Nonlinear Bank Capital Regulation

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Abstract

Prudential authorities mandate that banks hold equity capital exceeding a share of their risk-weighted assets. How should policymakers design this central tool of financial regulation? We develop a unifying framework for nonlinear bank capital regulation that nests canonical models of financial intermediation. Using a perturbation approach, we characterize the positive and normative effects of reforming risk weights through a small set of sufficient statistics, including credit-supply elasticities and welfare externalities from financial intermediation. These statistics are informative about credit-market frictions and the degree of competition. We estimate them in administrative data and apply the framework to evaluate the Federal Reserve's recent proposal to flatten the risk-weight schedule. Our analysis reveals nonlinear effects on credit allocation, leading to a moderate decline in total credit supply but a rise in bank equity, enhancing financial stability. Finally, we derive new sufficient-statistics formulas for the (constrained) Pareto-optimal risk weights that correct credit-supply externalities while accommodating market imperfections. Calibrated to empirical moments, the optimal schedule balances efficiency gains in production against the risk externalities of credit supply arising from government guarantees. Numerical simulations indicate that the Fed's proposed weights are close to optimal, generating sizable welfare gains for households at the expense of banks and entrepreneurs.

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1 Introduction

In the aftermath of the global financial crisis, bank regulation has placed renewed emphasis on financial stability. At the center of these reforms lies the minimum capital requirement. This core instrument, codified in the Basel Accords, mandates that a bank's equity capital must exceed a share of its risk-weighted assets. Because the requirement is risk-sensitive, greater risk-taking translates into higher capital requirements through larger risk weights. Two components are therefore central: the capital adequacy ratio, which sets the minimum equity share, and the nonlinear risk-weight schedule, which assigns a weight to each type of exposure.

Regulatory authorities worldwide introduced model-based capital requirements with strong risk sensitivity under Basel II. By contrast, the Federal Reserve's 2023 "Basel Endgame" proposal marks a shift toward a revised standardized approach with much flatter risk weights, intended to reduce complexity and enhance comparability across banks. Hence, regulators disagree on the optimal degree of risk sensitivity in bank capital requirements that balances efficient credit allocation and financial stability. Despite its first-order importance, the positive and normative effects of nonlinear bank-capital regulation remain largely unexplored.

Our paper fills this gap. First, we develop a unified framework for analyzing nonlinear capital requirements that nests a wide range of canonical models from the banking literature. We introduce the perturbation approach, widely used in public economics, into the analysis of risk-sensitive bank regulation. This approach allows us to characterize both the corrective effects and behavioral distortions of nonlinear reforms in terms of a small set of empirically observable sufficient statistics. These statistics provide structural insights into the nature of credit-market frictions and the degree of market power. Second, we estimate these sufficient statistics using administrative data from the German credit registry. We then quantify the nonlinear responses to reforms of the current risk-weight schedule, such as the Federal Reserve's proposal, and derive the optimal design of regulatory risk weights. We show that the Fed's proposal is close to a constrained-efficient, Pigouvian optimum that corrects the welfare-relevant credit-supply externalities.

Framework. The paper proceeds as follows. We first develop a unified model of financial intermediation that nests a wide range of canonical environments differing in technology, information, and market structure. The representative intermediary chooses its equity, credit portfolio, and additional environment-specific variables such as loan pricing or monitoring intensity. The bank is subject to three sets of constraints: one describing the endogenous formation of credit risk, one ensuring market or contractual feasibility, and, most importantly, the risk-sensitive regulatory capital requirement. Non-financial agents include firms that invest capital in their businesses and households that supply deposits, both subject to feasibility conditions. This environment encompasses, among others, the seminal models of Kim and Santomero (1988), Boyd and De Nicolo (2005), Stiglitz and Weiss (1981), Holmstrom and Tirole (1997), and Gale and Hellwig (1985).

To study regulatory counterfactuals, we then adopt the perturbation, or variational, approach, widely used in public finance (see Chetty (2009); Kleven (2021)), to the context of bank regulation. We are the first to characterize both the positive and normative effects of nonlinear capital reforms in terms of observable sufficient statistics. This method provides a transparent characterization of equilibrium responses to risk-weight changes that jointly reflect the behavior of financial and non-financial agents.[1]

Positive analysis. The behavioral responses to risk weights can be summarized by four elasticities. (i) The semi-elasticity of credit supply with respect to risk weights – the level effect. (ii) The semi-elasticity of credit supply with respect to marginal risk weights – the slope effect. (iii) The elasticity of credit supply with respect to default risk – the risk channel. (iv) The elasticity of credit risk with respect to bank lending – the risk feedback from leverage. Together, these sufficient statistics determine the equilibrium effects of arbitrary nonlinear reforms to the risk-weight schedule at the loan level.

Characterizing these elasticities across the benchmark models yields three general empirical predictions. First, credit-supply responses to risk weights would reveal a binding capital constraint, indicating deviations from the Modigliani and Miller (1958) capital-structure irrelevance principle. Second, credit-risk responses are informative about the nature of loan-market frictions: a positive effect of leverage on default risk is consistent with entrepreneurial moral hazard, whereas a negative relationship points to adverse selection. Third, the perturbation approach brings a novel channel to light. If credit supply responds to *marginal* risk weights, banks internalize how their lending decisions affect default risk and thereby the regulatory capital constraint. This channel arises under market power but is absent in competitive credit markets.

Aggregating loan-level responses across the observed distribution of credit risk yields bank-level adjustments in total credit and equity. The equity response combines two components: a mechanical effect, as higher risk weights directly raise required capital, and a behavioral effect, as they increase the marginal cost of lending and induce portfolio adjustments.

Normative analysis. We next turn to the normative evaluation of bank capital regulation. By an envelope argument, bank welfare is locally unaffected by changes in credit supply or equity and thus responds only mechanically to regulatory reforms. By contrast, firms and depositors experience welfare externalities – typically, firms through changes in production scale and depositors through fiscal costs associated with government guarantees. Marginal welfare externalities from credit supply and bank equity, therefore, serve

[1]In contrast to structural approaches in the banking literature (e.g., Corbae and D'Erasmo (2021); Begenau and Landvoigt (2022)), the sufficient-statistics approach does not rely on parametric assumptions about preferences, technologies, or market structure.

as sufficient statistics for welfare analysis.

As a byproduct of this reform analysis, we derive sufficient-statistics formulas for optimal risk weights. In the first-best benchmark, all regulatory instruments are chosen to correct welfare externalities in the spirit of Pigou (1920). The Pareto-optimal risk-weight schedule addresses marginal externalities across the credit-risk distribution. In the presence of loan-market frictions, however, the choice of risk weights is constrained because credit risk itself responds to lending. When the regulatory domain becomes endogenous, the weights in the capital constraint turn linearly dependent, rendering the unconstrained correction infeasible (see Tinbergen (1952)). As a result, the constrained Pareto-optimal schedule is flatter than its first-best counterpart.

Empirical implementation. A central empirical challenge is to disentangle supply from demand-side responses in the loan market. We identify credit-supply elasticities using quasi-random variation in banks' internal estimates of firm credit risk. For the same borrower, differences in estimated default probabilities across banks generate variation in assigned risk weights, allowing us to isolate credit-supply responses to perceived risk and regulatory capital requirements. As an alternative strategy, we exploit a regulatory reform that introduced a nonlinear reduction in risk weights for small and medium-sized firms below a cutoff firm size, using the reform as an instrument in a two-step IV framework.

To identify the elasticity of credit risk with respect to bank lending, we use a shift–share design based on the 2011 German bank levy. The identifying variation arises from pre-reform firm–bank relationships that generate heterogeneous exposure to the tax-induced downward shift in the credit-supply curve. This quasi-experimental setting allows us to trace how exogenous changes in lending affect subsequent borrower risk. For robustness, we re-estimate the sufficient statistics using data from the U.S. syndicated loan market.

The results show economically meaningful credit-supply responses to both risk and risk weights, rejecting the Modigliani and Miller (1958) capital-structure irrelevance hypothesis. We find limited responses to marginal risk weights, consistent with a competitive loan market. Across the risk distribution, credit-risk responses vary systematically: for the majority of loans (prime to medium-grade ratings), leverage increases default probabilities, consistent with borrower moral hazard (see Allen and Gale (2000); Boyd and De Nicolo (2005)) or costly state verification (see Gale and Hellwig (1985)). At the top of the risk distribution (speculative and non-rated loans), we observe a negative relationship between leverage and default risk, consistent with adverse selection (see Stiglitz and Weiss (1981)).

Quantitative application: Basel Endgame. Using the estimated elasticities and the observed credit distribution, we evaluate the Federal Reserve's proposal to abolish the highly risk-sensitive Internal Ratings-Based Approach and replace it with a two-bracket risk-weight schedule. The proposal increases risk weights for low-risk loans to 65% and reduces them to 100% for high-risk loans. This reform generates strongly nonlinear incentive effects: lending declines for low-risk loans and expands for high-risk loans, both in substantial magnitude (20–40%). Overall, total credit supply falls moderately by 3.57%. Because many low-risk exposures experience a sharp increase in risk weights, bank equity rises by 4.33%. Roughly three-quarters of this adjustment reflects behavioral (incentive) effects, while one-quarter reflects mechanical (inframarginal) effects. In sum, the proposal makes banks safer: the probability of a bailout declines by 1.4 percentage points, equivalent to an 8% increase in the capital adequacy ratio.

To quantify welfare implications, we estimate credit-supply spillovers on entrepreneurial income and measure household externalities through deposit insurance, following standard approaches in the literature (e.g., Corbae and D'Erasmo (2021); Dávila and Walther (2021); Oehmke and Opp (2022)). For comparison, we also simulate the (constrained) Pareto-optimal risk-weight schedule and find striking similarities with the flatter schedule proposed by the Fed. Both the Fed proposal and the optimal schedule produce comparable welfare patterns. Bank welfare declines because banks must hold more costly equity, while entrepreneurs experience, on average, reduced income due to lower credit supply. Welfare losses are concentrated among low-risk firms, whereas high-risk firms benefit from reallocation. Households, by contrast, gain as expected bailout costs decline. We therefore conclude that the Fed's proposed risk-weight schedule is close to optimal.

Robustness and extensions. To be added

Related literature. We contribute to several strands of literature. First, the banking literature has emphasized the importance of capital regulation since the 1970s (e.g., Santomero and Watson (1977); Koehn and Santomero (1980)). The rise in bank failures during the 1980s shifted attention toward the need for risk-sensitive regulation (see, e.g., Kim and Santomero (1988); Rochet (1992)).[2] In this area, our work relates to the theoretical analysis of optimal risk weights (e.g., Rochet (1992); Greenwood et al. (2017)).

Second, we connect to the recent literature in financial economics that applies the sufficient-statistics approach to regulatory design, pioneered in public finance (e.g., Chetty (2009)). Similar to Dávila and Walther (2021), Dávila and Goldstein (2023), and Van den Heuvel (2022), we characterize optimal financial regulation in terms of a small set of empirically observable measures. Our paper is most closely related to Dávila and Walther (2021), who analyze corrective taxation of investors under regulatory imperfections. We instead focus on quantity regulation in credit markets and empirically estimate the sufficient statistics for nonlinear risk weights.

A complementary strand of work analyzes capital regulation using quantitative structural models (e.g., Begenau and Landvoigt (2022); Corbae and D'Erasmo (2021); Elenev et al. (2021)). Whereas the sufficient-

[2] Santos (2001) provides an excellent overview of the early literature.

statistics approach focuses on parsimonious empirical moments and permits flexible nonlinear policies, the structural approach is informative about further channels of bank regulation, such as business-cycle effects.

Finally, we contribute to the empirical banking literature on risk-sensitive capital regulation. Meiselman et al. (2023) show that profitability predicts bank risk better than regulatory ratios; Plosser and Santos (2018), Behn et al. (2022), and Sizova (2023) document systematic underreporting in internal risk models; and Acharya et al. (2014) emphasize that risk weights are not necessarily forward-looking. Building on this work, we identify credit-supply and risk responses in credit-registry data and link them to core credit-market frictions to estimate policy counterfactuals.

Outline of the paper. Section 2 develops a general model of banking and defines the elasticities and welfare externalities that serve as sufficient statistics for capital regulation. Section 3 introduces the variational approach for analyzing nonlinear risk weights and derives both the positive and normative effects of regulatory reforms, characterizing optimal bank regulation in terms of these sufficient statistics. Section 4 estimates the key statistics using German administrative data and U.S. syndicated loan data. Section 5 uses these estimates to quantitatively evaluate nonlinear risk-weight reforms and the optimal design of regulatory risk weights. Section 6 discusses model extensions, and Section 7 concludes.

2 Framework

We develop a unified model of financial intermediation that serves as a common structure for a range of canonical banking models. The formulation is deliberately general so that specific assumptions on technology, information, and market structure reproduce well-known benchmark environments such as Kim and Santomero (1988), Boyd and De Nicolo (2005), Stiglitz and Weiss (1981), Holmstrom and Tirole (1997), or Gale and Hellwig (1985).

In the following, we describe the model in detail. Throughout, we define elasticities and distributional statistics that determine the positive effects of risk-sensitive bank capital regulation and marginal welfare externalities, capturing normative effects. These can be measured in the data and are sufficient statistics for regulatory policies. Their definition will not rely on a specific model structure or parametric assumptions.

2.1 Environment

There are three types of agents $x \in \{E, H, B\}$: a continuum of entrepreneurs $\theta \in [0,1] \equiv \Theta$ (borrowers), households (depositors), and a representative bank (financial intermediary). The bank chooses credit supply $k \equiv \{k_\theta\}_{\theta\in\Theta}$, equity $\mathcal{E}$, and has additional decision variables $z \equiv \{z_\theta\}_{\theta\in\Theta}$, such as loan pricing or monitoring intensity. We denote the complete bank choice set by $x \equiv (\, k,\, \mathcal{E},\, z\,)$. Depositors (households) make choices c, such as consumption. Each entrepreneur θ takes individual decisions d_θ, for example, effort, project scale, or risk-taking. We collect these in $d \equiv \{d_\theta\}_{\theta\in\Theta}$. The vector $p \equiv \{p_\theta\}_{\theta\in\Theta}$ summarizes ex-ante credit risk for each loan. It may depend on borrower characteristics, choices, and the equilibrium credit supply through leverage or contractual channels. This relationship is captured by a general risk-formation constraint

$$\mathcal{P}(p_\theta, k_\theta, z_\theta; d_\theta, \theta) = 0, \qquad \forall\, \theta \in \Theta, \tag{1}$$

that defines the mapping between credit supply, contractual variables, and ex-ante default risk. In practice, p_θ refers to the regulatory (reported) probability of default, which banks determine through their internal models. The risk-formation constraint $\mathcal{P}$ thus describes how these reported PDs depend on banks' choices and borrower characteristics. While the baseline analysis treats p_θ as reflecting true credit risk, the framework readily nests settings with risk-weight manipulation, in which banks influence reported PDs through misreporting or model adjustments.

The bank's feasibility constraints summarize incentive compatibility, borrowing limits, or market-structure restrictions:

$$\mathcal{C}(x, p; d) \leq 0. \tag{2}$$

Capital regulation imposes a requirement linking bank equity and risk-weighted assets,

$$\mathcal{R}(k, \mathcal{E}, p) \equiv \int_\Theta \omega(p_\theta) k_\theta \, d\theta - \mathcal{E}/\mathit{\Omega} \leq 0, \tag{3}$$

where $\mathit{\Omega} > 0$ is the capital-adequacy ratio and $\omega(p_\theta) \geq 0$ denotes the nonlinear risk weight on assets of risk p_θ. For now, we restrict the regulatory domain to a one-dimensional measure of ex-ante credit risk, but the analysis readily extends to cases where risk weights are tagged to additional observables, such as loss-given-default. For tractability, $\omega(p_\theta)$ is assumed to be twice continuously differentiable.[3] The marginal risk weights $\omega'(p_\theta)$ capture how capital requirements change with credit risk. Whereas the structural effects of the capital ratio $\mathit{\Omega}$ have received recent attention in the literature (see Van den Heuvel (2022), Begenau and Landvoigt (2022)), nonlinear risk weights $\omega(p_\theta)$ are studied rarely.

Entrepreneurs and depositors. Each entrepreneur θ chooses d_θ to maximize expected utility,

$$\mathcal{V}^E_\theta = \max_{d_\theta} U^E_\theta(d_\theta; k_\theta, z_\theta) \quad \text{s.t.} \quad \mathcal{C}^E_\theta(d_\theta; k_\theta, z_\theta) \leq 0, \tag{4}$$

[3]Note the difference between credit rating and actual default probabilities. While we require differentiability in the latter, risk weights may still be step-functions of credit ratings.

where $U^E_\theta(\cdot)$ denotes expected utility from the project, typically increasing in expected returns. $\mathcal{C}^E_\theta(\cdot)$ summarizes technological or incentive limits, such as participation, collateral, or limited-liability constraints. Given x, the entrepreneur's optimal choices induce an equilibrium default probability p_θ satisfying (1). Analogously, depositors make choices to maximize their expected utility subject to a set of constraints

$$\mathcal{V}^H = \max_c U^H(c;x) \quad \text{s.t.} \quad \mathcal{C}^H(c;x) \leq 0. \tag{5}$$

Bank. The representative bank chooses (x, p) to maximize expected utility

$$\mathcal{V}^B = \max_{x,p} U^B(x,p) \quad \text{s.t.} \quad \mathcal{P}(p_\theta, k_\theta, z_\theta; d_\theta, \theta) = 0,\ \forall\theta, \quad \mathcal{C}(x,p;c,d) \leq 0, \quad \mathcal{R}(x,p) \leq 0. \tag{6}$$

$U^B(x,p)$ denotes the intermediary's expected net return or utility from its credit portfolio. The constraints represent, respectively, (i) the endogenous formation of credit risk $\mathcal{P}$, (ii) market or contractual feasibility $\mathcal{C}$, and (iii) the prudential capital requirement $\mathcal{R}$.

The role of the capital constraint depends on the cost of equity finance, captured by $\partial U^B(\cdot)/\partial\mathcal{E}$. If equity were frictionless, the Modigliani and Miller (1958) proposition would imply that $\mathcal{R}(\cdot) \leq 0$ is slack and that changes in risk weights $\omega(p_\theta)$ or the capital ratio Ω would have no real effects on lending. In practice, equity is costly, for instance, because of tax deductibility of debt interest or deposit insurance, so the constraint binds in equilibrium.

The problem can be viewed in two layers. The *inner problem* determines operational choices, such as pricing and monitoring, conditional on the portfolio $\{k_\theta\}$ and equity $\mathcal{E}$. The *outer problem* then governs optimal portfolio and equity choices subject to regulation. The inner problem yields functions $z(k,\mathcal{E},p)$ that describe the bank's optimal operating policy. The outer problem determines $(k,\mathcal{E},p)$ to equate the marginal benefit of credit expansion with its marginal regulatory cost. The corresponding first-order conditions are reported and discussed in Appendix A.1.

The outer problem implies two optimality conditions that can be expressed in equity units. The first defines the shadow value of bank equity associated with the binding capital requirement. The second equates the marginal benefit of expanding credit to type θ with the marginal regulatory cost. At the optimum, the bank increases lending up to the point where the utility gain from an additional unit of credit, including the induced change in default risk through pricing or monitoring, equals the cost of raising extra equity in proportion to the risk weight $\omega(p_\theta)$ and its slope $\omega'(p_\theta)$. Hence, both the level and the slope of the risk-weight schedule influence credit supply and, through the (potential) endogenous response of risk p_θ, feed back into the portfolio composition.

Equilibrium. An *equilibrium* consists of a collection (x, p, c, d) such that, for a given regulatory environment $(\Omega, \omega(\cdot))$, the following conditions are satisfied: (i) for each $\theta \in \Theta$, the entrepreneur's decision d_θ solves (4), given the bank's policy vector x, yielding p_θ consistent with (1); (ii) the depositors' decisions c solve (5), given the bank's policy vector x; (iii) given entrepreneurs' and depositors' best responses $d = \{d_\theta\}_{\theta\in\Theta}$, the bank chooses (x, p) to maximize (6); and (iv) all constraints are jointly satisfied and mutually consistent. The equilibrium thus determines a joint allocation of credit, risk, and equity that satisfies optimality for entrepreneurs and the bank, as well as all regulatory and market-structure restrictions. Depending on the constraint specification, this allocation may correspond to a competitive credit market, a monopolistic intermediary, or an optimal-contracting outcome.

Sufficient statistics. To characterize how the equilibrium allocation responds to small reforms of the capital requirement, it is useful to summarize the bank's behavioral responses by a set of loan-level elasticities.

We assume that bank equity $\mathcal{E}$ and other choices z_θ are separable. The separability assumption does not restrict the underlying mechanism (pricing or monitoring may still be the channels through which risk responds) but it rules out a direct dependence of loan-level credit risk on bank-level equity, once these choices are optimally adjusted. Absent any across-asset risk spillovers, the equilibrium default probability p_θ can be written in reduced form as a function of credit supply, $p_\theta = p_\theta(k_\theta; \theta)$. Around the equilibrium risk profile, loan-level elasticities are defined as

$$\zeta_\theta^{k,\omega} \equiv \left.\frac{\partial \log k_\theta}{\partial\, \omega(p_\theta)}\right|_{\bar{p}}, \qquad \zeta_\theta^{k,\omega'} \equiv \left.\frac{\partial \log k_\theta}{\partial\, \omega'(p_\theta)}\right|_{\bar{p}}, \qquad \zeta_\theta^{k,p} \equiv \frac{\partial \log k_\theta}{\partial \log \bar{p}_\theta}, \qquad \varepsilon_\theta^{p,k} \equiv \frac{d \log p_\theta}{d \log k_\theta}. \tag{7}$$

The first two elasticities measure how credit supply reacts to the *level* and *slope* of the risk-weight schedule, evaluated at the equilibrium distribution of risks $\bar{p}$. $\zeta_\theta^{k,p}$ captures how credit responds to a change in borrower risk for fixed regulatory risk weights, while $\varepsilon_\theta^{p,k}$ summarizes the endogenous feedback from credit supply expansion to risk. This reduced-form relation embeds all indirect channels, such as adjustments in pricing or monitoring. These elasticities measure the positive effects of regulatory reforms.

Beyond behavioral responses, the welfare effects of marginal policy reforms depend on how credit supply and equity affect indirect utility across types. We define *marginal welfare externalities* as

$$\chi^H \equiv -\frac{1}{\mu}\frac{d\mathcal{V}^H}{d\mathcal{E}} \qquad \xi_\theta^H \equiv -\frac{1}{\mu}\frac{d\mathcal{V}^H}{dk_\theta} \qquad \text{and} \qquad \xi_\theta^E \equiv -\frac{1}{\mu}\frac{d\mathcal{V}_\theta^E}{dk_\theta},\ \forall\theta. \tag{8}$$

ξ_θ^E and ξ_θ^H measure the external effects of an additional unit of credit to type θ on entrepreneurial and household welfare, holding regulation constant. χ^H captures the marginal welfare effect of an incremental unit of bank equity. The latter term reflects any pecuniary or non-pecuniary spillover associated with higher capital

Model	Pricing	Level effect	Slope effect	Risk channel	Risk feedback
Kim & Santomero [1988] bank portfolio choice	partial equilibrium	$\zeta_\theta^{k,\omega} < 0$	$\zeta_\theta^{k,\omega'} = 0$	$\zeta_\theta^{k,p} = \omega'_\theta p_\theta \zeta_\theta^{k,\omega}$	$\varepsilon_\theta^{p,k} = 0$
Boyd & De Nicolò [2005]	monopolistic pricing	$\zeta_\theta^{k,\omega} < 0$	$\zeta_\theta^{k,\omega'} = p_\theta \varepsilon_\theta^{p,k} \zeta_\theta^{k,\omega} < 0$	$\zeta_\theta^{k,p} \lesseqgtr 0$	$\varepsilon_\theta^{p,k} > 0$
entrepreneurial moral hazard	competitive pricing	$\zeta_\theta^{k,\omega} < 0$	$\zeta_\theta^{k,\omega'} = (1-p_\theta)\zeta_\theta^{k,\omega} < 0$	$\zeta_\theta^{k,p} \lesseqgtr 0$	$\varepsilon_\theta^{p,k} > 0$
Holmström & Tirole [1997]	fixed capital	$\zeta_\theta^{k,\omega} = 0$	$\zeta_\theta^{k,\omega'} = 0$	$\zeta_\theta^{k,p} = 0$	$\varepsilon_\theta^{p,k} = 0$
double moral hazard	competitive pricing	$\zeta_\theta^{k,\omega} < 0$	$\zeta_\theta^{k,\omega'} = 0$	$\zeta_\theta^{k,p} \lesseqgtr 0$	$\varepsilon_\theta^{p,k} = 0$
Stiglitz & Weiss [1981]	monopolistic pricing	$\zeta_\theta^{k,\omega} < 0$	$\zeta_\theta^{k,\omega'} = p_\theta \varepsilon_\theta^{p,k} \zeta_\theta^{k,\omega} > 0$	$\zeta_\theta^{k,p} \lesseqgtr 0$	$\varepsilon_\theta^{p,k} < 0$
adverse selection	competitive pricing	$\zeta_\theta^{k,\omega} < 0$	$\zeta_\theta^{k,\omega'} = 0$	$\zeta_\theta^{k,p} = \omega'_\theta p_\theta \zeta_\theta^{k,\omega}$	$\varepsilon_\theta^{p,k} < 0$
Gale & Hellwig [1985]	monopolistic contracting	$\zeta_\theta^{k,\omega} < 0$	$\zeta_\theta^{k,\omega'} = p_\theta \varepsilon_\theta^{p,k} \zeta_\theta^{k,\omega} < 0$	$\zeta_\theta^{k,p} \lesseqgtr 0$	$\varepsilon_\theta^{p,k} > 0$
costly state verification	competitive rates	$\zeta_\theta^{k,\omega} < 0$	$\zeta_\theta^{k,\omega'} = 0$	$\zeta_\theta^{k,p} \lesseqgtr 0$	$\varepsilon_\theta^{p,k} \lesseqgtr 0$

Table 1: Summary of elasticities across canonical banking models.

buffers, for instance, lower bailout probabilities or transfers from households to banks. The shadow price of bank equity μ normalizes all externalities. Together with the behavioral elasticities, these externalities are sufficient statistics for normative evaluation of reforms to bank-capital regulation.

2.2 Connection to Canonical Banking Models

The general framework nests a range of canonical models that differ in the structure of the risk-formation constraint $\mathcal{P}$ and the feasibility conditions $\mathcal{C}$. Each specification emphasizes a particular transmission channel (e.g., portfolio choice, moral hazard, adverse selection, or costly verification) that governs how credit supply, risk, and regulation interact. In addition to unifying these microfoundations, the comparison provides empirical underpinnings: the sufficient statistics defined in the general framework can be linked to specific frictions and market structures identified in the canonical models.

Table 1 summarizes how these elasticities vary across benchmark environments. For further model details and structural characterizations, see Appendix A.2. Three broad insights emerge. First, when the capital constraint binds, higher risk weights tighten the effective capital requirement and reduce credit supply. Thus, credit-supply responses to risk weights provide direct evidence of deviations from the Modigliani and Miller (1958) capital-structure irrelevance result. Second, credit supply responds to marginal risk weights only if banks internalize their effect on aggregate credit risk. Since this channel is typically absent under perfect competition, such responses serve as a diagnostic for market power or imperfect risk pricing. Third, the sensitivity of credit risk to entrepreneurial leverage reveals the nature of loan-market frictions. Positive correlations are consistent with moral hazard, negative ones with adverse selection, and a null relationship implies that risk reflects exogenous loan characteristics rather than endogenous borrower behavior. We now turn to the specific canonical models.

Kim and Santomero (1988): portfolio risk choice. In the Kim–Santomero portfolio model, the bank allocates its balance sheet across risky and safe assets subject to the capital requirement. Asset risk is a characteristic, so $\mathcal{P}$ is degenerate and there is no feedback from lending to risk ($\varepsilon^{p,k} = 0$). Under costly equity, risk weights act as a linear tax on risky assets. A higher level of weights compresses lending in proportion to $\omega(p_\theta)$, delivering $\zeta^{k,\omega} < 0$. Since risk is exogenous, the slope of the schedule $\omega'(p_\theta)$ has no effect, implying $\zeta^{k,\omega'} = 0$. The model isolates the pure portfolio channel of capital regulation, where capital affects portfolio choice but not risk-taking.

Boyd and De Nicolo (2005): entrepreneurial moral hazard. A later line of work has endogenized credit risk (see Allen and Gale (2000)). Extending this mechanism, Boyd and De Nicolo (2005) model entrepreneurs who obtain funding on the credit market and choose risk under a risk-return trade-off (see Corbae and D'Erasmo (2021) for a recent quantitative model). The risk-formation constraint $\mathcal{P}$ captures the relationship between loan size and project risk: due to limited liability, greater entrepreneurial leverage raises loans' default probabilities ($\varepsilon^{p,k} > 0$). Imperfect competition in the credit market further amplifies the effect of leverage through higher loan prices. Under both competitive and monopolistic loan pricing, higher risk weights make lending more costly, thereby reducing credit supply ($\zeta^{k,\omega} < 0$). While banks cannot directly control entrepreneurial risk-taking, they internalize the marginal effect of lending on credit risk and, in turn, on the equity required by risk weights. Consequently, a steeper schedule also depresses lending ($\zeta^{k,\omega'} < 0$), strengthening the transmission of capital regulation through the risk-taking channel.

