

Imperfect Information, Shock Heterogeneity and Inflation Dynamics*

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Abstract

We consider two canonical approaches to imperfect information based on either the presence of noise that blurs the observation of fundamental shocks or, alternatively, the existence of compounded noiseless shocks whose distinct realizations remain unknown to agents. We establish that compounded noiseless shocks replicate closely observed co-movements in survey data for the universe of Japanese firms across 20 sectors that the alternative approaches of information with noise and perfect information fail to reproduce. We incorporate information frictions based on compounded noiseless shocks in the Calvo model of nominal price rigidities and establish an inverse relationship between the degree of shock heterogeneity and the response of inflation to changes in demand. We use Japanese industry-level data to estimate the degree of shock heterogeneity by applying principal component analysis, and we show that the observed increase in shock heterogeneity plays a significant role for the reduced sensitivity of inflation to movements in real activity since the late 1990s.

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1 Introduction

A tenet of modern macroeconomics is the rational expectation paradigm, which postulates that agents make decisions based on current economic conditions and expectations of future economic outcomes.¹ Several recent studies show that information frictions that limit knowledge of fundamental shocks are central to the formation of expectations and critical for models of rational expectations to explain aggregate fluctuations.² Canonical approaches to introduce imperfect information in structural models are either the presence of noise that blurs the observation of fundamental shocks, or alternatively, the existence of compounded noiseless shocks whose separate realizations remain unknown to agents.³ In this paper, we assess the empirical relevance of these alternative approaches using unique survey data from Japanese firms. We develop a theoretical framework consistent with the empirical findings that establishes a negative relationship between the degree of shock heterogeneity and the sensitivity of inflation to real activity. We then empirically test and corroborate this relationship using Japanese industry-level data.

To formalize the alternative approaches of imperfect information and confront them with the empirical evidence, we assume that firms face a demand that comprises aggregate and idiosyncratic components. Firms can observe either a signal of each of the two separate components that is blurred by noise, or alternatively, a signal that compounds the noiseless components together, making each separate component indistinguishable. A direct method for assessing these alternative approaches to imperfect information requires quantitative surveys on firms' expectations about aggregate and sectoral demand. While such survey data is unavailable for the United States, it is available for the universe of firms across 20 sectors in Japan in the Annual Survey of Corporate Behavior since 2004.⁴ By comparing survey data with observed data on aggregate and sectoral demand, we establish that expectations about changes in aggregate and idiosyncratic components of demand are strongly correlated.

¹See seminal article by Muth (1961) and the recent review on the literature by Angeletos (2018).

²Seminal studies by Mankiw and Reis (2002), Sims (2003), and Woodford (2003) establish the importance of imperfect information. Coibion et al. (2018) provide a recent overview of the literature.

³Both approaches are used to include imperfect information in structural models. Lorenzoni (2009), Blanchard et al. (2013), and Coibion and Gorodnichenko (2015a), among others, show that the noise approach is successful to describe changes in expectations about inflation and output. The approach based on existence of compounded shocks in the absence of noise was originally formulated in Phelps (1969) and Lucas (1972, 1973), and subsequently used in several studies to formalize important aspects of information frictions, as in Amador and Weill (2010) and Gaballo (2018), among others.

⁴See Section 2 for a full description of the data.

We show that this finding is consistent with information frictions that originate from compounded, noiseless shocks. In the presence of separately indistinguishable noiseless shocks, firms cannot disentangle movements in the distinct aggregate and idiosyncratic components of sectoral demand. Consequently, they evenly attribute any expected change in sectoral demand to each of the two components. Therefore, *expected* changes in aggregate and idiosyncratic components of demand become strongly correlated, consistent with the empirical evidence. The alternative approaches of information with noise or perfect information fail to generate the observed strong co-movement between expectations about aggregate and idiosyncratic components of demand because firms' expectations about both components are independent of each other and firms more successfully disentangle the effect of the distinct shocks on sectoral demand, dampening the co-movement between the distinct components of demand.

We apply the empirically successful approach based on compounded noiseless shocks to investigate the diminished sensitivity of inflation to changes in real activity.⁵ We establish that compounded and noiseless shocks introduce a link between the degree of shock heterogeneity—represented by the ratio of the volatility of idiosyncratic shocks to the volatility of aggregate shocks—and the response of agents to exogenous disturbances, which can be directly measured in the data, without facing the difficult task of disentangling noise from fundamental shocks.⁶ We enrich an otherwise standard Calvo model of nominal price rigidities by assuming that each firm observes a sectoral demand that compounds distinct aggregate and idiosyncratic components, whose separate realizations remain unknown to the firm. The interplay between perfect unobservability of distinct shocks and nominal price rigidities makes changes in aggregate prices dependent on past prices. Their relevance is inversely proportional to the frequency of price adjustment, as in the standard Calvo model, and on present and past movements in aggregate demand, which the firms fail to observe and instead infer from changes in sectoral demand that reflect simultaneous movements in aggregate and idiosyncratic components. Because of compounded noiseless shocks, the larger the volatility of exogenous changes in idiosyncratic demand relative to the volatility of aggregate demand, the more each firm attributes observed changes in sectoral demand to movements

⁵See survey by Mavroeidis et al. (2014) for a recent review of the literature focused on U.S. data. Mourougane and Ibaragi (2004), Veirman (2007), Nishizaki et al. (2014), Kaihatsu et al. (2017) and Kaihatsu and Nakajima (2015) provide evidence on the reduced sensitivity of inflation to real activity in Japan.

⁶Fernandez-Villaverde et al. (2007) establish that noise prevents the precise identification of macroeconomic shocks.

in idiosyncratic demand that in expectations are equal to zero. Thus, the firm expects that aggregate prices remain almost unchanged. Therefore, the model predicts that an increase in shock heterogeneity decreases the response of inflation to changes in economic activity.

A critical dimension to empirically quantifying the relevance of this mechanism relies on the quantitative assessment of the degree of heterogeneity on changes of idiosyncratic demand relative to changes in aggregate demand, which requires sector-level data. Such data is available from the Ministry of Finance of Japan, which has comprised quarterly data on sector-level sales for the universe of Japanese firms in 29 sectors since 1975.⁷ Using principal component analysis, we disentangle the volatility of exogenous movements in idiosyncratic demand relative to the volatility of exogenous movements in aggregate demand. We establish that shock heterogeneity—proxied by the volatility of idiosyncratic shocks standardized by the volatility of aggregate shocks—has steadily increased over the sample period, with the ratio of variance of idiosyncratic demand relative to the variance of aggregate demand increasing from 2 in the mid-1970s to 4 in the late-2000s.⁸

The theoretical framework provides a robust and testable implication on the inverse relationship between the degree of shock heterogeneity and the sensitivity of prices to changes in demand, which rationalizes the observed reduction in the sensitivity of inflation to demand observed in Japan and several industrialized economies.⁹ To assess whether our proxy for shock heterogeneity is powerful for the empirical assessment of the positive effect of shock heterogeneity on reduction in the sensitivity of inflation to demand, we conduct a Monte Carlo experiment. We use the model to generate artificial data on inflation and aggregate demand for different degrees of shock heterogeneity and estimate the sensitivity of prices to aggregate demand with several representative versions of the Phillips curve (i.e., a New Keynesian Phillips curve with forward-looking expectations and a hybrid Phillips curve with both forward- and backward-looking expectations). The estimation on artificial data shows that estimates of Phillips curves robustly attribute an increase in shock heterogeneity to a significant reduction in the sensitivity of prices to aggregate demand, therefore validat-

⁷Comparable U.S. data are: (i) the Quarterly Financial Report, published by the Census Bureau that includes data on sector-level sales and (ii) GDP data provided by the Bureau of Economic Analysis that contains sector-level national income. Owing to significant changes in industry classification, the sample periods of both datasets are restricted to the sample period 2000-2017, which is short for reliable empirical inference.

⁸We show that the aggregate shock series extracted from sector-level data are consistent with a more classical measure of aggregate shocks, as proxied by the output gap (see Appendix E).

⁹See Mavroeidis et al. (2014) for a recent review of the evidence.

ing the approach of estimating a Phillips curve to assess the empirical link between shock heterogeneity and the diminished relationship between prices and aggregate demand.

We then empirically test the implication of the model by estimating standard Phillips curve regressions that include our estimated measure of shock heterogeneity, as extracted by principal component analysis. We establish that the data robustly support a significant inverse relationship between shock heterogeneity and the sensitivity of prices to movements in real activity. The empirical results show that the sensitivity of inflation to aggregate demand has halved since the late 1990s, coinciding with a period of substantial increase in shock heterogeneity.

Our analysis connects with two strands of literature on imperfect information. First, it relates to studies that develop models with nominal price rigidities and imperfect information such as those of Fukunaga (2007), Nimark (2008), Angeletos and La'O (2009), Melosi (2017), and L'Huillier (2017). However, our research differs from those studies by relaxing the assumption that imperfect information disappears after one period, and instead assuming persistent imperfect knowledge. Second, it relates to studies that assume long-lasting imperfect information under flexible prices such as those of Woodford (2003), Hellwig and Venkateswaran (2009), Mackowiak et al. (2009), Crucini et al. (2015), and Kato and Okuda (2017). Third, it relates to studies that allow for coexistence of idiosyncratic and aggregate shocks in presence of costly information acquisition, as in Veldkamp and Wolfers (2007) and Acharya (2017). Our focus is different from those studies, since we empirically assess alternative approaches to information frictions, and study the interplay between imperfect information and shock heterogeneity for the relationship between inflation and real activity.

The analysis relates to the literature that investigates the effect of imperfect information on the empirical performance of the Phillips curve. Mankiw and Reis (2002) and Dupor et al. (2010) develop sticky-information models to investigate the effect of informational frictions on the empirical performance of the Phillips curve. Coibion and Gorodnichenko (2015b) establish that information frictions are critical in generating an empirically-consistent formation of expectations that explains the missing disinflation between 2009 and 2011. Mackowiak and Wiederholt (2009) investigate the effect of rational inattention on the Phillips curve and they establish a positive relationship between the variance of aggregate demand and the sensitivity of inflation to real activity. Unlike these studies, our analysis focuses on the effect of imperfect information on the sensitivity of inflation to changes in real activity. In this

respect, our analysis is closely related to studies that investigate changes in the relationship between inflation and real activity, as generated by the anchoring effect of inflation targets (Roberts (2004)), increasing competition in the goods market (Sbordone (2008), Zanetti (2009), IMF (2016), and Riggi and Santoro (2015)), downward wage rigidities (Akerlof et al. (1996)), structural reforms (Thomas and Zanetti (2009), Zanetti (2011), Cacciatore and Fiori (2016)), and lower trend inflation (Ball and Mazumder (2011)). Unlike these studies, our focus is on the relationship between information frictions and the sensitivity of inflation to real activity.

The remainder of the paper is organized as follows. Section 2 provides evidence from survey data for Japan. Section 3 lays out and assesses the empirical performance of the model with unobservability of separate and noiseless shocks. Section 4 considers alternative models with noisy and perfect information. Section 5 introduces unobservability of noiseless shocks in a prototype Calvo model with nominal price rigidities to establish the theoretical relationship between shock heterogeneity and the sensitivity of inflation to real activity. It uses industry-level data to test theoretical predictions. Section 6 concludes.

2 Evidence from Survey Data

Assessing the empirical relevance of the different ways to include information frictions requires quantitative survey data of expectations on aggregate and sectoral demands. While such survey data is unavailable for the United States, the Annual Survey of Corporate Behavior—a survey conducted by the Cabinet Office of Japan across 20 sectors since 2004—makes this data available for Japanese firms.¹⁰ We measure aggregate and sectoral demand from yearly data on gross domestic product in the National Accounts of Japan.¹¹

We study key relationships in the co-movements between survey data on expectations in

¹⁰The Economic and Social Research Institute in the Cabinet Office of Japan directly surveys approximately 1,000 public-listed Japanese firms on nominal and real growth rates of the Japanese economy as well as nominal and real growth rates of demand in their respective sectors. Sectoral averages are publicly available at: <http://www.esri.cao.go.jp/en/stat/ank/ank-e.html>. The survey is conducted each January, and questionnaires are available at: http://www.esri.cao.go.jp/en/stat/ank/h28ank/h28ank_questionnaire.pdf. We proxy expectations on aggregate demand with survey data on expectations on one-year-ahead GDP growth, and we proxy expectations on sectoral demand with survey data on expectations on one-year-ahead growth rate in sectoral demand.

¹¹The national account data is available at: <http://www.esri.cao.go.jp/jp/sna/menu.html>. We use the GDP growth rate as a proxy for aggregate demand and the growth rate in sectoral gross output as a proxy for sectoral demand.

aggregate and sectoral demand and the equivalent series in national account data.¹² We base the empirical assessment on three critical statistics. First, we characterize the co-movement between observed changes in aggregate and sectoral demand by the correlation coefficient between changes in observed aggregate demand and sectoral demand from national account data (labeled ρ_1). Second, we characterize the co-movement between *expected* changes in aggregate demand and *expected* changes in sectoral demand by deriving the correlation coefficient between changes in *expected* aggregate demand and changes in *expected* sectoral demand (labeled ρ_2). Important for the analysis, comparison between the correlation coefficients ρ_1 and ρ_2 provides an assessment on the extent to which observed changes about the co-movement between sectoral and aggregate demand co-move with expectations of the change in these variables.

Panel (a) in Figure 1 shows the correlation coefficient of changes in aggregate demand with changes in sectoral demand (ρ_1) on the y-axis and the correlation coefficient of expected changes in aggregate demand with expected changes in sectoral demand (ρ_2) on the x-axis. The figure shows that the correlation coefficient ρ_1 is large across different sectors with median value equal to 0.74. The correlation coefficient ρ_2 that captures equivalent co-movements in the expectations of changes in aggregate demand and sectoral demand is larger, with a median value of 0.90. Comparison between the correlation coefficients ρ_1 and ρ_2 shows that the co-movement in expectations is stronger than the co-movement in the observed data (i.e. $\rho_2 > \rho_1$) since most of the values lie below the 45-degree line.

[Figure 1 about here.]

The inequality $\rho_2 > \rho_1$ indicates large and persistence co-movement in expectations about aggregate and sectoral demand compared to the similar co-movement in underlying corresponding variables in observed data. In principle, the large and persistence correlation between expectations in aggregate and sectoral demand may be a by-product of large persistence in aggregate shocks that induces expectations in sectoral demand strongly co-move with expectations in aggregate demand. To rule out this possibility and exclude that the large value for ρ_2 is driven by the persistence in exogenous disturbances, we derive the correlation coefficient between the *first difference* in expectations of changes in aggregate

¹²We focus on correlations between aggregate and sectoral demand instead of correlations between aggregate and idiosyncratic demand since the former correlations are immune from pessimism bias on their own sectoral demand conditions.

and sectoral demand that removes the influence of persistent shocks on the co-movement in expectations between aggregate and sectoral demand. We consider the correlation coefficient of the differences between expectations at period $t - 1$ about changes in aggregate demand and sectoral demand (i.e., ρ_2) and those at period $t - 2$ (i.e., ρ_3). If $\rho_3 < \rho_2$, the persistence of aggregate demand shocks is an important driver of the strong co-movement between changes in aggregate and sectoral demand whereas if $\rho_3 = \rho_2$, the persistence of shocks is unimportant. Likewise, the large correlation of ρ_2 is an empirically important feature for the formation of expectations.

Panel (b) in Figure 1 shows on the y-axes the correlation coefficient of expected changes in aggregate demand with expected changes in sectoral demand, ρ_2 , and on the x-axes the correlation coefficient of the difference in expected changes of aggregate demand with the difference in expected changes of sectoral demand, ρ_3 . The figure reveals that the values of the correlation coefficients ρ_2 and ρ_3 are large and generally equal to each other, with most of the observations close to the value of 1, implying that $\rho_2 \approx \rho_3 \approx 1$. The next proposition summarizes the empirical relationships across correlation coefficients of observed and survey data.¹³

Proposition 1 (Empirical relation across correlation coefficients). *Define the following correlation coefficients:*

ρ_1 : *Correlation of observed changes in aggregate and sectoral demand.*

ρ_2 : *Correlation of expected changes in aggregate and sectoral demand.*

ρ_3 : *Correlation of the difference in expected changes of aggregate and sectoral demand.*

Empirical evidence from observed and survey data shows the following fundamental relationships across correlation coefficients hold:

$$\rho_1 < \rho_2 = \rho_3.$$

3 A Model with Compounded Noiseless Shocks

This section lays out a flexible-price model with monopolistic competition and demand comprising noiseless aggregate and idiosyncratic shocks. The theoretical framework resembles Angeletos and La'O (2009), departing from it across two critical dimensions. First, instead of noise shocks, we assume that imperfect information arises from existence of fundamental

¹³Appendix D shows the correlation coefficients derived for real demand. Results remain unchanged.

shocks whose distinct realizations remain unknown to agents. We implement it by assuming that sectoral demand additively combines unobservable disturbances to aggregate and idiosyncratic demand. Second, we assume that distinct disturbances continue to remain unknown to agents in the aftermath of their realization instead of becoming perfectly known in the subsequent period, in line with the empirical evidence on large and persistent forecast errors in Coibion and Gorodnichenko (2015b), Melosi (2014), and Fuhrer (2017).

The economy is populated by a representative household and a continuum of firms that produce differentiated goods indexed by $i \in [0, 1]$. Each representative household consumes the whole income, and there is no saving in equilibrium. Time is discrete and indexed by $t \in \{0, 1, 2, \dots\}$.

Households. Preferences of the representative household are described over consumption, C_t , and labor, N_t , by the utility function:

$$\sum_{t=0}^{\infty} \beta^t (\log C_t - N_t),$$

where $\beta \in (0, 1)$ is the discount rate. The household's consumption is described by the CES consumption aggregator:

$$C_t = \left[\int_0^1 (C_t(i) \Theta_t(i))^{\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}},$$

where $\eta > 1$ is the elasticity of the substitution across goods, $C_t(i)$ is the consumption of each good i , and $\Theta_t(i)$ is the idiosyncratic preference shocks.

Firms. Preferences of the representative household imply the following demand for each firm i :

$$C_t(i) = \Theta_t^{\eta-1}(i) \left(\frac{P_t(i)}{P_t} \right)^{-\eta} C_t, \quad (1)$$

where $P_t \equiv \left[\int_0^1 P_t^{1-\eta}(i) \Theta_t^{\eta-1}(i) di \right]^{\frac{1}{1-\eta}}$ is the aggregate price index and the idiosyncratic preference shock, $\Theta_t(i)$, acts as an exogenous, idiosyncratic demand shifter.¹⁴

Each firm i manufactures a single good i , according to the production technology:

$$Y_t(i) = A L_t^\epsilon(i), \quad (2)$$

where A is aggregate productivity and $\epsilon \in (0, 1)$ is the degree of diminishing returns.

¹⁴See Appendix A for the derivation of the demand function for each firm i .

Market Clearing. Market clearing implies $Y_t(i) = C_t(i)$ for each firm $i \in [0, 1]$, and $Y_t = C_t$ in the economy. Aggregate nominal demand, Q_t , is given by the following cash-in-advance constraint:

$$Q_t = P_t C_t.$$

In the subsequent analysis, we use lower-case variables to indicate logarithms of the corresponding upper-case variables (i.e., $x_t \equiv \log X_t$).

Optimal Price Setting. We first derive the optimal price setting rule with flexible prices, assuming perfect information about the current nominal shocks. We then describe the change in the environment under imperfect information. During each period t , firm i sets the optimal price:

$$p_t(i) = \mu + mc_t(i), \quad (3)$$

where $\mu \equiv \eta/(\eta - 1) > 0$ is the mark-up and $mc_t(i)$ is the nominal marginal cost faced by firm i . The nominal marginal cost is the difference between the nominal wage, w_t , and the marginal product of labor:

$$mc_t(i) = w_t + (1 - \epsilon) l_t(i) - a - \log(\epsilon). \quad (4)$$

Using the production technology in equation (2), we express labor input as: $l_t(i) = [y_t(i) - a]/\epsilon$, which we use in equation (4) to rewrite the nominal marginal cost as:

$$mc_t(i) = w_t + \frac{1 - \epsilon}{\epsilon} y_t(i) - \frac{1}{\epsilon} a - \log(\epsilon).$$

The optimal labor supply condition for the representative household is:

$$w_t - p_t = c_t, \quad (5)$$

and the consumer demand in equation (1) is:

$$c_t(i) = -\eta (p_t(i) - p_t) + c_t + (\eta - 1) \theta_t(i), \quad (6)$$

where the idiosyncratic, demand shock, $\theta_t(i)$, is normally distributed with mean zero.

We derive the optimal price-setting rule for firm i by using equations (5), (6), the equilibrium conditions, $y_t(i) = c_t(i)$ and $y_t = c_t$, and the cash-in-advance constraint, $y_t = q_t - p_t$, which yields:¹⁵

$$p_t(i) = r p_t + (1 - r) [q_t + v_t(i)] + \xi, \quad (7)$$

¹⁵Appendix B shows the derivation of the price setting rule.

where

$$r \equiv \frac{(\eta - 1)(1 - \epsilon)}{\epsilon + \eta(1 - \epsilon)}, \quad (8)$$

$$\xi \equiv \left(\frac{\epsilon}{\epsilon + \eta(1 - \epsilon)} \right) (\mu - \log(\epsilon)) - \left(\frac{1}{\epsilon + \eta(1 - \epsilon)} \right) a, \quad (9)$$

$$v_t(i) \equiv (1 - \epsilon)(\eta - 1)\theta_t(i), \quad (10)$$

where $p_t = \int_0^1 p_t(i)di$.¹⁶ Equation (7) shows that the optimal pricing rule for firm i is a weighted average of aggregate prices and sectoral demand, which comprises aggregate and idiosyncratic components. The weight is determined by the parameter r , which reflects the dependence of the optimal decision rules on average prices, namely the degree of strategic complementarity among firms.¹⁷

We normalize the distribution of the idiosyncratic shock such that $v_t(i) \sim \mathcal{N}(0, \tau^2)$.¹⁸ The parameter ξ , defined in equation (9), is a linear transformation of the level of aggregate productivity, a , and without loss of generality we normalize aggregate productivity such that $\xi = 0$.

Shocks and Information Structure. Aggregate nominal demand, q_t , follows the random walk process:

$$q_t = q_{t-1} + u_t, \quad (11)$$

where $u_t \sim \mathcal{N}(0, \sigma^2)$ is the exogenous disturbance to nominal demand. We assume that each firm i observes a sectoral demand, $x_t(i)$, that comprises movements in aggregate demand, q_t , and idiosyncratic demand, $v_t(i)$, according to:

$$x_t(i) = q_t + v_t(i), \quad (12)$$

where $v_t(i) \sim \mathcal{N}(0, \tau^2)$ is the exogenous disturbance to sectoral demand.¹⁹

¹⁶Appendix C shows the derivation of the index of aggregate prices.

