The Value of Renegotiation Frictions: Evidence from Commercial Real Estate *

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Abstract

Loan modifications play an important role in easing borrowers' financial burdens and mitigating loan losses. However, if borrowers can strategically renegotiate loans, lenders may tighten underwriting to compensate. To analyze this tradeoff, we develop a dynamic model of loan underwriting with frictional renegotiation and calibrate it using loan-level CRE data. We find that modification frictions can account for observed differences in loan underwriting and performance across CRE lenders. High frictions to modifying securitized CRE loans increase debt capacity for these loans. Easing these frictions reduces welfare by restricting the menu of LTVs available in the market.

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1. INTRODUCTION

Deadweight losses in bankruptcy or foreclosure often induce lenders to renegotiate loans that are in distress. While loan modifications can help reduce losses if they replace more costly resolution methods, they may open the door to strategic behavior on the part of borrowers (Hart and Moore, 1998). If borrowers can strategically negotiate concessions that are not needed to prevent default, lenders may increase spreads or restrict other loan terms at origination to mitigate this risk.

Given these counteracting effects, whether borrowers benefit from the ability to renegotiate debt is ambiguous, and depends on how borrowers value favorable at-origination loan terms relative to the dynamic flexibility that comes from renegotiation. The surge in loan modifications during the COVID-19 pandemic and recent policy changes aimed at facilitating loan modifications in various markets make understanding of this tradeoff especially important.[1]

In this paper, we evaluate the equilibrium effects of renegotiation frictions by developing and calibrating a tradeoff model with frictional loan modifications. Lenders offer loan contracts—a schedule of loan spreads and sizes—taking into account borrowers' dynamic incentives to strategically renegotiate loan terms. Motivated by the fact that many markets feature competition between lenders that differ in their ability to modify loans, heterogeneous borrowers endogenously sort into lenders that vary in modification frictions.[2] Borrowers choose the lender and loan contract that provides the best mix of cost, leverage, and flexibility. We use the model to study how modification frictions affect loan performance, credit availability, at-origination loans terms, borrower-lender matching, and borrower welfare.

In the model, modification frictions have an ambiguous effect on loan performance. On one hand, modification frictions lead to worse outcomes for seriously stressed loans, as lenders are less able to prevent default. On the other hand, modification frictions reduce borrowers' expected re-

[1]Appendix A discusses policy statements and rule changes meant to facilitate modifications for bank loans and securitized CRE loans. An et al. (2022) and Kim et al. (2022) discuss recent forbearance policies for residential mortgages.

[2]Bank loans tend to be easier to modify than market-financed debt. This has been discussed in the context of residential mortgages (Piskorski et al., 2010; Agarwal et al., 2011; Adelino et al., 2013), commercial mortgages (Black et al., 2017, 2020), and corporate credit (Gertner and Scharfstein, 1991; Hackbarth et al., 2007).

turns from renegotiation, discouraging them from strategically defaulting at intermediate levels of stress. The disciplining effect of renegotiation frictions in turn increases credit availability. Lenders with higher modification frictions provide debt service concessions only at higher levels of distress and thus are able to provide higher leverage in equilibrium. We show analytically that modification frictions shift out the competitive loan offer curve, enabling borrowers to achieve higher leverage for any given spread. However, the effect modification frictions have on the particular lender and loan contract borrowers choose requires quantitative analysis. How borrowers evaluate the tradeoff between debt capacity and modification ability depends on borrower characteristics—most notably their demand for leverage—and the welfare effects of changing modification frictions depends on the equilibrium menu of loan terms available on the market.

We calibrate the model using loan-level data from the commercial real estate (CRE) market, which is an ideal setting for several reasons. First, contractual and regulatory restrictions lessen commercial mortgage-backed securities (CMBS) servicers' ability to modify loans relative to other CRE lenders. These restrictions contribute to lower modification rates and higher delinquency rates for CMBS relative to banks, especially during downturns (Figure 1). Second, the CRE market is a setting where foreclosure costs are high and borrowers are known to act strategically, making the mechanisms in the model particularly relevant.[3] Finally, the CRE loan market features cross-lender differences in underwriting terms and loan performance consistent with the model. We demonstrate that banks modify CRE loans more often and more preemptively, bolstering the performance of loans securing highly-stressed properties, but enabling concessions for some loans against less-stressed properties.[4] Altogether, the CRE market provides a setting where differences in modification frictions are stark, likely to be impactful, and plausibly a first-order driver of loan market outcomes.

In the calibrated model, borrowers with a higher demand for leverage disproportionately sort into CMBS, as the restrictions to modifying these loans enable borrowers to achieve higher loan-to-

[3] See Brown et al. (2006) for evidence of high foreclosure costs and Flynn Jr. et al. (2021) for evidence of strategic renegotiation.

[4] Additionally, the calibrated model can rationalize the fact that CMBS loans have higher LTVs and spreads compared to those from balance sheet lenders.

value (LTV) ratios.[5] CMBS loans, on average, carry higher spreads and LTVs than loans from balance sheet lenders, consistent with the data. Though balance sheet lenders require higher spreads for any given contract—compensation for expected future modification costs—they make fewer loans to borrowers that will pay a premium for leverage; these borrowers prefer CMBS.

Policies that reduce modification frictions have heterogeneous effects on borrower welfare, depending on borrowers' demand for leverage. Reducing modification frictions for CMBS loans makes CMBS financing comparatively more desirable for most borrowers. However, the benefit to these borrowers is modest, as there are already other lenders (e.g., banks) providing renegotiable debt. Borrowers seeking high LTV loans prefer to borrow from CMBS due to the greater available leverage on such loans. A reduction in CMBS modification frictions is detrimental enough for these borrowers to make welfare decline in aggregate. Our estimates thus imply that making permanent the COVID-era relaxation of CMBS modification restrictions could be detrimental to CRE borrowers, as it would restrict the range of loan terms available in the market.

To understand the generalizability of these results, we revisit some of the simplifying assumptions of the model, specifically that borrowers hold the bargaining power and renegotiate for purely strategic reasons. We view these baseline assumptions as broadly realistic for CMBS borrowers, as CMBS disproportionately cater to large, institutional sponsors with access to multiple lenders and funding sources beyond the cash flow from properties. Indeed, consistent with these patterns, CMBS take on a negligible market share in the model for borrowers with either low bargaining power or severe debt service constraints. Since these borrowers present less of a strategic renegotiation risk, they have little reason to borrow in CMBS markets, and their welfare is relatively insensitive to changes in CMBS lending policies. Altogether, these extensions allow us to further refine our understanding of which borrowers benefit from modification frictions—namely, those with high demand for leverage and an elevated risk of strategic renegotiation—but do not change the broader prediction that borrowers, on average, benefit from the existence of a high-modification-friction lender.

[5] As we model the benefits from leverage as coming from a tax shield, demand for leverage is determined by tax rates. Modification frictions are calibrated using observed delinquency-to-modification ratios in the data.

Our work contributes to multiple strands of the literature. Most broadly, we contribute to corporate finance literature studying the effects of renegotiation on loan contracting. This work shows that liquidation costs enable borrowers to negotiate concessions from creditors (Hart and Moore, 1994, 1998). Since, in equilibrium, the expectation for strategic default is reflected in ex-ante financing terms, firms face a tradeoff between arrangements that discourage strategic defaults and ones that reduce the cost of liquidity defaults (Bolton and Scharfstein, 1990). A large literature uses asset pricing models to evaluate capital structure decisions and debt and equity values in such an environment (see, for example, Anderson and Sundaresan 1996 or Mella-Barral and Perraudin 1997).[6]

We build on this work by incorporating modification frictions to the model. Most existing work assumes that renegotiation either is impossible or is completely costless. Our model nests the renegotiation-free debt of Leland (1994) and the frictionless renegotiation of bank lenders in Hackbarth et al. (2007), while allowing for intermediate degrees of frictions. This more granular measure of renegotiation frictions improves the quantitative relevance of the analysis, as bank and CMBS loans experience both modifications and delinquencies, just in different proportions.[7] Furthermore, this modeling decision allows us to produce comparative statics with respect to modification frictions and to evaluate counterfactual effects from reducing or increasing these frictions.

Our work also contributes to the literature studying mortgage modifications. Regarding commercial mortgages, Flynn Jr. et al. (2021) show that a 2009 easing of CMBS modification restrictions incentivized strategic default. Glancy et al. (2021) show that recourse mitigates strategic renegotiation risk for bank loans and expands the range of contracts available to borrowers. Black et al. (2017, 2020) provide evidence that banks' advantage in renegotiating loans causes them to

[6]Extensions to this work further evaluate implications of renegotiation for firms' dividend policies (Fan and Sundaresan, 2000), investment (Pawlina, 2010), and funding mix (Hege and Mella-Barral, 2005; Hackbarth et al., 2007). Additionally, François and Morellec (2004); Antill and Grenadier (2019) incorporate more realistic bankruptcy procedures to the model. Relatedly, Rajan (1992) analyzes financing decisions when banks have an informational advantage allowing them to extract surplus from the firm.

[7]Adding even modest modification frictions does away with a few counterfactual predictions of the frictionless baseline. Models with frictionless modifications typically predict that debt is issued up to capacity, immediately renegotiated, and never liquidated.

specialize in riskier loans.[8] Our model accounts for each of these mechanisms: borrowers endogenously choose when to renegotiate depending on modification frictions, recourse use differs across lenders and additionally affects renegotiation outcomes, and borrower endogenously sort into heterogeneous lenders. The empirically grounded, calibrated model thus allows us to assess the equilibrium implications of these differences in modification frictions

Lastly, our empirical analysis illustrates some key differences in modification patterns for commercial mortgages relative to residential mortgages. During the housing bust in the 2000s, residential mortgage modification rates were generally low, with only modest differences between securitized and portfolio loans (Agarwal et al., 2011; Adelino et al., 2013). We find much higher modification rates for bank CRE loans and much larger differences across lender types.[9] A possible explanation for this difference is greater asymmetric information for residential mortgages. Non-pecuniary factors can play a large role households' default decisions (Guiso et al., 2013), which can discourage loan modification since lenders cannot identify which loans are likely to cure without support (Adelino et al., 2013). If information asymmetries are indeed less pronounced for CRE than RRE loans, this could explain the higher modification rates and the larger role for institutional factors in determining modification outcomes.

The outline for the rest of the paper is as follows. In Section 2, we present empirical evidence on differences in CRE loan modification rates across lender types. In Section 3, we present the model and the theoretical results. In Section 4, we present the model calibration, quantitative results, welfare counterfactuals, and extensions. In Section 5, we conclude.

[8] Regarding earlier work on the renegotiation of CRE loans, Snyderman (1991) and Brown et al. (2006) use data from life insurers to examine renegotiation and default outcomes. Riddiough and Wyatt (1994a,b) examine equilibrium workout/default outcomes in games where foreclosure costs incentivize lenders to restructure loans and borrowers to strategically default.

[9] Over 27 percent of all bank CRE loan balances received payment modifications during 2020, compared to under 7 percent of CMBS loan balances. In contrast, under 10 percent of distressed residential mortgage loans were modified during the 2000s housing bust regardless of securitization (Adelino et al., 2013).

2. CRE LOAN MODIFICATIONS IN THE DATA

In this section, we lay out the key empirical findings that discipline our modeling choices. We use loan-level data from banks and CMBS to document how renegotiation frictions affect modification and delinquency patterns in the CRE market. We show that bank CRE loan modifications are both more preemptive (supporting the less-troubled loans that CMBS rarely modify) and more responsive to stress (expanding notably when strains emerge). Banks have lower delinquency rates on stressed loans, suggesting that these modifications help bolster loan performance.

2.1. Data Sources

We use two data sources: monthly data on CMBS loans from Trepp and quarterly data on CRE loans held by large US banks from Federal Reserve stress test (Y-14) data.[10] Each data source provides information on loan terms, property characteristics, and loan performance over time.

Our data consists of first-lien commercial loans secured by stabilized, non-owner-occupied, nonresidential properties in the United States.[11] We exclude construction and land development loans and owner-occupied CRE loans—loan types predominantly provided by banks—to maintain a similar sample of loans for banks and CMBS. We exclude loans secured by multifamily properties, as government-sponsored enterprises account for a large share of such lending and terms differ notably from those for other property types.[12] Finally, we exclude loans that are cross-collateralized or missing information on the location of the collateral. Table 1 provides information on origination characteristics for this sample of loans by property and lender type.

The identification of loan modifications differs for the two types of lenders. For CMBS, mod-

[10]The Y-14Q reporting panel consists of banks with consolidated assets of $50 billion ($100 billion starting in 2019). Banks report loans with a committed balance of $1 million or more. The data are at the facility level, but, as most facilities have only one loan, we treat the data as being at the loan level.

[11]We drop bank loans for which the reported property value is an estimate for the property once it is completed or stabilized (as opposed to the value being reported "as is"). Additionally, we limit our CMBS loan sample to conduit, single-asset single-borrower, or large loan deals.

[12]We also exclude some minor property types (for example, healthcare) for which there is no consistent categorization across banks and CMBS. These filters limit our sample to loans backed by industrial, lodging, office, and retail properties.

ification dates and some details on the type of modification are either directly reported by the servicers or derived by our data vendor (Trepp). This information includes whether the modification involved a maturity date extension, a principal reduction, a rate reduction, the capitalization of interest or principal payments, forbearance, or a combination of various modification types. For banks, we impute modifications by identifying changes in loan terms over time, similar to the methodology of Adelino et al. (2013). Specifically, a loan is considered modified if it switched from being amortizing to interest only, if the interest rate changes on a fixed rate loan, if the committed balance rises (indicating interest payments are added to the loan balance as part of a forbearance plan), if the committed balance falls in tandem with a positive cumulative charge-off (indicating a write-off), if the maturity date is extended (outside of a pre-negotiated renewal), or if the loan enters troubled debt restructuring.[13]

For both lender types, we subdivide modifications into two broad types: those that result in a reduction in payments and those that do not. The latter category is largely made up of loan extensions, but can also include less common changes, such as adding or removing recourse or cross-collateralization from a loan.[14] Modifications that result in payment changes include interest rate changes, changes in the amortization schedule (including a switch to interest only), forbearance, and more substantial loan restructurings, such as A/B splits for CMBS loans.[15] While we provide descriptive information for overall modification rates, we focus most of our attention on payment modifications. Nonpayment modifications—most notably, extensions—might occur for reasons besides preventing default. For example, banks might be willing to extend a loan at the end of its term because it has favorable risk characteristics.

The two performance measures of interest are whether a loan is modified or goes 90 days delinquent in a quarter. Delinquency and modification rates are not always directly comparable across lenders: a single bank modification can appear multiple times (for example, if a forbearance

[13]Additionally, we consider changes in origination dates, which—per reporting guidelines—occur when there is a substantial change in a loan's terms.

[14]Extensions allow borrowers to avoid needing to refinance to make a balloon payment at maturity.

[15]Online Appendix Figure C.2 plots the share of outstanding loan balances that have received modifications by lender type and modification type.

period spans quarter-end), and delinquency rates are affected by the duration with which delinquent loans are reported. For this reason, our primary analysis predicts whether loans that had not been previously modified or 90 days delinquent become so in a given quarter. This measure of first modification or delinquency is not sensitive to reporting differences and thus better reflects the rate at which such events occur.

We analyze how loan performance across lenders differs by the degree of stress the loan is experiencing. In the time-series analysis, this amounts to analyzing increases in modifications and delinquency during the pandemic (covering 2020:Q1 to 2021:Q3).[16] In the cross-sectional analysis, we study how changes in current LTV affect loan performance. LTV is defined as the ratio of the current loan balance to the current property value, where the property value is estimated by interpolating between appraisals and extrapolating from the last appraisal using local property type-specific price indices from CBRE. We also examine how income-based measures of stress relate to loan performance, using current debt serve coverage ratios (DSCR). DSCR, defined as the ratio of the collateral's net operating income (NOI) to annual debt service obligations, measures how well the property can support the debt service costs associated with the loan.

2.2. *Modification and Delinquency Rates Over Time*

From Figure 1, we see that CRE loans held in banks' portfolios are modified much more frequently than loans in CMBS pools. In the quarters leading up to the pandemic, banks modified loans at a rate of about 1.5 percent per quarter—a rate over 20 times that of CMBS.[17] CMBS loans, by contrast, are more likely to become 90 days delinquent.

During the pandemic, these differences widened in absolute terms. Transitions into delinquency spiked for CMBS, reaching a peak of around 5 percent per quarter in 2020:Q3, while remaining well under 1 percent for bank loans.[18] Bank loans instead saw a spike in modification

[16]We choose this time period to identify the pandemic because it was during this period that REMIC rules were changed temporarily to allow for more modifications.

[17]Figure 1 plots transitions into modification/delinquency. The shares of outstanding loans by lender type that are delinquent or have been previously modified are provided in Online Appendix Figures C.1 and C.2, respectively.

[18]Since we define a loan as delinquent when it is 90+ days past due, loans entering delinquency in the third quarter

rates. The modification rate for bank loans rose to over 8 percent per quarter in 2020:Q1 and 2020:Q2, and remained elevated into 2021. Meanwhile, the CMBS modification rate only rose to 4 percent per quarter in 2020:Q2, and remained under 2 percent in other quarters.

Table 2 disaggregates the information in Figure 1 by property type and modification type. Banks are much more likely to modify loans across property types, with modification rates in 2018 and 2019 that range from 1.2 to 2.9 percent across property types, compared to under 0.1 percent for CMBS. Banks also experienced a larger increase in their modification rates during the pandemic, driven predominantly by payment modifications (mainly forbearances). The modification rate for bank lodging loans rose to 11 percent per quarter during the first year and a half of the pandemic. For other property types, modification rates still rose to near 5 or 6 percent per quarter. Meanwhile, for CMBS, modification rates only rose to around 4 percent for lodging loans while remaining under 1 percent for other property types. In the last column of Table 2, we show the share of loans that received either a payment modification or became 90 days delinquent, thus measuring the share of loans not making promised payments either due to delinquency or modification.[19] Modifications for bank loans are high enough that these overall distress rates are much higher than those for loans in CMBS pools, both before and during the pandemic, despite the higher delinquency rates for CMBS.

To get a more accurate estimate of the difference in the probability of receiving a modification for bank portfolio loans versus those in CMBS, we pool data across lenders and estimate linear probability models predicting modification and delinquency with lender type, while controlling for an array of risk characteristics. Our regressions take the following form:

$$\begin{aligned} \text{Mod}_{i,t} \times 100 = \quad & \beta_1 \text{CMBS}_i + \beta_2 \text{CMBS}_i \times \text{COVID}_t \\ & + \alpha_1 \mathbf{X}_{i,t} + \alpha_2 \mathbf{X}_{i,t} \times \text{COVID}_t + \gamma_{o(i)} + \delta_{s(i),t} + \zeta_{p(i),t} + \varepsilon_{i,t}, \end{aligned} \tag{1}$$

generally started to miss payments in the second quarter.

[19] Since loans can both be modified and become 90 days delinquent within a quarter, this rate may not be the exact sum of the rate of payment modifications and delinquencies.

where CMBS_i and COVID_t are indicators of whether loan i is funded by CMBS and whether quarter t is 2020:Q1 or later, respectively. $\mathbf{X}_{i,t}$ contains the following loan-level controls: log origination amount, term in years, an indicator for whether the loan is interest only, current LTV, and LTV and DSCR at origination. We also include originator by origination year fixed effects ($\gamma_{o(i)}$), state by quarter fixed effects ($\delta_{s(i),t}$), and property type by quarter fixed effects ($\zeta_{p(i),t}$). The dependent variable is multiplied by 100, so that the coefficients provide predicted effects in percentage points.

The left-hand-side variables are indicators for whether loan i was modified or became 90 days delinquent in quarter t. To account for differences in the reporting of modifications or delinquency, in each regression we remove observations after the first instance of the outcome of interest. That is, delinquency regressions predict whether previously performing loans first become seriously delinquent in time t, and modification regressions similarly predict the occurrence of a modification for previously unmodified loans.[20] As a result, our sample size varies somewhat in each column.

