se for AD: $\gamma(n) \uparrow$ at first
   $\rightarrow$ Effect present under RE but not with purely backward expectations ($\rho = 0$)
2. The + hike is delayed, the + expectations have de-anchored: $\gamma(n) \downarrow$ ultimately
   $\rightarrow$ Effect present under backward expectations ($\rho = 0$) but not under RE

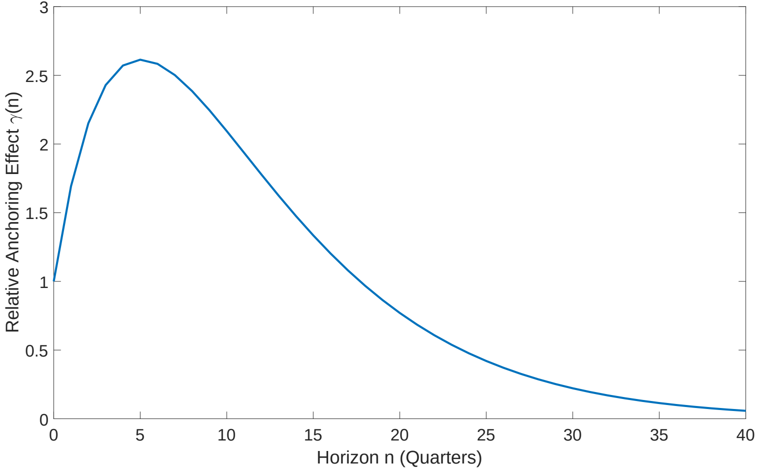
Bayesian Estimation

- Observables: inflation, CBO output gap, Fed Funds rate, SPF 1y inflation expect.
- Sample: 1984-2007

Parameter	Prior Distribution			Posterior Distribution		
	Distribution	Parameter 1	Parameter 2	Mode	Mean	St. Dev.
σ	Gamma	2.00	0.50	3.19	3.51	0.55
κ	Gamma	0.05	0.10	0.04	0.05	0.03
ϕ_π	Gamma	1.50	0.10	1.42	1.45	0.10
ϕ_y	Gamma	0.25	0.25	0.92	0.95	0.22
ρ	Uniform	0.00	1.00	0.61	0.54	0.10
μ	Uniform	0.00	1.00	0.95	0.95	0.01

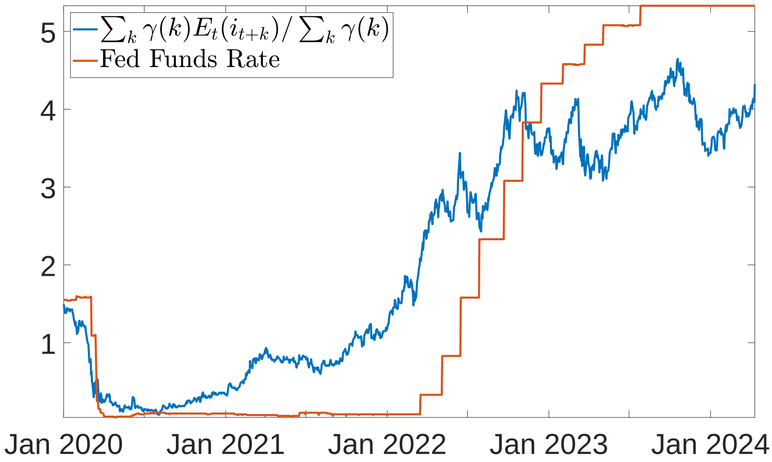
▸ More ▸ Output Growth

The Effect of Interest Rates of Different Horizons



Comparative Statics

Was the Fed Lagging Behind?