Holmstrom and Tirole (1997): double moral hazard. In Holmstrom and Tirole (1997), firms require external finance because project returns are not fully pledgeable. Funding comes from two sources: informed insiders (entrepreneurs and banks) who exert effort and monitor, and uninformed outside investors who lend without monitoring but whose claims must remain incentive compatible. Both parties face incentive problems: entrepreneurs must exert effort, and banks must monitor prudent behavior. The feasibility constraints $\mathcal{C}$ embed joint incentive-compatibility conditions that limit the amount of outside finance relative to total inside capital. The default probability p_θ is treated as a primitive, reflecting that risk is determined by incentive feasibility rather than by the scale of lending. Therefore, default risk is fixed ($\varepsilon^{p,k} = 0$).

Two polar cases illustrate how regulation transmits. When inside finance is scarce, the volume of intermediation is pinned down by available bank capital. Banks then earn a scarcity rent, and the regulatory constraint is slack, implying $\zeta^{k,\omega} = 0$ and $\zeta^{k,\omega'} = 0$. When the supply of inside capital is elastic and loan rates satisfy a zero-profit condition within $\mathcal{C}$, higher risk weights increase the cost of intermediation and raise loan interest rates. As a result, the amount of lending that satisfies incentive compatibility declines ($\zeta^{k,\omega} < 0$). Since credit risk is exogenous, marginal risk weights have no local effect ($\zeta^{k,\omega'} = 0$).

Stiglitz and Weiss (1981): adverse selection. Borrowers differ in project quality, which is private information. Loan pricing, therefore, shapes the composition of applicants: higher rates screen out safer types and attract riskier ones. In our mapping, the risk-formation constraint $\mathcal{P}$ captures this selection mechanism, while the feasibility set $\mathcal{C}$ imposes participation and, where relevant, zero-profit conditions. When lending expands because loan rates decline, the borrower pool improves, and average default risk falls, yielding a negative risk feedback ($\varepsilon^{p,k} < 0$). While the original model emphasizes selection across firms, a similar mechanism applies within firms if each finances multiple projects with unobserved risk. Lower funding costs then induce safer marginal projects, reducing the firm's average default probability.

The transmission of capital regulation depends on market structure. With monopolistic pricing, the bank internalizes the effect of interest rates on borrower composition. Higher risk weights raise the cost of capital and, through higher loan prices, reduce lending ($\zeta^{k,\omega} < 0$). At the same time, the bank recognizes that lower rates improve the applicant pool and thereby reduce the equity required by risk weights. A steeper schedule amplifies this channel, reinforcing the downward pressure on rates and expanding credit ($\zeta^{k,\omega'} > 0$). Under competitive pricing, the loan rate is pinned by the zero-profit condition, so the slope has no local bite ($\zeta^{k,\omega'} = 0$), while the level effect remains negative ($\zeta^{k,\omega} < 0$). Relative to entrepreneurial moral hazard, the key difference is the sign of the feedback: adverse selection makes risk fall when credit expands.

Gale and Hellwig (1985): costly state verification. In Gale and Hellwig (1985), project returns are privately observed, and lenders can verify returns only by paying an audit cost that depends on loan size and project realizations. Loan contracts must therefore ensure the borrower's incentive to repay in non-verified states. The incentive-compatible contract takes the form of standard debt, where borrowers repay a fixed amount when they can, and auditing occurs only upon default. In our mapping, the feasibility constraint $\mathcal{C}$ captures the borrower's and lender's participation conditions, taking into account the expected verification cost. The risk-formation constraint $\mathcal{P}$ describes how the probability of default p_θ depends on loan size k_θ through the threshold state that triggers default.

Larger loans increase the expected number of audits and the total monitoring cost. Whether this also raises default probability depends on the market structure. Under monopolistic contracting, the bank operates at the borrower's participation constraint. A higher loan allows greater surplus extraction through a larger repayment obligation, thereby raising the bankruptcy threshold. This implies a positive risk feedback ($\varepsilon_\theta^{p,k} > 0$). Under competitive rates, entrepreneurs borrow freely at posted loan rates that adjust to satisfy the bank's zero-profit condition. For a given rate, leverage and default risk are positively related. Yet the equilibrium rate adjusts so that the marginal product of capital equals its average product, rendering the feedback ambiguous ($\varepsilon_\theta^{p,k} \gtrless 0$). Capital regulation affects lending by interacting with contractual costs. Under monopolistic contracting, higher risk weights raise the cost of intermediation and reduce bank lending incentives ($\zeta_\theta^{k,\omega} < 0$). A steeper risk-weight schedule further penalizes risky loans, tightening credit supply ($\zeta_\theta^{k,\omega'} < 0$). Under competitive lending, the loan rate satisfies a zero-profit condition, so the slope of the schedule has no local effect ($\zeta_\theta^{k,\omega'} = 0$), while the level effect remains negative ($\zeta_\theta^{k,\omega} < 0$).

3 Analysis of Risk Weights

We now return to our general framework and study the loan- and bank-level effects of changing risk weights, i.e., the incidence of risk weights. We introduce a perturbation, or variational, approach for studying such reforms to bank capital regulation. This allows us to express the regulatory incidence on bank equity and credit supply in terms of the defined elasticities and the observed credit distribution. Then, we turn to a normative analysis of risk-weight schedules. We show that the marginal credit-supply and equity externalities characterize the welfare effects of risk-weight reforms and, finally, provide sufficient statistics formulas for optimal nonlinear risk weights that address the corrective motives of bank capital regulation.

3.1 Perturbation Approach

We start by analyzing the effects of reforming the existing risk-weight schedule. Consider an arbitrary initial risk-weight scheme ω and suppose the regulator implements a reform $\hat{\omega}$ that changes risk weights to $\omega + \epsilon\hat{\omega}$, where $\epsilon \to 0$. Formally, $\hat{\omega}$ is the infinite-dimensional direction of the reform, and ϵ parametrizes the reform size.[4] Similarly, we can study directional reforms to the capital adequacy ratio from Ω to $\Omega + \epsilon\hat{\Omega}$. The Gateaux derivative of a functional $\mathcal{F} : \mathcal{C}([0,1]) \to \mathbb{R}$ in the direction $\hat{\omega}$ is defined as[5]

$$\hat{\mathcal{F}}(\hat{\omega}) \equiv \frac{d}{d\epsilon}\mathcal{F}(\omega + \epsilon\hat{\omega})\bigg|_{\epsilon=0}.$$

Following a long tradition in the public finance literature (see Piketty (1997), Saez (2001), Golosov et al. (2014)), this approach allows us to derive the positive and normative effects of reforms in terms of empirically observable sufficient statistics. These statistics are evaluated around the current regulatory policies and can be measured in the data.[6] We do not need to rely on a specific model structure or parameterization, unifying a large class of banking models as described in Section 2.2. Recent contributions to the financial economics literature also follow a sufficient statistics approach to characterize, e.g., optimal bankruptcy exemptions (see Dávila (2020)) and prudential regulations (see Dávila and Walther (2021, 2023)).

[4] Suppose throughout that $\hat{\omega}$ is in the Banach space of continuously differentiable functions with bounded first derivative.
[5] To save on notation, we drop in the following the dependence of Gateaux derivatives on $\hat{\omega}$.
[6] The usual caveat applies that the considered reforms are small.

3.2 Loan-Level Incidence

We start by examining the effects of risk weights on each individual loan type $\theta \in \Theta$. One can characterize the equilibrium changes in type-specific credit supply and credit risk as follows (see Appendix B.1):

$$\frac{\hat{k}_\theta}{k_\theta} = \underbrace{\frac{\zeta_\theta^{k,\omega}}{1-\varepsilon_\theta^{k,p}\varepsilon_\theta^{p,k}}}_{\bar{\zeta}_\theta^{k,\omega}} \hat{\omega}(p_\theta) + \underbrace{\frac{\zeta_\theta^{k,\omega'}}{1-\varepsilon_\theta^{k,p}\varepsilon_\theta^{p,k}}}_{\bar{\zeta}_\theta^{k,\omega'}} \hat{\omega}'(p_\theta) + \Psi_\theta(\{\hat{\omega}(p_\theta), \hat{\omega}'(p_\theta)\}_{\theta\in\Theta}) \quad \text{and} \quad \frac{\hat{p}_\theta}{p_\theta} = \varepsilon_\theta^{p,k}\frac{\hat{k}_\theta}{k_\theta}. \tag{9}$$

Recall that corporate loans and entrepreneurial risk can depend on both marginal *and* average risk weights. Consequently, any rise in either average or marginal risk weights distorts lending incentives, typically reducing credit supply and the level of credit risk. These partial-equilibrium responses are proportional to $\zeta_\theta^{k,\omega}$ and $\zeta_\theta^{k,\omega'}$, respectively. The term $(1-\varepsilon_\theta^{k,p}\varepsilon_\theta^{p,k})^{-1}$ captures a feedback correction that accounts for the potential endogenous response of credit risk to changes in loan supply, and vice versa.[7]

Besides these direct effects of reforming an asset's risk weight (own-price responses), credit supply may also respond indirectly through asset substitution. For instance, optimal investment in one asset may depend on investments in other assets due to cross-asset covariance in returns (e.g., Kim and Santomero (1988)). In such cases, reforming the risk weight for a particular loan type or asset class affects not only that asset but also the entire composition of the bank's credit portfolio (cross-price effects). These portfolio adjustments, in turn, feed back into the demand for the reformed asset class. The term $\Psi_\theta(\cdot)$ captures this entire sequence of circular adjustments within the portfolio (see B.3.2 for a structural example).

3.3 Bank-Level Incidence

Having derived the loan-level incidence, we can now investigate the aggregate implications at the bank level. Proposition 1 characterizes the first-order responses of bank-level variables to an arbitrary reform of the risk-weight schedule.

Proposition 1. *The incidence of reforming an initial risk-weight scheme $\omega(p_\theta)$ in the direction $\hat{\omega}(p_\theta)$ on bank credit supply and equity is given by*

$$\hat{\mathcal{L}} = \int_\theta \hat{k}_\theta d\theta \tag{10}$$

and

$$\hat{\mathcal{E}} = \Omega \int_\theta \hat{\omega}(p_\theta) k_\theta d\theta + \Omega \int_\theta [\omega(p_\theta) + \omega'(p_\theta) p_\theta \varepsilon_\theta^{p,k}] \hat{k}_\theta d\theta, \tag{11}$$

where (9) describes the changes to the bank credit portfolio.

[7] This feedback effect is conceptually similar to those found in the public finance literature, e.g., for the income taxation of superstar managers (see Scheuer and Werning (2017)) and for scale-dependent investment return rates in the context of capital taxation (see Schulz (2021)).

Proof. Appendix B.2. □

The equilibrium response of bank credit supply in (10) is straightforward. Any increase in marginal or average risk weights of a given loan raises the bank's effective funding cost and thereby distorts its lending decisions. This distortionary effect reduces the aggregate credit supply $\mathcal{L}$ accordingly.

Recalling that the bank capital requirement is binding in equilibrium, bank equity $\mathcal{E}$ explicitly depends on the entire risk-weight schedule. A rise in risk weights increases the level of risk-weighted assets for a given credit supply. In other words, higher risk weights mechanically raise the amount of equity needed to satisfy the capital requirement, which is the first term in equation (11). This mechanical, or *inframarginal*, component adds to the behavioral adjustments arising from changes in loan size and risk (second term in (11)).

As discussed above, credit supply and loan risk are jointly shaped by both average and marginal risk weights, giving rise to two channels of behavioral adjustment: (i) changes in credit supply affect required equity for a given set of risk weights, and (ii) changes in credit supply alter risk weights indirectly through their effect on default probabilities. For $\omega \geq 0$ and $\omega' \varepsilon^{p,k} \geq 0$, a negative credit-supply response reduces the value of risk-weighted assets and thus the amount of equity needed to meet the capital requirement. In Section 5, we provide a quantification of these effects.

3.4 Elementary Reforms

In this section, we focus the exposition on regulatory reforms that raise all risk weights above a threshold $p^* \equiv p_{\theta^*}$ and have a mechanical effect of 1\$ on required equity capital. We may express such reforms as

$$\hat{\omega}(p^*) = \frac{\mathbb{1}[p > p^*]}{\varOmega \int_{p>p^*} k_p dR\,(p)}. \tag{12}$$

This class of reforms (see Saez (2001)) can be referred to as elementary reforms since any reform can be represented as a combination of such reforms (see Golosov et al. (2014)). Studying elementary reforms is, therefore, without loss of generality. Figure 1 illustrates the class of elementary reforms of risk weights.

We characterize the effects of elementary reforms in terms of empirically observable sufficient statistics: i) the equilibrium semi-elasticities that capture the size of regulatory distortions, $\bar{\zeta}_p^{k,\omega}$ and $\bar{\zeta}_p^{k,\omega'}$, and ii) a distributional statistic, the hazard rate of the credit risk distribution, $r(p)/[1 - R(p)]$, measuring the granularity of credit risk. For transparency, we abstract from cross-price effects.

Corollary 1. *The incidence of an elementary reform of risk weights $\hat{\omega}(p^*)$ on bank credit supply and equity*

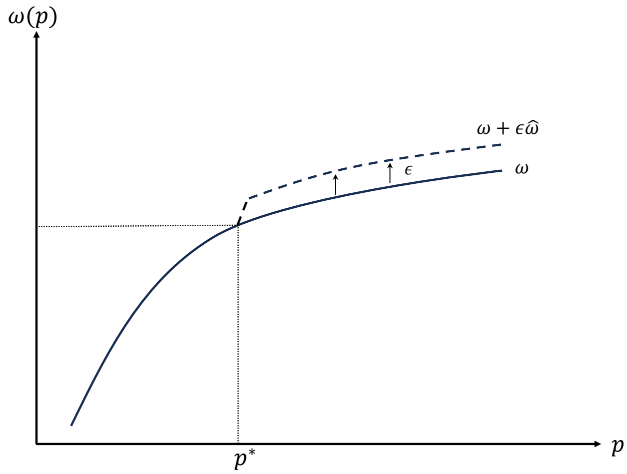

Figure 1: Elementary reforms of regulatory risk weights.

can be decomposed into:

$$\hat{\mathcal{L}}\left(p^*\right) = \int_{p>p^*} \frac{k_p \overline{\zeta}_p^{k,\omega}}{\varOmega\mathbb{E}\left(k_{p'}|p' > p^*\right)} \frac{dR\left(p\right)}{1-R\left(p^*\right)} + \frac{k_{p^*}\overline{\zeta}_{p^*}^{k,\omega'}}{\varOmega\mathbb{E}\left(k_{p'}|p' > p^*\right)} \frac{r\left(p^*\right)}{1-R\left(p^*\right)} \tag{13}$$

and

$$\hat{\mathcal{E}}\left(p^*\right) = 1 + \int_{p>p^*} \frac{k_p[\omega(p) + \omega'(p)p\varepsilon_p^{p,k}]\overline{\zeta}_p^{k,\omega}}{\mathbb{E}\left(k_{p'}|p' > p^*\right)} \frac{dR\left(p\right)}{1-R\left(p^*\right)} + \frac{k_{p^*}[\omega(p^*) + \omega'(p^*)p^*\varepsilon_{p^*}^{p,k}]\overline{\zeta}_{p^*}^{k,\omega'}}{\mathbb{E}\left(k_{p'}|p' > p^*\right)} \frac{r\left(p^*\right)}{1-R\left(p^*\right)}. \tag{14}$$

Proof. Appendix B.3. □

Elementary reforms change the risk-weight schedule in two respects. First, they increase the risk weights for loans with default risk above $p > p^*$. This mechanically raises the size of risk-weighted loans before behavioral effects materialize (first term in (14), respectively). Note that we have constructed the elementary reform to have a mechanical effect of 1\$. Moreover, the equilibrium lending and credit risk decline proportionally to the elasticity $\overline{\zeta}_p^{k,\omega}$. The first term in (13) and the second term in (14) express the overall impact on a bank's unweighted and risk-weighted loans.

Second, the reform raises the marginal risk weight for loans with an equilibrium default risk of $p = p^*$. This distorts lending and risk choices proportionally to $\overline{\zeta}_{p^*}^{k,\omega'}$. To obtain the size of this distortion, we need to weigh the behavioral responses by the hazard rate of the credit risk distribution, which compares the relative magnitude of entrepreneurs at $p = p^*$ to those at $p > p^*$. The larger the mass of entrepreneurs at the perturbed point of the credit risk distribution, the larger the distortions from changes to marginal risk weights. The final terms in (13) and (14) collect the resulting responses of bank credit supply and equity.

In Appendix B.3.2, we set up a fully structural model, where bailout transfers protect depositors and create a fiscal externality as in Dávila and Walther (2021). Banks raise deposits, allocate credit across heterogeneous entrepreneurs, and jointly determine their loan portfolio $\{k_\theta\}$ and the solvency cutoff $\upsilon^\star$ that triggers a bailout. The model delivers closed-form credit-supply elasticities and shows how the endogenous bailout cutoff links lending across assets, generating circularities from across-asset substitution. Under a flat baseline risk-weight scheme, these circularities can be absorbed by redefining the credit-supply semi-elasticities so that all incidence expressions continue to apply. Alternatively, one can construct a perturbation that cancels out all cross-price effects, thus eliminating any circularity.[8]

3.5 Normative Analysis

Besides characterizing the positive effects of risk-weight reforms, we now specify a *normative objective* for bank capital regulation. This allows us to study the incidence of risk weights on welfare and to derive a formula for the optimal nonlinear risk-weight schedule as a byproduct of our perturbation approach. In the context of income taxation, this procedure typically yields formulas for optimal marginal tax rates (see Diamond (1998); Saez (2001)). In that literature, the planner's objective is either tax revenue (Werning (2007)) or an aggregation of household utilities (Saez and Stantcheva (2016)).

The banking literature has not reached a consensus on the specification of a normative objective. Early contributions relied on mean–variance credit portfolio analysis (e.g., Kim and Santomero (1988); Rochet (1992); Gjerde and Semmen (1995)) or on value-at-risk approaches (e.g., Gordy (2003)), emphasizing risk–return trade-offs and financial stability. More recent structural models, such as Corbae and D'Erasmo (2021) and Begenau and Landvoigt (2022), derive welfare explicitly from the allocation of resources and the behavior of intermediaries. Following Dávila and Walther (2021), we adopt a welfarist perspective and evaluate the welfare effects of capital regulation by aggregating the utility changes of both financial and non-financial agents. Formally, we define the social welfare function as

$$\mathcal{W} = W\left(\mathcal{V}^B, \mathcal{V}^H, \{\mathcal{V}^E_\theta\}_{\theta\in\Theta}\right),$$

where $\mathcal{V}^B$, $\mathcal{V}^H$, and $\mathcal{V}^E_\theta$ denote the indirect utilities of the bank, the representative household (depositor), and entrepreneurs (borrowers) of type θ, respectively, as defined in Section 2. Let $\alpha^X \equiv \partial W/\partial \mathcal{V}^X > 0$ denote the Pareto weight assigned to agent $X \in \{B, H, E\}$, normalized such that $\alpha^B + \alpha^H + \int_\Theta \alpha^E_\theta d\theta = 1$.

[8] This result is related to the idea that bank capital regulation may not affect bank default risk if bank managers adjust their risk-taking so that, for any required level of capital, the probability of default remains constant (see Dick-Nielsen et al. (2023) and references therein).

Incidence on welfare. As a first step, we characterize the first-order welfare effects induced by an arbitrary risk-weight reform. The derivation follows from standard envelope arguments.

Lemma 1. *For a perturbation of the initial schedule $\omega(p_\theta)$ in the direction $\hat{\omega}(p_\theta)$, the first-order welfare effect on a monopolistic bank is*

$$\hat{\mathcal{V}}^B = -\,\mu \int_\Theta \hat{\omega}(p_\theta)\, k_\theta \, d\theta,$$

while under perfect competition (zero profits at all the loan level), loan rates adjust so that $\hat{\mathcal{V}}^B = 0$. The first-order welfare effects on households and entrepreneurs are

$$\hat{\mathcal{V}}^H = -\,\mu\, \chi^H \hat{\mathcal{E}} + \mu \int_\Theta \xi_\theta^H \, \hat{k}_\theta \, d\theta \qquad \text{and} \qquad \hat{\mathcal{V}}_\theta^E = -\,\mu\, \xi_\theta^E \, \hat{k}_\theta \,,$$

where the marginal welfare externalities are defined in (8).

Proof. See Appendix B.4. □

By the envelope theorem (see Milgrom and Segal (2002)), all bank choices are locally optimal under the initial policy, so risk-weight reforms generate no first-order behavioral effect on the bank's welfare. Under monopolistic banking, the welfare impact of raising risk weights is purely mechanical and proportional to the shadow cost of equity μ. Under perfect competition on the loan market, price adjustments fully offset this mechanical effect, implying a zero welfare effect.

In contrast, the welfare effects on households and entrepreneurs are structurally independent of the degree of competition. The coefficients χ^H, ξ_θ^H, and ξ_θ^E capture, in cost units of bank equity μ, the pecuniary and non-pecuniary externalities arising from changes in bank equity and credit supply. Competitive loan-market adjustments redistribute surplus between borrowers and lenders. These distributive pecuniary externalities (see Dávila and Korinek (2018)) are characterized in the Appendix. They operate as transmission channels for welfare responses and are embedded in the definitions of marginal welfare externalities.

Optimal nonlinear risk-weight schedule. As a byproduct of our perturbation approach, we can derive the optimal capital adequacy ratio and risk-weight schedule that maximizes aggregate social welfare:

$$\max_{\{\omega(p_\theta)\}_{\theta\in\Theta},\varOmega} \mathcal{W}(\{\omega(p_\theta)\}_{\theta\in\Theta}, \varOmega). \tag{15}$$

Formally, we choose the risk-weight scheme ω and capital adequacy ratio $\varOmega$ such that no reform, e.g., elementary (12), improves the regulatory objective function: $\hat{\mathcal{W}} = 0$. Observe that our environment features various potential sources of regulatory constraints: i) market failures and ii) imperfect competition. Therefore, it will be useful to characterize optimal bank capital regulation under each competition scenario in two steps. First, we derive the unconstrained Pareto-optimal regulation when there are no market failures (first-best allocation). Then, we consider the constrained efficient allocation where the regulatory authority faces the same market imperfections as all the agents (second-best allocation). For transparency, we assume in the main text that distributive pecuniary externalities from loan price responses are second order.

First-best allocation. Suppose a planner can freely choose the risk-weight schedule $\omega(\cdot)$ and the capital-adequacy ratio Ω. Moreover, abstract from market frictions that link credit supply and risk, so that $\varepsilon_\theta^{p,k} = 0$ for all θ. This delivers a standard benchmark.

Proposition 2. *Let the regulator maximize aggregate social welfare. In the absence of market frictions ($\varepsilon_\theta^{p,k} = 0$), the optimal capital-adequacy ratio is $1/\Omega = -(\alpha^H/\alpha^B)\chi^H$ and the optimal risk-weight scheme satisfies*

$$\omega(p) = (\alpha_p^E/\alpha^B)\xi_p^E + (\alpha^H/\alpha^B)\xi_p^H. \tag{16}$$

Proof. Appendix B.5. □

The optimal policy follows a Pigouvian logic: each instrument corrects the relevant marginal welfare externality at each point of the credit-risk distribution (see Pigou (1920); Sandmo (1975); Dixit (1985); Rothschild and Scheuer (2016); Dávila and Walther (2021)). While elasticities govern behavioral (loan- and bank-level) responses, aligning private and social incentives requires only the marginal welfare externalities of equity and credit. The larger the wedges—measured by χ^H and, respectively, $\{\xi_p^E, \xi_p^H\}$—the stronger the corrective terms in the optimum. This finding holds under any degree of loan-market competition.

Because equity is costly, the optimal risk-weight schedule acts as a type-specific tax on the bank's credit supply. The Pareto-weighted sum of credit-supply externalities determines the optimal correction for each loan type. Moreover, from the capital requirement (3), one unit of equity expands risk-weighted capacity by Ω; equivalently, equity is effectively "subsidized" at rate $1/\Omega$. At the Pareto-optimal regulation, the bank fully internalizes the marginal effect of an extra dollar of equity on household welfare. Finally, by setting $\chi^H = 0$, $\xi_p^E = 0$, and $\xi_p^H = 0$ for all p, one recovers the efficiency of the frictionless economy.

Second-best allocation. Whenever credit risk and leverage are linked by market failures, $\varepsilon_p^{p,k} > 0$ for some p, the planner's instruments become interdependent even when both the risk-weight schedule and the capital-adequacy ratio can be freely chosen. The planner faces the same informational or contractual frictions as the agents in the economy and solves a constrained Pareto problem.

Proposition 3. *Suppose the regulator maximizes aggregate social welfare and that market failures link credit risk and leverage, i.e., $\varepsilon_p^{p,k} > 0$ for some p. Then, the optimal capital-adequacy ratio is $1/\Omega =$*

$-(\alpha^H/\alpha^B)\chi^H$. *Defining* $\gamma(p';p) \equiv \exp[\int_p^{p'} 1/(\varepsilon_{p''}^{p,k} p'')dp'']$, *the optimal risk-weight schedule satisfies*

$$\omega(p) = b(p) - \int_0^p b'(p')\, \gamma(p';p)\, dp', \tag{17}$$

where $b(p) \equiv (\alpha_p^E/\alpha^B)\xi_p^E + (\alpha^H/\alpha^B)\xi_p^H$ *denotes the welfare externalities from credit supply.*

Proof. Appendix B.6. □

As in the first best, the optimal risk-weight schedule (17) corrects the marginal credit-supply externalities $b(p)$; for $\varepsilon_p^{p,k} = 0$ for all p, it collapses to (16). However, the presence of market failures makes the problem more intricate. Connecting to the Tinbergen (1952) rule, the number of linearly independent instruments is now smaller than the number of regulatory targets, even though $|\{\{\omega(p_\theta)\}_{\theta\in\Theta}, 1/\Omega\}| = |\{\{k_\theta\}_{\theta\in\Theta}, \mathcal{E}\}|$. The reason is that credit supply reacts to both average and marginal risk weights, rendering the instruments linearly dependent. As shown in Appendix B.6, the optimal schedule must therefore satisfy a differential equation balancing welfare gains and losses from adjusting average and marginal risk weights: $\omega(p) + \omega'(p)p\varepsilon_p^{p,k} = b(p)$.[9]

This dependence introduces an endogenous adjustment in the domain of risk weights. Whenever the planner seeks to impose a stronger correction for higher-risk loans ($b'(p) > 0$), the marginal risk weights become positive. Such nonlinearities induce behavioral distortions through credit-risk responses, which the planner must internalize. Conversely, when welfare weights and marginal externalities are uniform, market failures cease to affect the optimal schedule. In that case, there is no rationale for introducing curvature in $\omega(p)$, as shown in the following corollary.

Corollary 2. *Suppose welfare weights and marginal welfare externalities are uniform:* $\alpha_p^E = \alpha^E$, $\xi_p^E = \xi^E$, *and* $\xi_p^H = \xi^H$ *for all* p. *Then, the first-best and second-best capital regulations coincide.*

Therefore, when heterogeneity in welfare externalities is present, market failures modify the optimal risk weights. The adjustment term in (17) aggregates the nonlinear welfare effects of supplying credit to entrepreneurs with risk p and below, weighted by the strength of the market failure, captured by $\gamma(p';p)$. Consequently, the second-best schedule is flatter than in the first best: market failures reduce the slope of optimal risk weights across the risk distribution.

Finally, Appendix B.7 extends the analysis to a third-best setting in which the capital-adequacy ratio is imperfectly chosen, linking the results to Dávila and Walther (2021). In this case, the optimal risk-weight schedule jointly corrects credit-supply externalities and the equity wedge arising from suboptimal bank capitalization, both of which interact through the regulatory constraint. Put differently, risk weights

[9] To pin down the constant of the general solution, we assume that the lower bound of credit risk converges to zero, i.e., $\underline{p} \to 0$.

exhibit a leakage effect onto imperfectly chosen bank equity. As in Dávila and Walther (2021), the relevant leakage elasticities (here: the credit-supply elasticities) determine the effectiveness of addressing the equity wedge. The sufficient-statistics formula in Proposition 4 thus encompasses both credit-market failures and regulatory imperfections within a unified corrective framework.

4 Empirical Implementation

4.1 Data

To discipline the model and estimate the sufficient statistics, we draw on several administrative datasets provided by the Deutsche Bundesbank. Our primary source is the German credit registry (*Millionenkreditevidenz*), which reports quarterly exposures for all bank–borrower relationships in Germany with total credit exceeding €1.5 million (€1 million from 2015 onward). The sample covers 2000Q1–2022Q4 at a quarterly frequency.

