Understanding Persistent ZLB: Theory and Assessment

Pablo Cuba-Borda *  Sanjay R. Singh ‡

*Federal Reserve Board of Governors
‡University of California, Davis

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the threat of japanification

Benhabib, Schmitt-Grohé & Uribe (2001); Bullard (2010); Mertens & Ravn (2014); Schmitt-Grohé & Uribe (2017); Aruoba, Cuba-Borda & Schorfheide (2018); Lansing(2019); Ascari and Bonchi (2020); Nakata and Schmidt (forthcoming)
persistent ZLB hypotheses

*expectations-driven liquidity traps*


*secular stagnation*


considered in separate frameworks!!
**Question 1:** is it possible to reconcile secular stagnation and expectation-driven traps within an unified framework?

- allow movements in long-run real rate to kill strong Fisherian effects
- assumptions on nominal rigidity key as to which hypothesis emerges

**Question 2:** why does it matter?

- contrasting policy implications depending on hypothesis at play
- identify hypotheses using data → bayesian prediction-pools
  - Substantial uncertainty
  - Need many years of data
- need for robust policies
Outline

1. simple model
2. analytical results
3. quantitative evaluation
A simple new-Keynesian model: ingredients

- Euler equation with endogenous discounting: Uzawa (1968), Epstein (1983)

\[
1 = \delta_t \tilde{C}_t \beta \left[ \frac{C_t}{C_{t+1}} \frac{R_t}{\Pi_{t+1}} \right], \quad 0 < \beta < 1; \quad \delta_t > 0
\]  


- Nominal rigidity in prices: Bhattarai, Eggertsson, Gafarov (2019)

\[
\Pi_t = \kappa Y_t + (1 - \kappa \bar{Y}), \quad \kappa \equiv \frac{\alpha_p}{1 - \alpha_p}
\]

- Monetary policy rule subject to ZLB constraint

\[
R_t = \max \left\{ 1, (1 + r_t^*) \Pi^* \left( \frac{\Pi_t}{\Pi^*} \right)^{\phi_\pi} \right\}, \quad 1 + r_t^* \equiv \frac{1}{\delta_t \beta}
\]
Steady state representation $\rightarrow AD - AS$

**Aggregate Demand:**

$$Y_{AD} = \frac{1}{\beta \delta} \left\{ \begin{array}{ll}
\frac{1}{(1+r^\ast)\Pi^{\phi\pi} - 1}, & \text{if } R > 1 \\
\Pi, & \text{if } R = 1
\end{array} \right.$$  

- $R > 1$: negative relation between $Y$ and $\Pi$
- $R = 1$: positive relation between $Y$ and $\Pi$

**Aggregate Supply:**

$$\Pi = \kappa Y + (1 - \kappa)$$

- $\kappa$: degree of nominal rigidity
- $1 - \kappa$: lower bound on inflation
Result 1 (disarming the perils)

Let $\Pi^* = 1$, $0 < \delta < \frac{1}{\beta}$, and when prices are rigid enough, $\kappa < 1$, there exists a globally unique steady state, called the targeted-inflation steady state that features $Y = 1$, $\Pi = \Pi^* = 1$, $R = \frac{1}{\beta \delta} > 1$.
Result 2 (expectations-traps)

Let $\Pi^* = 1$, $0 < \delta < \frac{1}{\beta}$, but let prices more flexible $\kappa > 1$. There exist two steady states:

1. The targeted-inflation steady state $Y = 1, \Pi = \Pi^* = 1$, and $R > 1$.

2. An expectations-driven trap steady state with $Y = \frac{1 - \kappa}{\beta \delta - \kappa} < 1$, $\Pi = \frac{\beta \delta (1 - \kappa)}{\beta \delta - \kappa} < 1$ and $R = 1$. The local dynamics in a neighborhood around this steady state are locally indeterminate.
Result 3 (secular stagnation)

Let $\Pi^* = 1$, $\delta > \frac{1}{\beta}$, and $\kappa < 1$. There exists a unique, secular stagnation steady-state, with $Y = \frac{1-\kappa}{\beta \delta - \kappa} < 1$, $\Pi = \frac{\beta \delta (1 - \kappa)}{\beta \delta - \kappa} < 1$ and $R = 1$. The equilibrium dynamics in this steady state's neighborhood are locally determinate.
Take An and Schorfheide (2006):

- Add bonds-in-utility
- Monetary policy always constrained by the zero lower bound
- Structural shocks: government spending \( g \), technology growth \( z \), markups \( \nu \)

Use likelihood-based methods to compare secular stagnation vs BSGU traps (Geweke and Amissano 2011, Del Negro et al. 2016)


Details:

- Match -1.06% annualized inflation under secular stagnation and expectations-trap
- Natural interest rate: secular stagnation = -1.1% vs expectations-trap = 0%
expectations traps or secular stagnation?