▸ Yields

▸ Forwards

Hike Now or Play on Expectations of Future High Rates? Optimal Policy

Optimal Policy Characterization: A Generalized Target Criterion

- Consider loss function with given weight ω on the output gap ▸ TR ▸ RE

$$E_0 \sum_{t=0}^{\infty} \frac{1}{2} \beta^t \left(\pi_t^2 + \omega (y_t - y_t^e)^2 \right)$$

Proposition

The optimal MP under commitment is characterized by

$$\zeta_t^{\pi} + \frac{1}{\kappa}(\zeta_t^{y} - \rho \zeta_{t-1}^{y}) = 0$$

The optimal MP under discretion is characterized by

$$\zeta_t^{\pi} + \frac{1}{\kappa}\zeta_t^{y} = 0$$

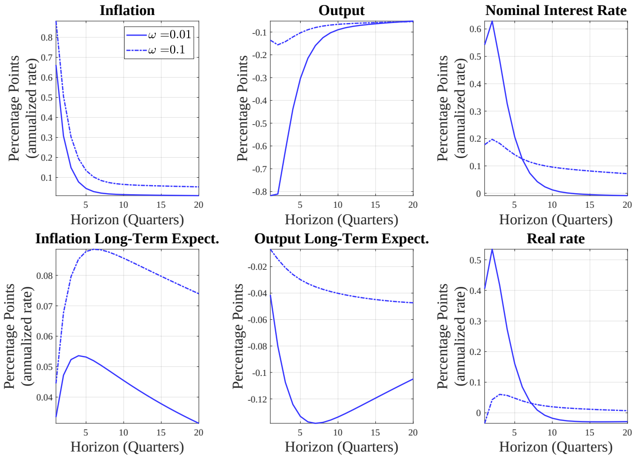
where

$$\zeta_t^{\pi} = \pi_t + (1-\mu)\gamma_t^{\pi^*} \text{ marg. cost of higher inflation}$$

$$\zeta_t^{y} = \omega(y_t - y_t^e) + (1-\mu)\gamma_t^{y^*} \text{ marg. cost of higher output gap}$$

$$(\gamma_t^{y^*}, \gamma_t^{\pi^*})' = E_t \sum_{k=0}^{\infty} (\beta D_0')^k \beta M_0' \Omega (Y_{t+k+1} - Y_{t+k+1}^e) \text{ marg. cost of higher LT-expect.}$$

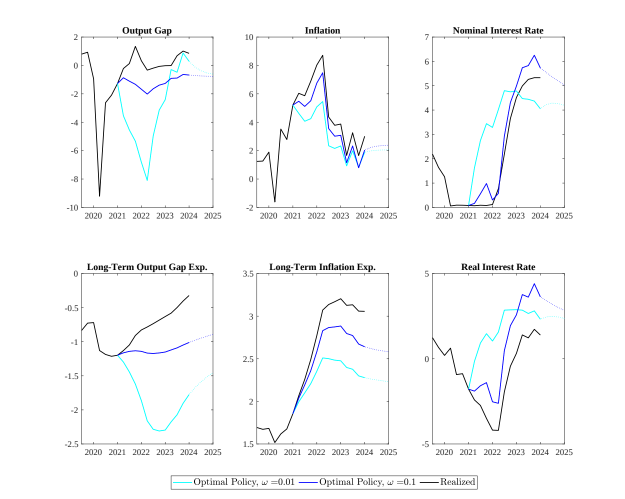
Higher Rates Now or Later?



Discretion RE

The 2021-2022 Inflation Surge

- Recover shocks under assumption Fed follows the estimated Taylor rule ▸ Shocks
- Counterfactual path under optimal policy from 2021Q1, for small/large ω ▸ 3/2022



Conclusion

- Risk of passive monetary policy away from RE is inflation spirals
- Against inflation spirals, possible to define active MP beyond Taylor rules
- Hike less now and rely more on higher future rates, the higher the output weight

APPENDIX

FG Puzzle and Stability Condition under FPH alone

1. **Finite Planning Horizons**
    - Agents perceive shocks and reason through GE only until h periods ahead
    - Beyond h, assume all variables beyond their control back to steady-state

$$Y_t^h = E_t \sum_{j=0}^{h} A^j b\nu_{t+j} + A^{h+1} Y_{t-1}^*$$

    - Assuming geometric distribution of planning horizons, aggregate economy satisfies

$$Y_t = \rho A E_t(Y_{t+1}) + b\nu_t$$

    - Solves FGP iff $\rho < \rho^*$; but also $\rho < \rho^*$ iff determinacy under a peg FGP