The registry contains unique identifiers at both the bank and borrower levels, which we use to merge loan-level exposures with borrower balance-sheet data from the *JANIS* database. At the credit-relationship level, we observe detailed loan characteristics, including loan volume, collateralization, maturity, and, starting in 2008, loan-specific risk weights and internal probabilities of default (PDs).

The PD variable, reported by banks, provides a direct measure of the internal estimate of ex-ante credit risk, p_θ. Combining these sources enables us to construct a representative joint distribution of credit supply and credit risk in the German economy.

Table 2 reports summary statistics for all main variables used in the regressions below, including variables at the credit-relationship level (Panel A), the bank level (Panel B), and the firm level (Panel C). Figure 2 displays the kernel density estimate of credit-risk estimates p in the German economy.

This distribution serves as a sufficient statistic in our framework, enabling the aggregation of loan-level responses into bank-level effects.

Appendix C presents a complementary estimation of the sufficient statistics using U.S. data from the syndicated loan market. While the U.S. sample is less representative, covering only large banks and firms and lacking direct information on regulatory risk, it demonstrates the applicability of our approach across datasets and regulatory environments. The exercise also provides additional, albeit more limited, policy insights into the Federal Reserve's reform proposal.

Panel A: Loan-Level Characteristics.

Variable	Mean	1%-Q	25%-Q	50%-Q	75%-Q	99%-Q
Log Credit	14.767	7.601	13.732	14.942	16.301	19.695
p (PD estimate)	0.016	0.000	0.001	0.004	0.012	0.200
log p	-5.519	-9.210	-6.725	-5.547	-4.382	-1.609
ω (Risk weight)	0.516	0.006	0.152	0.404	0.908	1.592

Panel B: Bank Characteristics.

Variable	Mean	1%-Q	25%-Q	50%-Q	75%-Q	99%-Q
Log Total Assets	16.567	10.959	15.000	16.810	18.462	21.033
Deposits-to-Assets	0.228	0.000	0.084	0.193	0.311	0.788
Equity-to-Assets	0.074	0.009	0.033	0.045	0.082	0.547
NPL-to-Loans	0.004	0.000	0.000	0.000	0.003	0.050

Panel C: Firm Characteristics.

Variable	Mean	1%-Q	25%-Q	50%-Q	75%-Q	99%-Q
Log Total Assets	10.004	5.858	8.912	9.917	11.010	14.522
Log Sales	9.824	4.654	8.752	9.973	11.027	14.188
Net Income	2872.87	-23408.00	33.00	464.00	2011.00	79411.00
Cash-to-Assets	0.074	0.000	0.004	0.026	0.088	0.624
Leverage	0.578	0.026	0.385	0.596	0.787	1.000

Table 2: Summary statistics (German credit registry).

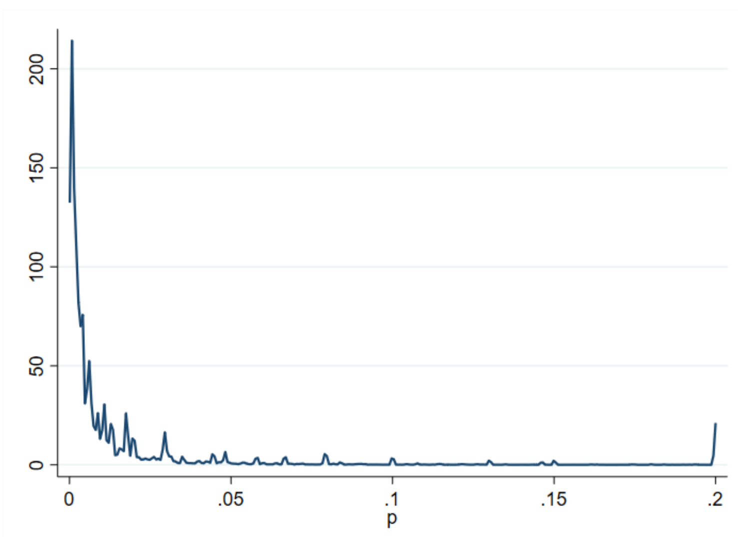

Figure 2: Kernel density estimate of regulatory credit risk (German credit registry).

4.2 Current Bank Capital Regulation

Risk-based capital regulation is the cornerstone of modern banking supervision. Since the first Basel Accord (1988), banks have been required to maintain minimum equity buffers relative to risk-weighted assets, as specified in Equation (3). Under Basel III, the minimum capital adequacy ratio Ω is set at 8%, absent additional requirements linked to systemic importance or the credit cycle of the jurisdiction. Risk weights used to compute risk-weighted assets depend on various borrower and loan characteristics, most importantly the asset class and the estimated probability of default.[10]

When the probability of default is proxied by external credit ratings, the framework is referred to as the *standardized approach* (SA). Table 3 reports the corresponding mapping between corporate credit ratings and regulatory risk weights.

Credit Rating	AAA–AA–	A+–A–	BBB+–BB–	Below BB–	Unrated
Risk Weight	20%	50%	100%	150%	100%

Table 3: Risk weights under the standardized approach (SA).

The second Basel Accord (Basel II, 2007) introduced an alternative methodology, the *Internal Ratings-Based Approach* (IRBA), which allows banks to use their own models to estimate default probabilities. Each estimated p_θ is then mapped into a regulatory risk weight through a continuous function prescribed by the Basel framework. Conditional on supervisory approval, banks thus effectively determine their own risk weights.[11] The mapping function under Basel III is given by:

$$\omega(p_\theta) = 12.5 \cdot LGD \left[G_{\mathcal{N}} \left(\frac{G_{\mathcal{N}}^{-1}(p_\theta) + G_{\mathcal{N}}^{-1}(0.999)\sqrt{a(p_\theta)}}{\sqrt{1 - a(p_\theta)}} \right) - p_\theta \right] \cdot \frac{1 + (M - 2.5)b(p_\theta)}{1 - 1.5b(p_\theta)},$$

where LGD is the loss given default, $G_{\mathcal{N}}^{-1}(\cdot)$ the inverse cumulative standard normal distribution function, and M the loan maturity in years. The Basel III correlation and maturity adjustments are defined as:

$$a(p_\theta) = 0.12 \cdot \frac{1 - \exp(-50p_\theta)}{1 - \exp(-50)} + 0.24 \cdot \left(1 - \frac{1 - \exp(-50p_\theta)}{1 - \exp(-50)} \right), \qquad b(p_\theta) = (0.11852 - 0.05478 \cdot \log(p_\theta))^2 .$$

Figure 3 plots the risk-weight schedule implied by the Basel III IRBA formula. The schedule is relatively favorable for low-risk exposures, with risk weights below 20%, and less favorable for high-risk loans, with weights exceeding 150%. Its concave shape highlights the asymmetric impact of risk-weight reforms across the credit-risk distribution. Because the IRBA substantially reduces capital requirements for low-risk borrowers, and since internal models allow banks to influence their estimated default probabilities, many

[10] Additional determinants include collateralization, exposure size, currency denomination, and other loan-specific features.

[11] Banks must obtain an IRBA license from the supervisory authority. Supervisors validate the internal credit risk model at initial approval and during ongoing review.

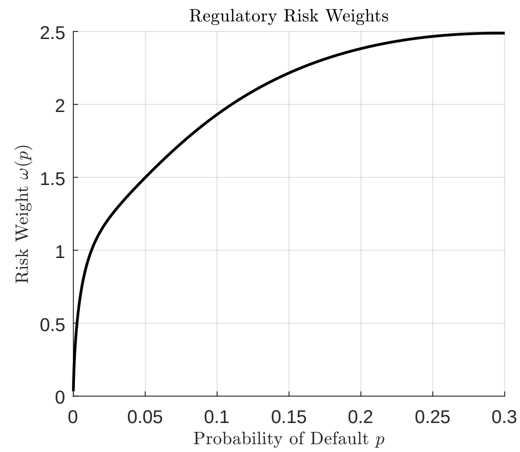

Figure 3: Risk weights under the internal ratings-based approach (IRBA).

Note: The figure illustrates the mapping between estimated default probabilities for corporate loans and regulatory risk weights under the IRBA. We set the loss given default to 45% and the loan maturity to five years, consistent with our empirical baseline.

large banks have adopted this framework (see Plosser and Santos (2018) and Behn et al. (2022)). We use the IRBA schedule as the empirical baseline for our simulations, as it best reflects the prevailing regulatory constraints faced by large German banks.

Importantly, this nonlinear schedule serves as the policy benchmark. Our central objective is to evaluate, in positive terms, the effects that reforms to this schedule have on credit markets and, in normative terms, to characterize the optimal risk-weight design. The current Basel formula was derived from credit-risk modeling techniques centered on single-factor value-at-risk calculations.[12] As a result, it does not account for market characteristics such as the empirical distribution of credit risk, the estimated credit-supply elasticities, or the externalities created by bank lending, which are central to both positive and normative analysis. Our paper provides the first rigorous attempt to derive a risk-weight schedule that maximizes the regulator's welfare objective.

4.3 Estimation

4.3.1 Credit-Risk Elasticity

To estimate $\varepsilon_p^{p,k}$, we identify the response of firm default probabilities to an exogenous shift in credit supply. This poses three major challenges.

First, observed credit quantities reflect equilibrium outcomes in the corporate loan market. Our goal is to isolate the credit-supply component determined by banks, as outlined in Section 2. We follow the approach

[12] See the explanatory note of the BCBS for details: `https://www.bis.org/bcbs/irbriskweight.pdf`.

of Khwaja and Mian (2008), estimating a bank–firm–quarter panel with firm-by-quarter fixed effects. These fixed effects absorb all borrower-specific variation, such as credit demand, profitability, alternative financing, and other firm-level determinants of lending. Identification therefore relies on firms that borrow from multiple banks in the same quarter. The administrative nature of our dataset, which covers all bank–firm relationships in Germany, provides a rich set of such multiple-bank observations.

Second, firm default probabilities are not directly observable. A large body of literature and a major segment of the financial industry are devoted to measuring this quantity as accurately as possible. In our data, we observe each bank's internal estimate of the probability of default, p_θ, which is used for regulatory capital calculation. These estimates combine hard and soft information, are back-tested by supervisors, and determine the capital charge. Banks thus have strong incentives to provide unbiased and accurate measures of credit risk.[13] Since all firms in our sample borrow from at least two banks, we observe multiple default-probability estimates per borrower, further improving measurement precision.

Third, even after isolating the credit-supply component, endogeneity concerns remain because bank lending decisions may still respond to unobserved firm or sector conditions. For credible identification, we require an exogenous shock to the firm-specific component of credit supply that is unrelated to borrower characteristics or systematic bank behavior. Crises or stress-test interventions are unsuitable for this purpose, as they typically target specific risk groups, sectors, or asset classes. We instead exploit the introduction of the German bank levy in 2011 (see Buch et al. (2016)). The levy was designed to internalize systemic risk in the banking sector and imposed a progressive charge based on bank size and involvement in derivatives, identified by German authorities as the two main systemic-risk factors. The policy's precise configuration was announced only shortly before its implementation, and the first contributions were assessed on 2010 balance sheets, making anticipatory adjustments highly unlikely. The levy effectively penalized balance-sheet size, shifting banks' credit-supply schedules inward across all borrowers. Because it depended solely on size, banks had no incentive to reallocate credit selectively across risk categories, industries, or borrower types, supporting our identification assumption.

The estimation of the credit-risk elasticity with respect to leverage proceeds in two stages. In the first stage, we implement a shift–share instrumental variable design based on pre-existing bank–firm relationships, which are highly persistent in Germany (see Huber (2021)). We combine this design with the Khwaja and Mian (2008) methodology to estimate the effect of the introduction of the bank levy on bank–firm-specific credit supply. In the second stage, we aggregate the predicted credit-supply shocks to the firm level and relate them to changes in the average default-risk estimates across the firm's lending banks:

[13] These models are calibrated at the portfolio level rather than the firm level. As a result, two banks can assign different, yet unbiased, default-probability estimates to the same borrower. This feature is central to our identification strategy.

	Homogeneous (1)	Risk Category 1 (2)	Risk Category 2 (3)	Risk Category 3 (4)	Risk Category 4 (5)
$\log(\widehat{credit}_{i,t-1})$	0.019***	0.019***	0.011***	0.023***	−0.024***
	(0.002)	(0.004)	(0.002)	(0.002)	(0.004)
Firm FE	Y	Y	Y	Y	Y
Time FE	Y	Y	Y	Y	Y
Obs.	280,171	7,334	30,392	213,491	26,528
R^2	0.693	0.733	0.536	0.632	0.635

Table 4: Estimated credit-risk elasticities (German credit registry).

$$\log(credit_{i,b,t}) = \gamma(BankLevy_{b,t-1} \times LevyIntroduction_t) + p_{i,b,t-1} + \beta X_{b,t-1} + \mu_{i,t} + \mu_b + v_{i,t}$$

and

$$p_{i,t} = \zeta_p^{p,k} \log(\widehat{credit}_{i,t-1}) + \beta X_{i,t-1} + \mu_i + \mu_t + u_{i,t}.$$

Here, $\log(credit_{i,b,t})$ denotes the logarithm of credit extended by bank b to firm i at quarter t. The firm–time fixed effect $\mu_{i,t}$ absorbs borrower-specific variation, while μ_b captures time-invariant bank characteristics. $BankLevy_{b,t-1}$ measures the levy charged to bank b in year t, based on its balance-sheet composition in year $t-1$, and $LevyIntroduction_t$ is a dummy variable equal to one for quarters from 2011Q4 onward. $p_{i,b,t}$ is bank b's estimate of firm i's probability of default, and $p_{i,t}$ is the average of these estimates across all banks lending to firm i in quarter t.

In the first stage, the vector $X_{b,t-1}$ includes lagged bank-level controls: the logarithm of total assets, the equity-to-assets ratio, the deposits-to-assets ratio, and the ratio of non-performing loans to total loans. In the second stage, $X_{i,t-1}$ collects lagged firm-level controls: the logarithm of total assets, sales, and net income, as well as the cash-to-assets ratio and leverage. The coefficient $\zeta_p^{p,k}$ measures the semi-elasticity of credit risk with respect to credit supply. The first-stage results confirm that the instrument is strongly correlated with bank credit supply, and the first-stage F-statistic ($F > 30$) rules out weak-instrument concerns.

The first column of Table 4 reports the estimates under the assumption of a homogeneous credit-risk elasticity across firms. We obtain a semi-elasticity of default risk with respect to credit supply of 0.019. This implies that a 10% increase in credit supply raises the probability of default by approximately 0.2 percentage points. Given the low baseline levels of firms' default probabilities (see Table 2), the effect is both statistically and economically significant. The estimate is consistent with prior evidence highlighting leverage as a key predictor of firm default (see Traczynski (2017), Campbell et al. (2008), and Cathcart et al. (2020)).

A positive elasticity of credit risk with respect to credit supply is incompatible with the predictions of the adverse-selection framework of Stiglitz and Weiss (1981). In this setting, an increase in credit supply lowers equilibrium interest rates and expands the borrower pool to include safer projects. As a result, the average default risk in borrowers' portfolios declines, contrary to our empirical finding. By contrast, a positive elasticity aligns with the moral-hazard framework of Boyd and De Nicolo (2005). In that model, greater credit supply increases leverage, reduces borrowers' "skin in the game," and thereby amplifies risk taking under limited liability. The costly-state-verification model of Gale and Hellwig (1985) offers a similar implication. Under monopolistic contracting, the bank operates at the borrower's participation constraint: a larger loan allows greater surplus extraction through a higher repayment obligation, which raises the bankruptcy threshold and increases default risk. Hence, both the moral-hazard and costly-state-verification frameworks can rationalize the positive credit-risk elasticity we estimate.

Heterogeneity. Columns (2) to (5) of Table 4 report heterogeneous credit-risk elasticities across firm-risk categories. We define four risk groups corresponding to the rating buckets used in the standardized approach (see Table 3): (1) AAA to AA$-$, (2) A$+$ to A$-$, (3) BBB$+$ to BB$-$, and (4) below BB$-$. We map these categories into probabilities of default using the S&P Global Fixed Income Tables (S&P Global (2024)).

For risk categories 1–3, the estimated elasticities are comparable in sign, magnitude, and statistical significance. By contrast, for risk category 4 we find a negative and statistically significant elasticity, suggesting that these firms are credit-constrained and improve their financial position in response to an outward shift in credit supply. Within the context of our benchmark models, this negative elasticity for high-risk borrowers aligns with the predictions of the adverse-selection framework of Stiglitz and Weiss (1981). This interpretation is natural, as adverse-selection frictions are likely to be most severe among the riskiest firms, where high risk premia exacerbate selection effects.

4.3.2 Credit-Supply Elasticities

The elasticities $\varepsilon_p^{k,p}$ and $\zeta_p^{k,\omega}$ measure the response of bank credit supply to changes in firm default risk and regulatory risk weights, respectively. Cleanly identifying these parameters poses several empirical challenges.

As before, isolating bank decisions from equilibrium credit-market outcomes is crucial. We address this issue using the Khwaja and Mian (2008) approach, which the administrative nature of our dataset makes feasible. A more fundamental challenge is that default probabilities p are themselves endogenous to lending outcomes k, as documented above. In equilibrium, p and k are jointly determined, making it difficult to separately identify the effect of p on k while also estimating the effect of k on p, as in the previous subsection. To overcome this simultaneity problem, we exploit the fact that, under the Khwaja and Mian (2008) framework, each firm in our sample borrows from at least two banks that independently report internal estimates of the probability of default. Banks have strong incentives, both for internal risk management and supervisory compliance, to provide unbiased and accurate estimates. Hence, small discrepancies in $p_{i,b}$ across banks for the same borrower reflect model uncertainty rather than true differences in credit risk (see Fraisse et al. (2020)). These differences arise because banks must estimate a single credit-risk model for their entire loan portfolio and cannot fine-tune predictions to individual firms. Crucially, banks do not observe other banks' p estimates and thus adjust their credit supply solely based on their own internal risk assessment, which we observe.

We therefore exploit cross-bank variation in $p_{i,b,t}$ for the same firm i to identify the elasticity of credit supply with respect to perceived borrower risk, $\varepsilon_p^{k,p}$. Conversely, shift–share variation in k, unrelated to firm-specific p, identifies the reverse elasticity $\varepsilon_p^{p,k}$ estimated in the previous subsection. This dual-source identification strategy allows us to disentangle supply-side responses from jointly determined equilibrium outcomes in the credit market.

We now turn to the identification of the elasticity of credit supply with respect to risk weights. As discussed in Section 4.2, regulatory risk weights are a deterministic function of default-risk estimates p. If this mapping were linear, it would be infeasible to estimate a linear specification that includes both p and the corresponding risk weight $\omega(p)$ as explanatory variables, since the two would be perfectly collinear. At the same time, joint estimation is essential given the tight economic link between the two variables, which necessitates partialing out individual effects.

However, the mapping from p to $\omega(p)$ is highly nonlinear under the Basel framework. Consequently, within-firm, across-bank variation in p translates into heterogeneous variation in $\omega(p)$, depending on the level of p. This nonlinearity generates identifying variation in risk weights: a given change in p leads to different changes in $\omega(p)$ across the risk distribution, implying different regulatory equity costs. The shape of the mapping function, calibrated in the Basel Accord, is fixed and exogenous to individual bank–firm relationships. Since the underlying variation in p stems from model uncertainty across banks and is unrelated to firm or bank fundamentals, the identification of the credit-supply elasticity with respect to risk weights exploits exogenous, nonlinear differences in regulatory equity requirements.

Formally, we estimate:

$$\log(credit_{b,i,t}) = \varepsilon_p^{k,p} \log(p_{i,b,t-1}) + \zeta_p^{k,\omega} \omega_{i,b,t-1} + X_{b,t-1} + \mu_b + \mu_{i,t} + u_{i,t}, \tag{18}$$

where $\log(credit_{b,i,t})$ denotes the logarithm of the credit amount extended by bank b to firm i at time t. The vector $X_{b,t-1}$ includes lagged bank-level controls: the logarithm of total assets, the equity-to-assets ratio, the deposits-to-assets ratio, and the ratio of non-performing loans to total loans. μ_b and $\mu_{i,t}$ denote

	Homogeneous (1)	Risk Category 1 (2)	Risk Category 2 (3)	Risk Category 3 (4)	Risk Category 4 (5)
$\log(p_{i,b,t-1})$	-0.063^{***}	-0.006	0.045	-0.049^{**}	-0.129^{***}
	(0.017)	(0.091)	(0.050)	(0.018)	(0.027)
$\omega_{i,b,t-1}$	-0.759^{***}	-1.289^{***}	-1.430^{***}	-0.732^{***}	-0.271^{***}
	(0.124)	(0.370)	(0.280)	(0.121)	(0.052)
Bank FE	Y	Y	Y	Y	Y
Borrower x Time FE	Y	Y	Y	Y	Y
Obs.	1,616,772	82,962	213,888	909,427	62,196
R^2	0.677	0.605	0.646	0.692	0.719

Table 5: Estimated credit-supply elasticities (German credit registry).

bank and firm–time fixed effects, respectively, the latter absorbing firm-specific demand shocks as in the specifications above.

The first column of Table 5 reports the estimates for the homogeneous credit-supply response. We find a semi-elasticity of credit supply with respect to the average risk weight of -0.76. That is, a 10 percentage point increase in the risk weight reduces credit supply by about 7.6%. Comparing this estimate to the range of -0.23 to -0.45 obtained from French administrative data by Fraisse et al. (2020) suggests that German banks exhibit somewhat greater sensitivity to regulatory capital costs.[14] The negative and economically sizable coefficient provides clear evidence for a marginal-cost channel of capital in banks' credit-supply decisions, as predicted by most of our benchmark models. By contrast, the alternative hypothesis of scarce bank capital, as in Holmstrom and Tirole (1997), appears inconsistent with our estimate.

The credit-supply elasticity with respect to default risk is estimated at -0.063. That is, a 1% increase in the borrower's probability of default reduces credit supply by only 0.06%. This small magnitude has two explanations. First, most default probabilities in the sample are low, implying that a 1% change is economically negligible and should elicit only a minor supply response. Second, the elasticity is identified jointly with risk weights that are functionally linked to default risk; hence, the estimate holds the regulatory capital cost constant. If banks can charge higher interest rates to resolve their risk-return trade-off, the quantity decision might be only marginally affected. Overall, the modest relationship between borrower default risk and credit supply aligns with prior evidence: Pool et al. (2015) document weak aggregate sensitivity of credit to risk, and Jiménez et al. (2017) show that, conditional on capital requirements, borrower default risk has little explanatory power for loan-level credit supply.

Qualitatively interpreting this empirical finding in light of our benchmark models provides only limited

[14]Another relevant comparison comes from studies that estimate credit-supply responses to changes in the capital-adequacy ratio Ω. A 10 percentage point increase in the risk weight, at an average capitalization level of 10%, corresponds to a 1 percentage point change in Ω. Both Bridges et al. (2014) and Jiménez et al. (2017) would predict a reduction in corporate credit of roughly 4% for such a change in capital requirements.

scope for differentiation among them. Most frameworks yield ambiguous predictions regarding the sign of the risk channel. Only two cases imply an unambiguously negative relationship between credit supply and borrower risk: the model of Kim and Santomero (1988) and the competitive-pricing environment of Stiglitz and Weiss (1981). Consistent with the data, these models predict that when the slope of the risk-weight function is strictly positive, higher borrower risk leads to lower credit supply.

As an additional test, we augment Specification (18) by including the marginal risk weight $\omega'_{i,b,t-1}$ to capture potential nonlinear responses to the slope of the risk-weight schedule. The estimated coefficient on $\omega'_{i,b,t-1}$ is statistically indistinguishable from zero and leaves the estimated elasticities with respect to default risk and risk weights virtually unchanged. This finding suggests that banks operate in a largely competitive environment, where marginal risk-weight adjustments do not materially affect lending margins.

Heterogeneity. Columns (2) to (5) of Table 5 report heterogeneous credit-supply elasticities by borrower risk category, defined as in the previous section. The sensitivity of credit supply to borrower default risk is particularly pronounced among riskier firms. This pattern is consistent with a scale effect: a 1% increase in default risk is economically less relevant for low-risk borrowers (categories 1 and 2) than for high-risk ones (categories 3 and 4). The sensitivity to risk weights also displays pronounced heterogeneity. While the elasticity lies between -1.3 and -1.4 for low-risk borrowers, it declines to -0.7 for risk category 3 and -0.27 for the riskiest firms. When interpreting these estimates, it is important to note that we directly control for the level of firm risk using each bank's own estimate $p_{i,b,t}$. Hence, the variation in risk weights partly reflects the nonlinearity of the mapping function $\omega(p)$. Given the strong concavity of this function, banks are expected to respond more strongly to risk-weight changes at the lower end of the risk distribution, where $\omega(p)$ is steep, than at higher p levels, where the schedule flattens out.[15]

Alternative estimation. As an alternative strategy to identify the credit-supply elasticity with respect to risk weights, we exploit the implementation of the SME Supporting Factor as a reform-based instrument. The SME Supporting Factor modifies the calculation of risk weights for firms with annual turnover below €50 million. After banks estimate p and map it into a regulatory risk weight using the Basel function, the resulting weight is multiplied by either 0.7619 or 0.85, with smaller loans receiving stronger relief. The reform followed a long, well-announced timeline with an implementation initially scheduled for 2021Q3. However, when the European economy was hit by the COVID-19 shock, the EU introduced a "quick fix" advancing the effective date to 2020Q3. This abrupt policy change was entirely unanticipated, leaving firms

[15] To illustrate, suppose Bank A estimates $p_{i,A,t} = 0.01$ and Bank B $p_{i,B,t} = 0.02$ for the same borrower. Because we control for $p_{i,b,t}$ in the regression, these level differences do not affect the estimate of $\zeta_p^{k,\omega}$. However, given the steepness of $\omega(p)$ at low p, a small change in p (e.g., an increase of 0.001) raises the capital cost of the loan more strongly at $p = 0.01$ than at $p = 0.02$. Consequently, banks are more sensitive to risk-weight changes induced by variation in p—the second moment—at the lower end of the risk distribution.

insufficient time to adjust reported turnover relative to the €50 million threshold. Because the supporting factor scales risk weights independently of p, it introduces exogenous variation in ω unrelated to borrower risk. We exploit this quasi-experimental shock to identify the elasticity of credit supply with respect to risk weights.

To estimate the elasticity, we implement a two-stage instrumental-variable design. We first restrict the sample to firms with annual turnover between €25 million and €75 million, creating a set of comparable borrowers around the €50 million threshold.[16]

In the first stage, we predict the post-reform risk weight applied to each borrower (from 2020Q3 onward) using the pre-reform risk weight in 2020Q2, interacted with an indicator variable equal to one if the firm's turnover falls below €50 million. This specification captures the mechanical adjustment introduced by the SME Supporting Factor, which reduced risk weights only for firms below the threshold. The instrument, therefore, exploits policy-driven variation in regulatory risk weights that is independent of borrower fundamentals and banks' internal models.

In the second stage, we re-estimate Specification (18) using the instrumented risk weights within this restricted subsample.[17] The first-stage results confirm a strong correlation between the instrument and actual risk weights, with an F-statistic exceeding 30, ruling out weak-instrument concerns.

We obtain a highly statistically significant elasticity with respect to risk weights of -0.389. Quantitatively, this estimate is slightly smaller in magnitude than that from the baseline specification but remains economically meaningful and consistent with previous findings in the literature. Overall, both the reform-based identification using the SME Supporting Factor and the broader estimation strategy yield qualitatively similar conclusions: higher regulatory risk weights significantly reduce bank credit supply.

4.4 Entrepreneurial Profit Responses

We estimate the entrepreneurial externality from credit supply, ξ_p^E, by measuring the sensitivity of firm profits to changes in credit supply. Empirically, we employ the same identification strategy as above for the elasticity of default risk with respect to credit supply. The first stage again uses the bank levy as a shift–share instrument for credit supply. The second stage is estimated as

$$\Delta \Pi_{i,t} = \gamma^E \, \Delta \log(\widehat{\text{credit}}_{i,t}) + \Delta X_{i,t} + \mu_i + \mu_t + u_{i,t},$$

where $\Pi_{i,t}$ denotes firm i's net income, $\Delta X_{i,t}$ is a vector of firm-level control variables, μ_i and μ_t are firm and time fixed effects, and $u_{i,t}$ is an error term.

[16] The results are not sensitive to the choice of bandwidth around the €50 million cutoff.

[17] Because the sample narrows substantially, we cannot estimate heterogeneity across firm-risk categories in this setting.

