Constructing the Term Structure of Uncertainty from the Ragged Edge of SPF Forecasts

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RESEARCH AGENDA

Setup

We observe predictions from the SPF
(or similar sources)

in form of **point and/or density** forecasts

for fixed horizons and/or fixed events

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...by filling in missing values as needed

Throughout we look at average SPF responses

OUR PAPER

1) State space model

Maps arbitrary sets of SPF point forecasts

(fixed-event & -horizon)

into term structures of expectations and uncertainty

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- Predictive densities reflect historical forecast errors

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2) We match the SPF histograms with entropic tilting

We replicate the entire "bin" structure

Robustness check: Tilting to moments from distributions fitted to SPF histograms

RELATED LITERATURE

Survey uncertainty based on past forecast errors

- Reifschneider & Tulip (2007/19), Clark, McCracken & Mertens (2020)
- Lahiri & Sheng (2010), Knüppel (2014), Jo & Sekkel (2019)

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Entropic tilting: recent applications

- Krüger, Clark & Ravazzolo (2017), Tallman & Zaman (2020)
- Galvao, Garratt, & Mitchell (2021), Ganics & Odendahl (2021) Banbura, Brenna, Parades & Ravazzolo (2021)

AGENDA

1. SPF data
2. State space model for forecasts
3. Densities from SPF histograms and model
4. Effects of entropic tilting on predictive densities
5. Conclusions

OUR DATA: U.S. SPF

1) Point forecasts

2) Probabilistic forecasts (histograms)

to be discussed later

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- “Fixed horizons:” Quarters 0 to 4, since 1968Q4
- “Fixed events:” Calendar years 1 to 3, since 1981Q3 (or 2009Q2)

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Today:
Focus on GDP growth results (RGDP)
w/others shown in paper

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MODEL OVERVIEW

❶ **Map observed outcomes** and SPF point forecasts $\boldsymbol{Z_t}$ **into latent state vector** of fixed-horizon forecasts $\boldsymbol{Y_t}$

$$\boldsymbol{Z_t} = \boldsymbol{C_t} \; \boldsymbol{Y_t}$$

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$$\boldsymbol{Y_t = F \, Y_{t-1} + \eta_t}$$

   with $\boldsymbol{F}$ known, and $\boldsymbol{\eta_t}$ a vector of forecast updates

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❸ **Specify DGP for η_t, options:**

a) Baseline: Martingale difference

b) Alternative: Persistent process

with SV or CONST shock variances

STATE EQUATION

Collect definitions of nowcast error, forecast updates and change in long-run forecast:

$$\begin{bmatrix} y_{t-1} \\ \cdots\cdots\cdots \\ y_{t|t} \\ y_{t+1|t} \\ \vdots \\ y_{t+H-1|t} \\ \cdots\cdots\cdots \\ y_{t+H|t} \end{bmatrix} = \begin{bmatrix} y_{t-1|t-1} \\ \cdots\cdots\cdots \\ y_{t|t-1} \\ y_{t+1|t-1} \\ \vdots \\ y_{t+H-1|t-1} \\ \cdots\cdots\cdots \\ y_{t+H-1|t-1} \end{bmatrix} + \begin{bmatrix} e_{t-1} \\ \cdots\cdots\cdots \\ \mu_{t|t} \\ \mu_{t+1|t} \\ \vdots \\ \mu_{t+H-1|t} \\ \cdots\cdots\cdots \\ \mu_t^* \end{bmatrix}$$

which can be cast in recursive form (with F known)

$$Y_t \quad = \quad F\, Y_{t-1} \quad + \quad \eta_t$$

Baseline model: $\eta_t \sim N(0, \Sigma_t)$

SPECIFICATION CHOICES

Baseline: forecast updates are unpredictable

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$$E_t y_{t+h} = y_{t+h|t}$$

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- We consider SV and CONST specifications for $\mathrm{Var}_t\, \eta_{t+1}$

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- We consider SV and CONST specifications for $\text{Var}_t\, \eta_{t+1}$

Alternative: persistent forecast updates

- $E(\eta_t) = 0$: model's prior is centered on SPF
- $\eta_t \sim \text{VAR}(p)$
- Imputed bias: $b_{t+h|t} = y_{t+h|t} - E_t y_{t+h}$

Martingale-difference case for forecast updates

Trend and gap shocks with SV

Decompose updates into long-run shifts and cyclical gaps

$$\eta_t = \begin{bmatrix} \tilde{\eta}_t + 1 \cdot \mu_t^* \\ \mu_t^* \end{bmatrix}$$

$$\mu_t^* \sim N(0, \sigma_*^2)$$

$$\tilde{\eta}_t \sim N\left(0, \lambda_t \cdot \tilde{\Sigma}\right) \qquad \log \lambda_t \sim AR(1) \text{ (scalar)}$$

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- Combines slow-moving endpoint of term structure with time-varying volatility over near-/medium term
- Low-order factor structure suited for handling of missing observations
- Scale SV invariant to reordering variables in $\tilde{\eta}_t$ (Carriero, Clark & Marcellino, 2016; Chan, 2020)

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- Y_t: latent vector of quarterly forecasts y_{t+h} and y_{t-1} where y_t is quarterly growth (annualized rate)
- C_t: known, reflects definition of forecast targets, e.g., growth in annual average level of GDP

$$\hat{y}_t = \frac{y_t + 2y_{t-1} + 3y_{t-2} + 4y_{t-3} + 3y_{t-4} + 2y_{t-5} + y_{t-6}}{16}$$

- As in Mariano & Murasawa (2003), Patton & Timmermann (2011), Aruoba (2020)

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- As in Mariano & Murasawa (2003), Patton & Timmermann (2011), Aruoba (2020)
- In Q4: next-year forecasts omitted (since spanned by quarterly forecasts)

ESTIMATION SETUP

1. Model applied **separately for each outcome variable** (RGDP, PGDP, CPI, UNRATE, TBILL)

2. Estimated with **MCMC over growing samples** of **real-time data and SPF** that start in 1968Q3

   (FRB Phil.'s Real-Time Data Set for Macroeconomists)