- **Solution to the Forward Guidance Puzzle**
    - FGP: Impact effect on π and y of announcing rate cut in n diverges to ∞ as $n \to \infty$
    - Model not subject to Forward Guidance Puzzle iff $\rho < \rho^*$

$$\rho^* = \frac{1 + \sigma\kappa + \beta - \sqrt{(1 + \sigma\kappa + \beta)^2 - 4\beta}}{2\beta}$$

Back

Stability Condition with FPH and Long-Term Learning

Lemma

The roots of the FPH-learning NK economy are the functions of the RE roots $\lambda_i^*(\phi)$

$$\lambda_i^f(\phi) = \frac{1}{\rho}\lambda_i^*(\phi)$$

$$\lambda_i^b(\phi) = \mu + (1-\mu)(1-\rho)\left(\frac{1}{\lambda_i^*(\phi) - \rho}\right)$$

- FPH-learning economy has 2 state and 2 jump variables
- For $\rho < |\lambda_2^*(\phi)|$, both $|\lambda_i^f(\phi)|$ are always > 1
- But Taylor principle determines whether both $|\lambda_i^b(\phi)|$ are < 1

Back

Paul Krugman @paulkrugman · 16 mars 2022

First question is which maturity of rates to look at. The answer there is clearly longish: interest rates mainly matter for long-lived investments, so it's something like the 10-year rate rather than Fed funds that matters for the real economy 7/

2 19 78

Opinion | My inflation warnings have spurred questions. Here are my answers.

By Lawrence H. Summers

Contributing columnist

April 5, 2022 at 4:40 p.m. EDT

A footnote: Yes, I've seen Paul Krugman's new theory of only future rates being relevant to spending. I'd remind him of his stricture about inventing new economic theories to fit one's prejudices, as well as of the decades-long tradition of using real Treasury bill or Fed funds rates to index monetary policy.

▸ Back

Bayesian Estimation: Full Results

Parameter	Prior Distribution			Posterior Distribution		
	Distribution	Parameter 1	Parameter 2	Mode	Mean	St. Dev.
$\bar{g}_Y$	Normal	0.50	0.10	0.50	0.50	0.01
$\bar{\pi}^E$	Normal	2.00	1.00	1.77	1.80	0.12
σ	Gamma	2.00	0.50	3.16	3.46	0.55
κ	Gamma	0.05	0.10	0.02	0.03	0.02
ϕ_π	Gamma	1.50	0.10	1.40	1.43	0.10
ϕ_y	Gamma	0.25	0.25	0.80	0.84	0.21
ρ	Uniform	0.00	1.00	0.64	0.58	0.09
μ	Uniform	0.00	1.00	0.94	0.94	0.01
ρ_ξ	Beta	0.50	0.10	0.93	0.92	0.02
ρ_i	Beta	0.50	0.10	0.88	0.88	0.03
ρ_p	Beta	0.50	0.10	0.48	0.51	0.08
σ_ξ	InvGamma	1.00	4.00	0.55	0.61	0.09
σ_i	InvGamma	1.00	4.00	0.46	0.49	0.11
σ_p	InvGamma	1.00	4.00	0.14	0.15	0.01
σ_e	InvGamma	1.00	4.00	0.32	0.33	0.03