	Homogeneous (1)	Risk Category 1 (2)	Risk Category 2 (3)	Risk Category 3 (4)	Risk Category 4 (5)
$\Delta\log(\widehat{credit}_{i,t-1})$	55.494***	−13.088	21.638	63.723***	−1.107
	(19.618)	(169.987)	(71.759)	(23.495)	(77.822)
Firm FE	Y	Y	Y	Y	Y
Time FE	Y	Y	Y	Y	Y
Obs.	287,096	6,775	28,358	197,447	24,676
R^2	0.021	0.068	0.071	0.030	0.123

Table 6: Estimated effect of credit supply on firm net income (German credit registry).

As reported in Table 6, Column (1), we obtain a statistically and economically significant estimate of $\gamma^E = 55.5$, indicating that a 10% increase in credit supply raises firm net income by roughly €5,500. Given an average credit volume of €2.5 million, this corresponds to an increase of about two euro cents in net income for each additional euro of credit extended. This estimate is both economically meaningful and consistent with the notion that easier credit conditions relax financing constraints and enhance firm profitability.

This relationship is not particularly stable across the risk-category distribution, as shown in Columns (2) to (5) of Table 6. However, the estimate for risk category 3, representing roughly two-thirds of the firms in our sample, is very close in magnitude to the aggregate effect and remains highly statistically significant. A positive relationship between credit supply and firm profitability is consistent with the empirical literature on the real effects of credit supply (disruptions) (see Chava and Purnanandam (2011); Ongena et al. (2015); Degryse et al. (2019)). Mechanisms linking credit availability to profits include effects on employment (Chodorow-Reich, 2014), investment (Amiti and Weinstein, 2018), and export activity (Paravisini et al., 2015).

5 Quantitative Application: Basel Endgame

In this section, we apply our sufficient-statistics framework to evaluate, both positively and normatively, the recent "Basel Endgame" reform proposal by the U.S. bank regulatory agencies: the Federal Reserve, the Federal Deposit Insurance Corporation, and the Office of the Comptroller of the Currency. A central element of the proposal is a revision of the regulatory risk-weight schedule. The reform abolishes the Internal Ratings-Based Approach (IRBA) and replaces it with standardized risk weights. Specifically, exposures rated AAA–BBB are assigned a weight of 65%, while all other corporate exposures receive a weight of 100% (see Board of Governors of the Federal Reserve System (2023)).

The left panel of Figure 4 displays the baseline risk-weight schedule and the counterfactual risk weights

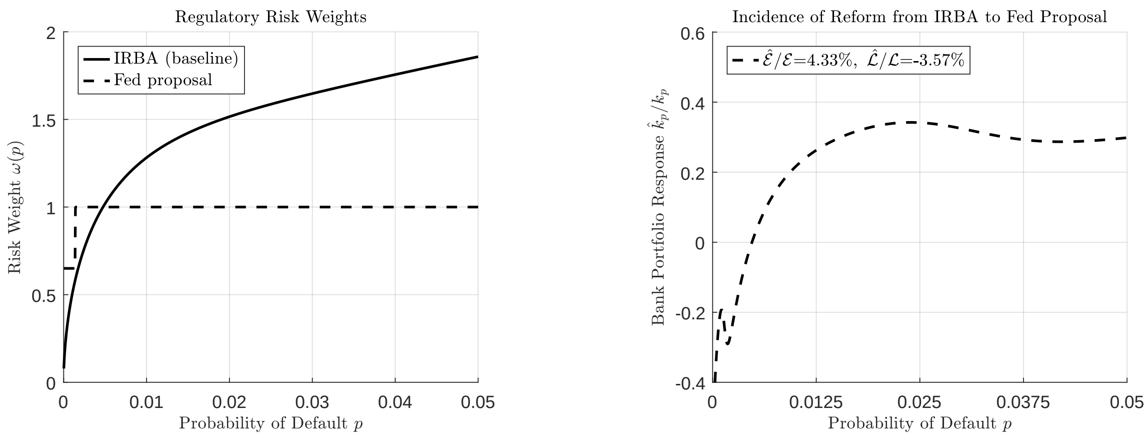

Figure 4: Left panel: risk-weight reform proposal. Right panel: loan-level responses.

under the Fed proposal. The reform increases risk weights for loans with low default probability and reduces them for high-risk exposures, resulting in a two-bracket risk-weight schedule that partially preserves the nonlinearity of the IRBA. Even though the main text focuses on the Fed proposal, our framework can flexibly accommodate any regulatory reform. For illustration, Appendix D.5 quantitatively evaluates the class of elementary risk-weight reforms.

5.1 Calibration

We map credit ratings into default probabilities using the S&P Global Fixed Income Tables (see S&P Global (2024) for the latest installment). To avoid discontinuous responses, we smooth the discrete jumps in the proposed risk-weight schedule, $\hat{\omega}$. This step also reflects current implementation practices, as a significant share of loans is extended to unrated firms, leading to an imperfect mapping between credit ratings and default risk. Under the Fed proposal, these unrated exposures are assigned a risk weight of 100%.

To capture cross-sectional heterogeneity in default probabilities and loan sizes, we estimate flexible skewed generalized t (SGT) distributions for both variables. We then map percentiles of the default-risk distribution to corresponding percentiles of the loan-size distribution to reproduce the empirically observed inverse relationship between credit risk and average loan size (see Figure 7). Behavioral responses are quantified using a smoothed version of the credit-supply and risk elasticities estimated for each risk group in the preceding section.

For the normative evaluation, we draw on our estimates of profit responses to changes in credit supply. We use the estimated semi-elasticity and divide it by the average level of credit supply to obtain a uniform measure of the marginal welfare externality on entrepreneurs, $d\mathcal{V}_p^E/dk_p$.

Evaluating welfare externalities on households requires structural assumptions about the underlying mechanism. Following the literature on bank regulation (Corbae and D'Erasmo (2021); Dávila and Walther (2021); Oehmke and Opp (2022), among others), we focus on financial externalities arising from government guarantees.[18] As detailed in Appendix B.3.2, financial intermediation is costly for households, as they finance deposit insurance or, equivalently, bailouts that guarantee deposit repayment in adverse states of the economy. Denote $\upsilon \in [\underline{\upsilon}, \overline{\upsilon}] \sim F(\upsilon)$ as a macroeconomic shock to the bank value. Depending on the shock realization, taxpayers cover the residual between deposit promises $\mathcal{D}$ and the state-contingent value of the bank $\Pi^B(\cdot, \upsilon)$. Letting $\kappa_{\mathcal{T}} = 25\%$ denote the deadweight cost of deposit insurance, the total fiscal externality is

$$-(1+\kappa_{\mathcal{T}})\mathcal{T}, \quad \text{where} \quad \mathcal{T} \equiv \max\{0, \mathcal{D} - \Pi^B(\cdot, \upsilon)\}.$$

The state-contingent going concern value of the bank consists of total loan repayments (loan rate $r = 4.68\%$; see Corbae and D'Erasmo (2021)), recovered loans in default (recovery rate $\phi = 0.6030$; see Corbae and D'Erasmo (2021)), and the post-liquidation value of the loan portfolio (depreciation rate $\iota = 0.1965$; see Corbae and D'Erasmo (2021)):

$$\Pi^B(\cdot, \upsilon) = \int_\Theta \left[\upsilon(1+r)(1-p_\theta) + (1-\phi)p_\theta - \iota\right] k_\theta \, d\theta.$$

This structure allows us to recover the monetary cost of deposit insurance and, in turn, compute the marginal welfare externalities on households, $d\mathcal{V}^H/dk_p$ and $d\mathcal{V}^H/d\mathcal{E}$. Using the observed credit distribution, we can further identify the threshold shock $\upsilon^\star$ that triggers deposit insurance, i.e., the bailout cutoff. As a proxy for aggregate shocks to loan repayments, we use quarterly DAX stock index returns from 2005Q1 to 2022Q4, assuming $\upsilon \overset{d}{=} 1 + R_{DAX}$.[19] Finally, we calibrate the shadow cost of bank equity such that the current capital adequacy ratio is optimal ($\mu = 0.48\%$).

Figure 8 depicts the estimated marginal welfare externalities from credit supply. A one-dollar reduction in credit supply lowers deposit insurance costs by approximately 2¢ (growing in credit risk) but reduces entrepreneurial profits by 1.5¢. In contrast, cutting bank equity by one dollar lowers household welfare by about 6¢. Altogether, the model provides the classic regulatory trade-off between productivity-enhancing credit supply and fiscal externalities from government guarantees, leading to excessive risk-taking in the banking sector. Capital regulation is therefore designed to align private with social costs and benefits of financial intermediation.

[18] Related frameworks modeling deposit insurance or bailouts as sources of financial externalities include Farhi and Tirole (2012), Bianchi (2016), Chari and Kehoe (2016), Keister (2016), Cordella et al. (2018), Dávila and Walther (2020), and Dovis and Kirpalani (2020).

[19] We obtain a cutoff shock of $\upsilon^\star = 0.9038$, an average bailout shock of $\mathbb{E}(\upsilon \mid \upsilon \leq \upsilon^\star) = 0.8454$, and a bailout probability of $F(\upsilon^\star) = 0.05$.

5.2 Positive Effects

The right panel of Figure 4 depicts the loan-level responses to the Fed's Basel Endgame proposal. Consistent with the direction of risk-weight changes, the distribution of credit responses is markedly nonlinear: credit supply contracts for loans with low default risk (-20% to -40%) and expands for loans with higher risk (+20% to +40%). Despite substantial reallocation along the risk distribution, aggregate bank credit supply decreases only modestly, by 3.57%. At the same time, bank equity rises by 4.33%. This increase is intuitive, given the high share of loans with very low default probabilities. These exposures initially carry near-zero risk weights, whereas the Fed proposal assigns them a weight of 65%. This mechanically raises required equity and, at the same time, attenuates the negative credit-supply responses at the bottom of the risk distribution. As a result, inframarginal and behavioral effects operate in the same direction, explaining roughly 25% and 75% of the total equity response, respectively.

In line with the proposal's stated objective (Board of Governors of the Federal Reserve System (2023)), banks become safer: the simulated bailout probability declines by 1.4 percentage points. To assess the strength of this effect, we compute how much the capital adequacy ratio would need to adjust downward to raise the bailout probability back to its intial level. For this we calculate a counteracting lump-sum perturbation preserving the nonlinear shape of the credit responses. Using equation (20), we obtain $\hat{\omega}_c = -0.08$. Hence, keeping the bailout probability constant after the Fed reform would require reducing the capital adequacy ratio to $\Omega = 0.07$. Conversely, the regulator would need to raise the capital adequacy ratio by 8% to achieve the same decline in bailout risk as implied by the Fed proposal (i.e., from 8% to 8.64%).

Simulations based on the U.S. sample. Having estimated the elasticities and distributional moments in the syndicated loan market, we can also evaluate the Fed proposal using these U.S.-based estimates. The left panel of Figure 11 shows the response of the bank credit portfolio representative of the syndicated loan market. Once again, we observe a pronounced reallocation from low- to high-risk loans. Overall credit supply declines moderately by 1.18%. However, due to the reform's mechanical effects on the regulatory constraint, bank equity rises by 5.48%. As a result, the bailout probability declines by one percentage point. Notice that these numbers are economically comparable to the German simulation highlighting the robustness of our approach.

Bank-size heterogeneity. to be added

5.3 Normative Effects

As a next step, we employ our estimates of marginal welfare externalities to evaluate the regulatory counterfactual normatively. The left panel of Figure 10 depicts each agent's money-metric welfare response along the credit-risk distribution. Because equity is costly, the Fed's proposal reduces bank welfare; however, the decline is small.

Entrepreneurs, on average, experience welfare losses due to reductions in credit supply induced by higher risk weights. In line with the nonlinear reallocation of credit, the welfare gradient is also highly nonlinear: low-risk firms experience substantial welfare losses, whereas high-risk firms benefit from easier access to credit. By contrast, the reform is welfare-improving for households. As shown in Figure 10, most of the welfare gains originate from the bottom of the risk distribution: greater bank equity and thus lower bank leverage reduce the fiscal cost of deposit guarantees.

The total welfare effect depends on the social welfare function. Following the banking literature, we focus on a utilitarian welfare objective ($\alpha^H = \alpha^B = \alpha^E_\theta, \forall\theta$). Under this criterion, the Fed proposal increases aggregate welfare: the welfare gains for households outweigh the losses incurred by banks and entrepreneurs.[20]

5.4 Optimal Nonlinear Risk Weights

We now illustrate the optimal nonlinear risk-weight schedule quantitatively, as derived in Propositions 2 and 3. While we estimate uniform marginal welfare externalities on entrepreneurs, the welfare externalities on households increase with credit risk (see Figure 8). As a result, the unconstrained and constrained Pareto optima differ, reflecting the linear dependence of policy instruments in the second-best environment.

In the left panel of Figure 5, we display the optimal risk-weight schedules. Optimal risk weights are slightly increasing with credit risk, with the constrained Pareto optimum being slightly flatter than the unconstrained one.

The near-linearity of the optimal schedule reflects the homogeneity of entrepreneurial profit responses and the residual value accounting by deposit insurance in bankruptcy. This residual value corresponds to the sum of repayments on the bank's credit portfolio. Since expected loan repayments decline with credit risk, high-risk loans are more costly to insure, but this effect is small. What matters most for expected deposit-insurance costs is the overall size of the lending portfolio rather than its composition. As a result, the marginal social cost of credit rises roughly proportionally with default risk, yielding an optimal schedule that is close to linear and relatively flat.

Altogether, both optimal schedules are strikingly close to the risk weights proposed by the Fed. Con-

[20] The welfare simulations based on the U.S. estimates deliver very similar predictions (see the right panel of Figure 11).

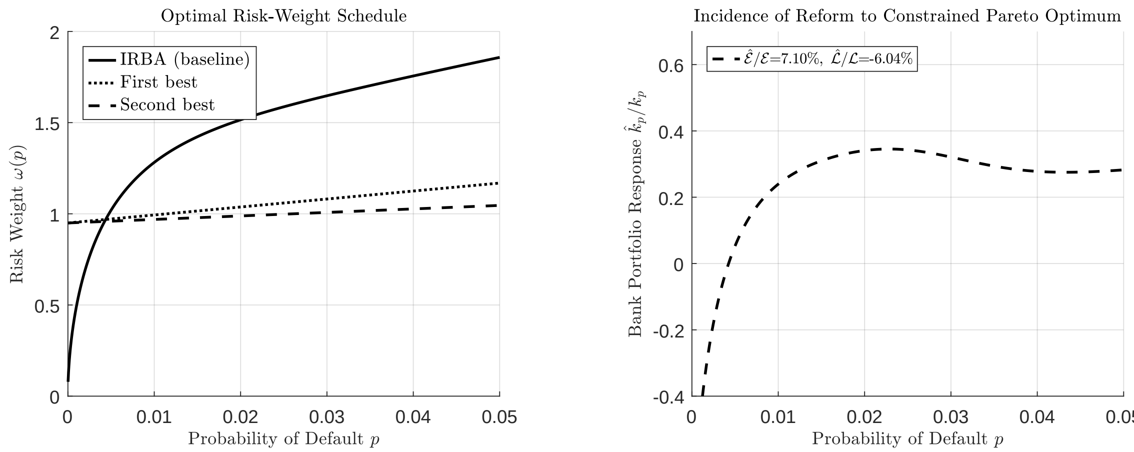

Figure 5: Optimal nonlinear risk weights.

Note: We parameterize a utilitarian social welfare function: $\alpha^H = \alpha^B = \alpha^E_\theta, \forall\theta$.

sequently, the responses of the credit portfolio and bank-level aggregates, depicted in the right panel of Figure 5, are comparable in direction and magnitude to those observed under the Fed proposal. Moreover, the welfare effects of moving to the (constrained) Pigouvian optimum are similar, as shown in the right panel of Figure 10. Both reforms increase household welfare at the expense of entrepreneurs and banks. We therefore conclude that the risk-weight schedule proposed by the Fed is close to optimal.

Optimal bank-specific risk weights. to be added

6 Extensions

Non-smooth risk-weight schedule. to be added

Large reforms. to be added

Multidimensional heterogeneity. to be added

Dynamic setting. to be added

7 Conclusion

This paper introduces the perturbation approach to the analysis of risk-sensitive bank capital regulation. We express both the positive and normative effects of nonlinear regulation through a small set of sufficient statistics, such as credit-supply elasticities and marginal welfare externalities, thereby unifying a broad class of banking models. The sufficient statistics we derive are informative about the nature of credit-market frictions and the degree of competition.

We estimate these statistics using administrative data and quantitatively evaluate reforms of nonlinear risk weights in bank capital requirements, focusing on the Fed's proposal for coarser risk weights. Despite a moderate impact on aggregate credit supply, the reform generates pronounced nonlinear effects across the firm distribution and increases bank equity, thereby improving financial stability.

Furthermore, we derive new sufficient-statistics representations of optimal nonlinear risk weights. Numerical simulations of the optimum reveal close similarities to the Fed's proposal. We expect the perturbation approach to be applicable beyond capital regulation, for instance by analyzing other dimensions of bank regulation (e.g., liquidity requirements) or alternative credit-market structures (e.g., shadow banking).

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A Appendix for Section 2

A.1 General Framework

This appendix provides the derivations underlying the general framework. We formalize the bank's optimization, derive the corresponding first-order conditions, and show how they give rise to the sufficient statistics introduced in Section 2.1.

Denote the choices by entrepreneurs as $d_\theta = d(k_\theta, z_\theta; \theta)$. Denoting the vector of Lagrange multipliers on the respective constraints as ρ_θ, optimal choices satisfy the following first-order conditions:

$$\partial_{d_\theta} U^E_\theta(d_\theta; k_\theta, z_\theta) + \rho^T_\theta\, \partial_{d_\theta} \mathcal{C}^E_\theta(d_\theta; k_\theta, z_\theta) = 0, \quad \rho^T_\theta \mathcal{C}^E_\theta(d_\theta; k_\theta, z_\theta) = 0.$$

Similarly, defining the vector of Lagrange multipliers for the depositor constraints as γ, the optimal depositor choices $c = c(x)$ are characterized by:

$$\partial_c U^H(c; x) + \gamma^T \partial_c \mathcal{C}^H(c; x) = 0, \quad \gamma^T \mathcal{C}^H(c; x) = 0.$$

Inner problem. Let λ denote the vector of multipliers on the feasibility constraints and η_θ the multipliers on the risk-formation constraints. The inner Lagrangian is

$$L^{\text{inner}} = U^B(k, \mathcal{E}, z, p) - \lambda^T\, \mathcal{C}(k, \mathcal{E}, z, p; c, d) - \int_\Theta \eta_\theta\, \mathcal{P}(p_\theta, k_\theta, z_\theta; d_\theta, \theta)\, d\theta.$$

First-order conditions define the optimal operational choices $z(k, \mathcal{E}, p)$:

$$U^B_{z_\theta}(k, \mathcal{E}, z, p) = \lambda^T\, \mathcal{C}_{z_\theta}(k, \mathcal{E}, z, p; c, d) + \eta_\theta\, \mathcal{P}_{z_\theta}(p_\theta, k_\theta, z_\theta; d_\theta, \theta),$$

together with $\lambda^T\, \mathcal{C}(k, \mathcal{E}, z, p; c, d) = 0$ and $\mathcal{P}(p_\theta, k_\theta, z_\theta; d_\theta, \theta) = 0$.

Outer problem. Let μ denote the multiplier on the regulatory constraint. All multipliers in the outer problem are evaluated at the inner optimum $z^\star = z(k, \mathcal{E}, p)$ delivered by the inner problem; for simplicity we retain the same symbols (λ, η). The outer Lagrangian is

$$L = U^B(k, \mathcal{E}, z, p) - \lambda^T\, \mathcal{C}(k, \mathcal{E}, z, p; c, d) - \int_\Theta \eta_\theta\, \mathcal{P}(p_\theta, k_\theta, z_\theta; d_\theta, \theta)\, d\theta - \mu\left(\int_\Theta \omega(p_\theta) k_\theta\, d\theta - \mathcal{E}/\Omega\right).$$

The first-order conditions for equity, credit supply, and risk are

$$U^B_{\mathcal{E}}(k, \mathcal{E}, z, p) - \lambda^T\, \mathcal{C}_{\mathcal{E}}(k, \mathcal{E}, z, p; c, d) = -\mu/\Omega,$$

$$U^B_{p_\theta}(k,\mathcal{E},z,p) - \lambda^T\,\mathcal{C}_{p_\theta}(k,\mathcal{E},z,p;c,d) = \eta_\theta \mathcal{P}_{p_\theta}(p_\theta,k_\theta,z_\theta;d_\theta,\theta) + \mu\,\omega'(p_\theta)k_\theta, \quad \forall\theta,$$

$$U^B_{k_\theta}(k,\mathcal{E},z,p) - \lambda^T\,\mathcal{C}_{k_\theta}(k,\mathcal{E},z,p;c,d) = \mu\,\omega(p_\theta) + \eta_\theta \mathcal{P}_{k_\theta}(p_\theta,k_\theta,z_\theta;d_\theta,\theta), \quad \forall\theta,$$

together with the complementary slackness condition $\mu(\int_\Theta \omega(p_\theta)k_\theta\,d\theta - \mathcal{E}/\Omega) = 0$.

Optimality conditions and elasticities. Combining these expressions yields the marginal rate of substitution between credit supply and equity under a binding capital requirement:

$$MRS_\theta \equiv \frac{U^B_{k_\theta}(\cdot) - \lambda^T\,\mathcal{C}_{k_\theta}(\cdot) + \left[U^B_{p_\theta}(\cdot) - \lambda^T\,\mathcal{C}_{p_\theta}(\cdot)\right]\mathcal{P}_{k_\theta}(\cdot)/\mathcal{P}_{p_\theta}(\cdot)}{U^B_{\mathcal{E}}(\cdot) - \lambda^T\,\mathcal{C}_{\mathcal{E}}(\cdot)} = -\Omega\left[\omega(p_\theta) + \omega'(p_\theta)p_\theta\mathcal{P}^{p,k}_\theta\right], \quad \forall\theta,$$

where $\mathcal{P}^{p,k}_\theta = -[\partial\mathcal{P}(\cdot;d_\theta,\theta)/\partial\log k_\theta]/[\partial\mathcal{P}(\cdot;d_\theta,\theta)/\partial\log p_\theta]$ captures the internalization of risk responses by the bank.

The loan-level elasticities with respect to risk weights defined in the main text follow as

$$\zeta^{k,\omega}_\theta = -\Omega\left/\frac{\partial MRS_\theta}{\partial\log k_\theta}\right., \qquad \zeta^{k,\omega'}_\theta = -\Omega p_\theta\mathcal{P}^{p,k}_\theta\left/\frac{\partial MRS_\theta}{\partial\log k_\theta}\right..$$

Similarly, the elasticity of bank lending with respect to credit risk can be computed from

$$\zeta^{k,p}_\theta = -\left(\Omega\frac{\partial[\omega(p_\theta) + \omega'(p_\theta)p_\theta\mathcal{P}^{p,k}_\theta]}{\partial\log p_\theta} + \frac{\partial MRS_\theta}{\partial\log p_\theta}\right)\left/\frac{\partial MRS_\theta}{\partial\log k_\theta}\right..$$

The elasticity of individual credit risk with respect to leverage follows from the risk-formation constraint. Absent direct effects from bank-level equity ($\partial_{\mathcal{E}} z_\theta(\cdot) = 0$), it can be expressed as:

$$\varepsilon^{p,k}_\theta = -\frac{k_\theta}{p_\theta}\frac{\mathcal{P}_{k_\theta}(\cdot) + \mathcal{P}_{z_\theta}(\cdot)\partial_{k_\theta}z_\theta(\cdot) + \mathcal{P}_{d_\theta}(\cdot)\partial_{k_\theta}d_\theta(\cdot)}{\mathcal{P}_{p_\theta}(\cdot)}.$$

A.2 Canonical Banking Models

A.2.1 Kim and Santomero (1988): Bank Portfolio Choice

The general framework nests Kim and Santomero (1988) as a special case. Denote by r_θ and σ_θ the expected return and the standard deviation of the risky asset θ the bank invests in, and assume that across-asset covariances are zero, as in the original model. Let $1 + r_{\mathcal{E}}$ and $1 + r_{\mathcal{D}}$ denote the interest rate on equity $\mathcal{E}$ and deposits $\mathcal{D}$, where equity is relatively costly $r_{\mathcal{E}} > r_{\mathcal{D}}$. The bank's balance-sheet constraint is $\mathcal{L} = \int_\Theta k_\theta\,d\theta = \mathcal{D} + \mathcal{E}$.

Banks choose their portfolio $\{k_\theta\}_{\theta\in\Theta}$ to maximize expected utility under a mean–variance trade-off:

$$U^B(k,\mathcal{E},p)=\int_\Theta r_\theta k_\theta\,d\theta-\frac{\gamma}{2}\int_\Theta \sigma_\theta^2 k_\theta^2\,d\theta-(1+r_\mathcal{D})\mathcal{L}-(r_\mathcal{E}-r_\mathcal{D})\mathcal{E},$$

subject to the capital-requirement constraint and the feasibility condition

$$k_\theta\geq 0,\quad \forall\theta.$$

Credit risk p_θ is summarized by a single exogenous risk factor that affects only the regulatory weights. There are no productive entrepreneurs receiving the bank investment or moral-hazard frictions, so risk is technologically fixed and $\varepsilon_\theta^{p,k}=0$.

Pointwise optimization over θ gives the bank's portfolio choice:

$$k_\theta=\max\left\{0,\ \frac{r_\theta-(1+r_\mathcal{D})-(r_\mathcal{E}-r_\mathcal{D})\Omega\omega(p_\theta)}{\gamma\sigma_\theta^2}\right\},\quad \forall\theta.$$

The bank equates the marginal risk-adjusted return to the cost of equity implied by the binding capital requirement. Since the regulatory risk weight $\omega(p_\theta)$ enters linearly, a higher level of weights acts as a tax on the corresponding assets and compresses credit supply proportionally.

At interior points ($k_\theta>0$), the portfolio elasticities with respect to the risk-weight parameters are

$$\zeta_\theta^{k,\omega}=-\frac{1}{k_\theta}\frac{\Omega(r_\mathcal{E}-r_\mathcal{D})}{\gamma\sigma_\theta^2}<0,\qquad \zeta_\theta^{k,\omega'}=0,\qquad \zeta_\theta^{k,p}=\omega'(p_\theta)p_\theta\zeta_\theta^{k,\omega}.$$

Hence, the Kim–Santomero model isolates the pure portfolio channel of capital regulation: risk weights alter the composition of assets without affecting risk-taking or default probabilities.

A.2.2 Boyd and De Nicolo (2005): Entrepreneurial Moral Hazard

Boyd and De Nicolo (2005) extend the model of Allen and Gale (2000) by allowing for entrepreneurial risk-taking on the loan market. Entrepreneurs choose their ex-ante level of credit risk p_θ under a classical risk–return trade-off for a given loan contract, while banks choose their lending, taking into account entrepreneurial moral hazard and its effect on loan pricing. We consider a generalized version with a continuum of entrepreneurs, each receiving a loan k_θ at interest rate r_θ.

In case of success, the project yields $(1+p_\theta)$ per unit of capital; in failure, output is zero and the entrepreneur is protected by limited liability. An entrepreneur of ability θ (as in Lucas, 1978) operates a production function $y_\theta(k_\theta)$ that exhibits decreasing returns to scale. Her expected utility is

$$U_\theta^E(p_\theta; k_\theta) = (1 - p_\theta)\big[(1 + p_\theta)y_\theta(k_\theta) - r_\theta k_\theta\big].$$

Optimal risk-taking satisfies the first-order condition

$$p_\theta = \frac{r_\theta k_\theta}{2y_\theta(k_\theta)}, \qquad \bar{\varepsilon}_\theta^{p,k} \equiv \frac{\partial \log p_\theta}{\partial \log k_\theta} = \frac{y_\theta(k_\theta) - y_\theta'(k_\theta)k_\theta}{y_\theta(k_\theta)} > 0,$$

so that risk-taking increases with leverage and the loan rate.

Denote by $1 + r_\mathcal{E}$ and $1 + r_\mathcal{D}$ the gross returns on equity and deposits, where $r_\mathcal{E} > r_\mathcal{D}$. Using the balance-sheet identity $\int_\Theta k_\theta \, d\theta = \mathcal{D} + \mathcal{E}$, the bank's objective and risk-formation constraints are

$$U^B(k, \mathcal{E}, p) = \int_\Theta (1 - p_\theta) r_\theta k_\theta \, d\theta - (1 + r_\mathcal{D}) \int_\Theta k_\theta \, d\theta - (r_\mathcal{E} - r_\mathcal{D})\mathcal{E},$$

$$\mathcal{P}(p_\theta, k_\theta, z; d_\theta, \theta) = p_\theta - \frac{r_\theta k_\theta}{2y_\theta(k_\theta)} = 0, \quad \forall\theta.$$

For transparency, suppose the deposit rate $r_\mathcal{D}$ is fixed. As in Boyd and De Nicolo (2005), an inverse demand function $r_\theta = r(k_\theta)$ determines the loan rate. Banks internalize the marginal effect of credit supply on pricing, $\varepsilon_\theta^{r,k} \equiv \partial \log r_\theta / \partial \log k_\theta \geq 0$, which generates monopolistic rents. The Lagrangian is given by

$$L = U^B(k, \mathcal{E}, p) - \mu \left(\int_\Theta \omega(p_\theta) k_\theta \, d\theta - \mathcal{E}/\Omega \right),$$

where μ is the multiplier on the regulatory constraint, and the bank's objective internalizes entrepreneurial risk-taking.

