Credit Surfaces and Economic Uncertainty

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1. Introduction

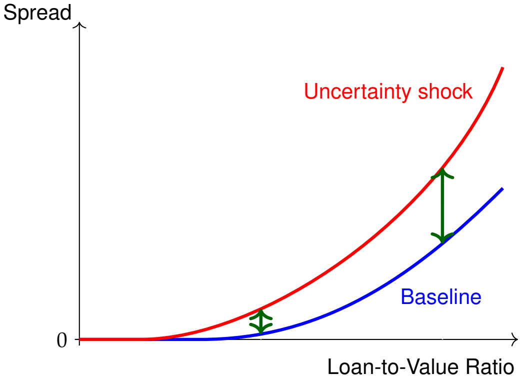
- **Credit conditions** are central to economic activity and economic policy (MP, MPP, housing, etc.)

- Credit conditions are traditionally described by the risk-free rate and a *single* credit spread

- But spreads depend on *multiple* credit terms
  - Leverage, rating, maturity, covenants, . . .

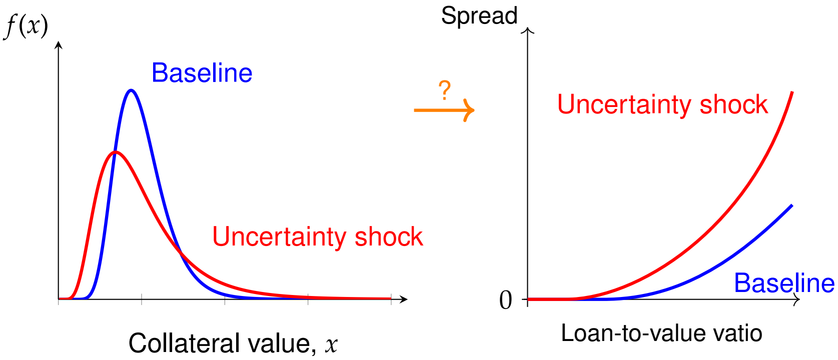
    → **credit surface**

Uncertainty Shocks and Credit Surfaces



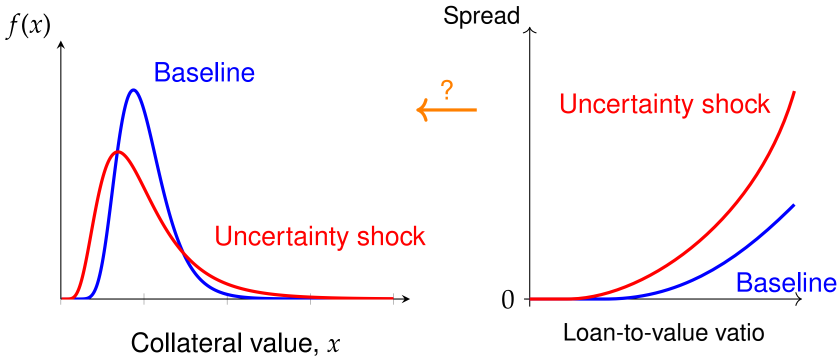
- **Steepening** and credit conditions: the higher the leverage, the higher the increase in spread with **uncertainty shocks**

What Uncertainty Shocks Steepen the Credit Surface?



- What type of **uncertainty shocks** steepen the credit surface?
- For example, if collateral value x is log-normal with std. dev. v. Does an increase in v steepen the credit surface?

What Do Credit Surfaces Reveal about Uncertainty?



- How can we use credit surfaces (bond prices) to **measure uncertainty** about collateral values?
- What does the steepening of **the credit surface reveals** about perception of **uncertainty** about collateral values? firm returns?

Related Literature

- Long tradition understanding the effect of uncertainty shocks on economic activity and prices

  Black and Scholes (1973); Merton (1974); Geanakoplos (1997, 2003, 2010); Fostel-Geanakoplos (2008, 2015); Bloom (2009); Brunnermeier and Pedersen (2009); Adrian and Boyarchenko (2012); Adrian and Shin (2013); . . .

- More recently how financial conditions amplify and make more persistent effect of uncertainty shocks on economic fluctuations

  Christiano, Motto and Rostagno (2014); Gilchrist, Sim and Zakrajšěk (2017); Arellano, Bai and Kehoe (2019); Alfaro, Bloom and Lin (2024)

- Well understood that volatility shocks increase credit spreads

- What is the effect of volatility and other uncertainty shocks on the credit surface?

