# The Scars of Supply Shocks

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**Abstract**

We study the effects of supply disruptions - for instance caused by the emergence of a pandemic - in an economy with Keynesian unemployment and endogenous productivity growth. By negatively affecting investment, even purely transitory negative supply shocks generate permanent output losses. The associated negative wealth effect depresses consumers' demand, which may even fall below the exogenous fall in supply. In this case, the optimal monetary policy response flips relative to conventional wisdom, as monetary expansions are needed to fight negative output gaps. If monetary policy is not expansionary enough a supply-demand doom loop emerges, causing a recession characterized by unemployment and weak productivity growth. Innovation policies, by fostering firms' investment, can restore full employment and healthy growth.

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# 1 Introduction

The ongoing Covid-19 pandemic - by forcing factories to shut down and disrupting global supply chains - is causing substantial damage to the productive capacity of the economy. Moreover, many commentators and policy institutions expect the Covid-19 shock to leave deep scars, by inducing a very persistent drop in output below its pre-pandemic trend (e.g., Wolf, 2020; IMF, 2020).[1] These facts have renewed interest in the macroeconomic implications of negative supply shocks. How do supply disruptions affect aggregate demand? What are the optimal monetary and fiscal responses to negative supply shocks? These questions are at the forefront of the current debate.

Much of the conventional thinking about these issues builds on the New Keynesian paradigm (Galí, 2009). In the New Keynesian model, following a negative supply shock demand contracts less than supply, and so the natural interest rate rises. The optimal policy response then entails a monetary tightening, to prevent the economy from experiencing excessive demand, overheating and inflation above target. The New Keynesian framework, however, assumes that after the shock dissipates the economy quickly bounces back to its pre-shock trend, and so does not allow for the possibility that supply disruptions might have scarring effects.

This paper provides a theory in which negative supply shocks may leave persistent scars on the economy, and shows that this effect might radically change the macroeconomic implications of supply disruptions relative to the traditional view. Our idea is that negative supply shocks - even if purely transitory - induce firms to reduce investment, and thus destroy the future productive capacity of the economy. The associated drop in wealth depresses consumers' demand, which may even fall below the reduction in supply. In this case, the natural interest rate declines and the optimal monetary policy response flips relative to conventional wisdom, as monetary expansions are needed to fight insufficient demand and negative output gaps. Moreover, fiscal interventions aiming at sustaining investment may be a crucial complement to monetary policy in stabilizing aggregate demand.

To formalize these insights, we provide a *Keynesian growth* framework with two key features. First, as in standard models of vertical innovation (Aghion and Howitt, 1992), firms invest in innovation in order to appropriate future monopoly rents. Second, as in the New Keynesian tradition, the presence of nominal rigidities implies that output may deviate from its potential level. Our theory thus combines the Keynesian insight that unemployment may arise due to weak aggregate demand, with the notion, developed by the endogenous growth literature, that sustained productivity growth is the result of investment in innovation by profit-maximizing agents.

We study the response of the economy to supply disruptions, modeled as standard negative

[1] For instance, the Covid-19 pandemic has lead the International Monetary Fund to revise substantially downward its forecast for the growth rate of global real GDP per capita for the 2019-2025 period, relative to its pre-pandemic projections. In its October 2020 World Economic Outlook (IMF, 2020), the IMF writes: "The medium-term projections also assume that the economies will experience scarring from the depth of the recession". Interestingly, the IMF has recently revised upward its projections for medium run growth in the United States, in response to the fiscal expansion envisaged by the Biden's administration (see the April 2021 World Economic Outlook). Underpinning this revision is the idea that fiscal stimulus might reverse the scarring effects triggered by the Covid-19 recession, and lead to persistent improvements in potential output.

TFP shocks. In our framework, negative supply shocks drive down the return to investment, by reducing firms' market size and thus their monopoly profits. The result is lower investment and slower productivity growth. Once the shock dissipates, investment recovers and productivity growth returns to its pre-shock level, but output falls permanently below its pre-shock trend.

These scars of supply shocks matter critically for the response of aggregate demand. When productivity growth is exogenous, households' permanent income falls by little in response to transitory supply disruptions. In our endogenous growth model, instead, the drop in wealth caused by negative supply shocks is amplified by the associated reduction in investment and productivity growth. In fact, we show that the decline in demand following negative supply shocks can be large enough to depress the natural interest rate. In contrast with conventional wisdom, the optimal monetary policy response then consists in cutting the policy rate to stimulate demand and maintain the economy at full employment.

If monetary policy is not expansionary enough, supply disruptions trigger recessions characterized by the emergence of a supply-demand doom loop. The reason is that lower demand reduces firms' profits and their incentives to invest. In turn, the consequent slowdown in investment and productivity growth causes another round of fall in aggregate demand. A too tight monetary stance, which fails to stabilize demand and to counteract this vicious spiral, may then greatly amplify the direct impact of supply disruptions on employment and productivity growth. This feature of the model is consistent with the empirical evidence provided by Garga and Singh (2020), Moran and Queralto (2018) and Jordà et al. (2020), suggesting that monetary policy tightenings can have a long-lasting negative impact on productivity growth by depressing firms' investment in innovation.

The supply-demand doom loop manifests itself in full force during liquidity traps, when monetary policy is constrained by the zero lower bound. Indeed, if a supply disruption hits an economy stuck in a liquidity trap it can generate particularly large drops in demand, employment and growth, effectively pushing the economy into a stagnation trap. This result contrasts with the predictions of the New Keynesian framework, in which temporary negative supply shocks tend to be expansionary during liquidity traps (Eggertsson et al., 2014). It is, however, consistent with recent empirical evidence suggesting that negative supply shocks are contractionary at the zero lower bound (Wieland, 2019).

Our model also yields novel insights on the behavior of inflation. Supply disruptions, in fact, are commonly thought to be inflationary because they are associated with increases in firms' marginal costs. In our model, instead, inflation might rise or fall during a supply disruption. This is the result of two opposing effects. On the one hand, lower productivity tends to push up firms' production costs and inflation. On the other hand, involuntary unemployment drives down nominal wages, putting downward pressure on inflation. It is very well possible, therefore, that a supply disruption might generate a recession featuring inflation below target. But it might also be the case that - following a supply disruption - negative output gap and inflation above target coexist. This result sounds a note of caution on the use of high inflation as an indicator of overheating. It might also help to rationalize the experience of the 1970s oil shocks, which were followed by high inflation, high unemployment, as well as low productivity growth (Blanchard and Gali, 2007).

We finally turn to fiscal interventions aiming at boosting future potential output, such as the forthcoming Biden's infrastructure plan. We show that these policies can complement monetary interventions as demand stabilization tools. By stimulating firms' investment and productivity growth, in fact, these fiscal interventions also foster consumers' demand. Through this force, a policy typically associated with the supply side of the economy can help sustain demand and employment. In fact, we show that when monetary policy does not stimulate demand enough to close the output gap, as it is the case during liquidity traps, it is optimal for the government to foster investment and future potential output, in order to bring the economy closer to full employment.

This paper is related mainly to two strands of the literature. First, it is connected to a recent literature, motivated by the Covid-19 epidemic, revisiting the macroeconomic implications of supply disruptions. Guerrieri et al. (2020) study an economy with multiple consumption goods. In their model, a shock reducing the supply of some goods may induce consumers to cut spending also on those goods not directly affected by the shock. If this effect is strong enough, aggregate demand falls by more than supply. They dub supply shocks with this property *Keynesian supply shocks*. Baqaee and Farhi (2020) derive a similar result in an economy with production networks and multiple intermediate goods. Caballero and Simsek (2020) show that supply shocks can be Keynesian due to spillovers between asset prices and aggregate demand. In Bilbiie and Melitz (2020) supply disruptions depress demand by inducing firms' exit, while in L'Huillier et al. (2021) Keynesian supply shocks emerge due to the presence of diagnostic expectations.[2] Our paper studies a different - and complementary - channel through which supply shocks can become Keynesian, based on the endogenous response of investment and productivity growth.[3]

Second, this paper is related to the literature unifying the study of business cycles and endogenous growth. Some examples of this literature are Fatas (2000), Comin and Gertler (2006), Benigno and Fornaro (2018), Moran and Queralto (2018), Anzoategui et al. (2019), Bianchi et al. (2019), Queralto (2019), Garga and Singh (2020), Cozzi et al. (2021) and Vinci and Licandro (2020).[4] Among this body of work, this paper is closest to Benigno and Fornaro (2018) and Garga and Singh (2020). Our paper builds on the framework introduced by Benigno and Fornaro (2018), who study an endogenous growth model with vertical innovation and nominal wage rigidities. They show that in this Keynesian growth framework fluctuations can be driven by animal spirits, and derive the optimal monetary and fiscal policy. Garga and Singh (2020) derive, in a similar Keynesian growth mode, the optimal monetary policy response to fundamental demand shocks. Our paper, instead, employs a Keynesian growth model to study supply shocks. To the best of our

[2]See Bilbiie (2008) for an early model in which supply shocks have Keynesian features.

[3]In our own earlier work (Fornaro and Wolf, 2020), we argued that permanent negative supply shocks can be Keynesian. In this paper, we move beyond the analysis in Fornaro and Wolf (2020) by providing a Keynesian growth model in which productivity growth is the result of firms' investment. We show that temporary negative supply shocks can be Keynesian by triggering endogenous drops in investment and productivity growth.

[4]See Cerra et al. (2020) for a recent survey of this literature.

knowledge, we are the first to show that the scars of supply shocks may change dramatically the macroeconomic implications of supply disruptions.

The rest of the paper is composed of five sections. Section 2 describes the baseline model. Section 3 derives the optimal monetary policy, and shows that the natural interest rate may fall after a supply disruption. Section 4 discusses the implications of imperfect monetary stabilization and the supply-demand doom loop. Section 5 studies optimal innovation policies. Section 6 concludes. Appendix A provides the proofs of all propositions, while Appendix B contains additional derivations, as well as model extensions.

# 2 Baseline model

This section lays down our baseline *Keynesian growth* model. The economy has two key elements. First, the rate of productivity growth is endogenous, and it is the outcome of firms' investment. Second, the presence of nominal wage rigidities implies that output and employment can deviate from their potential levels. In order to illustrate transparently our key results, the framework in this section is kept voluntarily simple. Throughout the paper, however, we will extend this baseline framework in several directions.

Consider an infinite-horizon closed economy. Time is discrete and indexed by $t \in \{0, 1, 2, ...\}$. The economy is inhabited by households, firms, and by a government that sets monetary and fiscal policy. For simplicity, we focus on a perfect foresight economy.

## 2.1 Households

There is a continuum of measure one of identical households deriving utility from consumption of a homogeneous "final" good. The lifetime utility of the representative household is

$$\sum_{t=0}^{\infty} \beta^t \log C_t, \tag{1}$$

where $C_t$ denotes consumption and $0 < \beta < 1$ is the subjective discount factor.

Each household is endowed with $\bar{L}$ units of labor and there is no disutility from working. However, due to the presence of nominal wage rigidities to be described below, a household might be able to sell only $L_t < \bar{L}$ units of labor on the market. Moreover, households can trade in one-period, non-state contingent bonds $B_t$. Bonds are denominated in units of currency and pay the nominal interest rate $i_t$. Finally, households own all the firms and each period they receive dividends $D_t$ from them.

The problem of the representative household consists in choosing $C_t$ and $B_{t+1}$ to maximize expected utility, subject to a no-Ponzi constraint and the budget constraint

$$P_t C_t + \frac{B_{t+1}}{1 + i_t} = W_t L_t + B_t + D_t,$$

where $P_t$ is the nominal price of the final good, $B_{t+1}$ is the stock of bonds purchased by the household in period $t$, and $B_t$ is the payment received from its past investment in bonds. $W_t$ denotes the nominal wage, so that $W_t L_t$ is the household's labor income.

The optimality conditions are

$$\lambda_t = \frac{1}{C_t P_t} \tag{2}$$

$$\lambda_t = \beta(1 + i_t)\lambda_{t+1}, \tag{3}$$

where $\lambda_t$ denotes the Lagrange multiplier on the budget constraint, plus the standard transversality condition.

## 2.2 Final good production

The final good is produced by competitive firms using labor and a continuum of measure one of intermediate inputs $x_{j,t}$, indexed by $j \in [0, 1]$. Denoting by $Y_t$ the output of the final good, the production function is

$$Y_t = (Z_t L_t)^{1-\alpha} \int_0^1 A_{j,t}^{1-\alpha} x_{j,t}^{\alpha} dj, \tag{4}$$

where $0 < \alpha < 1$, and $A_{j,t}$ is the productivity, or quality, of input $j$.[5] $Z_t$, instead, is an exogenous productivity shock, which we refer to as the "supply shock" in our model. This term captures all the factors affecting labor productivity not directly linked to firms' investment. For instance, a reduction in $Z_t$ could capture the fact that during a pandemic some firms cannot operate in order to preserve public health. Or, as we show in Appendix B.7, a fall in $Z_t$ has a similar impact on the economy as an exogenous rise in the price of oil.

Profit maximization implies the demand functions

$$P_t(1 - \alpha) Z_t^{1-\alpha} L_t^{-\alpha} \int_0^1 A_{j,t}^{1-\alpha} x_{j,t}^{\alpha} dj = W_t \tag{5}$$

$$P_t \alpha \left(Z_t L_t\right)^{1-\alpha} A_{j,t}^{1-\alpha} x_{j,t}^{\alpha-1} = P_{j,t}, \tag{6}$$

where $P_{j,t}$ is the nominal price of intermediate input $j$. Due to perfect competition, firms in the final good sector do not make any profit in equilibrium.

## 2.3 Intermediate goods production and profits

Every intermediate good is produced by a single monopolist. One unit of final output is needed to manufacture one unit of the intermediate good, regardless of quality. In order to maximize profits, each monopolist sets the price of its good according to

$$P_{j,t} = \frac{P_t}{\alpha}. \tag{7}$$

[5] More precisely, for every good $j$, $A_{j,t}$ represents the highest quality available. In principle, firms could produce using a lower quality of good $j$. However, the structure of the economy is such that in equilibrium only the highest quality version of each good is used in production.

This expression implies that each monopolist charges a constant markup $1/\alpha > 1$ over its marginal cost.

Equations (6) and (7) imply that the quantity produced of a generic intermediate good $j$ is

$$x_{j,t} = \alpha^{\frac{2}{1-\alpha}} A_{j,t} Z_t L_t. \tag{8}$$

Combining equations (4) and (8) gives

$$Y_t = \alpha^{\frac{2\alpha}{1-\alpha}} A_t Z_t L_t, \tag{9}$$

where $A_t \equiv \int_0^1 A_{j,t} dj$ is an index of average productivity of the intermediate inputs. Hence, production of the final good is increasing in the average productivity of intermediate goods, in the exogenous component of labor productivity, and in the amount of labor employed in production.

The profits earned by the monopolist in sector $j$ are given by

$$(P_{j,t} - P_t) x_{j,t} = P_t \varpi A_{j,t} Z_t L_t,$$

where $\varpi \equiv (1/\alpha - 1)\alpha^{2/(1-\alpha)}$. According to this expression, the producer of an intermediate input of higher quality earns higher profits. Moreover, profits are increasing in $Z_t L_t$ due to the presence of a market size effect. Intuitively, high production of the final good is associated with high demand for intermediate inputs, leading to high profits in the intermediate sector.