3. Generate **out-of-sample predictive densities** from 1992Q1 onwards

4. Predictions **evaluated against 2nd release** outcomes for RGDP and PGDP and latest data for CPI, UNRATE, TBILL

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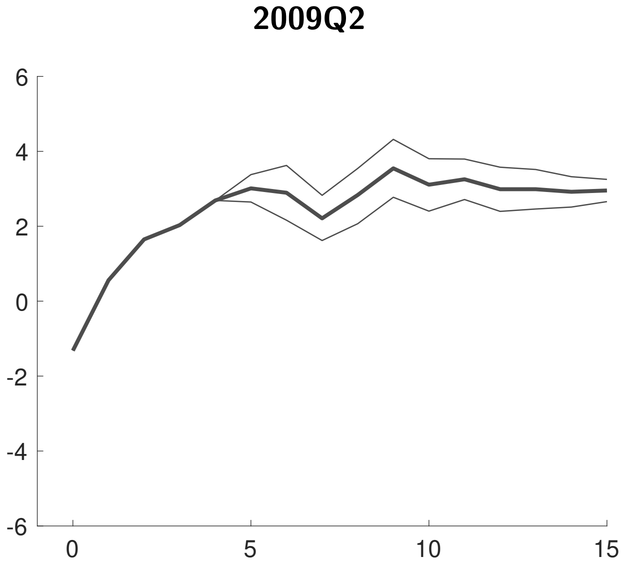
TERM STRUCTURE OF GDP GROWTH EXPECTATIONS

Quarterly real-time estimates w/68% bands for unobserved values



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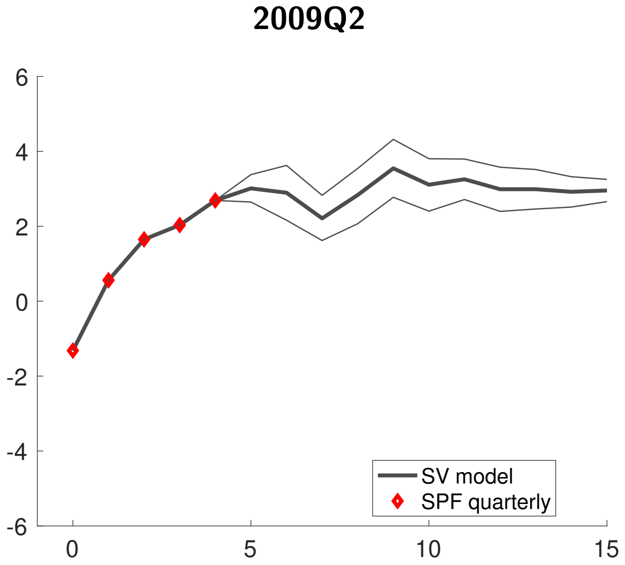

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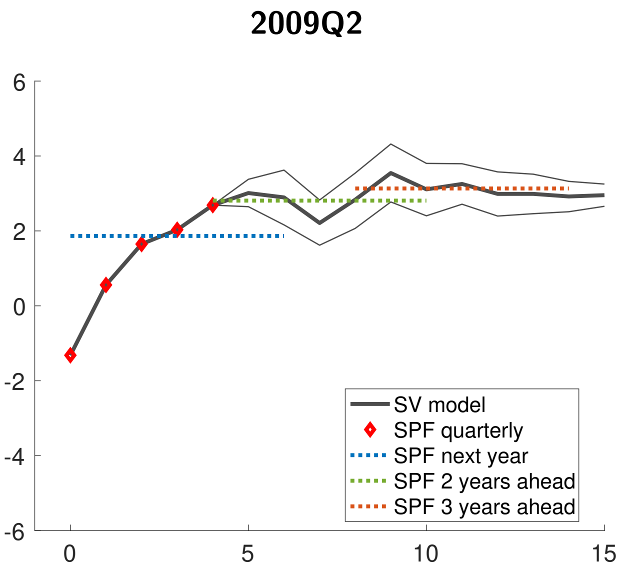

Dotted lines: quarters included in tent-shaped mapping from annual-average to quarterly changes

TERM STRUCTURES OF GDP GROWTH EXPECTATIONS

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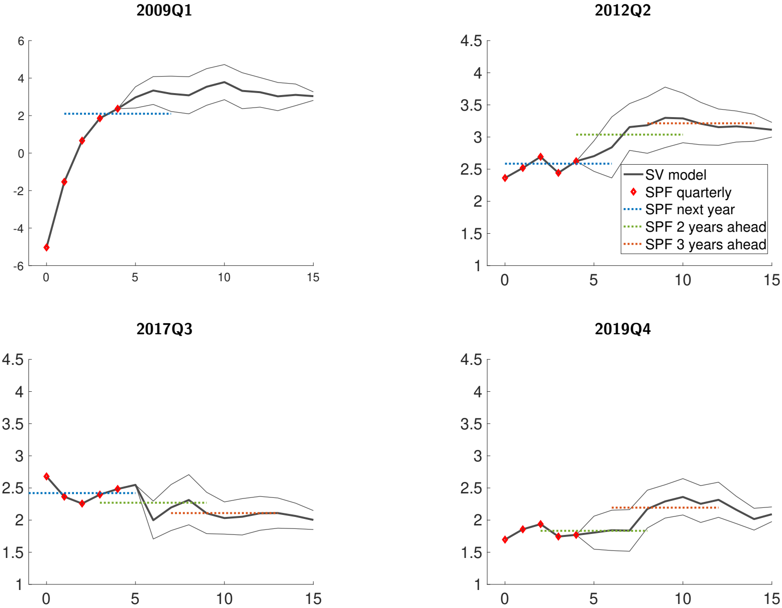

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FITTED TERM STRUCTURES OF EXPECTATIONS

Key feature

We can perfectly match any shape of the term structure of expectations that could be seen in the data

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Relaxation of MDS assumption

- Persistent forecast errors instead of $E_{t-1}\eta_t = 0$
- Transformation from Y_t to η_t still useful: motivates shrinkage to VAR(1)

$$Y_t = FY_{t-1} + \eta_t$$
$$\eta_t = G\eta_{t-1} + \varepsilon_t\,, \quad \varepsilon_t \sim N(0, \mathrm{Var}_{t-1}\,\varepsilon_t)$$

NON-MDS FORECAST UPDATES

Extended model

Relaxation of MDS assumption

- Persistent forecast errors instead of $E_{t-1}\eta_t = 0$
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$$
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\end{aligned}
$$

Results:

- **Similar avg forecast performance** (relative to MDS)
- **Persistence in η_t matters most at turning points**
- … and is **hard to predict in real time**

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2) Probabilistic forecasts (histograms)

- Fixed-event only, calendar years 1 to 3
- Using only predictions since 1992 (b/o data issues)
- To match SPF, we transform draws from log-linear model to actual annual-average changes

