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Discussion of:

Quantile density combination: An application to US GDP forecasts

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*The views expressed in the paper are those of the authors. No responsibility for them should be attributed to the Bank of Canada

Basic idea

- Nice paper, with a intuitive new idea
- Combine quantiles forecasts with quantile specific weights
  - Usually researchers apply combination weights to full distribution
- They find:
  - Quantile combinations perform better in the tails
  - And, surprisingly, across the whole distribution
  - Simulations show it is not just a small sample result – results get better with more data

Contribution

Straddle two literatures:

1. Growth at Risk:
   - Recent interest in non-gaussian forecasts starting with Adrian et al. (2019)
   - Lots of approaches to do this, but something with a lot of promise and under-investigated is forecast combination
2. Density forecasting and combinations :
   - Some work on assessing density performance across different parts of the distribution (e.g. Gneiting and Ranjan (2011) and Diks et al. (2011))
   - While Loaiza-Maya et al. (2021) create predictions ‘focused’ on regions of the predictive distribution
   - Since models can perform differently at each quantile why not leverage this with forecast combination?

Econometric Approach

1. Generate forecasts from a set of Quantile Regression models

2. Evaluate each forecast by quantile score

3. Use inverse quantile score as combination weights at each quantile

Quantile Regression Models

Quantile predictions are produced from the following set of models

$$y_{t+h,q,k} = x'_{t,k}\beta_q + \epsilon_{t+1}$$

Where,

h is forecast horizon {1, 4}

q are quantiles: {10, 25, 50, 75, 90}

k is the set of variables: {NFCI, Consumer Confidence, Credit Spread, Unemployment rate, real Residential Investment}

x includes lagged GDP and variable k (with lags)

Scoring Quantile Forecasts

The authors measure forecast accuracy with the Continuous Rank Probability Score (CRPS)

$$CRPS(f_{t+h,k}, y_{t+h}) = -\int_{-\infty}^{\infty} (F_{t+h,k} - \mathbb{I}(F_{t+h,k} \geq y_{t+h}))^2 dy$$

It can be decomposed into the quantile score

$$CRPS_{t+h,k} = \int_0^1 QS_{t+h,k}(q) dq$$

Which is used to evaluate the accuracy of a predictive distribution for each quantile

$$QS_q(F^{-1}_{t+h,k}(q), y_{t+h}) = 2\left(\mathbb{I}\{y_{t+h} \leq F^{-1}_{t+h,k}(q)\} - q\right)(F^{-1}_{t+h,k}(q) - y_{t+h})$$

Combining Quantile Forecasts

- Weights for each quantile forecast are calculated by:

$$\underset{K\times Q}{w_{t+h}(k,q)} = \frac{\sum_{t=m}^{m+n-h} 1/QS_{t,k,q}}{\sum_{k=1}^{K}\sum_{t=m}^{m+n-h} 1/QS_{t,k,q}}$$

- The combined quantile forecasts (for a given q) are then:

$$y_{t+h,q}^{c} = \sum_{k=1}^{K} w_{t+h,q,k} \times y_{t+h,q,k}$$

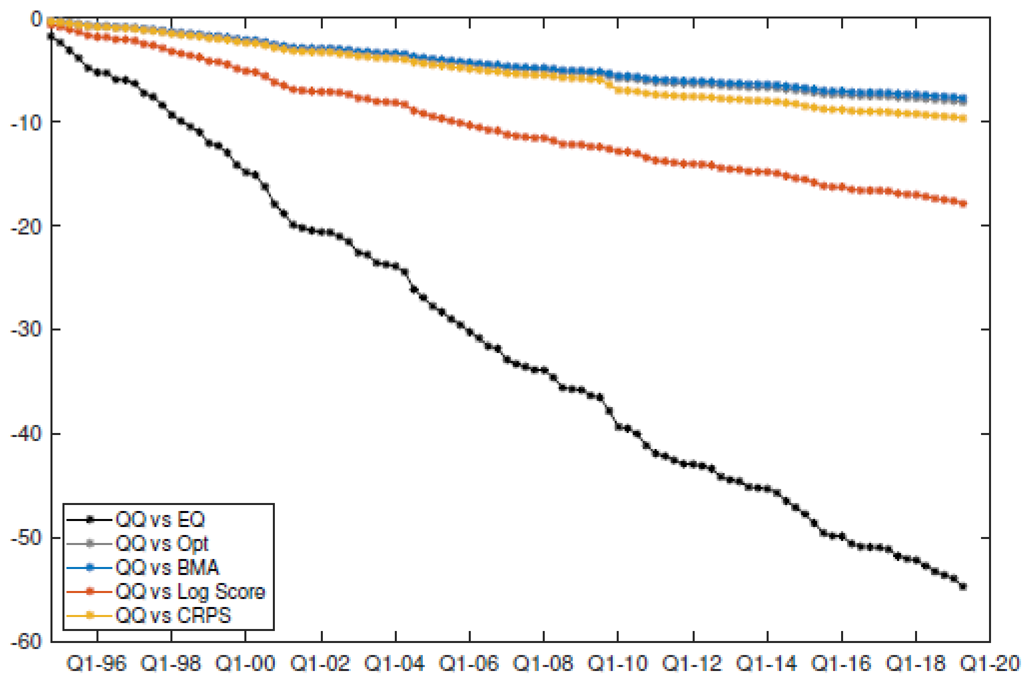
- Results in a combined forecasts for each of the 5 quantiles

