Tokenomics and Platform Finance[*]

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Abstract

We develop a dynamic model of platform economy where tokens derive value by facilitating transactions among users and the platform conducts optimal token-supply policy to finance investment in platform quality and to compensate platform owners. Even though token price is endogenously determined in a liquid market, the platform's financial constraint generates an endogenous token issuance cost that causes under-investment and conflicts of interest between insiders (owners) and outsiders (users). The franchise value (seigniorage) incentivizes the owners to buy back and burn tokens out of circulation, reducing token price volatility. Blockchain technology is crucial for token-based platforms because it enables platform owners to commit to predetermined rules of token supply that can significantly improve efficiency by addressing platform owners' time inconsistency and mitigating under-investment.

Keywords: Blockchain, Cryptocurrency, Dynamic Corporate Financing, Financial Constraint, Gig Economy, Token and Coin Offerings, Optimal Token Supply.

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1   Introduction

Over the past decades, digital platforms and online networks have received mounting interest, and are reshaping the organization of economic activities. Many successful platforms rely heavily on payment innovations (e.g., Alibaba and eBay) because the lack of trust among dispersed agents presents a major obstacle for economic exchanges. The blockchain technology rises to this challenge by allowing the creation of crypto-tokens as the local means of payment among platform users and providing consensus on transaction records to overcome problems of digital currency such as double-spending. Proven payment innovations (e.g., escrow account) can also be conveniently implemented via smart contracting, facilitating not only the economic exchanges among platform users but also the recruitment of on-demand labor and resources by the platform for its ongoing development.

Despite practitioners' enthusiasm, the economic trade-offs in managing platform tokens remain unclear. How are token prices determined by user activities (trading and usage) and platforms' token supply? How much should platforms invest in its productivity and pay contributors with tokens? How do platform owners profit from token-supply management? What do blockchains and smart contracting bring to both centralized and decentralized platforms? To take a first step in answering these questions, we propose a tractable dynamic framework for designing tokens and managing platform development.

Specifically, we build a continuous-time model of platform economy that captures the monetary, asset-pricing, and corporate-finance aspects of tokens. Because the platform is a unique marketplace for certain transactions, platform users demand tokens for a convenience yield. Platform owners manage the token supply. Token supply increases when the platform owners issue tokens to reward themselves or to pay decentralized contributors to enhance platform productivity. Contributors sell tokens to users (the ultimate token holders) for consumption goods. Token supply decreases when the owners raise external financing (consumption goods) to buy back and burn tokens out of circulation. Token buyback incurs a financing cost, so the platform operates in a realistic environment of investment, financing, and payout akin to Bolton, Chen, and Wang (2011). Our model differs because financial slack comes from token-supply management instead of cash management, and the token price is endogenously determined as the users' demand meets the owners' supply. Our model also differs from a majority of monetary-policy models because tokens as means of payment are issued to finance investment as in Bolton and Huang (2017) rather than stimulates nominal aggregate demand or alleviates liquidity shortage.

To capture users' network effects, a distinguishing feature of platform businesses, we allow tokens' convenience yield to depend on the aggregate number of users ("user base"). The user base evolves endogenously because users' participation depends positively on the platform's productivity (through the convenience yield) and their expectation of token price change. An intertemporal complementarity in user base arises — when potential users expect the platform to improve and more users to join in future, they expect token price to appreciate and thus participate more now.

Tokens not only allow users to transact with each other, but also allow the platform to access a pool of dispersed contributors for their effort and resources that can improve platform productivity. Because the contributors sell tokens to users who are the natural buyers of tokens for their convenience yield, the amount of resources the platform can raise through token payment depends on the users' valuation of tokens. Intertemporal complementarity in user base makes it crucial for the platform owners to invest in productivity growth.

The owners' value is the present value of all tokens paid to themselves net of the costs of token buy-back. In a Markov equilibrium, it is a function of the current platform productivity and token supply, which are the two state variables. The marginal value of productivity is positive, capturing the equilibrium dynamics of users' adoption and valuation of tokens. The marginal value of increasing token supply is negative because of the downward pressure on equilibrium token price. Therefore, when deciding on token issuance, the platform owners face a trade-off akin to that of durable-good producers. The distinction is that our model features endogenous investment and, as will be emphasized shortly, financial frictions. The resistance against excessive supply also applies to the platform's payout policy, i.e., tokens paid to the owners themselves, and encourages the owners to buy back tokens and burn them out of circulation in order to protect their continuation or franchise value.[1] In equilibrium, the ratio of token supply to platform productivity emerges as a key signal that drives platform investment. The payout and buyback impose two reflecting boundaries on the ratio.

A key inefficiency in our model is that when buying back tokens, the platform owners have to raise costly external funds. While token buyback occurs occasionally, the associated financing cost propagates into a *dynamic token issuance cost* because the platform owners

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[1]Burning cryptocurrencies or tokens means sending them to a public "eater address" from which they can never be used again because the private key of such an address is unobtainable. Practitioners often burn tokens to boost token value and reward token holders (e.g., Binance and Ripple). Some also use Proof-of-Burn as an environmentally friendly alternative to Proof-of-Work to generate consensus (e.g., Counterparty (XCP) blockchain), or destroy unsold tokens or coins after an ICO or token sale to maintain fair play (e.g., Neblio's burning of unsold NEBL tokens).

foresee costly buyback in future. Specifically, the owners' cost of issuing one more token, i.e., the decline of their franchise value, is larger than the users' valuation of tokens. This wedge causes the platform to under-invest in productivity enhancement, which in turn leads to lower user welfare. This divergence of owners' and users' interests is quite intuitive. From the platform owners' perspective, more tokens in circulation now implies a higher likelihood of token buyback and incidence of financing cost in the future. The benefit of token issuance for productivity enhancement is shared with users via a higher convenience yield of tokens, while the benefit of token buyback is shared with users via token price appreciation. Yet platform owners fully bear the financing cost.

After characterizing the platform owners' optimal token-management strategy (i.e., investment, payout, and buyback), we formally analyze the value added of blockchain technology in our setting. A defining feature of blockchain is the irreversible record-keeping, which enables the encoding of predetermined and invariable token-supply rules. Such commitment is valuable in the presence of conflicts of interest between users and platform owners. Specifically, motivated by Ethereum (one of the most popular blockchain applications by the token market capitalization), we consider a constant growth of token supply that finances decentralized contributions for productivity enhancement.

We find that commitment mitigate the under-investment problem by severing the state-by-state linkage between platform investment and the dynamic token issuance cost. While the increased amount of token issuance for productivity enhancement results in more frequent costly token buyback, the owners' value is higher than the no-commitment case due to the increase of token price driven by a fast trajectory of productivity and user-base growth. Our analysis of the commitment value from blockchain technology highlights the key differences between blockchain-based platforms and traditional centralized platforms, and thus partially explain why tokens become popular after the blockchain technology becomes available.

Our model also provides insights on the design of stablecoin. Different from the existing approaches based on open market operation or collateralization, the platform owners in our setting support their franchise value by occasionally burning tokens out of circulation. When the token supply is high relative to the platform productivity — precisely at the moment that token price is low but the marginal value of reducing token supply is high — the platform owners burn tokens. Consequently, token supply dynamics moderate price fluctuations. In our model solution, even though we allow the platform quality to randomly evolve with an annual volatility of 200%, the volatility of token price change is less than 10%. Committing to token-supply rules via blockchain further stabilizes token price.

Overall, by characterizing the token-based interactions among users and platform owners, our model reveals key trade-offs in the dynamic allocation of tokens. It also provides a guiding framework for practitioners: The various token offering schemes observed in real life can be viewed as special (sub-optimal) cases under (perhaps unnecessary) restrictions. Many existing platform businesses can also benefit from tokenization as discussed in this paper.[2]

We recognize the control right of platform owners and their discretion over token management and platform development. Our focus on platform owners is natural because without token distributions to themselves, the founders would either not launch the platform in the first place or design it in a way that allows future profits in other forms (e.g., stealing tokens from the users or financial contributions from the workers). In other words, token dividends compensate the founders' essential human capital.[3] Therefore, platform design, and in particular, the token supply, should be incentive-compatible and time-consistent, allowing the owners to dynamically extract rents in the form of token dividends. Such design is the only credible design to users (and regulators) who form expectation of token usage and valuation.