¹⁷Equation (7) shows that if the production technology converges to constant returns (i.e., $\epsilon \rightarrow 1$), average prices become less important in the determination of the price for firm i (i.e., $r \rightarrow 0$) since the marginal cost converges to the aggregate nominal wage across firms (i.e., $mc_t(i) \rightarrow w_t$) and heterogeneity in firms' prices decreases. The magnitude of the idiosyncratic shock decreases (i.e., $v_t(i) \rightarrow 0$) as the production technology converges to constant returns (i.e., $\epsilon \rightarrow 1$). As a result, in the limiting case of a linear production technology (i.e., $\epsilon = 1$), the optimal pricing rule is $p_t(i) = q_t + \xi$.

¹⁸The normalization implies $\theta_t(i) \sim \mathcal{N}(0, (1 - \epsilon)^{-2}(\eta - 1)^{-2}\tau^2)$.

¹⁹To retain direct comparability with related studies and maintain analytical tractability, we assume no persistence in the shocks. An appendix that shows the robustness of results to the inclusion of persistent shocks is available from the authors on request.

The information structure prevents each firm i from separately observing aggregate demand, q_t , and idiosyncratic demand, $v_t(i)$, which are central to the price setting rule in equation (7). Indistinguishability of distinct fundamental shocks requires inference of these variables from the observation of sectoral demand, $x_t(i)$, as described in equation (12), which has distribution:

$$x_t(i) \sim \mathcal{N}(q_t, \tau^2),$$

where $x_t(i)$ is the unbiased signal of aggregate demand.²⁰ To retain indistinguishability of distinct shocks, we assume that distinct realizations of q_t and $v_t(i)$ remain unknown, and the information set of firm i in period t comprises the infinite sequence of present and past realizations of sectoral demand (i.e., $\mathcal{H}_t(i) \equiv \{x_{t-s}(i)\}_{s=0}^\infty$). This assumption requires us to track the infinite sequence of changes in past sectoral demand.

Mapping between model and data. We now describe the mapping between the model and measurement in the data. Equation (11) shows that the exogenous disturbance to aggregate demand, u_t , is tracked by changes in aggregate demand (i.e., $u_t = q_t - q_{t-1}$), and similarly, we assume that the exogenous disturbance to idiosyncratic demand is tracked by changes in idiosyncratic demand (i.e., $\tilde{v}_t(i) = v_t(i) - v_{t-1}(i)$). To retain a general specification of exogenous disturbances, we allow persistence in changes of aggregate demand and idiosyncratic demand by assuming:

$$u_t = \rho_u u_{t-1} + e_t, \tag{13}$$

$$v_t(i) = \rho_v v_{t-1}(i) + \epsilon_t(i), \tag{14}$$

where $0 \leq \rho_u < 1$, $0 \leq \rho_v < 1$, and $e_t \sim \mathcal{N}(0, \sigma^2)$. Equation (14) implies the following change in idiosyncratic demand:

$$\tilde{v}_t(i) = \rho_v \tilde{v}_{t-1}(i) + \tilde{\epsilon}_t(i), \tag{15}$$

where $\tilde{\epsilon}_t(i) = \epsilon_t(i) - \epsilon_{t-1}(i)$, and $\tilde{\epsilon}_t(i) \sim \mathcal{N}(0, \tau^2)$. We make standard assumptions that exogenous innovations e_t and $\tilde{\epsilon}_t(i)$ are independent of each other, and $\tilde{\epsilon}_t(i)$ is independent from $\tilde{\epsilon}_{t-1}(i)$. Using equation (12) and equations (13)-(15), the change in sectoral demand,

²⁰We derive the signal structure as follows. From equation (12), we know that $q_t = x_t(i) - v_t(i)$. Since agents observe $x_t(i)$ and the average of $v_t(i)$ is zero, $\mathbb{E}[q_t | x_t(i)] = x_t(i)$. Therefore $\mathbb{V}[q_t | x_t(i)] = \mathbb{V}[v_t(i)] = \tau^2$.

$\tilde{x}_t(i)$, depends on the change in aggregate demand, u_t , and the change in idiosyncratic demand, $\tilde{v}_t(i)$, according to:

$$\tilde{x}_t(i) = u_t + \tilde{v}_t(i). \quad (16)$$

Equation (16) provides a direct mapping between the model and observed variables. We measure observed changes in aggregate demand, u_t , and observed changes in sectoral demand, $\tilde{x}_t(i)$, with yearly data on gross domestic product from the National Accounts of Japan, and we measure expectations about sectoral demand, $\mathbb{E}_{t-1}[\tilde{x}_t(i)]$, and expectations about aggregate demand, $\mathbb{E}_{t-1}[u_t]$, with survey data from the Annual Survey of Corporate Behavior.²¹

Correlation Coefficients under Noiseless Shocks. This section investigates whether the model with noiseless shocks replicates the observed co-movements across correlation coefficients reported in Proposition 1. When distinct realizations of shocks remain constantly unknown to agents, the formation of expectations is complex, requiring each firm to infer the value of distinct shocks from observing changes in sectoral demand, $\tilde{x}_t(i)$, that fail to reveal the separate realizations of aggregate and idiosyncratic demand. The next proposition characterizes the recursive law of motion that tracks expectations on changes in aggregate and idiosyncratic demand, $\mathbb{E}_t[u_t]$ and $\mathbb{E}_t[\tilde{v}_t(i)]$, respectively.²²

Proposition 2 (Expectations on changes in aggregate and idiosyncratic demand).

Define the vector of expectations on changes in aggregate and idiosyncratic demand as: $E_t \equiv \begin{bmatrix} \mathbb{E}_t[u_t] \\ \mathbb{E}_t[\tilde{v}_t(i)] \end{bmatrix}$. Expectations on aggregate and idiosyncratic demand follow the law of motion:

$$E_t = \Lambda \tilde{x}_t(i) + A E_{t-1} = \Lambda \tilde{x}_t(i) + A \Lambda \tilde{x}_{t-1}(i) + A^2 \Lambda \tilde{x}_{t-2}(i) + \dots, \quad (17)$$

where $\Lambda \equiv \begin{bmatrix} \lambda \\ 1 - \lambda \end{bmatrix}$ tracks the Kalman gains for changes in aggregate and idiosyncratic demand, and $A \equiv \begin{bmatrix} (1 - \lambda) \rho_u & -\lambda \rho_v \\ -(1 - \lambda) \rho_u & \lambda \rho_v \end{bmatrix}$ tracks the dependency of expectations on aggregate and idiosyncratic demand, $\mathbb{E}_t[u_t]$ and $\mathbb{E}_t[\tilde{v}_t(i)]$, on the ex-ante expectations of these components, $\mathbb{E}_{t-1}[u_t]$ and $\mathbb{E}_{t-1}[\tilde{v}_t(i)]$.

²¹The national account data is available at: <http://www.esri.cao.go.jp/jp/sna/menu.html>. We use GDP growth rate as a proxy for u_t , and the growth rate in sectoral gross output as a proxy for $\tilde{x}_t(i)$. We proxy $\mathbb{E}_{t-1}[u_t]$ with survey data on expectations on one-year ahead GDP growth, and we proxy $\mathbb{E}_{t-1}[u_t + \tilde{v}_t(i)]$ with survey data on expectations on one-year ahead growth rate in sectoral demand.

²²We assume the situation that the information structure converges to the steady state.

Proof: See Appendix F.1. \square

Proposition 2 shows that in the filtering process, the firm forms current expectations on aggregate and idiosyncratic demand, E_t , by updating ex-ante expectations, E_{t-1} , with information on the change in sectoral demand, $\tilde{x}_t(i)$, which comprises changes in aggregate demand, u_t , and idiosyncratic demand, $\tilde{v}_t(i)$. If changes in u_t and $\tilde{v}_t(i)$ have no persistence (i.e., $A = 0$ and $E_t = \Lambda\tilde{x}_t(i)$), the vector of expectations depends on the observation of current sectoral demand. The presence of persistence in u_t and $\tilde{v}_t(i)$, outlined in equations (13) and (14), generates two complementary channels whereby changes in sectoral demand (i.e., $\tilde{x}_{t-s}(i)$, for $s \in \{1, 2, 3, \dots\}$) influence current expectations, E_t . The first channel involves the mapping from past changes in aggregate and idiosyncratic demand (i.e., u_{t-s} and $\tilde{v}_{t-s}(i)$, for $s \in \{1, 2, 3, \dots\}$) to current changes in those demands, as revealed by iterating equations (13) and (14) backwards, yielding: $u_t = \rho_u^s u_{t-s} + \sum_{j=0}^{s-1} \rho_u^j e_{t-j}(i)$ and $\tilde{v}_t(i) = \rho_v^s \tilde{v}_{t-s}(i) + \sum_{j=0}^{s-1} \rho_v^j \epsilon_{t-j}(i)$ for $s \in \{1, 2, 3, \dots\}$. This channel creates a strong link between expectations of current and past changes in aggregate and idiosyncratic demand (the diagonal elements of the matrix A), respectively. The second channel depends on the relevance of *ex ante* knowledge of u_t and $\tilde{v}_t(i)$. Since $u_t = \tilde{x}_t(i) - \tilde{v}_t(i)$, the firm uses *ex ante* expectations $\mathbb{E}_{t-1}[\tilde{v}_t(i)] = \rho_v \mathbb{E}_{t-1}[\tilde{v}_{t-1}(i)]$ to extract accurate information on u_t from $\tilde{x}_t(i)$. Similarly, the firm forms expectations on $\tilde{v}_t(i)$ with *ex ante* expectations $\mathbb{E}_{t-1}[u_t] = \rho_u \mathbb{E}_{t-1}[u_{t-1}]$ to extract accurate information on $\tilde{v}_t(i)$ from $\tilde{x}_t(i)$. This channel generates a strong link between expectations of current changes in aggregate (idiosyncratic) demand and past changes in idiosyncratic (aggregate) demand (the non-diagonal elements of matrix A). The matrix A in equation (17) captures these two complementary channels.

Proposition 2 shows that perfect unobservability of indistinguishable fundamental shocks generates serially-correlated expectations on changes in aggregate and idiosyncratic demand, $\mathbb{E}_t[u_t]$ and $\mathbb{E}_t[\tilde{v}_t(i)]$. The general intuition for this result is straightforward. Equation (17) shows that under perfect unobservability of indistinguishable fundamental shocks each firm i forms expectations about distinct shocks using information contained in present and past changes of sectoral demand (i.e., $\tilde{x}_{t-s}(i)$, for $s \in \{0, 1, 2, \dots\}$). The inference process generates a strong co-movement between *expected* changes in aggregate demand and idiosyncratic demand. To outline the intuition, consider the following simple example. Suppose that the firm at time $t - 1$ expects exogenous disturbances in aggregate and idiosyncratic demand to

be equal to zero at time t (i.e., $\mathbb{E}_{t-1}[u_t] = \mathbb{E}_{t-1}[\tilde{v}_t(i)] = 0$), and instead the realizations for these exogenous changes are $u_t > 0$ and $\tilde{v}_t(i) = 0$, respectively. In this example, the change in sectoral demand is positive (i.e., $\tilde{x}_t(i) > 0$). Once the firm updates expectations on the distinct components, it attributes positive values to both components $\mathbb{E}_t[u_t]$ and $\mathbb{E}_t[\tilde{v}_t(i)]$ despite the realized value for the single idiosyncratic component $\tilde{v}_t(i)$ being zero. Perfect unobservability of indistinguishable fundamental shocks generates a strong co-movement between *expected* changes in aggregate demand and idiosyncratic demand.

The complexity of the system prevents the derivation of a general solution for the correlation coefficients $\{\rho_1, \rho_2, \rho_3\}$. Thus, we derive an analytical solution for the correlation coefficients $\{\rho_1, \rho_2, \rho_3\}$ in the special case of equal persistence in changes in aggregate and idiosyncratic demand (i.e., $\rho_u = \rho_v$) and then resort to numerical simulations to study the more general case (i.e., $\rho_u \neq \rho_v$).

Corollary 1 (Correlation coefficients under perfect unobservability of indistinguishable fundamental shocks for $\rho_u = \rho_v$). *Assume the same persistence in changes of aggregate and idiosyncratic demand, $\rho_u = \rho_v$. The correlation coefficients $\{\rho_1, \rho_2, \rho_3\}$ have the following relationship:*

$$\rho_1 < 1 \quad \text{and} \quad \rho_2 = \rho_3 = 1,$$

which is consistent with the empirical relationships across correlation coefficients in Proposition 1.

Proof: See Appendix F.2. \square

Corollary 1 shows that in the special case of the same persistence in changes in aggregate and idiosyncratic demand (i.e., $\rho_u = \rho_v$), the relationships across correlations coefficient $\{\rho_1, \rho_2, \rho_3\}$ are consistent with the fundamental relationship outlined in Proposition 1 (i.e., $\rho_3 = \rho_2 > \rho_1$).

To assess the quantitative performance of the model and ensure results hold for the general case of $\rho_u \neq \rho_v$, we resort to numerical simulations. We choose numerical values for the persistence of changes in aggregate and idiosyncratic demand, ρ_u and ρ_v , respectively, to ensure the critical condition $\rho_1 < \rho_2$ holds in the model of perfect information and information with noise (i.e., $\rho_u \in [\rho_v, 0.99]$, see Proposition 4 and 3). We calibrate the set of parameters

$\{\tau/\sigma, \rho_v\} \in \{0.5, 1, 2, 3, 4, 5\} \times \{0.01, 0.02, \dots, 0.98, 0.99\}$ to minimize the distance between the observed correlation coefficients $\{\rho_1, \rho_2\}$ in panel (a) of Figure 1 and the equivalent statistics in the model.²³ In the case of perfect unobservability of indistinguishable fundamental shocks, the calibration procedure shows that the model replicates the observed statistics in panel (a) for a wide range of parameter values, which is consistent with the finding that unobservability of indistinguishable fundamental shocks without presence of noise generates a high correlation between expectations about changes in sectoral and aggregate demand. Thereby, we set the benchmark value of $\tau/\sigma = 3$ and $\rho_v = 0.44$ to facilitate comparison with the alternative information structures considered in the next section.

Figure 2 shows numerical simulations. Panels (a) and (b) show that the predictions from the simulated model are consistent with the observed relation across correlation coefficients in the data ($\rho_1 < \rho_2 = \rho_3$) in Proposition 1.

[Figure 2 about here.]

4 Information with Noise and Perfect Information

In this section, we use the empirical results established in Proposition 1 to assess whether the alternative approaches of information with noise or perfect information replicate the empirical correlation coefficients.

Information with Noise. In an environment containing information with noise, each firm $h \in [0, 1]$ in industry i observes aggregate demand (u_t) with a noise ($\delta_{1,t}(i, h)$) and idiosyncratic demand ($\tilde{v}_t(i)$) with a noise ($\delta_{2,t}(i, h)$), such that the signals observed by the firm are:

$$y_t(i, h) = u_t + \delta_{1,t}(i, h), \quad (18)$$

$$z_t(i, h) = \tilde{v}_t(i) + \delta_{2,t}(i, h), \quad (19)$$

where $\delta_{1,t}(i, h) \sim \mathcal{N}(0, \sigma_y^2)$ and $\delta_{2,t}(i, h) \sim \mathcal{N}(0, \tau_z^2)$ are noise shocks orthogonal to each other.²⁴

²³We normalize the parameter σ equal to one. Excluding several outliers, the number of samples for the calibration is 15 sectors.

²⁴This approach to introduce information noise, in which the values of noises are heterogeneous across firms, is similar to Woodford (2003), Angeletos and La'O (2009), or rational inattention models by Mackowiak and Wiederholt (2009).

The information structure in equations (18) and (19) requires each firm i to infer the precise realization of fundamental shocks, u_t and \tilde{v}_t , from observed noisy signals $y_t(i, h)$ and $z_t(i, h)$, respectively. We solve the inference problem by applying the Kalman filter. Defining the Kalman gains for the filtering processes for u_t and $\tilde{v}_t(i)$ as $\tilde{\lambda}_u$ and $\tilde{\lambda}_v$, respectively, the following analytical results hold.

Proposition 3 (Correlation coefficients under information with noise). *If $\tilde{\lambda}_u = \tilde{\lambda}_v \equiv \tilde{\lambda}$, the values for the correlation coefficients $\{\rho_1, \rho_2, \rho_3\}$ satisfy the following relations:*

$$\begin{aligned} \rho_1 < \rho_2 > \rho_3 & \text{ if } \rho_u > \rho_v, \\ \rho_1 = \rho_2 = \rho_3 & \text{ if } \rho_u = \rho_v, \\ \rho_1 > \rho_2 < \rho_3 & \text{ if } \rho_u < \rho_v, \end{aligned}$$

which are equivalent to the perfect information case in Proposition 4 and inconsistent with empirical evidence in Proposition 1. If $\tilde{\lambda}_u < \tilde{\lambda}_v$, the values for the correlation coefficients $\{\rho_1, \rho_2, \rho_3\}$ satisfy the following relations:

$$\rho_1 < \rho_2 > \rho_3 \quad \left. \vphantom{\rho_1} \right\} \text{ if } \rho_u > \rho_v + \varepsilon,$$

$$\left. \begin{array}{l} \rho_1 \geq \rho_2 > \rho_3, \\ \rho_1 \geq \rho_2 = \rho_3, \\ \rho_1 \geq \rho_2 < \rho_3, \end{array} \right\} \text{ or } \left. \vphantom{\rho_1} \right\} \text{ where } \varepsilon > 0 \text{ satisfies } \rho_1 < \rho_2 \text{ under } \tilde{\lambda}_u < \tilde{\lambda}_v \text{ and is increasing if } \rho_u \leq \rho_v + \varepsilon$$

in $\tilde{\lambda}_v$ and decreasing in ρ_v and $\tilde{\lambda}_u$. These predictions are inconsistent with the empirical evidence in Proposition 1.

Proof: See Appendix F.4. \square

Proposition 3 shows that in a noise information environment with $\tilde{\lambda}_u = \tilde{\lambda}_v \equiv \tilde{\lambda}$, the relationship among correlation coefficients $\{\rho_1, \rho_2, \rho_3\}$ is the same as in the case of imperfect information. In the limiting case of the Kalman gains converging to unity (i.e. $\tilde{\lambda}_u = \tilde{\lambda}_v \equiv \tilde{\lambda} \rightarrow 1$), the information with noise produces the same correlation coefficients $\{\rho_1, \rho_2, \rho_3\}$ as in the case of perfect information.

Proposition 3 shows that if $\tilde{\lambda}_u < \tilde{\lambda}_v$, the noise information model also is unable to replicate the observed correlation coefficients $\{\rho_1, \rho_2, \rho_3\}$ in Proposition 1. The assumption of $\tilde{\lambda}_u < \tilde{\lambda}_v$ is strongly supported by the data across several dimensions. It is consistent with studies on rational inattention, which establish that firms are more attentive to idiosyncratic shocks than aggregate shocks, as shown in Mackowiak and Wiederholt (2009). Foerster et al.

(2011) and Garin et al. (2018) show that idiosyncratic shocks are more volatile and play a critical role in explaining fluctuations in real activity in U.S. data, and that firms' pricing decisions are more responsive to idiosyncratic shocks than aggregate shocks. Kumano et al. (2014) establish that sectoral idiosyncratic shocks are critical in explaining fluctuations in Japanese industrial production.²⁵

Figure 3 shows numerical simulations for the correlation coefficients under information with noise. We set $\sigma_y^2 = 1$ and $\tau_z^2 = 1$ to equate the degree of noise on current changes in aggregate and idiosyncratic demand and ensure condition $\tilde{\lambda}_u < \tilde{\lambda}_v$ holds. The minimum distance estimator selects $\tau/\sigma = 3$ and $\rho_v = 0.43$, and the figure shows numerical simulation for values of ρ_u within the range $[\rho_v, 0.99]$. Panel (a) shows the relationship between ρ_1 (y-axes) and ρ_2 (x-axes), and panel (b) shows the relationship between ρ_2 (y-axes) and ρ_3 (x-axes). By comparing the entries against those in Figure 1, we see that the model of information with noise consistently generates co-movements between correlation coefficients $\{\rho_1, \rho_2, \rho_3\}$ that are inconsistent with the empirical relationships outlined in Proposition 1.

[Figure 3 about here.]

Perfect Information. Under perfect information, each firm i perfectly observes the sequence of changes in aggregate and sector-specific demand u_{t-s} and $\tilde{v}_{t-s}(i)$, for $s \in \{0, 1, 2, \dots\}$, respectively. The persistence of the autoregressive parameters ρ_u and ρ_v , described in equations (13) and (15), is critical in replicating the empirical values of correlation coefficients $\{\rho_1, \rho_2, \rho_3\}$, as outlined in the next proposition.

Proposition 4 (Correlation coefficients under perfect information). *Under perfect information, the values for the correlation coefficients $\{\rho_1, \rho_2, \rho_3\}$ satisfy the following relations:*

$$\begin{aligned} \rho_1 < \rho_2 > \rho_3 & \text{ if } \rho_u > \rho_v, \\ \rho_1 = \rho_2 = \rho_3 & \text{ if } \rho_u = \rho_v, \\ \rho_1 > \rho_2 < \rho_3 & \text{ if } \rho_u < \rho_v. \end{aligned}$$

²⁵Consistent with this evidence, sectoral price changes in Japan are largely heterogeneous. In the economy, the proportion of goods that increase prices from the previous year minus the proportion of goods that decrease prices from the previous year falls within the range of only 40 to -40 percent. See https://www.boj.or.jp/en/research/research_data/cpi/cpirev.pdf

Proof: See Appendix F.3. \square

Proposition 4 shows that under perfect information, the model fails to replicate the fundamental relation across correlation coefficients outlined in Proposition 1. If the persistence of the change in aggregate demand is larger than the persistence of the change in idiosyncratic demand (i.e., $\rho_u > \rho_v$), changes in aggregate demand are important for expectations in the next period and therefore $\rho_1 < \rho_2$. However, under the same condition aggregate shocks are less relevant for the correlation coefficient for the difference in expected changes of aggregate and sectoral demand, such that $\rho_3 < \rho_2$, which is inconsistent with the empirical findings in Proposition 1. An equivalent result holds for the case of a smaller persistence of change in aggregate demand than persistence of change in idiosyncratic demand (i.e., $\rho_u < \rho_v$). In summary, the model with perfect information is inconsistent with the empirical relation across correlation coefficients outlined in Proposition 1.²⁶

Figure 4 shows numerical simulations for the model with perfect information. The minimum distance estimator selects $\tau/\sigma = 3$ and $\rho_v = 0.44$, and the figure shows numerical simulations for the values of ρ_u within the range $[\rho_v, 0.99]$. Panel (a) shows the relation between ρ_1 (y-axes) and ρ_2 (x-axes), and panel (b) shows the relation between ρ_2 (y-axes) and ρ_3 (x-axes). Comparing the entries against those in Figure 1 shows that the model with perfect information consistently generates the observed relation, $\rho_1 < \rho_2$, as reported in panel (a), but it fails to generate the observed relation, $\rho_2 = \rho_3$. The model with perfect information fails to replicate the co-movements in the data.

[Figure 4 about here.]