Results Summary

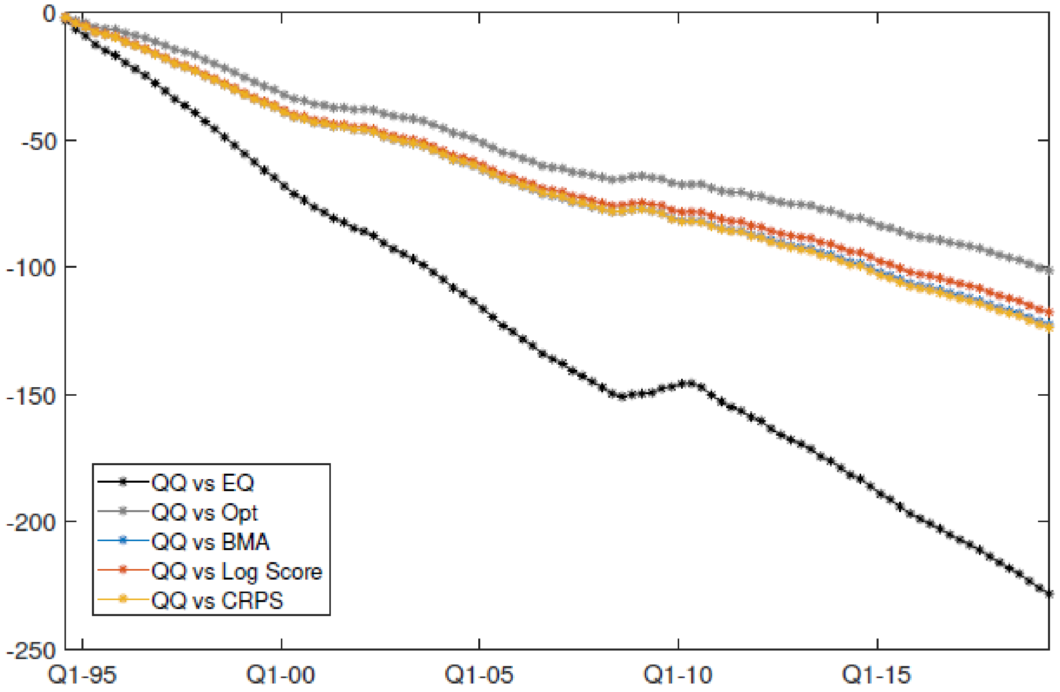
- Quantile Score weights beat other combination methods most of the time
  - Some exceptions depending on forecast horizon and CRPS weighting
- QS weights perform relatively well against individual models
  - Credit Spreads are very difficult to beat at one quarter ahead
- Simulations are used to validate the results
  - Results hold with larger sample size, more quantiles, and more models
  - As you add more quantiles you need more data to beat benchmarks

Quantile Combinations beat benchmarks

Cumulative CRPS against benchmarks (h=1)



Cumulative CRPS against benchmarks (h=4)



Against individual models quantile combinations…

Are competitive at t+1

Table 3: Average CRPS values with emphasis on specific regions of the distribution, one-quarter ahead forecasts.

Emphasis	Uniform	Centre	Tails	Right Tail	Left Tail	Heavy Tails
$\nu(q)$	$\nu_0 = 1$	$\nu_1 = q(1-q)$	$\nu_2 = (2q-1)^2$	$\nu_3 = q^2$	$\nu_4 = (1-q)^2$	$\nu_5 = (2q-1)^4$
Q comb	0.336	0.068	0.064	0.096	0.104	0.021
GDP	0.342	0.07	0.063	0.098	0.104	0.02
NFCI	0.361	0.073	0.067	0.099	0.115	0.022
ICS	0.334	0.068	0.063	0.095	0.103	0.02
U	0.367	0.075	0.069	0.104	0.114	0.022
CR Spread	**0.317**	**0.065**	**0.059**	**0.093**	**0.095**	**0.019**
ResInv	0.343	0.07	0.064	0.103	0.1	0.021

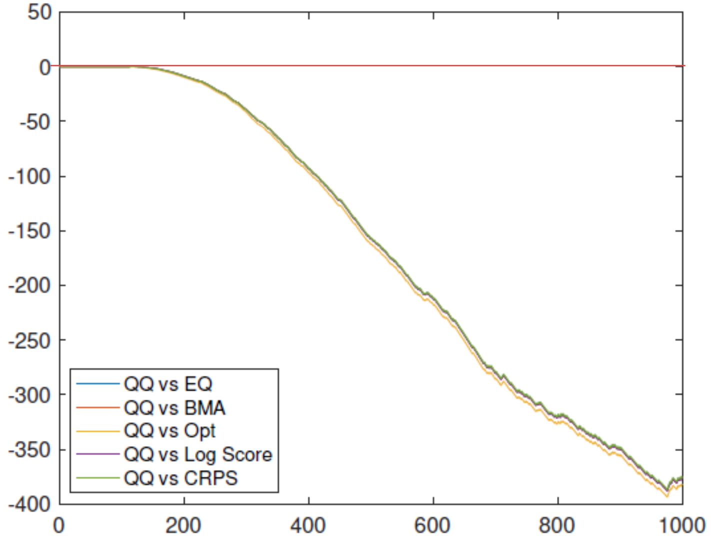
And win the horse race at t+4

Table 4: Average CRPS values with emphasis on specific regions of the distribution. four-quarter ahead forecasts.