### 2.4 Investment and productivity growth

Firms operating in the intermediate sector can invest in innovation in order to improve the quality of their products. In particular, a firm that invests $I_{j,t}$ units of the final good sees its productivity evolve according to

$$A_{j,t+1} = A_{j,t} + \chi I_{j,t}, \tag{10}$$

where $\chi > 0$ determines the productivity of investment.

Innovation-based endogenous growth models typically assume that knowledge is only partly excludable. For instance, this happens if inventors cannot prevent others from drawing on their ideas to innovate. For this reason, in most endogenous growth frameworks, the social return from investing in innovation is higher than the private one.[6] A simple way to introduce this effect in the model is to assume that every period, after production takes place, there is a constant probability $1 - \eta$ that the incumbent firm dies, and is replaced by another firm that inherits its technology. This assumption encapsulates all the factors that might lead to the termination of the rents from innovation, including patent expiration and imitation by competitors.

Firms producing intermediate goods choose investment in innovation to maximize their dis-

[6] See for instance Aghion and Howitt (1992).

counted stream of profits net of investment costs

$$\sum_{t=0}^{\infty} \frac{(\beta\eta)^t}{P_t C_t} \left( P_t \varpi A_{j,t} Z_t L_t - \eta P_t I_{j,t} \right), \tag{11}$$

subject to (10) and given the initial condition $A_{j,0} > 0$. Since firms are fully owned by domestic households, they discount profits using the households' discount factor $\beta^t/(P_t C_t)$, adjusted for the survival probability $\eta$.

From now on, we assume that firms are symmetric and so $A_{j,t} = A_t$. Moreover, in our baseline model we focus on equilibria in which investment in innovation is always positive. Optimal investment in research then requires[7]

$$\frac{1}{\chi} = \frac{\beta C_t}{C_{t+1}} \left( \varpi Z_{t+1} L_{t+1} + \frac{\eta}{\chi} \right). \tag{12}$$

Intuitively, firms equalize the marginal cost from performing research $1/\chi$, to its marginal benefit discounted using the households' discount factor. The marginal benefit is given by the increase in next period profits ($\varpi Z_{t+1} L_{t+1}$) plus the savings on future research costs $1/\chi$, adjusted for the firm survival probability $\eta$.

## 2.5 Aggregation and market clearing

Market clearing for the final good implies[8]

$$Y_t - \int_0^1 x_{j,t} dj = C_t + I_t, \tag{13}$$

where $I_t \equiv \int_0^1 I_{j,t} dj$. The left-hand side of this expression is the GDP of the economy, while the right-hand side captures the fact that all the GDP has to be consumed or invested. Using equations (8) and (9) we can write GDP as

$$Y_t - \int_0^1 x_{j,t} dj = \Psi A_t Z_t L_t, \tag{14}$$

where $\Psi \equiv \alpha^{2\alpha/(1-\alpha)}(1-\alpha^2)$.

The assumption about labor endowment implies that $L_t \leq \bar{L}$. Since labor is supplied inelastically by the households, $\bar{L} - L_t$ can be interpreted as the unemployment rate. For future reference, when $L_t = \bar{L}$ we say that the economy is operating at full employment, while when $L_t < \bar{L}$ the

[7] See Appendix B.1 for the derivation of equation (12).

[8] The goods market clearing condition can be derived by combining the households' budget constraint with the expression for firms' profits

$$D_t = \underbrace{P_t Y_t - W_t L_t - P_t \frac{1}{\alpha} \int_0^1 x_{j,t} dj}_{\text{profits from final goods sector}} + \underbrace{P_t \int_0^1 \left( \frac{1}{\alpha} - 1 \right) x_{j,t} dj - P_t \int_0^1 I_{j,t} dj}_{\text{profits from intermediate goods sector}}.$$

We also use the equilibrium condition $B_{t+1} = 0$, which is implied by the assumption of identical households.

economy operates below capacity and there is a negative output gap.

Long run growth in this economy takes place through increases in the quality of the intermediate goods, captured by increases in the productivity index $A_t$. Defining $g_t \equiv A_t/A_{t-1}$, we can write equation (10) as

$$g_{t+1} = 1 + \chi \frac{I_t}{A_t}. \tag{15}$$

This expression implies that higher investment in research in period $t$ is associated with faster productivity growth between periods $t$ and $t+1$. More precisely, the rate of productivity growth is determined by the ratio of investment in innovation $I_t$ over the existing stock of knowledge $A_t$. In turn, the stock of knowledge depends on all past investment in innovation, that is on the R&D stock. Hence, there is a positive link between R&D intensity, captured by the ratio $I_t/A_t$, and future productivity growth.

### 2.6 Wages, prices and monetary policy

We consider an economy with frictions in the adjustment of nominal wages.[9] The presence of nominal wage rigidities plays two roles in the model. First, it creates the possibility of involuntary unemployment, by ensuring that nominal wages remain positive even in presence of unemployment. Second, it opens the door to a stabilization role for monetary policy. Indeed, as we will see, prices inherit part of wage stickiness, so that the central bank can affect the real interest rate of the economy through movements in the nominal interest rate.

In the baseline model, we assume that nominal wage growth is indexed to the rate of productivity growth

$$\frac{W_t}{W_{t-1}} = g_t \frac{Z_t}{Z_{t-1}}. \tag{16}$$

This formulation of wage rigidities is particularly convenient, for reasons that we will explain shortly. However, the results that follow do not rely at all on this extreme form of wage stickiness. Indeed, in Section 4.4 we consider an economy in which wages are allowed to respond to fluctuations in employment, giving rise to a wage Phillips curve, and explore different assumptions about the relationship between wage inflation and productivity growth. We will also consider the case of a New Keynesian wage Phillips curve derived from the presence of wage adjustment costs à la Rotemberg (1982).

Turning to prices, combining equations (5) and (8) gives

$$P_t = \frac{1}{1-\alpha} \left(\frac{1}{\alpha}\right)^{\frac{2\alpha}{1-\alpha}} \frac{W_t}{Z_t A_t}. \tag{17}$$

Intuitively, prices are increasing in the marginal cost of firms producing the final good. An increase

[9] A growing body of evidence emphasizes how nominal wage rigidities represent an important transmission channel through which monetary policy affects the real economy. For instance, this conclusion is reached by Olivei and Tenreyro (2007), who show that monetary policy shocks in the US have a bigger impact on output in the aftermath of the season in which wages are adjusted. Micro-level evidence on the importance of nominal wage rigidities is provided by Fehr and Goette (2005), Gottschalk (2005), Barattieri et al. (2014) and Gertler et al. (2020).

in wages puts upward pressure on marginal costs and leads to a rise in prices, while a rise in productivity reduces marginal costs and prices. This expression, combined with the law of motion for wages, can be used to derive an equation for price inflation

$$\pi_t \equiv \frac{P_t}{P_{t-1}} = 1, \tag{18}$$

which implies that gross price inflation is constant and normalized to 1. This property of the baseline model is convenient, because it will allow us to focus attention on the two key variables at the heart of our analysis: employment and productivity growth. Later on, in Section 4.4, we will consider the implications of our model for inflation.

Monetary policy controls the nominal rate $i_t$. For now, we leave the central bank behavior unspecified. Throughout the paper, we will consider different monetary policy regimes.

## 2.7 Equilibrium

Given a path for $i_t$, the equilibrium of the economy can be described by three simple equations. The first equation captures the *demand* side of the economy. Start with the Euler equation, which determines households' consumption decisions. Combining households' optimality conditions (2) and (3) gives

$$\frac{1}{C_t} = \beta(1+i_t)\frac{1}{C_{t+1}\pi_{t+1}}.$$

According to this expression, demand for consumption is increasing in future consumption and decreasing in the real interest rate, $(1+i_t)/\pi_{t+1}$.

To understand how productivity growth relates to demand for consumption, it is useful to combine the previous expression with $A_{t+1}/A_t = g_{t+1}$ and $\pi_{t+1} = 1$ to obtain

$$c_t = \frac{g_{t+1}c_{t+1}}{\beta(1+i_t)}, \tag{19}$$

where we have defined $c_t \equiv C_t/A_t$ as consumption normalized by the productivity index. This equation implies a positive relationship between productivity growth and present demand for consumption. The reason is that faster productivity growth is associated with higher future wealth. This wealth effect leads households to increase their demand for current consumption in response to a rise in productivity growth.

The second key relationship in our model is the *growth* equation, which is obtained by combining equation (2) with the optimality condition for investment in research (12)

$$g_{t+1} = \beta\frac{c_t}{c_{t+1}}\left(\chi\varpi Z_{t+1}L_{t+1} + \eta\right). \tag{20}$$

This equation implies a positive relationship between growth and future market size. Intuitively, a rise in $Z_{t+1}L_{t+1}$ is associated with higher future monopoly profits. In turn, higher profits induce entrepreneurs to invest more in research, leading to a positive impact on the growth rate of the economy. This is the classic market size effect emphasized by the endogenous growth literature.[10] At the same time, growth depends inversely on the growth rate of normalized consumption $c_{t+1}/c_t$. This is a cost of funds effect: when today's consumption is low, relative to future consumption, firms pay out dividends to households rather than invest. Both the market size effect and the cost of funds effect will play an important role in mediating the impact of supply shocks on investment and growth.

The third equation combines the goods market clearing condition (13), the GDP equation (14) and the fact that $I_t/A_t = (g_{t+1} - 1)/\chi$

$$\Psi Z_t L_t = c_t + \frac{g_{t+1} - 1}{\chi}. \tag{21}$$

Keeping output constant, this equation implies a negative relationship between productivity-adjusted consumption and growth, because to generate faster growth the economy has to devote a larger fraction of output to investment, reducing the resources available for consumption.

We are now ready to define an equilibrium as a set of sequences $\{g_{t+1}, L_t, c_t\}_{t=0}^{+\infty}$ satisfying the three equations (19), (20) and (21), as well as $L_t \leq \bar{L}$, $g_{t+1} > 1$ and $c_t > 0$ for all $t \geq 0$, given paths for monetary policy $\{i_t\}_{t=0}^{+\infty}$ and the supply shock $\{Z_t\}_{t=0}^{+\infty}$.

## 2.8 The balanced growth path

Before studying the implications of the model, let us spend a few words on the balanced growth path - or steady state - of the economy. A steady state is characterized by constant values for $g_{t+1}$, $L_t$, $c_t$, $i_t$ and $Z_t$ that satisfy equations (19)-(21). For most of the paper, we will be studying economies that fluctuate around a full employment steady state. From now on, we will denote the value of a variable in this steady state with an upper bar, and we normalize steady state productivity to $\bar{Z} = 1$. We now make some assumptions to ensure that a full employment steady state exists.

**Proposition 1** *Suppose that the parameters satisfy*

$$\beta(\chi \varpi \bar{L} + \eta) > 1, \tag{22}$$

*and that monetary policy is such that*

$$1 + \bar{i} = \chi \varpi \bar{L} + \eta. \tag{23}$$

*Then there exists a unique full employment steady state. Moreover, this steady state is characterized by $\bar{g} > 1$.*

[10] To be clear, what matters for our results is that productivity growth is increasing in employment relative to potential. This means that our key results would also apply to a setting in which scale effects related to population size were not present. For instance, in the spirit of Young (1998) and Howitt (1999), these scale effects could be removed by assuming that the number of intermediate inputs is proportional to population size.

Intuitively, condition (22) guarantees that in the full employment steady state the return to investment is sufficiently high that growth is positive. Condition (23), instead, ensures that the central bank policy is consistent with the existence of a full employment steady state.

# 3 Supply disruptions and optimal monetary policy

We start by characterizing the response of the economy to a supply disruption when monetary policy is conducted optimally. Throughout the paper, we will assume that the government has no commitment power, and so we derive the optimal policy under discretion. As stated in the following proposition, it turns out that the objective of monetary policy is quite simple. The central bank, in fact, needs to act so as to ensure that the economy operates at full employment at all times.

**Proposition 2** *Consider a central bank that operates under discretion and maximizes households' expected utility, subject to (19), (20), (21) and $L_t \leq \bar{L}$. The solution to this problem satisfies $L_t = \bar{L}$ for all $t \geq 0$.*

The intuition behind this result is straightforward. Since there is no disutility from working, it is optimal for households to work their full labor endowment. This corresponds to the allocation under flexible wages, so the optimal monetary policy effectively offsets the distortions due to nominal rigidities by keeping the output gap closed in every period.[11]

What is interesting, however, is how this objective is attained. To implement the optimal policy, the policy rate needs to track its natural counterpart, which is defined precisely as the interest rate that prevails in an economy without nominal rigidities. But, as we explain next, the response of the natural rate to a supply disruption depends heavily on the behavior of investment and productivity growth.

## 3.1 An exogenous growth benchmark

Let us first consider a counterfactual economy in which there is no investment and hence no productivity growth. As is well known from the New Keynesian literature (Galí, 2009), in this case the natural rate rises after a negative supply shock. The central bank should then react to supply disruptions by tightening monetary policy.

[11] To be precise, the economy is subject to three sources of inefficiency. First, involuntary unemployment is possible. Second, due to monopolistic competition production of intermediate goods is inefficiently low. Third, investment in innovation is subject to intertemporal spillovers, since when a firm dies its stock of knowledge is appropriated by another firm. This effect implies that in the competitive equilibrium under flexible wages investment is lower than in the social planner allocation. However, as shown in the proof to Proposition 2, interest rate policy can only seek to correct the first distortion. The economic intuition for this result is that, since consumption and investment in research are both decreasing in the interest rate, interest rate policy cannot affect the allocation of output between consumption and investment, and hence cannot correct for the inefficiencies due to the intertemporal spillover effect. We will go back to this point in Section 5, once we discuss innovation policies.

More precisely, assume that firms' investment has no impact on productivity growth ($\chi = 0$), and that $Z_t$ follows the process

$$\log Z_t = \rho \log Z_{t-1}, \tag{24}$$

where $0 \leq \rho < 1$ determines the persistence of the exogenous component of productivity.

Now suppose that the economy is hit by a previously unexpected negative supply shock, which corresponds to the initial condition $Z_0 < 1$. The path of the interest rate that implements the allocation under the optimal policy is then given by[12]

$$1 + i_t = \frac{Z_{t+1}}{\beta Z_t} = \frac{Z_0^{\rho^t(\rho-1)}}{\beta}. \tag{25}$$

Since $Z_0 < 1$, this expression implies that the policy rate increases in response to a temporary supply disruption. In absence of a monetary policy tightening, in fact, the negative supply shock would trigger excess demand for consumption.

This result forms part of the conventional wisdom on the optimal conduct of monetary policy. As we will see, however, this conventional wisdom might fail once the impact of supply shocks on investment and productivity growth is taken into account.

### 3.2 Back to the Keynesian growth framework

We now present our first, and perhaps most striking, result: in an economy in which productivity growth is driven by firms' investment, a negative supply shock might trigger a demand shortage that is larger than the supply disruption itself. When this happens, the natural rate declines in response to the shock. The optimal monetary policy response therefore consists in cutting the policy rate.