At an interior optimum (under standard regularity), optimal credit supply satisfies

$$MRS_\theta = \frac{(1 - p_\theta) r_\theta (1 + \varepsilon_\theta^{r,k}) - p_\theta r_\theta (\bar{\varepsilon}_\theta^{p,k} + \varepsilon_\theta^{r,k}) - (r_\mathcal{E} - r_\mathcal{D})}{-(r_\mathcal{E} - r_\mathcal{D})/\Omega} = -\Omega \big[\omega(p_\theta) + \omega'(p_\theta) p_\theta (\bar{\varepsilon}_\theta^{p,k} + \varepsilon_\theta^{r,k})\big],$$

where the left-hand side represents the marginal rate of substitution between credit supply and equity.

Monopolistic loan pricing. Under monopoly, a bank internalizes the full effect of loan pricing on risk-taking. The overall elasticity of default risk with respect to leverage combines price and quantity channels:

$$\varepsilon_\theta^{p,k} = \frac{d \log p_\theta}{d \log k_\theta} = \bar{\varepsilon}_\theta^{p,k} + \varepsilon_\theta^{r,k} > 0.$$

The first-order condition above implicitly defines k_θ. The corresponding credit-supply elasticities are

$$\zeta_\theta^{k,\omega} = -\frac{\Omega}{\frac{\partial MRS_\theta|_{p_\theta}}{\partial \log k_\theta} - \Omega\omega'(p_\theta)p_\theta \frac{\partial \varepsilon_\theta^{p,k}}{\partial \log k_\theta}} < 0, \qquad \zeta_\theta^{k,\omega'} = p_\theta \varepsilon_\theta^{p,k} \zeta_\theta^{k,\omega} < 0.$$

The elasticity of bank lending with respect to credit risk follows as

$$\zeta_\theta^{k,p} = \Big([\omega'(p_\theta)(1+\varepsilon_\theta^{p,k}) + \omega''(p_\theta)p_\theta\varepsilon_\theta^{p,k}]p_\theta + \Omega^{-1}(1+\bar{\varepsilon}_\theta^{p,k})p_\theta r_\theta \Big) \zeta_\theta^{k,\omega} \lesseqgtr 0.$$

Under monopolistic pricing, the level and slope of the risk-weight schedule affect credit supply both directly and through the induced response of entrepreneurial risk. A higher level of risk weights increases the cost of equity funding and compresses credit supply. A steeper schedule penalizes risky loans disproportionately, which further reduces credit supply and amplifies the risk-taking feedback. Consequently, capital requirements transmit more strongly in settings where banks possess market power and borrowers respond elastically to lending conditions.

Competitive loan pricing. Under perfect competition, a single bank's credit supply does not affect pricing, and loan rates satisfy the zero-profit condition

$$r_\theta = \min\{r \geq 0 \mid (1-p_\theta)r - (1+r_{\mathcal{D}}) - (r_{\mathcal{E}} - r_{\mathcal{D}})\Omega\omega(p_\theta) = 0\}.$$

The equilibrium risk elasticity coincides with the pure quantity channel, $\varepsilon_\theta^{p,k} = \bar{\varepsilon}_\theta^{p,k} > 0$. The corresponding marginal-rate-of-substitution condition simplifies to

$$\widetilde{MRS}_\theta = \frac{(1+r_{\mathcal{D}})\big(1 - p_\theta/[(1-p_\theta)\varepsilon_\theta^{p,k}]\big)}{-(r_{\mathcal{E}} - r_{\mathcal{D}})} = -\Omega[\omega(p_\theta)/(1-p_\theta) + \omega'(p_\theta)]p_\theta\varepsilon_\theta^{p,k}, \quad \forall\theta.$$

Hence, the credit-supply elasticities with respect to risk weights are

$$\zeta_\theta^{k,\omega} = -\frac{\Omega p_\theta/[(1-p_\theta)\varepsilon_\theta^{p,k}]}{\frac{\partial \widetilde{MRS}_\theta|_{p_\theta}}{\partial \log k_\theta} - \Omega[\omega(p_\theta)p_\theta/(1-p_\theta) + \omega'(p_\theta)p_\theta]\frac{\partial \varepsilon_\theta^{p,k}}{\partial \log k_\theta}} < 0, \qquad \zeta_\theta^{k,\omega'} = (1-p_\theta)\zeta_\theta^{k,\omega} < 0.$$

As in the monopolistic case, the elasticity of credit supply with respect to credit risk depends on the curvature of the risk-weight schedule:

$$\zeta_\theta^{k,p} = \Big[\frac{\omega(p_\theta)}{1-p_\theta} + \omega'(p_\theta) + \omega''(p_\theta)(1-p_\theta)p_\theta + (1-p_\theta)^{-1}\Omega^{-1}(r_{\mathcal{E}} - r_{\mathcal{D}})^{-1}\Big]\zeta_\theta^{k,\omega} \lesseqgtr 0.$$

Under competition, risk weights transmit primarily through balance-sheet costs: banks cannot adjust loan pricing to absorb tighter regulation, so changes in $\omega(p_\theta)$ directly shift the supply of funds. Since individual banks take prices as given, the feedback from entrepreneurial risk is muted compared to the monopolistic case. Nevertheless, both the level and slope of the risk-weight schedule influence equilibrium credit volumes through their impact on the effective cost of capital.

Welfare effects. Finally, applying the Envelope theorem, credit-supply externalities in both competitive and monopolistic environments consist of scale-induced changes in entrepreneurial profits and loan-price responses:

$$\frac{dU_\theta^E(p_\theta;k_\theta)}{dk_\theta} = (1-p_\theta)\Big[(1+p_\theta)y_\theta'(k_\theta) - r_\theta - k_\theta\frac{dr_\theta}{dk_\theta}\Big].$$

Under imperfect competition, the last term reflects the elasticity of the loan-demand curve $\varepsilon_\theta^{r,k}$, whereas under perfect competition the zero-profit condition directly links loan rates to capital requirements. The model thus captures how capital regulation interacts with market structure: monopolistic banks internalize pricing and risk feedback, magnifying policy effects, while competitive markets transmit them more mechanically through the balance-sheet channel.

A.2.3 Holmstrom and Tirole (1997): Double Moral Hazard

Following the notation in Freixas and Rochet (2008), projects require I units of funding and yield a verifiable return y in case of success and zero otherwise. Firms differ in initial wealth $A \sim G(A)$. There are two types of projects: good projects succeed with probability $1-p_\theta$, and bad projects succeed with probability $1-\bar{p}$, with $p_\theta < \bar{p}$ for all θ. Bad projects yield private benefits B to the firm when not monitored, while monitoring by the bank reduces these benefits to $b < B$. Banks monitor at cost c.[21] The risk-free rate is $1+r$, and the bank's gross return on loans is denoted by β_θ.

Borrower contracts and participation. Firms with sufficient capital can obtain direct funding from depositors, borrowing $I-A$ and promising a repayment R_θ^D. Borrower incentive compatibility requires

$$(1-p_\theta)(y-R_\theta^D) \;\geq\; (1-\bar{p})(y-R_\theta^D)+B \quad\iff\quad R_\theta^D \leq y - \frac{B}{\bar{p}-p_\theta}.$$

Depositor participation implies that only firms with sufficient own capital obtain direct finance:

$$(1-p_\theta)R_\theta^D \geq (1+r)(I-A) \quad\iff\quad A \geq \bar{A}_\theta(r) \equiv I - \frac{1-p_\theta}{1+r}\left(y - \frac{B}{\bar{p}-p_\theta}\right).$$

[21] Martinez-Miera and Repullo (2017) analyze a continuous-monitoring extension of Holmstrom and Tirole (1997).

Intermediated funding and bank monitoring. Entrepreneurs with lower wealth ($A < \bar{A}_\theta(r)$) require bank intermediation. They borrow I_θ^D from depositors and $I_\theta^B = I - I_\theta^D - A$ from the bank, promising a repayment $R_\theta^B = \beta_\theta I_\theta^B/(1-p_\theta)$.

Entrepreneurial incentive compatibility now requires

$$(1-p_\theta)(y - R_\theta^D - R_\theta^B) \geq (1-\bar{p})(y - R_\theta^D - R_\theta^B) + b,$$

which, using depositor participation, can be written as

$$(\bar{p} - p_\theta)\left[y - \frac{(1+r)(I - I_\theta^B - A)}{1-p_\theta} - R_\theta^B\right] \geq b.$$

Bank monitoring requires its own incentive constraint:

$$(1-p_\theta)R_\theta^B - c \geq (1-\bar{p})R_\theta^B,$$

which implies $c(1-p_\theta)/(\bar{p}-p_\theta) = \beta_\theta I_\theta^B$. Combining the two constraints yields the threshold for firm participation:

$$A \geq \underline{A}_\theta(\beta_\theta, r) \equiv I - \frac{1-p_\theta}{\bar{p}-p_\theta}\frac{c}{\beta_\theta} - \frac{1-p_\theta}{1+r}\left[y - \frac{b+c}{\bar{p}-p_\theta}\right].$$

Hence, moral hazard induces credit rationing: only firms with $A \in [\underline{A}_\theta, \bar{A}_\theta]$ receive intermediated credit, while poorer firms cannot commit to prudent behavior compatible with uninformed depositors' participation.

Entrepreneurial welfare. Individual welfare is therefore defined piecewise:

$$U_\theta^E(k, A) = (1+r)A + \mathbf{1}_{\{\underline{A}_\theta \leq A \leq \bar{A}_\theta\}}\big[(1-p_\theta)y - (1+r)(I - I_m(\beta_\theta))\big] + \mathbf{1}_{\{A \geq \bar{A}_\theta\}}\big[(1-p_\theta)y - (1+r)(I-A)\big],$$

where $I_m(\beta_\theta)$ denotes the bank-financed share of investment.

Bank optimization. The bank's profit function is

$$U^B(k, \mathcal{E}, p) = \int_\Theta k_\theta \pi_\theta^B(\beta_\theta)\, d\theta - (1+r)\mathcal{L} - (r_\mathcal{E} - r)\mathcal{E},$$

where we have plugged in the balance-sheet identity $\mathcal{L} = \int_\Theta k_\theta\, d\theta = \mathcal{D} + \mathcal{E}$, and defined the per-loan margin as

$$\pi_\theta^B(\beta_\theta) = \frac{(1-p_\theta)R_\theta^B - c}{I_\theta^B} = \beta_\theta \frac{1-\bar{p}}{1-p_\theta}.$$

Feasibility requires

$$k_\theta \le I_\theta^B \big[G(\bar{A}_\theta(r)) - G(\underline{A}_\theta(\beta_\theta, r))\big], \qquad \forall\theta.$$

Because default risk p_θ is a model primitive (following Holmstrom and Tirole (1997)), the credit-risk elasticity is zero:

$$\varepsilon_\theta^{p,k} = 0.$$

Two cases describe the transmission of capital regulation: fixed bank capital and competitive loan pricing.

Fixed bank capital. When informed capital is scarce, as in the benchmark case of Holmstrom and Tirole (1997), the bank's credit supply is determined by the available informed capital $k_\theta = \bar{k}_\theta$ and unaffected by regulation:

$$\zeta_\theta^{k,\omega} = 0, \quad \zeta_\theta^{k,\omega'} = 0, \quad \zeta_\theta^{k,p} = 0.$$

Banks earn a scarcity rent on capital, and the prudential constraint is slack.

Competitive loan pricing. When informed capital is in excess supply, the return rate β_θ adjusts to ensure zero profits on each loan:

$$\underline{\beta}_\theta = \min\{\beta_\theta \ge 0 \mid \pi_\theta^B(\beta_\theta) - (1+r) - (r_\mathcal{E} - r)\Omega\omega(p_\theta) = 0\}.$$

Since $\underline{\beta}_\theta$ rises with $\omega(p_\theta)$,

$$\frac{\partial \underline{\beta}_\theta}{\partial \omega(p_\theta)} = \frac{(r_\mathcal{E} - r)\Omega(1-p_\theta)}{1-\bar{p}} > 0,$$

higher risk weights increase the cost of intermediation and lower credit supply:

$$\zeta_\theta^{k,\omega} = -\frac{g(\underline{A}_\theta(\underline{\beta}_\theta, r))}{G(\bar{A}_\theta(r)) - G(\underline{A}_\theta(\underline{\beta}_\theta, r))} \frac{\partial \underline{\beta}_\theta}{\partial \omega(p_\theta)} < 0, \quad \zeta_\theta^{k,\omega'} = 0.$$

Because credit supply depends on borrower wealth, the relationship between credit and risk may be positive or negative:

$$\zeta_\theta^{k,p} = \frac{p_\theta}{1-p_\theta} + p_\theta \frac{\Big[g(\bar{A}_\theta(r)) - g(\underline{A}_\theta(\underline{\beta}_\theta, r))\Big]\Big[\tfrac{1}{1+r}\Big(y - \tfrac{B}{\bar{p}-p_\theta}\Big) + \tfrac{1-p_\theta}{1+r}\tfrac{B}{(\bar{p}-p_\theta)^2}\Big] + g(\underline{A}_\theta(\underline{\beta}, r))\tfrac{1-p_\theta}{1+r}\tfrac{B-(b+c)}{(\bar{p}-p_\theta)^2}}{G(\bar{A}_\theta(r)) - G(\underline{A}_\theta(\underline{\beta}_\theta, r))}$$
$$- p_\theta \frac{\partial \underline{\beta}_\theta}{\partial p_\theta}\left[1 + \frac{g(\underline{A}_\theta(\underline{\beta}_\theta, r))}{G(\bar{A}_\theta(r)) - G(\underline{A}_\theta(\underline{\beta}_\theta, r))}\right] \lesseqgtr 0$$

where

$$\frac{\partial \underline{\beta}_\theta}{\partial p_\theta} = (r_{\mathcal{E}} - r)\Omega\, \omega'(p_\theta)\frac{1-p_\theta}{1-\bar{p}} - \frac{\beta_\theta}{1-p_\theta} \lesseqgtr 0.$$

This reflects offsetting effects from leverage (intensive margin), borrower selection (extensive margin), and pricing adjustments.

Welfare effects. Finally, credit-supply externalities stem solely from loan pricing. There are no additional externalities from bank-level variables:

$$\frac{dU_\theta^E(k, A)}{dk_\theta} = \mathbf{1}_{\{\underline{A}_\theta \le A \le \bar{A}_\theta\}}(1+r)\frac{1}{k_\theta/I_\theta^B + g(\underline{A}_\theta(\underline{\beta}_\theta, r))I_\theta^B} > 0.$$

A.2.4 Stiglitz and Weiss (1981): Adverse Selection

Consider the adverse-selection model of Stiglitz and Weiss (1981), adopting the notation of Freixas and Rochet (2008). Entrepreneurs θ undertake risky projects that require I units of funding and yield a random return $y \ge 0$. Borrowers possess private information about project quality $q \sim H(q)$. Project quality affects the distribution of returns $F(y; q, \theta)$ via a mean-preserving spread. Collateral is denoted c and the loan interest rate $1 + r$. Since banks cannot observe q, they offer a single contract per type θ, knowing that it will endogenously screen borrowers by quality.

Entrepreneurs decide whether to participate in the credit market, $d_\theta \in \{0, 1\}$, to maximize

$$U_\theta^E(d_\theta; x) = d_\theta \cdot \pi^I(r, q, \theta),$$

where entrepreneurial expected profits $\pi^I(r; q, \theta)$ are

$$\int_y \max[-c,\, y-(1+r)I]\, dF(y; q, \theta) = -c\int_0^{(1+r)I-c} dF(y; q, \theta) + \int_{(1+r)I-c}^{\infty} [y-(1+r)I]\, dF(y; q, \theta).$$

Assuming profits increase in project quality q, there exists a cutoff q_θ^* such that $\pi^I(r; q_\theta^*, \theta) = 0$. Entrepreneurs with $q > q_\theta^*$ participate. The cutoff rises with the interest rate:

$$\frac{dq_\theta^*}{dr} = \frac{I\,[1-F((1+r)I-c; q_\theta^*, \theta)]}{\pi_q^I(r; q_\theta^*, \theta)} > 0,$$

so higher rates exclude low-risk borrowers and worsen the applicant pool (adverse selection).

Let $1 + r_{\mathcal{E}}$ and $1 + r_{\mathcal{D}}$ denote the gross returns on equity and deposits, with $r_{\mathcal{E}} > r_{\mathcal{D}}$. Using the balance-sheet identity $\int_\Theta k_\theta\, d\theta = \mathcal{D} + \mathcal{E}$, the bank chooses its interest-rate schedule $z = \{r_\theta\}_{\theta\in\Theta}$ and equity $\mathcal{E}$ to maximize

$$U^B(r,k,\mathcal{E}) = \int_\Theta k_\theta \pi_\theta^B(r_\theta)\, d\theta - (1+r_\mathcal{D})\mathcal{L} - (r_\mathcal{E} - r_\mathcal{D})\mathcal{E},$$

where per-loan expected repayments are

$$\pi_\theta^B(r_\theta) = \mathbb{E}_q\left[\int_y \min\{y+c,\ (1+r_\theta)I\}\, dF(y;q,\theta)\ \middle|\ q \geq q_\theta^*\right],$$

and the ex-ante credit-risk probability is

$$\mathcal{P}(p_\theta, k_\theta, z; d_\theta, \theta) = p_\theta - \mathbb{E}_q[F((1+r_\theta)I - c; q, \theta)]\,.$$

Feasibility is defined by

$$k_\theta \leq I\,[1 - H(q_\theta^*)], \qquad \forall\theta,$$

i.e. credit supply cannot be greater than total loan demand at the rate r_θ.

As in Stiglitz and Weiss (1981), $\pi_\theta^B(r_\theta)$ is concave in the interest rate. The intuition is that higher rates raise repayments by existing borrowers but worsen pool quality, as low-risk applicants exit. The bank's optimal rate depends on its capital cost: it supplies credit if the expected return per loan exceeds the cost of deposits plus the risk-weighted equity charge. In particular, the bank does not lend to θ if

$$\max_r \Big[\pi_\theta^B(r) - (1+r_\mathcal{D}) - (r_\mathcal{E} - r_\mathcal{D})\varOmega\, \omega(p_\theta)\Big] < (=)\, r_\mathcal{D}.$$

We abstract from such corner cases of redlining and partial rationing.[22] Hence, credit supply equals the number of active borrowers at rate r_θ, so the interest rate directly pins down credit volume:

$$k_\theta = I\,[1 - H(q_\theta^*)].$$

Monopolistic loan pricing. Under monopoly, the bank internalizes how r_θ affects both loan profits and borrower participation. The optimal rate satisfies

$$\left.\frac{d\pi_\theta^B(r)}{dr}\right|_{r=r_\theta} + \Big[\pi_\theta^B(r_\theta) - (1+r_\mathcal{D}) - (r_\mathcal{E} - r_\mathcal{D})\varOmega\big(\omega(p_\theta) + p_\theta\, \varepsilon_\theta^{p,k} \omega'(p_\theta)\big)\Big] \left.\frac{d\log[1-H(q_\theta^*)]}{dr}\right|_{r=r_\theta} = 0.$$

The elasticity of credit risk with respect to credit supply follows from the risk-formation constraint:

$$\varepsilon_\theta^{p,k} = \frac{\mathbb{E}_q[f((1+r_\theta)I - c; q, \theta)]\,/p_\theta}{d\log[1-H(q_\theta^*)]/dr\big|_{r=r_\theta}} < 0.$$

[22] Riley (1987) notes that the measure of rationed borrowers vanishes as the number of observable characteristics increases.

Expected credit risk improves as lower interest rates expand credit supply.

The concavity of per-loan returns at the optimum ($SOC_\theta < 0$) implies the following credit-supply elasticities:

$$\zeta_\theta^{k,\omega} = \left(\frac{d \log[1 - H(q_\theta^*)]}{dr} \bigg|_{r=r_\theta} \right)^2 \frac{(r_\mathcal{E} - r_\mathcal{D})\Omega}{SOC_\theta} < 0, \qquad \zeta_\theta^{k,\omega'} = p_\theta \, \varepsilon_\theta^{p,k} \, \zeta_\theta^{k,\omega} > 0,$$

and

$$\zeta_\theta^{k,p} = p_\theta \big[\omega'(p_\theta) + p_\theta \, \varepsilon_\theta^{p,k} \omega''(p_\theta)\big] \zeta_\theta^{k,\omega}.$$

A higher level of risk weights raises funding costs and lowers credit supply due to a rise in interest rates. Because banks internalize that lower rates improve pool quality (reducing p_θ), a steeper schedule mitigates this effect and expands credit supply.

Competitive loan pricing. Under competition, the equilibrium interest rate satisfies the zero-profit condition:

$$r_\theta = \min\Big\{ r \geq 0 \,\Big|\, \pi_\theta^B(r) - (r_\mathcal{E} - r_\mathcal{D})\Omega \, \omega(p_\theta) = r_\mathcal{D} \Big\}, \quad \forall \theta.$$

At that rate, marginal profitability is positive. Otherwise, banks would have an incentive to further cut interest rates:

$$\frac{d\pi_\theta^B(r)}{dr} \bigg|_{r=r_\theta} - (r_\mathcal{E} - r_\mathcal{D})\Omega \, \omega'(p_\theta) \mathbb{E}_q[f((1 + r_\theta)I - c; q, \theta)] \, I > 0.$$

Thus, the credit-supply elasticities can be expressed as:

$$\zeta_\theta^{k,\omega} = \frac{d \log[1 - H(q_\theta^*)]/dr\big|_{r=r_\theta} \cdot (r_\mathcal{E} - r_\mathcal{D})\Omega}{d\pi_\theta^B(r)/dr\big|_{r=r_\theta} - (r_\mathcal{E} - r_\mathcal{D})\Omega \, \omega'(p_\theta) \mathbb{E}_q[f((1 + r_\theta)I - c; q, \theta)] \, I} < 0, \qquad \zeta_\theta^{k,\omega'} = 0,$$

and

$$\zeta_\theta^{k,p} = \omega'(p_\theta) p_\theta \zeta_\theta^{k,\omega}.$$

The risk elasticity again satisfies the risk-formation constraint:

$$\varepsilon_\theta^{p,k} = \frac{\mathbb{E}_q[f((1 + r_\theta)I - c; q, \theta)] \, / p_\theta}{d \log[1 - H(q_\theta^*)]/dr\big|_{r=r_\theta}} < 0.$$

Welfare effects. In both cases, there are no bank-level externalities. However, credit supply affects borrower welfare via the interest-rate channel: a higher k_θ (lower rate) raises expected returns inframarginally,

$$\frac{dU_\theta^E(d_\theta; x)}{dk_\theta} = \frac{I \int_{q_\theta^*}^\infty [1 - F((1 + r_\theta)I - c; q, \theta)] \, \frac{h(q)}{h(q_\theta^*)} dq}{dq_\theta^*/dr\big|_{r=r_\theta}} > 0.$$

A.2.5 Gale and Hellwig (1985): Costly State Verification

Instead of ex-ante adverse selection, private information on the loan market may be ex-post, giving rise to non-verifiability of outcomes. In that case, banks may audit and the loan contract must be incentive compatible, as in Gale and Hellwig (1985). There are two dates $t = 0, 1$. At $t = 0$, an entrepreneur θ invests $k_\theta \geq 0$ units in a project; at $t = 1$ the project yields $f(k_\theta; s)$, where $s \in [0, \infty)$ is a privately observed state drawn from $H_\theta(s)$ with density $h_\theta(s)$. The production function is increasing and concave in scale: $f'(\cdot; s) > 0$, $f''(\cdot; s) < 0$, and $\partial_s f(\cdot; s)$, $\partial_s f'(\cdot; s) > 0$. The bank can observe the state by paying a weakly convex audit cost $c_1(k_\theta; s)$; auditing may also impose a non-pecuniary penalty $c_0 \geq 0$ on the entrepreneur. The risk-free deposit rate is $1 + r_{\mathcal{D}}$ and the return on equity is $1 + r_{\mathcal{E}}$. Entrepreneurs start with no assets or debt.

A loan contract for type θ is a quadruple $\big(k_\theta, C_{1,\theta}(s), W_{1,\theta}(s), B_\theta(s)\big)$ consisting of the loan size, the bank's state-contingent repayment, the entrepreneur's payoff, and an audit indicator $B_\theta(s) \in \{0, 1\}$. The optimal per-type contract maximizes bank profits subject to (i) entrepreneurial participation $\mathbb{E}_s[W_{1,\theta}(s) - c_0 B_\theta(s)] \geq \bar{U}_\theta$, (ii) non-negativity $W_{1,\theta}(s) \geq 0$ and $k_\theta \geq 0$, (iii) resource feasibility $C_{1,\theta}(s) + W_{1,\theta}(s) \leq f(k_\theta; s) - c_1(k_\theta; s) B_\theta(s)$, and (iv) incentive compatibility (truthful reporting without audit). An allocation is incentive compatible iff repayments are constant in non-audited states, $C_{1,\theta}(s) = R_{1,\theta}$ whenever $B_\theta(s) = 0$. One can further show that in audited states ($B_\theta(s) = 1$), the verification cost needs to be covered, so $R_{1,\theta} - C_{1,\theta}(s) \geq c_0 + c_1(k_\theta; s)$.

Under a mild regularity condition, the optimal contract is a standard debt contract: (i) $C_{1,\theta}(s) = R_{1,\theta}$ if $B_\theta(s) = 0$; (ii) bankruptcy (audit) occurs iff $f(k_\theta; s) < R_{1,\theta}$; (iii) in bankruptcy, the bank recovers $C_{1,\theta}(s) = f(k_\theta; s) - c_1(k_\theta; s)$. Hence there is a default threshold γ_θ with $f(k_\theta; \gamma_\theta) = R_{1,\theta}$, and defaults occur for $s < \gamma_\theta$. The default probability is $p_\theta = H_\theta(\gamma_\theta)$.

Using the notation by Gale and Hellwig (1985), one can define the following payment functions:

$$f_{\gamma_\theta}(k_\theta; s) \equiv f(k_\theta; s) - \mathbf{1}_{\{s<\gamma_\theta\}}\big(c_0 + c_1(k_\theta; s)\big), \quad g_{\gamma_\theta}(k_\theta; s) \equiv \mathbf{1}_{\{s\geq\gamma_\theta\}} f(k_\theta; \gamma_\theta) + \mathbf{1}_{\{s<\gamma_\theta\}}\big(f(k_\theta; s) - c_1(k_\theta; s)\big).$$

Imposing the balance-sheet identity $\mathcal{L} = \int_\Theta k_\theta \, d\theta = \mathcal{D} + \mathcal{E}$, the bank chooses credit $\{k_\theta\}_{\theta\in\Theta}$, equity $\mathcal{E}$, and default thresholds $z = \{\gamma_\theta\}_{\theta\in\Theta}$ to maximize

$$U^B(k, \mathcal{E}, z) = \int_\Theta \mathbb{E}_s[g_{\gamma_\theta}(k_\theta; s)] \, d\theta - (1 + r_{\mathcal{D}}) \, \mathcal{L} - (r_{\mathcal{E}} - r_{\mathcal{D}}) \, \mathcal{E},$$

subject to regulation. We now distinguish two contracting environments.

Monopolistic contracting. If banks choose the entire loan contract, entrepreneurial utility equals

$$U_\theta^E(k_\theta, \gamma_\theta) = \mathbb{E}_s[f_{\gamma_\theta}(k_\theta; s)] - \mathbb{E}_s[g_{\gamma_\theta}(k_\theta; s)],$$

and the participation constraint $U_\theta^E \geq \bar{U}_\theta$ binds in optimum. This pins down the risk feedback from credit supply:

$$\varepsilon_\theta^{p,k} = \frac{h_\theta(\gamma_\theta)\, k_\theta}{H_\theta(\gamma_\theta)} \; \frac{\displaystyle\int_{s \geq \gamma_\theta} \big[f'(k_\theta; s) - f'(k_\theta; \gamma_\theta)\big]\, dH_\theta(s)}{(1 - H_\theta(\gamma_\theta))\, \partial_{\gamma_\theta} f(k_\theta; \gamma_\theta) + h_\theta(\gamma_\theta)\, c_0} \; > \; 0.$$

Intuitively, with standard debt, a larger loan raises the audit region (more states fall below the fixed face value), increasing expected verification and the default probability.