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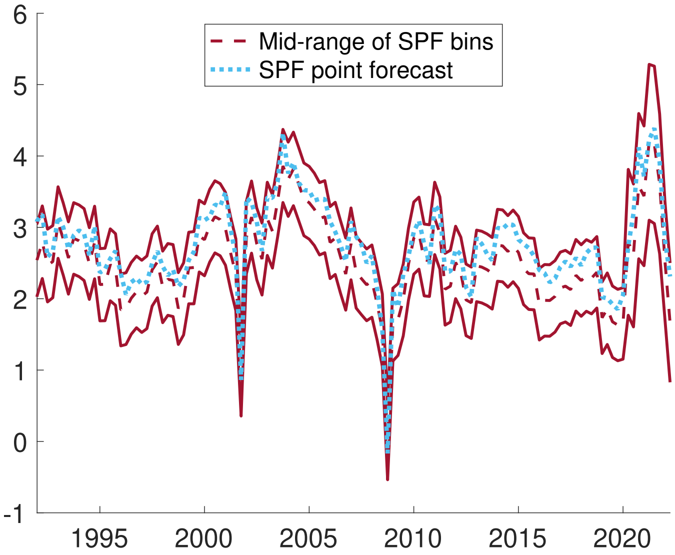
Focus on GDP growth results (RGDP)

w/others shown in paper

CONSISTENCY OF POINT AND DENSITY FORECASTS

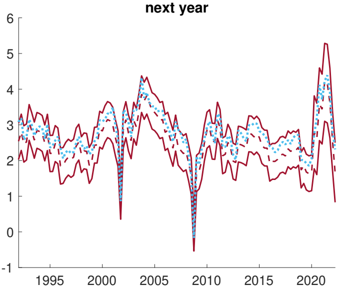
GDP growth next year

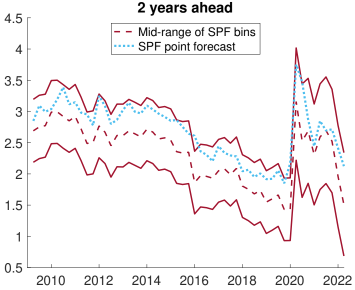


Ranges of histogram-consistent mean forecasts computed by placing mass for each bin on left / right edges

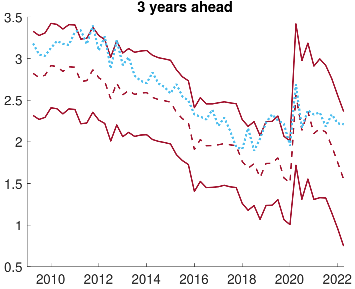
CONSISTENCY OF POINT AND DENSITY FORECASTS

Point vs. ranges of mean forecasts consistent with the SPF histograms





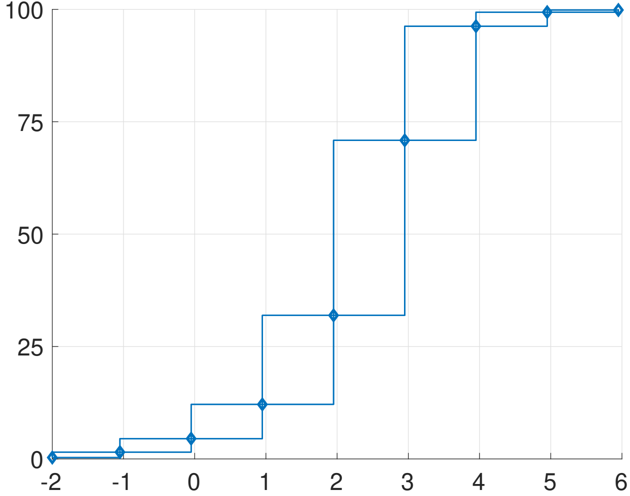
- Point forecasts almost always consistent with histograms
- Ranges of histogram-consistent mean forecasts computed by placing mass for each bin on left / right edges
- GDP growth



CDF IMPLIED BY SPF HISTOGRAMS

Next-year GDP growth

SPF histograms pin down selected CDF values:

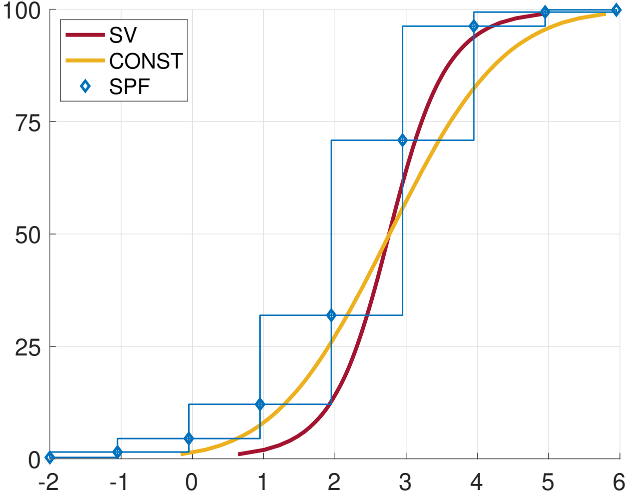


Growth rates (x-axis) and probabilities (y-axis) in percentage points

MODELS VS SPF

Cumulative densities for next-year GDP growth

By construction,
SV and CONST model densities have same median . . .



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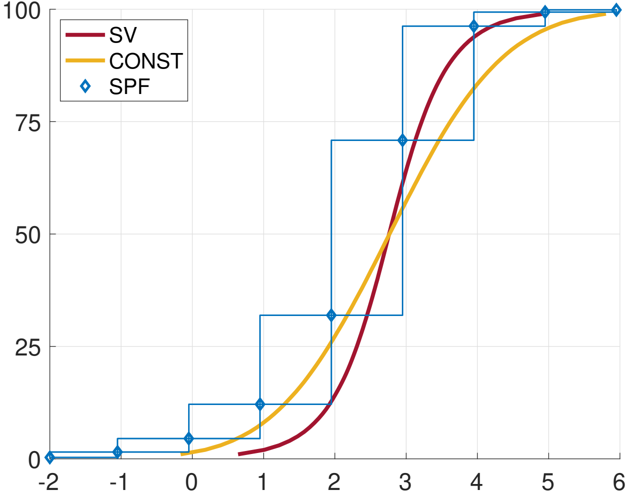

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SCORES FOR HISTOGRAM EVALUATIONS