Literature.   Our paper contributes to the literature on digital platforms. Classical studies (e.g., Rochet and Tirole, 2003) do not consider the use of tokens as platform-specific means of payment. We build on Cong, Li, and Wang (2018) that studies dynamic token valuation and inter-temporal linkages in user adoption. Our model differs by allowing endogenous platform productivity and endogenous token supply so that we can analyze the dynamic investment, financing, and payout policies of platforms and how blockchain technology adds value by enabling commitment. We share the view on platform token with Brunnermeier, James, and Landau (2019) – a platform is a currency area where a unique set of economic activities take place and its tokens derive value by facilitating the related transactions. Beyond this, we emphasize that a platform can invest in its quality, for example, payment efficiency (Duffie, 2019), and thereby, raise token value. Our paper is the first to formally analyze how platforms manage their investment and payout through token supply, and provide insights into the incentives and strategies of platform businesses.

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[2] The video game industry presents a case in which dispersed agents such as production companies, development companies, console makers, mobile phone makers, writers, composers, and importantly, gamers, all contribute to the evolving community. The traditional use of royalties for compensating such decentralized contribution is difficult to manage and subject to various agency issues. Microsoft and Ernst & Young (EY) recently designed a blockchain to address the problems and provide transparency (Roberts, 2018).

[3] The strategies of a platform, or in general a company, lies in the entrepreneurs' distinctive point of view on how to create and capture value – it is cannot entirely be outsourced to decentralized contributors (Felin and Zenger, 2018). While ideology might drive some innovation, economists should all recognize that innovations on digital platforms is ultimately driven by entrepreneurs' incentives and strategies.

The inter-temporal trade-off that platform owners face in deciding the token supply resembles that of durable goods producers: A platform owner is concerned that an increase of circulating token depresses the current and future token price, and thus, erode the franchise value. Unlike in the durable goods monopolist literature (Coase, 1972; Bulow, 1982; Stokey, 1981), our model features endogenous investment in platform productivity, which maps to endogenous product quality of durable goods, and financial constraints.

We connect platform economics to asset pricing by demonstrating that the token price is the present value of platform-specific convenience yield. Our treatment of convenience yield is related to Krishnamurthy and Vissing-Jorgensen (2012) and earlier studies.[4] Our contribution is to incorporate user network effects in convenience yield and study their implications on user adoption, platform investment, and platform token-supply policies. Relating to Brunnermeier and Sannikov (2014), we emphasize agent heterogeneity and its asset-pricing implications. Instead of balance-sheet crises, we model the endogenous formation of user base given the users' heterogeneity in deriving convenience yield from tokens.

We also connect the literature on platform economics to dynamic corporate finance, especially that emphasizing the role of financial slack (e.g., Bolton, Chen, and Wang, 2011). Instead of cash management for investment and payout, we analyze platforms' token-supply strategies when platform investment induces user network effects and, very importantly, the token price varies endogenously as users respond to token-supply variation.[5] From a methodological perspective, our paper is most related to Li (2017) who studies banks' dynamic issuance of inside money as the price of money fluctuates endogenously in response to the variation in banks' supply and producer's demand. The distinction is that tokens are outside money (not liabilities of the issuing platform).

Our paper clearly adds to emerging studies on blockchains and cryptocurrencies. We take the blockchain functionality as given and add to studies examining the dual role of tokens both among the users and among the miners (e.g., Sockin and Xiong, 2018; Pagnotta, 2018). Relative to other dynamic token valuation models with exogenous token supplies (Cong, Li, and Wang, 2018; Fanti, Kogan, and Viswanath, 2019), we endogenize the token supply. We are thus the first to study platforms' optimal monetary, investment, and payout policies with both endogenous token pricing and user adoption.[6] Importantly, we highlight

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[4]Baumol (1952) and Tobin (1956) emphasize the transactional convenience of money holdings.

[5]The platform owner essentially maximizes the present value of all future seigniorage, our model shares this basic insight of Cagan (1956).

[6]Related are discussions on the design of cryptocurrencies and tokens as private money (e.g., Chiu and Wong, 2015; Chiu and Koeppl, 2017). Almost all prior studies focus on Bitcoin or Proof-of-work protocols

the commitment value of blockchains and smart contracting.[7]

Furthermore, our paper adds to the discussion on token price volatility and stablecoins. On the demand side, high token price volatility (e.g., Liu and Tsyvinski, 2018) could be an inherent feature of platform tokens due to technology uncertainty and endogenous user adoption (Cong, Li, and Wang, 2018). Saleh (2018) emphasizes that cryptocurrency supply under proof-of-burn (PoB) protocols can reduce price volatility. We endogenize both the demand for platform tokens (as opposed to general payment private/decentralized money) driven by users' transaction needs and dynamic adoption, and the supply of tokens for entrepreneurs to extract rent and incentivize decentralized contribution. In particular, we show that the entrepreneurs' optimal dynamic token allocation stabilizes token price.

Finally, our paper is broadly related to the literature on crowdsourcing and the gig economy. While recordkeeping or consensus provision in the form of cryptocurrency mining are typical forms of decentralized on-demand contributions, resources raised through initial coin offerings (ICOs) is another salient type. Existing studies on ICOs and crowdfunding focus on one-time issuance of tokens before the platform launches (e.g., Garratt and Wallace, 2018; Chod and Lyandres, 2018; Canidio, 2018), yet platforms increase token supply on an on-going basis (e.g., Bitcoin and Kin). They also center on entrepreneurs' hidden effort or asymmetric information, whereas we emphasize decentralized contributors' effort that is highly relevant for digital platforms and the gig economy.[8]

2 Institutional Background

To understand tokenomics and its role in platform finance, some institutional background is necessary. In this section we discuss how in practice platforms, blockchains, and the use of tokens are all connected. We also use the Kik/Kin system and various other examples in real life to illustrate that (i) the use of platform tokens as local media of exchange and forms

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with exogenous user network.

[7]In practice the reliability of blockchain and its commitment can be part of the platform productivity. As analyzed by Biais, Bisière, Bouvard, and Casamatta (2017), proof-of-work protocols can lead to competing records of transactions ("forks"). Saleh (2017) analyzes the forks of proof-of-stake blockchains.

[8]Beside newly minted tokens from platforms, decentralized contributors such as miners can also receive market-based compensation, as seen in the transaction fees users attach (Easley, O'Hara, and Basu, 2019; Basu, Easley, O'Hara, and Sirer, 2019; Huberman, Leshno, and Moallemi, 2019). Note that entrepreneurs in most ICO models simply issue tokens to raise financing of products which they sell to earn future profit, thus do not concern the erosion of franchise value. Canidio (2018) is a rare exception and complements our paper in that he focuses on entrepreneurs' effort provision before a platform launch.

of compensation for dispersed agents is a hallmark feature of modern platforms, especially those blockchain-based, (ii) platform designers increasingly utilize token design and supply policy to manage platform development dynamically, (iii) platform owners derive benefit by issuing themselves tokens and actively managing the token amount in circulation. Our model is set up to fully capture these salient features.

Platform economy. Many of the largest companies in the world such as Alibaba, Amazon, Apple, Facebook, Microsoft, and Tencent are all platform businesses that match dispersed users with product/service providers, maintain digital-network infrastructures, and facilitate third-party innovations within their ecosystems.[9] The most salient and defining feature of platform businesses is network effect (e.g., Cusumano, Yoffie, and Gawer, 2019). Beside, digital platforms inevitably entail the concept of community/ecosystem/supply chain and give rise to the "gig economy" wherein on-demand labor and resources from dispersed or even anonymous agents in the network play important roles in the evolution of platforms.[10]

Platform businesses therefore face three critical issues: (i) The positive network externality among users of a platform implies scale and first-mover advantage are crucial. Platforms have to devote significant resources to user acquisition and retention. (ii) Platforms have to incentivize and compensate participants to innovate and grow the system, yet bilateral long-term employment contracts prove too costly when the participants and contributors are too dispersed (sometimes anonymous and the participation ephemeral). (iii) Like in any other business, platforms are firms that have to make dynamic decisions on financing and investment in infrastructure or talent.

Trust and blockchain solution. The issues above concern not only platform owners. Users unknown to one another have to trust the platform to conduct transactions on the network; open source contributors and third party innovators (e.g., app developer on the Android system) have to trust the platform that they would be fairly and timely compensated; investors and key personnel for the platform have to watch out for dilution of their

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[9]Even many traditional firms such as General Electric are exploring ways to adopt platform thinking to speed up growth and improve performance (Cusumano, Yoffie, and Gawer, 2019).