Related Literature, Continued

- **Models of credit spreads**

  Contingent claim models: Merton (JF 1974); Longstaff & Schwartz (JF 1995); Duffie & Lando (ECTA 2001); Collin-Dufresne & Goldstein (JF 2001); . . .

  GE models: Geanakoplos (1997; 2003; 2010); David (RFS 2007); Fostel & Geanakoplos (AER 2008; ECTA 2015); Simsek (ECTA 2013); Chatterjee, Corbae, Dempsey & Rios-Rull (ECTA 2023); Diamond & Landvoigt (JFE 2022); Ottonello & Winberry (ECTA 2020); Dávila & Walther (AER 2023); . . .

- **Comparative Statics**

  Robust: Milgrom & Roberts (AER 1990); Milgrom & Shannon (ECTA 1994); Athey (QJE 2002); . . .

  Uncertainty: Rothschild & Stiglitz (JET 1970); Levy (IER 1973); Glassermand & Pirjol (2023), . . .

Outline

2. Evidence from Corporate Bond Surfaces

- Data:
  - Option adjusted spreads (OAS) for constituents of investment grade and high yield bond indeces from ICE Data Indices, LLC
  - Leverage ratios from Compustat and CRSP
- Sample: Domestic, non-financial firms rated CCC- or above, bonds with 7-10 years remaining maturity
- Uncertainty shocks—months when VIX is in top decile of its distribution
- Nonparametric estimation of credit spread surface as function of leverage ratio
  - Consider rating groups: AAA and A-; BBB+ and BBB-; BB+ and BB-; B+ and B-; and CCC+ and CCC-
  - For each rating group w and uncertainty regime v, we fit the nonparametric function m_{wv} (> 9,000 obs, 200 bonds)

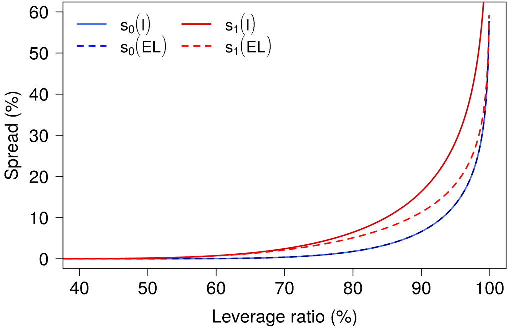
$$s_{it} = m_{wv}(\tilde{j}_{it}) + \varepsilon_{it} \quad \text{with} \quad i \in \mathcal{I}_w \quad \text{and} \quad t \in T_v$$

Empirical Leverage Ratio

- Following empirical corporate finance literature calculate

$$\text{Empirical Leverage (EL)} = \frac{\text{Debt}_B}{\text{Assets}_B - \text{Equity}_B + \text{Equity}_M}$$

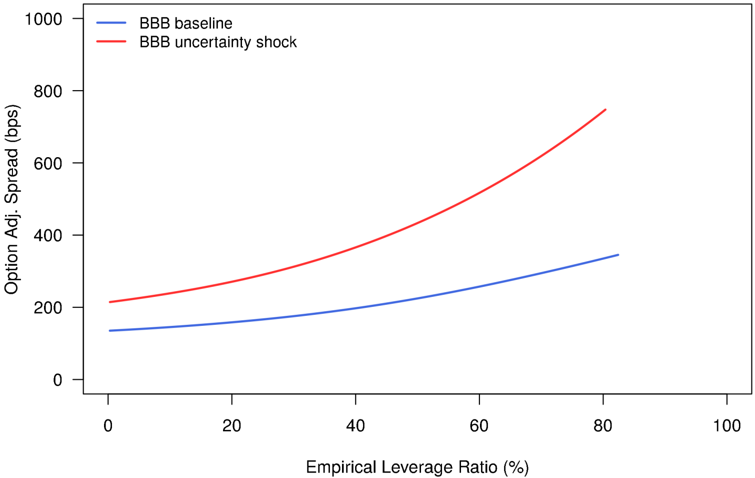
  - Book value equals market value at issuance but evolves independently of subsequent credit conditions



Notes: Log-normal corresponds to $\log X_0 \sim N(-0.02, 0.2^2)$ and $X_1 \sim N(-0.045, 0.5^2)$. Simulation assumes an instantaneous increase in uncertainty after issuance.