Setup

- Let $b_{j,t}$ denote the upper edge of SPF bin j (at t)
- Histogram provides discrete-valued CDF:

$$P_{j,t} = \textbf{Prob}_t(y_{t+h} \leq b_{j,t})$$

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Discrete Ranked Probability Scores

$$\textbf{DRPS}_t = \sum_j \left(P_{j,t} - \mathbb{1}\left(y^o_{t+h} \leq b_{j,t}\right)\right)^2$$

where y^o_{t+h} denotes the observed value

- Measures accuracy of predictions to fall into SPF bins
- Depends on specification of SPF bins ($b_{j,t}$)

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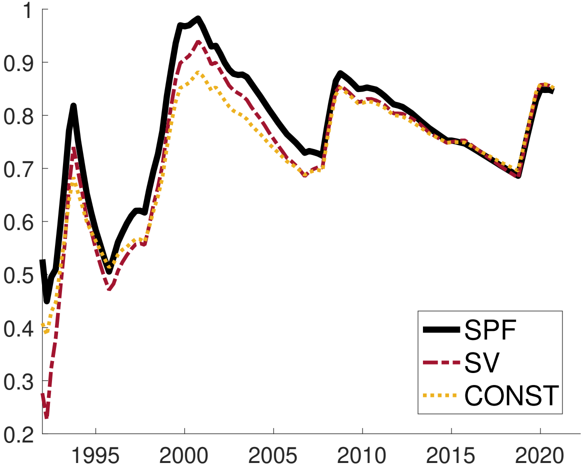
- Depends on specification of SPF bins ($b_{j,t}$)
- Bin-specific analogue to CRPS

ACCURACY OF PREDICTIONS FOR BIN EVENTS

Avg DRPS scores over growing samples, next-year GDP growth

Models better than SPF pre GFC and on par over full sample



Lower score is better

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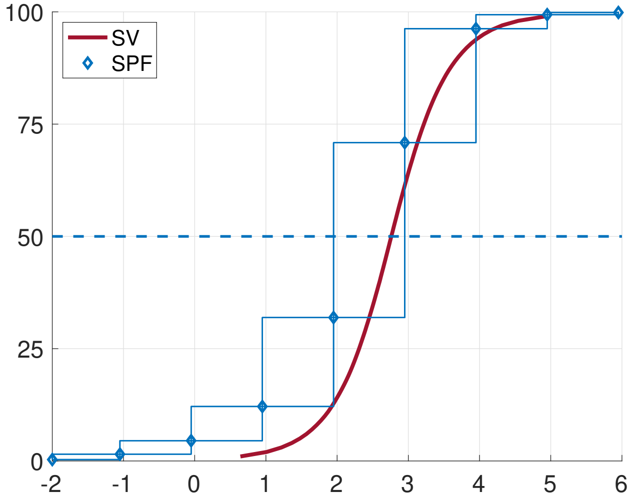
CDF BEFORE ENTROPIC TILTING

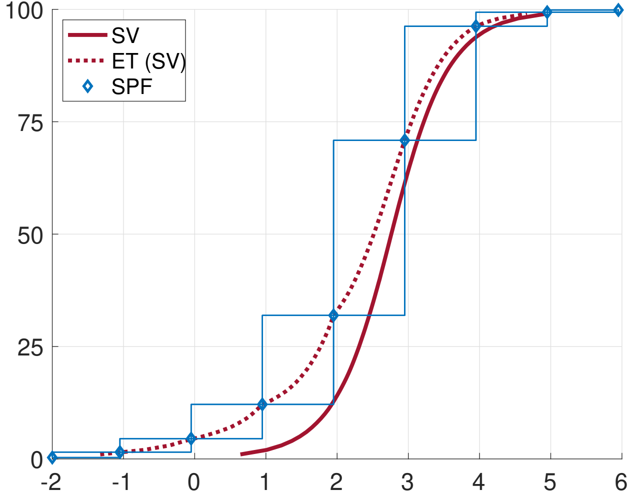
Cumulative densities for next-year GDP growth

Our state space model matches SPF point forecasts, but not generally the histogram bins



Growth rates (x-axis) and probabilities (y-axis) in percentage points

ET reweighs MCMC output to match bin probabilities while minimizing KL divergence



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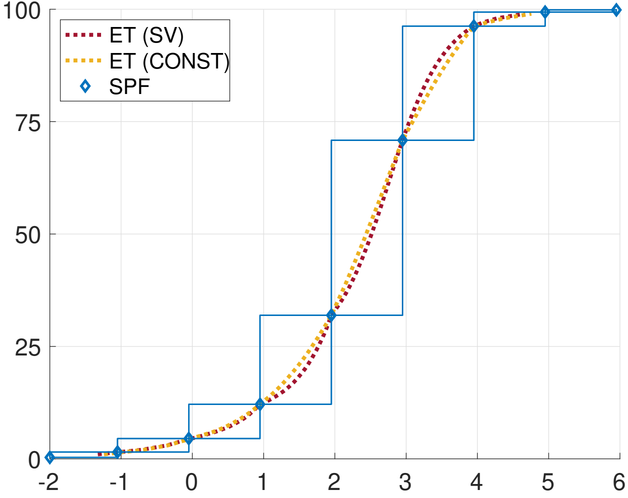
TILTED MODELS VS SPF

Cumulative densities for next-year GDP growth

After tilting, SV and CONST densities similar, but not identical:



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ENTROPIC TILTING

Generic setup

- **Given:** predictive density draws $f := \{y^i_{t+h}\}_{i=1}^M$
- **Target:** moment conditions $E[g(y_{t+h})] = \bar{g}$
- **Tilting problem:** Reweigh draws from f into $\tilde{f}$ to match $\bar{g}$ while minimizing KL divergence

$$\min_{\tilde{f}\in\mathbb{F}} \text{KL}(\tilde{f}, f) \text{ subject to } E_{\tilde{f}}\left[g(y_{t+h})\right] = \bar{g}$$

Key insight for our application

Bin probabilities are predictive moments

for example:

$$\text{Prob}_t\left(2.5 < y_{t+h} \leq 3.0\right) = E_t\left(\mathbb{1}\left(2.5 < y_{t+h} \leq 3.0\right)\right)$$

We target all bin probabilities

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   - Entropic tilting method
   - Average forecast performance w/and w/o entropic tilting
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POINT FORECAST PERFORMANCE

RMSE relative to SV

SV w/ET	CONST	CONST w/ET

Stars indicate Diebold-Mariano significance. Green/red colors indicate gains/losses.