**Proposition 3** *Assume that $Z_t$ is governed by the process (24), and that $Z_0 < 1$. If $\rho > 0$, the path of interest rates that implements the allocation under optimal policy is characterized by $i_t < \bar{i}$ for all $t \geq 0$. If $\rho = 0$, optimal policy implies $i_t = \bar{i}$ for all $t \geq 0$.*

To understand Proposition 3, let us start by studying the behavior of investment and productivity growth. By equation (20), under the optimal monetary policy productivity growth evolves according to

$$g_{t+1} = \beta \frac{c_t}{c_{t+1}} \left( \chi \varpi Z_{t+1} \bar{L} + \eta \right). \tag{26}$$

There are two channels through which the supply shock reduces growth. First, a transitory drop in $Z_t$ leads to a drop in $c_t / c_{t+1}$. This corresponds to an increase in the rate at which households discount future profits, reducing firms' incentives to invest. Moreover, if the shock is persistent, the fall in $Z_{t+1}$ lowers the profits that firms appropriate by investing in innovation. Both effects point toward a negative impact of supply disruptions on investment and productivity growth.

[12] To derive this expression, notice that if $\chi = 0$ then $g_{t+1} = 1$ and $c_t = \Psi Z_t L_t$ for every $t$.

What are the implications for aggregate demand? The fall in investment constitutes a drag on demand in itself. Quantitatively, this effect is stronger the higher the share of investment in GDP. Now imagine that, following standard practice in the endogenous growth literature, we interpret firms' investment in innovation as their expenditure in R&D. Given that spending in R&D represents a small fraction of GDP, in this case the direct impact of fluctuations in investment on innovation on aggregate demand will be small.

There is, however, a second channel through which a fall in investment depresses aggregate demand. Lower investment drives down productivity growth and so agents' future income. This negative wealth effect causes a drop in consumption demand. This second effect, on its own, might be strong enough to reverse the response of the natural rate to a supply shock, relative to the case in which productivity growth is exogenous. To see this point most clearly, consider the limit case where the investment share of GDP goes to zero, so that $c_t \approx \Psi Z_t \bar{L}$. From (19), the natural interest rate then evolves according to

$$1 + i_t = \frac{g_{t+1} c_{t+1}}{\beta c_t} = \frac{g_{t+1} Z_0^{\rho^t (\rho - 1)}}{\beta}. \tag{27}$$

This expression shows how the endogenous drop in $g_{t+1}$ puts downward pressure on the natural rate. As we show in Proposition 3, as long as the supply disturbance has some persistence ($\rho > 0$), this effect is strong enough to induce a drop in the natural rate after a negative supply shock.

To further illustrate this point, we resort to a simple numerical simulation.[13] We choose the length of a period to correspond to one year. We set $\chi$, $\beta$ and $\alpha$ by targeting three moments of the full employment steady state. $\chi$ is set to 1.95 so that steady state productivity growth is equal to 2%, while we choose $\beta = 0.995$ so that the real interest rate in steady state is equal to 2.5%. We set the labor share in gross output to $1 - \alpha = 0.86$, to match a ratio of spending in investment on innovation to GDP of 2%, close to the GDP share of business spending in R&D observed in the United States. This calibration choice implies a small direct impact of investment fluctuations on aggregate demand. To set the firm survival probability we follow Benigno and Fornaro (2018) and set $\eta = 0.9$. Finally, the shock is parametrized so that on impact GDP drops by 3% under the optimal monetary policy, and its persistence is set to $\rho = 0.75$.

Figure 1 illustrates how the economy responds to a supply disruption when monetary policy is conducted optimally. Under the exogenous growth counterfactual optimal policy would prescribe a monetary tightening. But once the endogenous response of productivity growth is taken into account the optimal policy entails a fall in the interest rate. Moreover, the figure highlights another interesting aspect of the optimal policy. While the optimal monetary policy involves closing the output gap, it does not steer output back to its pre-shock trend. Therefore, a permanent drop in the level of output - i.e. the scars of supply shocks - reflects an efficient response of the economy.

[13] To be clear, our objective is not to provide a careful quantitative evaluation of the framework or to replicate any particular historical event. In fact, both of these tasks would require a much richer model. Rather, our aim is to show how the model behaves for some reasonable parameter values.

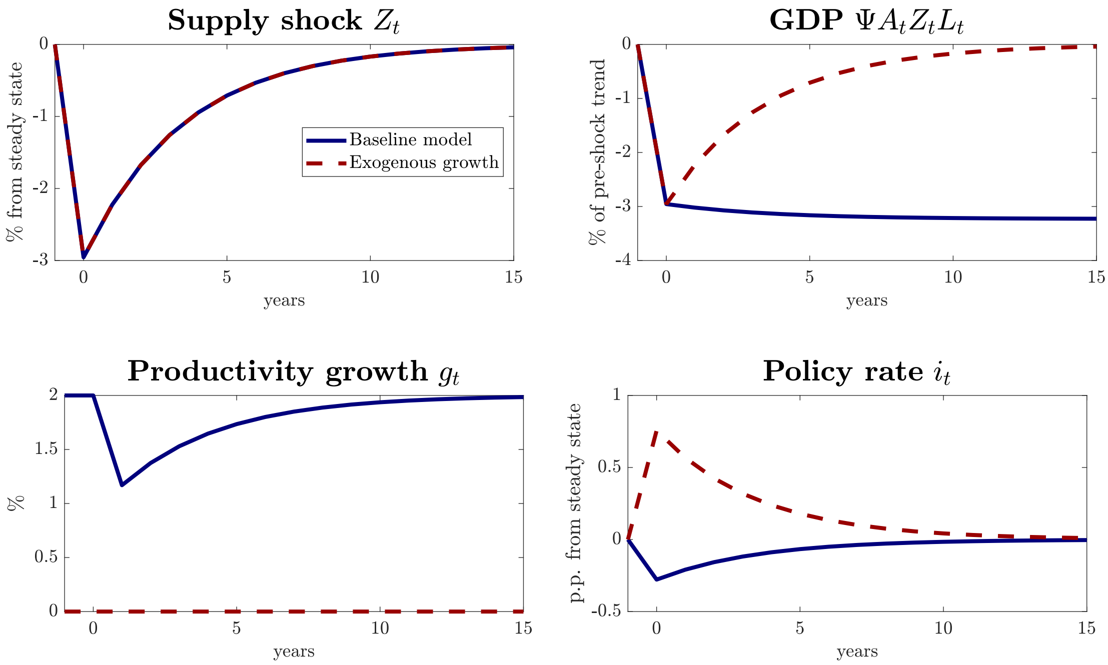



**Figure 1: Optimal monetary policy response to a supply disruption.**

**Some remarks on the investment function.** In our model, there is a linear relationship between investment in innovation and productivity growth. This is a common assumption in the theoretical literature on endogenous growth, since it is consistent with free entry in the research sector (Aghion and Howitt, 1992). Quantitative analyses, however, often assume that investment in research is subject to diminishing returns - to capture the existence of adjustment costs in investment in innovation (Acemoglu and Akcigit, 2012). With decreasing returns to investment, the response of productivity growth to a supply disruption is more muted - and it is no longer true that the natural rate unambiguously falls after a persistent negative supply shock. Rather, as we discuss in Appendix B.2, with a concave investment function the natural rate drops if the supply disruption is sufficiently persistent.[14]

Summing up, by negatively affecting investment and productivity growth, even purely transitory supply shocks generate permanent output losses. The associated negative wealth effect induces consumers to cut on their demand, which may even fall below the exogenous drop in supply. In this case, the optimal policy response to supply shocks flips relative to conventional wisdom, since a monetary expansion is needed to sustain demand at a level consistent with full employment.

[14] Even if one maintains the assumption of a linear investment function, there is a case in which the natural rate might rise after a negative supply disruption. This might happen if the shock is large enough to drive investment in innovation, and the endogenous component of productivity growth, to zero. The reason is simple. Once firms stop investing in innovation, further drops in $Z_t$ no longer depress the endogenous component of productivity growth - and the associated drag on consumers' demand becomes muted. This means that the impact of supply disruptions on the natural rate might become non-linear. In particular, the natural rate might rise in response to a negative supply shock severe enough to drive investment in innovation to zero.

# 4 Imperfect stabilization and the supply-demand doom loop

We have seen that under the optimal policy the central bank lowers the policy rate and stimulates aggregate demand in response to supply disruptions. But what happens if monetary policy does not impart enough stimulus to the economy? We now show that, as a result, supply shocks can give rise to recessions characterized by lack of demand, and that the impact of these shocks on output and productivity growth is amplified by a supply-demand doom loop.

One could consider several deviations of monetary policy from its optimal stance. To keep the analysis simple, we start by assuming that monetary policy follows the simple rule

$$1 + i_t = (1 + \bar{i}) \left(\frac{L_t}{\bar{L}}\right)^{\phi}. \tag{28}$$

Under this rule the central bank seeks to stabilize output around its potential level, by cutting the interest rate in response to falls in employment. In addition to condition (23), we assume that

$$\phi > \frac{\chi\varpi\bar{L}}{\chi\varpi\bar{L} + \eta}, \tag{29}$$

which implies that the steady state under the rule (28) is locally determinate.[15]

While focusing on this particular deviation from the optimal policy is analytically convenient, much of the results extend to other cases as well. For instance, monetary policy might end up being constrained by the zero lower bound when trying to pursue the optimal policy. As we discuss later on, the key results obtained under rule (28) also hold when monetary policy is constrained by the zero lower bound.

## 4.1 An insightful case: a permanent supply disruption

While our focus is on temporary supply disruptions, it is useful to first study a case in which $Z_t$ drops permanently to a lower level. The advantage is that, by focusing on a permanent shock, we can illustrate the key forces at the heart of the model using a simple graphical analysis.

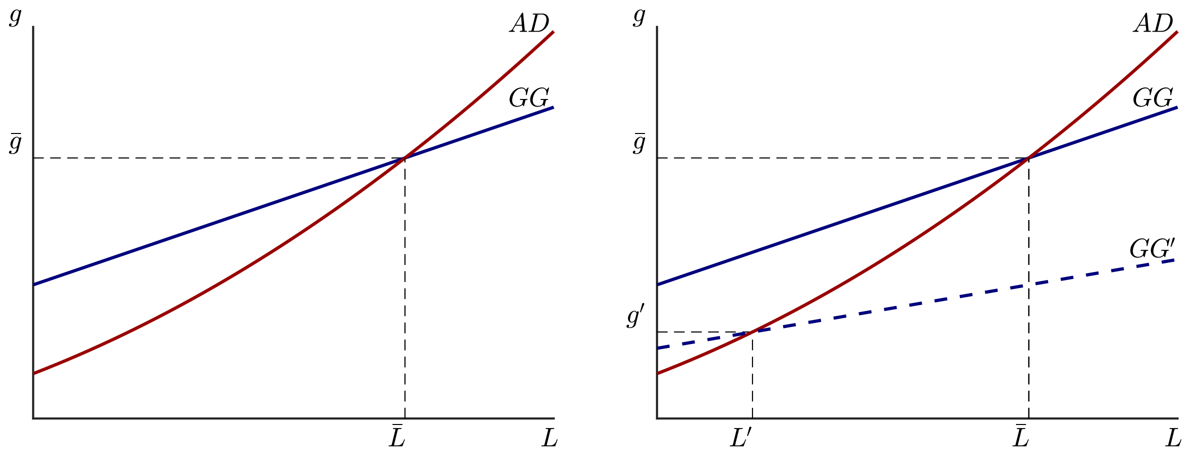
Figure 2 shows how $L$ and $g$ are determined in steady state. The (GG) schedule corresponds to the growth equation (20) evaluated in steady state, given by

$$g = \beta(\chi\varpi ZL + \eta). \tag{GG}$$

The (GG) schedule implies a positive relationship between $g$ and $L$. Intuitively, an increase in employment - and so in market size - is associated with a rise in the return from investing in innovation. Naturally, firms respond by increasing investment and productivity growth accelerates.

The (AD) curve, instead, summarizes the aggregate demand side of the model. It is obtained

[15] See Appendix B.3 for a proof.




**(a) The $L - g$ diagram.** **(b) A permanent supply disruption.**

**Figure 2: The $L - g$ diagram and permanent supply disruptions.**

by combining equations (19) and (28), evaluated in steady state

$$g = \beta(1 + \bar{i}) \left( \frac{L}{\bar{L}} \right)^{\phi}. \tag{AD}$$

This equation implies a positive relationship between $g$ and $L$. To understand the intuition behind this equation, consider what happens after a rise in productivity growth. Due to the associated positive wealth effect, households respond by increasing their demand for borrowing and consumption. Higher consumption, in turn, puts upward pressure on employment. The central bank reacts to the rise in employment by increasing the interest rate, which cools down households' demand for borrowing and restores equilibrium on the credit market.

A steady state equilibrium corresponds to an intersection of the (AD) and (GG) curves. Under our assumption about $\phi$, there is only one intersection between the two curves satisfying $L \leq \bar{L}$, meaning that the steady state exists and is unique. The steady state shown in the left panel of Figure 2 corresponds to the full employment steady state.

Now imagine that we start from the full employment steady state, and a previously unexpected permanent fall in $Z$ occurs. As shown in the right panel of Figure 2, the decline in $Z$ induces a downward shift of the GG curve. As already explained, the exogenous fall in labor productivity depresses firms' profits and their incentives to invest. Firms react by reducing investment and so, holding constant $L$, productivity growth $g$ drops. The fall in productivity growth, through its negative wealth effect, translates into lower aggregate demand. The central bank reacts by cutting the policy rate, but not by enough to prevent unemployment from arising. The result is a drop in employment below its efficient level ($L < \bar{L}$). Therefore, the negative supply shock gives rise to a drop in aggregate demand and involuntary unemployment.[16]

[16] This effect is well known from the literature on news shocks (e.g., Lorenzoni, 2009).

This is not, however, the end of the story. Lower demand further reduces firms' profits and their incentives to invest. This happens because firms' investment is sensitive to demand through the market size $L$. This effect generates another round of drops in investment and productivity growth. Lower productivity growth, in turn, induces a further cut in demand, which again lowers investment and growth. This vicious spiral, or supply-demand doom loop, amplifies the impact of the initial supply shock on employment and labor productivity growth.

It is possible to derive an expression for the elasticity of the endogenous component of productivity growth with respect to the supply shock. Combining (GG) and (AD) and differentiating gives

$$\left(\frac{\partial g}{\partial Z}\right)\frac{Z}{g}\bigg|_{Z=1} = \frac{1-\eta\beta/\bar{g}}{1-\frac{1-\eta\beta/\bar{g}}{\phi}}. \tag{30}$$

In this expression, the numerator captures the direct impact of a change in $Z$ on $g$. In this simple model, this direct effect is large when the externalities associated with innovation activities are substantial (i.e. when $\eta$ is small). The denominator, instead, captures the multiplier effect associated with the supply-demand doom loop. This multiplier effect is decreasing in $\phi$.[17] As it is intuitive, a smaller response of monetary policy to changes in employment amplifies the impact of supply shocks on productivity growth.

Therefore, in our framework monetary policy has an impact on the endogenous component of productivity growth. In particular, a tighter monetary policy stance - compared to the optimal one - leads to a slowdown in investment in innovation and productivity growth. This feature of the model is consistent with a recent body of empirical evidence (Garga and Singh, 2020; Moran and Queralto, 2018; Jordà et al., 2020), suggesting that monetary policy tightenings induce firms to cut investment in innovation, such as R&D, and have a long lasting negative impact on productivity and potential output.