Corporate Bond Surface and Uncertainty Shocks

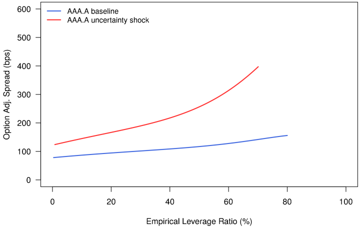
Ratings BBB+ to BBB-



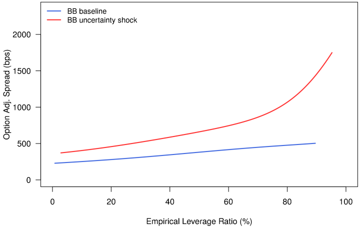
Source: Own elaboration using ICE Bond Indices, CRSSP, CRSP/Compustat, Compustat, and BLS.

Corporate Bond Surface and Uncertainty Shocks

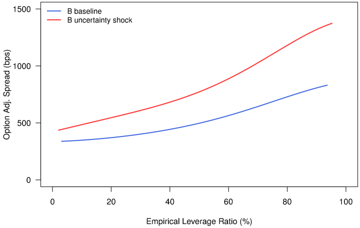
(a) Ratings AAA to A-



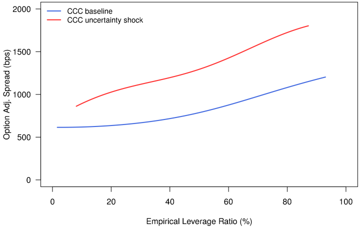
(b) Ratings BB+ to BB-



(c) Ratings B+ to B-



(d) Ratings CCC+ to CCC-



Source: Own elaboration using ICE Bond Indices, CRSSP, CRSP/Compustat, Compustat, and BLS.

3. Uncertainty, Pricing, and Credit Surfaces

- The economy has two periods $t = 0,\ 1$
- In period $t = 0$, agents trade financial assets and risky credit contracts
- Each contract is described by a promised amount $j \geq 0$ to be paid in period $t = 1$ and by the collateral backing the promise
- We assume that there is a risk-neutral measure that will price the collateral's payoffs and the bonds it backs
- Denote the associated cumulative distribution function of X by $F(x)$, the probability that X is x or less
- The distribution F determines the forward price of collateral

$$e \equiv e_F \equiv \int_0^\infty x\, dF(x)$$

Bond Pricing Function

- **Proposition:** Given any distribution F, the bond price function

$$\pi(j) \equiv \pi_F(j) \equiv \int_0^\infty \min\{j, x\}\, dF(x) = j - \int_0^j F(x)\, dx$$

- Is (1) continuous, with $\pi(0) = 0$; (2) concave; (3) continuously differentiable; (4) there are $m < M$ such that $\pi(j) = j$ on $[0, m)$ and π is strictly increasing on $[0, M)$, and flat afterward

- Moreover, F can be recovered using

$$F(j) \equiv 1 - \pi'_+(j)$$

- **Expected loss** of promise j: EL$(j) \equiv \mathbb{E}[j - \min\{j, X\}] = \int_0^j F(x)\, dx$

- The **equity value conditional on it being positive** $n(j) = \mathbb{E}[X - j | X > j]$ (the *mean residual life* in reliability theory)

Leverage Ratios and Dispersion of Collateral Values

- Three standard ratios between amount of debt and value of collateral:

  1. Promise-to-value $PTV_F(j) \equiv y_F(j) = \dfrac{j}{e_F}$

  2. Loan-to-value $LTV_F(j) \equiv \ell_F(j) \equiv \dfrac{\pi_F(j)/(1+r)}{e_F/(1+r)} = \dfrac{\pi_F(j)}{e_F}$

  3. Leverage (A2E) $Lev_F(j) = \dfrac{e_F}{e_F - \pi_F(j)} = \dfrac{1}{1 - \pi_F(j)/e_F} = \dfrac{1}{1 - \ell_F(j)}$

- Standard way of quoting bonds is promised yield or spread $S_F(j)$ (promised return above the riskless rate)

$$1 + S_F(j) \equiv \frac{j/(1+r)}{\pi_F(j)/(1+r)} = \frac{j}{\pi_F(j)} = \frac{j/e_F}{\pi_F(j)/e_F} = \frac{y_F(j)}{\ell_F(j)}$$

- All unchanged if j and the collateral payoffs X are multiplied by a scalar

- Motivates defining the **dispersion** of X, distribution F^D of the normalized random variable X/e_F, $F^D(y) \equiv F(e_F y)$

Credit Surfaces and Dispersion

- Each leverage ratio gives rise to a credit surface—mapping from leverage ratio to spread
- Using the functions induced by the dispersion of collateral values $\ell = \ell_{F^D}(y) = \pi_{F^D}(y)$ and $y = y_{F^D}(\ell) = \pi^{-1}_{F^D}(\ell)$
- Then, the LTV- and PTV-credit surfaces are given by

$$1 + s^*_{F^D}(\ell) = \frac{\pi^{-1}_{F^D}(\ell)}{\ell} = \frac{y_{F^D}(\ell)}{\ell} \qquad \text{and} \qquad 1 + s_{F^D}(y) = \frac{y}{\pi_{F^D}(y)} = \frac{y}{\ell_{F^D}(y)}$$