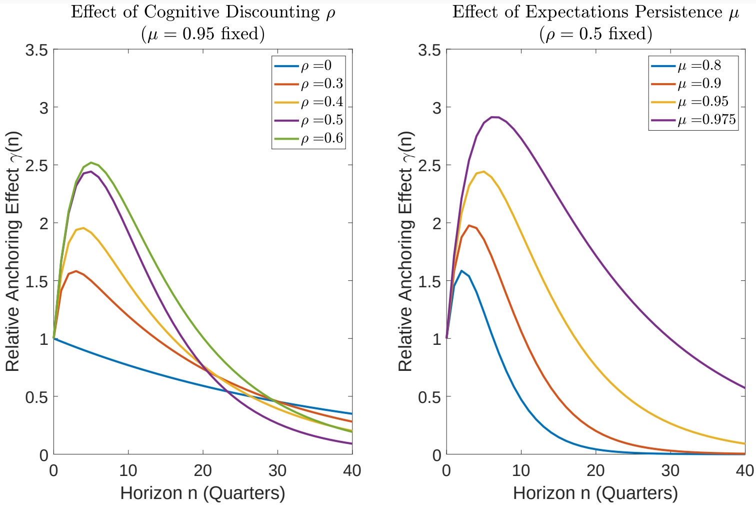
Back

Bayesian Estimation: Output Growth as Observable

Parameter	Prior Distribution			Posterior Distribution		
	Distribution	Parameter 1	Parameter 2	Mode	Mean	St. Dev.
$\bar{g}_Y$	Normal	0.50	0.10	0.50	0.50	0.01
$\bar{\pi}^E$	Normal	2.00	1.00	1.75	1.76	0.12
σ	Gamma	2.00	0.50	3.81	3.92	0.56
κ	Gamma	0.05	0.10	0.04	0.04	0.01
ϕ_π	Gamma	1.50	0.10	1.41	1.43	0.09
ϕ_y	Gamma	0.25	0.25	0.73	0.79	0.19
ρ	Uniform	0.00	1.00	0.54	0.52	0.05
μ	Uniform	0.00	1.00	0.95	0.95	0.01
ρ_ξ	Beta	0.50	0.10	0.91	0.90	0.02
ρ_i	Beta	0.50	0.10	0.87	0.87	0.03
ρ_p	Beta	0.50	0.10	0.53	0.53	0.08
σ_ξ	InvGamma	1.00	4.00	0.65	0.68	0.09
σ_i	InvGamma	1.00	4.00	0.43	0.47	0.09
σ_p	InvGamma	1.00	4.00	0.15	0.15	0.01
σ_e	InvGamma	1.00	4.00	0.31	0.32	0.02

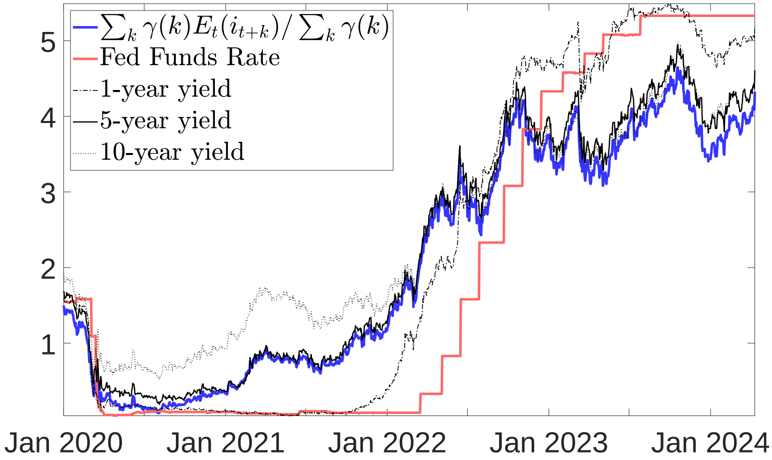
Back

The Effect of Interest Rates of Different Horizons



Back

Was the Fed Lagging Behind?



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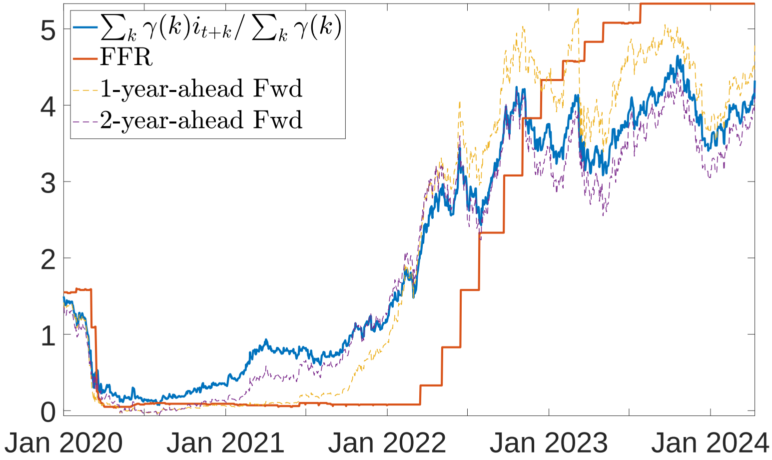
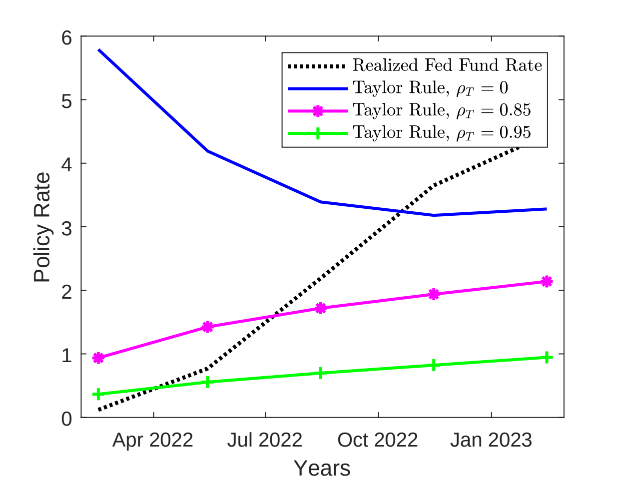