## 4.2 A temporary supply disruption

We turn back to the case of temporary shocks, by again assuming that $Z_t$ evolves according to the process (24). In the case of temporary shocks, deriving analytic results is more challenging. However, some insights can be obtained by combining (19), (20) and (28) as follows

$$(1+\bar{i})\left(\frac{L_t}{\bar{L}}\right)^{\phi} = \chi\varpi Z_{t+1}L_{t+1} + \eta. \tag{31}$$

Since $L_{t+1} \leq \bar{L}$, this equation directly implies that if $Z_{t+1} < 1$, then there is a negative output gap in period $t$ ($L_t < \bar{L}$). Of course, a lower $L_t$ leads to a reduction in the incentives to invest in period $t-1$, which lowers aggregate demand and employment in period $t-1$, and so on. Thus in

[17] Notice that $\bar{g} = \beta(\chi\varpi\bar{L} + \eta)$. The denominator can thus be written as

$$1 - \frac{\chi\varpi\bar{L}}{\chi\varpi\bar{L}+\eta}/\phi.$$

By assumption (29), the denominator is therefore positive for any permissible level of $\phi$.

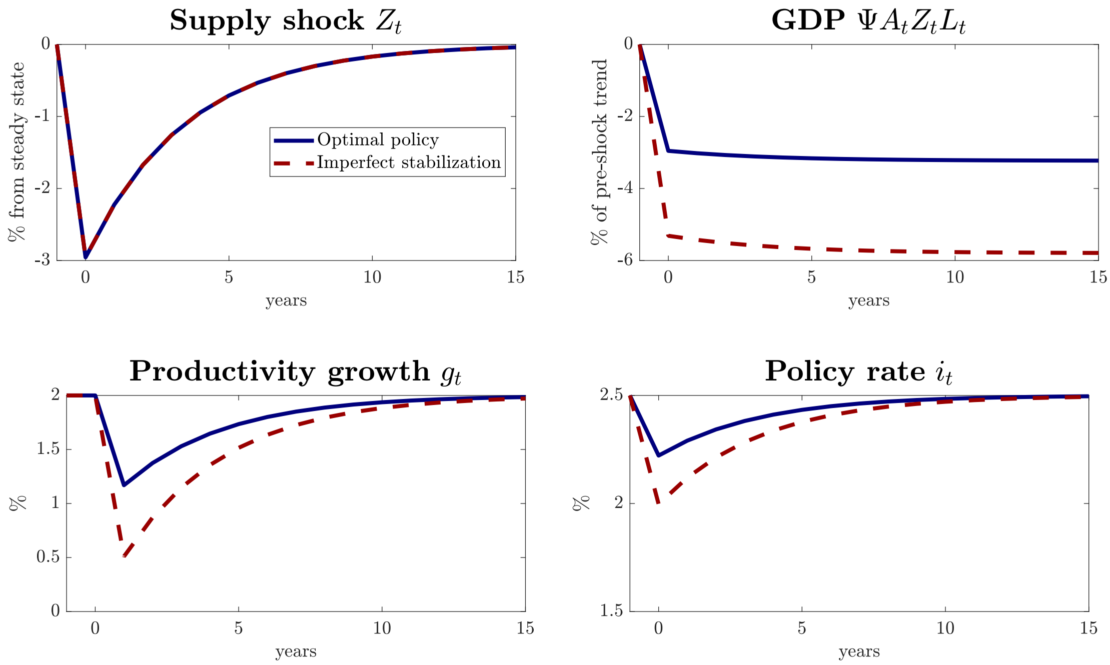



**Figure 3: Imperfect monetary stabilization.**

the case of temporary shocks, an intertemporal supply-demand doom loop emerges.

Figure 3 shows the response of the economy to a negative supply shock, both when monetary policy is conducted optimally and when monetary policy follows the rule (28). We set the response of the policy rate to the output gap to the illustrative value of $\phi = 0.2$. The other model parameters are kept as in Section 3.

As shown by the top right panel, under the simple rule on impact GDP falls by an additional 2.5% compared to optimal policy, so that the recession has a substantial component due to weak aggregate demand. Moreover, since weak aggregate demand drags down the endogenous component of productivity growth, the hysteresis effect is also larger under the interest rate rule compared to the optimal policy (lower left panel). Interestingly, the shock has a bigger impact on the economy under rule (28), in spite of the fact that on impact the policy rate in that case falls more compared to the optimal policy benchmark. This is due to agents' forward looking behavior, coupled with the fact that in the long run monetary policy is more expansionary under the optimal policy compared to rule (28).

Summing up, there are two lessons to be learned from this analysis. First, a negative shock which originates from the supply side of the economy can give rise to a slump characterized by lack of demand. Second, lack of demand feeds back into lower investment dragging down productivity growth, which further depresses consumers' demand. This supply-demand doom loop may greatly amplify the direct impact of negative supply shocks on employment and output growth.

### 4.3 Supply disruptions and liquidity traps

Another instance of imperfect monetary stabilization occurs when the central bank becomes constrained by the zero lower bound and the economy enters a liquidity trap. This might happen following a negative supply shock since, as we saw in Section 3, the optimal monetary policy response to supply disruptions might consist in lowering the policy rate. We give a full treatment of this case in Appendix B.4, here we just outline some key insights, and in particular we show that supply disruptions can give rise to stagnation traps.

Imagine that monetary policy is run optimally, but that the policy rate cannot fall below zero ($i_t \geq 0$). The solution to the optimal policy problem can be simply represented as

$$i_t(L_t - \bar{L}) = 0. \tag{32}$$

The economy thus operates at potential ($L_t = \bar{L}$) whenever the zero lower bound does not bind. But when $i_t = 0$ the economy might experience unemployment due to weak aggregate demand ($L_t \leq \bar{L}$). Of course, this happens when the natural interest rate becomes negative, and the zero lower bound prevents monetary policy from providing enough stimulus to attain full employment.

A large enough negative supply shock, due to its negative impact on consumers' demand, can therefore make the zero lower bound bind and plunge the economy into a liquidity trap. Moreover, when the lower bound binds the supply-demand doom loop becomes particularly severe, since monetary policy stops responding to changes in employment and aggregate demand altogether.[18,19] Even though monetary policy is conducted optimally, the economy then experiences a recession characterized by weak aggregate demand, involuntary unemployment and low productivity growth, in short a stagnation trap. Indeed, as observed by Benigno and Fornaro (2018), these are typical features of the liquidity trap episodes that have characterized several advanced economies in recent times. This property of the model is also in line with the empirical evidence provided by Wieland (2019), who shows that negative supply shocks hitting Japan during its liquidity trap - such as the 2011 earthquake and oil price hikes - led to contractions in output.

### 4.4 Implications for inflation

Inflation is commonly used as a key input in constructing output gap measures. Inflation above the central bank target, in fact, is seen as a sign of overheating, while inflation undershooting its target is normally associated with slack and insufficient demand. We now show that a supply disruption, contrary to this logic, might generate both inflation above target and unemployment

[18] In fact, in our simple model, when the zero lower bound binds investment goes all the way to zero. This strong response of investment is due to our assumption of linear investment technology. In presence of diminishing returns to investment on innovation, investment would remain positive even when the zero lower bound binds.

[19] As shown by Benigno and Fornaro (2018), when the zero lower bound binds the supply-demand doom loop can be so strong as to generate fluctuations purely driven by animal spirits. In this paper we abstract from this source of multiplicity, by assuming that agents never coordinate their expectations on the stagnation traps steady state described by Benigno and Fornaro (2018). As shown in Benigno and Fornaro (2018), this is the case if an appropriately designed system of strong countercyclical subsidies to investment is in place.

due to weak aggregate demand.[20] We will start by making this point using an ad-hoc wage Phillips curve. Later on, we will consider the case of a New Keynesian wage Phillips curve derived from the presence of wage adjustment costs à la Rotemberg (1982).

Let us start by replacing the wage equation (16) with the more general formulation

$$\frac{W_t}{W_{t-1}} = \left(g_t \frac{Z_t}{Z_{t-1}}\right)^{\omega} \left(\frac{L_t}{\bar{L}}\right)^{\xi}, \tag{33}$$

where $\omega \leq 1$ and $\xi \geq 0$. When $\omega < 1$ wages are only partially indexed to productivity growth. Instead, when $\xi > 0$ a Phillips curve component arises, since an increase in involuntary unemployment puts downward pressure on wage growth. Our baseline model corresponds to the case $\omega = 1$ and $\xi = 0$.

Under this formulation of wage rigidities, inflation is given by

$$\pi_t = \left(\frac{Z_t}{Z_{t-1}} g_t\right)^{\omega-1} \left(\frac{L_t}{\bar{L}}\right)^{\xi}. \tag{34}$$

Equation (34) implies that there are two channels through which a supply disruption affects marginal costs and inflation. First, lower labor productivity growth points toward rising marginal costs and inflation. Second, if demand is not perfectly stabilized, the supply disruption gives rise to involuntary unemployment. The result is lower wage growth, which exerts downward pressure on inflation.

The degree of nominal wage rigidities is the key determinant of whether inflation rises or falls in response to a supply disruption. Imagine that nominal wages are fully rigid, so that $\omega = \xi = 0$. Then expression (34) implies that inflation unambiguously rises after a negative supply shock, even if the shock generates a negative output gap. In this case the economy experiences a recession with high inflation and weak aggregate demand. Instead, the larger the response of wages to labor market slack, i.e. the larger $\xi$, the more likely it is that inflation will fall after a supply disruption.[21] A priori, it is then hard to say whether inflation will rise or fall following a negative supply shock. This makes inflation a poor guide of whether demand is strong enough to maintain the economy at full employment in our framework.

We illustrate these insights in Figure 4. To construct the figure, we assume that, in the economy with the wage Phillips curve (33), the central bank implements the same real interest rate path following the negative supply shock as shown in Figure 3. This implies that the path for all real variables is identical in equilibrium, such that equation (34) merely determines the equilibrium rate of inflation.

[20] In contrast, the New Keynesian model predicts a positive relationship between inflation and the output gap in response to the standard productivity shocks considered in this paper.

[21] When wages are fully flexible, the inflation response is determined by the fact that the real interest rate equals the natural rate, $1 + r_t^n = (1 + i_t)/\pi_{t+1}$, in combination with a monetary policy rule for $1 + i_t$, for instance the rule $1 + i_t = (1 + \bar{i})\pi_t^{\phi}$ with $\phi > 1$. Combining both we obtain the difference equation $\pi_{t+1}(1 + r_t^n) = (1 + \bar{i})\pi_t^{\phi}$. Because $1 + r_t^n$ declines after a negative supply shock, as we saw in Section 3, this implies that inflation necessarily declines following a negative supply shock under flexible wages.

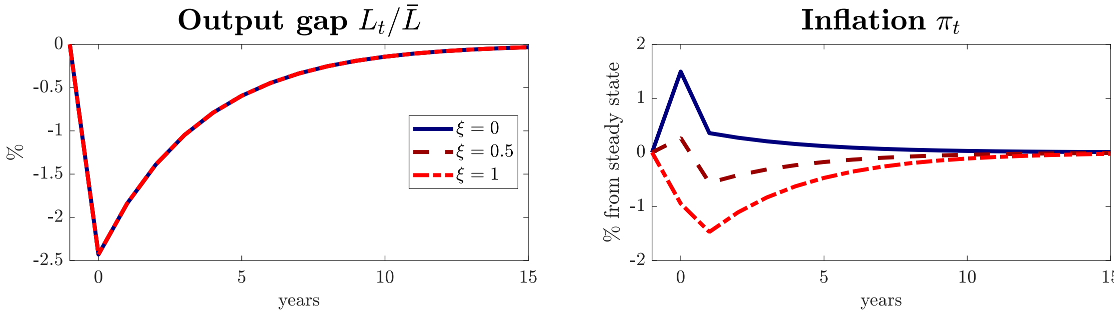



**Figure 4: Inflation response to supply disruptions.**

In the figure, we assume that $\omega = 0.5$, such that wages are partially indexed to productivity growth. Moreover, we contrast three different levels of wage rigidities. When wages are very rigid ($\xi = 0$, i.e. wages do not respond to the output gap), then inflation rises despite the fact that the output gap turns negative. In contrast, when wages are very responsive, then inflation declines alongside the output gap. An identical response of the output gap is therefore consistent with different equilibrium rates of inflation.[22]

Our results extend to the case of a New Keynesian wage Phillips curve. In Appendix B.5 we consider a case in which households are monopolistically competitive suppliers of labor, and where wage changes are subject to Rotemberg (1982)-type adjustment costs. This gives rise to a standard forward looking wage Phillips curve, of the type often considered in the New Keynesian literature. The two opposing forces mediating the impact of the supply shock on inflation are also present in this case. As it was the case in this section, the degree of wage rigidities turns out to be the key determinant of the inflation response to supply shocks. The more rigid the wages, the higher the likelihood that a supply disruption gives rise to a recession characterized by weak aggregate demand and inflation above target.

To conclude this section, we note that our model could thus help to rationalize the fact that the 1970s oil price shocks were accompanied by high unemployment and inflation above target.[23] Interestingly, in fact, this period was also characterized by a sustained productivity growth slowdown. The role of wage rigidities in mediating the impact of oil shocks on inflation is also consistent with the evidence provided by Blanchard and Gali (2007), who suggest that the hikes in oil prices of the 1970s had a particularly large impact on inflation due to the high degree of wage rigidities characterizing that period.

[22]It follows that also the equilibrium response of the nominal interest rate is different among economies. In fact, it can be the case that the equilibrium nominal rate rises, despite the negative output gap, and despite the fact that the real interest rate necessarily falls. This happens when the inflation response following the negative supply shock is sufficiently positive.

[23]In Appendix B.7, we show that oil price shocks enter the model in an isomorphic way as reductions in $Z_t$.

# 5 Innovation policies

An interesting question to ask is whether the government should employ other forms of policy interventions, if monetary policy does not optimally stabilize aggregate demand. A natural candidate is fiscal policy. In this paper, we focus on fiscal interventions fostering firms' investment in innovation and productivity growth, in short, on innovation policies. But the results of this section apply more generally to fiscal policies aiming at boosting future potential output, such as Biden's infrastructure plan. The key message of this section is that these policies, besides correcting the usual externalities associated with the growth process, can also play a role in stabilizing aggregate demand and closing the output gap.

Imagine that the government subsidizes investment in innovation at rate $s_t$. With this subsidy in place, firms' investment problem (11) is replaced by

$$\sum_{t=0}^{\infty} \frac{(\beta\eta)^t}{P_t C_t} \left( P_t \varpi A_{j,t} Z_t L_t - (1-s_t)\eta P_t I_{j,t} \right), \tag{35}$$

where $s_t$ denotes the subsidy. The subsidy is financed by a lump-sum tax on firms. As a result, the growth equation (20) is now given by

$$\frac{1-s_t}{\chi} = \beta \frac{c_t}{c_{t+1} g_{t+1}} \left( \varpi Z_{t+1} L_{t+1} + \eta \left( \frac{1-s_{t+1}}{\chi} \right) \right). \tag{36}$$

Naturally, a rise in the subsidy to innovation (i.e. an increase in $s_t$) leads firms to invest more which generates faster productivity growth.

## 5.1 Optimal monetary and innovation policies

We start by considering a scenario in which both monetary and innovation policies are chosen optimally by the government. Intuitively, the model embeds two sources of inefficiencies that these policy tools can correct. First, as it should be clear by now, it is inefficient to let employment deviate from its natural level $\bar{L}$. Second, the presence of knowledge spillovers means that firms do not fully appropriate the social return from investing in innovation, leading to an inefficiently low rate of productivity growth under laissez faire. This second source of inefficiency is typical of endogenous growth models.