The bank's per-type Lagrangian is defined as

$$L_\theta = \mathbb{E}_s[g_{\gamma_\theta}(k_\theta; s)] - (1 + r_{\mathcal{D}})\, k_\theta - (r_{\mathcal{E}} - r_{\mathcal{D}})\, \Omega\, \omega(p_\theta)\, k_\theta + \lambda_\theta \Big[\bar{U}_\theta - \mathbb{E}_s[f_{\gamma_\theta}(k_\theta; s)] + \mathbb{E}_s[g_{\gamma_\theta}(k_\theta; s)]\Big],$$

yielding the first-order condition for k_θ:

$$\mathbb{E}_s[g'_{\gamma_\theta}(k_\theta; s)] - (1 + r_{\mathcal{D}}) - (r_{\mathcal{E}} - r_{\mathcal{D}})\, \Omega\, \omega(p_\theta) + p_\theta \varepsilon_\theta^{p,k} \left[\frac{\mathbb{E}_s[\partial_{\gamma_\theta} g_{\gamma_\theta}(k_\theta; s)]}{k_\theta h_\theta(\gamma_\theta)} - (r_{\mathcal{E}} - r_{\mathcal{D}})\, \Omega\, \omega'(p_\theta)\right] = 0.$$

Perturbing around the optimum and assuming concavity gives the credit-supply elasticities:

$$\zeta_\theta^{k,\omega} = \frac{(r_{\mathcal{E}} - r_{\mathcal{D}})\, \Omega / k_\theta}{\mathbb{E}_s[g''_{\gamma_\theta}(k_\theta; s)] + p_\theta\, \partial_{k_\theta} \Big\{\varepsilon_\theta^{p,k} \, \frac{\mathbb{E}_s[\partial_{\gamma_\theta} g_{\gamma_\theta}(k_\theta; s)]}{k_\theta h_\theta(\gamma_\theta)}\Big\}} < 0, \qquad \zeta_\theta^{k,\omega'} = p_\theta\, \varepsilon_\theta^{p,k}\, \zeta_\theta^{k,\omega} < 0.$$

The elasticity with respect to default risk depends on the curvature of the risk-weight schedule and the shape of the production function

$$\begin{aligned}\zeta_\theta^{k,p} = \Bigg(&\mathbb{E}_s[\partial_{\gamma_\theta} g'_{\gamma_\theta}(k_\theta; s)]\, h_\theta(\gamma_\theta) - (r_{\mathcal{E}} - r_{\mathcal{D}})\, \Omega \Big[\big(1 + \varepsilon_\theta^{p,k} + p_\theta\, \partial_{\gamma_\theta} \varepsilon_\theta^{p,k}\, h_\theta(\gamma_\theta)\big)\omega'(p_\theta) + p_\theta \varepsilon_\theta^{p,k} \omega''(p_\theta)\Big] \\ &+ \frac{p_\theta}{k_\theta}\, \varepsilon_\theta^{p,k}\, \mathbb{E}_s[g'_{\gamma_\theta}(k_\theta; s)] \left[\partial_{\gamma_\theta} \log \varepsilon_\theta^{p,k} + \frac{\mathbb{E}_s[\partial_{\gamma_\theta} g'_{\gamma_\theta}(k_\theta; s)]}{\mathbb{E}_s[g'_{\gamma_\theta}(k_\theta; s)]} - \frac{h'_\theta(\gamma_\theta)}{h_\theta(\gamma_\theta)}\right]\Bigg) \frac{p_\theta\, \zeta_\theta^{k,\omega}}{(r_{\mathcal{E}} - r_{\mathcal{D}})\, \Omega} \; \lesseqgtr \; 0.\end{aligned}$$

Competitive rates. Under posted rates, entrepreneurs choose k, taking the loan rate r_θ as given. In equilibrium, each rate satisfies the bank's zero-profit condition, and borrowers choose scale optimally. Entrepreneurial expected utility is

$$U_\theta^E(k_\theta; r_\theta, \gamma_\theta) = \mathbb{E}_s[\max\{\, f(k_\theta; s) - (1 + r_\theta) k_\theta,\, 0\,\}] - c_0\, H_\theta(\gamma_\theta),$$

The constraints are summarized by three conditions: (i) borrower demand pins down credit supply $k_\theta \leq \arg\max_{k\geq 0} U^E_\theta(k; r_\theta, \gamma_\theta)$, (ii) banks make zero profits on each loan

$$\gamma_\theta = \min\Big\{\gamma \geq 0 \;\Big|\; \mathbb{E}_s[g_\gamma(k_\theta; s)] = (1 + r_{\mathcal{D}})\, k_\theta + (r_{\mathcal{E}} - r_{\mathcal{D}})\, \Omega\, \omega(p_\theta)\, k_\theta\Big\},$$

and (iii) the face value is consistent with the posted rate

$$r_\theta = \min\{\, r \geq 0 \;\big|\; f(k_\theta; \gamma_\theta) = (1 + r)\, k_\theta \,\}, \qquad \forall\theta.$$

Combining these conditions, the equilibrium allocation $(k_\theta, \gamma_\theta)$ solves

$$\frac{\mathbb{E}_s[g_{\gamma_\theta}(k_\theta; s)]}{k_\theta} = (1 + r_{\mathcal{D}}) + (r_{\mathcal{E}} - r_{\mathcal{D}})\, \Omega\, \omega(p_\theta), \qquad \mathbb{E}_s\big[f'(k_\theta; s) \,\big|\, s \geq \gamma_\theta\big] = \frac{f(k_\theta; \gamma_\theta)}{k_\theta}.$$

Rewriting the second condition as an implicit relation for $\gamma_\theta(k_\theta)$, the risk feedback is case-specific

$$\varepsilon^{p,k}_\theta = \frac{k_\theta h_\theta(\gamma_\theta)}{p_\theta}\, \frac{\mathbb{E}_s\big[f''(k_\theta; s) \,\big|\, s \geq \gamma_\theta\big] - \big[f'(k_\theta; \gamma_\theta) - f(k_\theta; \gamma_\theta)/k_\theta\big]/k_\theta}{\partial_{\gamma_\theta} f(k_\theta; \gamma_\theta)/k_\theta - \partial_{\gamma_\theta}\mathbb{E}_s\big[f'(k_\theta; s) \,\big|\, s \geq \gamma_\theta\big]} \;\lesseqgtr\; 0,$$

i.e. scale effects (decreasing returns) and threshold effects (moving the default boundary) can offset or reinforce each other.

Using the zero-profit condition, the credit-supply elasticities are given by

$$\zeta^{k,\omega}_\theta = -\frac{(r_{\mathcal{E}} - r_{\mathcal{D}})\, \Omega}{\mathbb{E}_s[g_{\gamma_\theta}(k_\theta; s)]/k_\theta - \mathbb{E}_s[g'_{\gamma_\theta}(k_\theta; s)]} < 0, \qquad \zeta^{k,\omega'}_\theta = 0,$$

and

$$\zeta^{k,p}_\theta = -\frac{(r_{\mathcal{E}} - r_{\mathcal{D}})\, \Omega\, \omega'(p_\theta)\, p_\theta - \mathbb{E}_s[\partial_{\gamma_\theta} g_{\gamma_\theta}(k_\theta; s)]\, \frac{p_\theta}{k_\theta h_\theta(\gamma_\theta)}}{\mathbb{E}_s[g_{\gamma_\theta}(k_\theta; s)]/k_\theta - \mathbb{E}_s[g'_{\gamma_\theta}(k_\theta; s)]} \;\lesseqgtr\; 0.$$

Welfare effects. Under monopolistic contracting, welfare is pinned down by the participation constraint. Under posted rates, borrower welfare moves with the loan price and with the non-pecuniary default cost. Because of decreasing returns, a larger scale tends to reduce the implied rate (via the face value), improving welfare; but if k_θ raises default risk, the expected bankruptcy cost may increase. Using $k_\theta\, dr_\theta/dk_\theta = f'(k_\theta; \gamma_\theta) - f(k_\theta; \gamma_\theta)/k_\theta + \partial_{\gamma_\theta} f(k_\theta; \gamma_\theta)\, d\gamma_\theta/dk_\theta$, we obtain

$$\frac{dU^E_\theta(k_\theta; r_\theta, \gamma_\theta)}{dk_\theta} = -\, p_\theta\, k_\theta\, \frac{dr_\theta}{dk_\theta} - c_0\, h_\theta(\gamma_\theta)\, \frac{d\gamma_\theta}{dk_\theta}.$$

B Appendix for Section 3

B.1 Loan-Level Incidence

Arbitrary reforms. We consider an arbitrary risk-weight reform that changes the schedule from ω to $\omega + \epsilon\hat{\omega}$ with $\epsilon \to 0$. Using the optimality condition from Section A.1 and the definitions of elasticities, we observe that the equilibrium loan-level response can be expressed as

$$\frac{\hat{k}_\theta}{k_\theta} = \zeta_\theta^{k,\omega}\hat{\omega}(p_\theta) + \zeta_\theta^{k,\omega'}\hat{\omega}'(p_\theta) + \varepsilon_\theta^{k,p}\frac{\hat{p}_\theta}{p_\theta} - \frac{\partial MRS_\theta}{\partial \mathcal{E}}\frac{\hat{\mathcal{E}}}{k_\theta} - \int_{\theta'}\left(\frac{\partial MRS_\theta}{\partial k_{\theta'}}\frac{\hat{k}_{\theta'}}{k_\theta} + \frac{\partial MRS_\theta}{\partial \bar{p}_{\theta'}}\frac{\hat{p}_{\theta'}}{k_\theta}\right)d\theta'. \quad (19)$$

The change to credit risk follows from the entrepreneurial risk functional: $\hat{p}_\theta/p_\theta = \varepsilon_\theta^{p,k}(\hat{k}_\theta/k_\theta)$. Thus, there are direct own-price effects from reforming the risk weight and indirect responses from across-asset substitution. The latter originate from circular responses in credit supply and changes to bank equity (see below).

As a next step, we plug the credit risk response into (19) to obtain a characterization of the equilibrium credit-supply response to risk weights

$$\frac{\hat{k}_\theta}{k_\theta} = \frac{\zeta_\theta^{k,\omega}}{1-\varepsilon_\theta^{k,p}\varepsilon_\theta^{p,k}}\hat{\omega}(p_\theta) + \frac{\zeta_\theta^{k,\omega'}}{1-\varepsilon_\theta^{k,p}\varepsilon_\theta^{p,k}}\hat{\omega}'(p_\theta) + \Psi_\theta(\{\hat{\omega}(p_\theta), \hat{\omega}'(p_\theta)\}_{\theta\in\Theta}),$$

where, assuming invertibility, the across-asset substitution term reads as

$$\Psi_\theta(\cdot) \equiv -\frac{\partial MRS_\theta}{\partial \mathcal{E}}\frac{\hat{\mathcal{E}}/k_\theta}{1-\varepsilon_\theta^{k,p}\varepsilon_\theta^{p,k}} - \int_{\theta'\in\Theta}\left(\frac{\partial MRS_\theta}{\partial k_{\theta'}} + \frac{p_{\theta'}}{k_{\theta'}}\frac{\partial MRS_\theta}{\partial \bar{p}_{\theta'}}\varepsilon_{\theta'}^{p,k}\right)\frac{\hat{k}_{\theta'}/k_\theta}{1-\varepsilon_\theta^{k,p}\varepsilon_\theta^{p,k}}d\theta'.$$

Counteracting reforms. Suppose the cross derivatives of MRS_θ are independent from θ. Then, we may construct a lump-sum perturbation $\hat{\omega}_c$, with $\hat{\omega}'_c = 0$, that counteracts all responses from across-asset substitution to $\hat{\omega}$. Intuitively, $\hat{\omega}_c$ is chosen such that all bank-level aggregates in the portfolio problem remain unchanged. Defining

$$\Lambda_{\theta'} \equiv -\Omega[\omega(p_{\theta'}) + \omega'(p_{\theta'})p_{\theta'}\varepsilon_{\theta'}^{p,k}] + \left(\frac{\partial MRS_\theta}{\partial k_{\theta'}} + \frac{p_{\theta'}}{k_{\theta'}}\frac{\partial MRS_\theta}{\partial \bar{p}_{\theta'}}\varepsilon_{\theta'}^{p,k}\right)\Big/\frac{\partial MRS_\theta}{\partial \mathcal{E}},$$

the lump-sum reform can be written as:

$$\hat{\omega}_c = -\frac{\int_\theta \hat{\omega}(p_\theta)(\Lambda_\theta\bar{\zeta}_\theta^{k,\omega} - \Omega)k_\theta d\theta + \int_\theta \hat{\omega}'(p_\theta)\Lambda_\theta\bar{\zeta}_\theta^{k,\omega'}k_\theta d\theta}{\int_\theta(\Lambda_\theta\bar{\zeta}_\theta^{k,\omega} - \Omega)k_\theta d\theta}. \quad (20)$$

Then, the across-asset substitution response in (9) becomes zero, $\varPsi_\theta(\cdot) = 0$, and the change in equilibrium credit supply is given by

$$\frac{\hat{k}_\theta}{k_\theta} = \frac{\zeta_\theta^{k,\omega}}{1 - \varepsilon_\theta^{k,p}\varepsilon_\theta^{p,k}}\left[\hat{\omega}(p_\theta) + \hat{\omega}_c\right] + \frac{\zeta_\theta^{k,\omega'}}{1 - \varepsilon_\theta^{k,p}\varepsilon_\theta^{p,k}}\hat{\omega}'(p_\theta).$$

Elementary reforms. We now specialize our analysis of credit-supply incidence by considering the class of elementary risk-weight reforms defined by (12). Changing from unobserved types to observable credit risk, the first-order effects on credit supply and risk read as

$$\frac{\hat{k}_p}{k_p} = \frac{\zeta_p^{k,\omega}}{1 - \varepsilon_p^{k,p}\varepsilon_p^{p,k}}\frac{\mathbb{1}[p > p^*]}{\varOmega\int_{p>p^*} k_p dR\,(p)} + \frac{\zeta_p^{k,\omega'}}{1 - \varepsilon_p^{k,p}\varepsilon_p^{p,k}}\frac{\delta(p - p^*)}{\varOmega\int_{p>p^*} k_p dR\,(p)} + \varPsi_p(p^*), \tag{21}$$

where $\delta(p - p^*)$ denotes the Dirac delta function.

B.2 Proof of Proposition 1

We now derive the bank-level responses to an arbitrary reform $\hat{\omega}$. First, we perturb the bank credit-supply equation $\mathcal{L} = \int_\theta k_\theta d\theta$:

$$\hat{\mathcal{L}} = \int_{\theta\in\Theta} \hat{k}_\theta d\theta,$$

where the behavioral response in the bank's portfolio is given by (9). Similarly, we compute the response of equity $\mathcal{E} = \varOmega\int_{\theta\in\Theta}\omega(p_\theta)k_\theta d\theta$:

$$\hat{\mathcal{E}} = \varOmega\int_{\theta\in\Theta}\hat{\omega}(p_\theta)k_\theta d\theta + \varOmega\int_{\theta\in\Theta}[\omega(p_\theta) + \omega'(p_\theta)p_\theta\varepsilon_\theta^{p,k}]\hat{k}_\theta d\theta.$$

Thus, in addition to an incentive effect from portfolio adjustments, bank equity depends directly on risk weights leading to an inframarginal response.

B.3 Proof of Corollary 1

B.3.1 General Environment

We now derive the bank-level incidence of elementary reforms. We plug the reform-specific loan-level responses into the expressions from Proposition 1. Changing from types to credit risk and ignoring cross-price effects, bank credit supply and equity respond according to

$$\hat{\mathcal{L}}\,(p^*) = \int_{p>p^*}\frac{k_p\overline{\zeta}_p^{k,\omega}}{\varOmega\int_{p'>p^*}k_{p'}dR\,(p')}dR\,(p) + \frac{k_{p^*}\overline{\zeta}_{p^*}^{k,\omega'}}{\varOmega\int_{p'>p^*}k_{p'}dR\,(p')}r\,(p^*)$$

and

$$\hat{\mathcal{E}}\left(p^{*}\right)=1+\int_{p>p^{*}}\frac{k_{p}[\omega(p)+\omega'(p)p\varepsilon_{p}^{p,k}]\overline{\zeta}_{p}^{k,\omega}}{\int_{p'>p^{*}}k_{p'}dR\left(p'\right)}dR\left(p\right)+\frac{k_{p^{*}}[\omega(p^{*})+\omega'(p^{*})p^{*}\varepsilon_{p^{*}}^{p,k}]\overline{\zeta}_{p^{*}}^{k,\omega'}}{\int_{p'>p^{*}}k_{p'}dR\left(p'\right)}r\left(p^{*}\right).$$

B.3.2 Structural Example for Across-Asset Substitution

Setup. As in the main text, the economy consists of three agent groups: banks (B), households (H), and entrepreneurs (E). Firms demand credit and undertake risky projects, while households supply deposits and ultimately bear the cost of deposit guarantees. Each agent $X \in \{B, H, E\}$ receives an endowment n_t^X in each period and consumes a single good c_t^X. Let $\beta^X \in (0,1]$ denote the agent's discount factor. All agents are risk-neutral, so their utility is

$$U^X(\cdot) = c_0^X + \beta^X \mathbb{E}_\upsilon[c_1^X(\upsilon)],$$

where $\upsilon \in [\underline{\upsilon}, \overline{\upsilon}] \sim F(\upsilon)$ is an aggregate shock realized in the second period.

In the first period, banks provide credit to entrepreneurs. Given bank credit supply, each entrepreneur chooses an ex-ante probability of failure p under a standard risk–return trade-off (see, e.g., Allen and Gale (2000); Boyd and De Nicolo (2005); Corbae and D'Erasmo (2021)). In case of success, the project yields $(1+p)$ per unit of capital invested; in case of failure, output is zero. Expected project returns, therefore, are equal $(1-p)(1+p)$ per unit.

Each entrepreneur θ operates a production function $f(k_\theta, \theta) = \theta k_\theta^{1-a}$, where $a \in (0,1)$ captures decreasing returns to scale. The aggregate shock υ is realized only after project and funding choices are made. If successful, the entrepreneur repays an interest rate r_θ per unit of bank credit. If unsuccessful, limited liability prevents repayment. Production capital depreciates at a rate δ.

The entrepreneur's budget constraints are:

$$c_0^E = n_0^E(\theta), \qquad c_1^E(\upsilon) = n_1^E(\theta) + (1-p)\big[(1+p)\theta k_\theta^{1-a} - (r_\theta+\delta)k_\theta\big].$$

Optimal risk-taking satisfies $p_\theta = (r_\theta+\delta)k_\theta^a/(2\theta)$, which increases with leverage, implying $\varepsilon_\theta^{p,k} = a$.

Banks raise deposits $\mathcal{D}$ from households at price $q_{\mathcal{D}}$ and choose a credit portfolio $\{k_\theta\}_{\theta\in\Theta}$ in the first period. As in Tobin's q models of investment, banks face convex adjustment costs $k_\theta + (b_\theta/2)k_\theta^2$ for each unit of capital they extend. In the second period, they receive state-contingent loan repayments:[23]

$$\Pi^B(\cdot,\upsilon) = \int_\Theta (1-p_\theta)\,\upsilon\,(1+r_\theta)\,k_\theta\,d\theta, \qquad \mathbb{E}_\upsilon[\upsilon] = 1.$$

In adverse states, banks may default on their debt. To prevent household losses, taxpayers provide a deposit

[23]The term $\Pi^B(\cdot,\upsilon)$ can capture "forced safety" effects as in Bahaj and Malherbe (2020).

or bailout guarantee $\mathcal{T}(\cdot, \upsilon)$ financed by households.

The budget constraints are:

$$c_0^B = n_0^B + q_{\mathcal{D}}\mathcal{D} - \mathcal{L} - \int_{\Theta} \frac{b_\theta}{2} k_\theta^2 \, d\theta, \qquad c_1^B(\upsilon) = n_1^B + \max\{\Pi^B(\cdot, \upsilon) + \mathcal{T}(\cdot, \upsilon) - \mathcal{D}, 0\}, \quad \forall \upsilon.$$

Banks repay their debt and consume their residual claim in states where loan repayments plus bailouts exceed the face value of debt; otherwise, they default. Because the profit function is asymmetric (unbounded above but bounded below), banks do not fully internalize the social cost of credit supply, which leads to excessive risk-taking. Capital regulation can mitigate these incentives.

We consider a bailout policy (deposit insurance) that fully prevents bank default in bad states:

$$\mathcal{T}(\cdot, \upsilon) = \max\{\mathcal{D} - \Pi^B(\cdot, \upsilon), 0\}, \quad \forall \upsilon.$$

Define $\upsilon^\star \in (\underline{\upsilon}, \overline{\upsilon})$ as the cutoff shock below which banks receive a bailout. It represents the lowest realization of υ for which the bank remains solvent. For $r_\theta = 0$, the cutoff equals the book value of debt relative to total expected loan repayments:

$$\upsilon^\star = \frac{\mathcal{L} - \mathcal{E}}{\mathcal{L} - \mathcal{K}}, \qquad \mathcal{K} \equiv \int_{\Theta} p_\theta k_\theta \, d\theta,$$

where $\mathcal{L}$ denotes total bank lending and $\mathcal{E}$ the equity position of the bank.

Elasticities. The optimal credit portfolio is then given by

$$k_\theta = \frac{\beta^B \big[1 - (1+a)p_\theta\big] (1 + r_\theta) \int_{\upsilon^\star}^{\overline{\upsilon}} \upsilon \, dF(\upsilon) - \mu\big[\omega(p_\theta) + a p_\theta \omega'(p_\theta)\big] - (1 + \beta^B)}{b_\theta}, \qquad \forall \theta.$$

The first term captures expected marginal loan returns weighted by the survival probability above the bailout cutoff. The second term represents the regulatory cost from risk weights, and the last term reflects the adjustment cost of capital creation. Here, $\mu/\varOmega = q_{\mathcal{D}} - \beta^B\big(1 - F(\upsilon^\star)\big)$ denotes the shadow price of bank equity. The corresponding credit-supply elasticities are:[24]

$$\zeta_\theta^{k,\omega} = -\frac{\mu}{b_\theta k_\theta} < 0, \quad \zeta_\theta^{k,\omega'} = a p_\theta \zeta_\theta^{k,\omega}, \quad \varepsilon_\theta^{k,p} = -(1+a)p_\theta \frac{\omega'(p_\theta)\mu + \beta^B(1 + r_\theta) \int_{\upsilon^\star}^{\overline{\upsilon}} \upsilon \, dF(\upsilon)}{b_\theta k_\theta} < 0.$$

These expressions describe how lending responds to changes in average and marginal risk weights, as well as to shifts in credit risk.

[24]For transparency, we assume a locally flat risk-weight schedule, $\omega''(p_\theta) = 0$, in the second expression.

Equilibrium. To close the model, consider households. They supply deposits at price $q_{\mathcal{D}}$ and derive second-period utility $\psi \geq 1$ per deposit unit, capturing a liquidity preference. They also bear the fiscal cost of government guarantees, $(1+\kappa_{\mathcal{T}})\mathcal{T}(\cdot, \upsilon)$, where $\kappa_{\mathcal{T}} \geq 0$ denotes the deadweight loss from fiscal intervention. Hence, the presence of a deposit insurance introduces a financial externality analogous to that in Dávila and Walther (2021) or Corbae and D'Erasmo (2021). The household budget constraints are:

$$c_0^H = n_0^H - q_{\mathcal{D}}\mathcal{D}, \qquad c_1^H(\upsilon) = n_1^H + \psi\mathcal{D} - (1+\kappa_{\mathcal{T}})\mathcal{T}(\cdot, \upsilon).$$

The equilibrium deposit price is determined by $q_{\mathcal{D}} = \beta^H \psi$. Assuming that households are at least as patient as banks ($\beta^H \geq \beta^B$; cf. Dávila and Walther (2023)) implies a positive cost of bank equity ($\mu > 0$). The existence of bailouts ($F(\upsilon^\star) > 0$) breaks the Modigliani–Miller capital-structure irrelevance result (Modigliani and Miller 1958), even when there is no liquidity preference ($\psi = 1$) or discount-rate differential ($\beta^H = \beta^B$).

Across-asset substitution. We now extend the analysis to allow for cross-asset interactions in the bank's portfolio choice. In this case, changes in the bailout cutoff affect lending across assets through the substitution term

$$\varPsi_p\left(p^*\right) = \frac{\partial \log k_p / \partial \upsilon^\star}{1 - \varepsilon_p^{k,p} \varepsilon_p^{p,k}} \, \hat{\upsilon}^\star\left(p^*\right).$$

For $f(\upsilon^\star) \to 0$, loans are unaffected by circularities from the bailout cutoff, $\partial k_\theta / \partial \upsilon^\star = 0$. Otherwise, a binding bailout cutoff generates nontrivial portfolio adjustments due to across-asset substitution.

Using the aggregate solvency condition, the response of the bailout cutoff to an elementary reform at threshold p^* is given by

$$\hat{\upsilon}^\star\left(p^*\right) = \frac{\displaystyle\int_{p>p^*} \frac{k_p\big(\varLambda_p \overline{\zeta}_p^{k,\omega} - \varOmega\big)}{\varOmega\,\mathbb{E}\big(k_{p'}\,\big|\,p' > p^*\big)} \frac{dR(p)}{1-R(p^*)} + \frac{k_{p^*}\varLambda_{p^*}\overline{\zeta}_{p^*}^{k,\omega'}}{\varOmega\,\mathbb{E}\big(k_{p'}\,\big|\,p' > p^*\big)} \frac{r(p^*)}{1-R(p^*)}}{\displaystyle\frac{d\Pi_B(\{k_\theta\}_{\theta\in\Theta}, \upsilon^\star)}{d\upsilon} - \int_p \varLambda_p \frac{\partial \log k_p}{\partial \upsilon^\star} k_p \, dR(p)}. \tag{22}$$

Here, $R(p)$ denotes the cumulative distribution of loan risk types. The numerator collects the direct incidence effects for loans above and at the cutoff p^*, while the denominator adjusts for general-equilibrium feedback through the bank's portfolio re-optimization.

Elementary reforms. We now derive the bank-level incidence of elementary reforms in the presence of across-asset substitution. Plugging the loan-level incidence (9) into Proposition 1 and integrating over credit risk types yields the following expressions for the responses of aggregate credit supply and bank equity:

$$\hat{\mathcal{L}}\,(p^*) = \int_{p>p^*} \frac{k_p\big(1+\varphi_{\mathcal{L}}\Lambda_p - \varphi_{\mathcal{L}}\Omega/\overline{\zeta}_p^{k,\omega}\big)\overline{\zeta}_p^{k,\omega}}{\Omega\,\mathbb{E}\big(k_{p'}\,\big|\,p'>p^*\big)}\frac{dR(p)}{1-R(p^*)} + \frac{k_{p^*}\big(1+\varphi_{\mathcal{L}}\Lambda_{p^*}\big)\overline{\zeta}_{p^*}^{k,\omega'}}{\Omega\,\mathbb{E}\big(k_{p'}\,\big|\,p'>p^*\big)}\frac{r(p^*)}{1-R(p^*)},$$

$$\begin{aligned}\hat{\mathcal{E}}\,(p^*) &= \int_{p>p^*} \frac{k_p\big[\omega(p)+\omega'(p)p\varepsilon_p^{p,k}+\varphi_{\mathcal{E}}\Lambda_p-\varphi_{\mathcal{E}}\Omega/\overline{\zeta}_p^{k,\omega}\big]\overline{\zeta}_p^{k,\omega}}{\mathbb{E}\big(k_{p'}\,\big|\,p'>p^*\big)}\frac{dR(p)}{1-R(p^*)}\\ &\quad+\frac{k_{p^*}\big[\omega(p^*)+\omega'(p^*)p^*\varepsilon_{p^*}^{p,k}+\varphi_{\mathcal{E}}\Lambda_{p^*}\big]\overline{\zeta}_{p^*}^{k,\omega'}}{\mathbb{E}\big(k_{p'}\,\big|\,p'>p^*\big)}\frac{r(p^*)}{1-R(p^*)}.\end{aligned}$$

The correction terms $\varphi_{\mathcal{L}}$ and $\varphi_{\mathcal{E}}$ capture circularities arising from the feedback of the bailout cutoff:

$$\varphi_{\mathcal{L}} = \frac{\displaystyle\int_p k_p\,\frac{\partial\log k_p/\partial\upsilon^\star}{1-\varepsilon_p^{k,p}\varepsilon_p^{p,k}}\,dR(p)}{\dfrac{d\Pi_B(\{k_\theta\}_{\theta\in\Theta},\upsilon^\star)}{d\upsilon}-\displaystyle\int_p \Lambda_p\,\frac{\partial\log k_p}{\partial\upsilon^\star}\,k_p\,dR(p)}, \qquad \varphi_{\mathcal{E}} = \frac{\displaystyle\int_p \omega(p)\,k_p\,\frac{\partial\log k_p/\partial\upsilon^\star}{1-\varepsilon_p^{k,p}\varepsilon_p^{p,k}}\,dR(p)}{\dfrac{d\Pi_B(\{k_\theta\}_{\theta\in\Theta},\upsilon^\star)}{d\upsilon}-\displaystyle\int_p \Lambda_p\,\frac{\partial\log k_p}{\partial\upsilon^\star}\,k_p\,dR(p)}.$$

When $\partial k_p/\partial\upsilon^\star = 0$, all circularities disappear and $\varphi_{\mathcal{L}} = \varphi_{\mathcal{E}} = 0$.