5 Compounded Noiseless Shocks and Inflation

Motivated by the results in the proceeding sections, we now investigate the effect of compounded noiseless shocks for the sensitivity of price changes to movements in aggregate demand. We derive the standard New Keynesian Phillips curve under information frictions with noiseless shocks and study the theoretical link between the degree of information heterogeneity and the sensitivity of prices to aggregate demand. We use sector-level data for 29

²⁶Our findings based on survey data for Japan corroborate the recent results in Coibion and Gorodnichenko (2015a) on the large persistent in forecast errors on inflation based on survey data for the U.S. Several studies establish implausibility of perfect information. See, for example, Jonung (1981), Jonung and Laidler (1988), Mankiw et al. (2003), Carroll (2003), Kiley (2007), Fuhrer (2012), and Fuhrer et al. (2012).

major sectors in Japan to quantify the degree of shock heterogeneity and test the empirical relationship between the sensitivity of prices to changes in aggregate demand.

The optimal price-setting rule in equation (7) under imperfect information is:

$$p_t(i) = r\mathbb{E}[p_t|\mathcal{H}_t(i)] + (1-r)\mathbb{E}[q_t + v_t(i)|\mathcal{H}_t(i)] = r\mathbb{E}[p_t|\mathcal{H}_t(i)] + (1-r)x_t(i), \quad (20)$$

where $r \equiv \frac{(\eta-1)(1-\epsilon)}{\epsilon+\eta(1-\epsilon)}$ and $\mathbb{E}[\cdot|\mathcal{H}_t(i)]$ represents the expectations, which are conditional on the information set of firm i in period t . Equation (20) shows that the information set is critical in the formation of expectations, and it plays an important role in the optimal pricing rule.

To embed nominal price rigidities in the model, we follow Calvo (1983) and assume that each firm i retains the same price with an exogenous probability $\theta \in (0, 1)$. The optimal price for firm i (i.e., $p_t^*(i)$) is equal to current expectations of the weighted average of present and future prices, as expressed by the following pricing rule:

$$\begin{aligned} p_t^*(i) &= (1-\beta\theta) \sum_{j=0}^{\infty} (\beta\theta)^j \mathbb{E}[p_{t+j}(i)|\mathcal{H}_t(i)] \\ &= (1-\beta\theta) \sum_{j=0}^{\infty} (\beta\theta)^j [r\mathbb{E}[p_{t+j}|\mathcal{H}_t(i)] + (1-r)\mathbb{E}[x_{t+j}(i)|\mathcal{H}_t(i)]], \end{aligned} \quad (21)$$

where the second equation is derived by using the optimal pricing rule in equation (20). Equation (21) shows that each firm i sets prices as a weighted average of the firm's expectations about current and *expected* future prices (i.e., the optimal price under flexible prices) based on the current information set ($\mathcal{H}_t(i)$).

The Equilibrium Average Price. By solving equation (21), we derive the equilibrium average price. Since disturbances to present and past aggregate demand (i.e., q_{t-s} for $s = 1, 2, 3, \dots$) remain unknown to the firm in each period t , we infer expectations about q_t from present and the infinite past sectoral demand (i.e., $x_{t-s}(i)$ for $s = 0, 1, 2, \dots$). The next proposition characterizes the equilibrium average price.

Proposition 5 (Analytical solution to the equilibrium average price).

The equilibrium average price is given by

$$p_t^* = [\theta + (1-\theta)a_1]p_{t-1} + (1-\theta) \sum_{j=0}^{\infty} a_{2+j}q_{t-j}, \quad (22)$$

where for $j \in \{4, 5, 6, \dots\}$,

$$a_0(j) = 1 - [(1-\beta\theta)r + \beta\theta a_1](1-\theta)\lambda(1-\kappa_{j-2}),$$

$$\begin{aligned}
a_1 &= [\theta + (1 - \theta)a_1] [(1 - \beta\theta)r + \beta\theta a_1], \\
a_2 &= [a_0(2) - \beta\theta\lambda]^{-1} \left[\begin{aligned} &(1 - \beta\theta)(1 - r) + \beta\theta a_3 \\ &+ [(1 - \beta\theta)r + \beta\theta a_1] (1 - \theta) \sum_{m=1}^{\infty} a_{2+m} \kappa_m \frac{\lambda_0}{1 - \lambda_0} \prod_{k=1}^m (1 - \lambda_{k-1}) \end{aligned} \right], \\
a_3 &= a_0^{-1}(3) \left[\begin{aligned} &[(1 - \theta)a_2 [(1 - \beta\theta)r + \beta\theta a_1] + \beta\theta a_2] \lambda(1 - \lambda) + \beta\theta a_4 \\ &+ [(1 - \beta\theta)r + \beta\theta a_1] (1 - \theta) \sum_{m=2}^{\infty} a_{2+m} \kappa_m \frac{\lambda_1}{1 - \lambda_1} \prod_{k=2}^m (1 - \lambda_{k-1}) \end{aligned} \right], \\
a_j &= a_0^{-1}(j) \left[\begin{aligned} &[(1 - \theta)a_2 [(1 - \beta\theta)r + \beta\theta a_1] + \beta\theta a_2] \lambda(1 - \lambda)^{j-2} \\ &+ [(1 - \beta\theta)r + \beta\theta a_1] (1 - \theta) \sum_{k=1}^{j-3} a_{j-k} (1 - \kappa_k) \lambda(1 - \lambda)^{j-k-2} \\ &+ \beta\theta a_{j+1} + [(1 - \beta\theta)r + \beta\theta a_1] (1 - \theta) \sum_{m=j-1}^{\infty} a_{2+m} \kappa_m \frac{\lambda_{j-2}}{1 - \lambda_{j-2}} \prod_{k=j-1}^m (1 - \lambda_{k-1}) \end{aligned} \right], \\
\lambda &= \frac{-\frac{\sigma^2}{\tau^2} + \sqrt{\left(\frac{\sigma^2}{\tau^2}\right)^2 + 4\frac{\sigma^2}{\tau^2}}}{2}, \lambda_s = \frac{\lambda_{s-1}\tau^2 + \sigma^2}{\lambda_{s-1}\tau^2 + \sigma^2 + \tau^2}, \lambda_0 = 1, \kappa_s = \frac{\tau^2\lambda}{\tau^2\lambda_{s-1} + \sigma^2 + \tau^2\lambda}, \kappa_0 = 0.
\end{aligned}$$

Proof: See Appendix F.5. \square

Equation (22) shows that the equilibrium average price depends on the past equilibrium price (i.e., p_{t-1}) and the infinite sequence of present and past aggregate demand (i.e., q_{t-j} for $j \in \{0, 1, 2, \dots\}$). The relevance for current prices is determined by the degree of shock heterogeneity, which requires the characterization of the infinite sequence of coefficients a_{2+j} for $j = 0, 1, 2, \dots$. The parameter $\lambda \in (0, 1)$ is the Kalman gain on q_t in the filtering process that determines the informativeness of sectoral demand $x_t(i)$ about aggregate demand q_t and is inversely related to the ratio τ/σ .²⁷ Proposition 5 shows that past shocks to aggregate demand are important since their distinct realizations remain unknown to the firm and their relevance to current prices depends on the degree of shock heterogeneity.

Using the average price defined in equation (22), we derive the inflation rate, π_t , as the change in the average price from period t to period $t - 1$, which yields:

$$\pi_t = [\theta + (1 - \theta)a_1] \pi_{t-1} + (1 - \theta) \sum_{j=0}^{\infty} a_{2+j} u_{t-j}. \quad (23)$$

Equation (23) is the Phillips curve that accounts for imperfect information resulting from indistinguishability of distinct shocks. Since the infinite number of unknown coefficients $\{a_1, a_2, a_3, \dots\}$ are highly non-linear and mutually dependent, as shown in Proposition 5,

²⁷The negative relationship shows that if the ratio τ/σ is large, on average, a large portion of changes in $x_t(i)$ is caused by $v_t(i)$ and therefore unrelated to changes in aggregate demand q_t .

we cannot analytically characterize the relationship between the structure of information and inflation. We therefore resort to numerical approximation of the analytical solution in Proposition 5.

5.1 Quantitative Assessment

An analytical solution for the Phillips curve in equation (23) is not feasible since it involves tracking the infinite sequence of past demand shocks. To implement the solution in a tractable way, we assume that the effect of exogenous disturbances to changes in aggregate demand becomes negligible after a large but finite number of N periods. The assumption enables us to recursively solve the system with the algorithm described in the next corollary.

Corollary 2 (Numerical approximation of the equilibrium average price).

Suppose $a_{N+1} = 0$. The equilibrium average price is given by

$$p_t^* = [\theta + (1 - \theta)a_1] p_{t-1} + (1 - \theta) \sum_{j=0}^{N-2} a_{2+j} q_{t-j},$$

where

$$a_1 = [\theta + (1 - \theta)a_1] [(1 - \beta\theta)r + \beta\theta a_1],$$

$$\begin{bmatrix} a_2 \\ a_3 \\ a_4 \\ \dots \\ a_N \end{bmatrix} = \begin{bmatrix} 1 & -K_{2,3} & -K_{2,4} & \dots & \dots & -K_{2,N} \\ -K_{3,2} & 1 & -K_{3,4} & \ddots & \ddots & \vdots \\ -K_{4,2} & -K_{4,3} & 1 & -K_{4,5} & \ddots & \vdots \\ \vdots & \vdots & \ddots & 1 & \ddots & -K_{N-2,N} \\ \vdots & \vdots & \ddots & \ddots & 1 & -K_{N-1,N} \\ -K_{N,2} & -K_{N,3} & \dots & \dots & -K_{N,N-1} & 1 \end{bmatrix}^{-1} \begin{bmatrix} k_2(1 - \beta\theta)(1 - r) \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

and,

$$\begin{aligned} K_{2,3} &\equiv k_2 [\beta\theta + k_0(1 - \theta)\kappa_1\lambda_0], K_{2,4} \equiv k_2 k_0(1 - \theta)\lambda\kappa_2(1 - \lambda_1)\lambda_0, \\ K_{2,N} &= k_2 k_0(1 - \theta)\kappa_{N-2}(1 - \lambda_{N-3})\dots(1 - \lambda_1)\lambda_0, K_{3,2} \equiv k_3 k_1(1 - \lambda), \\ K_{3,4} &\equiv k_3 [\beta\theta + k_0(1 - \theta)\kappa_2\lambda_1], K_{4,2} \equiv k_4 k_1(1 - \lambda)^2, \\ K_{4,3} &\equiv k_4 k_0(1 - \theta)(1 - \kappa_1)\lambda(1 - \lambda), K_{4,5} \equiv k_4 [\beta\theta + k_0(1 - \theta)\kappa_3\lambda_2(1 - \lambda_2)(1 - \lambda_3)], \\ K_{N-2,N} &\equiv k_{N-2} k_0(1 - \theta)\kappa_{N-2}(1 - \lambda_1)\lambda_0, K_{N-1,N} \equiv k_{N-1} [\beta\theta + k_0(1 - \theta)\kappa_{N-2}\lambda_1], \\ K_{N,2} &\equiv k_N k_1(1 - \lambda)^{N-2}, K_{N,3} \equiv k_N k_0(1 - \theta)\lambda(1 - \lambda)^{N-3}, \\ K_{N,N-1} &\equiv k_N k_0(1 - \theta)(1 - \kappa_{N-3})\lambda(1 - \lambda). \end{aligned}$$

for

$$\begin{aligned}
k_0 &\equiv (1 - \beta\theta)r + \beta\theta a_1, k_1 \equiv [(1 - \theta)k_0 + \beta\theta] \lambda, \\
k_2 &= [a_0(2) - \beta\theta\lambda]^{-1}, k_3 = a_0^{-1}(3), k_j = a_0^{-1}(j), \\
a_0(j) &= 1 - [(1 - \beta\theta)r + \beta\theta a_1] (1 - \theta)\lambda (1 - \kappa_{j-2}), \\
\lambda &= \frac{-\frac{\sigma^2}{\tau^2} + \sqrt{\left(\frac{\sigma^2}{\tau^2}\right)^2 + 4\frac{\sigma^2}{\tau^2}}}{2}, \lambda_s = \frac{\lambda_{s-1}\tau^2 + \sigma^2}{\lambda_{s-1}\tau^2 + \sigma^2 + \tau^2}, \lambda_0 = 1, \kappa_s = \frac{\tau^2\lambda}{\tau^2\lambda_{s-1} + \sigma^2 + \tau^2\lambda}, \kappa_0 = 0.
\end{aligned}$$

Proof: See Appendix F.6. \square

Corollary 2 provides a numerical algorithm to approximate the analytical solution of the equilibrium average price in Proposition 5 that enables us to investigate numerically the interplay between shock heterogeneity, described by the ratio τ/σ , and the degree of nominal price rigidity, described by the parameter θ , on the slope coefficient of the Phillips curve. In the numerical implementation of the model, we set the length of the approximation equal to 98 quarters (i.e., $N = 100$) and describe the approximate equation (23) as:

$$\begin{aligned}
\pi_t &= [\theta + (1 - \theta)a_1] \pi_{t-1} + (1 - \theta)a_2 u_t + (1 - \theta) \sum_{j=1}^{98} a_{2+j} u_{t-j}, \\
\pi_t &= \alpha_1 \pi_{t-1} + \alpha_2 u_t + (1 - \theta) \sum_{j=1}^{98} a_{2+j} u_{t-j}, \tag{24}
\end{aligned}$$

where $\alpha_1 = \theta + (1 - \theta)a_1$ and $\alpha_2 = (1 - \theta)a_2$. We consider shocks that occurred earlier than 98 quarters as negligible to the change in the response of current inflation. Using equation (24), we investigate the effect of parameters τ/σ and θ on the coefficients α_1 and α_2 that determine the response of inflation to past inflation and movements in demand (given that equation (11) provides $u_t = q_t - q_{t-1}$), respectively.

The Slope of the Phillips Curve. To simulate the system and study the theoretical properties of the model, we calibrate the Phillips curve with standard parameter values. We set $\eta = 2$, $\epsilon = 2/3$, $r = [(\eta - 1)(1 - \epsilon)]/[\epsilon + \eta(1 - \epsilon)] = 0.5$, and $\beta = 0.99$. While we estimate the degree of shock heterogeneity (i.e., τ/σ) in the next section, to investigate the properties of the model, we allow the ratio τ/σ to cover the wide range of values $[0, 3]$, and similarly, we allow the degree of nominal price rigidity (i.e., θ) to cover the whole range of admissible values $[0, 1]$.

Panel (a) in Figure 5 shows the sensitivity of parameters α_1 and α_2 in the Phillips curve equation (24) to the degree of nominal price rigidity, θ . As expected from the Calvo price setting mechanism, the increase in nominal price rigidities generates a rise in the coefficient α_1 since a low frequency of price changes increases the importance of past inflation in the determination of current inflation.²⁸ The increase in the degree of nominal price rigidity also generates a decrease in the coefficient α_2 since the sensitivity of individual prices to the current aggregate shock is lowered by less sensitive average prices, and the sensitivity of average prices to the same shock is directly dampened by the increase in Calvo parameter (θ). Panel (b) shows that the coefficient α_2 depends on the changes in information (shock) structures (i.e., τ/σ). The larger the degree of dispersion in idiosyncratic demand shocks relative to aggregate demand shocks, the more each firm attributes changes in sectoral demand to movements in idiosyncratic demand that are expected to be equal to zero on average in the aggregate economy, and therefore unimportant in the determination of aggregate prices. Therefore, an increase in shock heterogeneity decreases the response of aggregate prices to changes in aggregate demand, as encapsulated by the negative relationship between τ/σ and α_2 . Individual prices become less sensitive to the current aggregate shock (i.e., α_2 decreases), and consequently, the average price becomes less sensitive to aggregate shocks.

[Figure 5 about here.]

Response of Inflation to Shocks. How does the degree of shock heterogeneity influence the response of inflation to aggregate demand? To address this question, we simulate the model to determine the response of inflation to a one-period, negative aggregate demand shock. Figure 6 shows that the degree of imperfect information is critical in the response of inflation to the aggregate shock. The larger the degree of information heterogeneity, as represented by the ratio τ/σ , the lower the response of inflation to changes in current demand. Indistinguishability of distinct shocks dampens the response of inflation to changes in aggregate demand and increases the persistence of the adjustment of inflation to the aggregate shock. The intuition behind this result is straightforward. Given the impossibility of disentangling changes in aggregate and idiosyncratic demand, each firm conjectures that changes in sectoral demand, caused fully by changes in aggregate demand in this case, are

²⁸Note that α_1 is independent from information frictions because firms' filtering process about q_t is assumed to depend only on $x_{t-s}(i)$ for $s \in \{0, 1, 2, \dots\}$, and firms are assumed not to extract information about q_t from p_{t-1} .

partially caused by changes in idiosyncratic demand. This misperception induces the firms to decrease the response to aggregate shocks. If the ratio of τ/σ is large, firms conjecture that a large portion of the sectoral demand shock occurs due to idiosyncratic shock and aggregate demand does not change. Consequently, firms expect that the average price in the period is almost the same as that in the previous period, and prices are less sensitive to aggregate shocks.

[Figure 6 about here.]

5.2 Empirical Assessment

In this section, we provide an empirical evaluation of the relevance of the degree of information rigidity to inflation dynamics using Japanese data. We investigate the effect of information frictions with noiseless shocks for the diminished response of inflation to changes in aggregate demand.

Monte Carlo Experiment. To ensure our empirical analysis is powerful in estimating the effect of shock heterogeneity on the sensitivity of inflation to real activity, we conduct a Monte Carlo experiment. We use the theoretical model as the data-generating process and feed the system with aggregate shocks, u_t , to generate data series for inflation, π_t , for 10,000 periods, allowing for different degrees of information heterogeneity, as represented by the ratio, τ/σ , within the range of values $[0, 3]$ and for the range of degrees of nominal price rigidity $[0.25, 0.5]$. To ensure results are consistent across alternative specifications of the Phillips curve, we estimate the slope coefficient that captures the sensitivity of prices to real activity for two representative versions of the Phillips curve. First, we examine a New Keynesian Phillips curve that features forward-looking expectations on inflation, and second, a hybrid Phillips curve with backward- and forward-looking expectations on inflation. The two specifications are:

$$\begin{aligned}\pi_t &= \beta \mathbb{E}[\pi_{t+1} | \mathcal{H}_t(i)] + \kappa y_t, \\ \pi_t &= (1 - \gamma) \mathbb{E}[\pi_{t+1} | \mathcal{H}_t(i)] + \kappa y_t + \gamma \pi_{t-1},\end{aligned}$$

where $y_t \equiv \sum_{j=0}^2 (u_{t-j} - \pi_{t-j})$, consistent with the definition of aggregate demand in equation (3).²⁹ Panels (a) and (b) in Figure 7 show estimates for the coefficient κ in the New Keynesian

²⁹In the estimation, we set $\beta = 0.99$ and estimate parameters γ and κ using GMM with lagged inflation. Although not the main focus of this study, γ changes only slightly along with τ/σ and θ .

and hybrid Phillips curve, respectively, for values of τ/σ within the range $[0, 3]$ (on the x-axes) and different degrees of nominal price rigidity (θ , different lines). For both specifications, the slope coefficient κ is monotonically decreasing in θ and τ/σ , indicating that econometric estimates correctly attribute the increase in information heterogeneity to a reduction in the sensitivity of inflation to real activity, irrespective of the degree of nominal price rigidity, as predicted by the theoretical model.

[Figure 7 about here.]

Estimation of Information Heterogeneity. The estimation of shock heterogeneity requires detailed sector-level data. We use the Financial Statements Statistics of Corporations by Industry, compiled by the Ministry of Finance of Japan, which provide publicly available quarterly data on sector-level sales of Japanese firms.³⁰ Data cover the sample period 1975:Q1-2017:Q3 for 29 major sectors in the economy and we proxy aggregate shocks by the principal component of movements in sales growth across sectors, similar to the approach in Foerster et al. (2011) and Garin et al. (2018). To implement the procedure, we estimate changes in aggregate sales, u_t , as the principal (first) component of $\tilde{x}_t(i)$ across sectors, $i \in \{1, 2, \dots, 29\}$, by calculating it as $u_t = \sum_{i=1}^{29} \Lambda_i \tilde{x}_t(i)$, where Λ_i is the loading factor of $\tilde{x}_t(i)$.³¹ We proxy changes in idiosyncratic demand, $\tilde{v}_t(i)$, by subtracting the estimated principal component from changes in sectoral demand:³² $\tilde{x}_t(i) - (\sum_{i=1}^{29} \Lambda_i)^{-1} u_t = \tilde{x}_t(i) - (\sum_{i=1}^{29} \Lambda_i)^{-1} \sum_{i=1}^{29} \Lambda_i \tilde{x}_t(i)$, where the term $(\sum_{i=1}^{29} \Lambda_i)^{-1}$ normalizes u_t .³³

We proxy the variance of the aggregate shock, σ^2 , with the average of the square of the extracted principle component for alternative moving windows of size $2k + 1$:

$$\sigma_t^2 = \frac{1}{2k + 1} \sum_{s=-k}^k \left(\sum_{i=1}^{29} \Lambda_i \tilde{x}_{t+s}(i) \right)^2. \quad (25)$$

³⁰The data is available at <http://www.mof.go.jp/english/pri/reference/ssc/index.htm>.

³¹The proportion of the variance of the first component is around 20%, which is considerably larger than the variance of the second component (7%), suggesting that the second principal component plays a limited role in aggregate shocks.

³²To ensure results are robust to alternative normalizations, we implement alternative specifications. First, we define $u_t = \sum_{i=1}^{29} \Lambda_i \tilde{x}_t(i)$ and $\tilde{x}_t(i) - u_t$, and second, we define $u_t = (\sum_{i=1}^{29} \Lambda_i)^{-1} \sum_{i=1}^{29} \Lambda_i \tilde{x}_t(i)$ and $\tilde{x}_t(i) - u_t$. Results remain unchanged across different normalization assumptions.

³³Since the proxy for aggregate shock is $u_t = \sum_{i=1}^{29} \Lambda_i \tilde{x}_t(i)$ and the sectoral shock is $\tilde{x}_t(i)$, the scale of aggregate shocks $\sum_{i=1}^{29} \Lambda_i$ may differ from the scale of sectoral shocks. Estimation results reveal that $\sum_{i=1}^{29} \Lambda_i \approx 4.7$, which we use to normalize u_t .

Similarly, we proxy the variance of the sectoral shock, τ^2 , with the average of the square of the proxy of idiosyncratic demand for alternative moving windows of size $2k + 1$:

$$\tau_t^2 = \frac{1}{2k + 1} \sum_{s=-k}^k \sum_{i=1}^{29} \left[\tilde{x}_{t+s}(i) - \left(\sum_{i=1}^{29} \Lambda_i \right)^{-1} \sum_{i=1}^{29} \Lambda_i \tilde{x}_{t+s}(i) \right]^2. \quad (26)$$

We compute the variance of each of the shocks in equations (25) and (26) using four alternative time windows: two-years ($k = 4$), three-years ($k = 6$), five-years ($k = 10$), and ten-years ($k = 20$), excluding the upper and lower 10% of the samples as outliers. We calculate the shock heterogeneity (τ_t/σ_t) as the ratio of the square root of the estimate of the variance of idiosyncratic shocks (τ_t^2) to that of aggregate shocks for each period (σ_t^2).