We present results from these regressions in columns (1)–(3) of Table 3. The results confirm the general patterns shown in Figure 1. Column (1) shows that after controlling for loan-level characteristics, CMBS have higher pre-COVID delinquency rates, with CMBS loans going delinquent at a rate that is 0.06 percentage points higher than banks. CMBS also see a much larger spike during the pandemic, with the delinquency rate rising 0.28 percentage points more than for banks. Column (2) shows that CMBS have modification rates that are almost 1 percentage points below banks in normal times, with the difference rising by an additional 1.5 percentage points during the pandemic. These results are qualitatively similar, but somewhat smaller, for payment modifications, shown in column (3).

[20] Allowing for multiple modifications results in larger differences between bank and CMBS modification rates.

2.3. *Modification and Delinquency Rates by Property Performance*

The time-series evidence suggests that banks modify loans more than CMBS, increase modifications more in times of stress, and provide more preemptive modifications—modifying loans even for less troubled property types. Here, we examine the extent to which such patterns hold in the cross-section by looking at the propensity of lenders to modify loans when the property securing them experiences stress.

Figure 2 presents semi-linear regression estimates of the relationship between current LTV and either the quarterly delinquency rate (left panel) or quarterly modification rate (right panel). Each panel presents estimates from regressing loan performance on LTV quantile dummies, controlling for the same variables and fixed effects as in Table 3. The sample includes observations through 2019, and thus reflects pre-COVID performance patterns.

The figure shows that both delinquency and modification are rare for loans with low current LTVs. When loan balances are well below the value of the property, lenders have little reason to modify loans as they would be able to recover the full loan balance by liquidating the property after foreclosure. Likewise, borrowers have little reason to go delinquent when they could sell the property for enough to pay off the loan. Consequently, for loans with estimated LTVs under 60 percent, delinquency rates are negligible for both banks and CMBS, and modification rates are negligible for CMBS loans. Though low LTV bank loans do occasionally get modified, these modification rates are typically estimated to be under a half percent per quarter.

More notable differences emerge between the performance of bank and CMBS loans at higher LTVs. As LTVs rise above 60 percent, predicted modification rates for bank loans start to climb from under a half percent per quarter at an LTV of 60 percent to around one percent at an LTV of 90. In contrast, CMBS modification rates remain close to zero at most LTVs before starting to edge up as LTVs surpass 90.

While banks see modifications pick up at higher LTVs, CMBS mostly experience rising delinquency rates. Estimated delinquency rates for both bank and CMBS loans remain very low for

LTVs under 80, but begin to pick up after that point. The deterioration in performance at high LTVs is more pronounced for CMBS loans, which transition into delinquency at well over twice the rate that bank loans do at these high LTVs.

Overall, these findings echo many of the time-series results. First, banks provide more modifications in general (i.e., across the LTV spectrum). Second, banks are more preemptive in their modifications, whereas modifications for CMBS do not rise until the top LTV quantile (and even then rise very little). Finally, stress (i.e., very high LTVs) mostly manifests in the form of modifications for bank loans, and delinquency for CMBS loans.[21]

These findings can also be demonstrated through regressions similar to those from equation (1) but with the CMBS dummy interacted with the loan's current LTV rather than the COVID dummy. Columns (4)–(6) of Table 3 present the results of this analysis.[22] The main findings from Figure 2 broadly hold. Higher LTVs raise the likelihood of delinquency for both banks and CMBS, but the effect is about twice as strong for CMBS loans. Meanwhile, higher LTVs only raise the likelihood of payment modification for bank loans.

2.4. Discussion

To summarize, relative to CMBS, we have shown that:

1. banks modify more loans overall;

2. banks modify loans more preemptively;

3. banks experience lower delinquency rates, but higher modification rates, for stressed loans.

Why would CRE lenders differ so substantially in the propensity to modify their loans? The differences in modification rates hold controlling for LTV and DSCR—the primary terms that

[21] We further assess the robustness of the results in the online appendix. Online Appendix Figure C.3 shows that low DSCRs predominantly increase modification rates for bank loans and delinquency rates for CMBS loans. We emphasize the findings based on LTV since valuations are forward looking, though DSCR has the advantage that incomes are updated more frequently than assessed property values. Online Appendix Figure C.4 shows results by LTV during COVID, which display broadly similar patterns to those in Figure 2.

[22] To focus on cross-sectional differences, we restrict the sample to the pre-COVID period. LTV is demeaned so that the coefficient on $CMBS_i$ reflects the predicted effect for a loan at the average LTV.

affect borrowers' willingness and ability to maintain loan payments—so the results seem unlikely to reflect differences in the inherent riskiness of loans. Moreover, the differences hold within originators, thus accounting for factors such as underwriting technology or lender risk tolerance.[23] Finally, the fact that high-modification lenders have lower delinquency rates indicates that lenders differ in their *ability* to modify loans rather than the *need* to modify loans; unlike differences in servicing technology, differences in risk would cause modification and delinquency rates to move in the same direction. Overall, the stark differences in modification patterns for loans with identical originators and similar risk characteristics seems best explained by variation in the ability to modify loans.

A review of institutional factors affecting the lenders supports this hypothesis. CMBS are restricted in their ability to modify loans by both their pooling and servicing agreements (which define the rights and responsibilities of the mortgage servicer) and by IRS policies (which define when a modified mortgage would constitute a new loan, thereby threatening the securitization vehicle's REMIC status and subjecting it to federal taxation).[24] In contrast, the other major CRE lenders are typically the sole debt holder, face minimal restrictions on loan modifications, and were encouraged by regulators to modify loans during the pandemic. In March 2020, banks' regulators issued a joint statement actively encouraging banks to take "proactive actions that can manage or mitigate adverse impacts [of COVID-19] on borrowers." Life insurers, which we emphasize less in the empirical work due to data limitations, but are also an important balance sheet lender, were similarly encouraged "to work with borrowers who are unable, or may become unable, to meet their contractual payment obligations because of the effects of COVID-19."[25] In short, institutional differences between CMBS and balance sheet lenders plausibly result in these lenders differing in loan modification technologies.

[23]Though securitization limits a bank's credit risk exposure to a loan, banks are still exposed to risk on the CMBS loans they originate due to the need to warehouse loans before securitization (Black et al., 2012) or meet risk retention requirements after securitization (Flynn Jr et al., 2020).

[24]We provide more details on the regulatory environment affecting modifications for banks and CMBS in Appendix A, including a more thorough discussion of how tax considerations restrict CMBS modification options.

[25]We do not emphasize life insurers in Section 2 because the limited detail on loan terms and low reporting frequency for life insurer data prevent us from accurately identifying modifications.

In the remainder of this paper, we explore how these differences in modification ability affect the broader CRE market. While the ability to modify loans may benefit borrowers in times of stress, the specter of strategic renegotiation may restrict the range of contracts that banks are willing to offer. That is, if bank borrowers cannot commit to not strategically negotiating lower loan payments, banks may restrict leverage to mitigate this modification risk. Indeed, Table 1 shows that CMBS loans have higher average LTVs than bank loans across property types, and Online Appendix Table C.1 demonstrates that these differences hold controlling for other observable characteristics.

3. MODEL

This section presents a trade-off theory model of loan underwriting and performance where lenders face frictions in renegotiating loans. The model generates modification and delinquency patterns that align with the empirical observations in Section 2 and provides a framework for analyzing the broader equilibrium implications of modification frictions.

We start by deriving expressions for the values of equity and debt in this environment. We then solve for the equilibrium modification strategies, the set of contracts (LTVs and spreads) offered by a competitive loan market, and the loan contracts optimally chosen by borrowers. We then derive how borrowers optimally sort into lenders, which differ in modification ability. Finally, we aggregate across heterogeneous borrowers to solve for lenders' equilibrium loan portfolios, accounting for both differences in loan offers across lenders and the endogenous sorting of borrowers into lenders.

3.1. Environment and Value Functions

We start by considering the problem of a particular property investor negotiating a loan contract from a particular lender. At time $t = 0$, the investor buys a property partially using perpetual, defaultable debt with a flow coupon payment of C (to be endogenized later). Let the after-tax net operating income from this property at time t (denoted X_t) follow a geometric-Brownian motion process:

$$\frac{dX_t}{X_t} = \mu dt + \sigma dZ_t.$$

Lenders and property investors are risk neutral and discount cash flows at the risk-free rate r. Therefore, the present value of promised coupon payments is $\frac{C}{r}$ and the present value of future income is $\frac{X_t}{r-\mu}$. Investors earn a flow return of $X_t - (1-\tau)C$, where τ is the effective tax rate that determines the tax advantage of debt and thus the demand for leverage.[26]

In the event of default at time t, the lender can foreclose on the property and recover the unleveraged property value, less a proportional foreclosure cost $\alpha^F \in [0,1)$. In addition, motivated by the finding that loans with recourse are less likely to be modified (Glancy et al., 2021), we allow for the availability of recourse to affect loan recoveries. Specifically, lenders can claim a fraction $\theta \in [0, 1-\tau)$ of the present value of promised debt payments from a deficiency judgment, paying a proportional cost of $\alpha^D \in [0,1]$.[27]

The recovery in the event of foreclosure, $R(X)$, is therefore

$$R(X) = (1-\alpha^F)\frac{X}{r-\mu} + (1-\alpha^D)\theta\frac{C}{r}.$$

The deadweight costs of foreclosure leave room for mutually beneficial loan modifications with the purpose of forestalling loan defaults. Following Hackbarth et al. (2007), borrowers can make a take-it-or-leave-it offer to the lender to lower their debt service at time t to some amount $S(X)$. In Section 4.4, we generalize this assumption to allow lenders to have some bargaining power. We show that borrowers with low bargaining power rarely borrow from CMBS, making the assumption that borrowers have the bargaining power realistic for those on the margin between choosing bank

[26]The effective tax rate, τ, is a standard parameter in trade-off theory models. τ determines the size of the tax shield and, hence, the demand for leverage. It can stand in more generally for other factors that affect the demand for leverage, such as liquidity needs or wedges in required returns between borrowers and lenders.

[27]$\theta = 0$ for non-recourse loans, such as most CMBS or life insurer loans. For recourse loans (the majority of bank loans), θ reflects how much borrowers actually expect to pay in a deficiency judgment. Even a full recourse loan would have a low θ if the borrower has few outside assets. θ is bounded above by $1-\tau$ to ensure that there exists a value of $X_t > 0$ such that borrowers choose to renegotiate their loan.

or CMBS financing.[28]

We make one key departure from Hackbarth et al. (2007) in how renegotiations work: while the loan is operating under modified terms and paying $S(X) < C$, negotiations break down at an exogenous rate λ, resulting in foreclosure. Similar to Davydenko and Strebulaev (2007), by varying the likelihood that renegotiation fails, we can study how differences in modification frictions affect outcomes in the market.

In equilibrium, the borrower optimally chooses when to renegotiate their loan and what debt service amount to offer. Since borrowers can make a take-it-or-leave-it offer, when they seek a modification, they choose a strategic debt service offer $S(X)$ so as to make the lender indifferent between foreclosing and accepting the modification. In Appendix B, we derive this equilibrium offer as

$$S(X) = (1-\alpha^F)X + (1-\alpha^D)\theta C.$$

Regarding when renegotiation occurs, after income falls below an endogenous threshold X_n, lenders become willing to accept a sufficiently low debt service payment for borrowers to choose to renegotiate their loan. As a result, there are two regions in the model: a low region (denoted L) where $X \leq X_n$ and lenders receive loan payments $S(X) < C$, and a high region (denoted H) where $X > X_n$ and lenders receive loan payments C.

In Appendix B, we derive the optimal renegotiation threshold and the following equations

[28] If lenders have more bargaining power, strategic renegotiation becomes less of a concern since borrowers gain less from the process. In turn, borrowers with low bargaining power have little reason to borrow from CMBS, as it no longer provides more favorable loan terms. The extension thus rationalizes why CMBS serve larger CRE investors and justifies our focus on borrowers with bargaining power when we are studying the selection of borrowers into CMBS.

defining the values of debt and equity, $D(X)$ and $E(X)$ in the H region:[29]

$$
\begin{aligned}
D_H(X;C) &= \frac{C}{r}\left(1-\left(\frac{\frac{X}{r-\mu}}{\rho(\lambda,\theta)\frac{C}{r}}\right)^{-\gamma}\chi(\lambda,\theta)\right)\\
E_H(X;C) &= \frac{X}{r-\mu}-(1-\tau)\frac{C}{r}+\left(\frac{\frac{X}{r-\mu}}{\rho(\lambda,\theta)\frac{C}{r}}\right)^{-\gamma}\omega(\lambda,\theta)\frac{C}{r}\\
\frac{X_n}{r-\mu} &= \rho(\lambda,\theta)\frac{C}{r}
\end{aligned}
\tag{2}
$$

where $\rho(\lambda,\theta)$, $\chi(\lambda,\theta)$, $\omega(\lambda,\theta)$ and γ, are all positive constants (with respect to X and C) that are defined in Appendix B. We will discuss the economic interpretation of these expressions each in turn, suppressing the arguments to these functions for the sake of readability.

First, ρ measures the renegotiation boundary. Specifically, borrowers choose to renegotiate their loan when the value of the unlevered property falls below a fraction ρ of the present value of promised debt payments. In Section A of the Online Appendix, we analytically characterize ρ. We show that the modification boundary is decreasing in λ, meaning that borrowers are more willing to continue making promised debt payments when modifications are less certain.[30] Figure 3 plots debt service payments as a function of X_t for two loans that are identical except for λ. Debt service payments jump down when current incomes fall below the renegotiation threshold, but this drop occurs earlier for the lender with lower modification frictions. The cross-hatched region shows the range of X_t such only low-λ loans are renegotiated.

What does this result mean for loan performance? The fact that $\frac{\partial\rho}{\partial\lambda} < 0$ means that the model broadly reproduces the modification and delinquency rates patterns shown in Figure 2. ρ determines (inversely) the threshold LTV above which borrowers modify loans. Since ρ decreases with

[29] Values in the low region are solved for as well. We present the functions for $X > X_n$ here because they are used to determine loan offers, optimal terms chosen, and borrower welfare.

[30] To be precise, ρ is weakly decreasing in λ. For renegotiation frictions near 0, modifications occur at the highest income such that lenders are willing to provide a modification (which does not depend on λ). When frictions are higher, modifications occur when the potential reduction in debt service is enough to compensate borrowers for the risk that modifications break down, and thus greater modification frictions discourage renegotiation. Numerically, the lenders participation constraint is not binding for any borrower in the calibration, so our discussion focuses on the case where $\rho_\lambda < 0$. (See Online Appendix Section B.2 for more detail.)

λ, there is an intermediate range of LTVs such that bank loans (low-λ loans) are modified, while CMBS loans continue receiving promised payments. Above this range, all borrowers renegotiate, but more negotiations fail for CMBS. In short, banks provide more preemptive loan modifications than CMBS (due to strategic renegotiation), but have lower delinquency rates for severely-stressed loans (since renegotiation is more successful), matching empirical observations from Section 2.

Turning now to χ and ω, χ measures the loss to a lender from renegotiating a loan (relative to the value of default- and renegotiation-free debt). Specifically,

$$\chi = \left(\frac{C}{r} - D(X_n)\right) \Big/ \frac{C}{r}.$$

This loss is higher when there are greater deadweight costs to foreclosure (α^F and α^D is high) or when borrowers renegotiate at lower property values (ρ is lower).

Lastly, ω reflects the value of the renegotiation option to the borrower. The option value is higher when negotiations are less likely to break down (λ is low) and when loans have minimal recourse (θ is low).

3.2. Lender Pricing of LTV

We assume that lenders operate in a competitive market, so that loans are initially priced at par. The initial loan balance therefore is $D_H(X_0;C)$, making the coupon payment $C = r^m D_H(X_0;C)$, where r^m is the mortgage rate. Evaluating (2) at X_0 and substituting in for C, the expression for $D_H(X_0;C)$ can be rearranged to express LTV as a function of loan rate spreads:

$$LTV(s) = \frac{s^{\frac{1}{\gamma}}(1-s)}{\chi^{\frac{1}{\gamma}}\rho}, \tag{3}$$

where $LTV \equiv \frac{D_H(X_0;C)}{X_0/(r-\mu)}$ is the ratio of loan size to the unlevered property value and $s \equiv \frac{r^m - r}{r^m} = \left(\frac{X_0}{r-\mu}/\rho\frac{C}{r}\right)^{-\gamma}\chi$ reflects the loan rate spread.[31]

[31] This concept of LTV and spreads is convenient for presenting the expressions that follow. When we take the model to the data, we use the more conventional spreads measure $r^m - r = \frac{s}{1-s}r$. We also allow the benefits of leverage to be

The above expression is effectively the credit supply curve: it determines the schedule of loan terms that lenders are willing to offer property investors. It is clear that lenders are willing to offer higher LTVs for a given spread when borrowers are more willing to maintain promised debt payments instead of seeking a modification (ρ is low) or when their losses from a modification are lower (χ is low). In Section A.2 of the Online Appendix, we present the comparative statics of this supply curve with respect to λ. We show that lenders are willing to offer higher LTVs for loans with higher modification frictions because of how modification frictions affect the modification boundary.

In short, while the ability to modify loans provides borrowers some insurance against downward income movements, this gain comes at a cost. Lenders anticipate losses from strategic modification requests and provide less favorable loan terms at origination. Lenders are unwilling to offer high-LTV, easily modified loans, as borrowers would immediately be able to negotiate more favorable terms. Borrowers thus need to provide some protection from strategic renegotiation, either through a high down payment or frictional modifications.

3.3. *Equilibrium Pricing, LTV, and Welfare*

Here, we solve for the LTV and spread from (3) that borrowers optimally choose and evaluate welfare at these contracts.

Firms choose the debt contract that maximizes firm value $v(X;C) = E_H(X;C) + D_H(X;C)$. From Equation (2), $v(x;C)$ can be written as:[32]

$$v(X;C) = \frac{X}{r-\mu} + \frac{\tau C}{r} - \left(\frac{\frac{X}{r-\mu}}{\rho \frac{C}{r}} \right)^{-\gamma} (\chi - \omega) \frac{C}{r}. \tag{4}$$

Taking the first-order condition of (4) with respect to C, we can show that borrowers choose

incorporated into property values, so that unlevered LTVs differ from actual LTVs.

[32] At origination, borrowers maximize the sum of the value of equity and the amount of loan proceeds. Since loans price at par, this amounts to maximizing the value of the firm.

the contract with a spread:

$$s^* = \left(\frac{X_0}{r-\mu} / \rho \frac{C^*}{r} \right)^{-\gamma} \chi = \frac{\tau\chi}{(1+\gamma)(\chi-\omega)}, \tag{5}$$

Having now found the optimal spread chosen by the borrower, we can close the model and present closed-form expressions for the LTV chosen by the borrower and for borrower welfare. To find the equilibrium LTV, evaluate the supply function from equation (3) at the chosen spread from equation (5) and obtain

$$LTV^* = \left(\frac{\tau}{(1+\gamma)(\chi-\omega)} \right)^{\frac{1}{\gamma}} \left(1 - \frac{\tau\chi}{(1+\gamma)(\chi-\omega)} \right) \rho^{-1}. \tag{6}$$

Substituting C^*, found from rearranging equation (5), into (4), we obtain the value of the property investment for the optimal loan contract:

$$v(X_0) = \frac{X_0}{r-\mu} \left[1 + \underbrace{\tau \frac{\gamma}{1+\gamma} \left(\frac{\tau}{(1+\gamma)(\chi-\omega)} \right)^{\frac{1}{\gamma}} \rho^{-1}}_{\equiv v(\lambda,\theta)} \right]. \tag{7}$$

3.4. Choice of Lenders

The results thus far determine how a particular borrower i, defined by a set of risk characteristics and leverage preferences, chooses loan terms from a particular lender j, defined by λ_j. In this subsection, we model the selection of borrowers into different lenders, effectively endogenizing λ as the optimal choice from a menu of contracts offered by different types of lenders.