[10]Uber, Handy, Upwork, and PeoplePerHour are among the thousands of platforms worldwide that emerged in recent years for freelance and short-term tasks. Digital platforms organize productive activities directly around their demand rather than through the intermediation of firms (Economist, 2018). Such activities account for significant portions of participating worker's total wages (Farrell, Greig, and Hamoudi, 2018). Taking the perspectives of traditional businesses, it is hard to imagine Uber owns almost no car and Airbnb owns almost no house.

shares and stakes in the platform.[11] Traditional solutions have relied on the reputation of centralized parties such as the platform owners. When a buyer puts her payment into an escrow account on Taobao.com to be released to the seller once she receives the goods in satisfactory conditions, she is placing good faith on Alibaba's being responsible.

Blockchains present an alternative that is more decentralized. Although not always necessarily required, a majority of blockchain applications entail the use of cryptocurrencies and crypto-tokens. In the past few years, over 1000 different cryptocurrencies have been introduced and many central banks are actively exploring cryptocurrency and blockchain for retail and payment systems.[12] In these payment and settlement applications including Bitcoin and Ripple, blockchains provide decentralized consensus that helps avoiding double-spending and enables digital currencies to act as media of exchange.

In terms of (crowd-based) financing, blockchain-based crypto tokens have also emerged as a popular means to raise funds for startups. In Initial Coin Offerings (ICOs), Security Token Offerings (STOs), and Initial Exchange Offerings (IEOs), entrepreneurs sell "tokens" or "AppCoins" to dispersed investors around the globe.[13] Moreover, tokens are routinely used as compensations for talents joining the startup teams.

Overall, blockchains help enable the use of platform tokens and many recent startups trigger interests in the economics of using tokens on general platforms, blockchain-based or not, because designing platform tokens essentially allows businesses to have a monetary policy tool at the firm/platform/ecosystem level.

Kik/Kin case. Kik Interative Inc. is a social media messaging company founded in Waterloo in 2009 and is currently under a well-publicized lawsuit by the SEC for violations of the Securities Act. Kik introduced a messaging app "Kik Messenger" in 2010 that later became one of the most popular social media applications (Brenner, 2018).

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[11]Various mechanisms for building trust such as seniority provisions, restrictive covenants, relationship finance, and collateralization have been introduced but are not fully satisfactory, as we discuss in Section 6.

[12]For example, People's Bank of China aims to develop a digital currency system; Bank of Canada and Singapore Monetary Authority use blockchain for interbank payment systems; Deutsche Bundesbank works on prototype of blockchain-based settlement systems for financial assets; in a controversial move, the government of Venezuela became the first federal government to issue digital currency and announced on Feb 20, 2018 the presale of its "petro" cryptocurrency — an oil-backed token as a form of legal tender that can be used to pay taxes, fees and other public needs.

[13]While the first ICO in 2013 raised a meager $500k and sporadic activities over the next two years. 2016 saw 46 ICOs raising about $100m and according to CoinSchedule, in 2017 there were 235 Initial Coin Offerings. The year-end totals came in over $3 billion raised in ICO. In August, 2017, OmiseGO (OMG) and Qtum passed a US$1 billion market cap today, according to coinmarketcap.com, to become the first ERC20 tokens built on the Ethereum network and sold via an ICO to reach the unicorn status.

To better compete with larger players in the advertising market and to allow its users monetize their participation, Kik launched Kik Points, a virtual currency within Kik Messenger in 2014. Advertisers can exchange fiat money with Kik for points to pay consumers for answering surveys and polls and consumers can pay for purchases and usage with the points. Kik Points while in operation created an average of 300,000 daily transactions, but advertisers and users are concerned that nothing prevents Kik from creating more Kik Points or stopping accepting them entirely in future.

The company therefore introduced a new blockchain-based platform, Kin, which issues native tokens for user transactions and compensations for dispersed contributors in an ecosystem for digital services and social network wherein Kik is a key founding member.[14] The Kin project is overseen by the Kin foundation and aims to offer Kik and similar developers a way to monetize their businesses which was previously difficult without large initial scale or abusing user data. Because the blockchain-based token can guarantee a fixed supply and goes beyound Kik Messenger to include unlimited number of applications, services, products from unlimited number of developers, Kin has the potential to overcome the dilution/inflation issue and the hazard of a single entity (such as Kik) destroying its value completely.

Importantly, even though the supply of Kin is capped at $10 trillion USD, the founding team and the Kin Foundation actively manage the dynamic token allocations (e.g., using smart contracts): 30% is pre-allocated to the original Kik platform for being a founding member of Kin and early adopter, 10% (1 trillion) of Kin tokens are issued to dispersed investors in a 2-week initial coin offering (ICO), raising about US$100 million, 60% is initially allocated to the Kin foundation and is to be gradually distributed to early users and contributors through the Kin Rewards Engine schema or used to cover operation and marketing expenses. Every year, 20% of the remaining token reserves at the Kin Foundation is released to corporate partners to be used, for example, as incentive payments. While it is unclear if Kik and Kin optimally designed the allocation, they apparently thought about issues related to promoting user adoption, incentivizing third-party contributors to the system, controlling inflation, and compensating key personnel and partners while growing the platform.[15]

The case of Kik/Kin leads to the following three observations that apply generally to (blockchain-based) platforms.

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[14]Facebook's role in the creation of Libra coins is similar.

[15]Interested readers can find out more at, e.g., https://www.reddit.com/r/KinFoundation/comments $/8r2ull/circulating_supply_and_allocation/and https://blog.usejournal.com/kins-10-trillion-supply-and-the-case-for-kinnies-a41e2037c64c$.

(i) Token embedding. It should be clear from the Kik/Kin case that tokens are used as a platform currency/local medium of exchange — the "Token Embedding" phenomenon first highlighted in Cong, Li, and Wang (2018). Indeed, in many existing blockchain applications, native tokens are the required or favored medium of exchange.[16] For example, it is cheaper to make international payments and settlements using Ripples (RXP) on the Ripple network; to make profit by providing validation services, OmiseGo (OMG) tokens are required as stakes on the OmiseGo blockchain; even though Ethereum platform allows other AppCoins and cryptocurrencies, many transactions and fundraising activities are still carried out using Ethers (ETH) because of the convenience and popularity. Moreover, platform owners actively design the rules for token supplies and attributes and use tokens to compensate investors and contributors as well as to manage the platform development.

We note that it is natural and common in practice to introduce platform tokens that agents hold and use because transfers in fiat currencies inevitably rely on centralized third parties such as banks that are subject to the confines of physical location and jurisdictions. Tokens, in contrast, can be used not only for peer-to-peer exchanges, but also for compensating miners, validators, and other contributors who work to improve the stability and functionality of the ecosystem. This is especially convenient because cryptocurrency miners who maintain network securities under PoWs and liquidity providers in a staking-based system are not long-term employees of the platform and demand on spot, reliable payments. Moreover, native coins can be directly linked to history of transactions and events on the blockchain, a feature other currencies cannot provide.[17]

One example is Filecoin (FIL) which is used as the sole means of payment in the network marketplace to reward miners for block creation in the Filecoin consensus process. Another example is Basic Attention Token (BAT). As Strategic Coin explains in its BAT token launch research report, also functions as a medium exchange between users, advertisers,

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[16]See also Brunnermeier, James, and Landau (2019) who conclude, "Payments are at the center of any economic platform, and all other activities would organize themselves around the central payment functionality." Even though policy makers sometimes generically refer to non-cash-flow-based tokens as "utility tokens," we note that the majority of them are not for redeeming a product or service from the platform owner per se at fixed price. They represent the right to use the platforms to conduct business. These include many of the highest-profile projects: Filecoin, Golem, 0x, Civic, Raiden, Basic Attention Token (BAT), and more. Prices are not pre-set but derive from market interactions. On the Kin platform, for example, sellers and buyers set prices themselves.

[17]Kocherlakota (1998) treats money as an object that does not enter utility or production functions, and is available in fixed supply. He then shows that from a technological point of view, money is equivalent to a primitive form of memory. Now with the blockchain technology, money is indeed memory. The coins can indeed be associated with the knowledge of the full histories of the agents the one of them has direct or indirect contact with.

and publishers who participate in the Brave browser ecosystem. Advertisers purchase ads using BAT tokens, which are then distributed among both publishers and browser users as compensation for hosting the ads and viewing them, respectively.