Panel (a) in Figure 8 shows the estimated series for shock heterogeneity, defined as the ratio of the variance of idiosyncratic shocks to the variance of aggregate shocks (τ_t/σ_t), for alternative time windows. Entries show that the degree of information heterogeneity has steadily increased throughout the sample period, with the ratio τ_t/σ_t rising from a value of 2 in the early 1980s to 4 in the mid-2000s, subsequently reaching a value of approximately 3 after 2010 in the 10-year window. Shorter time windows show similar dynamics, despite increasing volatility. Overall, the analysis detects a robust increase in information heterogeneity in the post-2008 period.³⁴

Estimation of the Phillips Curve. In this section, we use the proxy for information heterogeneity to assess the empirical importance of shock heterogeneity for the reduced sensitivity of inflation to changes in demand over time.

[Table 1 and 2 about here.]

To implement the estimation of the Phillips curve, we use insights from the theoretical model encapsulated in equation (24). Thus, we regress current inflation on past inflation (π_{t-1}), changes in current aggregate demand (u_t) and an interaction term between changes in current aggregate demand and the degree of shock heterogeneity ($u_t \times \tau_t/\sigma_t$) that captures the differential effect of shock heterogeneity for the effect of aggregate demand on current inflation. Since the response of prices to aggregate demand is dependent on past demand,

³⁴Movements in τ_t/σ_t are primarily driven by changes in τ_t since the value for σ_t remains broadly stable across the sample period, except during the period of the global financial crisis (2007:4Q to 2010:1Q). Appendix E shows that the aggregate shock series extracted from industry-level data are consistent with measures of aggregate shocks as proxied by the output gap.

we include aggregate demand with eight lags.³⁵ Table 1 shows the estimates for the Phillips curve with measures of shock heterogeneity based on time windows of two years (column (1)), three years (columns (2)), five years (column (3)) and ten years (column (4)), respectively. All entries show that current inflation is positively correlated with past inflation and current demand, in line with the fundamental prediction of the Phillips curve. The interaction term is negative, implying that a rise in shock heterogeneity reduces the positive correlation between inflation and aggregate demand, in accordance with the results of the analysis, showing that shock heterogeneity plays an empirically significant part in the reduced sensitivity of inflation to real activity.

The theoretical analysis in section 5.1 shows that the degree of nominal price rigidity is positively related to the flattening of the Phillips curve. To ensure empirical results are unbiased by the reduced degree of nominal price rigidity over the sample period, we control the estimation for the degree of nominal price rigidity by using a dummy variable equal to 1 for the period 2000-2017 when nominal price rigidities decreased (see evidence in Sudo et al. (2014) and Kurachi et al. (2016)). Table 2 reports the results. We enrich the estimation of the Phillips curve with two additional interaction terms. The first term interacts the dummy variable for nominal price rigidities with past inflation ($\pi_{t-1} \times dummy$) to capture the interplay between the degree of nominal price rigidity and the effect of past inflation on current inflation. The second term interacts the dummy variable for nominal price rigidities with current aggregate demand ($u_t \times dummy$) to capture the interplay between nominal price rigidities and current aggregate demand. Columns (1) to (4) show that the coefficient for the interaction term of past inflation with the dummy variable ($\pi_{t-1} \times dummy$) is negative, indicating that the positive correlation between current inflation and past inflation decreases with a decline in nominal price rigidities, in line with the predictions of our model outlined in section 5.1. The estimates for the interaction term of changes in demand with the dummy variable ($u_t \times dummy$) are either remarkably close to zero (column (1)) or insignificant (columns (2)-(4)), showing that the relationship between inflation and changes in aggregate demand remains broadly unchanged across periods with different degrees of nominal price rigidity. This finding corroborates the empirical evidence in Sudo et al. (2014) and Kurachi et al. (2016), which shows a consistent rise in the frequency of price adjustment across the universe of Japanese firms since the early 2000s. Important for our analysis, the interaction

³⁵The results continue to hold if we include no lags or if we include additional lags of aggregate demand.

term between aggregate demand and the degree of nominal price rigidity ($u_t \times \tau_t / \sigma_t$) remains negative and retains the same magnitude as the estimates in Table 1, showing that the effect of the degree of shock heterogeneity is broadly similar across periods with different degrees of nominal price rigidity.³⁶

6 Conclusion

This paper assesses the empirical relevance of two canonical approaches to imperfect information based on either presence of noise that blurs the observation of fundamental shocks or alternatively, the existence of compounded noiseless shocks whose distinct realizations remain unknown to agents. We find that information frictions, based on the indistinguishability of distinct shocks in absence of noise, outperform models with information noise and perfect information in replicating the persistence of expectations about changes in aggregate and sectoral demand from survey data for the universe of firms across 20 sectors in Japan.

We embed information frictions based on compounded noiseless shocks in an otherwise standard general equilibrium model with nominal price rigidities, and we establish a negative relationship between the degree of shock heterogeneity—represented by the ratio of the volatility of idiosyncratic shocks to the volatility of aggregate shocks—and the sensitivity of inflation to real activity. We test the theoretical implication using Japanese industry-level data, finding that the observed increase in shock heterogeneity plays a statistically significant role for the reduced sensitivity of inflation to changes in aggregate demand.

The analysis can be extended across several dimensions. Within the realm of models with information frictions, an interesting extension of this study would be to endogenize the acquisition of information, which is likely to interact with the degree of shock heterogeneity in determining the reaction of aggregate variables to exogenous disturbances, thereby having a non-trivial role for the sensitivity of inflation to aggregate demand. Two promising approaches to endogenize the information structure are the rational inattention approach (Mackowiak and Wiederholt (2009), Mackowiak et al. (2009), and Matejka et al. (2017)) and the choice of information acquisition (Hellwig and Veldkamp (2009) and Myatt and Wallace (2012)). Another interesting extension would be to use the theoretical framework to assess the effect of compounded noiseless shocks for optimal monetary policy to assess whether re-

³⁶Results continue to hold if we use changes in real aggregate demand. An appendix that details results is available from the authors on request.

sults are different from those of alternative models of imperfect information (Adam (2007), Lorenzoni (2010), Angeletos et al. (2016), and Tamura (2016)). Extending the analysis across these important dimensions remains an open task for future research.

References

- Acharya, S. (2017). Costly information, planning complementarities, and the phillips curve. *Journal of Money, Credit and Banking*, 49(4):823–850.
- Adam, K. (2007). Optimal monetary policy with imperfect common knowledge. *Journal of Monetary Economics*, 54(2):267–301.
- Akerlof, G., Dickens, W., and Perry, G. (1996). The macroeconomics of low inflation. *Brookings Papers on Economic Activity*, 27(1):1–76.
- Amador, M. and Weill, P.-O. (2010). Learning from Prices: Public Communication and Welfare. *Journal of Political Economy*, 118(5):866–907.
- Angeletos, G.-M. (2018). Frictional coordination. *Journal of the European Economic Association*, 16(3):563–603.
- Angeletos, G.-M., Iovino, L., and La’O, J. (2016). Real Rigidity, Nominal Rigidity, and the Social Value of Information. *American Economic Review*, 106(1):200–227.
- Angeletos, G.-M. and La’O, J. (2009). Incomplete Information, Higher-Order Beliefs and Price Inertia. *Journal of Monetary Economics*, 56(S):19–37.
- Ball, L. and Mazumder, S. (2011). Inflation Dynamics and the Great Recession. Economics Working Paper Archive 580, The Johns Hopkins University, Department of Economics.
- Blanchard, O. J., L’Huillier, J.-P., and Lorenzoni, G. (2013). News, Noise, and Fluctuations: An Empirical Exploration. *American Economic Review*, 103(7):3045–3070.
- Cacciatore, M. and Fiori, G. (2016). The Macroeconomic Effects of Goods and Labor Market Deregulation. *Review of Economic Dynamics*, 20:1–24.
- Calvo, G. (1983). Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics*, 12(3):383–398.
- Carroll, C. (2003). Macroeconomic Expectations of Households and Professional Forecasters. *The Quarterly Journal of Economics*, 118(1):269–298.

- Coibion, O. and Gorodnichenko, Y. (2015a). Information Rigidity and the Expectations Formation Process: A Simple Framework and New Facts. *American Economic Review*, 105(8):2644–2678.
- Coibion, O. and Gorodnichenko, Y. (2015b). Is the Phillips Curve Alive and Well after All? Inflation Expectations and the Missing Disinflation. *American Economic Journal: Macroeconomics*, 7(1):197–232.
- Coibion, O., Gorodnichenko, Y., and Kamdar, R. (2018). The Formation of Expectations, Inflation and the Phillips Curve. *Journal of Economic Literature*.
- Crucini, M. J., Shintani, M., and Takayuki, T. (2015). Noisy information, distance and law of one price dynamics across US cities. *Journal of Monetary Economics*, 74(C):52–66.
- Dupor, B., Kitamura, T., and Tsuruga, T. (2010). Integrating sticky prices and sticky information. *The Review of Economics and Statistics*, 92(3):657–669.
- Fernandez-Villaverde, J., Rubio-Ramirez, J., Sargent, T., and Watson, M. (2007). ABCs (and Ds) of Understanding VARs. *American Economic Review*, 97(3):1021–1026.
- Foerster, A., Sarte, P.-D., and Watson, M. (2011). Sectoral versus aggregate shocks: A structural factor analysis of industrial production. *Journal of Political Economy*, 119(1):1–38.
- Fuhrer, J. (2012). The Role of Expectations in Inflation Dynamics. *International Journal of Central Banking*, 8(1):137–165.
- Fuhrer, J. (2017). Japanese and U.S. Inflation Dynamics in the 21st Century. IMES Discussion Paper Series 17-E-05, Institute for Monetary and Economic Studies, Bank of Japan.
- Fuhrer, J., Olivei, G., and Tootell, G. (2012). Inflation Dynamics When Inflation Is Near Zero. *Journal of Money, Credit and Banking*, 44:83–122.
- Fukunaga, I. (2007). Imperfect Common Knowledge, Staggered Price Setting, and the Effects of Monetary Policy. *Journal of Money, Credit and Banking*, 39(7):1711–1739.
- Gaballo, G. (2018). Price Dispersion, Private Uncertainty, and Endogenous Nominal Rigidities. *Review of Economic Studies*, 85(2):1070–1110.

- Garin, J., Pries, M., and Sims, E. (2018). The relative importance of aggregate and sectoral shocks and the changing nature of economic fluctuations. *American Economic Journal: Macroeconomics*, 10(1):119–48.
- Hellwig, C. and Veldkamp, L. (2009). Knowing What Others Know: Coordination Motives in Information Acquisition. *Review of Economic Studies*, 76(1):223–251.
- Hellwig, C. and Venkateswaran, V. (2009). Setting the Right Prices for the Wrong Reasons. *Journal of Monetary Economics*, 56(S):57–77.
- IMF (2016). How Has Globalization Affected Inflation? In Fund, I. M., editor, *World Economic Outlook April 2006 Globalization and Inflation*, chapter 3. International Monetary Fund.
- Jonung, L. (1981). Perceived and Expected Rates of Inflation in Sweden. *American Economic Review*, 71(5):961–968.
- Jonung, L. and Laidler, D. (1988). Are Perceptions of Inflation Rational? Some Evidence for Sweden. *American Economic Review*, 78(5):1080–1087.
- Kaihatsu, S., Katagiri, M., and Shiraki, N. (2017). Phillips Curve and Price-Change Distribution under Declining Trend Inflation. Bank of Japan Working Paper Series 17-E-5, Bank of Japan.
- Kaihatsu, S. and Nakajima, J. (2015). Has Trend Inflation Shifted?: An Empirical Analysis with a Regime-Switching Model. Bank of Japan Working Paper Series 15-E-3, Bank of Japan.
- Kato, R. and Okuda, T. (2017). Market Concentration and Sectoral Inflation under Imperfect Common Knowledge. IMES Discussion Paper Series 17-E-11, Institute for Monetary and Economic Studies, Bank of Japan.
- Kiley, M. (2007). A Quantitative Comparison of Sticky-Price and Sticky-Information Models of Price Setting. *Journal of Money, Credit and Banking*, 39(s1):101–125.
- Kumano, Y., Muto, I., and Nakano, A. (2014). What explains the recent fluctuations in Japan’s output? a structural factor analysis of Japan’s industrial production. *Journal of the Japanese and International Economies*, 34:135 – 153.

- Kurachi, Y., Hiraki, K., and Nishioka, S. (2016). Does a Higher Frequency of Micro-level Price Changes Matter for Macro Price Stickiness?: Assessing the Impact of Temporary Price Changes. Bank of Japan Working Paper Series 16-E-9, Bank of Japan.
- L’Huillier, J.-P. (2017). Consumers’ imperfect information and price rigidities. Working papers series, Brandeis University.
- Lorenzoni, G. (2009). A Theory of Demand Shocks. *American Economic Review*, 99(5):2050–2084.
- Lorenzoni, G. (2010). Optimal Monetary Policy with Uncertain Fundamentals and Dispersed Information . *Review of Economic Studies*, 77(1):305–338.
- Lucas, R. (1972). Expectations and the Neutrality of Money. *Journal of Economic Theory*, 4(2):103–124.
- Lucas, R. (1973). Some International Evidence on Output-Inflation Tradeoffs. *American Economic Review*, 63(3):326–334.
- Mackowiak, B., Moench, E., and Wiederholt, M. (2009). Sectoral Price Data and Models of Price Setting. *Journal of Monetary Economics*, 56(S):78–99.
- Mackowiak, B. and Wiederholt, M. (2009). Optimal Sticky Prices under Rational Inattention. *American Economic Review*, 99(3):769–803.
- Mankiw, G. and Reis, R. (2002). Sticky Information versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve. *The Quarterly Journal of Economics*, 117(4):1295–1328.
- Mankiw, G., Reis, R., and Wolfers, J. (2003). Disagreement about inflation expectations. Working Paper 9796, National Bureau of Economic Research.
- Matejka, F., Mackowiak, B., and Wiederholt, M. (2017). The Rational Inattention Filter. Working Paper Series 2007, European Central Bank.
- Mavroeidis, S., Plagborg-Moller, M., and Stock, J. (2014). Empirical Evidence on Inflation Expectations in the New Keynesian Phillips Curve. *Journal of Economic Literature*, 52(1):124–88.

- Melosi, L. (2014). Estimating models with dispersed information. *American Economic Journal: Macroeconomics*, 6(1):1–31.
- Melosi, L. (2017). Signalling Effects of Monetary Policy. *Review of Economic Studies*, 84(2):853–884.
- Mourougane, A. and Ibaragi, H. (2004). Is There a Change in the Trade-Off Between Output and Inflation at Low or Stable Inflation Rates?: Some Evidence in the Case of Japan. OECD Economics Department Working Papers 379, OECD Publishing.
- Muth, J. (1961). Rational expectations and the theory of price movements. *Econometrica*, 29(3):315–335.
- Myatt, D. and Wallace, C. (2012). Endogenous Information Acquisition in Coordination Games. *Review of Economic Studies*, 79(1):340–374.
- Nimark, K. (2008). Dynamic Pricing and Imperfect Common Knowledge. *Journal of Monetary Economics*, 55(2):365–382.
- Nishizaki, K., Sekine, T., and Ueno, Y. (2014). Chronic Deflation in Japan. *Asian Economic Policy Review*, 9(1):20–39.
- Phelps, E. (1969). The New Microeconomics in Inflation and Employment Theory. *American Economic Review*, 59(2):147–160.
- Riggi, M. and Santoro, S. (2015). On the Slope and the Persistence of the Italian Phillips Curve. *International Journal of Central Banking*, 11(2):157–197.
- Roberts, J. (2004). Monetary Policy and Inflation Dynamics. Finance and Economics Discussion Series 2004-62, Board of Governors of the Federal Reserve System (U.S.).
- Sbordone, A. (2008). Globalization and inflation dynamics: the impact of increased competition. Staff Reports 324, Federal Reserve Bank of New York.
- Sims, C. (2003). Implications of rational inattention. *Journal of Monetary Economics*, 50(3):665–690.
- Sudo, N., Ueda, K., and Watanabe, K. (2014). Micro price dynamics under Japan’s lost decades. *Asian Economic Policy Review*, 9(1):44–64.

- Tamura, W. (2016). Optimal Monetary Policy and Transparency under Informational Frictions. *Journal of Money, Credit and Banking*, 48(6):1293–1314.
- Thomas, C. and Zanetti, F. (2009). Labor market reform and price stability: An application to the Euro Area. *Journal of Monetary Economics*, 56(6):885–899.
- Veirman, E. D. (2007). Which nonlinearity in the Phillips curve? The absence of accelerating deflation in Japan. Reserve Bank of New Zealand Discussion Paper Series DP2007/14, Reserve Bank of New Zealand.
- Veldkamp, L. and Wolfers, J. (2007). Aggregate shocks or aggregate information? costly information and business cycle comovement. *Journal of Monetary Economics*, 54:37–55.
- Woodford, M. (2003). Imperfect Common Knowledge and the Effects of Monetary Policy. in P. Aghion, R. Frydman, J. Stiglitz, and M. Woodford, eds., *knowledge, information, and expectations in modern macroeconomics: In honour of Edmund S. Phelps*, Princeton: Princeton University Press.
- Zanetti, F. (2009). Effects of product and labor market regulation on macroeconomic outcomes. *Journal of Macroeconomics*, 31(2):320–332.
- Zanetti, F. (2011). Labor market institutions and aggregate fluctuations in a search and matching model. *European Economic Review*, 55(5):644–658.

A Derivation of the Demand Function

Define the expenditure level by $Z_t \equiv \int_0^1 P_t(i)C_t(i)di$. We then set the Lagrangean as follows.

$$L = \left[\int_0^1 (C_t(i)\Theta_t(i))^{\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}} - \lambda \left(\int_0^1 P_t(i)C_t(i)di - Z_t \right).$$

The first-order conditions are,

$$C_t(i)^{-\frac{1}{\eta}} C_t^{\frac{1}{\eta}} (\Theta_t(i))^{\frac{\eta-1}{\eta}} = \lambda P_t(i).$$

Thus, for any two goods, the following relationship holds.

$$C_t(i) = C_t(j) \left(\frac{P_t(i)}{P_t(j)} \right)^{-\eta} \left(\frac{\Theta_t(i)}{\Theta_t(j)} \right)^{\eta-1}.$$

By substituting the equations into the expression for consumption expenditures, we have

$$\begin{aligned} \int_0^1 P_t(i) \left[C_t(j) \left(\frac{P_t(i)}{P_t(j)} \right)^{-\eta} \left(\frac{\Theta_t(i)}{\Theta_t(j)} \right)^{\eta-1} \right] di &= Z_t \\ \Leftrightarrow C_t(j) = P_t^{-\eta}(j) \Theta_t^{\eta-1}(j) Z_t \frac{1}{\int_0^1 P_t^{1-\eta}(i) \Theta_t^{\eta-1}(i) di}. \end{aligned}$$

Finally, using

$$\int_0^1 P_t(i)C_t(i)di = Z_t = P_t C_t,$$

we have

$$C_t(i) = \Theta_t^{\eta-1}(i) \left(\frac{P_t(i)}{P_t} \right)^{-\eta} C_t \frac{P_t^{1-\eta}}{\int_0^1 P_t^{1-\eta}(i) \Theta_t^{\eta-1}(i) di}.$$

Define $P_t \equiv \left[\int_0^1 P_t^{1-\eta}(i) \Theta_t^{\eta-1}(i) di \right]^{\frac{1}{1-\eta}}$. We then have,

$$C_t(i) = \Theta_t^{\eta-1}(i) \left(\frac{P_t(i)}{P_t} \right)^{-\eta} C_t.$$

B Derivation of the Price Setting Rule

From

$$p_t(i) = \mu + mc_t(i),$$

$$c_t(i) = -\eta(p_t(i) - p_t) + c_t + (\eta - 1)\theta_t(i),$$

and

$$mc_t(i) = w_t + \frac{1 - \epsilon}{\epsilon} y_t(i) - \frac{1}{\epsilon} a - \log(\epsilon),$$

$p_t(i)$ is given by,

$$\begin{aligned} p_t(i) &= \mu + mc_t(i) \\ &= \mu + y_t + p_t + \frac{1 - \epsilon}{\epsilon} [-\eta(p_t(i) - p_t) + y_t + (\eta - 1)\theta_t(i)] - \frac{1}{\epsilon} a - \log(\epsilon) \\ &= \left(\frac{\epsilon}{\epsilon + \eta(1 - \epsilon)} \right) (\mu - \log(\epsilon)) - \left(\frac{1}{\epsilon + \eta(1 - \epsilon)} \right) a + \left(\frac{1}{\epsilon + \eta(1 - \epsilon)} \right) q_t \\ &\quad + \left(\frac{(\eta - 1)(1 - \epsilon)}{\epsilon + \eta(1 - \epsilon)} \right) p_t + \left(\frac{(\eta - 1)(1 - \epsilon)}{\epsilon + \eta(1 - \epsilon)} \right) \theta_t(i). \end{aligned}$$

C Derivation of the Index of Aggregate Prices

First, $P_t \equiv \left[\int_0^1 P_t^{1-\eta}(i) \Theta_t^{\eta-1}(i) di \right]^{\frac{1}{1-\eta}}$ can be expressed as, $P_t = \left[\int_0^1 \left(\frac{P_t(i)}{\Theta_t(i)} \right)^{1-\eta} di \right]^{\frac{1}{1-\eta}} = \left[\int_0^1 \left(\tilde{P}_t(i) \right)^{1-\eta} di \right]^{\frac{1}{1-\eta}}$ where $\tilde{P}_t(i) \equiv \frac{P_t(i)}{\Theta_t(i)}$. We then define $p_t \equiv \int_0^1 \tilde{p}_t(i) di$. Finally,

$$p_t \equiv \int_0^1 \tilde{p}_t(i) di = \int_0^1 p_t(i) di - \int_0^1 \theta_t(i) di = \int_0^1 p_t(i) di,$$

holds because $\theta_t(i) \sim \mathcal{N}(0, (1 - \epsilon)^{-2} (\eta - 1)^{-2} \tau^2)$ results in $\int_0^1 \theta_t(i) di = 0$.

D Correlation Coefficients for Real Demand

In Figure 1, we examine the correlation coefficients for nominal demand and establish Proposition 1. Figure A1 conducts the same exercise for real demand and confirms that the results of Proposition 1 remain intact for real demand.

Panel (a) in Figure A1 shows that the correlation coefficient ρ_1 is large across different sectors with median value equal to 0.67. The correlation coefficient ρ_2 that captures equivalent co-movements in the expectations of changes in aggregate demand and sectoral demand

is larger, with a median value of 0.90. Comparison between the two correlation coefficients ρ_1 and ρ_2 shows that the co-movement in expectations is stronger than the co-movement in the observed data (i.e. $\rho_2 > \rho_1$), as most of the points lie below the 45-degree line. Panel (b) in Figure A1 shows that the values of the correlation coefficients ρ_2 and ρ_3 are large and generally equal to each other, with most of the points close to the value of 1, such that $\rho_2 = \rho_3 \approx 1$. These results are consistent with Proposition 1.

[Figure A1 about here.]