First, consider a borrower i with a particular set of characteristics $\mathbf{b_i} \equiv (\tau_i, \sigma_i, \mu_i, \alpha_i^F)$. This borrower needs to choose a particular lender $j \in J$ to borrow from, with each j defined by a particular (λ_j, θ_j).[33] The borrower does this so as to maximize the value of a property investment with a mortgage from j. From equation (7), this amounts to maximizing $v_{i,j} \equiv \tau_i \frac{\gamma_i}{1+\gamma_i} \left(\frac{\tau_i}{(1+\gamma_i)(\chi_{i,j}-\omega_{i,j})} \right)^{\frac{1}{\gamma_i}} \rho_{i,j}^{-1}$,

[33] Given banks are the main recourse lender in the CRE market, we make θ a lender characteristic.

where i and i,j subscripts refer to functions evaluated for borrower and borrower-lender characteristics, respectively.

In reality, one would not expect all sorting in the CRE market to be driven by differences in modification frictions or the use of recourse. CRE lenders may also differ in risk tolerance, desired investment horizons, or various other dimensions (Glancy et al., 2022). To reflect these unmodeled factors affecting sorting, we add unobserved heterogeneity in preferences for CRE lenders so that borrowers match to lenders probabilistically based on their value from borrowing from a particular lender (instead of matching perfectly to the lender with the highest $v_{i,j}$).

In particular, we assume that i chooses j if $v_{i,j}z_{i,j} \geq v_{i,k}z_{i,k}\ \forall k \in J$, where $z_{i,k}$ is an i.i.d., Fréchet distributed random variable reflecting unobserved preferences with CDF $P(Z < z) = \exp(-z^{-\varepsilon})$. With this setup, the probability that borrower i chooses lender j is[34]

$$\mathrm{P}_j(\mathbf{b}_i) = \frac{v_{i,j}^{\varepsilon}}{\sum\limits_{k \in J} v_{i,k}^{\varepsilon}}. \tag{8}$$

In short, $v_{i,j}$ determines the average benefit that i gets from obtaining a mortgage from j. This amount reflects how well a particular lender's available terms match a borrower's preferences. Borrowers seeking high-LTV loans (those with a high τ) may like the higher debt capacity that can be found from lenders with a higher λ, while other borrowers may prefer the downside protection offered by lenders with a lower λ.

3.5. Aggregation

Having now determined how borrowers sort into particular lenders, we can solve for the portfolio characteristics of different lenders. Let $f(\mathbf{b})$ denote the probability density function of borrower characteristics.[35] Given the sorting implied by equation (8), the distribution of borrower characteristics for the loans made by a particular lender j will be $f_j(\mathbf{b}) = \frac{P_j(\mathbf{b})f(\mathbf{b})}{\int P_j(\mathbf{b})f(\mathbf{b})d\mathbf{b}}$.

[34] Note that as $\varepsilon \to \infty$, the probability of choosing the lender with the highest $v_{i,j} \to 1$. That is, in the limit, this setup encapsulates the situation where lenders maximize welfare as measured in equation (7).

[35] In Section 4, we will quantitatively explore heterogeneity in τ to analyze the effects of sorting based on leverage demand. However, here we consider the more general case with heterogeneity in other borrower characteristics.

We obtain the average characteristics for the loans of a given lender by integrating over this distribution. For example, the average unlevered LTV for lender j would be $\int \text{LTV}_j^*(\mathbf{b}) f_j(\mathbf{b}) d\mathbf{b}$, where $\text{LTV}_j^*(\mathbf{b})$ comes from equation (6) evaluated at a particular set of borrower and lender characteristics.

This expression shows that lenders' portfolios will differ for two reasons. First, lenders offer different terms, reflecting differences in λ and the effect λ has on loan outcomes. That is, lenders differ in the types of loans that would be made to any particular borrower. Second, lenders differ in which borrowers they serve. Borrowers disproportionately sort into the lenders that better match their preferences, creating differences in, for example, borrowers' willingness to accept higher spreads to achieve higher leverage.

4. QUANTITATIVE RESULTS

We now examine the quantitative implications of the model. Lenders differ in their ability to modify loans, resulting in a varied willingness to make high-LTV loans. Borrowers are heterogeneous in their demand for debt, causing higher demand borrowers to sort into lenders offering higher debt capacity. The first section describes the calibration of the parameters in the model. The second section documents how underwriting terms differ across lenders in the model. The third section examines the welfare effects of counterfactual changes to modification frictions. The final section presents findings from two extensions to the model; these extensions generalize some simplifying assumptions in the model and provide additional color as to which types of borrowers value renegotiation frictions.

4.1. Calibration

To provide a broad overview of the calibration, we directly set μ, λ_j, and some parameters of $f(\mathbf{b})$ based on values from the data or the related literature. We then jointly calibrate the remaining parameters to match relevant moments in the data.

Regarding lender parameters, we consider borrowers as choosing between three lender types that broadly span the various kinds of credit available in the CRE market: $(\lambda_{\text{Bank}}, \theta_{\text{Bank}})$ represents modifiable, recourse loans such as typical bank loans; $(\lambda_{\text{CMBS}}, 0)$ represents low-modification, non-recourse loans such as CMBS loans; and $(\lambda_{\text{Bank}}, 0)$ represents modifiable, non-recourse loans such as those provided by life insurers and some banks.[36]. Namely, life insurers and CMBS loans are assumed to be non-recourse, and banks and life insurers are assumed to have the same modification frictions, leaving us with three lender parameters to calibrate.[37] Given an estimate for r, we set λ_{Bank} and λ_{CMBS} to equate $\frac{\lambda_j}{r}$ to their respective pre-COVID delinquency-to-modification ratios in Table 4 (0.79 for banks, 7.95 for CMBS). The calibration of r and θ_{Bank} (for brevity referred to as θ going forward) are discussed below.

Regarding the borrower parameters, we will start by discussing parameters related to the distribution of borrower characteristics, as other moments involve integrating over this distribution. We allow τ to be heterogeneous so as to study how borrowers sort into lenders based on their demand for debt. We assume that $\tau_i \sim \beta(a, b, \underline{\tau}, \overline{\tau})$ and calibrate these parameters to match the distribution of LTVs in CMBS pools, omitting the highest and lowest percentiles to reduce the effects of reporting errors and outliers.[38] $\underline{\tau}$ and $\overline{\tau}$ are set to match the lowest and highest CMBS LTVs in the data (30 percent and 75 percent, respectively). The shape parameters, a and b, come from the joint calibration, with the mean and residual standard deviation of CMBS LTV as the corresponding target moments.[39] We assume that the value from leverage is capitalized into appraisals and transaction prices, so that the true LTV for a property is $\frac{LTV^*_{i,j}}{1+\nu_{i,j}}$, where $LTV^*_{i,j}$ is the optimal unlevered LTV from equation (6), and $\nu_{i,j}$ is the markup on the property value due to the benefits of leverage

[36] For brevity, we refer to these three lenders as banks, CMBS, and life insurers, respectively, though banks provide both recourse and non-recourse credit. These three lenders account for the vast majority of U.S. CRE lending (Glancy et al., 2022)

[37] λ for balance sheet lenders is calibrated to bank data because the more detailed reporting allows for accurate measurement of delinquencies and modifications. Frictions at life insurers are reasonably expected to be comparable to banks; life insurers were similarly encouraged by regulators to provide accommodation to stressed borrowers, faced minimal restrictions to doing so, and thus saw little increase in loan delinquency during the pandemic. CMBS and life insurer loans are assumed to be non-recourse based on industry intelligence that this is almost exclusively the case.

[38] We focus on CMBS since the lack of recourse or relationship lending means the data-generating process for CMBS likely aligns best with the factors incorporated into the model, with loan underwriting and performance driven by the cash flows of the underlying property.

[39] Some variation in CMBS LTVs reflects factors that are not accounted for in the model, for example, differences in LTV limits by property types. Since all of the variation in CMBS LTVs in the model reflects borrower preferences, we target the RMSE from a regression predicting LTV with observable property characteristics.

from equation (7).

Turning to the remaining parameters, we set $\mu = .01$ so that average NOI growth matches the 1 percent average rent growth in An et al. (2016).[40] r is targeted to match the 5.5 percent national cap rates in CBRE Econometric Advisors data.[41] α^F is targeted to produce the 30 percent average foreclosure cost in Brown et al. (2006).[42] The recourse parameters, θ and α^D, target the effects of recourse on loan spreads and LTVs in Glancy et al. (2021). σ targets the 2.43 percent average spread on CMBS loans. Finally, we calibrate ε, which reflects the sensitivity of market shares to changes in $v_{i,j}$, to match the elasticity of CMBS market shares with respect to loan rates in Glancy et al. (2022).[43]

We present the results from our calibration in Table 5. The top panel reports parameters that are either directly set or exactly determined by other parameters, while the bottom panel reports parameters determined in the joint calibration.

τ is estimated to range from 0.05 to 0.56, with a distribution that is right-skewed. Given the estimated required return of 7 percent, the modification breakdown rates are calibrated as 0.06 and 0.56 for banks and CMBS, respectively. NOI is estimated as having a volatility of 26 percent, and the calibrated α^F implies that recoveries average 77 percent of the unlevered property value. The value for θ indicates that banks expect to recover about 8 percent of the present value of promised debt payments from a deficiency judgment upon foreclosure, while the value for α^D indicates that they expect to lose about 40 percent of this due to the costs of collecting a deficiency judgment.

The right-most columns indicate that the model is successful at fitting the targeted moments.

[40] An et al. (2016) use panel data on property-level rents from 2001:Q2 to 2010:Q2 to estimate their model. See Table 5 for the GLS estimate of long-term average rent growth we use.

[41] The mean national cap rate (NOI as a fraction of property value) in the CBRE data is 5.5 percent, averaging over property types and quarters from 2012 to 2019. The cap rate in the model is $\frac{r-\mu}{1+v_{i,j}}$. Since the numerator is heterogeneous, the target is the average over borrowers and lenders.

[42] Based on a sample of distressed life-insurer-owned commercial properties, Brown et al. (2006) find that sales prices were about 30 percent lower than transfer values after accounting for capital expenditures. Note that $1 - \alpha^F$ is the recovery as a share of the unlevered property value, so the foreclosure cost relative to the actual property value is $1 - \frac{1-\alpha^F}{1+v_{i,j}}$.

[43] In Table 6, the authors estimate that a 25 basis point increase in CMBS loan rates—equivalent to a 1 percentage point origination fee per a common heuristic—causes 37.5 percent of CMBS borrowers to switch to other lenders. We calibrate ε so that such a decline in the value of borrowing from CMBS reduces the CMBS market share by this amount.

The targeted moments in the joint calibration—cap rates, foreclosure costs, CMBS spreads, the mean and dispersion of CMBS LTVs, the sensitivity of CMBS market shares to rate shocks, and the effects of recourse on LTVs and spreads—are all hit within at least two decimal places.

4.2. Effects of Modification Frictions on LTVs and Spreads

With the calibrated model, we can now investigate how modification frictions affect CRE loan market outcomes. Figure 4 plots how market shares of banks, life insurers and CMBS (depicted by the blue, green and red areas, respectively) vary by τ. The figure additionally plots LTVs (left panel) and spreads (right panel) as functions of τ for both lenders (shown by the equivalent color lines). This figure therefore displays both how underwriting terms vary for a particular borrower (different terms given τ) and how borrowers sort into lenders (different market shares by τ).

The effects of tighter LTV limits at balance sheet lenders are on clear display in the left figure. While LTVs for all lenders increase monotonically in a borrower's demand for leverage (i.e., τ), the slope for banks and life insurers flattens substantially around $\tau = 0.15$. In contrast, LTVs at CMBS are more responsive to borrower demand, resulting in higher LTVs for CMBS relative to other lenders for high τ borrowers. Moreover, this higher debt capacity at CMBS is valued by these high demand borrowers. CMBS provide little credit to borrowers with a τ below 0.15, namely borrowers who appear to be less constrained by the limited debt capacity from balance sheet lenders. However, CMBS are the primary provider of credit for the high τ borrowers which would be most constrained by these limits. Simply put, the higher debt capacity at CMBS is valued by high-leverage-demand borrowers, causing CMBS to make proportionally more loans to these borrowers.

Cross-lender differences in spreads do not vary as much throughout the τ distribution. Balance sheet lenders require a premium in order to offset expected future declines in cash flow due to modifications. As a result, banks and life insurers charge higher spreads than CMBS for all values of τ. However, while balance sheet lenders require higher spreads for all borrowers, there are offsetting compositional effects. Spreads increase monotonically in τ since high τ borrowers choose high-spread, high-LTV loans. As CMBS make more loans to the types of borrowers that choose high-spread loans, they can still have higher spreads, on average, if the sorting effect is strong enough.

Regarding variation within the balance sheet lenders, recourse lenders provide higher LTVs and lower spreads than non-recourse lenders throughout the distribution. LTV limits for the recourse lender (i.e., banks) are less tight, enabling the origination of loans with LTVs above the maximum LTV provided by the non-recourse balance sheet lender (i.e., life insurers). In turn, this availability of higher-LTV loans allows the recourse lender to achieve a greater market share at intermediate levels of demand (though the highest-demand borrowers still predominantly go to CMBS).

What do these patterns mean for the average portfolio characteristics of the lenders? Table 6 shows the average LTVs and spreads by lender type in the calibrated model. Differences in LTVs in the model are as expected given the sorting effects displayed in Figure 4: CMBS have the highest LTVs at 64 percent, followed by banks at 58 percent, and life insurers at 52 percent. These differences match up well with the data; CMBS, banks, and life insurers have average LTVs of 64 percent, 58 percent, and 56 percent in the data, respectively. Patterns for loan rate spreads are also qualitatively similar: In both the model and the data, CMBS offer the highest spreads, followed by banks, and then life insurers. However, the differences in spreads in the model are somewhat larger than those seen in the data. Overall, the calibrated model is successful at capturing patterns in the data. Namely, the estimated effects of modification frictions are able to explain why CMBS loans tend to have higher LTVs and spreads relative to balance sheet lenders.

4.3. Counterfactuals

We now investigate how modification frictions affect borrower welfare. We first analyze how aggregate welfare is affected by changing modification frictions, averaging over borrowers with different leverage preferences (τ_i) and idiosyncratic lender preferences ($z_{i,j}$). We then examine how effects vary across borrowers with different demand for leverage.

In the model, welfare is reflected in $v_{i,j}$, that is, the increase in property value (relative to the unlevered value) achieved from taking out a loan from j. We consider the effect of modification frictions at three different levels of aggregation. First, and most disaggregated, we consider the individual $v_{i,j}$, as defined in equation (7), which reflects the benefit a particular borrower gets from a loan from a given lender, abstracting from the discrete-choice element of the model. Second, we consider the expected benefit a borrower gets from choosing their preferred lender from the set J, averaging over idiosyncratic preferences.[44] This average gives us $\overline{v}(\tau;J)$, defining the expected benefit a borrower with a given τ receives from having access to the set of lenders J. Finally, the most aggregated measure integrates this function over the population distribution, $f(\tau)$, to provide the average benefit across borrowers from having access to J.

4.3.1. Aggregate Welfare

We start by analyzing how modification frictions affect overall welfare. In the baseline calibration, project values are increased by 10.9%, on average, due to borrowers' ability to take out a loan against a property. Figure 5 presents estimates of how this value changes for different bank and CMBS modification-breakdown rates. Each cell displays the percent change in the average benefit to debt when modification frictions for banks and CMBS are multiplied by the factors on the x- and y-axis, respectively.

Relative to the baseline calibration, welfare is generally decreasing in bank modification frictions, and increasing in CMBS modification frictions. These different counterfactual effects of changing modification frictions reflect the fact that $v_{i,j}$ is generally decreasing in λ_j for the typical bank borrower and increasing in λ_j (in the neighborhood of λ_{CMBS}) for the typical CMBS borrower (see Online Appendix Figure C.5).

Though the largest gains come from reducing modification frictions at banks, perhaps the most

[44] Recall from Section 3.4 that borrowers maximize $z_{i,j}v_{i,j}$, where $z_{i,j}$ is a Fréchet distributed random variable. We are interested in $\overline{v}(\tau_i;J) = \mathbb{E}(\max_j\{v_{i,j}z_{i,j}\})$. Integrating over the idiosyncratic lender preferences, we get that the expected welfare for a borrower with a given τ is:

$$\overline{v}(\tau_i;J) = \mathbb{E}(\max_{j\in J}\{v_{i,j}z_{i,j}\}) = \Gamma(\frac{\varepsilon-1}{\varepsilon})(\sum_{j\in J} v_{i,j}^{\varepsilon})^{\frac{1}{\varepsilon}},$$

which is increasing in each $v_{i,j}$, with a greater influence from the lenders with a higher $v_{i,j}$.

pertinent counterfactual is the effect of reducing frictions at CMBS, as those frictions to some degree reflect policy choices that can be altered. Indeed, the IRS issued guidance to enable more modifications during the pandemic, likely contributing to the decline in the delinquency-to-modification ratio shown in Table 4 and the spike in CMBS forbearances shown in Online Appendix Figure C.2.[45] Were such an easing of modification restrictions to be made permanent, how would this affect the welfare of those subsequently seeking a commercial mortgage?

The top-middle cell of Figure 5 shows the effect of reducing modification frictions at CMBS by a factor of 5, a change that is modestly smaller than the nearly eight-fold decline in the delinquency-to-modification rate for CMBS loans experienced during COVID. The average benefit borrowers gain from credit access drops by about 1.4 percent due to the decline in CMBS modification frictions, equivalent to about a 15 basis point drop in property values.[46] For the typical collateral securing a CMBS loan (≈\$20m in value), this amounts to a modest \$30,000 decline in property values.

4.3.2. Heterogeneous Effects

While these aggregate welfare calculations are useful for determining the broad desirability of changing modification frictions, they obscure a substantial amount of heterogeneity in how borrowers are affected. The modest net effect of changing modification frictions for CMBS loans reflects the fact that some borrowers benefit from the change while others lose out. We now turn our attention to these distributional effects.

Figure 6 plots $v_{i,\text{Bank}}$ (in blue) and $v_{i,\text{CMBS}}$ (in red), normalized to $v_{i,\text{Life}}$, for different values of τ using the parameters in the baseline calibration. The dashed red line shows the relative value for CMBS after reducing λ_{CMBS} by a factor of 5. The difference between the solid and dashed red line thus shows how the desirability of CMBS financing changes for borrowers with different demand for leverage when modification frictions at CMBS decline.

[45] This guidance is discussed in Appendix A.1 .

[46] The figure reports changes to how much the benefit to leverage changes: $\frac{\overline{v}'-\overline{v}}{\overline{v}}$. The change in average property values, $\frac{\overline{v}'-\overline{v}}{1+\overline{v}}$, is about one-tenth this amount, since $\frac{\overline{v}}{1+\overline{v}} = \frac{.109}{1.109} \approx 0.1$.

The relative value of bank and CMBS loans are both increasing in τ, reflecting the fact that recourse and modification frictions both facilitate higher LTV lending by discouraging strategic default.[47] Reducing modification frictions at CMBS rotates their value function toward that of life insurers. This result means that reducing modification frictions for CMBS loans makes those loans more desirable to low-leverage-demand borrowers but less desirable for high-leverage demand borrowers.

If reducing modification frictions for CMBS loans makes those loans more desirable to so many borrowers, why is the aggregate welfare effect negative? The problem is, the borrowers that prefer lower modification frictions still tend to prefer banks and life insurers to CMBS (the CMBS value function for low-leverage-demand borrowers is still well below that of banks and life insurers). Meanwhile, the value of CMBS financing falls for the high-leverage-demand borrowers that are more likely to take out CMBS loans.