The fact that (blockchain-based) platform ecosystems tend to be mediated by native tokens is only one aspect of token embedding. In principle no one needs to *hold* the native token if its velocity is infinite, i.e., people can instantaneously exchange other currencies with the native tokens. The second aspect of token embedding is that agents actually need to hold the tokens to conduct transactions and perform economic activities. While this is also true for fiat money in practice, blockchain-based systems add at least three more reasons.

First, to incentivize and allocate service flows to decentralized miners or service providers, many tokens are designed such that these agents have to hold the native tokens to earn the right to perform work to maintain the system, be it service provision or recordkeeping. Proof-of-Stake protocols typically fall in this category. These tokens are sometimes referred to as work tokens or staking tokens, and notable implementations include Keep (off-chain private computation), Filecoin (distributed file storage), Truebit (off-chain computation), Livepeer (distributed video encoding), and Gems (decentralized mechanical Turk).[18]

Second, blockchains enable the use of smart contracts—digital contracts allowing terms contingent on decentralized consensus that are typically self-enforcing and tamper-proof through automated execution. Smart contracts need to automate transactions once certain contingencies are fulfilled, which in turn requires a certain amount of tokens to be "escrowed" during the episode that such contingencies may be triggered.

Third, because the generation of decentralized consensus takes time, there is a technical limit on how quickly transactions can be validated and recorded. While many protocols such as the Lightening Network and Ethereum process transactions significantly faster than Bitcoin (seconds versus 10-11 minutes), the decentralized nature of the validation means it takes time to ensure robustness and synchronization of the consensus. During the confirmation period, agents have to hold tokens.

(ii) Dynamic token supply and platform management. Admittedly, much of the discussion on cryptocurrency has focused on its role as a competitor for fiat currency for

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[18]To enforce a mechanism to penalize workers who fail to perform their job to some pre-specified standard, work tokens have to be held as collateral. For example, in Filecoin, service providers contractually commit to storing some data with 24/7 access and some minimum bandwidth guarantee for a specified period of time. During the contract term, service providers must "escrow" some number of Filecoin, which can be automatically slashed (taken away) should they fail to perform the service.

general payments. In a way, fiat currencies are also an extreme form of platform tokens in that people "join" the platform by recognizing their value and accepting them (Gans and Halaburda, 2015), and the platform being the broader economy. What distinguishes tokens on digital platforms is that the adoption is no longer dictated by physical constraints: it is way more costly to adopt a different fiat currency by emigrating to a different country, but switching among digital platforms are relatively easy, which implies that the adoption of digital platforms are more endogenous than the adoption of a fiat currency.

Not only user adoption on platforms is endogenous, the development of platforms is also endogenous. Just like executives manage firms' hiring and investment dynamically, tokens allow platform owners to manage the contributions from dispersed agents in the network and thus the evolution of the platform. It is not only about a one-time ICO.

For example, Kin tokens were issued to allow broad adoption and contribution by developers and users to foster a "virtuous cycle in which the ecosystem grows in both size and quality" (Livingston, 2017). Through measures such as capping individual purchases at $4,400, Kik structured the offering to encourage actual usage of the tokens as a medium of exchange instead of speculation. It is believed that an entire ecosystem of independent users and developers would have the right incentives to grow the platform. Indeed, by the end of 2018, Kin exceeded Ether and Bitcoin in blockchain user activities according to blocktivity.info. Both the Apple App and Google Play Stores accept Kin as a digital currency; the Kin ecosystem has also integrated dozens of third-party apps including Perfect 365, a top-rated AR beauty app with over 100 million users, and Nearby, a popular app with millions of users for meeting new people. The Kin Rewards Engine schema dynamically incentive agents in the system to grow the platform.

We should remind the readers that the use of platform tokens are not necessarily decentralized or blockchain-based. Even a traditional, centralized transaction platform may see the opportunity to create open interfaces - APIs (e.g., Uber have opened up APIs to enable third parties to add value) and use platform tokens. Kik was using local currency even though it was not blockchain-based; many community-based companies such as Facebook and Tencent, Microsoft, and Amazon have also introduced platform tokens before (see Gans and Halaburda (2015) for an excellent discussion). Other notable examples include Linden dollar for the game Second Life, WoW Gold for the game World of Warcraft. Gans and Halaburda (2015) and Halaburda and Sarvary (2016) provide excellent early discourses on virtual currencies on platforms.

What is different about blockchain-based platforms is that the distributed ledger tech-

nology brings immutability and decentralized consensus to enable smart contracting, which potentially allows commitment from the platform designers not seen before and not reliant on reputation alone.

(iii) Entrepreneurs' token dividends. How entrepreneurs get compensated through tokens is under-discussed in academic studies yet is important in practice. As seen in the Kik/Kin example, founders, early investors, key personnel get tokens which they can offload in secondary markets for consumption. This is a form of seigniorage.[19] What distinguishes blockchain-based platforms is that the issuance of tokens can be rule-based whereas inflation presents a perennial concern with fiat money and central banks. Indeed, many token designs specifically guard against inflation, for example, by capping the token supply. Bitcoin supply is capped at 21 million and Dfinity, at 469.21 million. Platforms like Kin also employs a token supply policy that targets at optimally incentivizing innovative proposals and growing the platform, as well as compensating the owners.

Precisely because tokens are used to compensate platform owners, entrepreneurs and token designers care about how "inflated" the ecosystem is because they care about the platform's franchise value going forward. This leads to many platforms also burning tokens to reduce inflation. Kin burned about 10% of tokens during the migration from Etheorem to its own blockchain.[20] Another recent example of discretionary reduction of token in supply is the Synereo team which burned 33% of its cryptocurrency reserves on March 31, 2018 after meeting development goals with fewer tokens (AMP) than initially project.[21]

3   Model

There are three types of agents in a continuous-time economy: an entrepreneur representing platform owners, a pool of competitive contributors, and a unit measure of users. The owner designs the platform's protocol. Contributors devote efforts and resources required for the operation and continuing development of the platform. Users conduct peer-to-peer transactions and realize trade surpluses on the platform. A generic consumption good serves

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[19]Entrepreneurs earn seigniorage for building infrastructure for and promoting platform networks. Even for stablecoins such as JPM Coin and Libra, the core networks of powerful institutions the founders foster add to attracting users. Key members in Libra are also compensated through Libra Investment Tokens.

[20]https://medium.com/kinblog/blockchain-strategy-update-next-steps-in-migrating-to-the-kin-blockchain-aed99e209654

[21]https://synereo.com/burn-amps/

as the numeraire. We first lay out the economic environment of the platform in question.

3.1 Platform Productivity and Contributors

We study a dynamically evolving platform whose productivity (quality), A_t, evolves as

$$\frac{dA_t}{A_t} = L_t dH_t, \tag{1}$$

where L_t is the resources invested into productivity growth by the platform and dH_t is a shock to investment efficiency

$$dH_t = \mu^H dt + \sigma^H dZ_t. \tag{2}$$

Here Z_t is a standard Brownian motion that generates the information filtration of this economy. The platform productivity A_t broadly captures platform matching technologies, network security, processing capacity, regulatory conditions, users' interests, the variety of activities feasible on the platform, etc. It therefore affects directly users' utility on the platform, which shall be made clear shortly. We consider a stochastic economy because we want to explore the model's implications on the volatility of token price.

Platform productivity grows through L_t, which may represent labor or capital inputs. Blockchain-based platforms often pay workers or investors on the spot with their tokens instead of a contract that promises the typical deferred compensation. Note that even in the simplest setting of production where firms combine labor and capital into output (often through a Cobb-Douglas function), there exists an implicit assumption that firms obtain inputs and then pay workers and investors the after production. The role of tokens as means of spot payment is especially salient on blockchain-based platforms.

Our focus is on the dynamic interaction between the entrepreneur and platform users, so we do not explicitly model contributors' decision-making but instead specify directly the required numeraire value of payment for L_t to be $F(L_t, A_t)$, which is increasing and convex in L_t and may also depend on A_t. Let P_t denote the unit price of token in terms of the numeraire (consumption goods). Given A_t, to gather the resources L_t, the platform issues $F(L_t, A_t)/P_t$ units of new tokens to workers.