Back

Limited Guidance from the Taylor Principle on How Fast to Hike



$$i_t = \rho_T i_{t-1} + (1 - \rho_T)(i^* + \phi_\pi(\pi_t - \pi^*) + \phi_x x_t)$$

Back

Optimal Policy Characterization: A Generalized Target Criterion

- **Generalizes the optimal MP under RE, modifying it in two ways**

$$\text{Under Commitment: } \pi_t + \frac{\omega}{\kappa}(x_t - x_{t-1}) = 0$$

$$\text{Under Discretion: } \pi_t + \frac{\omega}{\kappa}x_t = 0$$

1. Cost of higher inflation ζ_t^π includes cost $\gamma_t^{\pi^*}$ of causing higher LT expectations
   $\rightarrow$ $\gamma_t^{\pi^*}$ itself corresponds to cost of higher inflation and output tomorrow
2. Term $\rho\zeta_{t-1}^y$ capturing the commitment to past promises now discounted at rate ρ
   $\rightarrow$ Commitment only matters for forward-looking component of expectations

- **Generalizes the optimal MP under adaptive learning (Molnar Santoro 2014)**

$$\pi_t + \frac{\omega}{\kappa}\left(x_t - (1-\mu)E_t\left(\sum_{k=0}^{\infty}(\beta\mu)^k\beta^2 x_{t+k+1}\right)\right) = 0.$$

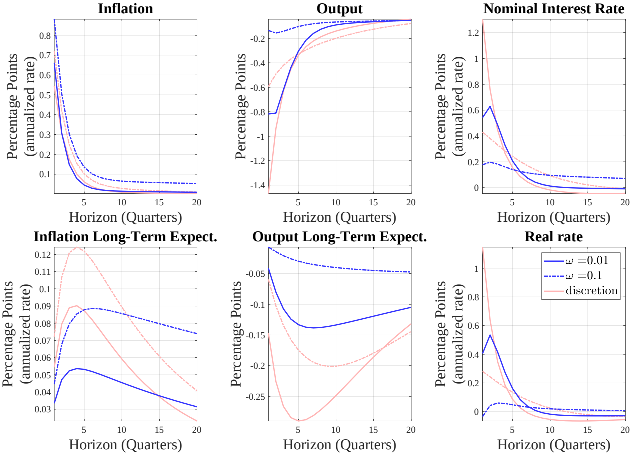
▸ Back

The Optimal Rate Policy is Self-Implementing

- Under RE, optimal equilibrium must still be implemented
  - Setting exogenous i_t^{optim} path leads to indeterminacy
  - Must be implemented through feedback rule of some form
- In FPH-learning, determinacy is guaranteed when setting exogenous i_t^{optim} path
  - No need for feedback rule to implement it: self-implementing!
  - Because risks are no longer indeterminacy risks
  - Risks of inflationary spirals already ruled out when choosing optimal eq.