This effect is seen more clearly in Figure 7, which plots how expected welfare is affected by reducing the modification breakdown rate at CMBS by a factor of 5. Specifically, it plots $\overline{v}(\tau;\tilde{J})/\overline{v}(\tau;J)$, where $\tilde{J}$ is the counterfactual set of lenders containing the lower-modification-friction CMBS lender. This average value reflects how much the value of borrowing from CMBS changes for a given τ and how likely CMBS are to lend to different borrowers.

The figure shows that while there is a welfare gain for low τ borrowers, the gain is small (well under 1 percent) since most of these borrowers will not choose CMBS loans. Welfare changes more notably for high τ borrowers, who are more reliant on CMBS. Welfare declines by around 5–10 percent for the borrowers with the highest demand for leverage.

Altogether, the welfare exercise demonstrates the importance of variety in loan underwriting. While most borrowers benefit from the ability to modify loans, CMBS serve an important niche in the market. Difficulties in modifying loans enable borrowers to achieve higher leverage than is available from lenders for which strategic renegotiation is more of a concern. Reducing CMBS' advantage in this regard is thus costly, especially for high-leverage-demand borrowers.

[47] The line for banks shows how borrowers value recourse (since banks and life insurers differ only in θ), while the line for CMBS shows how borrowers value modification frictions (since CMBS and life insurers differ only in λ).

4.4. *Extensions*

In this subsection, we present results from extensions that generalize the bargaining power assumption and incorporate financial constraints on the part of borrowers via debt servicing constraints. We view the simplifying assumptions that borrowers hold the bargaining power and are not debt service constrained as reasonable for the typical CMBS borrower, as CMBS investors generally have more financial resources and better diversified funding sources as compared with bank borrowers.[48] The extensions in this section allow us to evaluate these claims through the lens of the model.

We first detail the assumptions in each extension and then present results from an updated calibration of the model under these new assumptions. We leave the derivations of model outcomes for each extension to Section B of the Online Appendix.

In the first extension, we generalize the bargaining power assumption. Instead of $S(X)$ being determined by borrowers making a take-it-or-leave-it offer to the lender, we allow lenders to have some bargaining power $\beta \in [0,1]$. When β is higher, borrowers have less to gain from strategically renegotiating, which reduces the benefit of modification frictions.[49]

In the second extension, we allow for non-strategic renegotiation. Specifically, we assume that there is a threshold cash flow amount below which a borrower is unable to maintain loan payments, causing renegotiation even if borrowers would rather remain current (given the funds to do so). In this circumstance, the renegotiation threshold is given by $\rho(\lambda,\theta,\overline{\rho}) \equiv \max\{\overline{\rho},\rho(\lambda,\theta)\}$, where $\overline{\rho}$ is the ratio of unlevered property values to the present value of promised debt payments below which borrowers will renegotiate because they can no longer make the promised payment C, and $\rho(\lambda,\theta)$ is the unconstrained renegotiation boundary defined in Equation (13). When $\overline{\rho}$ is high enough, renegotiation occurs for non-strategic reasons, removing the benefit of modification

[48] Data from Real Capital Analytics, which covers CRE transactions over \$2.5 million in size, show that nearly half of post-GFC CMBS lending went to public or institutional buyers, compared to only 10 percent of bank lending. CMBS borrowers were also more likely to operate nationally scope (75 percent of CMBS lending vs. 50 percent for banks) and have multiple CRE loan relationships (95 percent of CMBS lending vs. about 75 percent for banks).

[49] When $\beta = 1$, lenders make a take-it-or-leave-it offer, meaning that borrowers gain nothing from renegotiation relative to foreclosure. At this extreme, the renegotiation threshold corresponds with the Leland (1994) threshold, regardless of λ.

frictions in discouraging renegotiation.

Figure 8 shows how lenders' market shares (top panel) and average LTVs (bottom panel) change as a function of β and $\overline{\rho}$, which are both 0 in the baseline model. Regarding market shares, CMBS' market shares decline to near zero for borrowers with no bargaining power or severe debt service constraints (top-left and top-right panels). As CMBS' advantage in the model comes from renegotiation frictions deterring strategic renegotiation, they make few loans to borrowers for which strategic renegotiation is less of a concern (either because borrowers lack the bargaining power to extract concessions or because cash flow constraints induce them to renegotiate before it is optimal to do so).

The bottom panel presents changes in average LTVs in these extensions. LTVs rise with lender bargaining power for low modification friction lenders, as bargaining power offsets the effects low frictions have on strategic renegotiation (bottom-left panel). At the same time, LTVs at CMBS are about unaffected. Regarding non-strategic defaults, once liquidity constraints become severe enough to be binding (meaning borrowers renegotiate before it would be optimal in the unconstrained model), LTVs drop at all lenders (bottom-middle panel). However, the decline in LTVs is most stark at CMBS, whose high modification frictions make them less efficient at mitigating the effects of liquidity strains on loan performance.

Overall, these extensions confirm that the effects of modification frictions are highly borrower dependent. For borrowers that are content with lower LTVs (those with a low τ) or do not strategically renegotiate loans (those with minimal bargaining power or severe debt service constraints), modifications frictions merely increase the risk of an inefficient resolution to stress. However, for borrowers that present more of a strategic renegotiation risk, modification frictions can mitigate this risk, increase debt capacity, and therefore benefit borrowers seeking higher leverage loans.

Consequently, while easing modification frictions could be beneficial overall—indeed, our findings suggest that facilitating bank CRE loan workouts is welfare enhancing—modification frictions can help CMBS serve a particular niche in the market. Our findings suggest that modification frictions enable CMBS to make high LTV loans that balance sheet lenders would be unwilling to make due to strategic renegotiation concerns. This outcome is consistent with the data; relative to banks, CMBS tend to provide loans with higher spreads and LTVs (consistent with their borrowers having high demand for leverage) and CMBS disproportionately serve large, institutional CRE investors (who would present more of a strategic renegotiation risk).

5. CONCLUSION

We investigate how renegotiation frictions affect loan outcomes. Empirically, we demonstrate that banks are more likely to modify loans than CMBS and are more willing to offer preemptive modifications. To better understand the equilibrium implications of these modification patterns, we build a tractable trade-off theory model adapted to the CRE market where modification frictions differ across lender types. We show that modification frictions discourage strategic renegotiation and facilitate higher LTV lending. In turn, borrowers demanding higher leverage disproportionately match to lenders with higher modification frictions. The model can thus explain why CMBS loans have higher average LTVs than bank loans. The model also allows us to evaluate the effects of changing modification frictions. Reducing modification frictions at CMBS constricts the range of contracts offered by CMBS and lowers welfare for borrowers seeking higher LTV loans.

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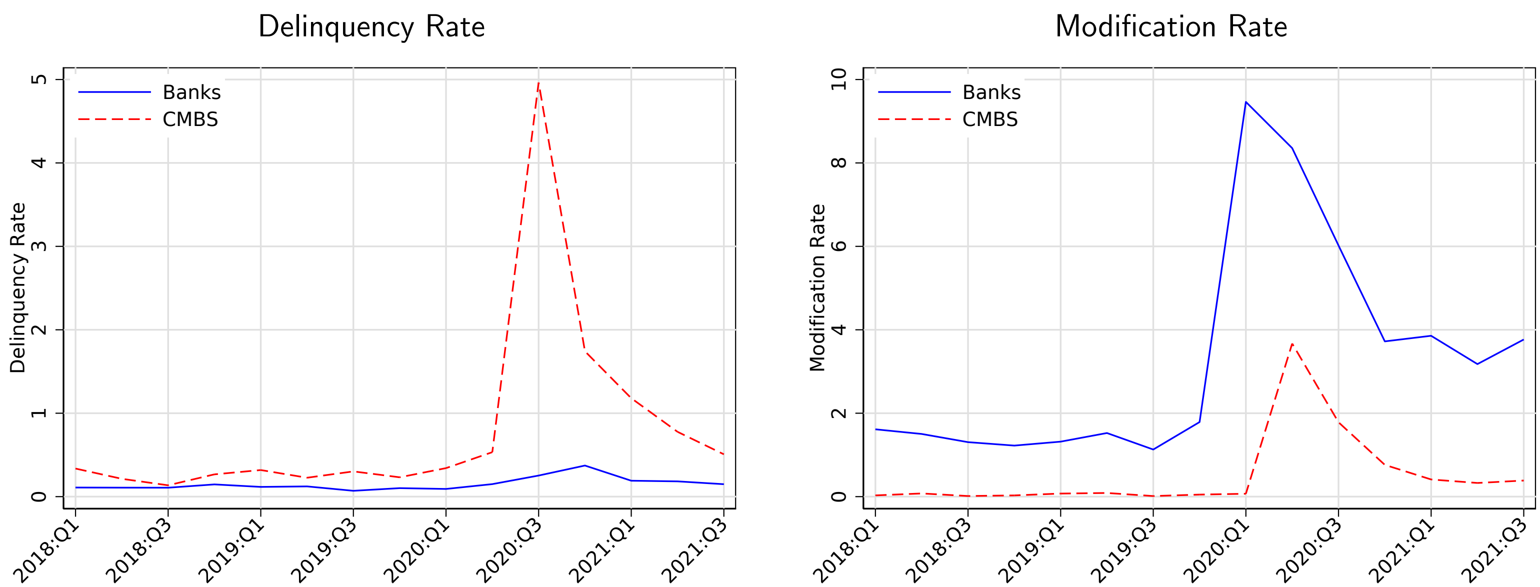

Figure 1: BANK AND CMBS DELINQUENCY AND MODIFICATION RATES. *Note:* Modifications include both payment and nonpayment modifications. Rates are calculated as the share of all outstanding loans in a given quarter that become 90 days delinquent or receive a modification (in percentage terms), where all loans more than 120 days delinquent have been removed from the sample. *Source:* Authors' calculations using Trepp CMBS data and Y-14 H.2 Schedule.

	Loans (#)	Orig. Amt (Mil.$)	Orig. LTV	Orig. DSCR	Rate Spread (percent)	Term	IO (percent)	Floating Rate (percent)	Recourse (percent)
Banks									
Industrial	3,829	13	57	2.6	2.25	7	20	51	77
Lodging	1,777	25	57	3.2	2.58	7	30	60	60
Office	6,946	21	60	2.8	2.22	7	32	55	67
Retail	9,382	9	57	2.4	2.26	8	20	47	75
CMBS									
Industrial	893	13	64	1.9	2.46	10	53	1	0.0
Lodging	2,801	21	62	2.2	2.75	9	23	1	0.0
Office	3,366	37	62	2.1	2.32	9	66	1	0.0
Retail	5,588	18	64	1.8	2.44	10	55	0	0.0

Table 1: LOAN ORIGINATION CHARACTERISTICS FOR BANK AND CMBS LOANS. *Note:* Limited to loans originated between 2012 and 2019 with non-missing values and with origination DSCRs greater than 1 and less than 20. Bank loans are limited to those originated after a lender begins reporting. All values are unweighted means. IO is interest-only. *Source:* Authors' calculations using Trepp CMBS data and Y-14 H.2 Schedule.

	2018:Q1–2019:Q4					2020:Q1–2021:Q3				
	Mod. Rate				Delinq. or Pay Mod.	Mod. Rate				Delinq. or Pay Mod.
	All	Pay	Other	Delinq. Rate		All	Pay	Other	Delinq. Rate	
Banks										
Industrial	1.17	0.49	0.68	0.09	0.58	4.89	3.97	0.93	0.06	4.02
Lodging	2.93	2.01	0.93	0.23	2.22	11.09	9.20	1.89	1.01	9.97
Office	1.59	0.73	0.86	0.11	0.83	6.14	5.02	1.12	0.14	5.14
Retail	1.21	0.50	0.71	0.11	0.60	4.65	3.29	1.36	0.21	3.48
CMBS										
Industrial	0.05	0.04	0.01	0.25	0.29	0.01	0.01	0.00	0.25	0.27
Lodging	0.04	0.02	0.02	0.29	0.31	3.76	2.40	1.36	4.11	6.43
Office	0.05	0.02	0.03	0.21	0.23	0.15	0.06	0.10	0.35	0.41
Retail	0.05	0.01	0.03	0.26	0.27	0.60	0.36	0.24	1.12	1.46

Table 2: MODIFICATION AND DELINQUENCY RATES. *Note:* Average quarterly modification and 90-day delinquency rates for bank and CMBS portfolios. Modification rates are calculated as the share of loans (in percentage terms) that are less than 120 days delinquent that receive a modification in a given quarter. Delinquency rates are calculated as the share of loans (in percentage terms) that are less than 120 days delinquent and become 90 days delinquent in the given quarter. *Source:* Authors' calculations using Trepp CMBS data and Y-14 H.2 Schedule.

	Delinquency	All Mods	Payment Mods	Delinquency	All Mods	Payment Mods
	(1)	(2)	(3)	(4)	(5)	(6)
CMBS	0.0594**	-0.968***	-0.386***	0.0741***	-1.522***	-0.864***
	(0.0289)	(0.0633)	(0.0464)	(0.0243)	(0.0533)	(0.0343)
CMBS × COVID	0.283***	-1.512***	-1.444***			
	(0.0462)	(0.102)	(0.0744)			
CMBS × LTV				0.0133***	-0.00369	-0.0105***
				(0.00103)	(0.00225)	(0.00145)
LTV	0.0202***	-0.00414**	0.00000742	0.0128***	-0.00267	0.00550***
	(0.000855)	(0.00187)	(0.00137)	(0.000904)	(0.00199)	(0.00128)
N	453,255	451,793	452,425	360,594	359,846	360,177
R2	0.03	0.05	0.05	0.02	0.03	0.03
Mean of Dep. Var. for Banks (%)	.11	1.51	.8	.09	1.14	.47
State × Qtr FEs	Y	Y	Y	Y	Y	Y
Property Type × Qtr FEs	Y	Y	Y	Y	Y	Y
Originator by Orig. Year FEs	Y	Y	Y	Y	Y	Y
Controls× COVID	Y	Y	Y	-	-	-
Sample	2012:Q1–2021:Q3	2012:Q1–2021:Q3	2012:Q1–2021:Q3	2012:Q1–2019:Q4	2012:Q1–2019:Q4	2012:Q1–2019:Q4

Table 3: LINEAR PROBABILITY REGRESSIONS. *Note:* All regressions are of the form described in equation (1). Data include loan-quarter observations from 2012q1 to 2019q4 that are not over 120 days delinquent, have an at-origination DSCR between 1 and 20, and have an at-origination LTV above 20 percent. The modification regressions predict first modification, so loan-quarter observations after a loan modification are removed from the sample, causing the observation numbers to vary across specifications. The dependent variables of interest are whether a loan goes 90 days delinquent (Columns 1 & 4), receives a modification (Columns 2 & 5), or receives a payment modification (Columns 3 & 6) in a quarter. Columns (1)–(3) include the interaction of the COVID and CMBS indicators. Columns (4)–(6) restrict the sample to the pre-COVID period and instead include the CMBS indicator interacted with the current LTV (which is demeaned and measured in percentage points). Dependent variables are multiplied by 100, so coefficients reflect predicted effects in percentage points. *,**,*** indicate significance at 10%, 5%, and 1%, respectively. *Source:* Authors' calculations using Trepp CMBS data and Y-14 H.2 Schedule.

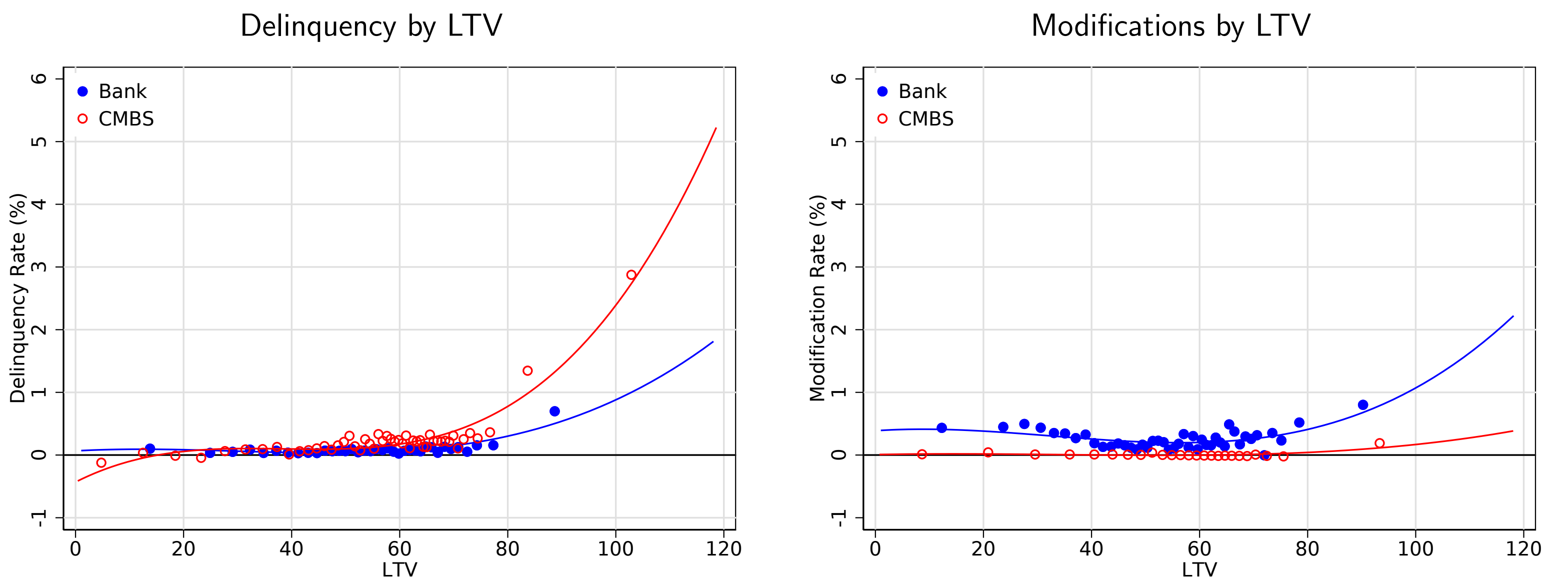

Figure 2: Delinquency and Modification Rates by Current LTV. *Note:* This figure presents semi-linear regression estimates of the relationship between current LTV and loan performance. The dots are bin scatter estimates for particular LTV quantiles, and the lines plot estimates derived from a global cubic polynomial on current LTV. Estimates are produced using the Stata command binsreg (see Cattaneo et al. 2019), controlling for origination DSCR, origination LTV, loan age in quarters, the log of the loan origination amount, the loan term, an indicator for whether the loan is interest only, and quarter by state, quarter by property type, and originator by origination year fixed effects. Data include loan-quarter observations from 2012q1 to 2019q4 that are not over 120 days delinquent, have an at-origination DSCR between 1 and 20, and have an at-origination LTV above 20 percent. Observations after the first delinquency or modification are removed. Modification and delinquency rates are quarterly rates in percentage points. Modification rates refer to modifications resulting in a change in payment. The current LTV is calculated as the ratio of the outstanding loan balance to the current property value. The current property value is linearly interpolated between reported assessed values and extrapolated from the last appraisal using the CBRE property price index for the property type of the loan in the loan's CBSA. *Source:* Authors' calculations using Trepp CMBS data, CBRE property price indices, and the Y-14 H.2 Schedule.

