Selection in Information Acquisition and Monetary Non-Neutrality

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Motivation

• The average firm is highly uncertain about economic outcomes.

• But there is a high degree of heterogeneity in subjective uncertainty.

• **This Paper:** Whose expectations matter for macroeconomic outcomes?

• **Summary:**
  • Subjective uncertainty is *positively* correlated w/ time since last price change (*selection*)
  • A model with *state-dependent information acquisition* explains this selection
  • Only *the most informed* firms’ expectations matter for output response
Motivation

Subjective uncertainty: standard deviation of belief about desired price change

There is a lot of heterogeneity in uncertainty across firms.
Motivation

Firms that changed their prices more recently have more accurate expectations.

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<th>(1)</th>
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<th>(3)</th>
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<tbody>
<tr>
<td><strong>Dependent variable:</strong> Subjective uncertainty about firms’ desired price changes</td>
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<tr>
<td>Dummy for price changes (last 12 months)</td>
<td>-0.112*</td>
<td>-0.210***</td>
<td>-0.265***</td>
<td></td>
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<tr>
<td></td>
<td>(0.057)</td>
<td>(0.063)</td>
<td>(0.056)</td>
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<tr>
<td>Time elapsed since price change</td>
<td></td>
<td></td>
<td></td>
<td>0.010*</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.005)</td>
</tr>
<tr>
<td>Observations</td>
<td>485</td>
<td>488</td>
<td>486</td>
<td>487</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.061</td>
<td>0.170</td>
<td>0.243</td>
<td>0.188</td>
</tr>
<tr>
<td>Industry fixed effects</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
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<td>Firm-level controls</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
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<tr>
<td>Manager controls</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
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</table>
Model: Rational Inattention + Calvo
Model: Firms, Shocks and Payoffs.

- Time is continuous and indexed by $t \geq 0$.
- There is a measure of price-setting firms indexed by $i \in [0, 1]$.
- $i$’s instantaneous profit:
  \[ \Pi - B(p_{i,t} - p_{i,t}^*)^2 \]
- Each firm follows an exogenous desired price:
  \[ dp_{i,t}^* = \sigma dW_{i,t} \]
- Price change opportunities arrive at Poisson rate $\theta$ (Calvo).
Model: Information Structure and Cost of Attention.

- Firm $i$ does not observe $p_{i,t}^*$ but see a signal process over time:
  \[ ds_{i,t} = p_{i,t}^* dt + \sigma_{s,i,t} dW_{s,i,t} \]

- Information sets:
  \[ S_{i,t} = \{s_{i,\tau}: 0 \leq \tau \leq t\} \cup S_{i,0}, \quad S_{i,0} \text{ given.} \]

- Attention problem: firm chooses $\sigma_{s,i,t} \in \mathbb{R}_+ \cup \{\infty\}$ for all $t \geq 0$.

- Cost of information increases with rate of reduction in differential entropy
  \[ C(\mathbb{P}(P_{i,t}^*; S_{i,t})): \quad C'(\cdot) \geq 0, \quad \mathbb{I}(P_{i,t}^*; S_{i,t}) \equiv h(P_{i,t}^*|S_{i,0}) - h(P_{i,t}^*|S_{i,t}) \]
$$\min_{\{\sigma_{s,i,t \geq 0, \tilde{p}_{i,t}: t \geq 0}\}} \mathbb{E} \left[ \int_0^\infty e^{-\rho t} \left( B(p_{i,t} - p_{i,t}^*)^2 dt + C(d\Pi(P_{i,t}^*, S_{i,t})) \right) \right] S_{i,0}$$

loss from mis-pricing

cost of information
$\min_{\{\sigma_{s,i,t} \geq 0, \tilde{p}_{i,t}: t \geq 0\}} \mathbb{E} \left[ \int_{0}^{\infty} e^{-\rho t} \left[ B(p_{i,t} - p_{i,t}^*)^2 dt + C(d\mathbb{H}(P_{i,t}; S_{i,t})) \right] dS_{i,0} \right]$ 

loss from mis-pricing \hspace{1cm} cost of information

s.t. \quad dp_{i,t} = (\tilde{p}_{i,t} - p_{i,t}) d\chi_{i,t}, \; \chi_{i,t} \sim \text{Poisson}(\theta)$