Let M_t denote the total amount of circulating tokens (i.e., token supply). The payment for L_t leads to new issuance that adds to the growth of token supply, dM_t, by $F(L_t, A_t)/P_t$.

The functional form of $F(\cdot)$ depends on the structure of labor or capital markets that a

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typical blockchain-based platform participates in. A distinguishing feature of labor supply in a "gig economy" is that workers provide on-demand contributions and receive token payments instead of long-term employment contracts.[22] Here tokens facilitate the acquisition of labor by allowing the platform to pay for labor *on the spot* and thereby avoiding the limited commitment on the part of platform that arises in the implementation of deferred compensation and may lead to complex legal enforcement issues especially when workers and the platform belong to different judicial areas. Moreover, since digital tokens are often programmable (via smart contracting), escrow accounts can be set up and enforced automatically so that tokens are released to workers only if their inputs (e.g., programming codes) are received. Therefore, tokens also reduce the platform's exposure to workers' limited commitment.[23] Finally, L_t also represents the capital received from crowd-based investors, for example through initial coin offerings (ICOs) or seasoned token offerings (STOs). Investors receive tokens immediately on the spot instead a promise of payoff in the future, and thus, avoid exposure to the platform's failure to deliver future returns.[24]

A new issue emerges – workers and investors who receive tokens as compensation may still worry about token depreciation because the platform may issue more tokens in the future. In other words, while tokens avoid limited commitments by facilitating spot payments, the platform's lack of commitment against excess token supply is still a concern. Our analysis of optimal token supply addresses this question, but first let us introduce platform users.

3.2 Platform Users

In our model, after receiving tokens, workers and investors may immediately sell tokens to users. Workers and investors who contribute L_t can also be users themselves, and the model is not changed at all as long as the utility from token usage and the disutility from contributing L_t (which gives rise to $F(\cdot)$) are additively separable.

As in Cong, Li, and Wang (2018), users can conduct transactions by holding tokens. We use $x_{i,t}$ to denote the value (real balance) of agent i's holdings in the unit of numeraire. By

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[22]Workers may represent miners in Proof-of-Work-based public blockchains or drivers on ride-sharing apps.

[23]Another reason to introduce tokens as means of payment for L_t is the heterogeneity in labor quality. Consider a subset of workers supply high-quality efforts because they understand better the technologies behind the platform. Naturally, these capable workers assign a higher value to tokens because they are not concerned about the adverse selection problem that low-quality workers face due to their lack of technological knowledge. In other words, in contrast to cash-based compensation, token-based compensation screens out high-type workers and thereby improves the match between employer (the platform) and employees (workers).

[24]In general, blockchain-based tokens have immutable transaction recording that finalizes the digital transactions among dispersed, untrusted agents.

facilitating transactions, these holdings generate a flow of utility (or convenience yield) over dt given by

$$x_{i,t}^{1-\alpha} \left(N_t^{\gamma} A_t u_i \right)^{\alpha} dt, \tag{3}$$

where N_t is the platform user base, u_i captures agent i's idiosyncratic needs for platform transactions, and $\alpha, \gamma \in (0, 1)$ are constants. We provide a theoretical foundation for this specification of transaction surplus in Appendix A. A crucial difference from Cong, Li, and Wang (2018) is that we endogenize A_t and the token supply M_t.

The flow utility of token holdings depends on N_t, the total measure of users on the platform with $x_{i,t} > 0$.[25] This specification captures the network externality among users, such as the greater ease of finding trading or contracting counterparties in a larger community.

We assume that users' transaction needs, u_i, are heterogeneous. Let $G_t(u)$ and $g_t(u)$ denote the cross-sectional cumulative distribution and density functions respectively that are continuously differentiable over a support $[\underline{U}_t, \overline{U}_t]$ and may vary over time. u_i can be broadly interpreted. For payment blockchains (e.g., Ripple and Bitcoin), a high value of u_i reflects user i's needs for international remittance. For smart-contracting platforms (e.g., Ethereum), u_i captures user i's project productivity, and token holdings facilitate contracting.[26] For decentralized computation (e.g., Dfinity) and data storage (e.g., Filecoin) applications, u_i corresponds to the need for secure and fast access to computing power and data.

As P_t denotes the unit price of token in terms of the numeraire, we let $k_{i,t}$ denote the units of token that user i holds so the *real balance*

$$x_{i,t} = P_t k_{i,t}. \tag{4}$$

To join the platform (i.e., $k_{i,t} > 0$), a user incurs a flow cost ϕdt. For example, transacting on the platform takes effort and attention. Therefore, agents with sufficiently high u_i choose to join the platform, while agents with sufficiently low u_i do not participate.

Let $y_{i,t}$ denote user i's *cumulative* utility from platform activities. As in Cong, Li, and Wang (2018), we assume that either the users are well-diversified so that their transaction surpluses on the platform are priced by a stochastic discount factor. Equivalently, we can

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[25]One is example involves a producer who accepts tokens as means of payment and earns net profits equal to the full transaction surplus. The profits depend on the scale of operation, i.e., the sales $x_{i,t}$, and variables that determine the profit margin, which include the total outreach, N_t, the platform efficiency A_t, and the producer's idiosyncratic productivity u_i.

[26]For example, in a debt contract, the borrower's Ethereum can be held in an escort or "margin" account, which is automatically transferred to the lender in case of default. Posting more Ethereum as margin allows for larger debt contracts, which in turn lead to projects of larger scale and profits.

write the optimization problem under the risk-neutral measure.[27] User i then maximizes life-time utility under the risk-neutral measure,

$$\mathbb{E}\left[\int_0^\infty e^{-rt}dy_{i,t}\right],\tag{5}$$

where we can write the incremental utility $dy_{i,t}$ as follows:

$$dy_{i,t} = \max\left\{0, \max_{k_{i,t}>0}\left[(P_t k_{i,t})^{1-\alpha}\left(N_t^\gamma A_t u_i\right)^\alpha dt + k_{i,t}\mathbb{E}_t\left[dP_t\right] - \phi dt - P_t k_{i,t} r dt\right]\right\}.\tag{6}$$

The outer "max" operator in (6) reflects user i's option to leave the platform and obtain zero surplus, and the inner "max" operator reflects user i's optimal choice of $k_{i,t}$.

Inside the inner max operator are four terms that add up to give the incremental transaction surpluses from platform activities. The first term corresponds to the blockchain trade surplus given in (3). The second term is the expected capital gains from holding $k_{i,t}$ units of tokens. Users care about the sum of the on-chain transaction surplus and the expected token appreciation given by the first two terms in (6). The third term is the participation cost, and the last term is the financing cost of holding $k_{i,t}$ units of tokens.

It is worth emphasizing that on this tokenized platform, users must hold tokens for at least an instant dt to complete transactions and derive utility flows. Section 2 contains motiving examples and institutional details. This holding period exposes users to token price change over dt.

3.3 The Entrepreneur

The entrepreneur represents the founding developers who own the platform and design its protocol that includes the development strategy $\{L_t, t \geq 0\}$. Over time, the entrepreneur receives a *cumulative* number of tokens D_t as dividends and evaluates the tokens with a risk-neutral utility function and time discount rate r:

$$\max_{\{L_t,D_t\}_{t\geq 0}}\int_{t=0}^{+\infty}\mathbb{E}\left[e^{-rt}P_t dD_t\left[\mathbb{I}_{\{dD_t\geq 0\}}+(1+\chi)\,\mathbb{I}_{\{dD_t<0\}}\right]\right].\tag{7}$$

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[27]For the special case where users are risk neutral, the risk-neutral measure is the same as the data-generating physical measure.

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When $dD_t > 0$, the entrepreneur receives token dividends that have a market value P_t per unit.[28] Note that in equilibrium, the entrepreneur immediately sells her tokens to users who derive an extra convenience yield from token holdings.

We allow the entrepreneur to buy back and burn tokens to reduce the token supply (i.e., $dD_t < 0$). By reducing token supply, the owner can boost token price, and as a result, increase the value of future token dividends. A higher token price also allows the platform to gather more resources for productivity growth in the future. When $dD_t < 0$, the entrepreneur raises external financing (numeraire goods) at a proportional cost χ and buy back tokens.[29] As will be shown below, introducing such financing costs creates a conflict of interest between the entrepreneur and platform users, and thereby, allows us to discuss the role of blockchain as commitment device and how it affects the entrepreneur and users' welfare.