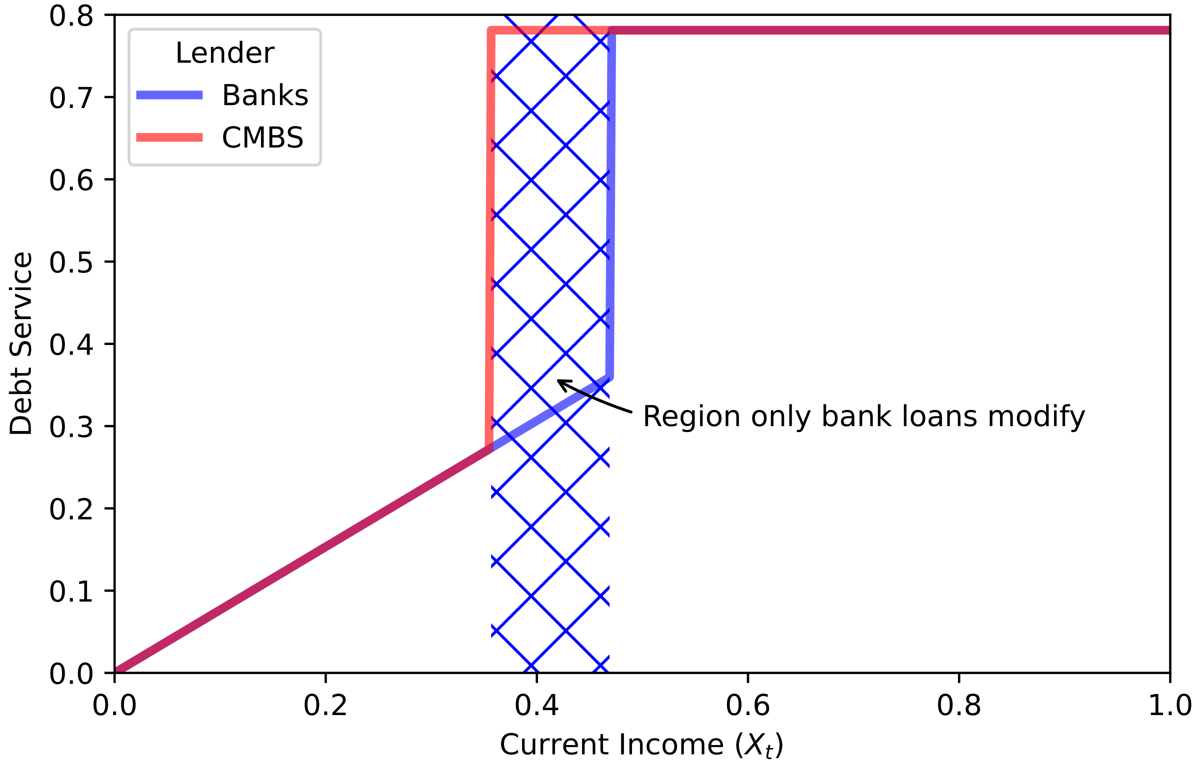

Figure 3: DEBT SERVICE COSTS BY CURRENT NOI. *Note:* This figure plots debt service costs as a function of current NOI (X_t) for two lenders with identical promised coupons but different λs. Payments for lender with a low λ ("banks") are shown in blue, and payments for the high λ lender ("CMBS") are shown in red. The cross-hatched region shows the range of incomes where only the low λ loan is modified.

	2012:Q1-2019:Q4	2020:Q1-2021:Q3
Banks	0.79	0.70
CMBS	7.95	1.02

Table 4: DELINQUENCY-TO-MODIFICATION RATIOS. *Note:* Values are the ratio of delinquency rates to modification rates by lender type and time period. We use these values to calibrate $\frac{\lambda_j}{r}$, reflecting the breakdown risk in the model. *Source:* Authors' calculations using Trepp CMBS data and Y-14 H.2 Schedule.

Estimated Parameters		Model Fit		
Parameter	Estimate	Moment	Target	Model
Directly Set				
μ	0.010	Rent Growth, An et al. (2016)	1%	1%
$\underline{\tau}$	0.051	Min CMBS LTV	30%	30%
$\overline{\tau}$	0.564	Max CMBS LTV	75%	75%
λ_{Bank}	0.055	$\frac{\lambda_{\text{Bank}}}{r}$ =Bank Delinquency-to-Mod Rate	0.79	0.79
λ_{CMBS}	0.558	$\frac{\lambda_{\text{CMBS}}}{r}$ =CMBS Delinquency-to-Mod Rate	7.95	7.95
Jointly Estimated				
r	0.070	Average Cap Rate, CBRE	5.50%	5.50%
α^F	0.233	30% Foreclosure Cost, Brown et al. (2006)	30%	30%
σ	0.255	Average CMBS Spread	2.43%	2.43%
ε	17.624	Effect of 25bp shock on CMBS share	-37.5%	-37.5%
a	1.109	Average CMBS LTV	0.64	0.64
b	2.670	Dispersion in CMBS LTV	0.06	0.06
θ	0.084	Effect of Recourse on LTV	2.90	2.90
α^D	0.401	Effect of Recourse on Spreads	-19bp	-19bp

Table 5: CALIBRATION RESULTS. *Note:* From left to right, this table presents (1) the variable to be calibrated, (2) the calibrated value, (3) a description of the corresponding target, (4) the targeted moment in the data, and (5) the value of that moment in the calibrated model.

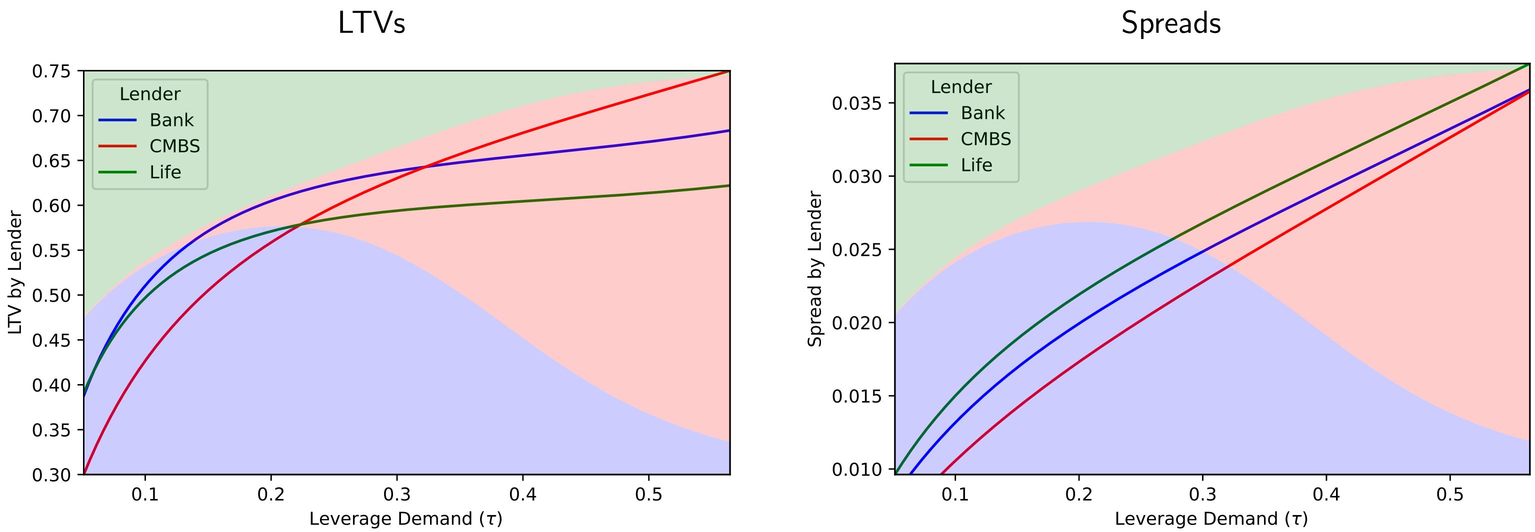

Figure 4: LTV AND SPREAD BY τ. *Note:* Lines show either the LTV (left) or loan rate spread (right) chosen by a borrower from a given lender at a given τ. The shaded regions show the market share by lender. Results for banks, CMBS, and life insurers are shown in blue, red, and green, respectively.

Lender	Model	Data
LTVs		
Banks	58	58
CMBS	64	64
Life	52	56
Spreads		
Bank	1.89	2.27
CMBS	2.43	2.43
Life	1.83	2.18

Table 6: AVERAGE LTVS AND SPREADS. *Note:* This table presents average LTV and spreads by lender in the model and the data. *Source:* Authors' calculations using Trepp CMBS data, NAIC, and Y-14 H.2 Schedule.

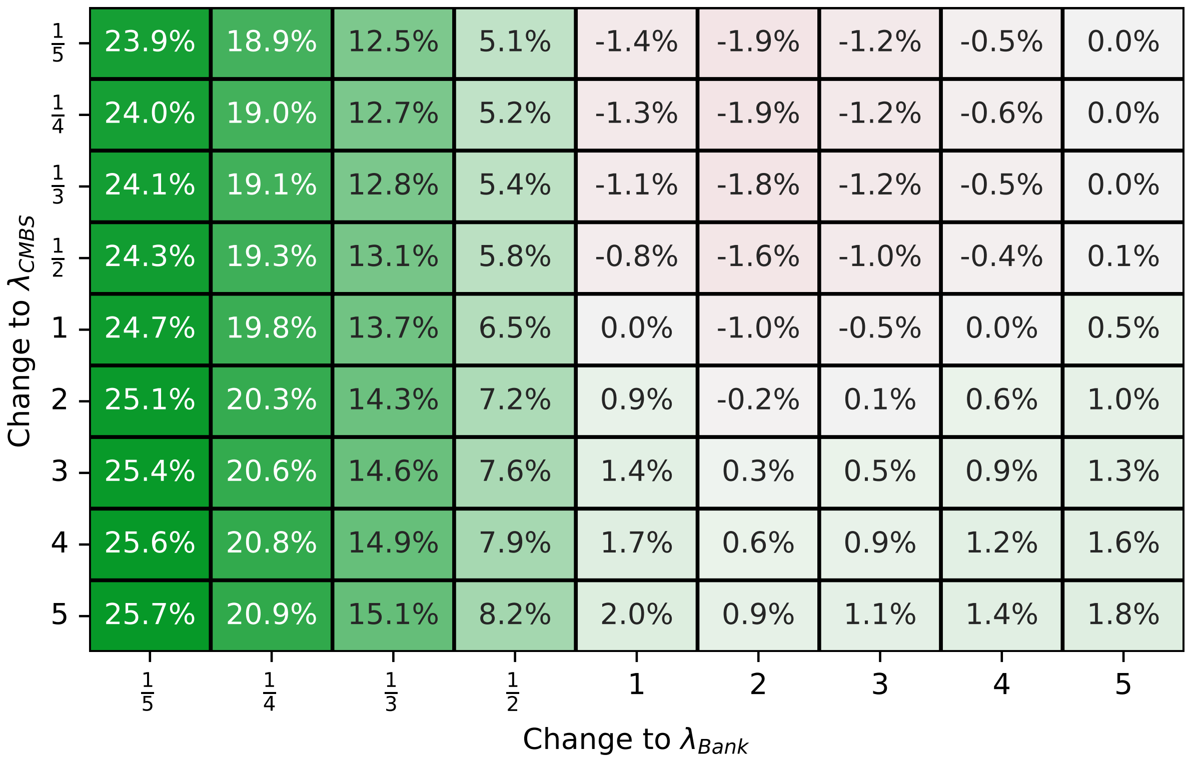

Figure 5: Change in Aggregate Welfare For Counterfactual Modification Frictions. *Note:* This heat map displays the percent change in average welfare (relative to the baseline calibration) for different combinations of bank and CMBS modification frictions. This welfare measure averages over borrowers by their demand for leverage (τ_i) and idiosyncratic lender preferences ($z_{i,j}$). Welfare in the baseline calibration is 0.109, meaning that, on average, the availability of CRE loans increases property values by 10.9% (relative to unlevered property values). Each cell gives the percent change in welfare relative to this baseline when modification frictions for banks and CMBS are multiplied by the factors on the x- and y-axis, respectively.

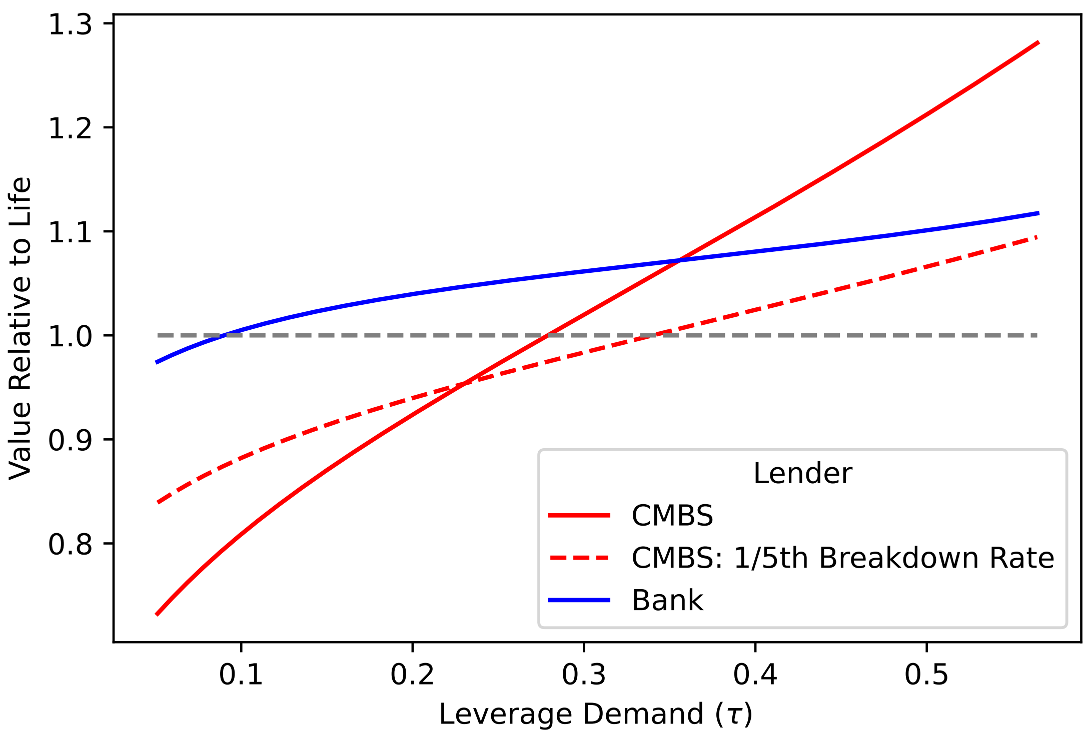

Figure 6: VALUES BY τ AND LENDER TYPE. *Note:* Solid lines show $v_{i,\text{Bank}}$ and $v_{i,\text{CMBS}}$, respectively, normalized to $v_{i,\text{Life}}$, for different values of τ. The dashed red line shows $v_{i,\text{CMBS}}$ normalized to $v_{i,\text{Life}}$, for different values of τ when λ is reduced by a factor of 5.

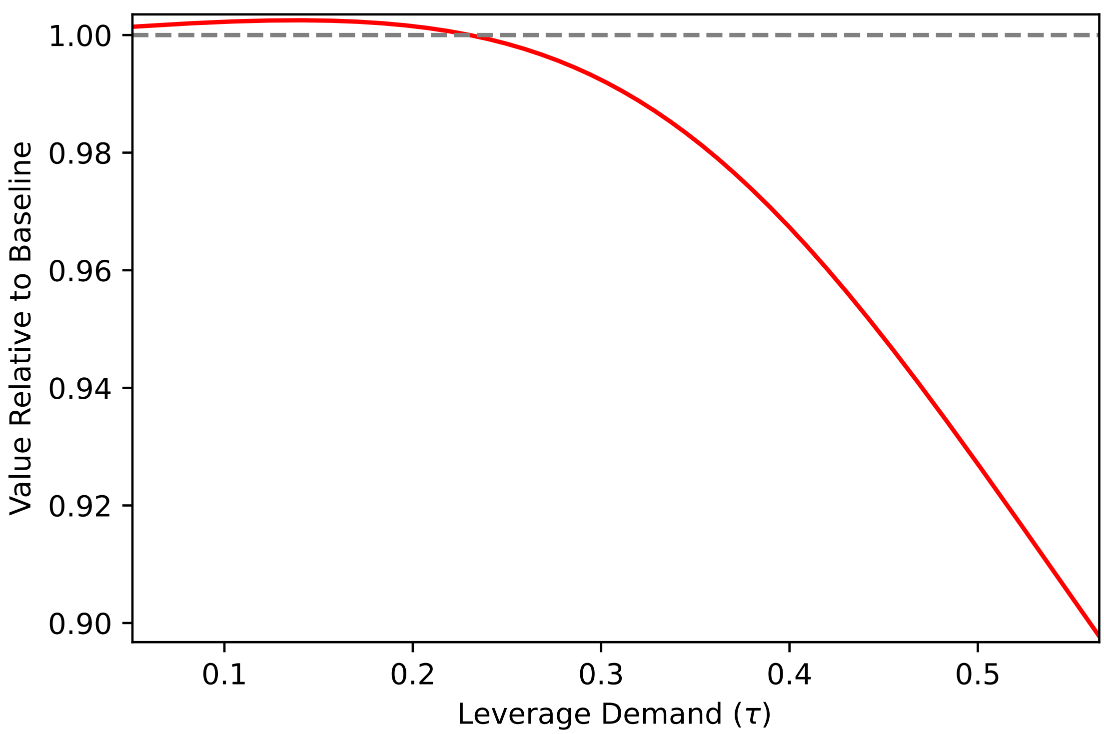

Figure 7: CHANGE IN WELFARE FROM REDUCING λ_{CMBS} BY FOUR-FIFTHS. *Note:* This figure plots $\overline{v}(\tau_i)$ over τ, normalized by its respective value for the baseline calibration, when λ is reduced by a factor of 5.

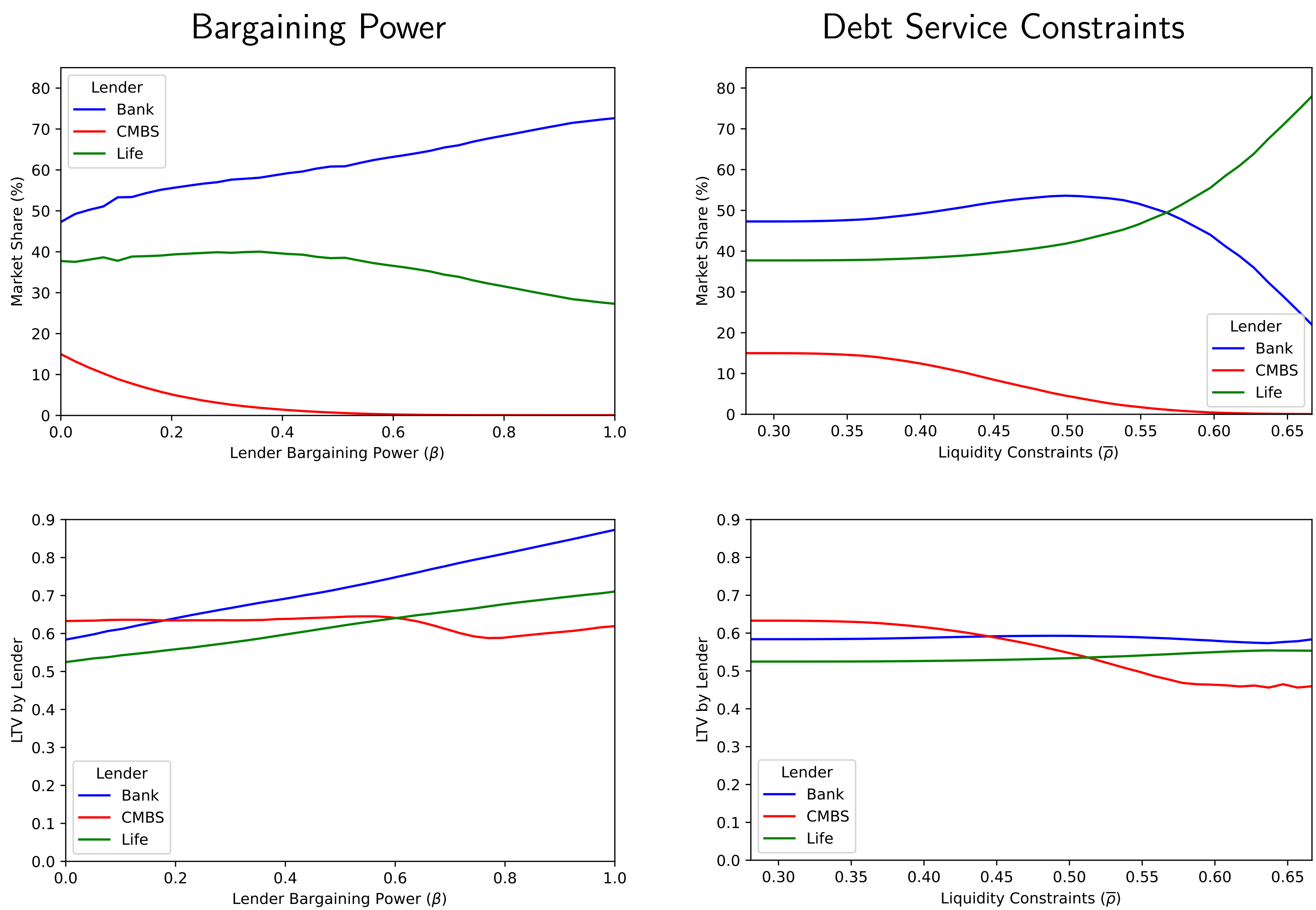

Figure 8: MODEL EXTENSIONS. *Notes:* This figure plots how market shares for each type of lender (top panel) or the average LTV for a particular lender's portfolio (bottom panel) changes as a function lender bargaining power (left), or debt service constraints (right). Other parameter values are the same as those in Table 5.