Under a flat baseline risk-weight scheme ($\omega(p) = 1$), the corrections in the equity and credit-supply responses are uniform $\varphi_{\mathcal{L}} = \varphi_{\mathcal{E}} = \varphi$. We can then redefine the semi-elasticities to incorporate circularities:

$$\tilde{\zeta}_p^{k,\omega} = \big(1+\varphi\Lambda_p-\varphi\Omega/\overline{\zeta}_p^{k,\omega}\big)\overline{\zeta}_p^{k,\omega}, \qquad \tilde{\zeta}_p^{k,\omega'} = \big(1+\varphi\Lambda_p\big)\overline{\zeta}_p^{k,\omega'},$$

yielding the same functional forms as in Corollary 1.

We also derive closed-form expressions for the response of the bailout cutoff. Differentiating the definition of $\upsilon^\star = (\mathcal{L}-\mathcal{E})/(\mathcal{L}-\mathcal{K})$ yields

$$\frac{\hat{\upsilon}^\star}{\upsilon^\star} = \frac{\hat{\mathcal{L}}-\hat{\mathcal{E}}}{\mathcal{L}-\mathcal{E}} - \frac{\hat{\mathcal{L}}-\hat{\mathcal{K}}}{\mathcal{L}-\mathcal{K}}.$$

Hence, the bailout probability decreases with higher bank equity and increases with higher expected credit losses. For an elementary reform and $\partial k_p/\partial\upsilon^\star = 0$, we obtain the following expression:

$$\begin{aligned}\hat{\upsilon}^\star(p^*)(\mathcal{L}-\mathcal{K}) &= -\int_{p>p^*}\frac{k_p\Big\{[1-(1+\varepsilon_p^{p,k})p]\,\upsilon^\star-\Omega[\omega(p)+\omega'(p)p\,\varepsilon_p^{p,k}]\,\overline{\zeta}_p^{k,\omega}\Big\}}{\Omega\,\mathbb{E}\big(k_{p'}\,\big|\,p'>p^*\big)}\frac{dR(p)}{1-R(p^*)}\\ &\quad-\frac{k_{p^*}\Big\{[1-(1+\varepsilon_{p^*}^{p,k})p^*]\,\upsilon^\star-\Omega[\omega(p^*)+\omega'(p^*)p^*\varepsilon_{p^*}^{p,k}]\,\overline{\zeta}_{p^*}^{k,\omega'}\Big\}}{\Omega\,\mathbb{E}\big(k_{p'}\,\big|\,p'>p^*\big)}\frac{r(p^*)}{1-R(p^*)}.\end{aligned} \tag{23}$$

The first term captures the mechanical effect of higher equity requirements, which reduces the likelihood of a bailout. The second and third terms summarize the behavioral effects of higher risk weights for loans with

$p > p^*$ and $p = p^*$, respectively, which may offset the mechanical effect depending on the slope of $\omega(p)$.

Counteracting reforms. Using Equation (20), we obtain a sequence of lump-sum perturbations $\hat{\omega}_c(p^*)$ that counteracts the effects of elementary reforms $\hat{\omega}(p^*)$ on the bailout cutoff and, thus, removes circularities in banks' responses:

$$\hat{\omega}_c(p^*) = -\frac{\displaystyle\int_{p>p^*} \frac{k_p\big(\Lambda_p \overline{\zeta}_p^{k,\omega} - \Omega\big)}{\Omega\,\mathbb{E}\big(k_{p'}\,\big|\,p' > p^*\big)} \frac{dR(p)}{1-R(p^*)} + \frac{k_{p^*}\Lambda_{p^*}\overline{\zeta}_{p^*}^{k,\omega'}}{\Omega\,\mathbb{E}\big(k_{p'}\,\big|\,p' > p^*\big)} \frac{r(p^*)}{1-R(p^*)}}{\displaystyle\int_p k_p\big(\Lambda_p \overline{\zeta}_p^{k,\omega} - \Omega\big)\,dR(p)}. \tag{24}$$

The combined reform $\hat{\omega}(p^*) + \hat{\omega}_c(p^*)$ has the following first-order impact on bank credit supply and equity:

$$\begin{aligned}
\hat{\mathcal{L}}_c\,(p^*) &= \int_{p>p^*} \frac{k_p\big(1 + \varphi_{\mathcal{L}}^c \Lambda_p - \varphi_{\mathcal{L}}^c\,\Omega/\overline{\zeta}_p^{k,\omega}\big)\overline{\zeta}_p^{k,\omega}}{\Omega\,\mathbb{E}\big(k_{p'}\,\big|\,p' > p^*\big)} \frac{dR(p)}{1-R(p^*)} + \frac{k_{p^*}\big(1 + \varphi_{\mathcal{L}}^c \Lambda_{p^*}\big)\overline{\zeta}_{p^*}^{k,\omega'}}{\Omega\,\mathbb{E}\big(k_{p'}\,\big|\,p' > p^*\big)} \frac{r(p^*)}{1-R(p^*)}, \\
\hat{\mathcal{E}}_c\,(p^*) &= \int_{p>p^*} \frac{k_p\big[\omega(p) + \omega'(p)\,p\,\varepsilon_p^{p,k} + \varphi_{\mathcal{E}}^c \Lambda_p - \varphi_{\mathcal{E}}^c\,\Omega/\overline{\zeta}_p^{k,\omega}\big]\overline{\zeta}_p^{k,\omega}}{\mathbb{E}\big(k_{p'}\,\big|\,p' > p^*\big)} \frac{dR(p)}{1-R(p^*)} \\
&\quad + \frac{k_{p^*}\big[\omega(p^*) + \omega'(p^*)\,p^*\,\varepsilon_{p^*}^{p,k} + \varphi_{\mathcal{E}}^c \Lambda_{p^*}\big]\overline{\zeta}_{p^*}^{k,\omega'}}{\mathbb{E}\big(k_{p'}\,\big|\,p' > p^*\big)} \frac{r(p^*)}{1-R(p^*)},
\end{aligned}$$

where the correction factors now include only own-price effects:

$$\varphi_{\mathcal{L}}^c \equiv -\frac{\displaystyle\int_p k_p\,\overline{\zeta}_p^{k,\omega}\,dR(p)}{\displaystyle\int_p k_p\big(\Lambda_p \overline{\zeta}_p^{k,\omega} - \Omega\big)\,dR(p)}, \qquad \varphi_{\mathcal{E}}^c \equiv -\frac{\displaystyle\int_p \omega(p)\,k_p\,\overline{\zeta}_p^{k,\omega}\,dR(p)}{\displaystyle\int_p k_p\big(\Lambda_p \overline{\zeta}_p^{k,\omega} - \Omega\big)\,dR(p)}.$$

Starting from a flat risk-weight scheme ($\omega(p) = 1$), the corrections are uniform, $\varphi_{\mathcal{L}}^c = \varphi_{\mathcal{E}}^c = \varphi^c$. Redefining the semi-elasticities to incorporate these corrections,

$$\tilde{\zeta}_p^{k,\omega} \equiv \big(1 + \varphi^c \Lambda_p + \varphi^c\,\Omega/\overline{\zeta}_p^{k,\omega}\big)\overline{\zeta}_p^{k,\omega}, \qquad \tilde{\zeta}_p^{k,\omega'} \equiv \big(1 + \varphi^c \Lambda_p\big)\overline{\zeta}_p^{k,\omega'},$$

yields the same expressions as in Corollary 1.

B.4 Proof of Lemma 1

To derive the welfare effects of risk-weight reforms, we apply the envelope theorem for arbitrary choice sets (Milgrom and Segal (2002)). Under monopolistic banking, credit supply, loan pricing, and all operational decisions are locally optimal, implying that behavioral adjustments are second order. However, the bank's welfare still depends mechanically on the equity required by capital regulation. Thus, a reform of the risk-weight schedule induces an inframarginal welfare response:

$$\hat{\mathcal{V}}^B = -\,\mu \int_\Theta \hat{\omega}(p_\theta)\, k_\theta \, d\theta.$$

Under competitive loan pricing, interest rates $\{r_\theta\}$ adjust such that bank profits are zero for all policy regimes. In this case, price responses exactly offset the mechanical effect of a regulatory reform:

$$\left(\frac{dL^{\text{inner}}}{dr_\theta}\right)\hat{r}_\theta = \mu\, \hat{\omega}(p_\theta)\, k_\theta, \quad \forall\theta.$$

Aggregating across loan types, the total change in bank welfare is therefore zero:

$$\hat{\mathcal{V}}^B = \int_\Theta \frac{dL^{\text{inner}}}{dr_\theta}\, \hat{r}_\theta \, d\theta - \mu \int_\Theta \hat{\omega}(p_\theta)\, k_\theta \, d\theta = 0.$$

For entrepreneurs, the welfare response follows from the same envelope logic. Since each d_θ is optimally chosen, the indirect effect of a reform operates only through induced changes in credit supply:

$$\hat{\mathcal{V}}^E_\theta = \Big(\frac{dU^E_\theta}{dk_\theta} + \rho_\theta^\top \frac{d\mathcal{C}^E_\theta}{dk_\theta}\Big)\hat{k}_\theta = -\,\mu\, \xi^E_\theta\, \hat{k}_\theta.$$

Interpreting loan rates as functions of credit supply, $r_\theta = r_\theta(k_\theta)$, this welfare effect also embeds loan-price adjustments. To highlight this mechanism, one can decompose the response into direct and interest-rate components:

$$\hat{\mathcal{V}}^E_\theta = \Big(\frac{\partial U^E_\theta}{\partial k_\theta} + \rho_\theta^\top \frac{\partial \mathcal{C}^E_\theta}{\partial k_\theta}\Big)\hat{k}_\theta + \Big(\frac{\partial U^E_\theta}{\partial r_\theta} + \rho_\theta^\top \frac{\partial \mathcal{C}^E_\theta}{\partial r_\theta}\Big)\frac{dr_\theta}{dk_\theta}\, \hat{k}_\theta = -\,\mu\, (\xi^E_{\theta,k} + \xi^E_{\theta,r})\, \hat{k}_\theta.$$

Hence, interest-rate movements redistribute surplus between entrepreneurs and banks, giving rise to *distributive pecuniary externalities* in the sense of Dávila and Korinek (2018). Their total welfare impact at the loan level can be expressed as

$$\alpha^B \left(\frac{dL^{\text{inner}}}{dr_\theta}\right)\hat{r}_\theta + \alpha^E_\theta \left(\frac{\partial U^E_\theta}{\partial r_\theta} + \rho_\theta^\top \frac{\partial \mathcal{C}^E_\theta}{\partial r_\theta}\right)\hat{r}_\theta.$$

Finally, household welfare responds through the same envelope logic, reflecting the externalities from changes in bank equity and credit supply:

$$\hat{\mathcal{V}}^H = \Big(\frac{dU^H}{d\mathcal{E}} + \gamma^\top \frac{d\mathcal{C}^H}{d\mathcal{E}}\Big)\hat{\mathcal{E}} + \int_\Theta \Big(\frac{dU^H}{dk_\theta} + \gamma^\top \frac{d\mathcal{C}^H}{dk_\theta}\Big)\hat{k}_\theta \, d\theta = -\,\mu\, \chi^H \hat{\mathcal{E}} - \,\mu \int_\Theta \xi^H_\theta \hat{k}_\theta \, d\theta.$$

B.5 Proof of Proposition 2

The unconstrained Pigouvian regulation maximizes (15) absent any market frictions, $\varepsilon_{\theta}^{p,k} = 0$, $\forall\theta$. Using Lemma 1, the welfare response to any reform $\hat{\omega}$ or $\hat{\Omega}$ can be expressed as

$$\hat{\mathcal{W}}/\mu = \int_{\Theta} \left[\alpha^B \omega(p_\theta) - (\alpha_\theta^E \xi_\theta^E + \alpha^H \xi_\theta^H)\right] \hat{k}_\theta d\theta - \left(\alpha^B/\Omega + \alpha^H \chi^H\right) \hat{\mathcal{E}}.$$

Under credit-market competition, distributive pecuniary effects add to these welfare responses. For clarity, we assume they are second-order for welfare. Therefore, the first-best regulation satisfies $\alpha^B/\Omega + \alpha^H\chi^H = 0$ and $\alpha^B\omega(p) - (\alpha_p^E\xi_p^E + \alpha^H\xi_p^H) = 0$.

B.6 Proof of Proposition 3

In the second-best case, the welfare effect of an arbitrary regulatory reform $\hat{\omega}$ or $\hat{\Omega}$ is given by

$$\hat{\mathcal{W}}/\mu = \int_{\theta\in\Theta} \left[\alpha^B[\omega(p_\theta) + \omega'(p_\theta)p_\theta\varepsilon_\theta^{p,k}] - (\alpha_\theta^E \xi_\theta^E + \alpha^H \xi_\theta^H)\right] \hat{k}_\theta d\theta - \left(\alpha^B/\Omega + \alpha^H \chi^H\right) \hat{\mathcal{E}}.$$

Again, we omit distributive pecuniary effects. While the condition for the optimal capital adequacy ratio remains unchanged relative to the first-best benchmark, the optimal risk-weight schedule follows an ordinary differential equation:

$$\omega(p) + \omega'(p)p\varepsilon_p^{p,k} = (\alpha_p^E/\alpha^B)\xi_p^E + (\alpha^H/\alpha^B)\xi_p^H \equiv b(p).$$

Using the variation of parameters, we obtain the general solution to the ODE

$$\omega(p) = \exp\left[-\int_{\underline{p}}^{p} 1/(p'\varepsilon_{p'}^{p,k})dp'\right]\left(C_0 + \int_{\underline{p}}^{p} b(p')/(p'\varepsilon_{p'}^{p,k}) \exp\left[\int_{\underline{p}}^{p} 1/(p''\varepsilon_{p''}^{p,k})dp''\right] dp'\right).$$

Accordingly, we define $\gamma(p';p) \equiv \exp[\int_{p}^{p'} 1/(\varepsilon_{p''}^{p,k}p'')dp'']$. For a constant risk-taking elasticity, $\varepsilon_p^{p,k} = \varepsilon^{p,k}$, it is easy to verify that $\gamma(p';p) = (p'/p)^{1/\varepsilon^{p,k}}$. Noting that $\omega(\underline{p}) = C_0$, the general solution reads as

$$\omega(p) = \int_{\underline{p}}^{p} b\left(p'\right)\gamma'(p';p)dp' + \gamma(\underline{p};p)\omega(\underline{p}).$$

To pin down the value of the constant C_0, we note from the ODE that the optimal bottom marginal risk weight is $\omega(\underline{p}) = b(\underline{p})$ for $\underline{p} \to 0$. Integrating by parts, the formula for the optimal risk-weight schedule simplifies to

$$\omega(p) = b(p) - \int_0^p b'(p')\gamma(p';p)dp'.$$

B.7 Third-Best Regulation

In addition to market failures, $\varepsilon_p^{p,k} > 0$ for some p, certain dimensions of bank regulation may be structurally constrained and, as a result, imperfectly chosen, as in Dávila and Walther (2021). In this spirit, we derive optimal risk weights under market imperfections when the capital adequacy ratio is exogenously fixed at a potentially suboptimal level Ω.

Proposition 4. *Suppose the regulator maximizes aggregate social welfare, market failures exist, and the capital-adequacy ratio is fixed at Ω. Defining $\gamma(p';p) \equiv \exp[\int_p^{p'} 1/(\varepsilon_{p''}^{p,k} p'')dp'']$, the optimal risk-weight schedule satisfies*

$$\omega(p) = b(p) - \int_0^p b'(p')\,\gamma(p';p)\,dp', \tag{25}$$

where

$$b(p) \equiv \frac{(\alpha_p^E/\alpha^B)\xi_p^E + (\alpha^H/\alpha^B)\xi_p^H}{-\Omega(\alpha^H/\alpha^B)\chi^H} + \frac{\Delta_\Omega}{-(\alpha^H/\alpha^B)\chi^H}\frac{\mathbb{E}[\gamma(p;p')k_{p'}/k_p|p' \geq p]}{\overline{\zeta}_p^{k,\omega'} r(p)/[1-R(p)]},$$

captures the welfare externalities from credit supply, and $\Delta_\Omega \equiv -(\alpha^H/\alpha^B)\chi^H - 1/\Omega$ is the Pigouvian wedge from bank equity.

The optimal risk weights correct the welfare externalities from credit supply, summarized by $b(p)$. As in the second-best setting, the presence of market failures implies an adjustment for linear dependence measured through $\varepsilon_\theta^{p,k}$. Compared to the second best, the externality term $b(p)$ now incorporates a correction for imperfectly chosen bank equity. When the equity subsidy rate is too low, a positive Pigouvian wedge $\Delta_\Omega > 0$ arises. In that case, banks fail to internalize the externality of their equity choice on households fully, and equilibrium bank equity is inefficiently low from a social perspective. Risk weights can mitigate this wedge by affecting bank equity in the regulatory constraint. The optimal risk-weight schedule is therefore adjusted to handle this policy imperfection. For $\Delta_\Omega = 0$, we recover (17).

While the adjustment term captures complex interactions between market and policy failures,[25] its properties are intuitive. Importantly, the size of adjustment decreases with the magnitude of policy distortions from risk weights, resembling inverse elasticity formulas in optimal taxation (see Diamond (1998); Saez

[25] The proof below shows that an integro-differential equation characterizes the optimal risk-weight schedule. It balances welfare gains and losses from changes in average and marginal risk weights, given an imperfectly set capital ratio. As before, we assume the lower bound of credit risk approaches zero ($\underline{p} \to 0$) to solve for the particular solution to the differential equation.

(2001)). This distortion is measured by the credit-supply semi-elasticity with respect to marginal risk weights. The elasticity is weighted by the hazard rate of credit risk, $r(p)/[1 - R(p)]$, to capture the relative magnitude of behavioral responses at each point in the risk distribution. A stronger behavioral response implies that risk weights are more effective at addressing welfare externalities from bank equity, thereby reducing optimal risk weights. Conversely, optimal risk weights rise with the share of high-risk exposures, $\mathbb{E}[\gamma(p;p')k_{p'}/k_p|p' \geq p]$. This measure of credit-risk granularity reflects the strength of market failures when correcting suboptimal bank equity.

Altogether, the regulatory optimum inherits central features of classical tax formulas in public finance and clarifies the main trade-off of bank capital regulation. On the one hand, regulation should correct all welfare-relevant externalities of credit supply. On the other hand, the corrective motives embedded in the risk-weight schedule must be balanced against its effectiveness in addressing regulatory and market imperfections.

Proof. We now prove Proposition 4. First, we derive welfare effects from an elementary risk-weight reform, showing that the optimal schedule satisfies a differential equation. We then solve this equation using the method of variation of parameters.

When the capital-adequacy ratio is chosen suboptimally, welfare effects from changes in bank equity must be accounted for. Using Proposition 1, the welfare response to a reform of the risk-weight schedule is

$$\begin{aligned}\hat{\mathcal{W}}/\mu = &\int_{\theta\in\Theta}\Big[-\alpha^H\chi^H\Omega[\omega(p_\theta)+\omega'(p_\theta)p_\theta\varepsilon_\theta^{p,k}] - (\alpha_\theta^E\xi_\theta^E + \alpha^H\xi_\theta^H)\Big]\hat{k}_\theta d\theta \\ &- \left(\alpha^B/\Omega + \alpha^H\chi^H\right)\Omega\int_{\theta\in\Theta}\hat{\omega}(p_\theta)k_\theta d\theta.\end{aligned}$$

To characterize the optimal nonlinear risk-weight schedule in sufficient-statistics form, we consider the class of elementary reforms (12) and abstract from across-asset substitution effects. The aggregate welfare effect of increasing the risk weight at p^* is then

$$\begin{aligned}\hat{\mathcal{W}}(p^*)/\mu = &-\left(\alpha^B/\Omega + \alpha^H\chi^H\right) + \frac{\int_{p>p^*} k_p\left[-\alpha^H\chi^H\Omega[\omega(p)+\omega'(p)p\varepsilon_p^{p,k}] - (\alpha_p^E\xi_p^E+\alpha^H\xi_p^H)\right]\overline{\zeta}_p^{k,\omega}dR(p)}{\Omega\int_{p>p^*}k_p dR(p)} \\ &+ \frac{k_{p^*}\left[-\alpha^H\chi^H\Omega[\omega(p^*)+\omega'(p^*)p^*\varepsilon_{p^*}^{p,k}] - (\alpha_{p^*}^E\xi_{p^*}^E+\alpha^H\xi_{p^*}^H)\right]\overline{\zeta}_{p^*}^{k,\omega'}r(p^*)}{\Omega\int_{p>p^*}k_p dR(p)}.\end{aligned}$$

Setting welfare effects to zero $\hat{\mathcal{W}}(p^*) = 0$, the optimal risk-weight schedule satisfies the following integro-differential equation:

$$
\begin{aligned}
&-\left[-\alpha^H\chi^H\Omega[\omega(p^*)+\omega'(p^*)p^*\varepsilon_{p^*}^{p,k}]-(\alpha_{p^*}^E\xi_{p^*}^E+\alpha^H\xi_{p^*}^H)\right]k_{p^*}\overline{\zeta}_{p^*}^{k,\omega}r\left(p^*\right)\\
&=\frac{\overline{\zeta}_{p^*}^{k,\omega}}{\overline{\zeta}_{p^*}^{k,\omega'}}\left[\int_{p>p^*}\left[-\alpha^H\chi^H\Omega[\omega(p)+\omega'(p)p\varepsilon_p^{p,k}]-(\alpha_p^E\xi_p^E+\alpha^H\xi_p^H)\right]k_p\overline{\zeta}_p^{k,\omega}dR\left(p\right)\right.\\
&\left.+\Omega\left(\alpha^B/\Omega+\alpha^H\chi^H\right)\int_{p>p^*}k_p dR\left(p\right)\right]
\end{aligned}
$$

This expression trades off welfare effects from marginal and average risk weights.

We can rewrite this equation as a first-order differential equation $K'(p^*)=D(p^*)[K(p^*)+C(p^*)]$ with boundary condition $K(\overline{p})=0$. Using the method of variation of constants, the solution is

$$
K(p^*)=-\int_{p^*}^{\overline{p}}D(p)C(p)\exp\left[-\int_{p^*}^{p}D(p')dp'\right]dp.
$$

Integration by parts yields

$$
K(p^*)=-\int_{p^*}^{\overline{p}}C'(p)\exp\left[-\int_{p^*}^{p}D(p')dp'\right]dp-C(p^*).
$$

Differentiating gives

$$
\begin{aligned}
K'(p^*)&=C'(p^*)\exp\left[-\int_{p^*}^{p^*}D(p')dp'\right]-D(p^*)\int_{p^*}^{\overline{p}}C'(p)\exp\left[-\int_{p^*}^{p}D(p')dp'\right]dp-C'(p^*)\\
&=-D(p^*)\int_{p^*}^{\overline{p}}C'(p)\exp\left[-\int_{p^*}^{p}D(p')dp'\right]dp.
\end{aligned}
$$

Recall that $C(p)=\Omega(\alpha^B/\Omega+\alpha^H\chi^H)\int_{p>p^*}k_p dR(p)$, so $C'(p)=-\Omega(\alpha^B/\Omega+\alpha^H\chi^H)k_p r(p)$, and that $D(p^*)=1/(\varepsilon_{p^*}^{p,k}p^*)$. Hence, the optimal risk-weight schedule satisfies

$$
\begin{aligned}
&-\left(-\alpha^H\chi^H\Omega[\omega(p^*)+\omega'(p^*)p^*\varepsilon_{p^*}^{p,k}]-(\alpha_{p^*}^E\xi_{p^*}^E+\alpha^H\xi_{p^*}^H)\right)k_{p^*}r(p^*)\\
&=\frac{\Omega(\alpha^B/\Omega+\alpha^H\chi^H)}{\overline{\zeta}_{p^*}^{k,\omega'}}\int_{p^*}^{\overline{p}}k_p\exp\left[-\int_{p^*}^{p}\zeta_{p'}^{k,\omega}/\zeta_{p'}^{k,\omega'}dp'\right]dR(p),
\end{aligned}
$$

which can be expressed as a first-order inhomogeneous differential equation:

$$
\omega'(p^*)+\omega(p^*)/(\varepsilon_{p^*}^{p,k}p^*)=b(p^*)\,/(\varepsilon_{p^*}^{p,k}p^*), \tag{26}
$$

where

$$b(p) \equiv \frac{\alpha_p^E \xi_p^E + \alpha^H \xi_p^H}{-\Omega \alpha^H \chi^H} - \frac{\alpha^B/\Omega + \alpha^H \chi^H}{-\alpha^H \chi^H \overline{\zeta}_p^{k,\omega'}} \frac{1-R(p)}{r(p)} \mathbb{E}\left[\gamma(p;p')\frac{k_{p'}}{k_p}\middle| p' \geq p\right].$$

Applying the variation of parameters yields the general solution to (26):

$$\omega(p^*) = \exp\left[-\int_{\underline{p}}^{p^*} 1/(\varepsilon_{p'}^{p,k} p')dp'\right]\left(C_0 + \int_{\underline{p}}^{p^*} b(p)/(\varepsilon_p^{p,k} p)\exp\left[\int_{\underline{p}}^{p} 1/(\varepsilon_{p'}^{p,k} p')dp'\right]dp\right).$$

As in the second best, define $\gamma(p';p) \equiv \exp[\int_p^{p'} 1/(\varepsilon_{p''}^{p,k} p'')dp'']$. Since $\omega(\underline{p}) = C_0$, we can rewrite the expression as

$$\begin{aligned}\omega(p^*) &= \exp\left[-\int_{\underline{p}}^{p^*} 1/(\varepsilon_{p'}^{p,k} p')dp'\right]\int_{\underline{p}}^{p^*} b(p)/(\varepsilon_p^{p,k} p)\exp\left[\int_{\underline{p}}^{p} 1/(\varepsilon_{p'}^{p,k} p')dp'\right]dp + \gamma(\underline{p};p^*)\,\omega(\underline{p}) \\ &= \int_{\underline{p}}^{p^*} b(p)\gamma'(p;p^*)dp + \gamma(\underline{p};p^*)\omega(\underline{p}).\end{aligned}$$

Letting $\underline{p} \to 0$, the bottom risk weight is $\omega(\underline{p}) = b(\underline{p})$. Integrating by parts then yields the closed-form expression for the optimal risk-weight schedule:

$$\omega(p^*) = b(p^*) - \int_0^{p^*} b'(p')\,\gamma(p';p^*)\,dp'.$$

□

C Appendix for Section 4

As a complementary exercise, we re-estimate the key elasticities using data from the U.S. syndicated loan market. While the German administrative data provide a more comprehensive coverage of all bank–firm relationships, the U.S. setting offers an informative benchmark, particularly for assessing the implications of the Federal Reserve's proposed reform.

C.1 Data

Syndicated loans are a primary source of corporate financing in the United States, with total committed credit exceeding \$6 trillion in 2023 (see Board of Governors of the Federal Reserve System (2024)). Already before the global financial crisis, 90% of the 500 largest U.S. public firms had obtained syndicated credit (Sufi (2007)); this share now approaches 100% as the market has expanded more than fivefold since then. We use data from LPC Loan Connector (formerly Dealscan) covering syndicated loan issuances, including

Panel A: Loan-Level Characteristics					
Variable	Mean	Median	St. Dev.	Min	Max
Loan volume (log)	19.14	19.23	1.50	11.51	24.62
Maturity (in months)	48	60	21.75	1	361
Interest spread (in bps)	211.40	175	147.15	17.5	750
Panel B: Borrower Characteristics					
Total assets (in billion \$)	7.35	1.17	21.2	0	158.69
Total debt (log)	18.93	19.30	2.78	11.35	24.37
Bank debt share (in %)	46.43	42.62	38.84	0	100
PoD	0.04	0	0.15	0	0.89
Risk weight (imputed)	1.01	1	0.18	0.2	1.5

Table 7: Summary statistics (U.S. syndicated loan market).

information on lender and borrower identities, loan volumes, pricing terms, and other contractual features. Unlike the German administrative data, the U.S. data do not include banks' internal default-probability estimates (p_θ). To enrich the dataset, we link borrowers to Compustat/CRSP and Capital IQ, which provide firm-level balance sheet information, debt structure, and performance measures.

