New Pricing Models, Same Old Phillips Curve?

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October 2021

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- **State dependent** [eg menu cost]: better micro fit, but hard to simulate
  - mostly characterize IRF of nominal price to permanent nominal MC shocks

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Properties:

1. nearly identical to (NK-PC) for some $\kappa > 0$ [Alvarez-Le Bihan-Lippi, Gertler-Leahy]
2. exactly equal to mixture of two time dependent models
3. entirely recoverable from distribution of price changes [Alvarez-Lippi-Oskolkov]

New and old pricing models

Random menu cost model

- Discrete time, quadratic approximation to firm i's objective function:

$$\min_{\{p_{it}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} \left(p_{it} - p_{it}^* - \widehat{MC}_t \right)^2 + \xi_{it} \mathbf{1}_{\{p_{it} \neq p_{it-1}\}} \right]$$

$$p_{it}^* = p_{it-1}^* + \sigma \epsilon_{it} \quad \epsilon_{it} \sim \mathcal{N}(0,1) \quad \text{(idiosyncratic shock)}$$

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- Aggregate price and inflation: $\log P_t = \int p_{it} di, \quad \pi_t = \log P_t - \log P_{t-1}$

General time dependent model

- Firm i re-adjusts according to exogenous probabilities
- After s periods, cumulative adjustment prob Φ_s. At any date t:

$$\min_{p_{it}} \mathbb{E}_0 \left[\sum_{s=0}^{\infty} \beta^s \Phi_s \frac{1}{2} \left(p_{it} - p^*_{it+s} - \widehat{MC}_{t+s} \right)^2 \right]$$

- **Calvo**: $\Phi_s = (1-\lambda)^s \lambda$ (constant adjustment hazard λ)
- Also nests other cases (e.g. increasing adjustment hazard, Taylor)

[Whelan, Sheedy, Carvalho-Schwartzman]

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- Define $J^{nom}_{t,s} \equiv \frac{\partial \log P_t}{\partial \log MC_s}$. The $\mathbf{J}^{nom}$ matrix is **the nominal price Jacobian**:

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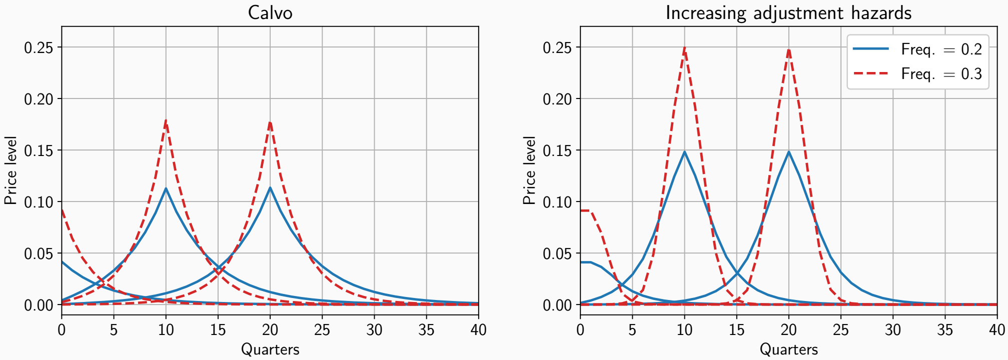
  - column s = IRF of price level to small aggregate nominal cost shock at date s
  - IRF to permanent shock: $\widehat{\mathbf{P}} = \mathbf{J}^{nom}\mathbf{1}$ [Golosov-Lucas, Alvarez-Le Bihan-Lippi, ...]
  - Flexible prices: $\mathbf{J}^{nom} = \mathbf{I}$

▸ Closed-form

- Time-dependent models have closed-form solution for $\mathbf{J}^{nom}$



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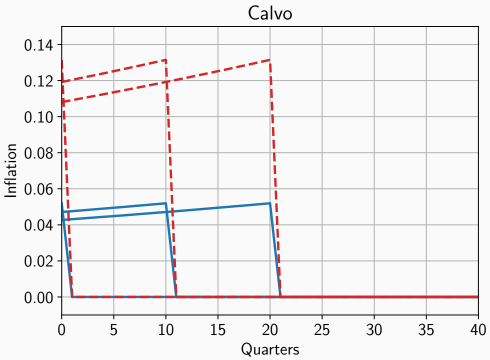
- To get inflation π use first difference matrix $\mathbf{D}$