A. INSTITUTIONAL OVERVIEW

In this appendix, we briefly review institutional factors that affect the willingness of different lenders to offer CRE loan modifications.

A.1. CMBS Modification Restrictions

CMBS have regulatory and contractual restrictions that complicate loan modifications and limit the types of modifications they can provide. Two of the most significant factors limiting loan modifications are tax rules, which prohibit "significant modifications," and pooling and servicing agreements (PSAs), which are contracts that limit the actions available to parties running the CMBS deal.[50]

Regarding the tax rules, most CMBS deals are structured as real estate mortgage investment conduits (REMICs). A REMIC is an entity satisfying certain criteria, including having effectively all of its investments in qualified mortgages and real estate property (including property in foreclosure). REMICs are exempt from federal income taxes. This exemption allows them to avoid double taxation when they issue pass-through securities to investors. Qualified mortgages must meet certain criteria, including being transferred to the REMIC on its start-up day.

To maintain REMIC status, the REMIC cannot add new loans or property in years subsequent to its start-up day. This can make loan modifications difficult to pursue as a substantially modified loan may be considered a new loan. This new loan will not have been transferred to the REMIC on its start-up day and, therefore, will jeopardize the entity's REMIC status. There are exceptions to this rule, including modifications that are "occasioned by default or a reasonably foreseeable default," but even if the modification falls into an exception, there is a danger that the modified loan will violate another REMIC requirement.

REMIC rules have changed over time. During the global financial crisis, the IRS updated the rules to allow more flexibility for modifications with the REMIC structure. The rule issued in 2009

[50] Other aspects of CMBS also make modifications more prohibitive. For example, CMBS receive credit ratings, and certain modifications will require a new rating.

relaxed the foreseeable default requirement to allow modifications if the servicer determines that "there is a significant risk of default of the pre-modification loan upon maturity of the loan or at an earlier date." Despite this more lenient rule, there were still concerns about making significant modifications to loans (see Flynn Jr. et al. 2021 for further details on this rule change and its effects). Another major rule change occurred in the pandemic when the IRS issued a statement temporarily allowing forbearances within the REMIC structure. This rule change led to a number of forbearances, which had previously been extremely uncommon for CMBS loans.[51]

Regarding PSAs, in addition to maintaining REMIC status, each CMBS pool has a PSA that outlines restrictions that the special servicer must abide by when modifying loans. For example, a 2016 PSA provides specific guidance on the special servicer's ability to defer interest:[52]

> The Special Servicer shall use its reasonable efforts to the extent possible to cause each Specially Serviced Loan to fully amortize prior to the Rated Final Distribution Date and shall not agree to a modification, waiver, or amendment of any term of any Specially Serviced Loan if such modification, waiver or amendment would . . . provide for the deferral of interest unless interest accrues on the related Mortgage Loan or the related Serviced Whole Loan at the related Mortgage Rate.

PSAs also outline other parties that must consent to modifications. These consent requirements can also complicate, or at least delay, the approval of modifications. This can be particularly problematic when the relevant parties are inundated with requests, as was the case early on in the pandemic.

[51] See IRS Rev. Proc 2020-26 at https://www.irs.gov/pub/irs-drop/rp-20-26.pdf.

[52] The text comes from Section 3.18 ("Modifications, Waivers, Amendments and Consents") in the pooling and servicing agreement for Morgan Stanley Bank of America Merrill Lynch Trust 2016-C31.

A.2. *Bank and Life Insurer Modification Encouragement*

In contrast to CMBS, where modifications can be curtailed by REMIC rules and PSA restrictions, portfolio lenders have fewer impediments to modification. Since banks and life insurers are typically the sole holder of loans, they rarely have conflicts of interest across dispersed investors to complicate loan negotiations.

Instead, modification decisions are more sensitive to lenders' assessments of how a modification would affect the likely recovery from a potentially distressed loan and by the views of supervisors as to the risks associated with such modifications. On this second point, regulatory agencies actively encouraged lenders to work with customers who were adversely affected by the pandemic.

A joint press release from US bank regulatory organizations in March 2020 read:[53]

> The agencies view prudent loan modification programs offered to financial institution customers affected by COVID-19 as positive and proactive actions that can manage or mitigate adverse impacts on borrowers, and lead to improved loan performance and reduced credit risk. ... Regardless of whether modifications are considered TDRs or are adversely classified, agency examiners will not criticize prudent efforts to modify terms on existing loans for affected customers.

A follow-up press release in April reaffirmed and further clarified this regulatory stance. Additional 2022 proposed guidance on "Prudential Commercial Real Estate Loan Accommodations and Workouts" discussed the importance of working with "CRE borrowers who are experiencing financial difficulty" as opposed to just "customers affected by COVID-19," highlighting that workouts are expected to be a an important part of banks' general resolution toolkit beyond the pandemic.[54]

[53]The text from the March 2020 press release is available at https://www.federalreserve.gov/newsevents/pressreleases/bcreg20200322a.htm, and a revision to that interagency statement pertaining to the CARES Act is available at https://www.federalreserve.gov/newsevents/pressreleases/bcreg20200407a.htm.

[54]The proposed guidance is available at https://www.federalregister.gov/documents/2022/09/15/2022-19940/policy-statement-on-prudent-commercial-real-estate-loan-accommodations-and-workouts

Life insurers, which we emphasize less in the empirical analysis due to data limitations, were similarly encouraged "to work with borrowers who are unable, or may become unable, to meet their contractual payment obligations because of the effects of COVID-19."[55]

B. THEORY APPENDIX

In this section we derive the equilibrium strategic debt service offer from renegotiations, the functions defining the values of debt and equity and the threshold NOI level below which borrowers renegotiate their loans.

B.1. Value Functions Solutions

Since lenders and borrowers are risk neutral, the value functions for debt and equity in the H and L regions must satisfy the ordinary differential equations (ODEs):

$$\begin{aligned} rD_H(X) &= C + \mu X D_H'(X) + \frac{1}{2}\sigma^2 X^2 D_H''(X) \\ rD_L(X) &= S(X) + \mu X D_L'(X) + \frac{1}{2}\sigma^2 X^2 D_L''(X) \\ rE_H(X) &= X - (1-\tau)C + \mu X E_H'(X) + \frac{1}{2}\sigma^2 X^2 E_H''(X) \\ rE_L(X) &= X - (1-\tau)S(X) + \mu X E_L'(X) + \frac{1}{2}\sigma^2 X^2 E_L''(X) \\ &\quad + \lambda(-\theta\frac{C}{r} - E_L(X)), \end{aligned} \tag{9}$$

where $\lambda(-\theta\frac{C}{r} - E_L(X))$ reflects the expected loss to equity holders from renegotiation breaking down.[56]

First, we determine $S(X)$ based on the equilibrium condition that lenders are indifferent between modification and foreclosure. We then solve this set of ODEs to find the resultant value

[55] The statement from the National Association of Insurance Commissioners is available at https://content.naic.org/sites/default/files/inline-files/INT%2020-03%20%20-%20TDR%20for%20COVID-19%3B%20Consolidated%20Appropriations%20Act%20Update.pdf.

[56] This term does not enter into $D_L(X)$ because $S(X)$ is set so that the lender is indifferent between continuation and foreclosure.

functions. Since borrowers make a take-it-or-leave-it offer to their lender, the value of debt must equal the recovery value from foreclosure: $D_L(X) = (1-\alpha^F)\frac{X}{r-\mu} + (1-\alpha^D)\theta\frac{C}{r}$. We can then substitute $D_L(X)$, $D'_L(X)$, and $D''_L(X)$ into the second line of equation (9) and solve for $S(X)$ as

$$S(X) = (1-\alpha^F)X + (1-\alpha^D)\theta C. \tag{10}$$

Once we substitute this expression for $S(X)$ into the fourth line of equation (9), we can see that the remaining three ODEs take the form

$$cV(X) = a + bX + V'(X)\mu X + \frac{1}{2}\sigma^2 X^2 V''(X),$$

which has solution

$$V(X) = \frac{a}{c} + \frac{b}{c-\mu}X + A_\gamma X^{-\gamma} + A_\zeta X^\zeta,$$

where $\gamma > 0$ and $\zeta > 1$ are functions of c, μ, and σ, and A_γ and A_ζ are constants to be pinned down by boundary conditions.

We can solve this set of ODEs as a function of the renegotiation boundary, X_n, using a set of value-matching and asymptotic conditions. Using the asymptotic conditions, we can show that

$$\begin{aligned} D_H(X) &= \frac{C}{r} + A_\gamma^D X^{-\gamma} \\ D_L(X) &= (1-\alpha^F)\frac{X}{r-\mu} + (1-\alpha^D)\theta\frac{C}{r} \\ E_H(X) &= \frac{X}{r-\mu} - (1-\tau)\frac{C}{r} + A_\gamma^E X^{-\gamma} \\ E_L(X) &= (1-\eta_x)\frac{X}{r-\mu} - (1-\tau-\eta_c)\frac{C}{r} + A_\zeta^E X^\zeta. \end{aligned} \tag{11}$$

where

$$\begin{aligned}
\eta_x &= \frac{\lambda + (1-\alpha^F)(1-\tau)(r-\mu)}{r+\lambda+\mu} \\
&= (1-\alpha^F)(1-\tau) + \frac{\lambda}{r+\lambda-\mu}\left(1-(1-\alpha^F)(1-\tau)\right) \\
\eta_c &= \frac{\lambda(1-\tau-\theta) + r(1-\tau)(1-(1-\alpha^D)\theta)}{r+\lambda} \\
&= (1-\tau)(1-(1-\alpha^D)\theta) - \frac{\lambda}{r+\lambda}(\tau+(1-\tau)\alpha^D)\theta \\
\gamma &= \left(\mu - .5\sigma^2 + \sqrt{(.5\sigma^2-\mu)^2 + 2\sigma^2 r}\right)/\sigma^2 > 0 \\
\zeta(\lambda) &= -\left(\mu - .5\sigma^2 - \sqrt{(.5\sigma^2-\mu)^2 + 2\sigma^2(r+\lambda)}\right)/\sigma^2 > 1.
\end{aligned} \tag{12}$$

The other nonlinear term in $D_H(X)$ is eliminated by the condition that $\lim_{X\to\infty} D_H(X) = \frac{C}{r}$. $D_L(X)$ is determined by the equilibrium condition that banks are indifferent between foreclosure and renegotiation. The other nonlinear term in $E_H(X)$ is eliminated by the condition that the value of the default option goes to 0 as $X \to \infty$. The other non-linear term in $E_L(X)$ is eliminated by the condition that $\lim_{X\to 0} E_L(X) = \frac{-\lambda\theta C}{r(r+\lambda)} - \frac{(1-\tau)(1-\alpha^D)\theta C}{r+\lambda}$.[57]

The remaining unknowns determining the value of equity are the constants A^E_γ and A^E_ζ, and the renegotiation boundary X_n. These are determined by the following conditions:

$$\text{Value Matching: } E_H(X_n) = E_L(X_n)$$

$$\text{Smooth Pasting: } E'_H(X_n) = E'_L(X_n)$$

$$\text{Super Contact: } E''_H(X_n) = E''_L(X_n)$$

Super contact is a condition for borrowers optimally choosing the renegotiation threshold that

[57] $\frac{\lambda\theta C}{r(r+\lambda)}$ is the present discounted value of a deficiency judgment payout of $\frac{\theta C}{r}$ with an exponentially distributed arrival time, and $\frac{(1-\tau)(1-\alpha^D)\theta C}{r+\lambda}$ is the present discounted value of debt payments (excluding tax shields) made before negotiation breaks down. Combined, they give the value of payments by the property investor—the only cash flow when the property is yielding no income. The second condition says that if negotiation breaks down immediately, the value in the renegotiation state is $\frac{-\theta C}{r}$, reflecting an immediate deficiency judgment.

maximizes the value of equity (see Dumas (1991) or Chapter 4.6 of Dixit (1993)).[58] Using this condition is appropriate under the assumption that lenders are willing to accept a modification rather the foreclose at the borrower's renegotiation boundary. For very low λs, the renegotiation boundary would be determined by the highest income at which lenders would accept a modification. For completeness, we derive the renegotiation boundary for the full range of λ in Section B.2. However, we find numerically in our calibration that modification frictions are sufficiently high for these to be the relevant equilibrium conditions, even for the comparatively low-friction lenders. Consequently, we focus here on the case where the lender participation constraint is not binding.

Noting that

$$E_H(X) - E_L(X) = \eta_x \frac{X}{r-\mu} - \eta_c \frac{C}{r} + A_\gamma^E X^{-\gamma} - A_\zeta^E X^\zeta$$

the equilibrium conditions are that:

$$\begin{aligned}
\text{Value Matching: } & \eta_x \frac{X_n}{r-\mu} - \eta_c \frac{C}{r} + A_\gamma^E X_n^{-\gamma} - A_\zeta^E X_n^\zeta = 0 \\
\text{Smooth Pasting: } & \eta_x \frac{1}{r-\mu} - \gamma A_\gamma^E X_n^{-\gamma-1} - \zeta A_\zeta^E X_n^{\zeta-1} = 0 \\
\text{Super Contact: } & \gamma(\gamma+1) A_\gamma^E X_n^{-\gamma-2} - \zeta(\zeta-1) A_\zeta^E X_n^{\zeta-2} = 0
\end{aligned}$$

We can use these to solve for the unknowns in the value of equity equations:

$$\begin{aligned}
\frac{X_n}{r-\mu} &= \underbrace{\frac{\gamma\zeta}{(1+\gamma)(\zeta-1)} \frac{\eta_c}{\eta_x}}_{\equiv \rho(\lambda,\theta)} \frac{C}{r} \\
A_\gamma^E X_n^{-\gamma} &= \left(\underbrace{\frac{\zeta}{\gamma+\zeta}\eta_c - \frac{\zeta-1}{\gamma+\zeta}\eta_x \rho}_{\equiv \omega(\lambda,\theta)} \right) \frac{C}{r} \\
A_\zeta^E X_n^\zeta &= \left(\frac{1+\gamma}{\gamma+\zeta}\eta_x \rho - \frac{\gamma}{\gamma+\zeta}\eta_c \right) \frac{C}{r}
\end{aligned} \tag{13}$$

[58] The value of equity is increasing in A_γ^E, which is a function of X_n, so the borrower's renegotiation decision is a matter of choosing X_n to maximize A_γ^E. Super contact holds when $\frac{\partial A_\gamma^E}{\partial X_n} = 0$.

where ρ measures the default boundary and ω measures the option value of renegotiation to the borrower.[59]

The unknown determining the value of debt in Equation (11), A^D_γ, is identified by the value matching conditions that $D_H(X_n) = D_L(X_n)$. For this equation to hold, we must have that:

$$\begin{aligned} -A^D_\gamma X_n^{-\gamma} &= \frac{C}{r} - D_L(X_n) \\ &= \left(\underbrace{1-(1-\alpha^F)\rho-(1-\alpha^D)\theta}_{\equiv \chi(\lambda,\theta)} \right) \frac{C}{r} \end{aligned} \tag{14}$$

where the second line comes from substituting in the function for $D_L(X)$ from Equation (14) and the renegotiation boundary X_n from Equation (13). Note that χ is percentage difference in value between a default free loan and a loan at the renegotiation boundary, and thus can be thought of as a measure of loss given renegotiation.

Substituting the constants and renegotiation boundary from Equations (13) and (14) into the value functions in Equation (11) gives the values of debt and equity as a function of X and C:

$$\begin{aligned} D_H(X;C) &= \frac{C}{r}\left(1-\left(\frac{\frac{X}{r-\mu}}{\rho\frac{C}{r}}\right)^{-\gamma}\chi\right) \\ D_L(X;C) &= (1-\alpha^F)\frac{X}{r-\mu}+(1-\alpha^D)\theta\frac{C}{r} \\ E_H(X;C) &= \frac{X}{r-\mu}-(1-\tau)\frac{C}{r}+\left(\frac{\frac{X}{r-\mu}}{\rho\frac{C}{r}}\right)^{-\gamma}\omega\frac{C}{r} \\ E_L(X;C) &= (1-\eta_x)\frac{X}{r-\mu}-(1-\tau-\eta_c)\frac{C}{r}+\left(\frac{\frac{X}{r-\mu}}{\rho\frac{C}{r}}\right)^{\zeta}\left(\frac{1+\gamma}{\gamma+\zeta}\eta_x\rho-\frac{\gamma}{\gamma+\zeta}\eta_c\right)\frac{C}{r} \end{aligned} \tag{15}$$

where ρ, χ, ω and ζ are functions of λ as before, though we suppress the arguments for readability.

[59] The last two expressions solve for the constants for an arbitrary $\rho = \frac{X_n}{r-\mu}/\frac{C}{r}$ without imposing super contact, which corresponds with the optimally chosen renegotiation boundary. Substituting in the expression for ρ from the top expression gives the value function at the optimal boundary.

B.2. Lenders' renegotiation decision

For a loan to be modified, the borrower needs the debt service reduction to be enough to compensate borrowers for the risk of negotiations breaking down and the costs of foreclosure must be high enough for lenders to prefer modification to foreclosure. The expressions in Section B.1 derive renegotiation boundary optimally chosen by the borrower, without confirming that the lender would accept the modification. We now solve for renegotiation outcomes accounting for both participation constraints.

The lender's participation constraint is that $D_H'(X_n) \geq D_L'(X_n)$. If this did not hold (namely if there was a concave kink at X_n), the value function would be below the recovery value around the renegotiation boundary and the lender would choose to foreclose instead. Differentiating D_H and D_L from Equation (11) with respect to X, evaluating at X_n, and using the value matching condition $D_H(X_n) = D_L(X_n)$ to remove the constant, we can express the inequality as:

$$\frac{X_n}{r-\mu} \leq \frac{\gamma}{1+\gamma}\frac{1-(1-\alpha^D)\theta}{1-\alpha^F}\frac{C}{r} \tag{16}$$

When $\frac{\gamma}{1+\gamma}\frac{1-(1-\alpha^D)\theta}{1-\alpha^F}$ exceeds the ρ derived in Equation (13), lenders' willingness to accept a modification becomes the binding constraint, and the renegotiation boundary is determined by this smooth pasting condition. This means that the full expression for the renegotiation boundary is:

$$\rho(\lambda,\theta) = \min\{\frac{\gamma}{1+\gamma}\frac{1-(1-\alpha^D)\theta}{1-\alpha^F}, \frac{\gamma\zeta}{(1+\gamma)(\zeta-1)}\frac{\eta_c}{\eta_x}\}. \tag{17}$$

In Online Appendix Section A.1, we show that the expression on the right is increasing in λ, is below the expression on the left at $\lambda = 0$, and is above the expression on the left for a sufficiently high λ. Consequently, there is a threshold level of renegotiation frictions above which renegotiation is determined by the borrower's super contact condition and below which renegotiation is determined by the lender's smooth-pasting condition. Intuitively, when negotiations are unlikely to break down, borrowers have little to lose from negotiating a debt service reduction, and thus do so at first opportunity. When frictions are higher, borrowers are deterred from renegotiating, and the factor determining when renegotiation occurs is when it is optimal for the borrower to do so.