E Aggregate Shocks and the Output Gap

To evaluate whether the extracted shock ($u_t = \sum_{i=1}^{29} \Lambda_i \tilde{x}_t(i)$) is a plausible measure of aggregate disturbances that is consistent with established measures, we compare the developments in the eight-quarters backward moving averages of the aggregate shocks, $\frac{1}{8} \sum_{s=0}^7 u_{t-s}$,³⁷ with the developments in output gap published by the Bank of Japan.³⁸

Figure A2 examines the relationship between the dynamics of the estimates for aggregate shocks and the output gaps in order to confirm the validity of our estimates of aggregate shocks. The figure shows that the two series are highly correlated, with a correlation coefficient equal to 0.71,³⁹ therefore suggesting that our identified measure for the aggregate shock is able to produce a plausible measure of aggregate shocks.

[Figure A2 about here.]

³⁷Our measure of the aggregate shock is a flow rather than stock concept. By comparing moving averages of the aggregate shocks (i.e., the averages of flow data) with the output gap (i.e. stock data), we ensure that our measure is consistent with conventional measures.

³⁸The series is available here. https://www.boj.or.jp/en/research/research_data/gap/index.htm/
The description of the methodology for the estimation is here https://www.boj.or.jp/en/research/brp/ron_2017/ron170531a.htm/.

³⁹We conduct the same exercise with the eight-quarters backward moving averages of the normalized real shocks, $\frac{1}{8} \sum_{s=0}^7 \left((\sum_{i=1}^{29} \Lambda_i)^{-1} u_{t-s} - \pi_{t-s} \right)$, and obtain the broadly same results (a correlation coefficient equal to 0.63).

F Proofs

F.1 Proof of Proposition 2

The filtering process by firms in industry i is given by,

$$\begin{aligned}
\mathbb{E}_t[u_t] &= \lambda [\tilde{x}_t(i) - \mathbb{E}_{t-1}[\tilde{v}_t(i)]] + (1 - \lambda) \mathbb{E}_{t-1}[u_t] \\
&= \lambda [\tilde{x}_t(i) - \rho_v \mathbb{E}_{t-1}[\tilde{v}_{t-1}(i)]] + (1 - \lambda) \rho_u \mathbb{E}_{t-1}[u_{t-1}] \\
&= \lambda \tilde{x}_t(i) - \lambda \rho_v \mathbb{E}_{t-1}[\tilde{v}_{t-1}(i)] + (1 - \lambda) \rho_u \mathbb{E}_{t-1}[u_{t-1}], \\
\mathbb{E}_t[\tilde{v}_t(i)] &= (1 - \lambda) [x_t(i) - \mathbb{E}_{t-1}[u_t]] + \lambda \mathbb{E}_{t-1}[\tilde{v}_t(i)] \\
&= (1 - \lambda) [\tilde{x}_t(i) - \rho_u \mathbb{E}_{t-1}[u_{t-1}]] + \lambda \rho_v \mathbb{E}_{t-1}[\tilde{v}_{t-1}(i)] \\
&= (1 - \lambda) \tilde{x}_t(i) - (1 - \lambda) \rho_u \mathbb{E}_{t-1}[u_{t-1}] + \lambda \rho_v \mathbb{E}_{t-1}[\tilde{v}_{t-1}(i)], \\
\mathbb{E}_t[u_t] + \mathbb{E}_t[\tilde{v}_t(i)] &= \tilde{x}_t(i),
\end{aligned}$$

where \mathbb{E}_t means the expectations on information set in period t and λ is determined as follows.

The relationship between $\mathbb{E}_t[u_t]$ and $\mathbb{E}_t[\tilde{v}_t(i)]$ are given by,

$$\begin{aligned}
\mathbb{E}_t[u_t] &= \lambda \tilde{x}_t(i) + \lambda(1 - \lambda)(\rho_u - \rho_v) \sum_{j=0}^{\infty} [(1 - \lambda)\rho_u + \lambda\rho_v]^j \tilde{x}_{t-j-1}(i) \\
\mathbb{E}_t[\tilde{v}_t(i)] &= (1 - \lambda)\tilde{x}_t(i) - \lambda(1 - \lambda)(\rho_u - \rho_v) \sum_{j=0}^{\infty} [(1 - \lambda)\rho_u + \lambda\rho_v]^j \tilde{x}_{t-j-1}(i) \\
&= \tilde{x}_t(i) - \mathbb{E}_t[u_t].
\end{aligned}$$

Denote $\mathbb{V}_t[u_t]$ in the steady state information structures by \mathbb{V}_u . Then, the steady state $\mathbb{V}_t[\tilde{v}_t(i)] \equiv \mathbb{V}_v$ is expressed as follows. From

$$\mathbb{E}_t[\tilde{v}_t(i)] = \tilde{x}_t(i) - \mathbb{E}_t[u_t] = \tilde{v}_t(i) + [u_t - \mathbb{E}_t[u_t]],$$

$\mathbb{V}_v = \mathbb{V}_u \equiv \mathbb{V}$ and $\frac{\partial \mathbb{E}_t[u_t]}{\partial \mathbb{E}_t[\tilde{v}_t(i)]} = -1$ hold. Next, given this relationship, we find the Kalman gain λ in the filtering process. λ minimizes the imprecision of

$$\begin{aligned}
\mathbb{E}_t[u_t] &= \lambda [\tilde{x}_t(i) - \mathbb{E}_{t-1}[\tilde{v}_t(i)]] + (1 - \lambda) \mathbb{E}_{t-1}[u_t] \\
&= u_t + \lambda [\epsilon_t(i) + \rho_v [\tilde{v}_{t-1}(i) - \mathbb{E}_{t-1}[\tilde{v}_{t-1}]]] + (1 - \lambda) [\rho_u (\mathbb{E}_{t-1}[u_{t-1}] - u_{t-1}) - e_t]
\end{aligned}$$

Therefore, λ minimize \mathbb{V} which is given by,

$$\mathbb{V} = \lambda^2 \tau^2 + (1 - \lambda)^2 \sigma^2 + [\lambda \rho_v + (1 - \lambda) \rho_u]^2 \mathbb{V} \quad (27)$$

By taking first derivative of \mathbb{V} with respect to λ , we obtain the optimal λ as follows.

$$\lambda = \frac{\sigma^2 + (\rho_u - \rho_v)\rho_u\mathbb{V}}{\tau^2 + \sigma^2 + (\rho_u - \rho_v)^2\mathbb{V}}$$

This equation can be expressed as,

$$\begin{aligned} & \mathbb{V} = \lambda^2\tau^2 + (1 - \lambda)^2\sigma^2 + [\lambda\rho_v + (1 - \lambda)\rho_u]^2\mathbb{V} \\ \Leftrightarrow & \lambda^2\tau^2 + \lambda^2\sigma^2 - 2\lambda\sigma^2 + \sigma^2 + \lambda^2(\rho_u - \rho_v)^2\mathbb{V} - 2\lambda(\rho_u - \rho_v)\rho_u\mathbb{V} + \rho_u^2\mathbb{V} - \mathbb{V} = 0 \\ \Leftrightarrow & (\tau^2 + \sigma^2 + (\rho_u - \rho_v)^2\mathbb{V})\lambda^2 - 2(\sigma^2 + (\rho_u - \rho_v)\rho_u\mathbb{V})\lambda + \sigma^2 + \rho_u^2\mathbb{V} - \mathbb{V} = 0 \\ \Leftrightarrow & \lambda^2 - \frac{2(\sigma^2 + (\rho_u - \rho_v)\rho_u\mathbb{V})}{\tau^2 + \sigma^2 + (\rho_u - \rho_v)^2\mathbb{V}}\lambda + \frac{\sigma^2 + \rho_u^2\mathbb{V} - \mathbb{V}}{\tau^2 + \sigma^2 + (\rho_u - \rho_v)^2\mathbb{V}} = 0 \\ \Leftrightarrow & \left(\lambda - \frac{\sigma^2 + (\rho_u - \rho_v)\rho_u\mathbb{V}}{\tau^2 + \sigma^2 + (\rho_u - \rho_v)^2\mathbb{V}}\right)^2 - \left(\frac{\sigma^2 + (\rho_u - \rho_v)\rho_u\mathbb{V}}{\tau^2 + \sigma^2 + (\rho_u - \rho_v)^2\mathbb{V}}\right)^2 + \frac{\sigma^2 + \rho_u^2\mathbb{V} - \mathbb{V}}{\tau^2 + \sigma^2 + (\rho_u - \rho_v)^2\mathbb{V}} = 0. \end{aligned}$$

Then, we obtain

$$\begin{aligned} 0 &= -\left(\frac{\sigma^2 + (\rho_u - \rho_v)\rho_u\mathbb{V}}{\tau^2 + \sigma^2 + (\rho_u - \rho_v)^2\mathbb{V}}\right)^2 + \frac{\sigma^2 + \rho_u^2\mathbb{V} - \mathbb{V}}{\tau^2 + \sigma^2 + (\rho_u - \rho_v)^2\mathbb{V}} \\ &= -(\sigma^2 + (\rho_u - \rho_v)\rho_u\mathbb{V})^2 + (\sigma^2 + \rho_u^2\mathbb{V} - \mathbb{V})(\tau^2 + \sigma^2 + (\rho_u - \rho_v)^2\mathbb{V}) \\ &= -\sigma^4 - (\rho_u - \rho_v)^2\rho_u^2\mathbb{V}^2 - 2\sigma^2(\rho_u - \rho_v)\rho_u\mathbb{V} \\ &\quad + \sigma^2\tau^2 + \rho_u^2\mathbb{V}\tau^2 - \mathbb{V}\tau^2 + \sigma^4 + \sigma^2\rho_u^2\mathbb{V} - \sigma^2\mathbb{V} \\ &\quad + \sigma^2(\rho_u - \rho_v)^2\mathbb{V} + \rho_u^2(\rho_u - \rho_v)^2\mathbb{V}^2 - (\rho_u - \rho_v)^2\mathbb{V}^2 \\ &= -(\rho_u - \rho_v)^2\mathbb{V}^2 + [-2\sigma^2(\rho_u - \rho_v)\rho_u + \sigma^2(\rho_u - \rho_v)^2 + (\tau^2 + \sigma^2)(\rho_u^2 - 1)]\mathbb{V} + \sigma^2\tau^2 \\ &= \mathbb{V}^2 + \frac{\sigma^2(\rho_u^2 - \rho_v^2) + (\tau^2 + \sigma^2)(1 - \rho_u^2)}{(\rho_u - \rho_v)^2}\mathbb{V} - \frac{\sigma^2\tau^2}{(\rho_u - \rho_v)^2} \\ &= \mathbb{V}^2 + \frac{\sigma^2(1 - \rho_v^2) + \tau^2(1 - \rho_u^2)}{(\rho_u - \rho_v)^2}\mathbb{V} - \frac{\sigma^2\tau^2}{(\rho_u - \rho_v)^2}. \end{aligned}$$

Therefore,

$$\mathbb{V} = \frac{-\frac{\sigma^2(1 - \rho_v^2) + \tau^2(1 - \rho_u^2)}{(\rho_u - \rho_v)^2} + \sqrt{\left(\frac{\sigma^2(1 - \rho_v^2) + \tau^2(1 - \rho_u^2)}{(\rho_u - \rho_v)^2}\right)^2 + \frac{4\sigma^2\tau^2}{(\rho_u - \rho_v)^2}}}{2}.$$

and

$$\lambda = \frac{\sigma^2 + (\rho_u - \rho_v)\rho_u\mathbb{V}}{\tau^2 + \sigma^2 + (\rho_u - \rho_v)^2\mathbb{V}}.$$

Note that

$$\begin{aligned} \mathbb{E}_t[\tilde{v}_t(i)] &= (1 - \lambda)[\tilde{x}_t(i) - \mathbb{E}_{t-1}[u_t]] + \lambda\mathbb{E}_{t-1}[\tilde{v}_t(i)] \\ &= \tilde{v}_t(i) + (1 - \lambda)[u_t - \mathbb{E}_{t-1}[u_t]] + \lambda[\mathbb{E}_{t-1}[\tilde{v}_t(i)] - \tilde{v}_t(i)] \\ &= \tilde{v}_t(i) + (1 - \lambda)[e_t + \rho_u(u_{t-1} - \mathbb{E}_{t-1}[u_{t-1}])] + \lambda[\rho_v(\mathbb{E}_{t-1}[\tilde{v}_{t-1}(i)] - \tilde{v}_{t-1}(i)) - \epsilon_t(i)] \end{aligned}$$

holds, and thus

$$\mathbb{V} = \lambda^2 \tau^2 + (1 - \lambda)^2 \sigma^2 + [\lambda \rho_v + (1 - \lambda) \rho_u]^2 \mathbb{V}$$

holds, which is consistent to $\mathbb{V}_v = \mathbb{V}_u$.

Therefore,

$$\begin{aligned} \mathbb{E}_t [u_t] &= \lambda \tilde{x}_t(i) - \lambda \rho_v ((1 - \lambda) \tilde{x}_{t-1}(i) - (1 - \lambda) \rho_u \mathbb{E}_{t-2} [u_{t-2}] + \lambda \rho_v \mathbb{E}_{t-2} [\tilde{v}_{t-2}(i)]) \\ &\quad + (1 - \lambda) \rho_u (\lambda \tilde{x}_{t-1}(i) - \lambda \rho_v \mathbb{E}_{t-2} [\tilde{v}_{t-2}(i)] + (1 - \lambda) \rho_u \mathbb{E}_{t-2} [u_{t-2}]) \\ &= \lambda \tilde{x}_t(i) + [(1 - \lambda) \rho_u \lambda - \lambda \rho_v (1 - \lambda)] \tilde{x}_{t-1}(i) \\ &\quad + [(1 - \lambda) \rho_u (1 - \lambda) \rho_u + \lambda \rho_v (1 - \lambda) \rho_u] \mathbb{E}_{t-2} [u_{t-2}] \\ &\quad - [\lambda \rho_v \lambda \rho_v + (1 - \lambda) \rho_u \lambda \rho_v] \mathbb{E}_{t-2} [\tilde{v}_{t-2}(i)] \\ &= \lambda \tilde{x}_t(i) + (1 - \lambda) (\rho_u - \rho_v) \lambda \tilde{x}_{t-1}(i) + (1 - \lambda) \rho_u [(1 - \lambda) \rho_u + \lambda \rho_v] \mathbb{E}_{t-2} [u_{t-2}] \\ &\quad - \lambda \rho_v [\lambda \rho_v + (1 - \lambda) \rho_u] \mathbb{E}_{t-2} [\tilde{v}_{t-2}(i)]. \end{aligned}$$

Define $E_t \equiv \begin{bmatrix} \mathbb{E}_t [u_t] \\ \mathbb{E}_t [\tilde{v}_t(i)] \end{bmatrix}$, $\Lambda \equiv \begin{bmatrix} \lambda \\ 1 - \lambda \end{bmatrix}$, $A \equiv \begin{bmatrix} (1 - \lambda) \rho_u & -\lambda \rho_v \\ -(1 - \lambda) \rho_u & \lambda \rho_v \end{bmatrix}$. Then, finally the expectations are obtained as,

$$\begin{aligned} E_t &= \Lambda \tilde{x}_t(i) + A E_{t-1} = \Lambda \tilde{x}_t(i) + A \Lambda \tilde{x}_{t-1}(i) + A A E_{t-2} \\ &= \Lambda \tilde{x}_t(i) + A \Lambda \tilde{x}_{t-1}(i) + A^2 \Lambda \tilde{x}_{t-2}(i) + \dots \square \end{aligned}$$

F.2 Proof of Corollary 1

We have the following equations:

$$\begin{aligned} A^k &= [(1 - \lambda) \rho_u + \lambda \rho_v]^{k-1} \begin{bmatrix} (1 - \lambda) \rho_u & -\lambda \rho_v \\ -(1 - \lambda) \rho_u & \lambda \rho_v \end{bmatrix}, \\ A^k \Lambda &= [(1 - \lambda) \rho_u + \lambda \rho_v]^{k-1} \begin{bmatrix} \lambda (1 - \lambda) (\rho_u - \rho_v) \\ -\lambda (1 - \lambda) (\rho_u - \rho_v) \end{bmatrix}. \end{aligned}$$

Thereby, we observe that $A^k \Lambda = 0$ for $k \in \{1, 2, 3, \dots\}$ if $\rho_u = \rho_v$, and then E_t depends only on $x_t(i)$.

In such a case,

$$\mathbb{E}_t [u_t] = \lambda \tilde{x}_t(i), \quad \mathbb{E}_t [\tilde{v}_t(i)] = (1 - \lambda) \tilde{x}_t(i), \quad \lambda = \frac{\sigma^2}{\tau^2 + \sigma^2},$$

holds. Therefore, correlations (ρ_2, ρ_3) are given as follows.

$$\begin{aligned}
\rho_2 &= \rho(\mathbb{E}_{t-1}[u_t], \mathbb{E}_{t-1}[u_t + \tilde{v}_t(i)]) = \rho(\rho_u \mathbb{E}_{t-1}[u_{t-1}], \rho_u \mathbb{E}_{t-1}[u_{t-1}] + \rho_v \mathbb{E}_{t-1}[\tilde{v}_{t-1}(i)]) \\
&= \rho(\rho_u \lambda \tilde{x}_{t-1}(i), [\rho_u \lambda + \rho_v(1-\lambda)] \tilde{x}_{t-1}(i)) \\
&= \frac{\rho_u \lambda [\rho_u \lambda + \rho_v(1-\lambda)] \mathbb{V}[\tilde{x}_{t-1}(i)]}{\sqrt{\rho_u^2 \lambda^2 \mathbb{V}[\tilde{x}_{t-1}(i)]} \sqrt{[\rho_u \lambda + \rho_v(1-\lambda)]^2 \mathbb{V}[\tilde{x}_{t-1}(i)]}} = 1.
\end{aligned}$$

$$\begin{aligned}
\rho_3 &= \rho(\mathbb{E}_{t-1}[u_t] - \mathbb{E}_{t-2}[u_{t-1}], \mathbb{E}_{t-1}[u_t + \tilde{v}_t(i)] - \mathbb{E}_{t-2}[u_{t-1} + \tilde{v}_{t-1}(i)]) \\
&= \rho(\rho_u [\mathbb{E}_{t-1}[u_{t-1}] - \mathbb{E}_{t-2}[u_{t-2}]], \rho_u [\mathbb{E}_{t-1}[u_{t-1}] - \mathbb{E}_{t-2}[u_{t-2}]] + \rho_v [\mathbb{E}_{t-1}[\tilde{v}_{t-1}(i)] - \mathbb{E}_{t-2}[\tilde{v}_{t-2}(i)]]) \\
&= \rho(\lambda \rho_u [\tilde{x}_{t-1}(i) - \tilde{x}_{t-2}(i)], [\lambda \rho_u + (1-\lambda)\rho_v] [\tilde{x}_{t-1}(i) - \tilde{x}_{t-2}(i)]) \\
&= \frac{\rho_u \lambda [\rho_u \lambda + \rho_v(1-\lambda)] \mathbb{V}[\tilde{x}_{t-1}(i) - \tilde{x}_{t-2}(i)]}{\sqrt{\rho_u^2 \lambda^2 \mathbb{V}[\tilde{x}_{t-1}(i) - \tilde{x}_{t-2}(i)]} \sqrt{[\rho_u \lambda + \rho_v(1-\lambda)]^2 \mathbb{V}[\tilde{x}_{t-1}(i) - \tilde{x}_{t-2}(i)]}} = 1. \square
\end{aligned}$$

F.3 Proof of Proposition 4

The Value of $\rho(u_t, u_t + \tilde{v}_t(i))$

$$\rho_1 = \rho(u_t, u_t + \tilde{v}_t(i)) = \frac{\mathbb{V}[u_t]}{\sqrt{\mathbb{V}[u_t]} \sqrt{\mathbb{V}[u_t + \tilde{v}_t(i)]}} = \sqrt{\frac{\sigma^2}{\sigma^2 + \frac{1-\rho_u^2}{1-\rho_v^2} \tau^2}}.$$

Note that this value is the same under any information structures because here we assume that the fluctuations are exogenous.