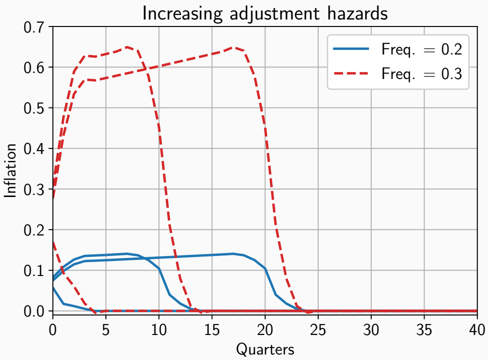
$$\pi = \mathbf{D}\left(\mathbf{I} - \mathbf{J}^{nom}\right)^{-1} \mathbf{J}^{nom} \cdot \widehat{\mathbf{mc}} \equiv \mathbf{J} \cdot \widehat{\mathbf{mc}}$$

- This gives the **Phillips curve Jacobian J**
    - sequence-space analogue of the (NK-PC)

Visualizing J for Calvo and general time-dependent model

- Calvo model: $\pi_t = \kappa \widehat{mc}_t + \beta \mathbb{E}_t \pi_{t+1}$



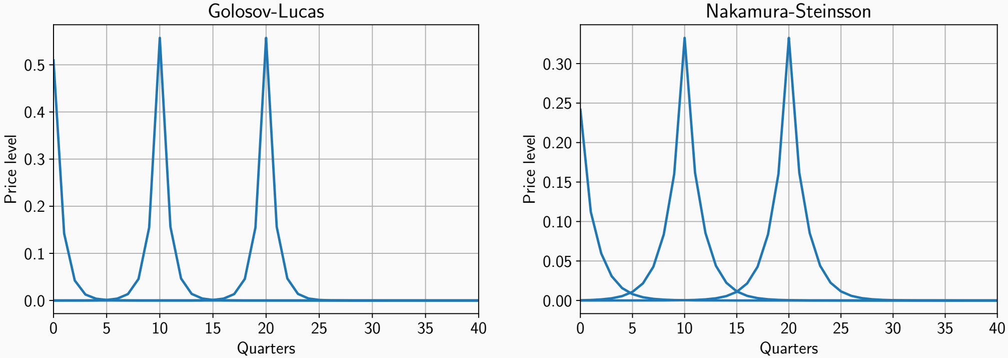


Observational equivalence

Calibration of random menu cost models

- Given λ; calibrate ξ, σ to match:
  - Average frequency of price change of 23.9% quarterly ("freq")
  - Median price adjustment of 8.5%
  - (regular price changes for median sector in U.S. CPI [Nakamura-Steinsson])
- Two benchmarks: $\lambda = 0$ (GL) and $\lambda = 0.75$ (NS)
- Note:
  - only two effective parameters are λ and ξ/σ^2, ξ then determines scale
  - for convenience will reparameterize by λ and freq (or duration=1/freq)

Nominal price Jacobians in our two menu cost models



[Fast computation with method in Auclert-Bardoczy-Rognlie-Straub]

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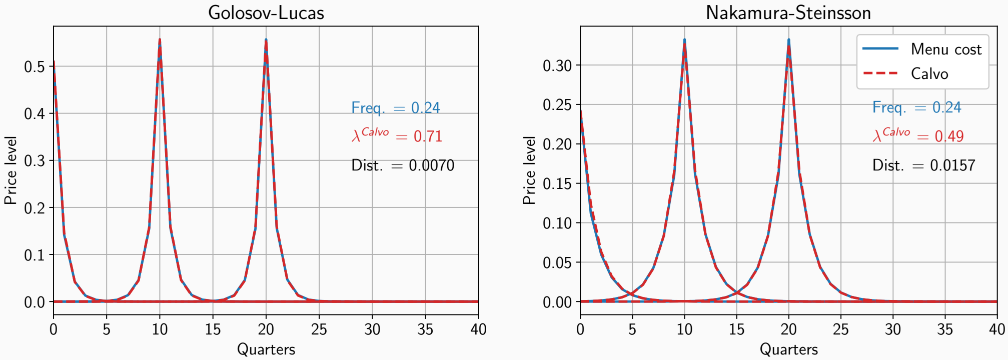
- Note: Two models with the same $\mathbf{J}^{nom}$ also share the same
  - … real Phillips curve price Jacobian $\mathbf{J}$
  - … IRF to any shock to MC or mc
  - … IRF to any fundamental shock once integrated in a broader macro model

Calvo approximates nominal price Jacobians



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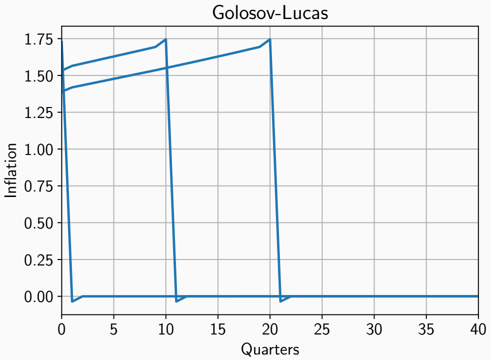