Table 7 summarizes the U.S. data, with loan characteristics reported in Panel A and borrower characteristics in Panel B. Panel A shows that the average loan size is roughly \$200 million, underscoring the importance of the syndicated loan market in corporate finance. The typical loan has a maturity of four to five years, and borrowers pay a spread of 175–225 basis points above the reference rate. Panel B indicates that the average firm in our sample has total assets of about \$7 billion. On average, 46% of these firms' debt is provided by banks—a share that varies widely across firms. The average probability of default (PD), described below, is around 4%, with a large mass of firms exhibiting near-zero default risk. Imputed risk weights based on external credit ratings average around 1.[26]

To obtain the PD estimates and thus empirically characterize the credit-risk distribution, we compute the distance to default ($DtD_{i,t}$) for each firm-quarter following the methodology of Bharath and Shumway (2008):

$$DtD_{i,t} = \frac{\log((E+F)/F) + (r_{i,t} - 0.5\sigma_v^{naive})}{\sigma_v^{naive}},$$

where E denotes the market value of equity, F the face value of debt, $r_{i,t}$ the stock return over the previous year, and σ_v^{naive} a proxy for firm-value volatility. We compute

$$\sigma_v^{naive} = \frac{E}{E+F}\sigma_E + \frac{F}{E+F}(0.05 + 0.25\sigma_E),$$

where σ_E is the standard deviation of past equity returns, 0.05 approximates term-structure volatility, and

[26]The imputation procedure is described in Section 4.2.

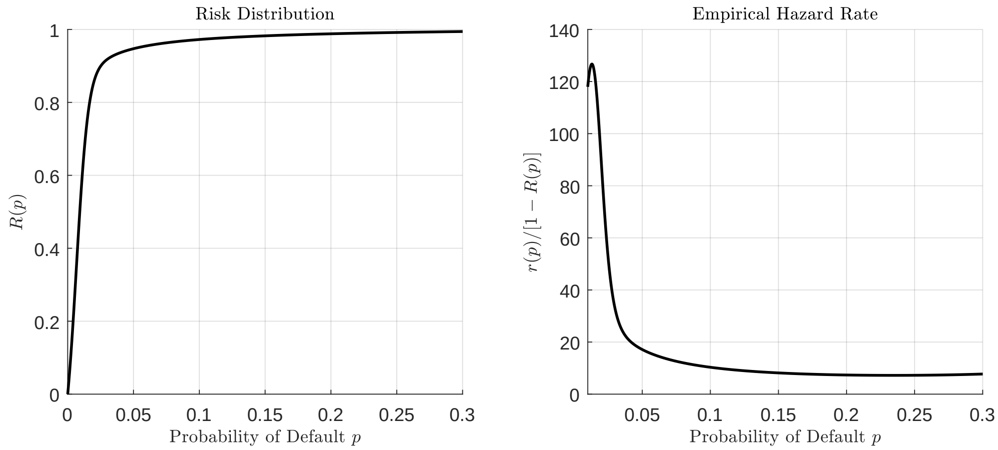

Figure 6: Credit risk distribution (U.S. syndicated loan market).

debt volatility is assumed to be one-quarter of equity volatility (see Bharath and Shumway (2008)). Because all inputs vary at a quarterly frequency, we obtain a time series of firm-level credit-risk distributions. We use 2019Q4 as the benchmark quarter to inform the model with a representative cross-section. Finally, we convert the distance-to-default measure into a probability of default, $p_\theta = G_{\mathcal{N}}(-DtD)$, where $G_{\mathcal{N}}$ is the standard normal cumulative distribution function. The resulting distribution of p_θ is shown in Figure 6.

C.2 Estimation

While the elasticity estimates based on the German administrative data serve as our benchmark, we complement them with estimates for the U.S. economy. The U.S. data, though less comprehensive, primarily capture the upper tail of the firm-size distribution and rely on market-based measures of default risk (see Section 4.1). Nonetheless, analyzing the U.S. syndicated loan market is informative, as it represents one of the key segments the Federal Reserve considers when calibrating its proposed reform of the risk-weight schedule evaluated below.

Credit-risk elasticity. To estimate $\varepsilon_p^{p,k}$ using U.S. data, we face the same identification challenges discussed in Section 4.3: isolating exogenous variation in credit supply to measure its effect on firm default probabilities. A direct analogue to the German bank levy, which shifted aggregate credit supply, is difficult to identify in the U.S. context. However, unlike the mostly bank-dependent firms in Germany, the large, publicly listed firms in our U.S. sample rely substantially on bond-market financing. We therefore exploit a credit-supply shock originating in the corporate bond market: the Federal Reserve's intervention following the COVID-19 shock. Beginning in 2020Q2, the Fed established special-purpose vehicles to purchase corporate bonds in both primary and secondary markets, shifting the demand curve for corporate bonds outward (Gilchrist et al. (2020)). This market-wide intervention was implemented in response to aggregate financial stress rather than firm-specific fundamentals, thus addressing the endogeneity concern discussed above.

Importantly, the intervention's impact was heterogeneous across firms: those more reliant on bond-market financing (and less reliant on bank loans) benefited more strongly. We capture this differential exposure through a shift-share design based on firms' pre-crisis debt composition. The first-stage and second-stage specifications are:

$$\log(debt_{i,t}) = \gamma\big(ShareOfBankDebt_{i,2019Q4} \times FedIntervention_t\big) + \beta X_{i,t-1} + \mu_i + v_{i,t},$$

$$\log(p_{i,t}) = \varepsilon_p^{p,k} \widehat{\log(debt_{i,t})} + \beta X_{i,t-1} + \mu_i + \mu_t + u_{i,t},$$

where $\log(debt_{i,t})$ is the logarithm of firm i's total debt, μ_i and μ_t denote firm and time fixed effects, $ShareOfBankDebt_{i,2019Q4}$ measures the share of bank-provided debt in 2019Q4, and $FedIntervention_t$ is a dummy equal to one for quarters starting in 2020Q2. The vector $X_{i,t-1}$ includes lagged firm-level controls: the logarithm of total assets, sales, and net income, the current ratio, the cash-to-assets ratio, and leverage. Probabilities of default, $p_{i,t}$, are obtained by translating firms' credit ratings into PDs using the S&P Global Fixed Income Tables (S&P Global (2024)).

Credit-supply elasticities. To identify $\varepsilon_p^{k,p}$ and $\zeta_p^{k,\omega}$, we generate exogenous variation in p and ω, as outlined in Section 4.3. In the U.S. setting, we cannot directly observe banks' internal default-risk estimates that feed into regulatory formulas. We therefore approximate p using market-implied probabilities of default and construct risk weights ω from the Basel Committee's standardized mapping of external credit ratings (see Table 3). Although these measures are coarser than those in the German dataset, systematic measurement error should not arise if market prices on average reflect default probabilities accurately and the Basel mapping correctly represents regulatory capital constraints.

Because the U.S. data are reported at the firm rather than the bank–firm level, the Khwaja and Mian (2008) identification strategy cannot be implemented. To disentangle supply from demand, we instead follow the approach proposed by Degryse et al. (2019). We include industry-by-time fixed effects ($\mu_s \times \mu_t$) and assume that firms within the same three-digit industry face a common demand for credit at each point in time. Variation in $\varepsilon_p^{k,p}$ then captures differences in credit supply across banks (bank fixed effects) to firms within the same industry that differ in their time-varying default probabilities, controlling for time-invariant firm characteristics (firm fixed effects). The coefficient $\zeta_p^{k,\omega}$ measures the differential credit supply by the same bank when facing two borrowers from the same industry at the same time with identical economic risk ($\log(p_{i,t-1})$), holding constant firm-specific heterogeneity.

Sufficient Statistic	Market Side	Estimate for the U.S.
elasticity of default risk w.r.t. credit supply	entrepreneurs	$\varepsilon_p^{p,k} = 0.177$
credit-supply elasticity w.r.t. default risk	banks	$\varepsilon_p^{k,p} = -0.002$
credit-supply semi-elasticity w.r.t. average risk weight	banks	$\zeta_p^{k,\omega} = -0.399$

Table 8: Overview of estimated elasticities (U.S. syndicated loan market).

To mitigate simultaneity between credit supply and default risk, we lag the probability of default by one quarter, ensuring that feedback effects through $\varepsilon_p^{p,k}$ do not bias our estimates. The estimation equation is:

$$\log(credit_{b,i,t}) = \varepsilon_p^{k,p} \log(p_{i,t-1}) + \zeta_p^{k,\omega} \omega_{i,t-1} + X_{b,t-1} + \mu_b + \mu_i + \mu_s \times \mu_t + u_{i,t},$$

where $\log(credit_{b,i,t})$ denotes the logarithm of the credit volume extended by bank b to firm i at time t. The vector $X_{b,t-1}$ includes lagged bank-level controls: the logarithm of total assets, the equity-to-assets ratio, the deposits-to-assets ratio, and the ratio of non-performing loans to total loans. μ_b, μ_i, and $\mu_s \times \mu_t$ denote bank, firm, and industry-by-time fixed effects, respectively, following the setup of Degryse et al. (2019).

In Table 8, we summarize the estimated elasticities. All parameters are statistically significant at the 1% level. We do not estimate heterogeneity by risk category because both the risk classification and key regressors originate from the same input (the credit rating) and subsample sizes are too small for reliable estimation.

We estimate an elasticity of default risk with respect to leverage of 0.177. A 1% increase in debt raises the probability of default by roughly 0.18%. This coefficient indicates a strong sensitivity of default risk to leverage among U.S. firms. The positive sign is consistent with the benchmark mechanisms of Boyd and De Nicolo (2005) and Gale and Hellwig (1985). Large U.S. firms appear to exhibit stronger risk-shifting or moral-hazard incentives, potentially due to greater convexity in shareholder payoffs or managerial compensation (Meckling and Jensen (1976)).

The credit-supply elasticity with respect to default risk is estimated at –0.002, implying that a 1% increase in borrower risk reduces credit supply by 0.002%. Although smaller in magnitude than the German estimate, it carries the same sign and supports the same benchmark models. Because probabilities of default in the U.S. data are typically an order of magnitude lower than in the German sample, this weaker elasticity is largely mechanical.

Finally, the credit-supply semi-elasticity with respect to the average risk weight is estimated at –0.421. A 10-percentage-point increase in risk weights reduces credit supply by approximately 4.2%. This magnitude aligns closely with the estimates from the French administrative credit registry reported by Fraisse et al.

(2020), who find effects between 2.3% and 4.5%. Hence, banks in the U.S. syndicated loan market exhibit a comparable sensitivity to regulatory capital costs, albeit slightly lower than that of German banks.

While measurement quality and identification strength are somewhat weaker in the U.S. setting, the estimated elasticities share the same signs, significance, and broad magnitudes as those derived from the German data. This consistency supports two conclusions. First, corporate loan markets across countries, despite institutional, scale, and compositional differences, can be described by a similar class of models. Second, our sufficient-statistics approach is robust and adaptable, delivering meaningful insights even in less comprehensive data environments.

D Appendix for Section 5

D.1 Calibrated Model

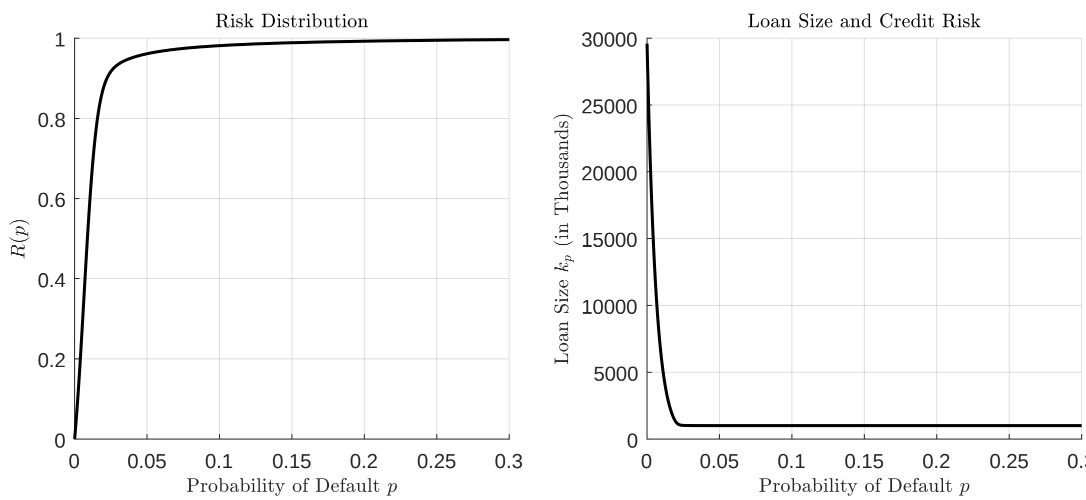

Figure 7: Estimated credit distribution (German credit registry).

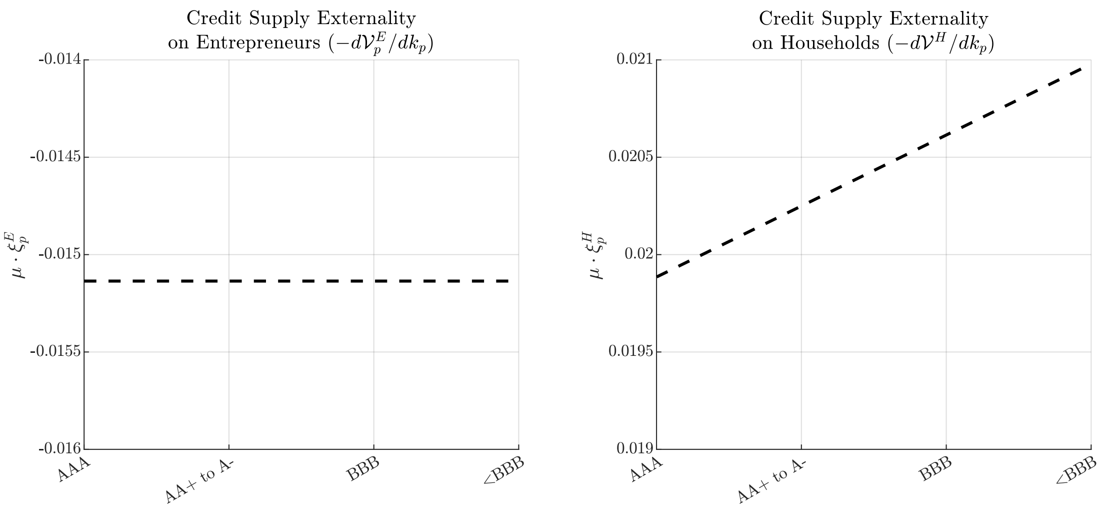

Figure 8: Estimated marginal welfare externalities (German credit registry).

D.2 Robustness

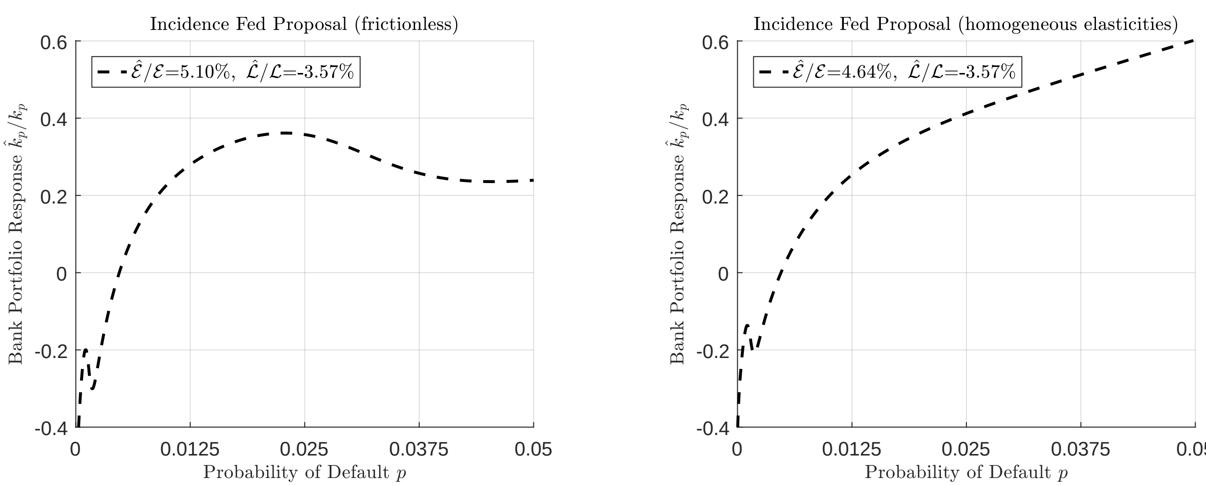

Figure 9: Robustness of Fed proposal incidence; left panel: no credit-market frictions; right panel: homogeneous elasticity estimates (German credit registry).

D.3 Welfare Decomposition

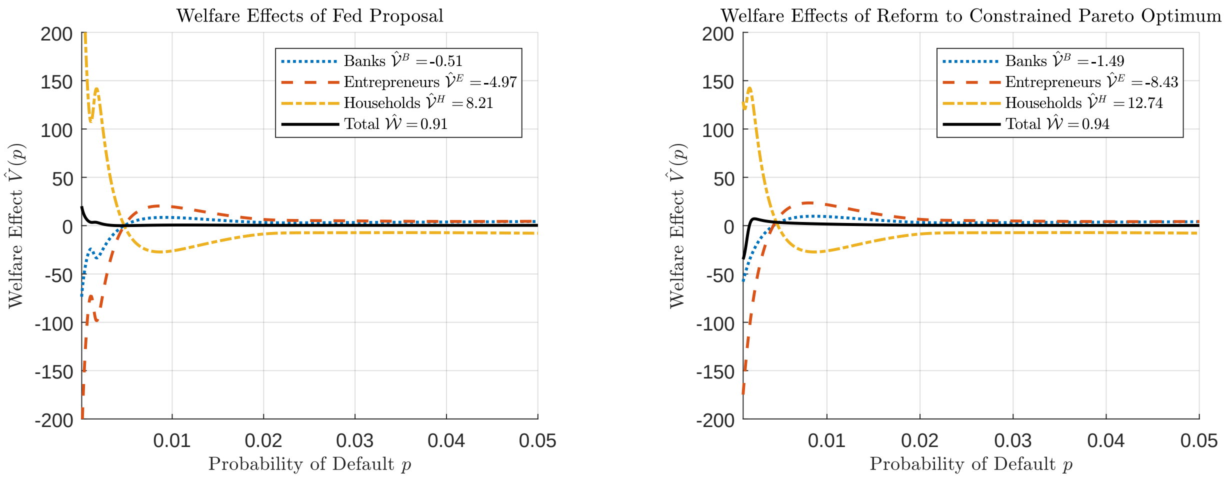

Figure 10: Decomposition of welfare effects (in thousands); left panel: Fed proposal; right panel: constrained Pareto optimum (German credit registry).

D.4 Additional Simulations Based on U.S. Estimates

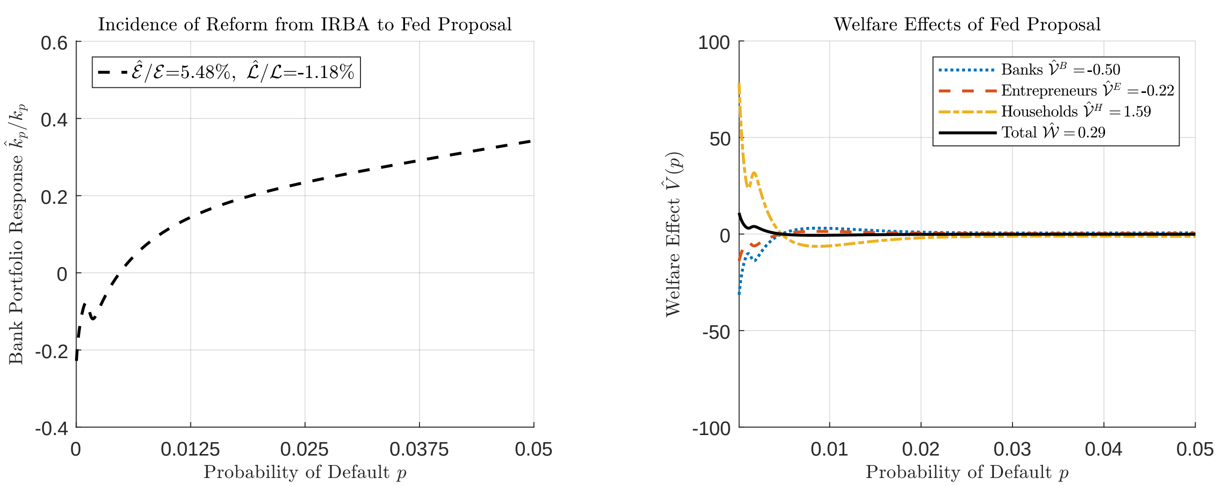

Figure 11: Positive and normative effects of Fed proposal based on U.S. estimates; left panel: loan- and bank-level incidence; right panel: welfare decomposition (in millions) (U.S. syndicated loan market).

D.5 Elementary Risk-Weight Reforms

While the main text focuses on the Fed proposal as a specific policy application, our framework allows for the analysis of arbitrary classes of regulatory reforms. In this section, we study the effects of elementary

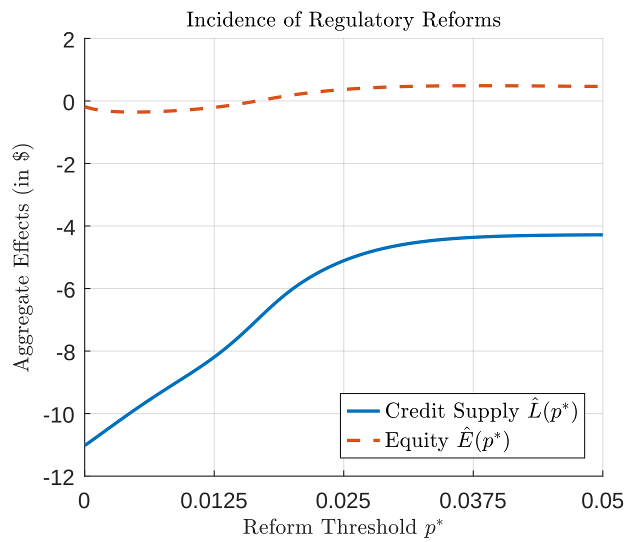

Figure 12: Bank-level effects of elementary reforms of risk weights (German credit registry).

reforms, as defined by equation (12), which raise risk weights at each point in the credit distribution. We employ the same model calibration as in the main text.

Bank-level effects. Figure 12 depicts the effects on bank credit supply (solid line) and equity (dashed line), as given by equations (13) and (14). Consider, for instance, a reform that raises risk weights for all loans with a default probability greater than or equal to 5% ($p^* = 0.05$). Recall that the reform is constructed to have a mechanical effect of one dollar on the bank's required equity. Absent any behavioral adjustments in bank credit supply, equity would therefore increase by one dollar. Hence, the observed value of 0.46 implies that 54¢ of this one-dollar increase in equity is offset by behavioral reductions in credit supply.

Since the capital constraint binds, the lending response on the asset side of the bank's balance sheet is a scaled version of the equity response (even if all risk weights were set to one), with the capital adequacy ratio Ω serving as the scaling factor. The corresponding total reduction in bank credit supply in this case amounts to approximately 4.28 dollars. Across the distribution, a clear pattern emerges: the magnitude of these behavioral offsets declines with credit risk. This pattern is driven by heterogeneous credit-supply responses, as shown in the following.

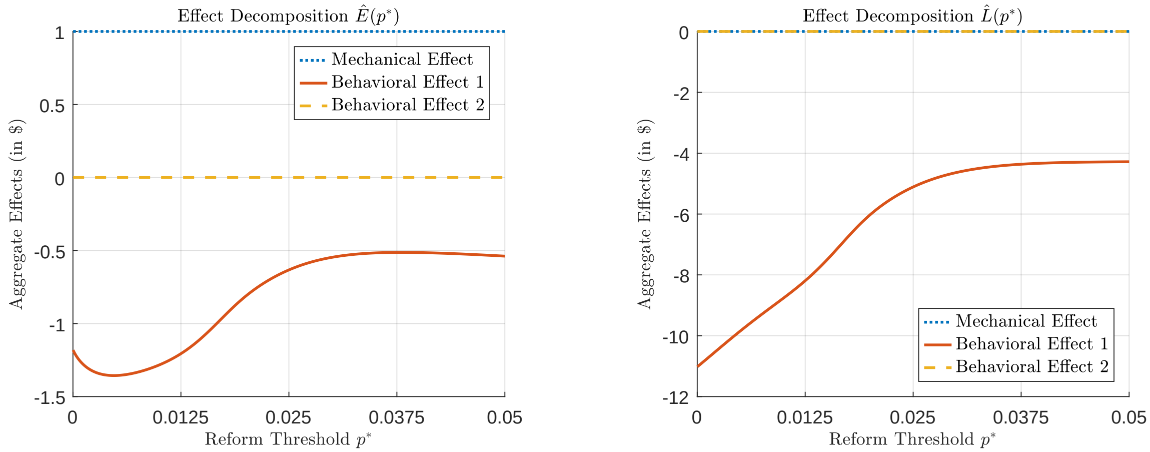

Figure 13: Decomposition of bank-level effects (German credit registry).

Decomposition. In Figure 13, we decompose the total equity and credit-supply responses into their mechanical and behavioral components. The left panel displays the responses of bank equity. Recall that, in general, behavioral effects arise from changes in risk-weight levels, with responses for $p > p^*$ (solid line), and from changes in marginal risk weights, with responses at $p = p^*$ (dashed line). However, since we detect no significant responses to the slope of the schedule, the behavioral adjustments are driven entirely by risk-weight levels.

While the mechanical response is uniform across all levels of credit risk, the behavioral responses exhibit pronounced nonlinearities throughout the risk distribution. As we discuss in more detail below, in addition to the declining credit-supply elasticity, a further source of nonlinearity in the behavioral responses is the curvature of the existing risk-weight schedule.

The right panel depicts the decomposition of credit-supply responses. Note that by construction, there is no mechanical response. Moreover, all behavioral responses are due to risk-weight levels, as for the equity response. However, there is no nonlinearity in behavioral responses inherited from the risk-weight scheme.

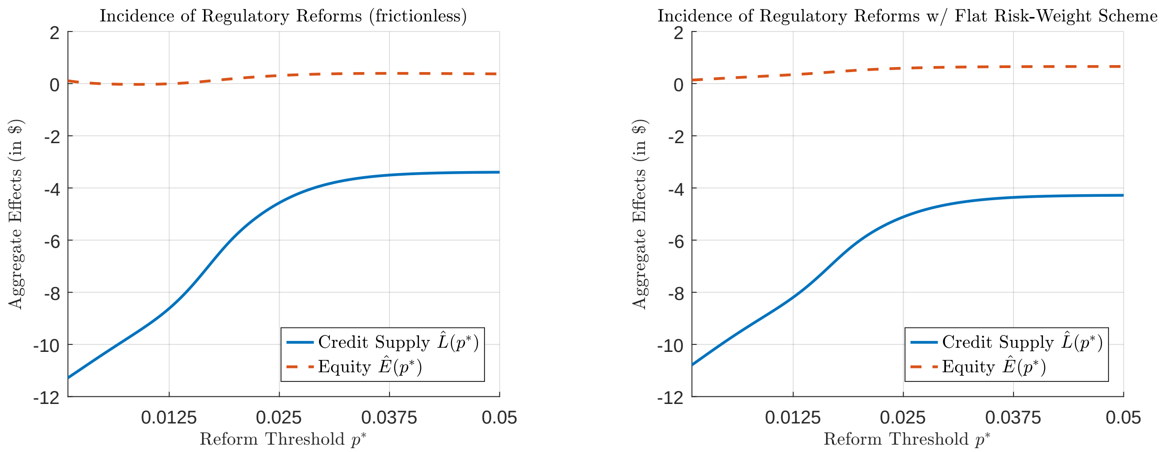

Figure 14: Robustness of elementary reform incidence; left panel: bank-level effects for $\varepsilon_p^{p,k} = 0$, $\forall p$; right panel: bank-level effects for $\omega(p) = 1$, $\forall p$ (German credit registry).

Robustness. While the qualitative features of elementary reforms are intuitive, their quantitative magnitude depends on the estimated credit-risk distribution, the specified elasticities, and the regulatory policies under which they are evaluated. Therefore, in Figure 14, we investigate robustness with respect to some of these measures.

The left panel examines the role of market frictions for the risk-weight incidence. Specifically, we re-estimate the bank-level responses while setting the elasticity of credit risk with respect to entrepreneurial leverage to zero. This corresponds to a frictionless environment in which the bank does not internalize endogenous risk choices when determining credit supply. In this case, credit risk becomes an entrepreneurial characteristic rather than an equilibrium object. We find that the absence of risk feedback dampens the behavioral responses of both equity and credit supply.

In the right panel, we display the regulatory incidence under a flat risk-weight schedule, similar to the standardized approach (SA). While the credit-supply response remains largely unaffected, the effect on equity becomes noticeably flatter. In particular, by attaching less weight to credit-supply changes at the bottom of the risk distribution, positive marginal risk weights in the baseline specification ($\omega' > 0$) attenuate the behavioral response of bank equity. The uniform risk-weight scheme shown in the right panel removes this channel, leading to a substantially flatter equity response.

E Appendix for Section 6

E.1 Non-Smooth Risk-Weight Schedule

To be added

E.2 Large Reforms

To be added

E.3 Multidimensional Heterogeneity

To be added

E.4 Dynamic Setting

To be added