The Value of $\rho(\mathbb{E}_{t-1}[u_t], \mathbb{E}_{t-1}[u_t + \tilde{v}_t(i)])$

$$\begin{aligned}
\rho_2 &= \rho(\mathbb{E}_{t-1}[u_t], \mathbb{E}_{t-1}[u_t + \tilde{v}_t(i)]) = \rho(\rho_u u_{t-1}, \rho_u u_{t-1} + \rho_v \tilde{v}_{t-1}(i)) \\
&= \frac{\rho_u^2 \mathbb{V}[u_t]}{\sqrt{\rho_u^2 \mathbb{V}[u_t]} \sqrt{\rho_u^2 \mathbb{V}[u_t] + \rho_v^2 \mathbb{V}[\tilde{v}_t(i)]}} = \sqrt{\frac{\sigma^2}{\sigma^2 + \frac{\rho_v^2}{\rho_u^2} \frac{1-\rho_u^2}{1-\rho_v^2} \tau^2}}.
\end{aligned}$$

The Value of $\rho(\mathbb{E}_{t-1}[u_t] - \mathbb{E}_{t-2}[u_{t-1}], \mathbb{E}_{t-1}[u_t + \tilde{v}_t(i)] - \mathbb{E}_{t-2}[u_{t-1} + \tilde{v}_{t-1}(i)])$

$$\begin{aligned}
\rho_3 &= \rho(\mathbb{E}_{t-1}[u_t] - \mathbb{E}_{t-2}[u_{t-1}], \mathbb{E}_{t-1}[u_t + \tilde{v}_t(i)] - \mathbb{E}_{t-2}[u_{t-1} + \tilde{v}_{t-1}(i)]) \\
&= \rho(\rho_u [\rho_u u_{t-2} + e_{t-1} - u_{t-2}], \rho_u [\rho_u u_{t-2} + e_{t-1} - u_{t-2}] + \rho_v [\rho_v \tilde{v}_{t-2}(i) + \epsilon_{t-1}(i) - \tilde{v}_{t-2}(i)]) \\
&= \frac{\rho_u^2 \mathbb{V}[e_{t-1} - (1-\rho_u)u_{t-2}]}{\sqrt{\rho_u^2 \mathbb{V}[e_{t-1} - (1-\rho_u)u_{t-2}]} \sqrt{\rho_u^2 \mathbb{V}[e_{t-1} - (1-\rho_u)u_{t-2}] + \rho_v^2 \mathbb{V}[\epsilon_{t-1}(i) - (1-\rho_v)\tilde{v}_{t-2}(i)]}} \\
&= \sqrt{\frac{\sigma^2}{\sigma^2 + \frac{\rho_v^2}{\rho_u^2} \frac{1-\rho_u^2}{1-\rho_v^2} \frac{1-\rho_v}{1-\rho_u} \tau^2}}.
\end{aligned}$$

Comparative Static Now we have

$$\rho_1 = \sqrt{\frac{\sigma^2}{\sigma^2 + \frac{1-\rho_u^2}{1-\rho_v^2}\tau^2}}, \rho_2 = \sqrt{\frac{\sigma^2}{\sigma^2 + \frac{\rho_v^2}{\rho_u^2} \frac{1-\rho_u^2}{1-\rho_v^2}\tau^2}}, \rho_3 = \sqrt{\frac{\sigma^2}{\sigma^2 + \frac{\rho_v^2}{\rho_u^2} \frac{1-\rho_u^2}{1-\rho_v^2} \frac{1-\rho_u}{1-\rho_u}\tau^2}}.$$

Therefore $\rho_1 \leq \rho_2$ if $\rho_v \leq \rho_u$ and $\rho_2 \geq \rho_3$ if $\rho_v \geq \rho_u$. \square

F.4 Proof of Proposition 3

Using the notation about $\mathbb{V}[\bar{\mathbb{E}}_t[\tilde{v}_t(i)]]$ and $\mathbb{V}[\bar{\mathbb{E}}_t[u_t]]$, we can express $\{\rho_1, \rho_2, \rho_3\}$ as follows.

$$\begin{aligned} \rho_1 &= \sqrt{\frac{1}{1 + \frac{\mathbb{V}[\tilde{v}_t(i)]}{\mathbb{V}[u_t]}}}, \rho_2 = \sqrt{\frac{1}{1 + \frac{\rho_v^2}{\rho_u^2} \frac{\mathbb{V}[\bar{\mathbb{E}}_t[\tilde{v}_t(i)]]}{\mathbb{V}[\bar{\mathbb{E}}_t[u_t]}}}, \\ \rho_3 &= \sqrt{\frac{1}{1 + \frac{\rho_v^2}{\rho_u^2} \frac{\mathbb{V}[\bar{\mathbb{E}}_t[\tilde{v}_t(i)] - \bar{\mathbb{E}}_{t-1}[\tilde{v}_{t-1}(i)]]}{\mathbb{V}[\bar{\mathbb{E}}_t[u_t] - \bar{\mathbb{E}}_{t-1}[u_{t-1}]}}} = \sqrt{\frac{1}{1 + \frac{\rho_v^2}{\rho_u^2} \frac{(1-\rho_{\bar{\mathbb{E}}[\tilde{v}(i)])\mathbb{V}[\bar{\mathbb{E}}_t[\tilde{v}_t(i)]]}{(1-\rho_{\bar{\mathbb{E}}[u]})\mathbb{V}[\bar{\mathbb{E}}_t[u_t]}}}, \end{aligned}$$

where $\rho_{\bar{\mathbb{E}}[\tilde{v}(i)]}$ and $\rho_{\bar{\mathbb{E}}[u]}$ indicate correlation of $(\bar{\mathbb{E}}_t[\tilde{v}_t(i)], \bar{\mathbb{E}}_{t-1}[\tilde{v}_{t-1}(i)])$ and that of $(\bar{\mathbb{E}}_t[u_t], \bar{\mathbb{E}}_{t-1}[u_{t-1}])$, respectively. Next, the following equalities hold.

$$\begin{aligned} \mathbb{V}[\bar{\mathbb{E}}_t[\tilde{v}_t(i)] - \bar{\mathbb{E}}_{t-1}[\tilde{v}_{t-1}(i)]] &= 2\mathbb{V}[\bar{\mathbb{E}}_t[\tilde{v}_t(i)]] - 2\mathbb{C}[\bar{\mathbb{E}}_t[\tilde{v}_t(i)], \bar{\mathbb{E}}_{t-1}[\tilde{v}_{t-1}(i)]] \\ &= 2(1 - \rho_{\bar{\mathbb{E}}[\tilde{v}(i)]})\mathbb{V}[\bar{\mathbb{E}}_t[\tilde{v}_t(i)]], \\ \mathbb{V}[\bar{\mathbb{E}}_t[u_t] - \bar{\mathbb{E}}_{t-1}[u_{t-1}]] &= 2(1 - \rho_{\bar{\mathbb{E}}[u]})\mathbb{V}[\bar{\mathbb{E}}_t[u_t]]. \end{aligned}$$

The average expectations across firms in one sector, $\bar{\mathbb{E}}_t[u_t]$ and $\bar{\mathbb{E}}_t[\tilde{v}_t(i)]$, are given as,

$$\begin{aligned} \bar{\mathbb{E}}_t[u_t] &= \tilde{\lambda}_u u_t + (1 - \tilde{\lambda}_u)\bar{\mathbb{E}}_{t-1}[u_t] = \tilde{\lambda}_u u_t + (1 - \tilde{\lambda}_u)\rho_u \bar{\mathbb{E}}_{t-1}[u_{t-1}] \\ &= \tilde{\lambda}_u u_t + (1 - \tilde{\lambda}_u)\rho_u \tilde{\lambda}_u u_{t-1} + (1 - \tilde{\lambda}_u)^2 \rho_u^2 \tilde{\lambda}_u u_{t-2} + \dots \\ &= \tilde{\lambda}_u e_t + \tilde{\lambda}_u \rho_u \left[1 + (1 - \tilde{\lambda}_u)\right] e_{t-1} + \tilde{\lambda}_u \rho_u^2 \left[1 + (1 - \tilde{\lambda}_u) + (1 - \tilde{\lambda}_u)^2\right] e_{t-2} + \dots \\ &= \sum_{j=0}^{\infty} \rho_u^j \left[1 - (1 - \tilde{\lambda}_u)^{j+1}\right] e_{t-j}, \bar{\mathbb{E}}_t[\tilde{v}_t(i)] = \sum_{j=0}^{\infty} \rho_v^j \left[1 - (1 - \tilde{\lambda}_v)^{j+1}\right] \tilde{e}_{t-j}(i), \end{aligned}$$

where

$$\begin{aligned} \tilde{\lambda}_u &= \frac{\rho_u^2 \mathbb{V}_{t-1}[u_{t-1}] + \sigma^2}{\sigma_y^2 + \rho_u^2 \mathbb{V}_{t-1}[u_{t-1}] + \sigma^2}, \tilde{\lambda}_v = \frac{\rho_v^2 \mathbb{V}_{t-1}[\tilde{v}_{t-1}(i)] + \tau^2}{\tau_z^2 + \rho_v^2 \mathbb{V}_{t-1}[\tilde{v}_{t-1}(i)] + \tau^2}, \\ \mathbb{V}_{t-1}[u_{t-1}] &= \frac{-[(1 - \rho_u^2)\sigma_y^2 + \sigma^2] + \sqrt{[(1 - \rho_u^2)\sigma_y^2 + \sigma^2]^2 + 4\sigma_y^2\sigma^2}}{2\rho_u^2}, \\ \mathbb{V}_{t-1}[\tilde{v}_{t-1}(i)] &= \frac{-[(1 - \rho_v^2)\tau_z^2 + \tau^2] + \sqrt{[(1 - \rho_v^2)\tau_z^2 + \tau^2]^2 + 4\tau_z^2\tau^2}}{2\rho_v^2}. \end{aligned}$$

Here, because

$$\begin{aligned}
\rho_{\mathbb{E}[u]} &= \frac{\mathbb{C} \left[\tilde{\lambda}_u u_t + \rho_u (1 - \tilde{\lambda}_u) \bar{\mathbb{E}}_{t-1} [u_{t-1}], \bar{\mathbb{E}}_{t-1} [u_{t-1}] \right]}{\mathbb{V} [\bar{\mathbb{E}}_t [u_t]]} = \rho_u \left[\tilde{\lambda}_u \frac{\mathbb{C} [u_t, \bar{\mathbb{E}}_t [u_t]]}{\mathbb{V} [\bar{\mathbb{E}}_t [u_t]]} + (1 - \tilde{\lambda}_u) \right], \\
\rho_{\mathbb{E}[\tilde{v}(i)]} &= \rho_v \left[\tilde{\lambda}_v \frac{\mathbb{C} [v_t(i), \bar{\mathbb{E}}_t [\tilde{v}_t(i)]]}{\mathbb{V} [\bar{\mathbb{E}}_t [\tilde{v}_t(i)]]} + (1 - \tilde{\lambda}_v) \right], \mathbb{V} [u_t] = \frac{1}{1 - \rho_u^2} \sigma^2, \mathbb{V} [\tilde{v}_t(i)] = \frac{1}{1 - \rho_v^2} \tau^2, \\
\frac{\mathbb{C} [u_t, \bar{\mathbb{E}}_t [u_t]]}{\mathbb{V} [\bar{\mathbb{E}}_t [u_t]]} &= \frac{\tilde{\lambda}_u}{1 - (1 - \lambda_u) \rho_u^2} \frac{\mathbb{V} [u_t]}{\mathbb{V} [\bar{\mathbb{E}}_t [u_t]]}, \frac{\mathbb{C} [\tilde{v}_t(i), \bar{\mathbb{E}}_t [\tilde{v}_t(i)]]}{\mathbb{V} [\bar{\mathbb{E}}_t [\tilde{v}_t(i)]]} = \frac{\tilde{\lambda}_v}{1 - (1 - \lambda_v) \rho_v^2} \frac{\mathbb{V} [\tilde{v}_t(i)]}{\mathbb{V} [\bar{\mathbb{E}}_t [\tilde{v}_t(i)]]}, \\
\mathbb{V} [\bar{\mathbb{E}}_t [u_t]] &= \sum_{j=0}^{\infty} \rho_u^{2j} \left[1 - (1 - \tilde{\lambda}_u)^{j+1} \right]^2 \sigma^2 \\
&= \left[\frac{1}{1 - \rho_u^2} - \frac{2(1 - \tilde{\lambda}_u)}{1 - \rho_u^2 (1 - \tilde{\lambda}_u)} + \frac{(1 - \tilde{\lambda}_u)^2}{1 - \rho_u^2 (1 - \tilde{\lambda}_u)^2} \right] \sigma^2, \\
\mathbb{V} [\bar{\mathbb{E}}_t [\tilde{v}_t(i)]] &= \sum_{j=0}^{\infty} \rho_v^{2j} \left[1 - (1 - \tilde{\lambda}_v)^{j+1} \right]^2 \tau^2 \\
&= \left[\frac{1}{1 - \rho_v^2} - \frac{2(1 - \tilde{\lambda}_v)}{1 - \rho_v^2 (1 - \tilde{\lambda}_v)} + \frac{(1 - \tilde{\lambda}_v)^2}{1 - \rho_v^2 (1 - \tilde{\lambda}_v)^2} \right] \tau^2,
\end{aligned}$$

the following relationship hold.

$$\begin{aligned}
\frac{\mathbb{V} [u_t]}{\mathbb{V} [\bar{\mathbb{E}}_t [u_t]]} &= \frac{1 - \rho_u^2 (1 - \tilde{\lambda}_u)}{\tilde{\lambda}_u \left[1 - \left(\frac{(1 - \rho_u^2)(1 - \tilde{\lambda}_u)}{1 - \rho_u^2 (1 - \tilde{\lambda}_u)^2} \right) \right]} = \frac{1 - \rho_u^2 (1 - \tilde{\lambda}_u)^2}{\tilde{\lambda}_u^2} \frac{1 - \rho_u^2 (1 - \tilde{\lambda}_u)}{1 + \rho_u^2 (1 - \tilde{\lambda}_u)}, \\
\frac{\mathbb{V} [\tilde{v}_t(i)]}{\mathbb{V} [\bar{\mathbb{E}}_t [\tilde{v}_t(i)]]} &= \frac{1 - \rho_v^2 (1 - \tilde{\lambda}_v)}{\tilde{\lambda}_v \left[1 - \left(\frac{(1 - \rho_v^2)(1 - \tilde{\lambda}_v)}{1 - \rho_v^2 (1 - \tilde{\lambda}_v)^2} \right) \right]} = \frac{1 - \rho_v^2 (1 - \tilde{\lambda}_v)^2}{\tilde{\lambda}_v^2} \frac{1 - \rho_v^2 (1 - \tilde{\lambda}_v)}{1 + \rho_v^2 (1 - \tilde{\lambda}_v)}, \\
1 - \rho_{\mathbb{E}[u]} &= 1 - \rho_u \left[\frac{\lambda_u^2}{1 - (1 - \tilde{\lambda}_u) \rho_u^2} \frac{\mathbb{V} [u_t]}{\mathbb{V} [\bar{\mathbb{E}}_t [u_t]]} + (1 - \tilde{\lambda}_u) \right] = 1 - \frac{\rho_u (2 - \tilde{\lambda}_u)}{1 - \rho_u^2 (1 + \tilde{\lambda}_u)}, \\
1 - \rho_{\mathbb{E}[\tilde{v}(i)]} &= 1 - \rho_v \left[\frac{\lambda_v^2}{1 - (1 - \tilde{\lambda}_v) \rho_v^2} \frac{\mathbb{V} [\tilde{v}_t(i)]}{\mathbb{V} [\bar{\mathbb{E}}_t [\tilde{v}_t(i)]]} + (1 - \tilde{\lambda}_v) \right] = 1 - \frac{\rho_v (2 - \tilde{\lambda}_v)}{1 - \rho_v^2 (1 + \tilde{\lambda}_v)}.
\end{aligned}$$

Proof of (i) Suppose $\tilde{\lambda}_u = \tilde{\lambda}_v \equiv \tilde{\lambda}$. We then have the following relationship.

$$\begin{aligned}
\frac{\mathbb{V}[\tilde{v}_t(i)]}{\mathbb{V}[u_t]} &\stackrel{\geq}{\leq} \frac{\rho_v^2 \mathbb{V}[\mathbb{E}_t[\tilde{v}_t(i)]]}{\rho_u^2 \mathbb{V}[\mathbb{E}_t[u_t]]} \Leftrightarrow \frac{\rho_v^2 \mathbb{V}[u_t]}{\rho_u^2 \mathbb{V}[\mathbb{E}_t[u_t]]} \stackrel{\geq}{\leq} \frac{\mathbb{V}[\tilde{v}_t(i)]}{\mathbb{V}[\mathbb{E}_t[\tilde{v}_t(i)]]} \\
&\Leftrightarrow \rho_v^2 \frac{1 - \rho_u^2 (1 - \tilde{\lambda}_u)^2}{\tilde{\lambda}_u^2} \frac{1 - \rho_u^2 (1 - \tilde{\lambda}_u)}{1 + \rho_u^2 (1 - \tilde{\lambda}_u)} \stackrel{\geq}{\leq} \rho_u^2 \frac{1 - \rho_v^2 (1 - \tilde{\lambda}_v)^2}{\tilde{\lambda}_v^2} \frac{1 - \rho_v^2 (1 - \tilde{\lambda}_v)}{1 + \rho_v^2 (1 - \tilde{\lambda}_v)} \Leftrightarrow \rho_u \stackrel{\geq}{\leq} \rho_v, \\
\rho_{\mathbb{E}[u]} \stackrel{\leq}{\geq} \rho_{\mathbb{E}[v(i)]} &\Leftrightarrow \frac{\rho_u (2 - \tilde{\lambda}_u)}{1 + \rho_u^2 (1 - \tilde{\lambda}_u)^2} \stackrel{\leq}{\geq} \frac{\rho_v (2 - \tilde{\lambda}_v)}{1 + \rho_v^2 (1 - \tilde{\lambda}_v)^2} \Leftrightarrow \rho_u \stackrel{\leq}{\geq} \rho_v.
\end{aligned}$$

Therefore, we have $\rho_1 \stackrel{\leq}{\geq} \rho_2 \stackrel{\geq}{\leq} \rho_3$ if $\rho_u \stackrel{\geq}{\leq} \rho_v$. Note that the first derivative of $\frac{\rho(2-\tilde{\lambda})}{1+\rho^2(1-\tilde{\lambda})^2}$ with respect to ρ is $\frac{(2-\tilde{\lambda})(1-\rho^2(1-\tilde{\lambda})^2)}{(1+\rho^2(1-\tilde{\lambda})^2)^2} > 0$. \square

Proof of (ii) Suppose $\tilde{\lambda}_u < \tilde{\lambda}_v$. In this case, to satisfy $\rho_1 < \rho_2$,

$$\begin{aligned}
\frac{\mathbb{V}[\tilde{v}_t(i)]}{\mathbb{V}[u_t]} &> \frac{\rho_v^2 \mathbb{V}[\mathbb{E}_t[\tilde{v}_t(i)]]}{\rho_u^2 \mathbb{V}[\mathbb{E}_t[u_t]]} \\
\Leftrightarrow \frac{1}{\rho_u^2} \frac{1 - \rho_u^2 (1 - \tilde{\lambda}_u)^2}{\tilde{\lambda}_u^2} \frac{1 - \rho_u^2 (1 - \tilde{\lambda}_u)}{1 + \rho_u^2 (1 - \tilde{\lambda}_u)} &< \frac{1}{\rho_u^2} \frac{1 - \rho_v^2 (1 - \tilde{\lambda}_v)^2}{\tilde{\lambda}_v^2} \frac{1 - \rho_v^2 (1 - \tilde{\lambda}_v)}{1 + \rho_v^2 (1 - \tilde{\lambda}_v)},
\end{aligned}$$

must hold. We derive the conditions to satisfy the inequality above. Define the function $f(\rho, \tilde{\lambda})$ as

$$f(\rho, \tilde{\lambda}) \equiv \frac{1}{\rho^2} \frac{1 - \rho^2 (1 - \tilde{\lambda})^2}{\tilde{\lambda}^2} \frac{1 - \rho^2 (1 - \tilde{\lambda})}{1 + \rho^2 (1 - \tilde{\lambda})}.$$

Then the following inequality hold:

$$\begin{aligned}
\frac{\partial f(\rho, \tilde{\lambda})}{\partial \lambda} &= \frac{-2(1 - \rho^2)(1 - (1 - \tilde{\lambda})^3 \rho^4)}{\tilde{\lambda}^3 \rho^2 (1 + (1 - \tilde{\lambda}) \rho^2)^2} < 0, \\
\frac{\partial f(\rho, \tilde{\lambda})}{\partial \rho} &= \frac{-1 - 2(1 - \tilde{\lambda})\rho - (1 - \tilde{\lambda})^2(2\tilde{\lambda} - 3)\rho^2}{\tilde{\lambda}^2 \rho^4 (1 + (1 - \tilde{\lambda}) \rho^2)^2} < 0.
\end{aligned}$$

Therefore, to satisfy $f(\rho_u, \tilde{\lambda}_u) < f(\rho_v, \tilde{\lambda}_v)$, if $\tilde{\lambda}_u < \tilde{\lambda}_v$ holds, $\rho_u = \rho_v + \varepsilon > \rho_v$ must hold. From the inequalities above, obviously ε is increasing in $\tilde{\lambda}_v$ and decreasing in ρ_v and $\tilde{\lambda}_u$.

However, under $\tilde{\lambda}_u < \tilde{\lambda}_v$ and $\rho_v + \varepsilon < \rho_u$

$$\rho_{\mathbb{E}[u]} > \rho_{\mathbb{E}[v(i)]} \Leftrightarrow \frac{\rho_u (2 - \tilde{\lambda}_u)}{1 + \rho_u^2 (1 - \tilde{\lambda}_u)} > \frac{\rho_v (2 - \tilde{\lambda}_v)}{1 + \rho_v^2 (1 - \tilde{\lambda}_v)}$$

holds. Note that both of the $\rho_{\mathbb{E}[u]}$ and $\rho_{\mathbb{E}[\tilde{v}(i)]}$ are monotonically decreasing in $\tilde{\lambda}_u$ and $\tilde{\lambda}_v$, respectively as

$$\frac{\partial \frac{\rho(2-\tilde{\lambda})}{1+\rho^2(1-\tilde{\lambda})}}{\partial \tilde{\lambda}} = -\frac{\rho(1-\rho^2)}{(1+\rho^2(1-\tilde{\lambda}))^2} < 0. \square$$

F.5 Proof of Proposition 5

We assume the information structure that firms can know past average prices but do not infer past aggregate demands from average prices.

First, we conjecture that $p_t^*(i)$ takes the following expression,

$$p_t^*(i) = a_1 p_{t-1} + \sum_{j=0}^{\infty} a_{2+j} x_{t-j}(i).$$

Given this guess, and given the fact that only a randomly selected fraction $1 - \theta$ of firms can adjust prices in any given period, we infer that the aggregate price level must satisfy,

$$p_t = \theta p_{t-1} + (1 - \theta) \int_0^1 p_t^*(i) di = b_1 p_{t-1} + \sum_{j=0}^{\infty} b_{2+j} \int_0^1 x_{t-j}(i) di = b_1 p_{t-1} + \sum_{j=0}^{\infty} b_{2+j} q_{t-j},$$

where $b_1 = \theta + (1 - \theta)a_1$, $b_{2+j} = (1 - \theta)a_{2+j}$.

Therefore, $p_t^*(i)$ is obtained as,

$$\begin{aligned} p_t^*(i) &= (1 - \beta\theta) [(1 - r)x_t(i) + r\mathbb{E}[p_t|\mathcal{H}_t(i)]] + \beta\theta\mathbb{E}[p_{t+1}^*(i)|\mathcal{H}_t(i)] \\ &= (1 - \beta\theta) \left[(1 - r)x_t(i) + r\mathbb{E}[b_1 p_{t-1} + \sum_{j=0}^{\infty} b_{2+j} q_{t-j} | \mathcal{H}_t(i)] \right] + \beta\theta\mathbb{E}[p_{t+1}^*(i)|\mathcal{H}_t(i)] \\ &= b_1 [(1 - \beta\theta)r + \beta\theta a_1] p_{t-1} + [b_2 [(1 - \beta\theta)r + \beta\theta a_1] + \beta\theta a_2] \mathbb{E}[q_t | \mathcal{H}_t(i)] \\ &\quad + [(1 - \beta\theta)r + \beta\theta a_1] \sum_{j=1}^{\infty} b_{2+j} \mathbb{E}[q_{t-j} | \mathcal{H}_t(i)] + [(1 - \beta\theta)(1 - r) + \beta\theta a_3] x_t(i) \\ &\quad + \beta\theta \sum_{j=2}^{\infty} a_{2+j} x_{t+1-j}(i). \end{aligned}$$

Here, $\mathbb{E}[q_{t-j} | \mathcal{H}_t(i)]$ is calculated as follows.

First, the following equations hold:

$$\mathbb{E}[q_t | \mathcal{H}_t(i)] = \lambda \sum_{s=0}^{\infty} (1 - \lambda)^s x_{t-s}(i), \quad \lambda = \frac{-\frac{\sigma^2}{\tau^2} + \sqrt{\left(\frac{\sigma^2}{\tau^2}\right)^2 + 4\frac{\sigma^2}{\tau^2}}}{2}.$$

Then, for $j \in \{1, 2, 3, \dots\}$, we have

$$\begin{aligned}
\mathbb{E}[q_{t-j}|\mathcal{H}_t(i)] &= \kappa_j \sum_{k=0}^{j-1} \frac{\lambda_k}{1-\lambda_j} \prod_{l=0}^{j-1-k} (1-\lambda_{j-l}) x_{t-k}(i) + (1-\kappa_j) \mathbb{E}[q_{t-j}|\mathcal{H}_{t-j}(i)] \\
&= \kappa_j \sum_{k=0}^{j-1} \frac{\lambda_k}{1-\lambda_j} \prod_{l=0}^{j-1-k} (1-\lambda_{j-l}) x_{t-k}(i) + (1-\kappa_j) \lambda \sum_{s=0}^{\infty} (1-\lambda)^s x_{t-s-j}(i), \\
\kappa_s &= \frac{\tau^2 \lambda}{\tau^2 \lambda_{s-1} + \sigma^2 + \tau^2 \lambda}, \lambda_s = \frac{\lambda_{s-1} \tau^2 + \sigma^2}{\lambda_{s-1} \tau^2 + \sigma^2 + \tau^2}, \kappa_0 = 0, \lambda_0 = 1.
\end{aligned}$$

Note that all of the noises in the first term and the second term are independent each other.