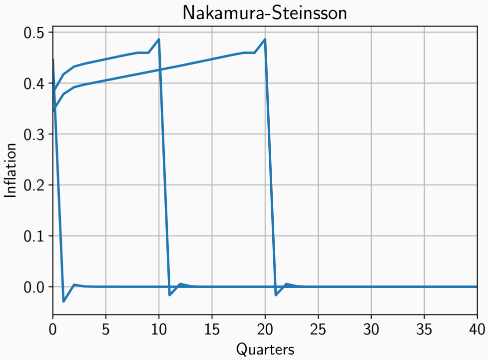

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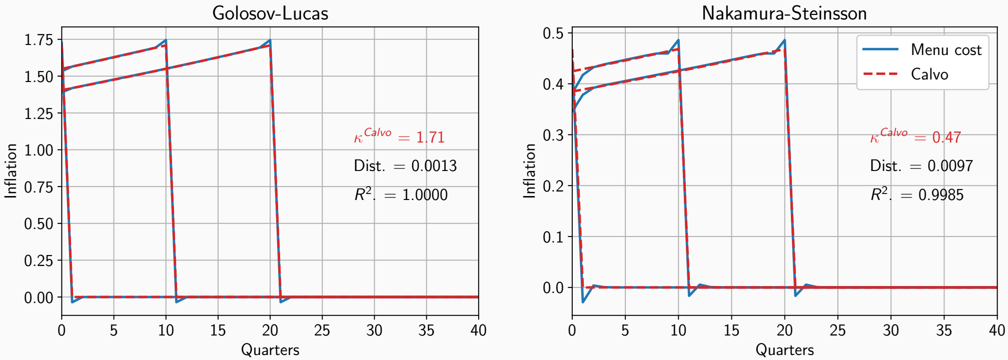
AR shocks | Smets-Wouters



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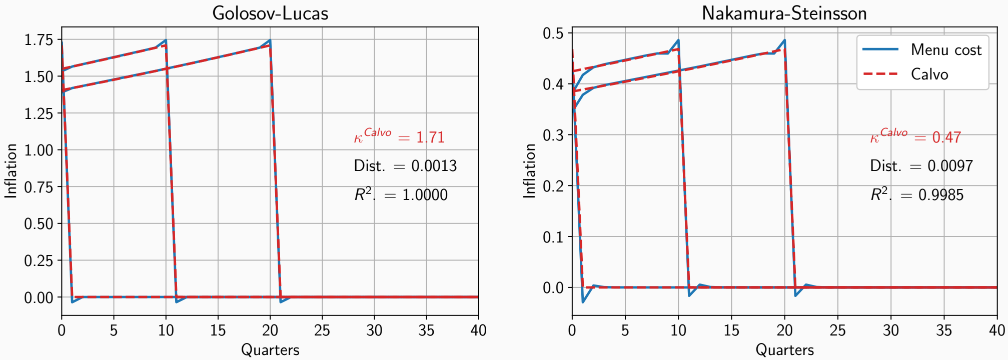

- What's κ^{Calvo}? Alvarez-Le-Bihan-Lippi holds approximately:

$$\frac{Kur^{Calvo}}{Freq^{Calvo}} = \frac{3\,(2-\lambda)}{\lambda} \simeq \frac{Kur}{Freq}$$

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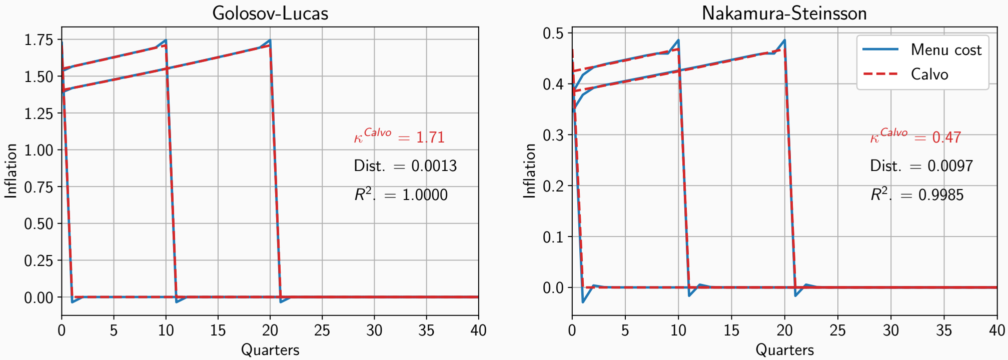