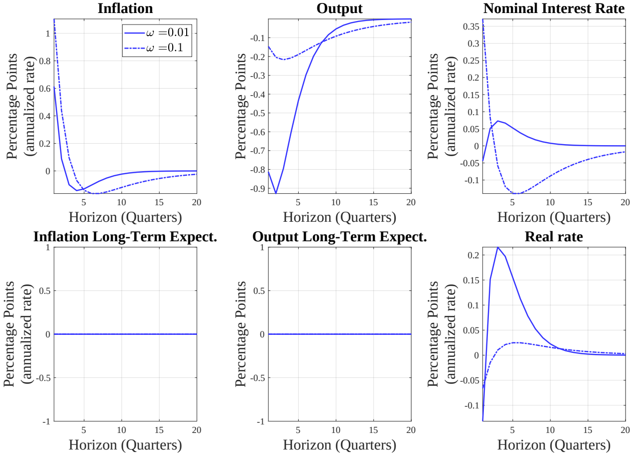
▸ Back

How Fast to Hike? Commitment vs. Discretion

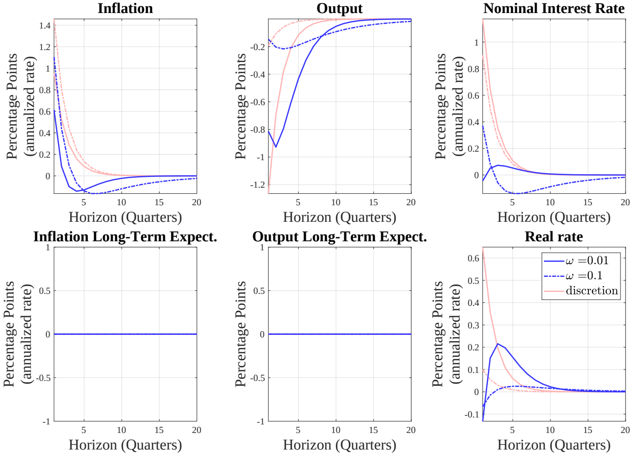


Back

How Fast to Hike? Rational Expectations



Back


Inflation
Output
Nominal Interest Rate
Inflation Long-Term Expect.
Output Long-Term Expect.
Real rate
Percentage Points
(annualized rate)
Horizon (Quarters)
ω =0.01
ω =0.1
discretion

Back

Shortcomings of Optimal MP under Rational Expectations

- Rational Expectations: Optimal MP under commitment characterized by

$$\pi_t + \frac{\omega}{\kappa}(x_t - x_{t-1}) = 0,$$

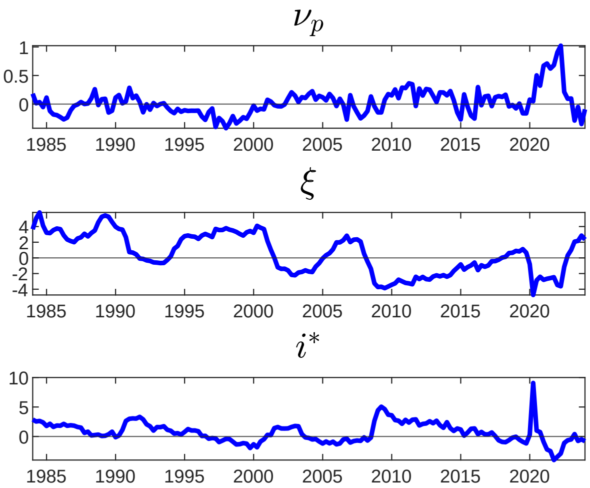
- History-dependent (close to Nominal-GDP-targeting)
  $\rightarrow$ Implies a gradual increase in interest rates IRF

- **Suspicion 1**: Relies heavily on strongly forward-looking expectations
  $\rightarrow$ In particular, $\uparrow r$ via expectations of below-target π

- **Suspicion 2**: RE do not capture the risk of inflation spirals
  $\rightarrow$ Distinct indeterminacy risk taken care of through implementation

Back

Shocks



ν_p
1
0.5
0
1985
1990
1995
2000
2005
2010
2015
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ξ
4
2
0
-2
-4
1985
1990
1995
2000
2005
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2015
2020
i^*
10
5
0
1985
1990
1995
2000
2005
2010
2015
2020

Back

The 2021-2022 Inflation Surge: Back to March 2022

- Recover shocks under assumption Fed follows the estimated Taylor rule ▸ Shocks
- Counterfactual path under optimal policy from 2021Q1, for small and large ω