- The credit surface is determined entirely by the dispersion F^D
- The credit surface reveals all there is to know about the dispersion F^D

$$F^D(y) = F^D(\ell(1 + s^*(\ell))) = 1 - \frac{1}{1 + s^*(\ell) + \ell s^{*\prime}_{+}(\ell)}$$

$$F^D(y) = 1 - \frac{1}{1 + s(y)}\left[1 - \frac{ys'(y)}{1 + s(y)}\right]$$

4. Leverage and The Shape of the Credit Surface

- **Lemma 1:** relates the curvature of g with the curvature of $h(x) = g(x)/x$ (or $= x/g(x)$)
  - If $g'' \geq 0$ (convex), then $h(x) = g(x)/x$ is increasing, if $g''' \geq 0$, then h is convex
  - If $g'' \leq 0$ (concave), then $h(x) = x/g(x)$ is increasing, inequality on g''', imply h is convex
- **Theorem 1:** The LTV-credit surface is increasing and convex if
  1. The density function f is log concave;
  2. The value of equity conditional on it being positive, n, is convex;
  3. For all $x \in (\underline{x}, M)$,
  $$\frac{df(x)}{dx} + \frac{3\,[f(x)]^2}{1 - F(x)} \geq 0\,.$$
  Moreover, condition (1) or (2) imply (3)

The Shape of the LTV-Credit Surface

- **Corollary 1:** The value of equity conditional on it being positive, $n(j)$ is convex, when $X \sim$ exponential, normal (truncated to prevent negative collateral values), power-law, and uniform
- In these cases, the LTV-credit surface is convex
- **Corollary 2:** The density function f is log concave, when $X \sim$ exponential, Gamma, normal, uniform, and Weibull (with shape parameter $k \geq 1$)
- In these cases, the LTV-credit surface is convex
- **Corollary 3:** When $X \sim$ lognormal convexity of the LTV-credit surface requires std. dev. not to be too large or $\log(j(\ell))$ to be sufficiently far from the mean of $\log(X)$

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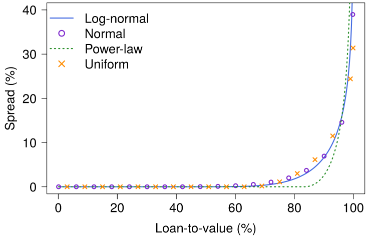
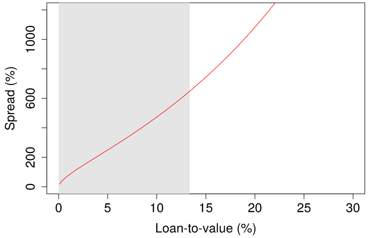
- We provide similar results for the PTV-credit surface in the paper
- The LTV-credit surface is convex under milder assumptions than the PTV- or leverage-credit surfaces

Numerical Simulations: LTV-Credit Surfaces

(a) LTV-Credit Surface, with Different Distribution Functions



(b) LTV-Credit Surface (Log-normal Distribution with High Volatility)



Notes: Left panel, represents log-normal with $\log X \sim N(-0.02, 0.2^2)$; normal with $X \sim N(0.9991, 0.2^2)$, truncated to the left at 3 standard deviations from the mean; power-law corresponds to $X \sim F(x) = 1 - x^{-(\beta-1)}$ for $x \geq 1$ and zero otherwise with $\beta = 7$; and uniform with $X \sim U[0.65, 1.35]$. Right panel, presents an example when the LTV-credit surface if not convex, using a log-normal with $\log X \sim N(-2.3, 3^2)$ (high volatility). Shaded area corresponds to LTV ℓ such that $(\log(j(\ell)) - \mu)/\sigma \in (-\sigma, \sigma/2)$.