When both monetary and innovation policies are set optimally, a clear separation of roles emerges. Monetary policy acts so as to close the output gap, by guaranteeing that employment is always equal to its efficient level. Innovation policies, instead, correct the knowledge externalities by subsidizing investment in innovation and thereby raising the growth rate of the economy. We summarize this result in the following proposition.

**Proposition 4** *Assume that fiscal and monetary policy operate under discretion, choosing $s_t$ and $i_t$ to maximize households' expected utility, subject to (19), (21), (36) and $L_t \leq \bar{L}$ for all $t \geq 0$.*

*The policy rate $i_t$ is adjusted so that $L_t = \bar{L}$ for all $t \geq 0$, while the subsidy $s_t$ is adjusted to ensure that $g^{op}_{t+1} \equiv \beta(\chi \Psi Z_t \bar{L} + 1)$ for all $t \geq 0$.*

Using the results stated in Proposition 4, one can show that the investment subsidy evolves according to

$$s_t = 1 - \frac{\chi \varpi Z_{t+1} \bar{L} + \eta(1 - s_{t+1})}{\chi \Psi Z_{t+1} \bar{L} + 1}.$$

Since $\varpi < \Psi$ and $\eta < 1$, one can then see that the optimal innovation policy entails a positive subsidy in every period - in order to induce firms to internalize the knowledge externalities that they produce.

### 5.2 Innovation policies as demand-stabilization tools

Innovation policies can also be used as demand-stabilization tools, when monetary policy deviates from its optimal stance. To illustrate this point, we study a simple but insightful case. Imagine that the central bank fixes the interest rate to its value in the steady state with optimal innovation policies (i.e. $i_t = \chi \Psi \bar{L}$ for all $t \geq 0$). This corresponds to an extreme case of imperfect monetary stabilization, as monetary policy fails completely to react to the business cycle. The optimal innovation policy then takes a very simple form.

**Proposition 5** *Consider a government that operates under discretion, choosing $s_t$ to maximize households' expected utility, subject to (19), (21), (36), as well as $i_t = \chi \Psi \bar{L}$ and $L_t \leq \bar{L}$ for all $t \geq 0$. The solution to this problem satisfies $L_t = \bar{L}$ for all $t \geq 0$.*

The key insight stated by Proposition 5 is that, as long as innovation policies are set optimally, the economy operates at full employment even if monetary policy holds the policy rate fixed. Consider the response of the economy to a supply disruption. Since monetary policy does not react to the shock, in absence of a fiscal response the economy would end up experiencing involuntary unemployment. Under the optimal policy, however, the government reacts by increasing the subsidy to firms' investment. This intervention stimulates aggregate demand through two channels. First, an increase in the subsidy induces firms to invest more, which directly increases demand for the final good. But more investment also crowds in consumption. The reason is that higher investment leads to faster productivity growth, generating a positive wealth effect which boosts households' demand for consumption. In fact, as highlighted by Proposition 5, it is optimal for the government to subsidize investment until demand is strong enough to support full employment.[24]

In light of the reasoning above, it is quite simple to understand why under the optimal innovation policy the economy always operates at full employment. When the output gap is negative,

[24] Things would be a bit different if we moved away from the case where monetary policy keeps the interest rate fixed at $\chi \Psi \bar{L}$. For example, under a Taylor rule, the central bank would react to the rise in employment triggered by the subsidy by increasing the interest rate. The rise in the interest rate, in turn, would depress demand for consumption. So the impact of a rise in the subsidy on consumption is ambiguous in this case. For this reason, it might not be optimal for the government to always keep the economy at full employment with the subsidy. Detailed results are available upon request.

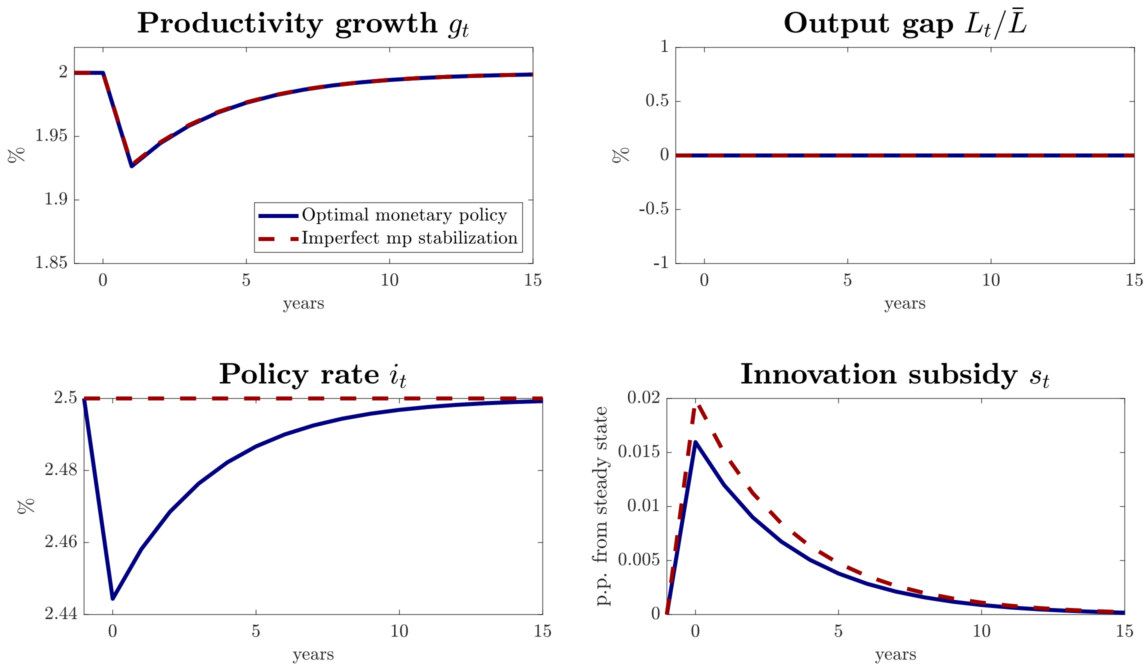



**Figure 5: Optimal innovation policies.** Notes. Blue solid: jointly-optimal monetary and innovation policies. Red dashed: optimal innovation policies under imperfect monetary stabilization.

a rise in the subsidy not only leads to an increase in investment, but also crowds in consumption. Therefore, subsidizing investment until the economy reaches full employment maximizes both investment and consumption. Since there is no disutility from working, this strategy delivers the highest utility.

Figure 5 illustrates these results graphically, by tracing the impact of a supply disruption when both monetary and innovation policies are run optimally (solid lines), and when monetary policy holds the policy rate fixed (dashed lines).[25] As mentioned above, under the optimal monetary policy the central bank lowers the interest rate to support demand at a level consistent with full employment. When the interest rate is held constant, instead, the same outcome is achieved through a rise in the subsidy to innovation. Interestingly, even a small rise in the subsidy can have a sizable impact on aggregate demand and employment (notice the small difference in the path of productivity growth and the subsidy between the two economies). This happens because the rise in investment stimulates consumers' demand and reverses the supply-demand doom loop.

To conclude this section, we briefly consider how innovation policies should be designed when monetary policy is run optimally, but might be constrained by the zero lower bound. We leave the technical derivations of this case to Appendix B.6, and here we just outline the main results from this analysis. When the zero lower bound does not bind, monetary policy keeps the economy at full employment, while innovation policies correct for the knowledge externalities. When the zero lower bound binds, however, the government increases the subsidy above the level needed to correct for the knowledge externalities, in order to stimulate aggregate demand and maintain the

[25] For this exercise, we set $\chi$ so as to generate a steady state growth rate of 2% under optimal innovation subsidies. All the other parameters are kept as in Section 3.

economy at full employment. The model thus suggests an active role for innovation policies to stimulate demand during liquidity traps.

Taking stock, innovation policies - a typical supply side intervention - can play a role in complementing monetary policy as an aggregate demand management tool. This property of innovation policies becomes particularly useful when monetary policy cannot perfectly stabilize demand, for instance during liquidity traps. These results suggest that supply side policies aiming at boosting future potential output, such as Biden's infrastructure plan, can have important effects both on the supply and on the demand side of the economy.

## 6 Conclusion

In this paper, we have revisited the macroeconomic implications of supply disruptions, through the lens of a Keynesian growth framework. In our model, negative supply shocks generate very persistent - or even permanent - drops in GDP below its pre-shock trend. These scars of supply shocks depress aggregate demand, which might even fall more than supply. In this case, the natural interest rate drops and monetary policy should expand to fight Keynesian unemployment. Innovation policies aiming at fostering firms' investment and productivity growth can usefully complement monetary policy, especially during liquidity traps.

We conclude by pointing out two avenues for future research left open by this paper. First, as mentioned in the introduction, a recent literature has emphasized different channels through which negative supply shocks can lead to more than proportional falls in demand. It would be interesting to study the extent to which these channels interact and amplify each other. Second, this paper shows that studying jointly business cycles and endogenous growth might change radically our view of the macroeconomic implications of supply shocks. An open question is whether this result applies to other sources of macroeconomic fluctuations as well. We hope that the Keynesian growth framework presented in this paper will provide a useful tool for future research, aiming at shedding light on these crucial issues.

# Appendix

## A Proofs of all propositions

### A.1 Proof of Proposition 1

**Proposition 1** *Suppose that the parameters satisfy*

$$\beta(\chi\varpi\bar{L}+\eta)>1,$$

*and that monetary policy is such that*

$$1 + \bar{i} = \chi\varpi\bar{L} + \eta.$$

*Then there exists a unique full employment steady state. Moreover, this steady state is characterized by $\bar{g} > 1$.*

**Proof.** The fact that growth is positive follows directly from the first condition in the proposition, because growth in the full employment steady state is given by $\bar{g} = \beta(\chi\varpi\bar{L} + \eta)$. Moreover, the assumption $1+\bar{i} = \bar{g}/\beta$ ensures that (19) is consistent with the full employment steady state. Note next that, by using (21),

$$\bar{c} = \Psi\bar{L} - \frac{\bar{g}-1}{\chi} = \Psi\bar{L} - \frac{\beta(\chi\varpi\bar{L}+\eta)-1}{\chi} = \bar{L}(\Psi - \beta\varpi) + \frac{1-\beta\eta}{\chi}.$$

Due to $\beta < 1$, $\eta \leq 1$ and $\varpi < \Psi$, this expression implies that steady state consumption is positive. To see that $\varpi < \Psi$, note that $\varpi/\Psi = \alpha/(1+\alpha) < 1$. These arguments also directly imply that the full employment steady state is unique. ■

### A.2 Proof of Proposition 2

**Proposition 2** *Consider a central bank that operates under discretion and maximizes households' expected utility, subject to (19), (20), (21) and $L_t \leq \bar{L}$. The solution to this problem satisfies $L_t = \bar{L}$ for all $t \geq 0$.*

**Proof.** Under discretion, in every period the central bank maximizes the representative household expected utility subject to the resource constraint of the economy, households' Euler equation and firms' optimality condition for investment, taking future variables as given. The period $t$ problem of the central bank can be written as:

$$\max_{\{L_t, c_t, g_{t+1}, i_t\}} \left\{ \sum_{\tau=t}^{\infty} \beta^{\tau} \log C_{\tau} = \beta^t \left( \frac{1}{1-\beta} \log A_t + \log c_t + \frac{\beta}{1-\beta} \log g_{t+1} \right) + \varsigma_t^1 \right\},$$

subject to

$$\Psi Z_t L_t = c_t + \frac{g_{t+1}-1}{\chi}$$

$$c_t = \frac{g_{t+1}}{1+i_t}\varsigma_t^2$$

$$1 + i_t = \varsigma_t^3$$

$$L_t \leq \bar{L}$$

and

$$\varsigma_t^1 = \sum_{\tau=t+1}^{\infty} \beta^{\tau} \log\left(c_{\tau} \Pi_{\hat{\tau}=t+2}^{\tau} g_{\hat{\tau}}\right)$$

$$\varsigma_t^2 = \frac{1}{\beta c_{t+1}^{-1}}$$

$$\varsigma_t^3 = \chi \varpi Z_{t+1} L_{t+1} + \eta.$$

The variables $\varsigma_t^1$, $\varsigma_t^2$ and $\varsigma_t^3$ are taken as given by the central bank, because they are a function of parameters and expectations about future variables only.

Notice that the objective function is strictly increasing in $c_t$ and $g_{t+1}$. Also notice that, from the first, second and third constraints, we can write $c_t = c(L_t)$ with $c'(L_t) > 0$ and $g_{t+1} = g(L_t)$ with $g'(L_t) > 0$. It follows that $L_t = \bar{L}$ under the optimal policy. ■

### A.3 Proof of Proposition 3

**Proposition 3** *Assume that $Z_t$ is governed by the process (24), and that $Z_0 < 1$. If $\rho > 0$, the path of interest rates that implements the allocation under optimal policy is characterized by $i_t < \bar{i}$ for all $t \geq 0$. If $\rho = 0$, optimal policy implies $i_t = \bar{i}$ for all $t \geq 0$.*

**Proof.** From Proposition 2 we know that $L_t = \bar{L}$ at all times under the optimal policy. In this case the system of equations (19)-(21) collapses to

$$c_t = \frac{g_{t+1} c_{t+1}}{\beta(1 + i_t)},$$

$$g_{t+1} = \beta \frac{c_t}{c_{t+1}} \left( \chi \varpi Z_{t+1} \bar{L} + \eta \right).$$

$$\Psi Z_t \bar{L} = c_t + \frac{g_{t+1} - 1}{\chi}.$$

Combining the first two equations reveals that

$$1 + i_t = \chi \varpi Z_{t+1} \bar{L} + \eta.$$

Using the process (24) and $Z_0 < 1$, we can write

$$1 + i_t = \chi \varpi Z_0^{\rho^{t+1}} \bar{L} + \eta. \tag{A.1}$$

For $\rho > 0$ ($\rho = 0$), this expression implies that $i_t < \bar{i}$ ($i_t = \bar{i}$) for all $t \geq 0$. ■

### A.4 Proof of Proposition 4

**Proposition 4** *Assume that fiscal and monetary policy operate under discretion, choosing $s_t$ and $i_t$ to maximize households' expected utility, subject to (19), (21), (36) and $L_t \leq \bar{L}$ for all $t \geq 0$. The policy rate $i_t$ is adjusted so that $L_t = \bar{L}$ for all $t \geq 0$, while the subsidy $s_t$ is adjusted to ensure that $g_{t+1}^{op} \equiv \beta(\chi \Psi Z_t \bar{L} + 1)$ for all $t \geq 0$.*

**Proof.** Under discretion, in every period the government maximizes the representative household expected utility subject to the resource constraint of the economy, households' Euler equation and firms' optimality condition for investment, taking future variables as given. Because $s_t$ and $i_t$ are both chosen freely, the Euler equation and the first order condition for investment do not enter as constraints in the maximization.

The period $t$ problem of the government can thus be written as (compare the proof of Proposition 2 in Appendix A.2):

$$\max_{\{L_t,c_t,g_{t+1}\}}\left\{\sum_{\tau=t}^{\infty}\beta^{\tau}\log C_{\tau}=\beta^{t}\left(\frac{1}{1-\beta}\log A_t+\log c_t+\frac{\beta}{1-\beta}\log g_{t+1}\right)+\varsigma_t\right\},$$

subject to

$$\Psi Z_t L_t = c_t + \frac{g_{t+1}-1}{\chi}$$

$$L_t \leq \bar{L}$$

where the variable $\varsigma_t$, is given by

$$\varsigma_t = \sum_{\tau=t+1}^{\infty}\beta^{\tau}\log\left(c_{\tau}\Pi_{\hat{\tau}=t+2}^{\tau}g_{\hat{\tau}}\right).$$

The variable $\varsigma_t$ is taken as given by the government, because it is a function of parameters and expectations about future variables only.