$ds_{i,t} = p_{i,t}^* dt + \sigma_{s,i,t} dW_{s,i,t}, \; S_{i,0}, p_{i,0} \text{ given.}$
Model

\[
\begin{align*}
\min_{\{\sigma_{s,i,t} \geq 0, \tilde{p}_{i,t} : t \geq 0\}} \mathbb{E}\left[ \int_0^\infty e^{-\rho t} \left( B(p_{i,t} - p_{i,t}^*)^2 dt + C(d\Pi(P_{i,t}^*; S_{i,t})) \right) | S_{i,0} \right] \\
\text{loss from mis-pricing} & \quad \text{cost of information} \\
\text{s.t. } dp_{i,t} = (\tilde{p}_{i,t} - p_{i,t}) d\chi_{i,t}, \chi_{i,t} \sim \text{Poisson}(\theta) \\
& \quad ds_{i,t} = p_{i,t}^* dt + \sigma_{s,i,t} dW_{s,i,t}, \quad S_{i,0}, p_{i,0} \text{ given.}
\end{align*}
\]

Today, two extremes of convexity for \( C(d\Pi) \):

- Linear: \( C_L(d\Pi) = \omega d\Pi \)
- Extremely Convex: \( C_F(d\Pi) = \begin{cases} 0 & d\Pi \leq \bar{\lambda} dt \\ \infty & d\Pi > \bar{\lambda} dt \end{cases} \)
Definition
We define firm i’s true price gap and perceived price gap, and subjective uncertainty as

\[ x_{i,t}^* \equiv p_{i,t}^* - p_{i,t}, \quad x_{i,t} \equiv \mathbb{E}[x_{i,t}^* | S_{i,t}], \quad z_{i,t} \equiv \text{Var}(x_{i,t}^* | S_{i,t}) \]

respectively.

State variables for firm’s problem: (belief distribution about \(x_{i,t}^*\))

- \(x_{i,t}\): how much firm thinks its price is from optimal price
- \(z_{i,t}\): subjective uncertainty
Results
Results

Theorem (Optimal Information Acquisition with Linear Cost)

1. It is optimal for firms to never acquire information in between price changes, and uncertainty grows linearly with time.

2. Upon the arrival of an opportunity for a price change, firm acquires enough information to reset their uncertainty to $Z^*$ where

$$\frac{1}{Z^*} = \frac{B}{\omega(\rho + \theta)} + \theta \int_0^{\infty} e^{-(\rho+\theta)h} \frac{1}{Z^* + \sigma^2 h} dh$$

(1)

Proposition (Optimal Information Acquisition with Convex Cost)

All firms have the same uncertainty, independent of their state:

$$z = \sigma^2 \bar{\lambda}$$
Theorem (Optimal Information Acquisition with Linear Cost)

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$$\frac{1}{Z^*} = \frac{B}{\omega(\rho + \theta)} + \theta \int_0^\infty e^{-(\rho+\theta)h} \frac{1}{Z^* + \sigma^2 h} dh \quad (1)$$

Proposition (Optimal Information Acquisition with Convex Cost)

All firms have the same uncertainty, independent of their state:

$$z = \frac{\sigma^2}{\lambda} \quad (2)$$
Proposition

The time invariant distribution of uncertainty

- with the convex cost is a univariate degenerate distribution at $\frac{\sigma^2}{\lambda}$. 
Proposition

The time invariant distribution of uncertainty

- with the convex cost is a univariate degenerate distribution at $\frac{\sigma^2}{\lambda}$.
- with the linear cost is an exponential with rate $\theta / \sigma^2$ shifted by $Z^\ast$.