By substituting the equations above into the condition for $p_t^*(i)$, we obtain

$$\begin{aligned}
p_t^*(i) &= b_1 [(1-\beta\theta)r + \beta\theta a_1] p_{t-1} + [b_2 [(1-\beta\theta)r + \beta\theta a_1] + \beta\theta a_2] \left(\lambda \sum_{s=0}^{\infty} (1-\lambda)^s x_{t-s}(i) \right) \\
&\quad + [(1-\beta\theta)r + \beta\theta a_1] \sum_{j=1}^{\infty} b_{2+j} \left(\begin{aligned} &\kappa_j \sum_{k=0}^{j-1} \frac{\lambda_k}{1-\lambda_j} \prod_{l=0}^{j-1-k} (1-\lambda_{j-l}) x_{t-k}(i) \\ &+ (1-\kappa_j) \lambda \sum_{s=0}^{\infty} (1-\lambda)^s x_{t-s-j}(i) \end{aligned} \right) \\
&\quad + [(1-\beta\theta)(1-r) + \beta\theta a_3] x_t(i) + \beta\theta \sum_{j=2}^{\infty} a_{2+j} x_{t+1-j}(i).
\end{aligned}$$

The conditions are given by,

$$\begin{aligned}
a_0(j) &= 1 - [(1-\beta\theta)r + \beta\theta a_1] (1-\theta) \lambda (1-\kappa_{j-2}), \\
a_1 &= [\theta + (1-\theta)a_1] [(1-\beta\theta)r + \beta\theta a_1], \\
a_2 &= [a_0(2) - \beta\theta\lambda]^{-1} \left[\begin{aligned} &(1-\beta\theta)(1-r) + \beta\theta a_3 \\ &+ [(1-\beta\theta)r + \beta\theta a_1] (1-\theta) \sum_{m=1}^{\infty} a_{2+m} \kappa_m \frac{\lambda_0}{1-\lambda_0} \prod_{k=1}^m (1-\lambda_{k-1}) \end{aligned} \right], \\
a_3 &= a_0^{-1}(3) \left[\begin{aligned} &[(1-\theta)a_2 [(1-\beta\theta)r + \beta\theta a_1] + \beta\theta a_2] \lambda (1-\lambda) + \beta\theta a_4 \\ &+ [(1-\beta\theta)r + \beta\theta a_1] (1-\theta) \sum_{m=2}^{\infty} a_{2+m} \kappa_m \frac{\lambda_1}{1-\lambda_1} \prod_{k=2}^m (1-\lambda_{k-1}) \end{aligned} \right], \\
a_j &= a_0^{-1}(j) \left[\begin{aligned} &[(1-\theta)a_2 [(1-\beta\theta)r + \beta\theta a_1] + \beta\theta a_2] \lambda (1-\lambda)^{j-2} \\ &+ [(1-\beta\theta)r + \beta\theta a_1] (1-\theta) \sum_{k=1}^{j-3} a_{j-k} (1-\kappa_k) \lambda (1-\lambda)^{j-k-2} \\ &+ \beta\theta a_{j+1} + [(1-\beta\theta)r + \beta\theta a_1] (1-\theta) \sum_{m=j-1}^{\infty} a_{2+m} \kappa_m \frac{\lambda_{j-2}}{1-\lambda_{j-2}} \prod_{k=j-1}^m (1-\lambda_{k-1}) \end{aligned} \right].
\end{aligned}$$

Then, given the equations

$$p_t^*(i) = a_1 p_{t-1} + \sum_{j=0}^{\infty} a_{2+j} x_{t-j}(i), b_1 = \theta + (1-\theta)a_1, b_{2+j} = (1-\theta)a_{2+j},$$

we can characterize the equilibrium. Here, $a_3 > a_4 > \dots > a_j > \dots$ holds and $a_j \rightarrow 0$ as $j \rightarrow \infty$. \square

F.6 Proof of Corollary 2

From Proposition 5, for $k_0 \equiv (1 - \beta\theta)r + \beta\theta a_1$, $k_1 \equiv [(1 - \theta)k_0 + \beta\theta] \lambda$ and $a_{N+1} = 0$, we have

$$A = C + KA \Leftrightarrow A = [I - K]^{-1} C,$$

where

$$A = \begin{bmatrix} a_2 \\ a_3 \\ a_4 \\ \dots \\ a_N \end{bmatrix}, C = \begin{bmatrix} k_2(1 - \beta\theta)(1 - r) \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, K = \begin{bmatrix} 0 & K_{2,3} & K_{2,4} & \dots & \dots & K_{2,N} \\ K_{3,2} & 0 & K_{3,4} & \ddots & \ddots & \vdots \\ K_{4,2} & K_{4,3} & 0 & K_{4,5} & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 & \ddots & K_{N-2,N} \\ \vdots & \vdots & \ddots & \ddots & 0 & K_{N-1,N} \\ K_{N,2} & K_{N,3} & \dots & \dots & K_{N,N-1} & 0 \end{bmatrix}$$

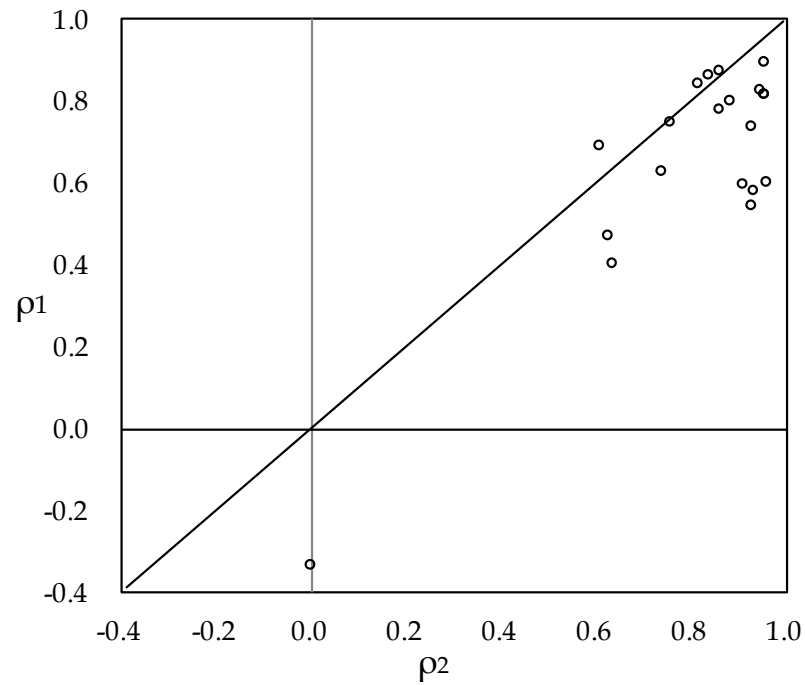
$$k_2 = [a_0(2) - \beta\theta\lambda]^{-1}, k_3 = a_0^{-1}(3), k_j = a_0^{-1}(j),$$

for

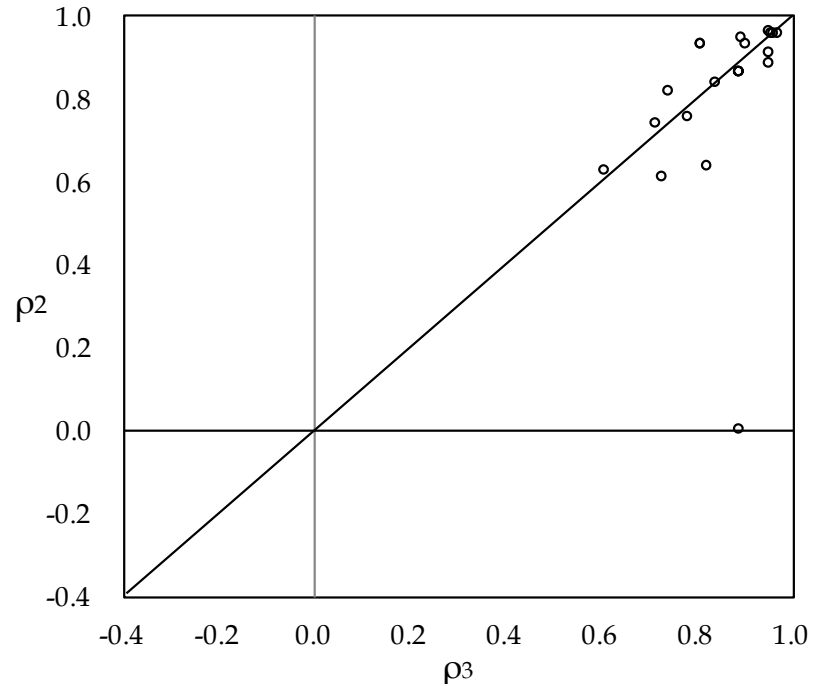
$$\begin{aligned} K_{2,3} &\equiv k_2 [\beta\theta + k_0(1 - \theta)\kappa_1\lambda_0], K_{2,4} \equiv k_2 k_0(1 - \theta)\kappa_2(1 - \lambda_1)\lambda_0, \\ K_{2,N} &\equiv k_2 k_0(1 - \theta)\kappa_{N-2}(1 - \lambda_{N-3})\dots(1 - \lambda_1)\lambda_0, K_{3,2} \equiv k_3 k_1(1 - \lambda), \\ K_{3,4} &\equiv k_3 [\beta\theta + k_0(1 - \theta)\kappa_2\lambda_1], K_{4,2} \equiv k_4 k_1(1 - \lambda)^2, \\ K_{4,3} &\equiv k_4 k_0(1 - \theta)(1 - \kappa_1)\lambda(1 - \lambda), K_{4,5} \equiv k_4 [\beta\theta + k_0(1 - \theta)\kappa_3\lambda_2(1 - \lambda_2)(1 - \lambda_3)], \\ K_{N-2,N} &\equiv k_{N-2} k_0(1 - \theta)\kappa_{N-2}(1 - \lambda_1)\lambda_0, K_{N-1,N} \equiv k_{N-1} [\beta\theta + k_0(1 - \theta)\kappa_{N-2}\lambda_1], \\ K_{N,2} &\equiv k_N k_1(1 - \lambda)^{N-2}, K_{N,3} \equiv k_N k_0(1 - \theta)(1 - \kappa_1)\lambda(1 - \lambda)^{N-3}, \\ K_{N,N-1} &\equiv k_N k_0(1 - \theta)(1 - \kappa_{N-3})\lambda(1 - \lambda). \square \end{aligned}$$

Figure 1: Empirical correlation coefficients for nominal demand

(a) Correlations of observed changes and expected changes in aggregate and sectoral demand (ρ_1 and ρ_2)



(b) Correlations of expected changes and the difference in expected changes in aggregate and sectoral demand (ρ_2 and ρ_3)

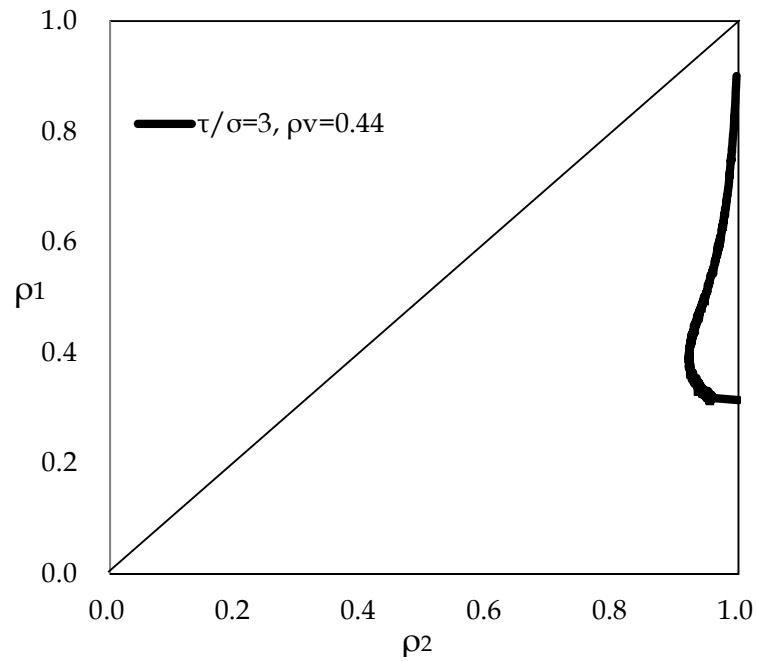


Note: Sample periods include 2004 to 2015 for observed and expected changes in aggregate and sectoral demand and 2005 to 2015 for the difference in expected changes in aggregate and sectoral demand. Expected changes are based on the calendar year while observed changes are based on the fiscal year.

Source: Cabinet Office of Japan "National Accounts of Japan," Cabinet Office of Japan "Annual Survey of Corporate Behavior."

Figure 2: Correlation coefficients under perfect inseparability of distinct shocks

(a) Correlation of observed changes and expected changes in aggregate and sectoral demand (ρ_1 and ρ_2)



(b) Correlation of expected changes and the difference in expected changes in aggregate and sectoral demand (ρ_2 and ρ_3)

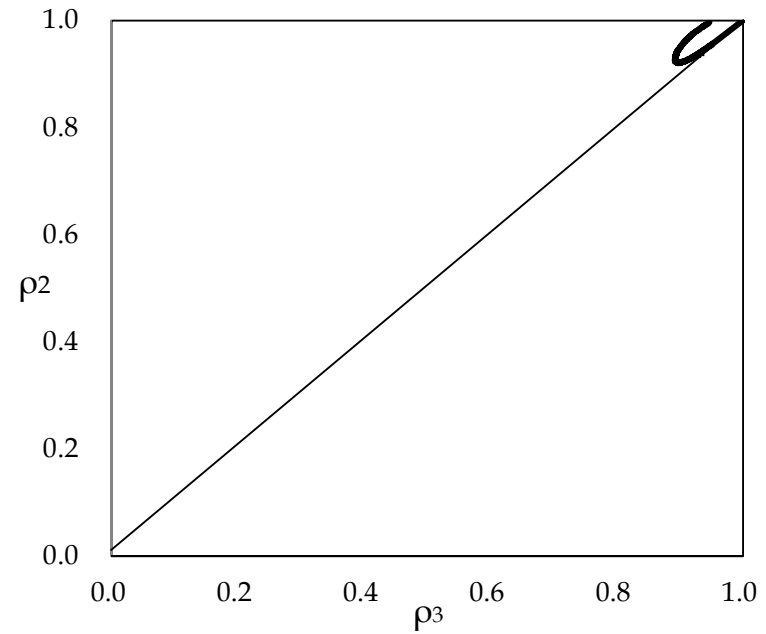
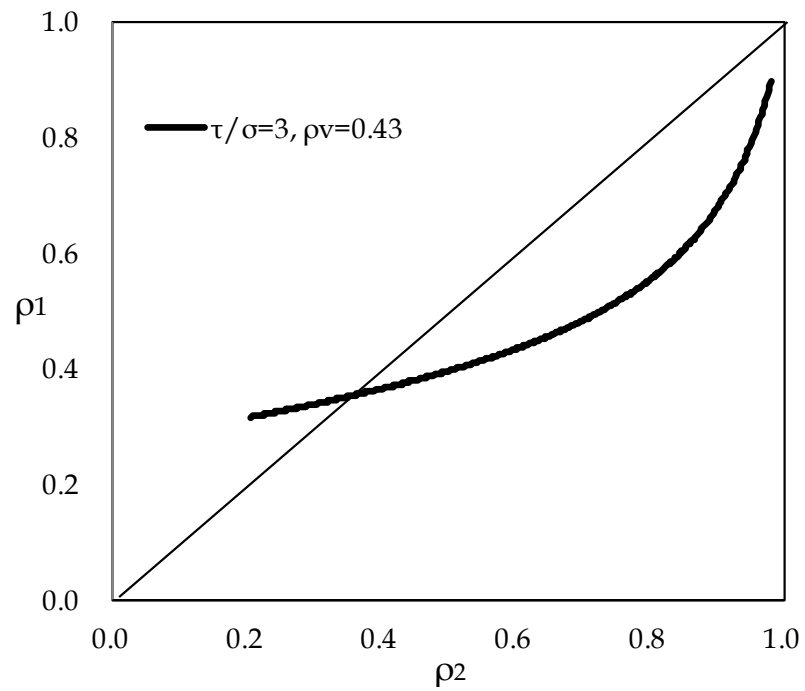


Figure 3: Correlation coefficients under information with noise

(a) Correlation of observed changes and expected changes in aggregate and sectoral demand (ρ_1 and ρ_2)



(b) Correlation of expected changes and the difference in expected changes in aggregate and sectoral demand (ρ_2 and ρ_3)

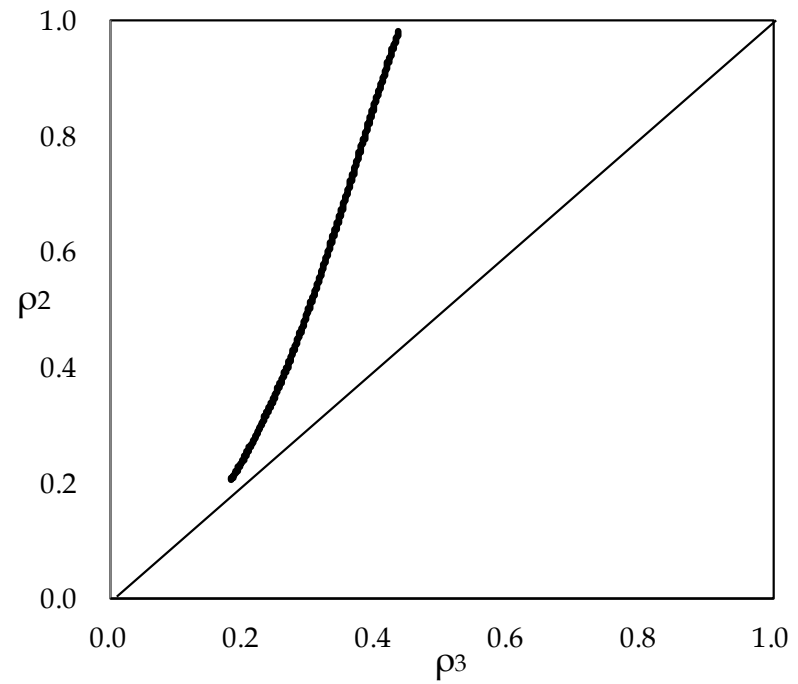
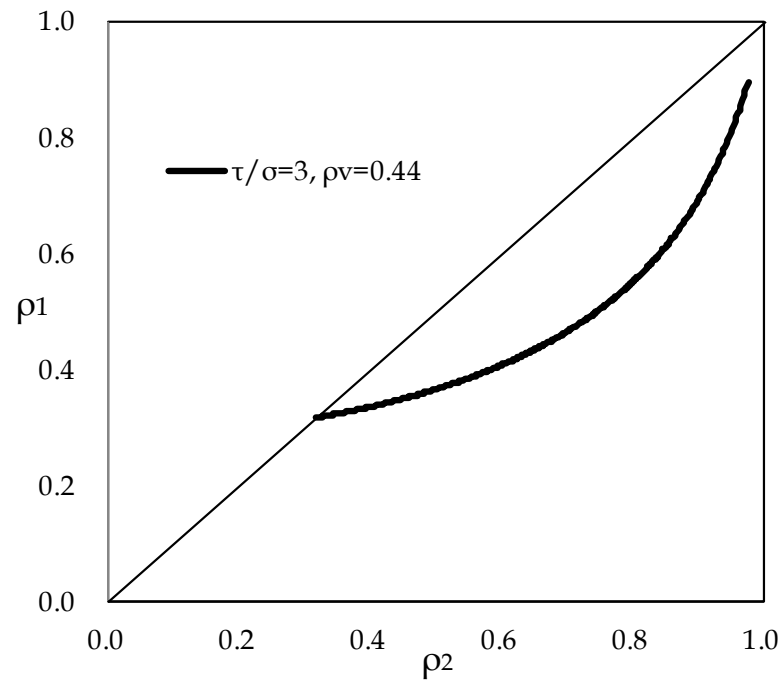


Figure 4: Correlation coefficients under perfect information

(a) Correlation of observed changes and expected changes in aggregate and sectoral demand (ρ_1 and ρ_2)



(b) Correlation of expected changes and the difference in expected changes in aggregate and sectoral demand (ρ_2 and ρ_3)

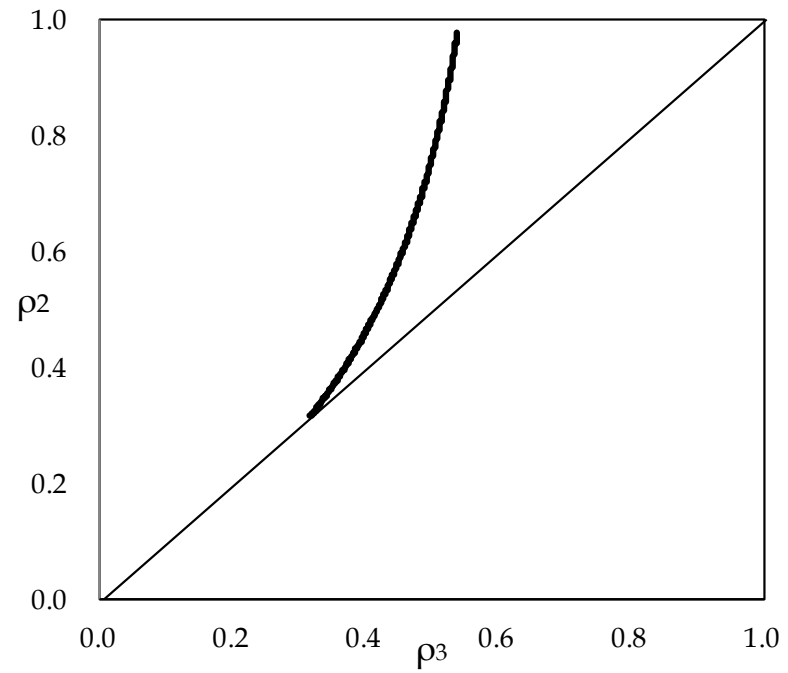
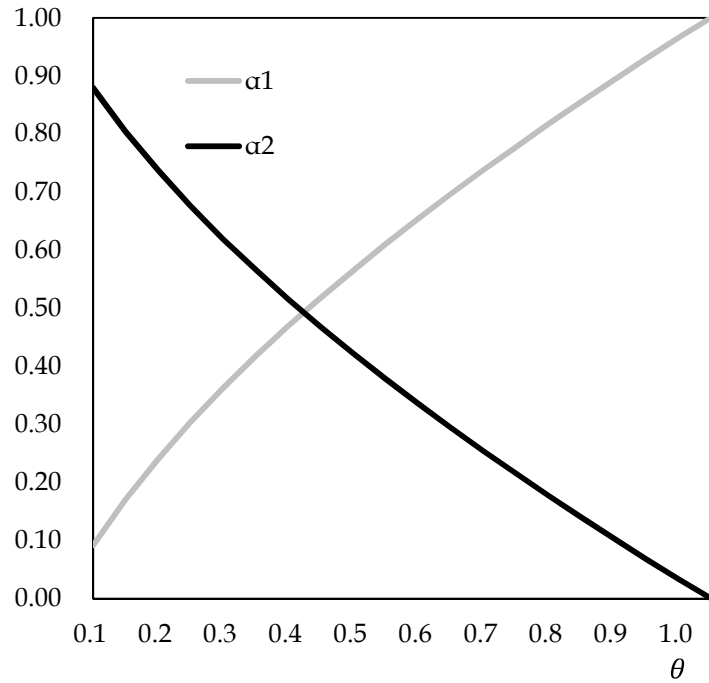
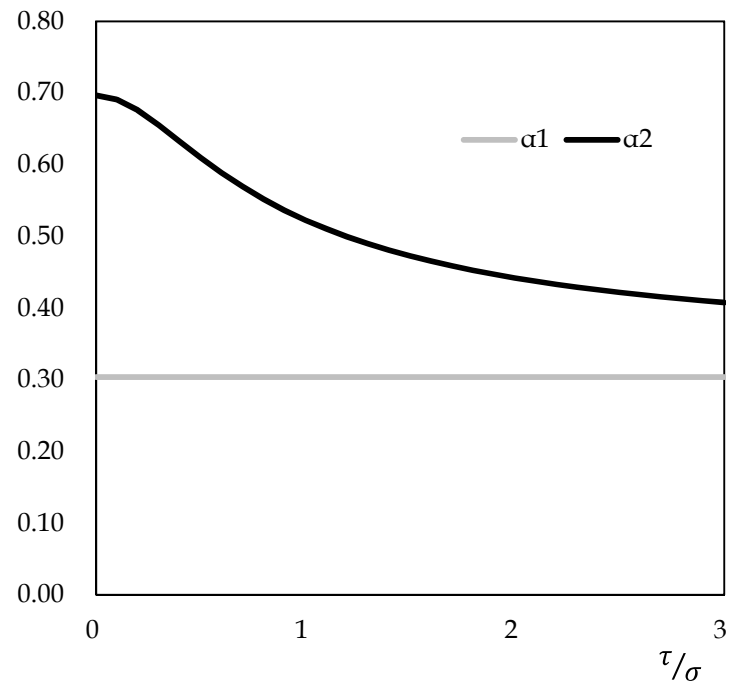