Considering both the token issued for investment L_t and the entrepreneur's dividend/buyback, we have the key accounting identity that describes the evolution of token supply:

$$dM_t = \frac{F(L_t, A_t)}{P_t} dt + dD_t. \tag{8}$$

The left side relates to tokens' function as means of payment while the right relates to financing and payouts. When the platform invests (first term of right side) or distributes token dividends (second term of right side), the total amount of tokens in circulation increases; the token supply decreases when the entrepreneur burns tokens out of circulation. Later we show that the platform owner's financial slack varies with M_t due to the financing cost χ, so the token stock is akin to the cash inventory in Bolton, Chen, and Wang (2011), except that the token price is endogenous and the financial slack decreases in M_t.

In what follows, we characterize a Markov equilibrium with the platform productivity A_t and the token supply M_t as state variables.

Definition 1. *A Markov equilibrium with state variable A_t and M_t is described by agents' decisions and equilibrium token price such that the token market clearing condition holds, users optimally decide to participate (or not) and choose token holdings, contributors supply*

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[28] For example, blockchain behemoth Bitmain Technologies Ltd and Founders Fund (known for early bets on SpaceX and Airbnb) invest in EOS and hold ownership stakes that entitle them for future token rewards. The gradual distribution of token dividends can be viewed as contingent vesting in reality – a certain amount of total tokens D_t have been allocated by time t but are distributed over time (via dD_t) depending on the stages of platform development and the tokens outstanding (i.e., different values of A_t and M_t).

[29] The parameter χ may also represent the forgone consumption and other investment opportunities. It is consistent with Bolton, Chen, and Wang (2011) who also model in reduced form the information, incentive, and transactions costs that a firm incurs whenever it raises real resources from external equity market.

resources for the compensation of $F(L_t, A_t)$ in numeraire value, and the platform strategies, i.e., L_t and D_t, are optimally designed to maximize the owner's value.

4 Dynamic Equilibrium

We first derive the platform owners' optimal investment and token dividend distribution, which in turn pin down the token supply. We then derive platform users' optimal decisions on adoption and token holding in order to aggregate token demand. Finally, token market clearing yields the equilibrium dynamics of token price, decentralized contribution, user adoption, and token allocation.

4.1 Optimal Token Supply

At time t, the owner's continuation or franchise value V_t (i.e., the time-t value fucntion) satisfies the following Hamilton–Jacobi–Bellman (HJB) equation

$$rV(M_t, A_t)\, dt = \max_{L_t, dD_t} P_t dD_t \left[\mathbb{I}_{\{dD_t \geq 0\}} + (1 + \chi) \mathbb{I}_{\{dD_t < 0\}} \right] + V_{M_t} \left[\frac{F(L_t, A_t)}{P_t} dt + dD_t \right] \quad (9)$$
$$+ V_{A_t} A_t L_t \mu^H dt + \frac{1}{2} V_{A_t A_t} A_t^2 L_t^2 (\sigma^H)^2 dt.$$

The first term in this HJB equation reflects the platform's dividend payout $(dD_t > 0)$ and buyback $(dD_t < 0)$. When there are more tokens circulating, the token price tends to be low, and so does the owner's continuation value. Therefore, we expect $V_{M_t} < 0$, as we later confirm in the numerical solution. Payout occurs only if $-V_{M_t} \leq P_t$, i.e., the marginal cost of increasing token supply is not greater than the market value of token. Buyback happens when $-V_{M_t} \geq P_t(1 + \chi)$, i.e., when the marginal benefit of decreasing token supply is not less than the cost of burning tokens. The second term is the product of marginal value of token supply, V_{M_t}, and the drift of token supply, which consists of tokens paid to contributors and tokens distributed to or burned by the entrepreneur. The third term is the marginal benefit of an A_t increase. The platform productivity increases in labor L_t, which is the mean productivity of $A_t \mu^H dt$. But hiring labor using tokens increases the token supply M_t, which has a marginal cost of $V_{M_t} \frac{F_{L_t}}{P_t} dt$ as $V_{M_t} < 0$. Moreover, labor productivity is uncertain, so the fourth term captures how such risk enters into the choice of L_t. The next proposition summarizes the platform's optimal policies.

Proposition 1 (**Optimal Token Supply**). *The optimal token supply strategy is given by (1) the optimal choice of L_t solved implicitly by the following equation*

$$V_{A_t} A_t \mu^H + V_{A_t A_t} A_t^2 L_t^* \left(\sigma^H\right)^2 = F_L \left(L_t^*, A_t\right) \left(\frac{-V_{M_t}}{P_t}\right) \tag{10}$$

and (2) the optimal choice of dD_t — the owner receives token dividends ($dD_t^* > 0$) only if $P_t \geq -V_{M_t}$, and buys back and burns tokens out of circulation ($dD_t^* < 0$) only if $-V_{M_t} \geq P_t (1 + \chi)$.

Equation (10) equates the marginal benefit of investment to the marginal cost. The left side is the marginal impact on the drift of A_t, evaluated by the entrepreneur's marginal value of A_t growth and adjusted for the risk of productivity shock via the second term.

The right side is the marginal cost of investment. Since the entrepreneur's marginal cost of token supply can be larger than the market value of tokens, the physical marginal cost F_L is multiplied by $-V_{M_t}/P_t$. This multiplier reflects a *token issuance cost*. Here the platform pays for investment with "undervalued" tokens. The payout/buy-back policy in Proposition 1 implies that $V_{M_t}/P_t \in [1, 1 + \chi]$. Because the entrepreneur incurs a financing cost $\chi > 0$ when burning tokens, there exists a region of (M_t, A_t) such that $V_{M_t}/P_t > 1$, i.e., a token issuance cost exists. As will be shown in the solution, the condition $-V_{M_t} \geq P_t$ holds in strict inequality almost everywhere. We have the following corollary that highlights the link between off-platform capital markets and the platform's token issuance cost.

Corollary 1 (**Token Issuance Cost**). *The entrepreneur's off-platform financing cost χ leads to a token issuance cost: When $\chi > 0$, there exists a positive measure of (M_t, A_t) such that the token issuance cost is positive, i.e., $-V_{M_t}/P_t > 1$. This issuance cost distorts the investment policy by amplifying the marginal cost of investment as shown in Equation (10).*

Our characterization of the optimal payout/buyback policy allays the concern over fraudulent designs by founding developers and the resulting explosion of token supply that destroys token value. One might worry that the entrepreneur has strong incentive and ample opportunity to build "back doors" in the protocol that allow them to steal tokens when tokens become valuable. The stolen tokens are likely to be sold in secondary markets for dollars, and thus, depress the token price. However, as shown in Proposition 1, our setup already allows the entrepreneur to extract token as dividends and the optimal payout policy maximizes the owner's value, so it is an incentive-compatible rewarding scheme for the founding designers. In other words, we characterize a subgame perfect equilibrium between a large

layer (the platform) and a continuum of small players (users). From a regulatory perspective, any proposal of blockchain or platform design should disclose the payout schemes to the founding developers and they should be broadly inline with the above characterization.

Corollary 2 (**Resistance Against Excess Token Supply**). *By Pontryagin's maximum principle, the optimal payout and investment policies in Proposition 1 maximize the entrepreneur's value at any t and rule out excess token supply from additional token issuance.*

4.2   Optimal Token Demand

We conjecture and later verify that in equilibrium, the token price, P_t, evolves as

$$dP_t = P_t \mu_t^P dt + P_t \sigma_t^P dZ_t, \tag{11}$$

where μ_t^P and σ_t^P are endogenously determined. Agents take the price process as given under rational expectation. Conditioning on joining the platform, user i chooses the optimal token holdings, $k_{i,t}^*$, by using the first order condition,

$$(1 - \alpha) \left(\frac{N_t^\gamma A_t u_i}{P_t k_{i,t}^*} \right)^\alpha + \mu_t^P = r, \tag{12}$$

which states that the sum of *marginal* transaction surplus on the platform and the expected token price change is equal to the required rate of return, r.

Rearranging this equation, we obtain the following expression for the optimal token holdings:

$$k_{i,t}^* = \frac{N_t^\gamma A_t u_i}{P_t} \left(\frac{1 - \alpha}{r - \mu_t^P} \right)^{\frac{1}{\alpha}}. \tag{13}$$

$k_{i,t}^*$ has several properties. First, agents hold more tokens when the common productivity, A_t, or agent-specific transaction need, u_i, is high, and also when the user base, N_t, is larger because it is easier to conduct trades on the platform. Equation (13) reflects an investment motive to hold tokens, that is $k_{i,t}^*$ increases in the expected token appreciation, μ_t^P.