Because the objective is increasing in both $c_t$ and $g_{t+1}$, and because both variables depend positively on $L_t$ in the resource constraint, it is optimal to set $L_t = \bar{L}$. To obtain the optimal level of growth, we set up the Lagrangian

$$\mathcal{L}_t = \left(\frac{1}{1-\beta}\log A_t+\log c_t+\frac{\beta}{1-\beta}\log g_{t+1}\right)+\varsigma_t^1+\lambda_t\left(\Psi Z_t\bar{L}-c_t-\frac{g_{t+1}-1}{\chi}\right).$$

The first order conditions are

$$\frac{1}{c_t}-\lambda_t = 0.$$

$$\frac{\beta}{1-\beta}\frac{1}{g_{t+1}}-\frac{\lambda_t}{\chi}=0.$$

Combining both to eliminate the multiplier reveals that $g_{t+1}^{opt} = \beta(\chi\Psi Z_t\bar{L}+1)$.

To see the separation of roles between monetary and innovation policies, consider that when innovation policies are equal to zero, monetary policy continues to close the output gap (see Proposition 2). Therefore, the addition of innovation subsidies to the policy problem has the effect of raising the growth rate in the economy. ■

## A.5 Proof of Proposition 5

**Proposition 5** *Consider a government that operates under discretion, choosing $s_t$ to maximize households' expected utility, subject to (19), (21), (36), as well as $i_t = \chi\Psi\bar{L}$ and $L_t \leq \bar{L}$ for all $t \geq 0$. The solution to this problem satisfies $L_t = \bar{L}$ for all $t \geq 0$.*

**Proof.** The problem of the fiscal authority is given by

$$\max_{\{L_t, c_t, g_{t+1}\}} \left\{ \sum_{\tau=t}^{\infty} \beta^{\tau} \log C_{\tau} = \beta^t \left( \frac{1}{1-\beta} \log A_t + \log c_t + \frac{\beta}{1-\beta} \log g_{t+1} \right) + \varsigma_t^1 \right\},$$

subject to

$$\Psi Z_t L_t = c_t + \frac{g_{t+1} - 1}{\chi}$$

$$c_t = \frac{g_{t+1}}{1 + \chi \Psi \bar{L}} \varsigma_t^2$$

$$L_t \leq \bar{L}$$

and

$$\varsigma_t^1 = \sum_{\tau=t+1}^{\infty} \beta^{\tau} \log \left( c_{\tau} \Pi_{\hat{\tau}=t+2}^{\tau} g_{\hat{\tau}} \right)$$

$$\varsigma_t^2 = \frac{1}{\beta c_{t+1}^{-1}}.$$

The variables $\varsigma_t^1$ and $\varsigma_t^2$ are taken as given by the fiscal authority, because they are a function of parameters and expectations about future variables only. In this problem, we have omitted (36) from the set of constraints because $s_t$ can be chosen freely by the policy maker.

For given $L_t$, the first two constraints already determine the equilibrium $c_t$ and $g_{t+1}$, as they constitute a system of two equations in two unknowns. Moreover, the first constraint defines a negative relationship between $c_t$ and $g_{t+1}$, whereas the second constraint defines a positive relationship. At the intersection, both $c_t$ and $g_{t+1}$ must therefore increase for a given increase in $L_t$, as this shifts the first constraint to the right. Because the objective is strictly increasing in $c_t$ and $g_{t+1}$, the optimal point is therefore reached when $L_t = \bar{L}$. This establishes that innovation policies are such that the output gap is closed at all times.

By combing the two constraints, and using the definition of $\varsigma_t^2$, we can also derive an expression for the implied growth rate of productivity

$$g_{t+1}^{ip} = g_{t+1}^{op} \left( 1 + \frac{\chi (c_{t+1} - c^{op})}{1 + \chi \Psi \bar{L}} \right)^{-1}, \tag{A.2}$$

where $g_{t+1}^{ip}$ denotes the growth rate under optimal innovation policies, $g_{t+1}^{op}$ denotes growth under optimal monetary and innovation policies (see Proposition 4) and where $c^{op} = \Psi \bar{L} - (g^{op} - 1)/\chi$ denotes steady state consumption under optimal monetary and innovation policies. What equation (A.2) reveals is that growth increases above the level under optimal monetary and innovation policies whenever consumption is expected to be below the steady state next period ($c_{t+1} < c^{op}$). Intuitively, an expected recession reduces today's consumption demand. In the absence of a monetary policy response, the economy would be characterized by involuntary unemployment. To raise demand, as we explain in more detail in Section 5.2, innovation subsidies are increased and growth is higher as this crowds in consumption. ■

# B Additional derivations

## B.1 Investment first order condition

In this Appendix we derive the investment first order condition (12). Firms producing intermediate goods choose investment in innovation to maximize

$$\sum_{t=0}^{\infty} \frac{(\beta\eta)^t}{P_t C_t} \left( P_t \varpi A_{j,t} Z_t L_t - \eta P_t I_{j,t} \right),$$

subject to

$$A_{j,t+1} = A_{j,t} + \chi I_{j,t}$$

$$I_{j,t} \geq 0,$$

given the initial condition $A_{j,0}$. The last constraint takes into account the fact that investment cannot be negative.

We define the following Lagrangian

$$\mathcal{L} = \sum_{t=0}^{\infty} \frac{(\beta\eta)^t}{P_t C_t} (P_t \varpi A_{j,t} Z_t L_t - \eta P_t I_{j,t}(1 - \chi \upsilon_{j,t})) + \gamma_t (A_{j,t+1} - A_{j,t} - \chi I_{j,t}).$$

Optimal investment satisfies

$$\frac{1}{\chi} - \upsilon_{j,t} = \beta \frac{C_t}{C_{t+1}} \left( \varpi Z_{t+1} L_{t+1} + \eta \left( \frac{1}{\chi} - \upsilon_{j,t+1} \right) \right)$$

$$\upsilon_{j,t} I_{j,t} = 0,$$

where $\upsilon_{j,t} \geq 0$.

Notice that if investment is always positive ($\upsilon_{j,t} = 0$ for all $t \geq 0$) the optimality condition reduces to equation (12). However, investing might not be profitable for firms. This happens if the marginal increase in profits obtained from investing is lower than its marginal cost. For instance, this happens in case the zero lower bound on the nominal interest rate becomes a binding constraint - a case analyzed in Appendices B.4 and B.6.

## B.2 Model with diminishing returns to investment

In this Appendix we study the case where firms face diminishing returns to investment. As discussed in Section 3, our key result is that in this case the natural interest rate falls as long as the supply disruption is sufficiently persistent.

Firms' investment technology is now

$$A_{j,t+1} = A_{j,t} + \chi I_{j,t}^{\xi} A_t^{1-\xi}, \tag{B.1}$$

where $0 < \xi \leq 1$ and where aggregate technology $A_t = \int_0^1 A_{j,t} dj$ is taken as given by the individual firm. Under this formulation, firms combine investment and the aggregate stock of knowledge to increase their future productivity.[26] This is a simple way to introduce diminishing returns from investment in innovation.

Firms choose investment in innovation to maximize their expected profits

$$\sum_{t=0}^{\infty} \frac{(\beta\eta)^t}{P_t C_t} \left(P_t \varpi A_{j,t} Z_t L_t - \eta P_t I_{j,t}\right),$$

subject to (B.1). The non-negativity constraint on investment $I_{j,t} \geq 0$ never binds in equilibrium, since the return to investment becomes infinity as $I_{j,t}$ approaches zero. The Lagrangian becomes

$$\mathcal{L} = \sum_{t=0}^{\infty} \frac{(\beta\eta)^t}{P_t C_t}(P_t \varpi A_{j,t} Z_t L_t - \eta P_t I_{j,t}) + \gamma_t(A_{j,t+1} - A_{j,t} - \chi I_{j,t}^{\xi} A_t^{1-\xi}).$$

The optimality condition for investment is

$$\frac{1}{\chi\xi}\left(\frac{I_{j,t}}{A_t}\right)^{1-\xi} = \beta \frac{C_t}{C_{t+1}}\left(\varpi Z_{t+1} L_{t+1} + \frac{\eta}{\chi\xi}\left(\frac{I_{j,t+1}}{A_{t+1}}\right)^{1-\xi}\right).$$

As in the baseline model, the equilibrium can be summarized by three equations. The first one is households' Euler equation (19). The growth equation (20) now becomes

$$\left(\frac{g_{t+1}-1}{\chi}\right)^{\frac{1-\xi}{\xi}} g_{t+1} = \beta \frac{c_t}{c_{t+1}}\left(\chi\xi\varpi Z_{t+1} L_{t+1} + \eta\left(\frac{g_{t+2}-1}{\chi}\right)^{\frac{1-\xi}{\xi}}\right). \tag{B.2}$$

Moreover, the market clearing condition (21) is replaced by

$$\Psi Z_t L_t = c_t + \left(\frac{g_{t+1}-1}{\chi}\right)^{\frac{1}{\xi}}. \tag{B.3}$$

These expressions generalize the ones from the baseline model to the case $0 < \xi < 1$.

We are interested in deriving the path of the interest rate under the optimal monetary policy. As in the baseline model, under the optimal policy the economy always operates at full employment ($L_t = \bar{L}$ for all $t \geq 0$), and the path of the interest rate corresponds to the one of the natural rate. To make progress, let us take a log-linear approximation of (B.2)-(B.3) around the full employment steady state to obtain

$$\bar{g}\left(\left[\frac{1-\xi}{\xi}\frac{\bar{g}}{\bar{g}-1}+1\right]\hat{g}_{t+1} - \hat{c}_t + E_t\hat{c}_{t+1}\right) = (\bar{g}-\beta\eta)\hat{Z}_{t+1} + \beta\eta\frac{1-\xi}{\xi}\frac{\bar{g}}{\bar{g}-1}\hat{g}_{t+2}.$$

$$\hat{Z}_t = s_c\hat{c}_t + (1-s_c)\frac{1}{\xi}\frac{\bar{g}}{\bar{g}-1}\hat{g}_{t+1},$$

where $s_c \equiv \bar{c}/\Psi\bar{L}$ is the consumption share of GDP in steady state. Here, $\hat{x}_t \equiv \log(x_t) - \log(\bar{x})$ for

[26] Assuming that firms combine investment with their individual stock of knowledge would not change any of the results that follow.

every variable $x_t$

Using $\hat{Z}_{t+1} = \rho \hat{Z}_t$, we can guess and verify that the solution to the model can be written as

$$\hat{c}_t = \gamma_c \hat{Z}_t$$

$$\hat{g}_{t+1} = \gamma_g \hat{Z}_t,$$

where the two parameters $\gamma_c$ and $\gamma_g$ are given by

$$\gamma_c = \frac{1}{s_c} \frac{\bar{g} + \frac{\bar{g}}{\bar{g}-1}\left(\frac{1-\xi}{\xi}(\bar{g} - \beta\eta\rho) - \frac{1-s_c}{\xi}(\bar{g} - \beta\eta)\rho\right)}{\bar{g} + \frac{\bar{g}}{\bar{g}-1}\left(\frac{1-\xi}{\xi}(\bar{g} - \beta\eta\rho) + \frac{1}{\xi}\bar{g}(1-\rho)\frac{1-s_c}{s_c}\right)}$$

$$\gamma_g = \frac{(\bar{g} - \beta\eta)\rho + \frac{\bar{g}(1-\rho)}{s_c}}{\bar{g} + \frac{\bar{g}}{\bar{g}-1}\left[\frac{1-\xi}{\xi}(\bar{g} - \beta\eta\rho) + \frac{1}{\xi}\bar{g}(1-\rho)\frac{1-s_c}{s_c}\right]}.$$

Log-linearizing (19) yields an equation for the interest rate

$$\hat{c}_t = \hat{g}_{t+1} - \hat{i}_t + \hat{c}_{t+1}.$$

Inserting the solution for $\hat{c}_t$ and $\hat{g}_{t+1}$, and rearranging we then obtain

$$\begin{aligned} \hat{i}_t &= (\gamma_g - (1-\rho)\gamma_c)\hat{Z}_t \\ &= \frac{(\bar{g} - \beta\eta)\rho + \frac{\bar{g}(1-\rho)}{s_c} - (1-\rho)\frac{1}{s_c}\left(\bar{g} + \frac{\bar{g}}{\bar{g}-1}\left[\frac{1-\xi}{\xi}(\bar{g} - \beta\eta\rho) - \frac{1-s_c}{\xi}(\bar{g} - \beta\eta)\rho\right]\right)}{\bar{g} + \frac{\bar{g}}{\bar{g}-1}\left[\frac{1-\xi}{\xi}(\bar{g} - \beta\eta\rho) + \frac{1}{\xi}\bar{g}(1-\rho)\frac{1-s_c}{s_c}\right]}\hat{Z}_t. \end{aligned}$$

Manipulating the expression above shows that the natural interest rate falls following a negative supply shock if and only if

$$\xi > \frac{\rho(\bar{g} - \beta\eta) + (1-\rho)\frac{1}{s_c}\bar{g}}{\frac{1}{s_c}(\bar{g} - \beta\eta\rho) + \frac{\rho}{1-\rho}\frac{\bar{g}-1}{\bar{g}}(\bar{g} - \beta\eta)} \equiv \bar{\xi}. \tag{B.4}$$

Moreover, differentiating the expression above with respect to $\rho$ gives

$$\frac{\partial \bar{\xi}}{\partial \rho} = -\frac{(\bar{g} - \beta\eta)\left(\frac{1-s_c}{s_c^2}\bar{g} + \frac{1}{1-\rho}\frac{\bar{g}-1}{\bar{g}}\left[\frac{\rho^2}{1-\rho}(\bar{g} - \beta\eta) + \frac{1}{s_c}\bar{g}(1+\rho)\right]\right)}{\left(\frac{1}{s_c}(\bar{g} - \beta\eta\rho) + \frac{\rho}{1-\rho}\frac{\bar{g}-1}{\bar{g}}(\bar{g} - \beta\eta)\right)^2} < 0.$$

This means that, the stronger the diminishing returns to investment (i.e. the lower $\xi$), the larger the shock persistence must be (i.e. the higher $\rho$) for the interest rate to fall following a negative supply shock.

It is instructive to look at two limiting cases. First, assume that the shock is purely transitory ($\rho = 0$). In this case, (B.4) reduces to $\bar{\xi} = 1$. From (B.1), this implies that when the shock is purely transitory, the natural rate unambiguously rises for any degree of curvature $\xi < 1$. The second limit case is the one of permanent shocks, in which case $\rho = 1$. In this case, (B.4) implies that $\bar{\xi} = 0$. Therefore, for permanent shocks the natural rate unambiguously declines for any degree of curvature $\xi < 1$.

In sum, when firms face diminishing returns to investment, the negative supply shocks must be persistent enough for the natural rate to decline after the shock.

### B.3 Determinacy of the model with interest rate rule

Here we establish that in the baseline model with interest rate rule (28), the full employment steady state is locally determinate under condition (29). We therefore complement the analysis from Section 4.