5. Uncertainty Shocks and Credit Surfaces

- The difference between the two credit surfaces equals

$$s_1^*(\ell) - s_0^*(\ell) = \frac{y_1(\ell) - y_0(\ell)}{\ell}$$

- From Lemma 1, need $y_1(\ell) - y_0(\ell)$ to be convex so that $s_1^*(\ell) - s_0^*(\ell)$ is increasing (steepening)

- **Theorem 3:**

  1. If F_1^D is a mean-preserving spread of F_0^D, then $y_1(\ell) - y_0(\ell)$ and $s_1^*(\ell) - s_0^*(\ell)$ are positive
  2. If $F_1^D \circ y_1$ is smaller in the hazard rate order than $F_0^D \circ y_0$, then $y_1(\ell) - y_0(\ell)$ is positive, increasing, and convex, and $s_1^*(\ell) - s_0^*(\ell)$ is positive and increasing

Differentiable Changes in Dispersion and LTV-Credit Surfaces

- **Lemma 2:** If, for each fixed v, the map

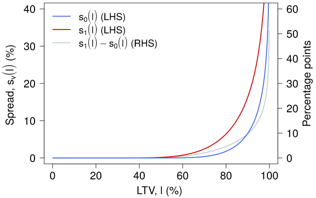
$$y \mapsto K(y,v) = \frac{1}{1 - F^D(y,v)} \int_{\underline{y}_v}^{y(\ell,v)} \frac{\partial F^D(x,v)}{\partial v}\, dx$$

  is convex, then $\ell \mapsto \partial y(\ell,v)/\partial v$ is convex and the LTV-credit surface steepens everywhere as v increases

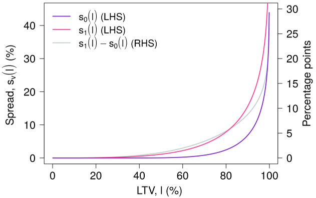
- **Theorem 4:** If Y_v is a proportional mean-preserving spread of Y ($Y_v = 1 + v(Y-1)$) then $K_{yy}(y,v) = n''(y)$, so if n is convex, then the LTV-credit surface steepens everywhere as v increases
  - **Corollary 5:** A proportional mean-preserving spread steepens the LTV-credit surface when $Y \sim$ normal, power-law, and uniform

- **Corollary 6:** The LTV-credit surface steepens everywhere for log-normal distributions as the std. dev. of the log increases, or for power-law distributions as the power coefficient decreases

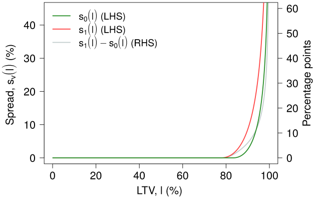
(a) Log-Normal Distributions



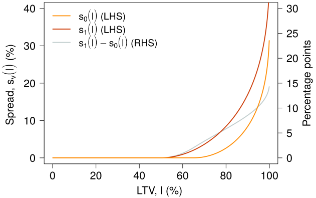
(b) Normal Distributions



(c) Power-Law Distributions

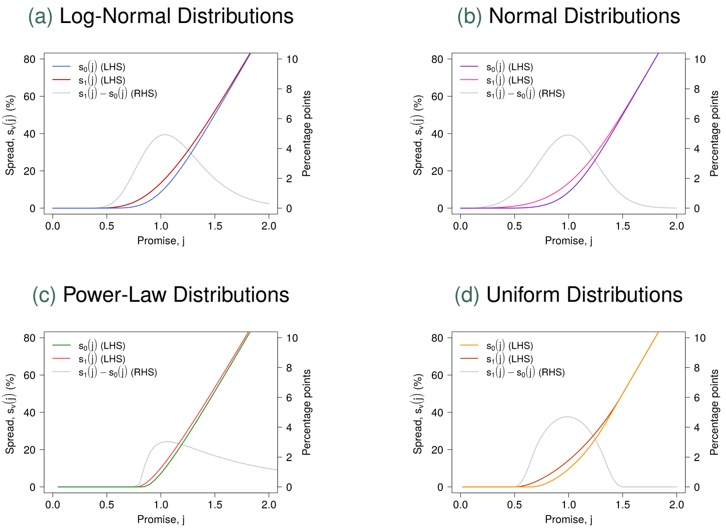


(d) Uniform Distributions



Uncertainty Shocks and PTV-Credit Surfaces

Numerical Illustrations



6. Conclusions

- We present evidence that the bond-credit surface not only moves up, but also steepens with economic uncertainty
- We present a simple model of collateral and equilibrium to explore the general shape of the credit surface along the leverage dimension
- Our results describe a new steepening channel through which uncertainty shocks affect the supply of credit and the macroeconomy. Our results suggest that macroeconomic policy could be more effective by targeting the entire credit surface, rather than its riskless perimeter
- In practice, financial contracts specify multiple credit terms, spanning other dimensions of the Credit Surface. Our analysis provides a suitable point of departure for future research in this area.

We are just starting to scratch the credit surface!

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Thank you!