Figure 5: The slope of the Phillips curve

(a) The degree of nominal price rigidity (θ)

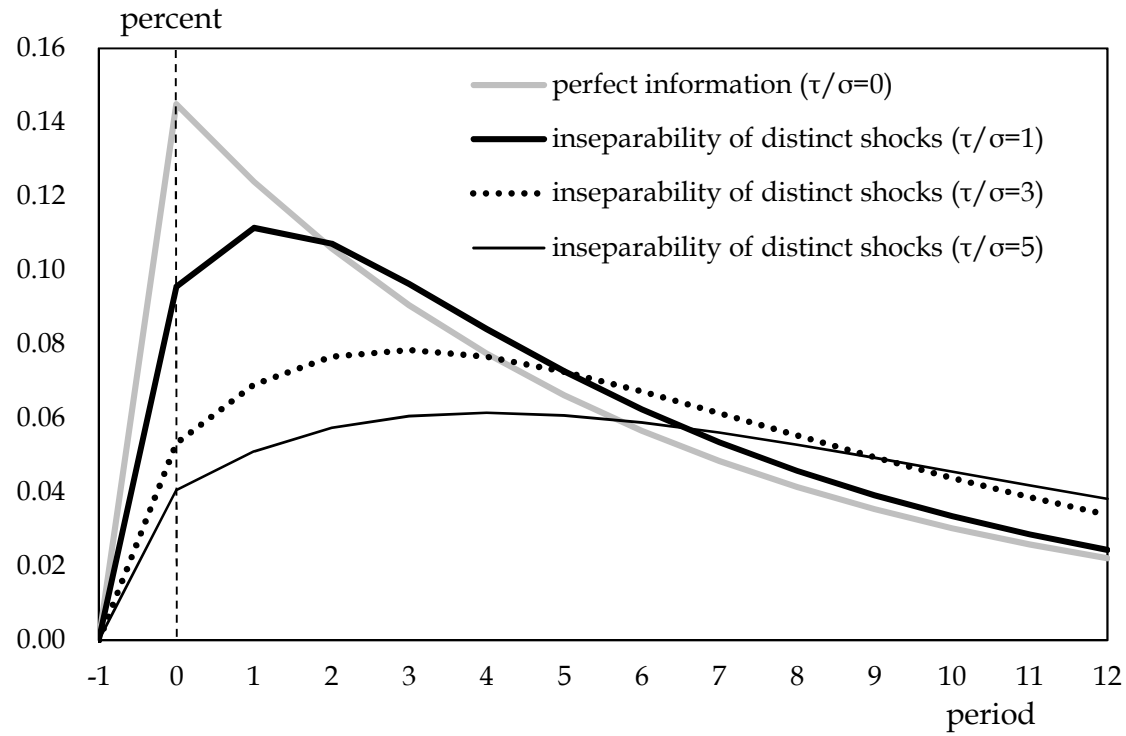


(b) The degree of shock heterogeneity (τ/σ)



Notes: Parameters are $\tau/\sigma = 0.2$, $r = 0.5$, $\beta = 0.99$ for (a), and $\theta = 0.2$, $r = 0.5$, $\beta = 0.99$ for (b). a_{101} is approximated to zero.

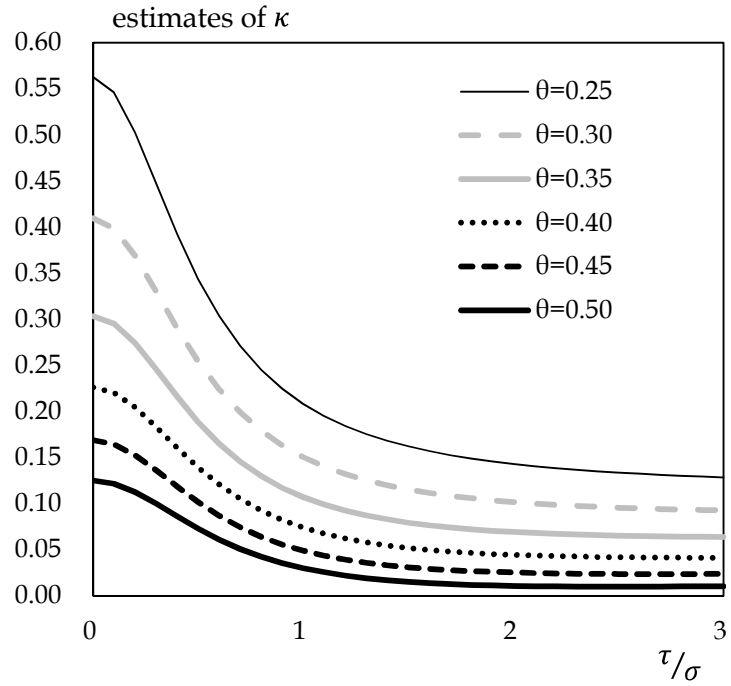
Figure 6: Responses of aggregate inflation to aggregate shocks (Simulation)



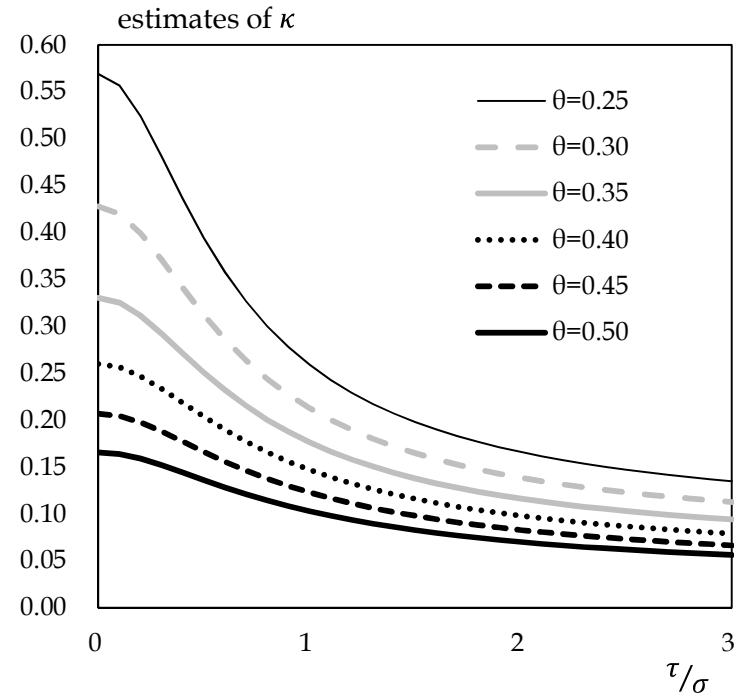
Notes: Parameters are $\theta = 0.8$, $r = 0.5$, $\beta = 0.99$. a_{101} is approximated to zero.

Figure 7: Estimates for the slope coefficient in the NKPC and the hybrid NKPC

(a) Estimates for the slope coefficient (κ) in the NKPC

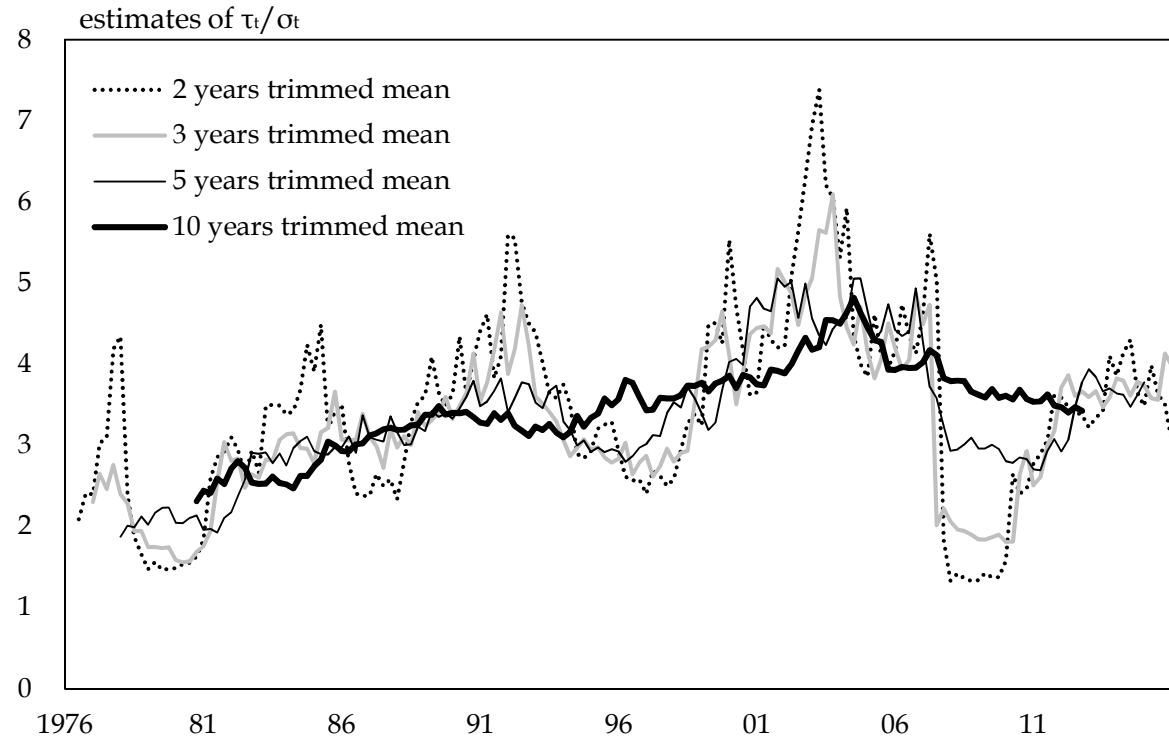


(b) Estimates for the slope coefficient (κ) in the hybrid NKPC



Notes: Parameters are $r = 0.5$, $\beta = 0.99$. a_{101} is approximated to zero.

Figure 8: Estimates of shock heterogeneity (τ_t/σ_t)

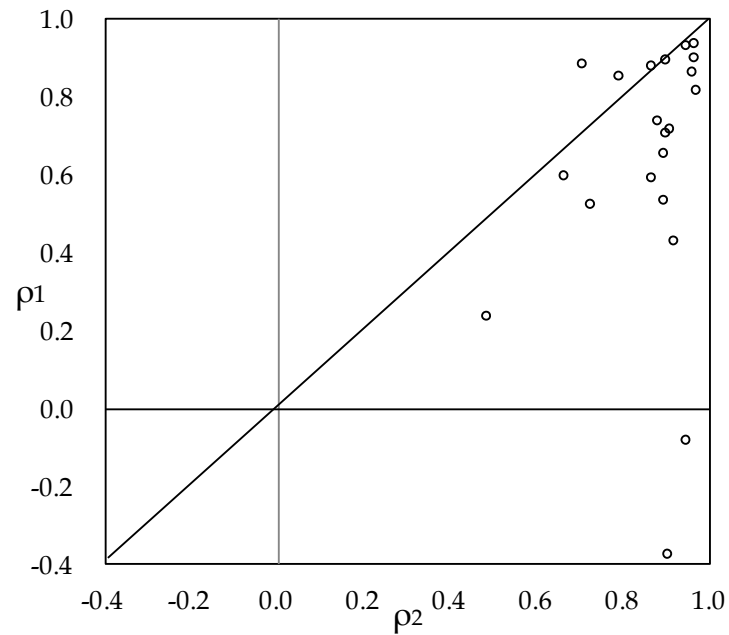


Notes: Upper and lower 10% of the samples are excluded in estimation.

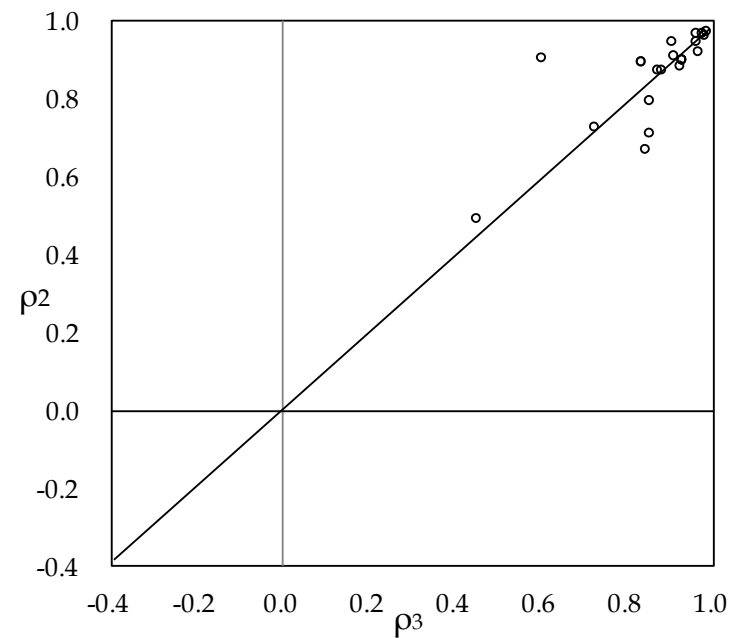
Source: Ministry of Finance "Financial statements statistics of corporations by industry".

Figure A1: Empirical correlation coefficients for real demand

(a) Correlation of observed changes and expected changes in aggregate and sectoral demand (ρ_1 and ρ_2)



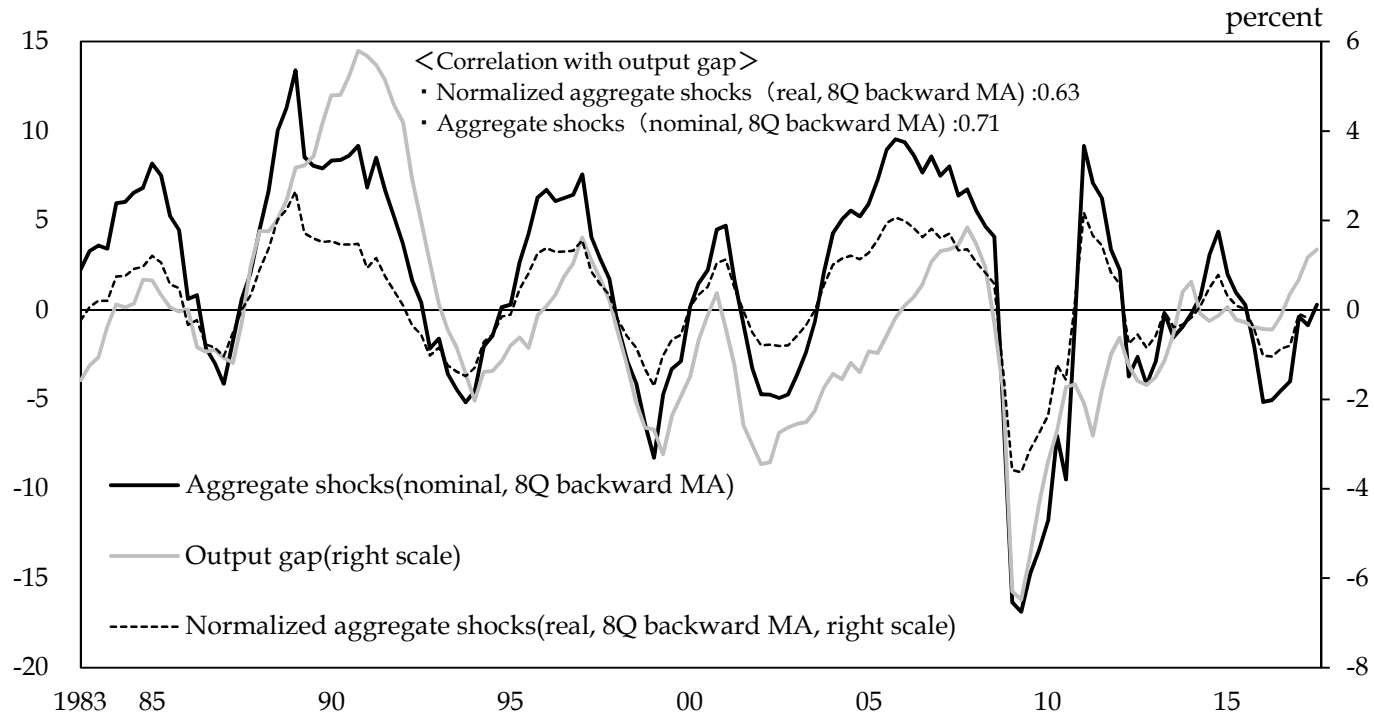
(b) Correlation of expected changes and the difference in expected changes in aggregate and sectoral demand (ρ_2 and ρ_3)



Note: Sample periods are 2004–2015 years for observed and expected changes in aggregate and sectoral demand and 2005–2015 years for the difference in expected changes in aggregate and sectoral demand. Expected changes are calendar year basis while observed changes are fiscal year basis.

Source: Cabinet Office of Japan “National Accounts of Japan”, Cabinet Office of Japan “Annual Survey of Corporate Behavior”.

Figure A2: Aggregate shocks and output gap



Sources: Ministry of Finance “Financial statements statistics of corporations by industry”,
Bank of Japan “Output Gap and Potential Growth Rate”.

Table 1: Estimation of the Phillips curve (part 1)

Dataset: Financial statement statistics of corporations by industry, consumer price index

All industries excluding the financial industry (29 industries), 1977/1Q-2017/3Q

Dependent Variable: Inflation rate (π_t , core consumer price index, seasonally adjusted, QoQ)

	<i>2 years trimmed mean</i>	<i>3 years trimmed mean</i>	<i>5 years trimmed mean</i>	<i>10 years trimmed mean</i>
<i>Constant</i>	0.041* (0.022)	0.037 (0.024)	0.029 (0.021)	0.026 (0.020)
<i>Lag of inflation (π_{t-1})</i>	0.574*** (0.055)	0.589*** (0.068)	0.539*** (0.064)	0.530*** (0.067)
<i>Aggregate shocks (u_t)</i>	0.024*** (0.005)	0.027*** (0.008)	0.042*** (0.015)	0.057*** (0.018)
<i>Aggregate shocks \times shock heterogeneity ($u_t \times \tau_t / \sigma_t$)</i>	-0.005*** (0.002)	-0.006** (0.003)	-0.010** (0.005)	-0.013** (0.005)
<i>Observations</i>	162	162	162	162
<i>Adjusted-R²</i>	0.68	0.67	0.69	0.69
<i>SE</i>	0.29	0.29	0.29	0.28

Note: Estimated by ordinary-least-squares. The standard errors are HAC estimators. Aggregate shocks are calculated as the developments in the principal component of 29 industries. First-to-eight lags of aggregate shocks are included in estimation as control variables. Data extrapolation using the values in the closest periods is applied for the missing values in the estimates of shock heterogeneity. The series for the core consumer price index is "all items, less fresh food".

**** Significant at the 1 percent level. ** Significant at the 5 percent level. * Significant at the 10 percent level.*

Table 2: Estimation of the Phillips curve (part 2)

*Dataset: Financial statement statistics of corporations by industry, consumer price index
All industries excluding the financial industry (29 industries), 1977/1Q-2017/3Q*

Dependent Variable: Inflation rate (π_t , core consumer price index, seasonally adjusted, QoQ)

	<i>2 years trimmed mean</i>	<i>3 years trimmed mean</i>	<i>5 years trimmed mean</i>	<i>10 years trimmed mean</i>
<i>Constant</i>	0.017 (0.029)	0.012 (0.029)	0.007 (0.028)	0.014 (0.026)
<i>Lag of inflation (π_{t-1})</i>	0.604*** (0.075)	0.630*** (0.076)	0.594*** (0.080)	0.572*** (0.075)
<i>Lag of inflation × time dummy (2000-2017) ($\pi_{t-1} \times 1_{\{2000-2017\}}$)</i>	-0.354** (0.154)	-0.384** (0.158)	-0.372** (0.154)	-0.376** (0.146)
<i>Aggregate shocks (u_t)</i>	0.028*** (0.007)	0.028*** (0.009)	0.037** (0.014)	0.065*** (0.021)
<i>Aggregate shocks × time dummy (2000-2017) ($u_t \times 1_{\{2000-2017\}}$)</i>	-0.008* (0.005)	-0.007 (0.005)	-0.002 (0.005)	0.008 (0.006)
<i>Aggregate shocks × shock heterogeneity ($u_t \times \tau_t / \sigma_t$)</i>	-0.005*** (0.002)	-0.005** (0.003)	-0.008* (0.004)	-0.018** (0.007)
<i>Observations</i>	162	162	162	162
<i>Adjusted-R²</i>	0.71	0.69	0.70	0.71
<i>SE</i>	0.28	0.28	0.28	0.27

Note: Estimated by ordinary-least-squares. The standard errors are HAC estimators. Aggregate shocks are calculated as the developments in the principal component of 29 industries. First-to-eight lags of aggregate shocks are included in estimation as control variables. Data extrapolation using the values in the closest periods is applied for the missing values in the estimates of shock heterogeneity. The series for the core consumer price index is “all items, less fresh food”.

**** Significant at the 1 percent level. ** Significant at the 5 percent level. * Significant at the 10 percent level.*