Using $k_{i,t}^*$, we obtain the following expression for the user's maximized profits conditional on participating on the platform:

$$N_t^\gamma A_t u_i \alpha \left(\frac{1 - \alpha}{r - \mu_t^P} \right)^{\frac{1 - \alpha}{\alpha}} - \phi. \tag{14}$$

User i only participates when the preceding expression is non-negative. That is, only those users with sufficiently large u_i participate. Let $\underline{u}_t$ denote the type of the marginal participant, then

$$\underline{u}_t = \underline{u}\left(N_t; A_t, \mu_t^P\right) = \frac{\phi}{N_t^\gamma A_t \alpha}\left(\frac{r - \mu_t^P}{1 - \alpha}\right)^{\frac{1-\alpha}{\alpha}}. \tag{15}$$

The adoption threshold $\underline{u}_t$ is decreasing in A_t because a more productive platform attracts more users. The threshold also decreases when agents expect a higher token price appreciation (i.e., higher μ_t^P). Because only agents with $u_i \geq \underline{u}_t$ participate, the user base is then

$$N_t = 1 - G_t\left(\underline{u}_t\right). \tag{16}$$

Equations (15) and (16) jointly determine the user base N_t given A_t and μ_t^P. Note that zero adoption is always a solution, and trivially leads to zero token price.

Proposition 2 (**Token Demand and User Base**). *Given A_t and μ_t^P, the platform has a positive user base if Equations (15) and (16) have solutions for $\underline{u}_t$ and N_t. Conditional on participating, User i's optimal token holding, $k_{i,t}^*$, is given by Equation (13). The token holding, $k_{i,t}^*$, decreases in P_t and increases in A_t, μ_t^P, u_i, and N_t.*

To obtain the numerical solution, we later specify $G_t(u)$ and explicitly derive N_t.

4.3 Token Market Clearing

Clearing the token market determines the token price. Let us define the *participants' aggregate transaction need* as

$$U_t := \int_{u \geq \underline{u}_t} u g_t(u)\, du, \tag{17}$$

the integral of u_i of participating agents.

The market clearing condition is

$$M_t = \int_{i \in [0,1]} k_{i,t}^* di. \tag{18}$$

Substituting optimal holdings in Equation (13) into the market clearing condition in Equation (18), we arrive at the *Token Pricing Formula*.

Proposition 3 (**Token Pricing**). *The equilibrium token price is given by*

$$P_t = \frac{N_t^\gamma U_t A_t}{M_t} \left(\frac{1-\alpha}{r - \mu_t^P} \right)^{\frac{1}{\alpha}}.$$ (19)

The token price increases in N_t – the larger the user base is, the higher trade surplus individual participants can realize by holding tokens, and stronger the token demand. The price-to-user base ratio increases in the platform productivity, the expected price appreciation, and the network participants' aggregate transaction need, while it decreases in the token supply M_t.[30] Equation (19) implies a differential equation for P_t on the state space of (M_t, A_t). This can be clearly seen once we apply the infinitesimal generator to the token price, $P_t = P(M_t, A_t)$, in the Markov equilibrium to express μ_t^P into a collection of first and second derivatives of P_t by Itô's lemma. Note that the equilibrium user base, N_t, is already a function of A_t and μ_t^P as shown in Proposition (2). Therefore, the collection of token market clearing condition at every t essentially solve the full dynamics of token price.

Together, Equation (8) and (18) describe the primary and secondary token markets. The change of M_t is a flow variable, given by Equation (8), that includes the new issuances from platform investment and payout and the repurchases by the entrepreneur. The token supply M_t is a stock variable, and through Equation (18), it equals the token demand of users.

5 Equilibrium Characterization

We further characterize the equilibrium by analytically deriving and numerically solving the system of differential equations concerning token price and platform owners' value function. To streamline exposition and focus on core economic insights, we make some simplifying parametric assumptions.

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[30]The formula reflects certain observations by practitioners, such as incorporating DAA (daily active addresses) and NVT Ratio (market cap to daily transaction volume) in token valuation framework, but instead of heuristically aggregating such inputs into a pricing formula, we solve *both* token pricing and user adoption as an equilibrium outcome. See, for example, *Today's Crypto Asset Valuation Frameworks* by Ashley Lannquist at Blockchain at Berkeley and Haas FinTech.

5.1 Parametric Choices

Following the literature on investment in finance and macroeconomics (e.g., Bolton, Chen, and Wang, 2011), we assume a convex (quadratic) investment cost function of F:

$$F\left(L_t, A_t\right) = \left(L_t + \frac{\theta}{2}L_t^2\right)A_t, \tag{20}$$

where $\theta > 0$ can depend on the elasticity of labor and capital supply. In comparison with traditional contracts, token-based compensation may enlarge the supply elasticity and thereby reduce θ by alleviating the problem of limited commitment as previously discussed.

Lemma 1 (Parameterized Optimal Investment). *Under the parametric restriction given by Equation (20), the platform's optimal investment is*

$$L_t^* = \frac{V_{A_t}\mu^H + \frac{V_{M_t}}{P_t}}{-\frac{V_{M_t}}{P_t}\theta - V_{A_t A_t}A_t\sigma^2}, \tag{21}$$

To obtain closed-form solutions, we assume that u_i follows the commonly used Pareto distribution on $[\underline{U}_t, +\infty)$ with cumulative probability function (c.d.f.) given by

$$G_t\left(u\right) = 1 - \left(\frac{\underline{U}_t}{u}\right)^{\xi}, \tag{22}$$

where $\xi \in (1, 1/\gamma)$ and $\underline{U}_t = 1/\left(\omega A_t^\kappa\right), \omega > 0, \kappa \in (0, 1)$. The cross-section mean of u_i is $\frac{\xi \underline{U}_t}{\xi - 1}$.

Note that $\underline{U}_t$ decreases in A_t, which reflects the competition from alternative platforms that are inspired by the platform's success (i.e., a high value of A_t). For example, this specification captures the reality that after the success of Bitcoin, alternative blockchains emerge as competitors in the area of payments. Similarly, there now exists alternative platforms to Ethereum for smart contracting. The effects of competition are small when ω is close to zero. As A_t increases, the mass of u_i is being shifted towards lower values.[31]

Lemma 2 (Parameterized User Base). *Given A_t and μ_t^P, from Proposition 2, we have a unique non-degenerate solution, N_t, for Equations (15) and (16) under the Pareto distri-*

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[31]Platform competition can also be captured by the depreciation of A_t. The current specification lends technical convenience.

bution of u_i given by Equation (22):

$$N_t = \left(\frac{A_t^{1-\kappa} \alpha}{\omega \phi} \right)^{\frac{\xi}{1-\xi\gamma}} \left(\frac{1-\alpha}{r - \mu_t^P} \right)^{\left(\frac{\xi}{1-\xi\gamma} \right)\left(\frac{1-\alpha}{\alpha} \right)}, \tag{23}$$

if $\underline{u}_t \geq \frac{1}{\omega A_t^\kappa}$, i.e., $A_t^{1-\kappa} (\frac{1-\alpha}{r-\mu_t^P})^{\frac{1-\alpha}{\alpha}} \leq \frac{\omega\phi}{\alpha}$; otherwise, $N_t = 1$.

Our later discussion focuses on $\xi\gamma < 1$ so that the user base, N_t, is increasing in the platform productivity in spite of the competition effects. This is realistic because a technology leader usually benefits from its innovation despite the presence of potential competing followers. Moreover, we focus on low values of A_t such that $N_t < 1$ in the Markov equilibrium so as to examine how token allocation interacts with user base dynamics. Under the Pareto distribution, the aggregate transaction need is given by

$$U_t = N_t \left(\frac{\xi \underline{u}_t}{\xi - 1} \right) = \left(\frac{\xi}{(\xi-1)\, \omega A_t^\kappa} \right) \left(\frac{A_t^{1-\kappa} \alpha}{\omega \phi} \right)^{\frac{\xi-1}{1-\xi\gamma}} \left(\frac{1-\alpha}{r - \mu_t^P} \right)^{\left(\frac{\xi-1}{1-\xi\gamma} \right)\left(\frac{1-\alpha}{\alpha} \right)}. \tag{24}$$

We adopt the following parameter restriction, which helps reduce the dimension of state space in our numerical analysis and convey economic intuitions.