ONLINE APPENDIX

A. COMPARATIVE STATICS AND ANALYTIC RESULTS

In this section, we analyze the comparative statics for key functions in the model.

A.1. Characterization of the Modification Boundary

We start by characterizing the function defining where renegotiation occurs. In Appendix B.2, we show that

$$\rho(\lambda,\theta) = \min\{\frac{\gamma}{1+\gamma}\frac{1-(1-\alpha^D)\theta}{1-\alpha^F}, \underbrace{\frac{\gamma\zeta}{(1+\gamma)(\zeta-1)}\frac{\eta_c}{\eta_x}}_{\tilde{\rho}(\lambda,\theta)}\}$$

where the left expression gives the threshold where lenders are first willing to accept a modification and $\tilde{\rho}$ is renegotiation boundary given in Equation (13), pertaining to the threshold at which borrowers would optimally modify a loan, assuming the lender is willing to accept the offer. η_c, η_x, γ and ζ are defined in Equation (12).

A.1.1. Comparative statics for modification frictions ($\frac{\partial\rho}{\partial\lambda}$)

The left expression in the expression for ρ is independent of λ; since lenders are indifferent between foreclosure and modification, the rate at which modifications break down does not affect their willingness to modify a loan. So, we instead focus on $\tilde{\rho}$, reflecting the borrower's preferred renegotiation threshold, which is a function of λ.

Differentiating $\tilde{\rho}$ with respect to λ gives that

$$\frac{\tilde{\rho}_\lambda}{\tilde{\rho}} = \frac{\frac{\partial}{\partial\lambda}\left(\frac{\zeta}{\zeta-1}\right)}{\left(\frac{\zeta}{\zeta-1}\right)} + \frac{\frac{\partial\eta_c}{\partial\lambda}}{\eta_c} - \frac{\frac{\partial\eta_x}{\partial\lambda}}{\eta_x}.$$

whose components can be solved as the following by differentiating the expressions in Equation (12) with respect to λ:

$$\frac{\frac{\partial}{\partial\lambda}\left(\frac{\zeta}{\zeta-1}\right)}{\left(\frac{\zeta}{\zeta-1}\right)} = -\frac{\frac{\partial\zeta}{\partial\lambda}}{\zeta} = -\left((.5\sigma^2-\mu)^2+2\sigma^2(r+\lambda)\right)^{\frac{-1}{2}}\Big/\zeta < 0$$

$$\frac{\partial\eta_c}{\partial\lambda} = \frac{-r}{(r+\lambda)^2}(\tau+(1-\tau)\alpha^D)\theta < 0$$

$$\frac{\partial\eta_x}{\partial\lambda} = \frac{r-\mu}{(r+\lambda-\mu)^2}\left(1-(1-\alpha^F)(1-\tau)\right) > 0.$$

All three terms in $\frac{\tilde{\rho}_\lambda}{\tilde{\rho}}$ are negative, meaning that the modification boundary is decreasing in λ for any λ such that the lender's participation constraint is not binding. That is, higher modification frictions cause borrowers to be willing to maintain promised debt payments for lower levels of NOI.

A.1.2. Characteristics of ρ in the limit

Since $\tilde{\rho}$ is monotonically decreasing in λ, and the lender's participation constraint is independent of λ, if $\rho = \tilde{\rho}$ for any given λ, they will also be equal for any break down rate above that level. Now we establish that the lender's participation constraint is binding for sufficiently low λs and not binding for sufficiently high λ. This means that there is a threshold breakdown rate at which the lender participation constraint ceases to be binding, and above which the dynamics are driven by the equations provided in the main text.

First, consider the case where negotiation frictions do not break down. Evaluating η_c and η_x from Equation (12) at $\lambda = 0$, we get that $\tilde{\rho}(0,\theta) = \frac{\gamma}{1+\gamma}\frac{\zeta(0)}{\zeta(0)-1}\frac{1-(1-\alpha^D)\theta}{1-\alpha^F} > \frac{\gamma}{1+\gamma}\frac{1-(1-\alpha^D)\theta}{1-\alpha^F}$. Since $\tilde{\rho}(0,\theta)$ is higher than the lender's participation constraint (i.e., borrowers would like to modify loans at incomes higher than lenders would accept), the modification boundary when there are no breakdowns is determined by that constraint.

Taking the limit of the expressions in Equation (12) as $\lambda \to \infty$, we get that $\lim_{\lambda\to\infty}\tilde{\rho} = \frac{\gamma}{1+\gamma}(1-\tau-\theta) < \frac{\gamma}{1+\gamma}\frac{1-(1-\alpha^D)\theta}{1-\alpha^F}$, meaning that the lender participation constraint does not bind, and the renegotiation boundary equals $\tilde{\rho}$. To summarize, the renegotiation boundary for extreme values of

λ are:

$$\begin{aligned} \rho(0,\theta) &= \frac{\gamma}{1+\gamma}\frac{1-(1-\alpha^D)\theta}{1-\alpha^F} \\ \lim_{\lambda\to\infty}\rho(\lambda,\theta) &= \frac{\gamma}{1+\gamma}(1-\tau-\theta). \end{aligned} \tag{1-1}$$

At the lower limit for λ, the renegotiation boundary is identical to Hackbarth et al. (2007) except for the term $(1-\alpha^D)\theta$, reflecting how much recourse affects the negotiation boundary when modifications never break down. Since negotiations never break down at the lower limit, recourse only affects modifications to the extent that it affects the lender's recovery in foreclosure (i.e., its disagreement payoff). Therefore, the boundary only shifts to the extent that lenders can recover losses. Higher foreclosure costs raise the renegotiation threshold because lenders are more willing to accept a lower debt service payment to avoid a foreclosure.

At the other limit, as $\lambda \to \infty$, negotiations break down immediately. In this case, the decision to renegotiate is a decision to accept foreclosure. This limit corresponds to the default threshold in Leland (1994)—shifted to reflect recourse—where firms are choosing an optimal default threshold instead of a renegotiation threshold. At this limit, the recourse share matters on its own, instead of the recourse share multiplied by the recovery rate. Without modifications, recourse affects the default boundary because it imposes losses on the borrower and discourages them from defaulting. This expression says that borrowers will be willing to maintain debt payments even when the present value of NOI falls below the present value of promised debt payments to preserve the option value of the loan (γ is decreasing in σ), to preserve their debt shield (the τ term), and to avoid a deficiency judgment (the θ term). Foreclosure costs no longer matter, as they affect the lender's recovery, not the borrower's loss.

At intermediate values of λ, both sets of mechanisms matter: lenders' potential recoveries affect borrowers' incentives to modify by changing the payments required on modified loans, while borrowers' losses in foreclosure affect incentives to modify by changing the cost of modifications breaking down. The extent to which each factor matters depends on how close λ is to either extreme.

A.2. *Comparative Statics for Supply Curves*

Here we analyze how recourse and modification frictions affect supply curves—that is, the LTVs that lenders are willing to offer for a given loan rate spread. Comparative statics with respect to θ and λ are similar, as both variables affect supply by changing the modification boundary. For this reason, we analyze the effects of these variables together.

Substituting χ from (14) into the supply curve defined in (3) and differentiating with respect to θ and λ, we can see that increasing the degree of recourse or of modification frictions induce lenders to offer higher LTVs for a given spread:

$$\begin{aligned}
\frac{\partial LTV(s;\theta,\lambda)}{\partial \lambda} &= LTV(s)\left(\underbrace{\frac{(1-\alpha^F)\rho-\gamma\chi}{\gamma\chi}}_{(-\text{ or }0)}\underbrace{\frac{\rho_\lambda}{\rho}}_{(-)}\right) > 0 \\
\frac{\partial LTV(s;\theta,\lambda)}{\partial \theta} &= LTV(s)\left(\underbrace{\frac{(1-\alpha^F)\rho-\gamma\chi}{\gamma\chi}}_{(-\text{ or }0)}\underbrace{\frac{\rho_\theta}{\rho}}_{(-)}+\underbrace{\frac{1-\alpha^D}{\gamma\chi}}_{(+)}\right) > 0,
\end{aligned} \tag{1-2}$$

where ρ_θ and ρ_λ are the partial derivatives of ρ with respect to θ and λ, respectively, which are all negative (see Appendix A.1).

As $1-\alpha^D$ and $\gamma\chi$ are positive, it is clear that the sign of the comparative statics depends critically on the sign of $(1-\alpha^F)\rho-\gamma\chi$. This expression measures the sensitivity of loan supply to changes in the modification boundary, accounting for both the direct effects of changing ρ in equation (3), and the effects operating through χ.[60]

[60] These effects work in opposite directions. A lower modification boundary directly increases allowable LTVs; however, as modifications occur at lower property values, loan losses when modifications do occur are higher. We show here that the first effect wins out.

Substituting in χ from Equation (14), we can show that

$$\begin{aligned}(1-\alpha^F)\rho(\lambda,\theta)-\gamma\chi &= (1+\gamma)(1-\alpha^F)\rho(\lambda,\theta)-\gamma(1-(1-\alpha^D)\theta) \\ &= (1+\gamma)(1-\alpha^F)\left(\rho(\lambda,\theta)-\rho(0,\theta)\right)\end{aligned} \tag{1-3}$$

where $\rho(0,\theta)$ comes from equation (1-1). This sensitivity is 0 when the lender's participation constraint is binding, and negative for λs that are sufficiently high for the lender's participation constraint to not be binding.

Having derived the direction of the effects of recourse and modification frictions on supply, we now discuss the economics involved. Focusing first on the top line of (1-2), which shows how modification frictions affect LTV, we can see that λ affects the supply curve entirely by shifting the modification boundary. When λ is higher, the renegotiation threshold (ρ) is lower, since the risk of negotiations breaking down discourages renegotiation at the margin. Lenders can therefore offer a higher original LTV and achieve the same risk of modification, and thus allow higher LTVs for a given spread. Overall, this term shows that increased modification frictions ($\lambda \uparrow$) lower the modification boundary ($\rho \downarrow$), which allows borrowers to take out a higher LTV for a given spread ($LTV(s) \uparrow$). That is, credit supply is increasing in λ.

The second line in (1-2) shows how the availability of recourse affects LTVs. The first term is similar to the previous expression. Increasing recourse shifts the supply curve out by lowering the modification boundary. However, there is one additional term, $\frac{1-\alpha^D}{\gamma\chi}$, which reflects the extent to which recourse reduces loss given default. Thus, recourse affects supply in two ways: first, it discourages borrowers from seeking modifications (as with increasing λ), and, second, it directly affects recoveries when lenders foreclose. Both of these forces contribute to a positive relationship between LTV and recourse.

B. EXTENSIONS

This section derives loan outcomes for the extensions described in Section 4.4. The first subsection derives outcomes when lenders have bargaining power when renegotiating loans. The second subsection derives outcomes when borrowers face debt service constraints.

B.1. Bargaining Power

In this subsection, we extend the model to allow lenders to have some bargaining power in loan modification negotiations. As before, borrowers choose the threshold at which to pursue a modification. However, instead of the modified payment being determined by a take-it-or-leave-it offer from the borrower, now $S(X)$ is determined by a more general bargaining process. Let β denote the bargaining power of the lender in modification renegotiations. When $\beta = 0$, the borrower has all of the power, and modification outcomes are as before: $S(X;\beta = 0)$ is as in Equation (10). When $\beta = 1$, the lender has all of the bargaining power and makes a take-it-or-leave-it offer to the borrower to modify the debt service amount, denoted $S(X;\beta = 1)$. Finally, when $\beta \in (0,1)$, the modified debt service amount is a weighted average of these two outcomes, with a weight of β on the outcome where the lender sets the offer:

$$S(X;\beta) = \beta S(X;\beta = 1) + (1-\beta)S(X;\beta = 0).$$

To determine the modified debt service amount for a given bargaining power, we thus need to solve for $S(X;\beta = 1)$. By a similar logic to what was laid out in Appendix B, given all the bargaining power, lenders would set $S(X)$ to make the borrower indifferent to foreclosure: $E_L(X) = -\theta\frac{C}{r}$. From equation (9), this value function would be satisfied for $S(X;\beta = 1) = \frac{X+\theta C}{1-\tau}$.

Combined with equation (10), we get that the modified debt service amount when lenders have bargaining power β is

$$S(X;\beta)=\left((1-\beta)(1-\alpha^F)+\frac{\beta}{1-\tau}\right)X+\left((1-\beta)(1-\alpha^d)+\frac{\beta}{1-\tau}\right)\theta C,$$

which is increasing in β, particularly when foreclosure costs are higher.

The differential equations defining debt and equity values in the modification region are:

$$\begin{aligned}
rD_L(X;\beta) &= S(X;\beta)+\mu X D_L'(X)+\frac{1}{2}\sigma^2X^2D_L''(X)\\
&\quad +\lambda(R(X)-D_L(X)),\\
rE_L(X;\beta) &= X-(1-\tau)S(X;\beta)+\mu X E_L'(X)+\frac{1}{2}\sigma^2X^2E_L''(X)\\
&\quad +\lambda(-\theta\frac{C}{r}-E_L(X)),
\end{aligned}$$

which are as before, besides the change to $S(X)$ and the fact that the $R(X)-D_L(X)$ does not drop out of the first equation (since lenders are no longer indifferent to foreclosure). The solutions to these equations for the new $S(X)$ function are:

$$\begin{aligned}
D_L(X;\beta) &= \left((1-\alpha^F)+\frac{\beta}{1+\frac{\lambda}{r-\mu}}(\frac{\tau}{1-\tau}+\alpha^F)\right)\frac{X}{r-\mu}\\
&\quad +\left((1-\alpha^D)+\frac{\beta}{1+\frac{\lambda}{r}}(\frac{\tau}{1-\tau}+\alpha^D)\right)\theta\frac{C}{r}+A_\zeta^DX^\zeta\\
E_L(X;\beta) &= \frac{1-\beta}{1+\frac{\lambda}{r-\mu}}(\alpha^F+\tau(1-\alpha^F))\frac{X}{r-\mu}\\
&\quad +\left(\frac{1-\beta}{1+\frac{\lambda}{r}}(\alpha^D+\tau(1-\alpha^D))-1\right)\theta\frac{C}{r}+A_\zeta^EX^\zeta.
\end{aligned}$$

The expressions for the high region are the same as in Equation (11), besides a change in the constants pinned down by boundary conditions. Since the basic structure is the same as in the baseline model, the outcomes continue to depend on the loadings on $\frac{X}{r-\mu}$ and $\frac{C}{r}$ in the functions

$E_H(X) - E_L(X)$ and $D_H(X) - D_L(X)$, which are now:[61]

$$\eta_{D,C} = 1 - \left((1-\alpha^D) + \frac{\beta}{1+\frac{\lambda}{r}} \left(\frac{\tau}{1-\tau} + \alpha^D \right) \right) \theta$$
$$\eta_{D,X} = (1-\alpha^F) + \frac{\beta}{1+\frac{\lambda}{r-\mu}} \left(\frac{\tau}{1-\tau} + \alpha^F \right)$$
$$\eta_{E,C} = 1 - \tau - \theta + \frac{1-\beta}{1+\frac{\lambda}{r}} \left(\alpha^D + \tau(1-\alpha^D) \right) \theta$$
$$\eta_{E,X} = 1 - \frac{1-\beta}{1+\frac{\lambda}{r-\mu}} \left(\alpha^F + \tau(1-\alpha^F) \right).$$

Consequently, the solution for the value of equity obtained in equation (2) still holds, but with ρ and ω updated with the new loadings:

$$\begin{aligned} \rho(\lambda,\theta,\beta) &= \frac{\gamma\zeta}{(1+\gamma)(\zeta-1)} \frac{\eta_{E,C}}{\eta_{E,X}} \\ \omega(\lambda,\theta,\beta) &= \frac{\zeta}{\zeta+\gamma}\eta_{E,C} - \frac{\zeta-1}{\zeta+\gamma}\eta_{E,X}\rho \end{aligned} \tag{1-4}$$

It is immediately clear from these equations that higher lender bargaining power discourages borrowers from renegotiating, i.e., ρ is decreasing in β. By shifting cash flows to lenders in the event of a modification, lender bargaining power prevents borrowers from renegotiating loans until they face a larger decline in cash flows. It is also readily apparent that lender bargaining power interacts with modification frictions. β and λ always appear together in these expressions, with β divided by $1+\frac{\lambda}{r-\mu}$ or $1+\frac{\lambda}{r}$. Therefore, as modification frictions get higher, the effects of bargaining power get smaller. Note that $\lim_{\lambda\to\infty} \rho(\lambda,\theta,\beta) = \rho(\lambda',\theta,1)\ \forall \lambda',\beta$. Namely, if lenders have full bargaining power, the effect of modification breakdowns on the renegotiation

[61] Specifically, these constants are such that:

$$E_H(X) - E_L(X) = \eta_{E,X}\frac{X}{r-\mu} - \eta_{E,C}\frac{C}{r} + A^E_\gamma X^{-\gamma} - A^E_\zeta X^\zeta$$

$$D_H(X) - D_L(X) = \eta_{D,C}\frac{C}{r} - \eta_{D,X}\frac{X}{r-\mu} + A^D_\gamma X^{-\gamma} - A^D_\zeta X^\zeta$$

boundary goes away. As borrowers realize no surplus from modifications, renegotiations occur at the point that a borrower would otherwise default in a model without modifications (the default threshold in Leland 1994).

Expressions for the value of debt are now complicated by the additional non-linear term in the low region. Consequently, we solve the model numerically. Specifically, we numerically solve the ODEs using finite differences for a given C and ρ. We then solve for the ρ that maximizes the value of equity for a given C, and the C that maximizes the value of the firm for a given ρ. The intersection of these equations provides the equilibrium C and ρ, and hence the equilibrium values of equity and debt for a particular parameterization.

B.2. Non-strategic Default

In this subsection, we outline how the model changes when we allow for non-strategic loan renegotiation. In the baseline model, borrowers are assumed to have access to outside funds and thus able to maintain loan payments in excess of property cash flows. While this assumption is likely reasonable for the larger sponsors that disproportionately utilize CMBS credit, it may not be true for many smaller borrowers. In this extension, we analyze what happens when some borrowers do not have access to outside funds and thus need to renegotiate earlier than those that renegotiate for purely strategic reasons.