POINT FORECAST PERFORMANCE
RMSE relative to SV

	SV w/ET		CONST		CONST w/ET	
h	92–22	92–16	92–22	92–16	92–22	92–16
0						
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						
11						
12						
13						
14						
15						

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POINT FORECAST PERFORMANCE

RMSE relative to SV

	SV w/ET		CONST		CONST w/ET	
h	92–22	92–16	92–22	92–16	92–22	92–16
0	1.01	1.01*	1.00	1.00	1.01**	1.01*
1	1.00	1.01	1.00	1.00	1.00	1.01
2	1.00	1.00	1.00	1.00	1.00	1.00
3	1.00	0.99	1.00	1.00	1.00	0.99
4	1.00	0.99	1.00	1.00	1.00	0.99
5	1.00	0.98	1.00	1.00	1.00	0.99
6	1.00	0.99	1.00	1.01	1.01	1.01
7	1.00	0.99	1.00	1.01	1.00	1.00
8	1.00	0.99	1.00	1.02	1.00	1.01
9	1.00	0.99	1.00	1.01	1.00	1.00
10	1.00	1.00	1.00	1.02	1.00	1.01
11	1.00	0.99	1.00	1.02*	1.00	1.01
12	1.00	1.00	1.00	1.02	1.00	1.01
13	1.00	1.01*	1.00	1.02	1.00	1.02
14	1.00	1.00	1.00	1.00	1.01	1.00
15	1.00	1.00	1.00	1.00	1.00	1.00

Stars indicate Diebold-Mariano significance. Green/red colors indicate gains/losses.

DENSITY FORECAST PERFORMANCE

CRPS relative to SV

	SV w/ET		CONST		CONST w/ET	
h	92–22	92–16	92–22	92–16	92–22	92–16
0	1.00	1.00				
1	1.00	1.00				
2	0.99	1.00				
3	0.99	0.99				
4	0.99	0.99				
5	0.99	0.98				
6	0.99	0.99				
7	1.00	0.99				
8	1.00	0.99				
9	1.00	0.99				
10	1.00	0.99				
11	0.99	0.98				
12	0.99	0.99				
13	1.00	1.00				
14	1.00	1.00				
15	0.99*	0.99				

Stars indicate Diebold-Mariano significance. Green/red colors indicate gains/losses.

DENSITY FORECAST PERFORMANCE

CRPS relative to SV

	SV w/ET		CONST		CONST w/ET	
h	92–22	92–16	92–22	92–16	92–22	92–16
0	1.00	1.00	0.99	0.99	1.00	0.99
1	1.00	1.00	1.04	1.01	1.04*	1.02
2	0.99	1.00	1.00	1.02	1.01	1.02
3	0.99	0.99	1.01	1.02	1.01	1.01
4	0.99	0.99	1.01	1.03*	1.01	1.01
5	0.99	0.98	1.01	1.04*	1.01	1.01
6	0.99	0.99	1.03	1.06**	1.03	1.04
7	1.00	0.99	1.03	1.06**	1.03	1.04
8	1.00	0.99	1.03	1.06**	1.03	1.04
9	1.00	0.99	1.03	1.06**	1.03	1.05
10	1.00	0.99	1.03**	1.07***	1.04**	1.06**
11	0.99	0.98	1.04**	1.07***	1.03	1.05*
12	0.99	0.99	1.04**	1.07***	1.03**	1.06**
13	1.00	1.00	1.04***	1.07***	1.04***	1.06***
14	1.00	1.00	1.04**	1.05***	1.04**	1.05**
15	0.99*	0.99	1.04**	1.04**	1.04**	1.04**

Stars indicate Diebold-Mariano significance. Green/red colors indicate gains/losses.

AGENDA

Our contributions:

Model that transforms an arbitrary set
of fixed-event/-horizon SPF data
into a consistent term structure

SUMMARY

Our contributions:

Model that transforms an arbitrary set
of fixed-event/-horizon SPF data
into a consistent term structure

- **Matches observed SPF**
- **Can be used to produce FOMC-like fan charts**
- **Incorporates all SPF bins with entropic tilting**

SUMMARY

Our contributions:

Model that transforms an arbitrary set
of fixed-event/-horizon SPF data
into a consistent term structure

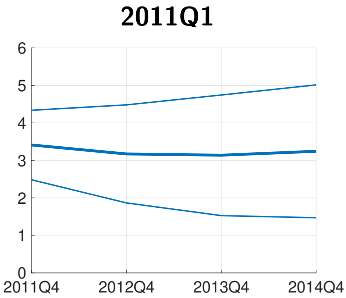
- Matches observed SPF
- Can be used to produce FOMC-like fan charts
- Incorporates all SPF bins with entropic tilting

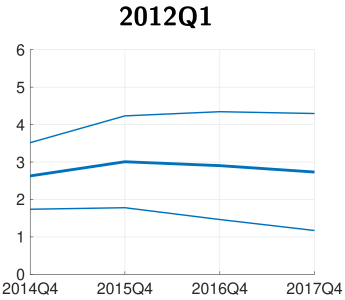
Findings

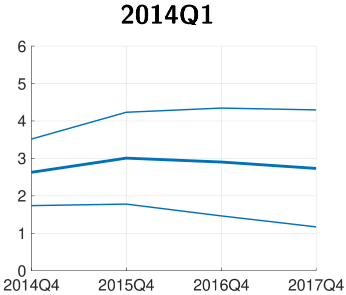
- Calendar-year **histograms add** some, but mostly **occasional value** ...
- ... *relative to model centered on SPF point forecasts*
- At onset of COVID-19, narrower uncertainty after tilting

APPENDICES

FAN CHARTS FOR Q4/Q4 GDP GROWTH







- In format of FOMC's SEP
- Generated by SV model
- Next: comparison against SEP uncertainty bands

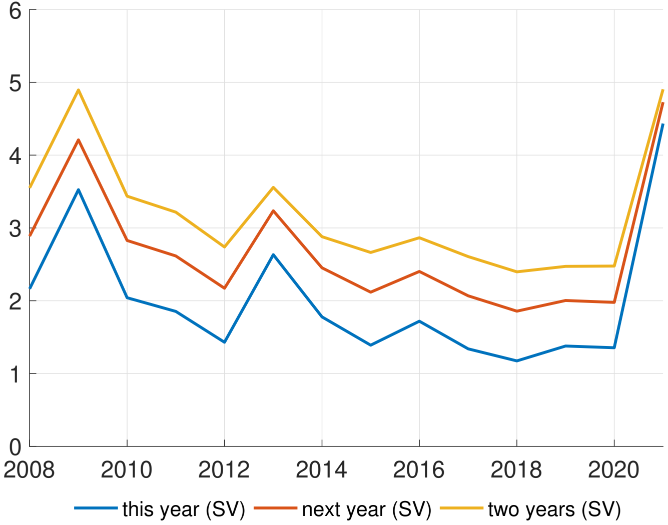
FAN CHART UNCERTAINTY: MODEL VS SEP

SEP setup

- **SEP** fan-chart bands based on historical forecast errors **assume constant variances over last 20-years**
- Uncertainty **bands reflect** $\pm$ **RMSE** around forecast
- . . . and can differ from FOMC's subjective assessments