The baseline model with interest rate rule can be summarized by the following set of equations

$$c_t = \frac{g_{t+1}}{\beta c_{t+1}^{-1}} \frac{\bar{\pi}}{(1+\bar{i})\left(\frac{L_t}{\bar{L}}\right)^{\phi}}.$$

$$g_{t+1} = \beta \frac{c_t}{c_{t+1}} \left(\chi \varpi Z_{t+1} L_{t+1} + \eta\right).$$

$$\Psi Z_t L_t = c_t + \frac{g_{t+1} - 1}{\chi}.$$

where the first equation is the combination of (19) and (28).

To show that the steady state is locally determinate, we take a log-linear approximation[27]

$$\hat{g}_{t+1} = \phi \hat{L}_t + \hat{c}_t - \hat{c}_{t+1}$$

$$\Psi \bar{L} \hat{L}_t = \bar{c}\hat{c}_t + \frac{\bar{g}}{\chi}\hat{g}_{t+1}$$

$$\hat{g}_{t+1} = \hat{c}_t - \hat{c}_{t+1} + \frac{\bar{g} - \beta\eta}{\bar{g}}\hat{L}_{t+1}$$

where $\hat{x}_t \equiv \log(x_t) - \log(\bar{x})$ for every variable $x_t$. This system can be written as:

$$\hat{L}_t = \xi_1 \hat{L}_{t+1} + \xi_2 \hat{g}_{t+2}$$

$$\hat{g}_{t+1} = \xi_3 \hat{L}_{t+1} + \xi_4 \hat{g}_{t+2},$$

where

$$\xi_1 \equiv \frac{1}{\phi}\frac{\bar{g} - \beta\eta}{\bar{g}}$$

$$\xi_2 \equiv 0$$

$$\xi_3 \equiv \frac{\bar{g} - 1}{\bar{g}\frac{\Psi\bar{L}}{\bar{c}} - 1}\left(\phi\xi_1 + \frac{\Psi\bar{L}}{\bar{c}}(\xi_1 - 1)\right)$$

[27] In the approximation, we keep $Z_t$ fixed at its steady state value $Z_t = 1$ as variation in this variable is irrelevant for the determinacy properties of the dynamic system.

$$\xi_4 \equiv \frac{\bar{g}\left(1 - \frac{\bar{c}}{\Psi \bar{L}}\right)}{\bar{g} - \frac{\bar{c}}{\Psi \bar{L}}}.$$

The system is determinate if and only if:[28]

$$|\xi_1 \xi_4 - \xi_2 \xi_3| < 1 \tag{B.5}$$

$$|\xi_1 + \xi_4| < 1 + \xi_1 \xi_4 - \xi_2 \xi_3. \tag{B.6}$$

Condition (B.5) holds if

$$\phi > \frac{(\bar{g} - \beta\eta)\left(1 - \frac{\bar{c}}{\Psi \bar{L}}\right)}{\bar{g} - \frac{\bar{c}}{\Psi \bar{L}}},$$

while condition (B.6) holds if

$$\phi > \frac{\bar{g} - \beta\eta}{\bar{g}}. \tag{B.7}$$

Because $\bar{g} > 1$, inserting $\bar{g} = \beta(\chi \varpi \bar{L} + \eta)$ in (B.7) shows that the steady state is locally determinate if and only if condition (29) holds.

### B.4 Monetary policy at the zero lower bound

In this Appendix we study optimal monetary policy in the baseline model by taking explicit account of the zero lower bound constraint.

In Appendix B.1, we show that by taking explicit account of the non-negativity constraint on investment, the investment first order condition is given by

$$\frac{1}{\chi} - \upsilon_t = \beta \frac{c_t}{c_{t+1} g_{t+1}} \left( \varpi Z_{t+1} L_{t+1} + \eta \left( \frac{1}{\chi} - \upsilon_{t+1} \right) \right) \tag{B.8}$$

$$\upsilon_t (g_{t+1} - 1) = 0, \tag{B.9}$$

where $\upsilon_t \geq 0$. In these expressions, $\upsilon_t$ denotes the non-negative multiplier associated with the non-negativity constraint for investment $g_{t+1} \geq 1$.

Following the same steps as Appendix A.2, the problem of the monetary authority under optimal discretion is given by

$$\max_{\{L_t, c_t, g_{t+1}, i_t\}} \left\{ \sum_{\tau=t}^{\infty} \beta^{\tau} \log C_{\tau} = \beta^t \left( \frac{1}{1-\beta} \log A_t + \log c_t + \frac{\beta}{1-\beta} \log g_{t+1} \right) + \varsigma_t^1 \right\},$$

subject to

$$\Psi Z_t L_t = c_t + \frac{g_{t+1} - 1}{\chi}$$

$$c_t = \frac{g_{t+1}}{1 + i_t} \varsigma_t^2$$

[28] See Bullard and Mitra (2002).

$$g_{t+1} = \begin{cases} [1, 1 + \chi \Psi Z_t L_t] & \text{if } 1 + i_t = \varsigma_t^3 \\ 1 & \text{if } 1 + i_t > \varsigma_t^3 \end{cases}$$

$$L_t \leq \bar{L}$$

$$i_t \geq 0$$

as well as

$$\varsigma_t^1 = \sum_{\tau=t+1}^{\infty} \beta^\tau \log \left( c_\tau \Pi_{\hat{\tau}=t+2}^{\tau} g_{\hat{\tau}} \right)$$

$$\varsigma_t^2 = \frac{1}{\beta c_{t+1}^{-1}}$$

$$\varsigma_t^3 = \chi \varpi Z_{t+1} L_{t+1} + \eta \left(1 - \chi \upsilon_{t+1}\right).$$

$\varsigma_t^1$, $\varsigma_t^2$ and $\varsigma_t^3$ are taken as given by the central bank, because they are a function of parameters and expectations about future variables only.

The third constraint captures the fact that investment might be unprofitable for firms. It is the combination of (19) and (B.8)-(B.9). When $1 + i_t = \varsigma_t^3$, then $\upsilon_t = 0$ such that firms are willing to undertake any amount of investment. Instead, when $1 + i_t > \varsigma_t^3$, then it must be the case that $\upsilon_t > 0$ such that $g_{t+1} = 1$.[29]

Notice that the objective function is strictly increasing in $c_t$ and $g_{t+1}$. Also notice that, from the second and third constraints, we can write $c_t = c(i_t)$ with $c'(i_t) < 0$ and $g_{t+1} = g(i_t)$ with $g'(i_t) \leq 0$. We can thus rewrite the problem of a central bank under discretion as

$$\min_{\{L_t, i_t\}} \; i_t,$$

subject to

$$\Psi Z_t L_t = c(i_t) + \frac{g(i_t) - 1}{\chi}$$

as well as the two inequality constraints

$$L_t \leq \bar{L}$$

$$i_t \geq 0.$$

In case the zero lower bound is slack, the solution to this problem is $L_t = \bar{L}$ as well as

$$1 + i_t = \max\left( \frac{\varsigma_t^2}{\Psi Z_t \bar{L}}, \varsigma_t^3 \right) \equiv 1 + i_t^*, \tag{B.10}$$

where we define $1 + i_t^*$ as the interest rate consistent with full employment. Note that, compared with Appendix A.2, there are two terms appearing in the nominal rate consistent with full employment, as not necessarily $1 + i_t^* = \varsigma_t^3$. This difference arises, because here we take explicit account

[29]The case $1 + i_t < \varsigma_t^3$ can not be an equilibrium, for it would imply that firms' demand for investment becomes unbounded.

of the possibility that the investment constraint $g_{t+1} \geq 1$ may bind.

If instead $1 + i_t^* < 1$, the lowest possible interest rate is $i_t = 0$. Thus we can write

$$1 + i_t = \max(1 + i_t^*, 1). \tag{B.11}$$

Combining (B.10) and (B.11) reveals that, whenever the zero lower bound binds, $1 + i_t = 1 > \varsigma_t^3$, which from previous arguments implies that $g_{t+1} = 1$ when the zero lower bound binds. Inserting this in the resource constraint yields the equilibrium amount of employment at the zero lower bound

$$\Psi Z_t L_t = c(0)$$

where $c_t = c(0) = \varsigma_t^2$ denotes consumption at the zero lower bound. From equation (B.10), when $1 + i_t^* < 1$, we know that $\varsigma_t^2 < \Psi Z_t \bar{L}$, implying $c(0) = \Psi Z_t L_t < \Psi Z_t \bar{L}$. Therefore, the zero lower bound spell is necessarily associated with a negative output gap $L_t < \bar{L}$. This implies that the outcome of the policy problem can be represented by a complementary slackness condition:

$$i_t(L_t - \bar{L}) = 0.$$

## B.5 New Keynesian wage Phillips curve

Here we augment the baseline model by a New Keynesian wage Phillips curve. Specifically, we assume that households are monopolistically suppliers of labor. Moreover, we assume that wage changes entail Rotemberg (1982)-type adjustment costs.

### B.5.1 Model

We continue to assume that the economy is populated by a unit mass of households, however, we now make the household index specific: $k \in [0, 1]$. Each household $k$ has utility

$$\sum_{t=0}^{\infty} \beta^t \left( \log(C_t(k)) - G(L_t(k)) \right),$$

where $G(L_t(k))$ is positive, increasing and convex in labor supplied $L_t(k)$. The budget constraint is given by

$$P_t C_t(k) + \frac{B_{t+1}(k)}{1 + i_t} = W_t(k) L_t(k) + B_t(k) + D_t.$$

Households' Euler equation is as in the baseline model. We discuss households' labor supply choice below.

Final goods firms' production technology is still given by

$$Y_t = (Z_t L_t)^{1-\alpha} \int_0^1 A_{j,t}^{1-\alpha} x_{j,t}^{\alpha} dj.$$

However, labor demand $L_t$ is now a composite of the labor supplied by the different households

$k \in [0, 1]$

$$L_t = \left( \int_0^1 L_t(k)^{\frac{\varepsilon-1}{\varepsilon}} dk \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{B.12}$$

where $\varepsilon > 1$ denotes the elasticity of substitution among different labor varieties. Firms' optimal labor demand solves

$$\min_{\{L_t(k)\}_{k\in[0,1]}} \int_0^1 W_t(k) L_t(k) dk \quad \text{subject to (B.12)},$$

taking as given $\{W_t(k)\}_{k\in[0,1]}$. The first order condition is

$$L_t(k) = \left( \frac{W_t(k)}{W_t} \right)^{-\varepsilon} L_t. \tag{B.13}$$

We now turn to labor supply. Each household maximizes utility subject to household-specific labor demand (B.13), subject to the budget constraint and subject to a wage adjustment cost. The optimization problem is to maximize the following objective function:

$$\sum_{t=0}^{\infty} \beta^t \left( \frac{1}{P_t C_t(k)} \left( W_t(k) L_t(k) - \frac{\theta}{2} A_t P_t \left( \frac{W_t(k)}{W_{t-1}(k)} - \bar{g} \right)^2 \right) - G(L_t(k)) \right)$$

subject to (B.13), where $\theta \geq 0$ is the adjustment cost parameter. The adjustment cost is multiplied with $A_t P_t$ to ensure the existence of a balanced growth path. Moreover, we assume that wage inflation is indexed to the growth rate of productivity in the full employment steady state, $\bar{g}$. This ensures that the wage rigidities does not bind in steady state.

The first order condition is

$$\begin{aligned} \frac{1}{P_t C_t(k)} \left( (1-\varepsilon) L_t(k) - \theta A_t P_t \left( \frac{W_t(k)}{W_{t-1}(k)} - \bar{g} \right) \frac{1}{W_{t-1}(k)} \right) - G'(L_t(k))(-\varepsilon) \frac{L_t(k)}{W_t(k)} \\ + \beta \frac{1}{P_{t+1} C_{t+1}(k)} \theta A_{t+1} P_{t+1} \left( \frac{W_{t+1}(k)}{W_t(k)} - \bar{g} \right) \frac{W_{t+1}(k)}{W_t(k)^2} = 0. \end{aligned}$$

We now assume that $W_{-1}(k) = W_{-1}$, that is, all households face identical initial conditions. Because households face identical problems, this implies that in equilibrium, households make identical decisions. From now on, we thus omit the household index $k$.

Denoting nominal wage inflation $\pi_t^W$ we can write

$$\frac{A_t}{C_t} \theta (\pi_t^W - \bar{g}) \pi_t^W - \beta \frac{A_{t+1}}{C_{t+1}} \theta (\pi_{t+1}^W - \bar{g}) \pi_{t+1}^W = \varepsilon L_t \left( G'(L_t) - \frac{\varepsilon - 1}{\varepsilon} \frac{W_t}{P_t} \frac{1}{C_t} \right). \tag{B.14}$$

This wage Phillips curve replaces the reduced-form rule (16) in the main text. The rest of the model is as before, except that a resource loss due to the wage adjustment cost appears in the market clearing condition.

### B.5.2 Equilibrium

Given a path for $\{Z_t\}_{t=0}^{+\infty}$, an equilibrium is a set of processes $\{c_t, L_t, g_{t+1}, i_t, \pi_t, \pi_t^W\}_{t=0}^{+\infty}$ satisfying $g_{t+1} > 1$ as well as the following equations for all $t \geq 0$. Households' Euler equation

$$\frac{g_{t+1}}{c_t} = \beta(1+i_t)\frac{1}{c_{t+1}\pi_{t+1}},$$

an equation linking price inflation to firms' marginal costs

$$\pi_t^W = \frac{Z_t}{Z_{t-1}} g_t \pi_t,$$

the evolution of wage inflation, obtained by combining (B.14) and (5),

$$(\pi_t^W - \bar{g})\pi_t^W - \beta\frac{c_t}{c_{t+1}}(\pi_{t+1}^W - \bar{g})\pi_{t+1}^W = \frac{\varepsilon}{\theta} L_t \left( \frac{G'(L_t)}{c_t^{-1}} - \frac{\varepsilon - 1}{\varepsilon}(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} Z_t \right), \tag{B.15}$$

the growth equation

$$g_{t+1} = \beta\frac{c_t}{c_{t+1}}\left(\chi\varpi Z_{t+1}L_{t+1} + \eta\right),$$

the market clearing condition

$$\Psi Z_t L_t = c_t + \frac{g_{t+1}-1}{\chi} + \frac{\theta}{2}(\pi_t^W - \bar{g})^2, \tag{B.16}$$

which includes the resource loss term, plus a rule for monetary policy. The flexible-wage allocation is nested once $\theta \to 0$. Denoting flexible-wage variables with a bar, note that in contrast to the baseline model, the flexible-wage level of employment $\bar{L}_t$ is now time-varying.

### B.5.3 Steady state

In steady state, $Z = 1$. We again focus on the full employment steady state $L_t = \bar{L}$. Moreover, in steady state we normalize $\pi_t = 1$, implying that $\pi_t^W = \bar{g}$. The system of equations collapses to

$$\begin{aligned}
\bar{g} &= \beta(1+\bar{i}) \\
\frac{G'(\bar{L})}{\bar{c}^{-1}} &= \frac{\varepsilon-1}{\varepsilon}(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} \\
\bar{g} &= \beta\left(\chi\varpi\bar{L} + \eta\right), \\
\Psi\bar{L} &= \bar{c} + \frac{\bar{g}-1}{\chi}.
\end{aligned}$$

Relative to the baseline model, $\bar{L}$ is thus not exogenous but an endogenous variable, determined in the previous system of equations. More precisely, the previous system determines $\{\bar{c}, \bar{L}, \bar{g}, \bar{i}\}$, for given parameters.