Liquidity constraints affect the location of the renegotiation boundary rather than outcomes within each region, so the value functions conditional on X_n are the same as before.[62] Borrowers will renegotiate loans if either it is optimal for them to do so or if cash flows fall below the amount required to service the promised debt payments. Denote $\overline{\rho}$ as the ratio between unlevered property values and the present value of promised debt payments below which borrowers will renegotiate

[62] Liquidity constraints could also affect outcomes within the renegotiation region if a borrower's maximum feasible debt service payment falls below $S(X_t)$, meaning that borrowers are unable to offer modified loan terms the bank prefers to foreclosure. We assume that this does not occur as a lender would only prefer foreclosure to receiving X_t if the benefit of a speedier deficiency judgment outweighs foreclosure costs. For this to hold, borrowers would need to have other assets of value, contradicting the idea that borrowers are constrained. More formally, if borrowers have outside assets $\frac{\theta C}{r}$ yielding a return r, then the maximum debt service amount would be $X_t + \theta C$, which necessarily is greater than $S(X_t) = (1-\alpha^F)X_t + (1-\alpha^D)\theta C$.

because they can no longer make the promised payment C. In this circumstance, the renegotiation threshold is given by $\frac{X_n}{r-\mu} = \rho(\lambda,\theta,\overline{\rho})\frac{C}{r}$, where $\rho(\lambda,\theta,\overline{\rho}) \equiv \max\{\overline{\rho},\rho(\lambda,\theta)\}$ and $\rho(\lambda,\theta)$ is the unconstrained renegotiation boundary.[63]

For sufficiently high liquidity constraints, lenders choose to foreclose rather than modify loans. This occurs when $\overline{\rho} > \frac{\gamma}{1+\gamma}\frac{1-(1-\alpha^D)\theta}{1-\alpha^F}$ (see Section B.2 for a discussion of lenders' participation constraints). In this case, modifications do not occur, and the model is equivalent to a Leland (1994) model with an exogenous default boundary. To maintain the focus on renegotiation, we consider in Figure 8 parameter values such that borrowers are able to make up debt service short falls enough that lenders are willing to consider a modification at the point non-strategic default occurs.

[63]For example, $\overline{\rho} = \frac{r}{r-\mu}$ would correspond with borrowers renegotiating when $X_t < C$ (when cash flow falls under debt service costs) and $\overline{\rho} = \frac{r}{r-\mu}(1-\theta)$ would correspond with borrowers renegotiating when $X_t + \theta C < C$ (when cash flow plus the return on outside assets covered by recourse pledges falls under debt service costs).

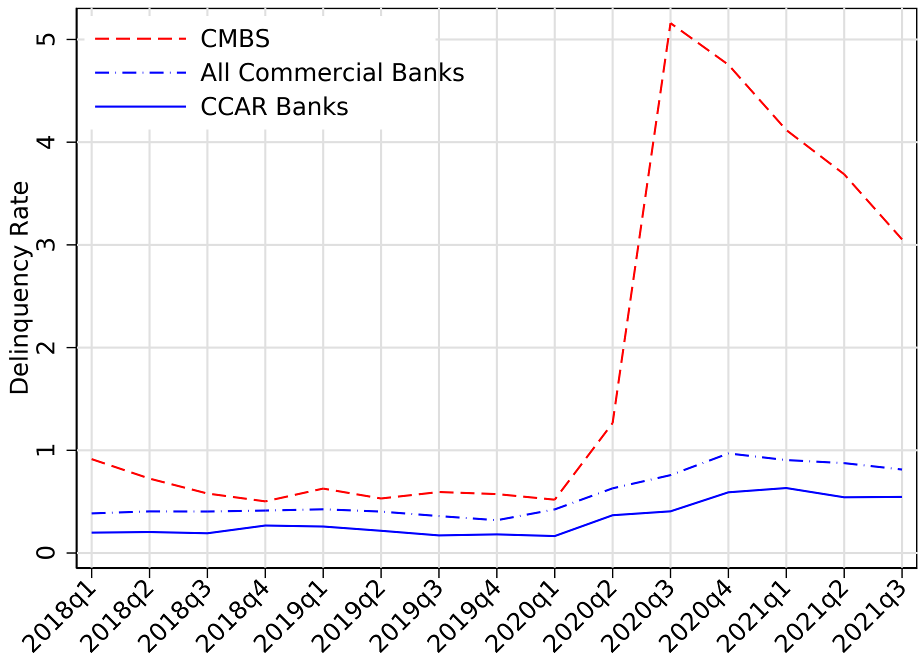

Figure C.1: SHARE OF BALANCES THAT ARE 90+ DAYS DELINQUENT OR NON-ACCRUAL. *Note:* Shares are in percentage points. *Source:* Authors' calculations using Trepp CMBS data, Call Reports, and Y-14 H.2 Schedule.

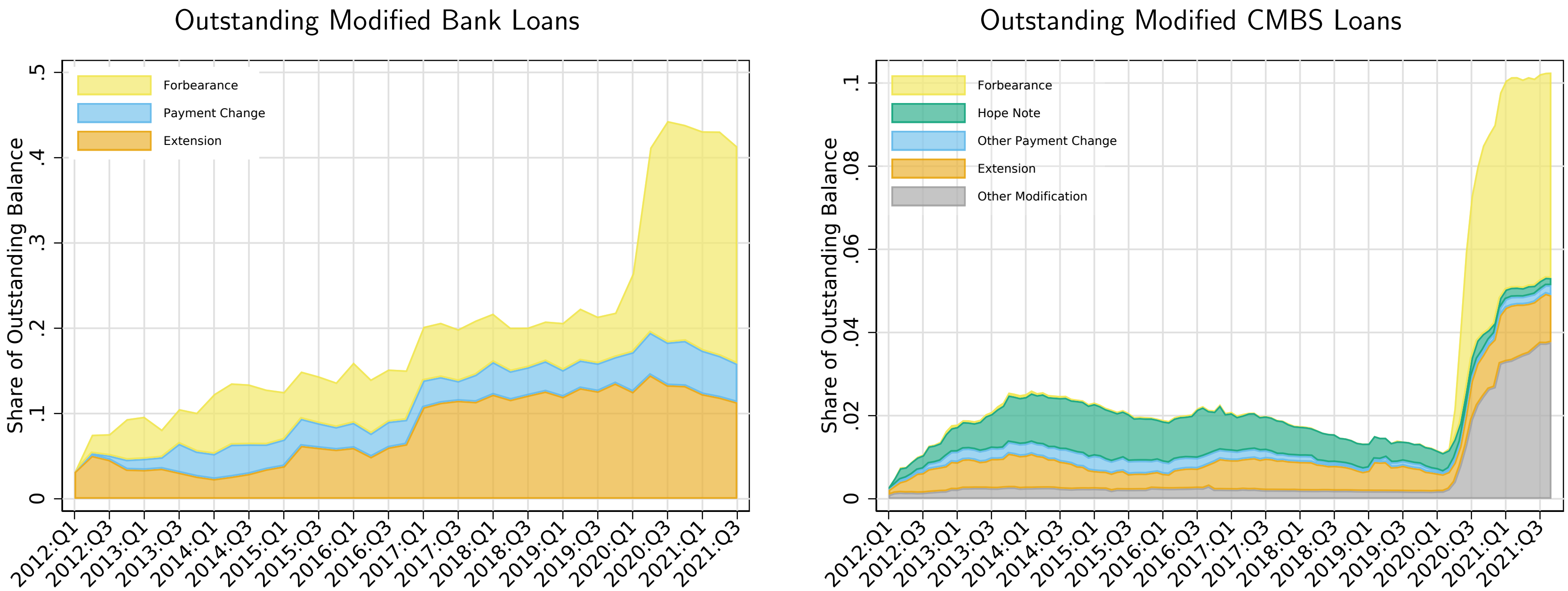

Figure C.2: BANK AND CMBS MODIFICATION TYPES. *Note:* Share of outstanding balances that have received a modification since January 2012. Outstanding balances are limited to loans that are current or less than 120 days delinquent. A "hope note" is a type of CMBS modification where an underwater loan is split into two pari passu pieces, generally also with an equity injection from the borrower, where the A piece is paid off as normal, and the B piece (or hope note) is only repaid if the property value recovers. *Source:* Authors' calculations using Trepp CMBS data and Y-14 H.2 Schedule.

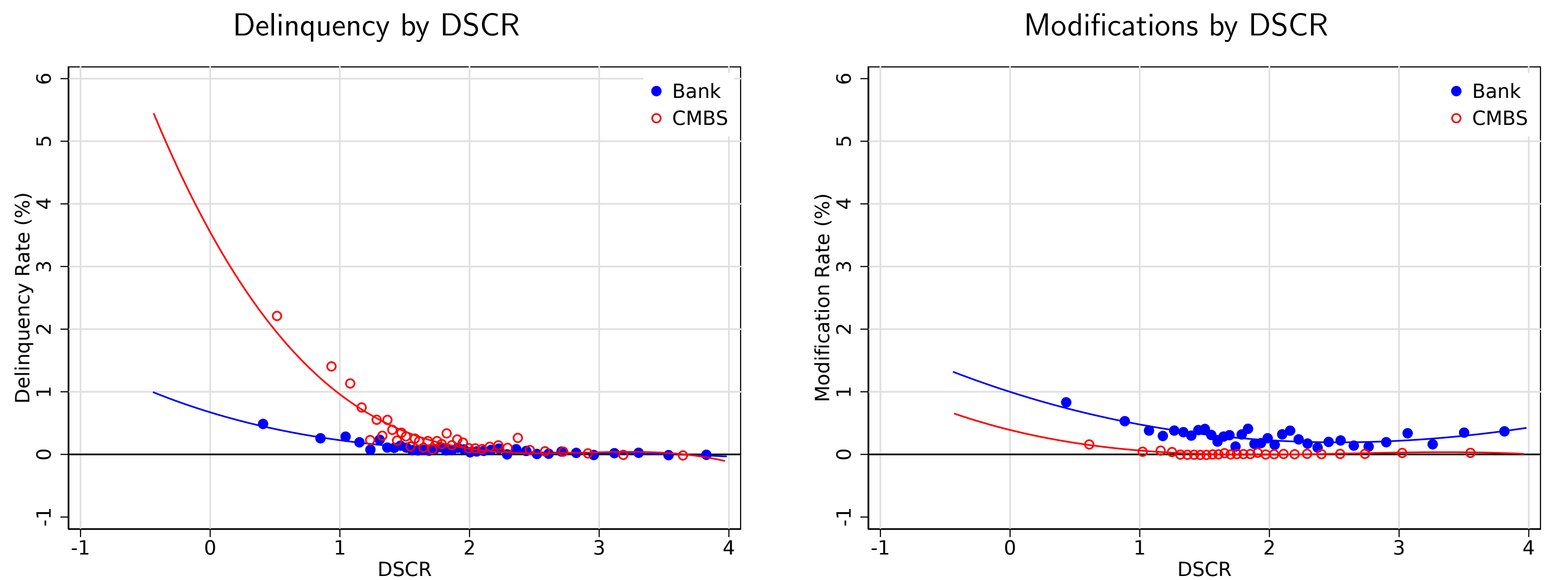

Figure C.3: Delinquency and Modification Rates by Current DSCR. *Note:* This figure presents semi-linear regression estimates of the relationship between current DSCR and the quarterly rate at which loans go delinquent (left) or get modified (right). Dots are bin scatter estimates for particular LTV quantiles, and the lines plot estimates derived from a global cubic polynomial in current LTV. Estimates are produced using the Stata command binsreg (see Cattaneo et al. 2019), controlling for origination DSCR, origination LTV, loan age in quarters, the log of the loan origination amount, the loan term, an indicator for whether the loan is interest only, and quarter by state, quarter by property type, and originator by origination year fixed effects. Data include loan-quarter observations from 2012q1 to 2019q4 that have an at-origination DSCR between 1 and 20, an at-origination LTV above 20 percent, and are not 120+ days delinquent. The current DSCR is calculated as the current NOI divided by the current loan payment. The current NOI is linearly interpolated between reported values and extrapolated using the CBRE NOI index for the property type of the loan in the loan's market. The current loan payment is reported in the CMBS data and is imputed using the loan terms in the bank data. *Source:* Authors' calculations using Trepp CMBS data, CBRE indices, and the Y-14 H.2 Schedule.

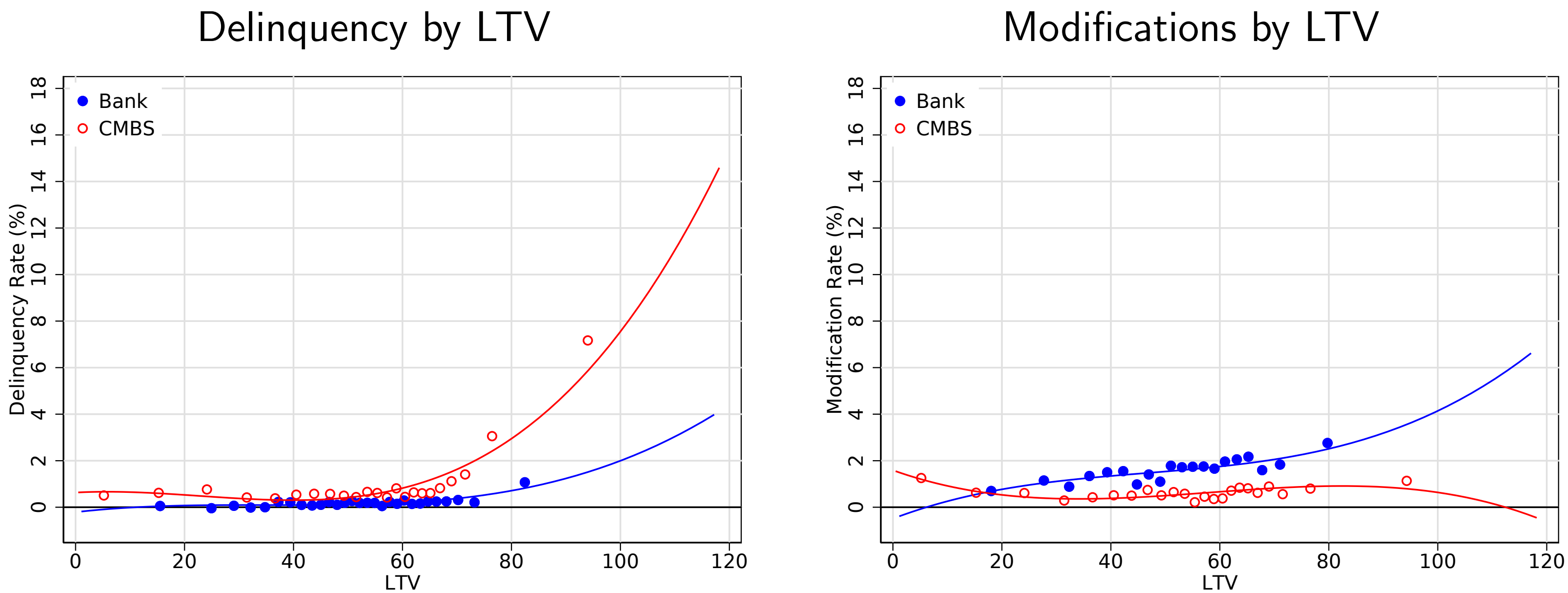

Figure C.4: Delinquency and Modification Rates by Current LTV During the COVID-19 Pandemic. *Note:* Data include loan-quarter observations from 2020q1–2021q3 that are less than 120 days delinquent, have a DSCR at origination greater than one and less than 20, and have an origination LTV greater than 20 percent. Observations after the first delinquency or modification are removed. Rates are in percentage points. Modification rates refer to modifications that result in a change in payment. The dots are coefficients from local linear regressions produced using the Stata command binsreg (see Cattaneo et al. 2019) with controls for origination DSCR, origination LTV, loan age in quarters, the log of the loan origination amount, the loan term, an indicator for whether the loan is interest only, and quarter by state, quarter by property type, and originator by origination year fixed effects. The line is derived from a cubic polynomial regression. The current LTV is calculated as the ratio of the outstanding loan balance to the current property value. The current property value is linearly interpolated between reported assessed values and extrapolated using the CBRE property price index for the property type of the loan in the loan's market. *Source:* Authors' calculations using Trepp CMBS data, CBRE property price indices, and the Y-14 H.2 Schedule.

	LTV (in percentage points)			
	Full Sample		Non-recourse loans	
	(1)	(2)	(3)	(4)
CMBS	2.435***	2.137***	2.649***	2.404***
	(0.295)	(0.291)	(0.335)	(0.331)
Interest Rate Spread		1.589***		3.031***
		(0.140)		(0.214)
N	36,854	34,850	14,856	14,433
R2	0.18	0.18	0.19	0.20
Originator × Orig. Year FEs	Y	Y	Y	Y
Property Type FEs	Y	Y	Y	Y
CBSA × State FEs	Y	Y	Y	Y

Table C.1: LTV REGRESSIONS. *Note:* Each column presents a regression predicting at-origination LTV with lender type for the combined sample of bank and CMBS loans. Columns (1) and (2) include all first-lien loans on stabilized properties in the sample, with column (2) adding a control for loan rate spreads. Columns (3) and (4) exclude bank loans with recourse from the sample, with column (4) including the spread control. *,**,*** indicate significance at 10%, 5%, and 1%, respectively. *Source:* Authors' calculations using Trepp CMBS data and Y-14 H.2 Schedule.

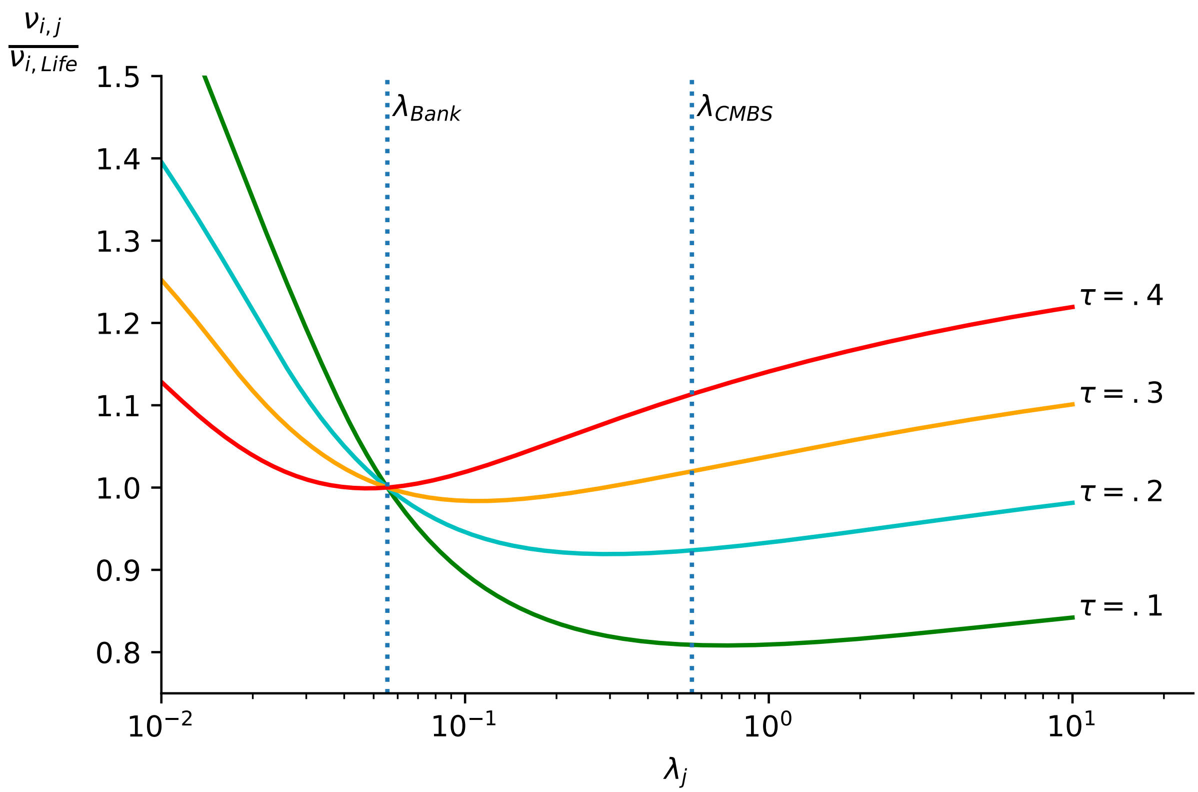

Figure C.5: WELFARE BY λ AND τ. *Note:* This chart plots $v_{i,j}$, normalized to $v_{i,\text{Life}}$, as a function of λ_j for different values of τ_i. The line thus represents the value of a nonrecourse loan for a particular modification friction, relative to a nonrecourse loan with $\lambda_j = \lambda_{\text{Bank}}$. Vertical lines show the modification frictions at banks and CMBS in the baseline calibration.