FAN CHART UNCERTAINTY OVER TIME

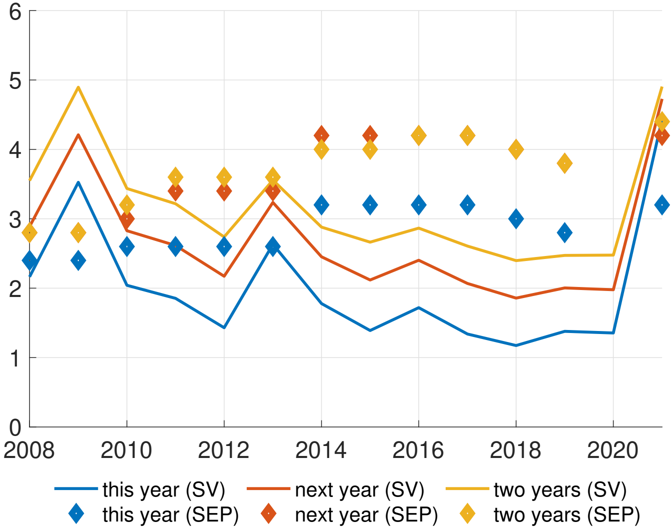
Width of 68% bands from SV model . . .



Width of 68% bands for Q4/Q4 forecasts

FAN CHART UNCERTAINTY OVER TIME

Width of 68% bands from SV model vs. SEP's RMSE-based bands



Width of 68% bands for Q4/Q4 forecasts

FAN CHART UNCERTAINTY: MODEL VS SEP

SEP setup

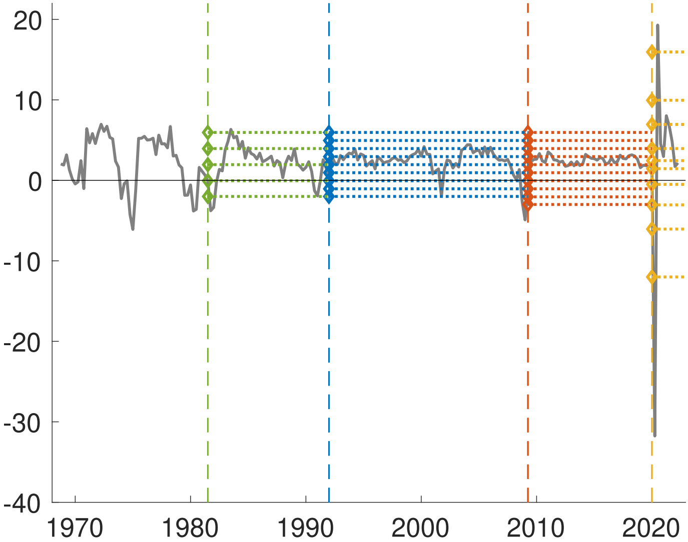
- **SEP** fan-chart bands based on historical forecast errors **assume constant variances over last 20-years**
- Uncertainty **bands reflect** $\pm$ **RMSE** around forecast
- ... and can differ from FOMC's subjective assessments

Takeaways

- **SV-model bands more nimble than SEP estimates**
- After GFC:
  - SV estimates returned to lower levels
  - while SEP remained elevated

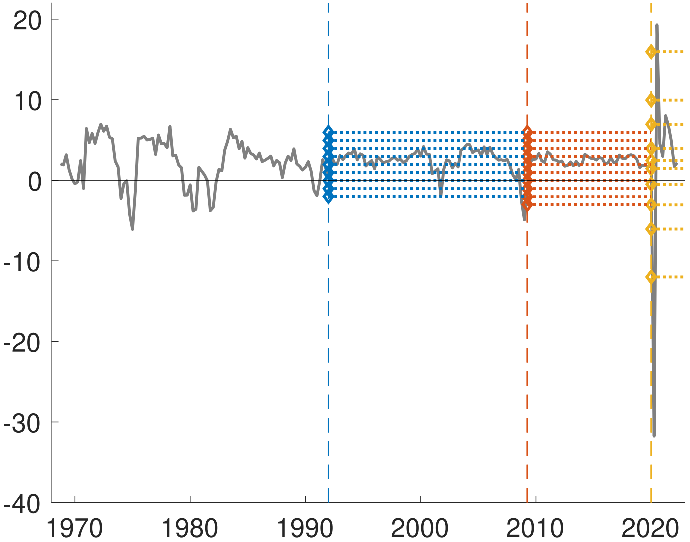
AVAILABILITY OF SPF DENSITY FORECASTS

Nowcast and widths of histogram bins



AVAILABILITY OF SPF DENSITY FORECASTS

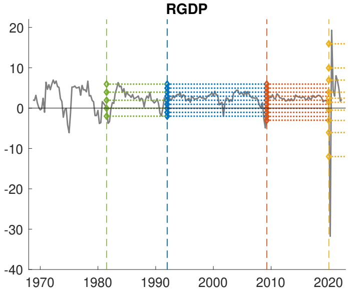
Nowcast and widths of histogram bins

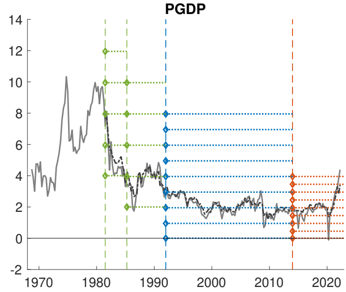


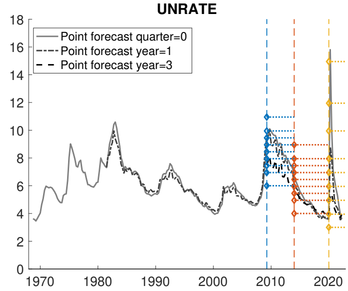
We consider only histograms as of 1992 (b/o data issues)

AVAILABILITY OF SPF PREDICTIONS

Real growth (RGDP), inflation (PGDP), unemployment rate (UNRATE)