Figure I: Distribution of Uncertainty Across Firms
Implications for Monetary Non-Neutrality
Monetary Non-Neutrality

• Consider a permanent shock to $x_{i,0}^*$ of size $\delta$, and define

$$M(x, z, \delta) = \int_0^\infty \mathbb{E}_0 \left[ y_{i,t} | x_{i,0}^* = x + \delta, z_{i,t} = z \right] \, dt, \quad M(\delta) = \int M(x, z, \delta) \tilde{F}(dx, dz)$$

Theorem (Sufficient statistic with linear cost)
Cumulative response of output to a 1 percent monetary shock (area under IRF):

$$M(1) = \frac{1}{\theta} + \frac{Z^*}{\sigma^2} \quad (3)$$

inverse frequency of price change \quad subjective (normalized) uncertainty of price-setters

• Main takeaway:

Only the most informed firms’ expectations matter for monetary non-neutrality
Evidence suggests there is selection in information acquisition.

This is consistent with a state-dependent information acquisition model.

Selection implies that only the most informed firms’ expectations matter for output response to monetary shocks.
Monetary Non-Neutrality

• Can we still identify non-neutrality of money from distribution of price changes?
Monetary Non-Neutrality

- Can we still identify non-neutrality of money from distribution of price changes? **No.**

**Proposition**

*The distribution of price changes is invariant to Z*. 
Monetary Non-Neutrality

- Can we still identify non-neutrality of money from distribution of price changes? No.

**Proposition**
The distribution of price changes is invariant to $Z^*$.

Intuition of Proof: take an arbitrary price change,

$$\Delta p_{i,t} = \lambda_{i,t}(p_{i,t}^* + \text{noise} - p_{i,t-h})$$  \hfill (4)

- Optimality of $\lambda_{i,t}$ implies $\text{var}(\Delta p_{i,t}) = \sigma^2 h$. 
Monetary Non-Neutrality

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• Optimality of $\lambda_{i,t}$ implies $\text{var}(\Delta p_{i,t}) = \sigma^2 h$.
• So $\Delta p_{i,t}$ is generated by a Brownian motion of scale $\sigma$. 
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$$\Delta p_{i,t} = \lambda_{i,t}(p_{i,t}^* + \text{noise} - p_{i,t-h})$$

• Optimality of $\lambda_{i,t}$ implies $\text{var}(\Delta p_{i,t}) = \sigma^2 h$.
• So $\Delta p_{i,t}$ is generated by a Brownian motion of scale $\sigma$.
• In hypothetical economy assign $\Delta p_{i,t}$ to a firm whose ideal price is $p_{i,t}$. 
Monetary Non-Neutrality

• Can we still identify non-neutrality of money from distribution of price changes? No.

**Proposition**

*The distribution of price changes is invariant to $Z^*$.*

Intuition of Proof: take an arbitrary price change,

$$\Delta p_{i,t} = \lambda_{i,t}(p_{i,t}^* + \text{noise} - p_{i,t-h})$$  \hspace{1cm} (4)

• Optimality of $\lambda_{i,t}$ implies $\text{var}(\Delta p_{i,t}) = \sigma^2 h$.
• So $\Delta p_{i,t}$ is generated by a Brownian motion of scale $\sigma$.
• In hypothetical economy assign $\Delta p_{i,t}$ to a firm whose ideal price is $p_{i,t}$.
• The hypothetical economy is as if it has no information frictions but has the same distribution of price changes.
Monetary Non-Neutrality

• Because it takes time for firms to become aware of the shock when it is unannounced:

\[ db = -\lambda(z)b + U, \]
\[ \lambda(z) = 1 - \frac{Z^*}{z} \]

• In fact:

\[ \mathcal{M}(F_b) - \mathcal{M}(F_x) = \frac{Z^*}{\sigma^2} \]

• Need to know uncertainty conditional on price change.