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$$\frac{Kur^{Calvo}}{Freq^{Calvo}} = \frac{3(2-\lambda)}{\lambda} \simeq \frac{Kur}{Freq} \quad \Rightarrow \quad \kappa^{Calvo} \simeq 4 \cdot \left(\left(\frac{1}{3} \frac{Kur}{Freq} \right)^2 - 1 \right)^{-1}$$

Extensions

- Arbitrary parameters →
- Steady state inflation →
- Infrequent shocks →
- Multi-product models →
- Multi-sector models →
- Large shocks →

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▸ $E^t(x)$ ▸ intuition

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Proposition

Any random menu cost model is equiv. to a mixture of two time-dep. models:

$$\mathbf{J}^{nom} = \underbrace{\alpha \mathbf{J}^{nom,td}(\Phi^e)}_{\text{“extensive margin”}} + \underbrace{(1-\alpha)\,\mathbf{J}^{nom,td}(\Phi^i)}_{\text{“intensive margin”}}$$

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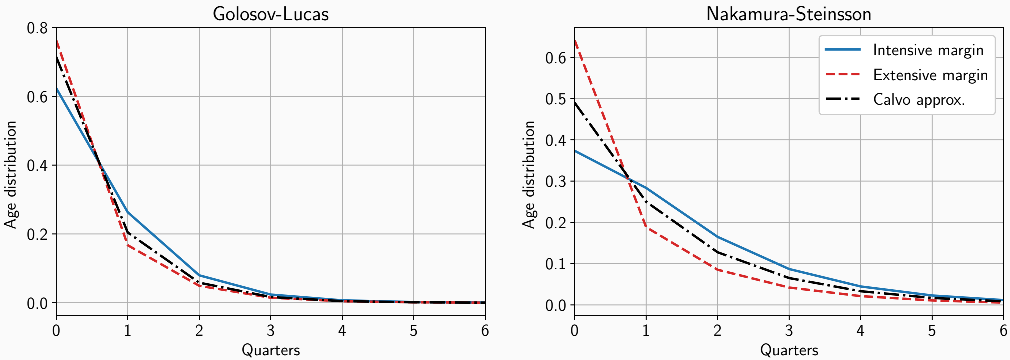
- Mixture represents two adjustment margins:
  - extensive margin: sS bands shift
  - intensive margin: reset point $p^*_{it} + \widehat{MC}_t$ shifts
- Can derive expressions straight from menu cost steady state:
  - based on expected future price gaps $E^t(x) \equiv \mathbb{E}\left[x_t | x_0 = x\right]$

Why does the Calvo approximation work?

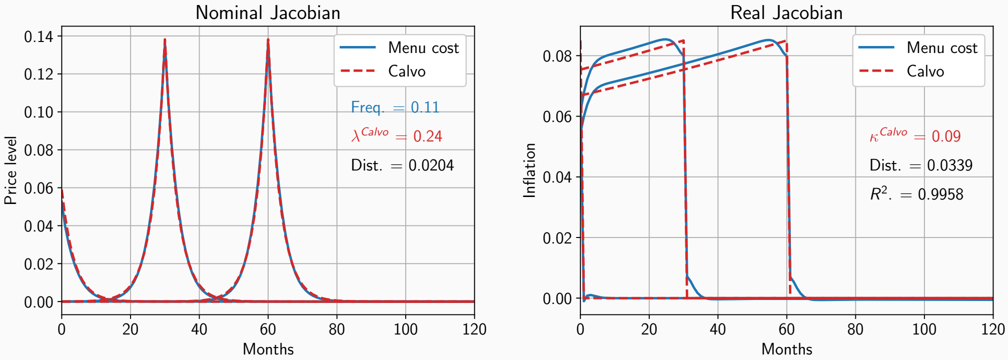


- Extensive and intensive margin offset each other.

Generalizing the approach

- Exact equivalence even holds with general distributions for ξ_{it}
  - now: mixture of many time-dep. models
- Can recover hazards and price gap distribution from data on price changes
  - similar to Alvarez-Lippi-Oskolkov
- Given equivalence, can directly compute entire nominal Jacobian!
  - not necessary to solve any model!

Implied nominal and real Jacobians vs. Calvo fit



Conclusion

New pricing models, old Phillips curve

- We obtain the **Phillips Curve of menu cost models**

  … observationally equivalent to Calvo (NK-PC) for a given κ

  … theoretically equal to mixture of time-dependent models

  … easy to embed in DSGE models w/ sufficient statistics formulas