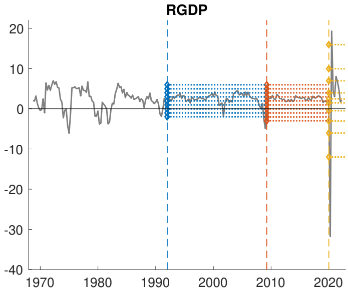


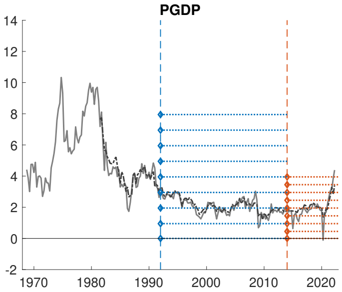
- Point forecasts since 1968
- Next-year bins since 1981 (and since 2009 for UNRATE)
- . . . beyond next year since 2009
- But, w/data issues prior 1992
- . . . and bin changes throughout

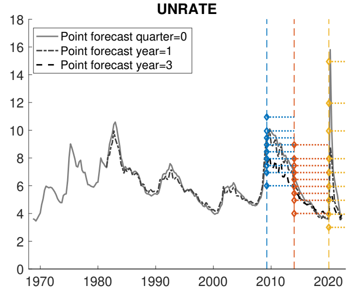
Point forecasts, and widths of histograms

AVAILABILITY OF SPF PREDICTIONS

Real growth (RGDP), inflation (PGDP), unemployment rate (UNRATE)







- Point forecasts since 1968
- Next-year bins since 1981 (and since 2009 for UNRATE)
- . . . beyond next year since 2009
- But, w/data issues prior 1992
- . . . and bin changes throughout
- Using only bin data since 1992

Point forecasts, and widths of histograms

STATE SPACE FOR FORECASTS AND THEIR UPDATES

1) Accounting identity from CMM for H steps ahead:

$$y_{t+H} = e_{t+H} + \sum_{i=1}^{H} \mu_{t+H|t+i} + y_{t+H|t}$$

2) Track changes in long-run forecasts

$$y_{t+H|t} = y_{t+H-1|t-1} + \mu_t^*$$

We obtain a state equation with known transition F

$$Y_t = F\, Y_{t-1} + \eta_t\,, \quad \eta_t \sim \text{TBD}$$

$$\underbrace{\begin{bmatrix} y_{t-1} \\ y_{t|t} \\ y_{t+1|t} \\ \vdots \\ y_{t+H|t} \end{bmatrix}}_{Y_t} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 0 & 1 & \dots & 0 \\ & & & & \ddots & \\ 0 & \dots & & 0 & & 0 \\ 0 & \dots & & \dots & 0 & 1 \\ 0 & \dots & & \dots & 0 & 1 \end{bmatrix}}_{F} Y_{t-1} + \underbrace{\begin{bmatrix} e_{t-1} \\ \mu_{t|t} \\ \mu_{t+1|t} \\ \vdots \\ \mu_t^* \end{bmatrix}}_{\eta_t}$$

Recall: $e_t = y_t - y_{t|t}$ and $\mu_{t+h|t} = y_{t+h|t} - y_{t+h|t-1}$

COMMENTS ON STATE EQUATION

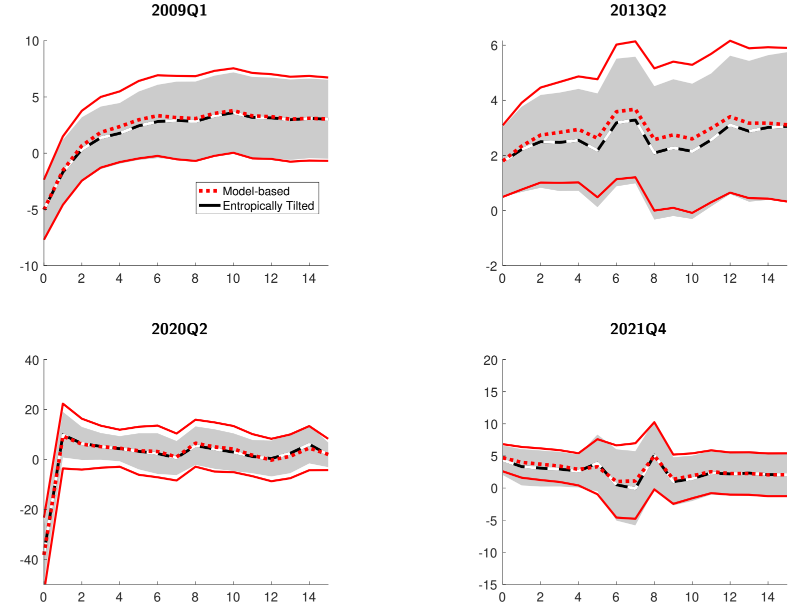
$$\boldsymbol{Y_t = F\, Y_{t-1} + \eta_t}$$

- All rows except last replicate CMM data for $\boldsymbol{\eta_t}$
- Transition matrix $\boldsymbol{F}$ is known
- All roots of $\boldsymbol{F}$ are zero except for one unit root
- $\boldsymbol{F}$ implies common trend in outcomes and forecasts (assuming stationary $\boldsymbol{\eta_t}$)
- $\mathbf{Var}\,(\boldsymbol{\mu_t^*}) \rightarrow \mathbf{0}$ captures (near) stationary $\boldsymbol{Y_t}$
- MDS assumption, $\boldsymbol{E_{t-1}\eta_t = 0}$, closes state space
- In extension, we consider VAR for $\boldsymbol{E_{t-1}\eta_t}$ (as in CMM)

Even if not literally true, MDS assumption provides useful shrinkage for VAR in η_t

FAN CHARTS FOR GDP GROWTH

SV model before (red) and after entropic tilting (black)

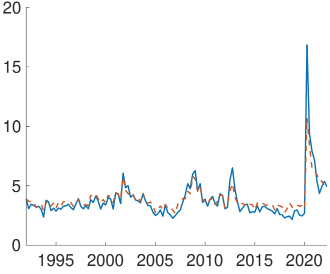


Predictive means and 68% uncertainty bands

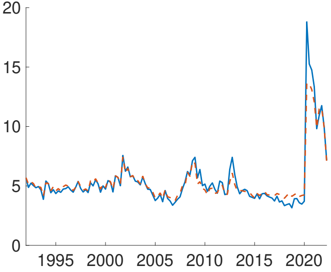
EFFECTS OF TILTING ON UNCERTAINTY

Real growth: SV model before (blue) and after ET (red)