- policymaker confronted with two different models: $\mathcal{M}_S$ vs $\mathcal{M}_B$
  \[ p(y_t \mid \lambda, \mathcal{P}) = \lambda p(y_t \mid y_{1:t-1}, \mathcal{M}_b) + (1 - \lambda)p(y_t \mid y_{1:t-1}, \mathcal{M}_s), \quad 0 \leq \lambda \leq 1 \]
- real-time assessment is highly uncertain
inspect the mechanism: theoretical moments

- Why are expectation-traps a better description of the Japanese experience?
- Equilibrium indeterminacy of expectation-trap central to model fit

(a) $\text{corr}(\Delta y_t^o, \pi_t^o)$

(b) $\sigma_{\pi_t^o} / \sigma_{\Delta y_t^o}$
provide first unified treatment of two sources of persistent ZLB: secular stagnation and expectation traps

tractable theoretical framework with analytical results and AS-AD representation

suggest robust policies to tackle contrasting policy implications

quantitative assessment of the best-fitting hypothesis in real time

equilibrium indeterminacy is central for quantitative performance

similar findings in medium-scale DSGE model
simple model: households

$$\max_{\{C_t, b_t\}} \sum_{t=0}^{\infty} \Theta_t [\log C_t - \chi h_t]$$

s.t. $$C_t + b_t = \frac{W_t}{P_t} h_t + \frac{R_{t-1}}{\Pi_t} b_{t-1} + \Phi_t + T_t$$

- endogenous discounting (Uzawa-Epstein): $$\Theta_0 = 1$$ and $$\Theta_{t+1} = \hat{\beta} \left( \tilde{C}_t \right) \Theta_t \forall t \geq 0$$
- $$\hat{\beta}(\tilde{C}) = \delta_t \beta \tilde{C}_t$$, where $$0 < \beta < 1 \rightarrow$$ analytical results
- $$\tilde{C}_t$$ is average consumption that the household takes as given
- $$\delta_t > 0$$ are exogenous shocks to the discount factor

$$1 = \delta_t \beta \tilde{C}_t \left[ \frac{C_t}{C_{t+1}} \frac{R_t}{\Pi_{t+1}} \right]$$ \hspace{1cm} (4)

simple model: producers

- Continuum of intermediate goods $Y_t(j), \ j \in [0, 1]$

$$\max_{P_t(j)} \Phi_t(j) = (1 + \tau) P_t(j) Y_t(j) - W_t h_t(j),$$

$$s.t. \ Y_t(j) = h_t(j), \ Y_t(j) = f(P_t(j)/P_t, Y_t)$$

- Labor services $h_t(j)$ bought from competitive labor market at nominal price $W_t$

- Bhattarai, Eggertsson, Gafarov (2019): fraction $\alpha_p$ of firms can adjust prices, $(1 - \alpha_p)$ indexation: $\frac{P_t^i}{P_t} = \Gamma_t \frac{P_{t-1}}{P_t}$

- Final-good producing firms: $Y_t = \left( \int_0^1 Y_t(j)^{1-v} dj \right)^{\frac{1}{1-v}}, v = 1/2 \rightarrow \text{linear Phillips curve}$

$$\Pi_t = \kappa Y_t + (1 - \kappa \bar{Y}), \ \kappa \equiv \frac{\alpha_p}{1 - \alpha_p} \quad (5)$$
simple model: government policies

fiscal policy

\[ B_t + T_t = R_{t-1} B_{t-1} \]

monetary policy

\[ R_t = \max \left\{ 1, (1 + r^*_t) \Pi^* \left( \frac{\Pi_t}{\Pi^*} \right)^{\phi_\pi} \right\} \]  \hspace{1cm} (6)

- natural rate = \( (1 + r^*_t) \equiv \frac{1}{\delta_0 \beta} \)
- inflation target = \( \Pi^* = 1 \)
- \( \phi_\pi > 1 \)

market clearing

\[ C_t = Y_t, \quad \text{and} \quad B_t = 0 \]
Simple model: competitive equilibrium