Emphasis	Uniform	Centre	Tails	Right Tail	Left Tail	Heavy Tails
$\nu(q)$	$\nu_0 = 1$	$\nu_1 = q(1-q)$	$\nu_2 = (2q-1)^2$	$\nu_3 = q^2$	$\nu_4 = (1-q)^2$	$\nu_5 = (2q-1)^4$
Q comb	**0.319**	**0.066**	**0.056**	**0.095**	**0.092**	**0.018**
GDP	0.393	0.079	0.076	0.117	0.118	0.025
NFCI	0.35	0.071	0.066	0.095	0.113	0.021
ICS	0.344	0.07	0.065	0.096	0.108	0.021
U	0.393	0.08	0.074	0.112	0.121	0.024
CR Spread	0.352	0.072	0.066	0.099	0.109	0.021
ResInv	0.329	0.067	0.062	0.097	0.099	0.02

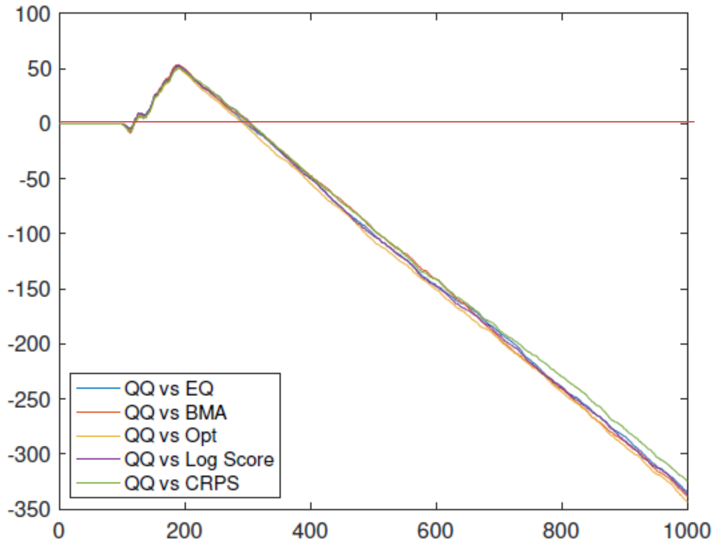
Simulations show…

As sample size increases quantile score combinations outperform



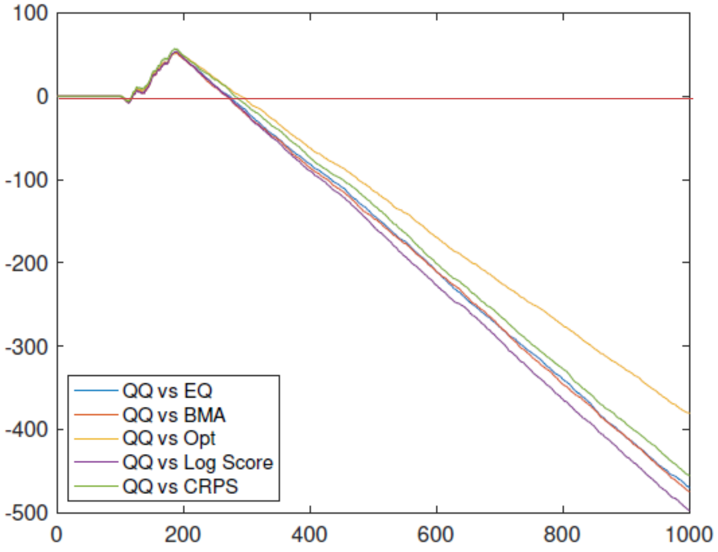
With more quantiles it takes data to see improvements

Q = 10



Adding more models doesn’t change results much

Q = 10, K = 10



Comments and extensions:

- Currently the authors predict 5 quantiles, why not the full predictive distribution?
  - A few ways to interpolate the full distribution:
    - Skew-t (Adrian, Boyarchenko, Giannone 2019)
    - Kernel smoothing (Gaglianone and Lima 2012, Korobilis 2017)
    - Non-parametrics (Mitchell, Poon, and Zhu, working paper)
    - Gaussian process (Rodrigues and Fan, 2017, Korobilis et al. 2021)
- Many Central Banks have a growth-at-risk tools, does your paper have any implications for the use of these tools?
  - Anything you can say about the relative importance of individual predictors?
- Could you use Bayesian Predictive Synthesis with quantile regression as a synthesis function?
  - Maybe that's the next paper!


Thanks for listening!