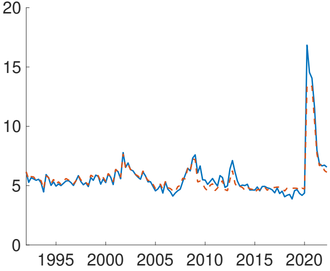
3 quarters ahead



7 quarters ahead



11 quarters ahead



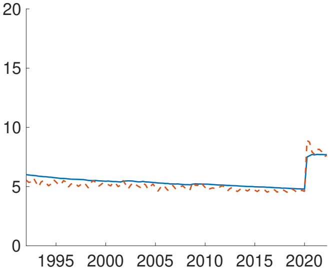
- Uncertainty measured by width of 68% bands
- Not much effect from ET
- Except for narrowing at onset of COVID-19
- Stronger effects on CONST (see next)

EFFECTS OF TILTING ON UNCERTAINTY

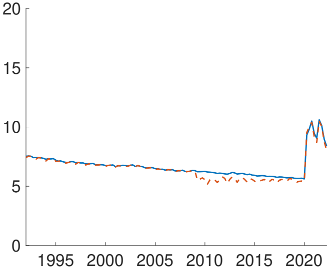
BACKUP

Real growth: CONST model before (blue) and after ET (red)

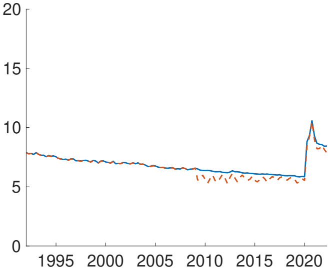
3 quarters ahead



7 quarters ahead



11 quarters ahead



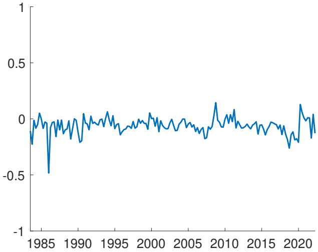
- **More visible effects of ET on CONST**
- Narrower until COVID
- Recall: Longer-run SPF histograms available only since 2009

SKEW INDUCED BY TILTING

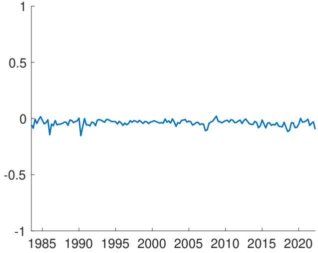
Bowley coefficient

- Our model has zero skew, only ET can induce skew
- *Some* skewness at targeted annual horizon
- But, w/o carrying over to quarterly term structure

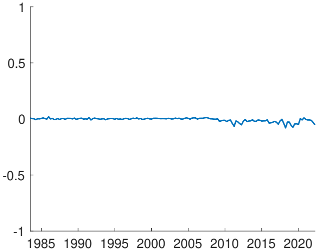
next year



3 quarters ahead



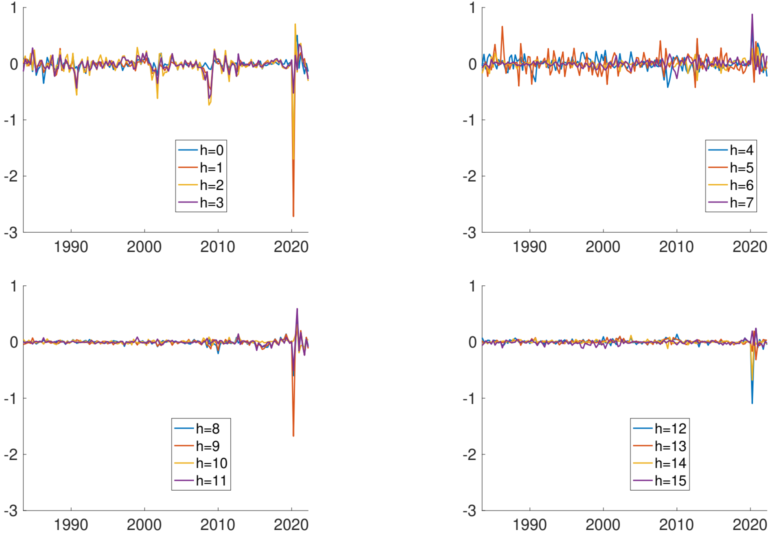
11 quarters ahead



APPENDICES

BIAS IN SPF EXPECTATIONS OF GDP GROWTH

$\text{Bias}_t = E_t y_{t+h} - y_{t+h|t}$ from non-MDS model



MDS VS NON-MDS MODEL: FORECAST PERFORMANCE

Relative RMSE and CRPS (MDS in denominator)

	RMSE				CRPS			
	SV		CONST		SV		CONST	
h	92–22	92–16	92–22	92–16	92–22	92–16	92–22	92–16
0	1.00	1.00	1.12	1.01	1.01	1.01	1.08	1.01
1	1.02	1.00	1.06	1.01	1.02	1.01	1.04	1.01
2	1.00	0.98*	1.00	0.97**	1.01	0.99*	1.00	0.99
3	1.00	1.00	1.00	0.99	1.02	1.01	1.00	1.00
4	1.00	1.00	1.00	0.99	1.02	1.01	1.01	1.01
5	1.00	1.00	1.00	1.01	1.01	1.00	1.01	1.00
6	1.00	1.00	1.00	1.02	1.00	0.99	1.01	1.01
7	1.00	1.00	1.00	0.99	1.00	0.99	0.99	0.99
8	1.00	1.01	1.00	1.00	1.00	0.99	0.99	0.99
9	1.00	1.01	1.01	1.01	1.00	0.99	1.01	1.01
10	1.00	1.01	1.01	1.02	0.99	0.98	1.01	1.01
11	1.00	1.01	1.00	1.02	1.00	0.98	1.01	1.02
12	1.00	1.00	1.00	1.02	0.99	0.97**	1.01	1.01
13	1.00	1.00	1.01	1.01	0.99	0.97**	1.00	1.00
14	1.00	1.00	1.00	1.00	0.99	0.98	1.00	1.01
15	1.00	1.00	1.00	1.00	0.98*	0.98**	1.00	1.00

Stars indicate Diebold-Mariano significance. Green/red colors indicate gains/losses.

Persistence in forecast updates
matters mostly at turning points

. . . and is hard to predict in real time

Croushore (2010), Mertens & Nason (2020),
Matthes & Foerster (2021), Hajdini and Kurmann (2022),
Farmer, Nakamura & Steinsson (2022),
Bianchi, Ludvigson & Ma (2022)