### B.5.4 Numerical example

To illustrate the properties of this version of the model, we resort to numerical simulations. First we make sure that the baseline model and the model presented in this section share the same steady state. To do so, we assume that $G(\cdot)$ is given by the conventional constant-Frisch elasticity function, that is

$$G(L_t) = \zeta \frac{L_t^{1+\varphi}}{1+\varphi},$$

where $\varphi > 0$ is the inverse Frisch elasticity of labor supply, and where $\zeta > 0$ is a parameter. For given other parameters, we choose $\zeta$ such that the full employment steady state is characterized by the same $\bar{L}$ as in the baseline model. This implies that the steady state values for all other variables are also identical. We thus set the same parameters $\alpha, \beta, \chi$ and $\eta$ as in the baseline model, to hit the same targets in steady state.

Moreover, we choose 3 additional parameters related with the wage Phillips curve. We set the value of some parameters in the range of those commonly used by the literature (e.g., Galí, 2011). First we assume that $\varphi = 3$. Second, we set the elasticity of substitution between different labor types to $\varepsilon = 10$.

To calibrate the wage stickiness parameter, we start by drawing a parallel between the wage Phillips curve implied by our model and the one that would emerge under Calvo frictions in wage adjustment. Up to a first-order approximation, the slope of the wage Phillips curve (B.15) is given by

$$\frac{\varepsilon - 1}{\theta} \bar{L}(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}.$$

In turn, in a comparable Calvo framework, the slope of the Phillips curve is given by (see Born and Pfeifer (2020))

$$\frac{(1-\theta_c)(1-\beta\theta_c)}{\theta_c(1+\varepsilon\varphi)},$$

where $\theta_c$ is the Calvo-type wage stickiness parameter.

Our model is calibrated at yearly frequency. In the numerical simulation below, we contrast two levels of wage rigidities: a baseline level of wage rigidities versus a case where wages are somewhat more flexible. Our baseline level of wage rigidities is given by $\theta_c = 0.3164$. This implies a per-quarter stickiness probability of $0.3164^{1/4} = 0.75$, and thus an average duration of a wage contract of $1/(1-0.75) = 4$ quarters - or one year. This level of wage duration is in line with the empirical evidence (e.g., Olivei and Tenreyro, 2007). We then solve for the implied $\theta$ in order to match the two slopes of the Phillips curve. This procedure implies $\theta = 84.45$. Instead, in the case of more flexible wages we target a wage duration of three quarters. This implies $\theta_c = 0.1975$, and therefore $\theta = 38.3$.

### B.5.5 Numerical results

We want to highlight the role of wage rigidities in leading to dramatically different inflation responses to a negative supply shock. To do so, we proceed in the same way as in the main text

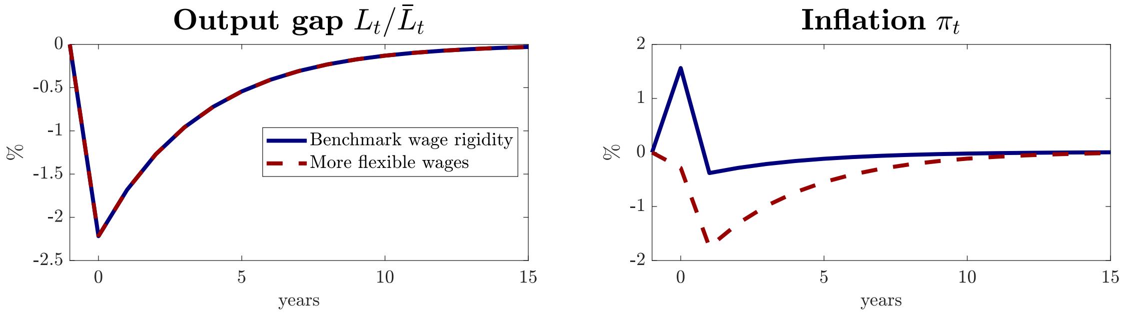



**Figure 6: Inflation response after supply disruption: microfounded wage Phillips curve.**

(see Section 4.4). We assume that, in the economy with the wage Phillips curve, the central bank implements the same real interest rate path following the negative supply shock as shown in Figure 3. This implies that the path for all real variables is identical in equilibrium. With this procedure, we can therefore study how the same response of the output gap can lead to different inflation responses depending on the degree of wage rigidities.

The result is shown in Figure 6. In the present model, the output gap is somewhat less negative following the negative supply shock, reflecting that the flexible-wage level of employment in this model is time-varying (compare Figure 4 in the main text). In turn, the inflation response is given in the right panel. Under our baseline calibration of wage rigidities, we find that inflation rises in response to the shock. Instead, when wages are more flexible, we find that inflation declines. We thus confirm, in the context of a microfounded wage Phillips curve, that the degree of wage rigidities is key in shaping the inflation response following a negative supply shock.

## B.6 Innovation policies at the zero lower bound

In this Appendix we study how monetary and innovation policies are set optimally jointly when the zero lower bound binds, complementing the analysis in Section 5.2. The case where monetary policy is conducted optimally at the zero lower bound (but where innovation policies are absent) is analyzed in Appendix B.4.

The key finding in this Appendix is that that there exists a separation of roles between monetary and innovation policies. When the zero lower bound is slack, fiscal policy sets the subsidy $s_t$ to ensure that the growth rate in the economy is efficient by correcting for the knowledge subsidies (see Section 5), whereas monetary policy sets the policy rate to ensure that the economy operates at full employment. However, when the zero lower bound binds, monetary policy sets the policy rate to zero, whereas fiscal policy increases the subsidy to ensure that the economy operates at full employment.

With innovation subsidies in place, and by taking account of the non-negativity constraint on investment, the investment first order condition by firms is given by (see also Appendix B.4)

$$\frac{1-s_t}{\chi} - \upsilon_t = \beta \frac{c_t}{c_{t+1}g_{t+1}} \left( \varpi Z_{t+1} L_{t+1} + \eta \left( \frac{1-s_{t+1}}{\chi} - \upsilon_{t+1} \right) \right)$$

$$\upsilon_t(g_{t+1}-1)=0,$$

where $\upsilon_t \geq 0$. Here, $s_t$ denotes the innovation subsidy, as in Section 5.

The problem of the policy maker is given by

$$\max_{\{L_t,c_t,g_{t+1},i_t,s_t\}} \left\{ \sum_{\tau=t}^{\infty} \beta^\tau \log C_\tau = \beta^t \left( \frac{1}{1-\beta} \log A_t + \log c_t + \frac{\beta}{1-\beta} \log g_{t+1} \right) + \varsigma_t^1 \right\},$$

subject to

$$\Psi Z_t L_t = c_t + \frac{g_{t+1}-1}{\chi}$$

$$c_t = \frac{g_{t+1}}{1+i_t} \varsigma_t^2$$

$$g_{t+1} = \begin{cases} [1, 1+\chi \Psi Z_t L_t] & \text{if } (1-s_t)(1+i_t) = \varsigma_t^3 \\ 1 & \text{if } (1-s_t)(1+i_t) > \varsigma_t^3 \end{cases}$$

$$L_t \leq \bar{L}$$

$$i_t \geq 0,$$

where the variables $\varsigma_t^1$, $\varsigma_t^2$, $\varsigma_t^3$ are given by

$$\varsigma_t^1 = \sum_{\tau=t+1}^{\infty} \beta^\tau \log \left( c_\tau \Pi_{\hat{\tau}=t+2}^{\tau} g_{\hat{\tau}} \right)$$

$$\varsigma_t^2 = \frac{1}{\beta c_{t+1}^{-1}}.$$

$$\varsigma_t^3 = \chi \varpi Z_{t+1} L_{t+1} + \eta \left(1 - s_{t+1} - \chi \upsilon_{t+1}\right).$$

The variables $\varsigma_t^1$, $\varsigma_t^2$ and $\varsigma_t^3$ are taken as given by the policy maker, because they are functions of parameters and expectations about future variables only.

The policy problem is identical compared to Appendix B.4, except that the policy maker has an additional instrument, the subsidy $s_t$. Because the subsidy can be chosen freely and appears only in the first order condition for investment, this implies that this condition can be omitted from the set of constraints in the optimization.

We thus simplify the optimization problem as follows

$$\max_{\{L_t,c_t,g_{t+1},i_t\}} \left\{ \sum_{\tau=t}^{\infty} \beta^\tau \log C_\tau = \beta^t \left( \frac{1}{1-\beta} \log A_t + \log c_t + \frac{\beta}{1-\beta} \log g_{t+1} \right) + \varsigma_t^1 \right\},$$

subject to

$$\Psi Z_t L_t = c_t + \frac{g_{t+1}-1}{\chi}$$

$$c_t = \frac{g_{t+1}}{1+i_t} \varsigma_t^2$$

$$g_{t+1} \geq 1$$

$$L_t \leq \bar{L}$$

$$i_t \geq 0.$$

We start by inspecting the case where the zero lower bound constraint is slack in equilibrium. In this case, the second constraint can be omitted from the set of constraints, as it merely determines the implied equilibrium nominal rate. Hence the policy problem is to maximize welfare subject to the resource constraint, plus the two inequality constraints $g_{t+1} \geq 1$ and $L_t \leq \bar{L}$.

Combining the first order conditions for $c_t$ and $g_{t+1}$ gives, for a given $L_t$

$$g_{t+1} = \max\left(\beta(\chi \Psi Z_t L_t + 1), 1\right)$$

$$c_t = \min\left((1-\beta)\left(\Psi Z_t L_t + \frac{1}{\chi}\right), \Psi Z_t L_t\right).$$

The first part of the max/min corresponds to the case in which the constraint $g_{t+1} \geq 1$ is slack, the second part to the case in which it binds such that $g_{t+1} = 1$. Because in these expressions, both $g_{t+1}$ and $c_t$ are weakly increasing in $L_t$, and because the objective is strictly increasing in $c_t$ and $g_{t+1}$, it is optimal to set $L_t = \bar{L}$

$$g_{t+1} = \max\left(\beta(\chi \Psi Z_t \bar{L} + 1), 1\right) \tag{B.17}$$

$$c_t = \min\left((1-\beta)\left(\Psi Z_t \bar{L} + \frac{1}{\chi}\right), \Psi Z_t \bar{L}\right). \tag{B.18}$$

From the second constraint, the implied nominal rate is then given by

$$1 + i_t^* = \max\left(\frac{\beta\chi}{1-\beta}\varsigma_t^2, \frac{1}{\Psi Z_t \bar{L}}\varsigma_t^2\right), \tag{B.19}$$

where we denote by $1 + i_t^*$ the nominal interest rate consistent with full employment when the zero lower bound is slack.

Assume next that $1 + i_t^* < 1$. In this case, the zero lower bound must bind ($i_t = 0$), such that for given $L_t$, the implied $g_{t+1}$ and $c_t$ can be computed directly from combining the first and second constraint in the optimization

$$g_{t+1} = \max\left(\frac{1}{\chi\varsigma_t^2 + 1}(\chi \Psi Z_t L_t + 1), 1\right)$$

$$c_t = \min\left(\frac{\chi\varsigma_t^2}{\chi\varsigma_t^2 + 1}\left(\Psi Z_t L_t + \frac{1}{\chi}\right), \Psi Z_t L_t\right).$$

As before, both $g_{t+1}$ and $c_t$ are weakly increasing in $L_t$, hence it is optimal to set $L_t = \bar{L}$. Inserting this in $g_{t+1}$, and using that $\varsigma_t^2 < \Psi Z_t \bar{L}$ because the zero lower bound binds, we observe that

$$\frac{1}{\chi\varsigma_t^2+1}(\chi\Psi Z_t\bar{L}+1) > \frac{1}{\chi\Psi Z_t\bar{L}+1}(\chi\Psi Z_t\bar{L}+1) = 1.$$

Hence full employment is feasible at the zero lower bound, and the investment constraint is necessarily slack such that growth is given by

$$g_{t+1} = \frac{1}{\chi\varsigma_t^2+1}(\chi\Psi Z_t\bar{L}+1), \tag{B.20}$$

with the implied level of consumption

$$c_t = \frac{\chi\varsigma_t^2}{\chi\varsigma_t^2+1}\left(\Psi Z_t\bar{L}+\frac{1}{\chi}\right). \tag{B.21}$$

Observe further that, because $\varsigma_t^2 < (1-\beta)/(\beta\chi)$ when the zero lower bound binds, it follows that

$$g_{t+1} = \frac{1}{\chi\varsigma_t^2+1}(\chi\Psi Z_t\bar{L}+1) > \beta(\chi\Psi Z_t\bar{L}+1)$$

$$c_t = \frac{\chi\varsigma_t^2}{\chi\varsigma_t^2+1}\left(\Psi Z_t\bar{L}+\frac{1}{\chi}\right) < (1-\beta)\left(\Psi Z_t\bar{L}+\frac{1}{\chi}\right).$$

Comparing this with the growth rate and consumption when the zero lower bound is slack in the region where growth is strictly positive (equations (B.17) and (B.18)) we find that when the zero lower bound binds, growth is necessarily higher and consumption is necessarily lower. Intuitively, when $i_t = 0$, additional innovation policies raise investment, as this increases aggregate demand to the level needed to maintain full employment (see also Section 5.2 for the same argument).

## B.7 Model with oil price shocks

In this Appendix, we consider a version of the model with oil price shocks. Our key result is that oil price shocks enter the model in a similar way as the TFP shocks considered in the main text.

Imagine that the production function is now given by

$$Y_t = o_t^\gamma\left((Z_tL_t)^{1-\alpha}\int_0^1 A_{j,t}^{1-\alpha}x_{j,t}^\alpha dj\right)^{1-\gamma},$$

where $o_t$ denotes the quantity of oil used in production and $0 \le \gamma < 1$. We denote the price of oil in real terms (i.e. normalized by $P_t$) as $p_t^o$. The price of oil is set exogenously on the global oil markets. The optimal demand for oil by firms implies

$$p_t^o o_t = \gamma Y_t. \tag{B.22}$$

We can then rewrite the production function as

$$Y_t = \left(\frac{\gamma}{p_t^o}\right)^{\frac{\gamma}{1-\gamma}} (Z_t L_t)^{1-\alpha} \int_0^1 A_{j,t}^{1-\alpha} x_{j,t}^{\alpha} dj. \tag{B.23}$$

Following the derivations in the main text, we can further write GDP as

$$Y_t - p_t^o o_t - \int_0^1 x_{j,t} dj = \left(\frac{\gamma}{p_t^o}\right)^{\frac{\gamma}{(1-\gamma)(1-\alpha)}} \tilde{\Psi} Z_t A_t L_t, \tag{B.24}$$

where we define $\tilde{\Psi} \equiv (1 - \alpha^2 - \gamma)\alpha^{\frac{2\alpha}{1-\alpha}}$.

To close the model, we assume that the country is in financial autarky, so that the trade balance is equal to zero period by period. In this case, the market clearing condition for the final good is

$$\left(\frac{\gamma}{p_t^o}\right)^{\frac{\gamma}{(1-\gamma)(1-\alpha)}} \tilde{\Psi} Z_t A_t L_t = C_t + I_t. \tag{B.25}$$

With these results, one can check that the response of the economy to an increase in the price of oil is qualitatively isomorphic to the response to a negative TFP shock.

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