- three endogenous processes \( \{Y_t, R_t, \Pi_t\} \) that satisfy:

\[
1 = \delta_t \beta Y_t \left[ \frac{Y_t}{Y_{t+1}} \frac{R_t}{\Pi_{t+1}} \right]
\]

\[
\Pi_t = \kappa Y_t + (1 - \kappa)
\]

\[
R_t = \max \left\{ 1, (1 + r^*_t) \Pi_t^{\phi\pi} \right\}
\]

- for a given exogenous \( \{\delta_t\}_{t=0}^{\infty} \), \( 1 + r^*_t = \frac{1}{\delta_t \beta} \) and initial price level \( P_{-1} \)

- focus on steady state representation \( \rightarrow AD - AS \) diagrams
why does it matter?
Example: exogenous aggregate demand shift (e.g. government policies)

- nature of stagnation matters, hence need to assess likelihood of $B$ vs $S$
- in the paper: fiscal policy, neofisherian exit, structural reforms
policy implications

- challenges in disentangling the source of liquidity trap, especially in real time
- need for developing policies that are robust to the source of recession
- two theoretical proposals in the paper
  - price indexation schemes in the presence of price adjustment frictions
  - minimum wage policies in the presence of downward nominal wage rigidity
- we do not analyze other potential trade-offs from these policies → future research
robust policy

Result 4 (Price Indexation)

Consider an indexation rule where non-optimizing firms index their prices to last period’s price level with indexation coefficient: $\Gamma_t = \frac{P_t}{Y_t^{t-1}(P_t - \lambda P_{t-1}) + P_{t-1}}$, then the price Phillips curve is given by: $\Pi_t = \kappa Y_t + (\lambda - \kappa \bar{Y})$

1. There does not exist expectations-driven liquidity trap $\forall \lambda > \kappa$.
2. Output and inflation under secular stagnation are increasing in $\lambda$.

Intuition:

- $\lambda$ can be interpreted like an increase in markups
  - sets a lower bound on deflation to eliminate expectations-trap steady state
  - increasing inflation stimulates output under secular stagnation

- other policies that restrict deflation have similar effects (e.g. minimum income)

- structural reforms that increase price flexibility make the economy vulnerable to expectation-driven traps
calibrated parameters

<table>
<thead>
<tr>
<th></th>
<th>$\beta$ (discount factor)</th>
<th>$\delta$ (Euler eq. wedge)</th>
<th>$\kappa$ (Slope of NKPC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>secular stagnation $(S)$</td>
<td>0.942</td>
<td>0.1132</td>
<td>0.0036</td>
</tr>
<tr>
<td>expectations trap $(B)$</td>
<td>0.942</td>
<td>0.1058</td>
<td>0.0019</td>
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</table>

Galí & Gertler (1999)

<table>
<thead>
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<th></th>
<th>natural rate</th>
<th>inflation</th>
<th>output gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>secular stagnation $(S)$</td>
<td>-1.1</td>
<td>-1.06</td>
<td>-7.6</td>
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<tr>
<td>expectations trap $(B)$</td>
<td>1.0</td>
<td>-1.06</td>
<td>-4.5</td>
</tr>
</tbody>
</table>

- natural rate:
- expectations trap: match real rate in data during sample period
- implied output gap for Japan: Haussman and Wieland (2014) $\sim$ 5%
why sunspots matter?

- Consider MSV solution criterion (McCallum, 2003)
- Prediction-pool cannot distinguish between hypothesis of stagnation under MSV
inspecting the mechanism

- equilibrium indeterminacy breaks positive relation between $\pi$ and $y$ at the ZLB

![Graphs showing response to shocks](image-url)
role of indeterminacy

- In an expectations trap $\zeta_t = \hat{\pi}_t - \hat{\pi}_t - 1$ not unique
- Use likelihood to pin down correlation of $\zeta$ with fundamental innovations
euro area: correlation $\pi, \Delta y$

- 10-year rolling correlation of GDP deflator inflation and GDP growth

![Graph showing the 10-year rolling correlation of GDP deflator inflation and GDP growth for the euro area. The graph illustrates a decrease in correlation from 2005 to 2020.](image-url)
which correlation matters?

$$\text{corr}(\zeta, g) = 0$$
which correlation matters?

$$corr(\zeta, z) = 0$$
which correlation matters?

\[ \text{corr}(\zeta, v) = 0 \]

\[ \rho(\zeta, v) > 0, \text{ induce } \rho(\pi, y) < 0, \text{ consistent with Wieland (2019)} \]