Assumption 1. $\frac{2\xi-1}{1-\xi\gamma} = \frac{\kappa}{1-\kappa}$.

The assumption implies $\kappa > 1/2$ given that $\gamma > 0$ and $\xi\gamma < 1$. Moreover, N_t increases in A_t while U_t decreases in A_t, and the effects of A_t on N_t and U_t exactly cancel out each other in the token pricing formula (Equation (19)). The intuition is that even though a better platform productivity induces more users, individual users' needs are now weaker because they have access to alternative platforms. The token pricing formula then simplifies.

Lemma 3 (**Parameterized Token Price**). *Under Assumption 1, the equilibrium token price in Proposition 3 when $N_t < 1$ is given by*

$$P_t = \frac{A_t}{M_t} \frac{\xi}{(\xi - 1)\, \omega^{\frac{1}{1-\kappa}}} \left(\frac{\alpha}{\phi} \right)^{\frac{\kappa}{1-\kappa}} \left(\frac{1-\alpha}{r - \mu_t^P} \right)^{\frac{1}{\alpha} + \left(\frac{1-\alpha}{\alpha} \right)\left(\frac{\kappa}{1-\kappa} \right)}. \tag{25}$$

In the following, we characterize a Markov equilibrium in a transformed state space. The equilibrium variables depend on (m_t, A_t), where

$$m_t = \frac{M_t}{A_t}. \tag{26}$$

By inspecting Equation (25), we can see that m_t is the only state variable driving the token price – in such an equilibrium μ_t^P is a function of m_t only and so does P_t.

For the parameters that affect user activities, we follow Cong, Li, and Wang (2018) to set $\alpha = 0.3$, $\phi = 1$, $r = 0.05$, and the volatility parameter, $\sigma^H = 2$. For the mean productivity growth, we set $\mu^H = 0.5$, which generates a μ_t^P in line with the values in Cong, Li, and Wang (2018). We set the rest of parameters to illustrate the qualitative implications of the model: $\theta = 10,000$ so the highest level of investment, L_t, is about 25% higher than the lowest level; $\xi = 2$, $\kappa = 0.8$ and $\omega = 100$ for the distribution parameters of u_i; $\chi = 20\%$ for the financing cost. The model's qualitative implications are robust to the choice of these parameters. Finally, we set $\gamma = 1/8$ to satisfy Assumption 1.

5.2 Solving the Equilibrium

We numerically solve the model in the space $(m_t = M_t/A_t, A_t)$ instead of the original space (M_t, A_t), because given the parametric choices in Section 5.1, m_t shall be the only state variable driving the token price P_t and the platform investment L_t. By Itô's Lemma, the dynamics of m_t is given by

$$\frac{dm_t}{m_t} = \frac{dM_t}{M_t} - \frac{dA_t}{A_t} + L_t^2(\sigma^H)^2 dt, \tag{27}$$

where the last term is a quadratic variation term from Itô's calculus. We conjecture that the value function $V(M_t, A_t) = v\left(\frac{M_t}{A_t}\right) A_t$, so the derivatives are given by

$$V_{M_t} = v'(m_t), \quad V_{A_t} = v(m_t) - v'(m_t) m_t, \quad V_{A_t A_t} = v''(m_t) \frac{m_t^2}{A_t}. \tag{28}$$

In the interior region $(dD_t = 0)$, the HJB equation is

$$rv(m_t) = \max_{L_t} v'(m_t) \frac{\left(L_t + \frac{\theta}{2} L_t^2\right)}{P_t} + [v(m_t) - v'(m_t) m_t] L_t \mu^H + \frac{1}{2} v''(m_t) m_t^2 L_t^2 (\sigma^H)^2. \tag{29}$$

Therefore, if P_t and L_t are all functions of m_t, and we confirm the value function conjecture.

Substituting these derivatives of $V(M_t, A_t)$ into the optimal investment, we have

$$L_t^* = \frac{[v(m_t) - v'(m_t) m_t] \mu^H + \frac{v'(m_t)}{P_t}}{-\frac{v'(m_t)}{P_t} \theta - v''(m_t) m_t^2 (\sigma^H)^2}. \tag{30}$$

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Therefore, L_t is a function of m_t as long as P_t is a function of m_t. Equation (25) implies that P_t is a function of m_t because when P_t is a function of m_t, so is μ_t^P. Hence the conjecture of m_t being the only the state variable driving P_t and L_t is internally consistent.

The optimality conditions for dD_t give us the boundary conditions to solve $v(m_t)$. We conjecture that $v' < 0$ because when m_t increases, more tokens are supplied and the current and future token price declines, which reduces the owner's continuation value. There exists a lower bound for m_t such that $m_t \geq \underline{m}$, $dD_t^* \geq 0$. At this payout boundary, the marginal value of retained token must be equal to the market value, i.e.,

$$-v'(\underline{m}) = P(\underline{m}). \tag{31}$$

Since the payout boundary is optimally chosen, we also have the usual "super contact" condition:

$$-v''(\underline{m}) = P'(\underline{m}). \tag{32}$$

Moreover, because the payout boundary is a reflecting boundary, to rule out arbitrage in the token market, we have:

$$P'(\underline{m}) = 0. \tag{33}$$

Intuitively, the distribution of token dividends happens when the token supply is sufficiently small relative to the platform productivity, i.e., low m_t.

The upper bound for m_t is the buyback boundary. As m_t increases, the token supply is large relative to the platform productivity, and the token price declines, so the owners buy back and burn tokens out of circulation. At $m_t = \overline{m}$,

$$-v'(\overline{m}) = P(\overline{m})(1+\chi). \tag{34}$$

Since the buyback boundary is optimally chosen, we also have the "super contact" condition:

$$-v''(\overline{m}) = P'(\overline{m})(1+\chi). \tag{35}$$

Moreover, because the payout boundary is a reflecting boundary, to rule out arbitrage in the token market, we have:

$$P'(\overline{m}) = 0. \tag{36}$$

At the boundaries, the amount of payout and buyback exactly offsets the variation in m_t

from L_t and the shock dZ_t that would otherwise drive it beyond the boundaries.

Proposition 4 (**Markov Equilibrium Solution**). *With Lemmas 1, 2, and 3, there exists a Markov equilibrium with A_t and $m_t = M_t/A_t$ as state variables and the following properties:*
(i) The token price $P(m_t)$ and the value function $V(M_t, A_t) = v(m_t) A_t$ uniquely solve the system of ordinary differential equations given by Equation (25) and (29) subject to boundary conditions given by Equations (31) to (36).
(ii) The token price P_t, platform investment L_t in Equation (30), and the payout/buy-back policy D_t, all depend on m_t only.
(iii) Users' optimal token holdings and participation decisions together with the user base depend on both m_t and A_t according to Proposition 2.

Discussion: Token Supply Limit. Blockchain platforms often feature a cap on token supply. One way to incorporate this is to have an absorbing upper bound of m_t, say $\widetilde{m}$. In such case, once reaching a multiple of the platform productivity, i.e., $\widetilde{m} A_t$, the supply would grow proportionally with A_t forever, and according to Lemma 3, token price will then be a constant. As for newly issued tokens, they are divided between the entrepreneur and contributors, and here the entrepreneur faces a standard consumption-savings trade-off – if she takes a larger share of the new tokens, the productivity grows slower.

5.3   Endogenous Platform Development

Panel A of Figure 1 plots $v(m_t)$. Because the value function, $V(M_t, A_t) = v(m_t) A_t$, Panel A shows that the platform owner's value, scaled by productivity, declines in the productivity-adjusted token supply (a form of platform "inflation"). Intuitively, when more tokens are circulating relative to platform productivity, it is more likely for the owner to reach the buyback (upper) boundary and pay the financing cost, and in the less likely event of token payout, the owner receives a lower value due to the depressed token price when m_t is high. Note that the value function is always positive in Panel A, suggesting that the platform owner never abandons the platform. The curve starts at the payout boundary and ends at the buyback boundary where the owner actively devotes real resources to buy back and burn tokens out of circulation in order to support token price and her franchise value.

Panel B of Figure 1 plots the optimal platform investment against the productivity-adjusted token supply. The declining pattern is largely driven by the rising cost of token
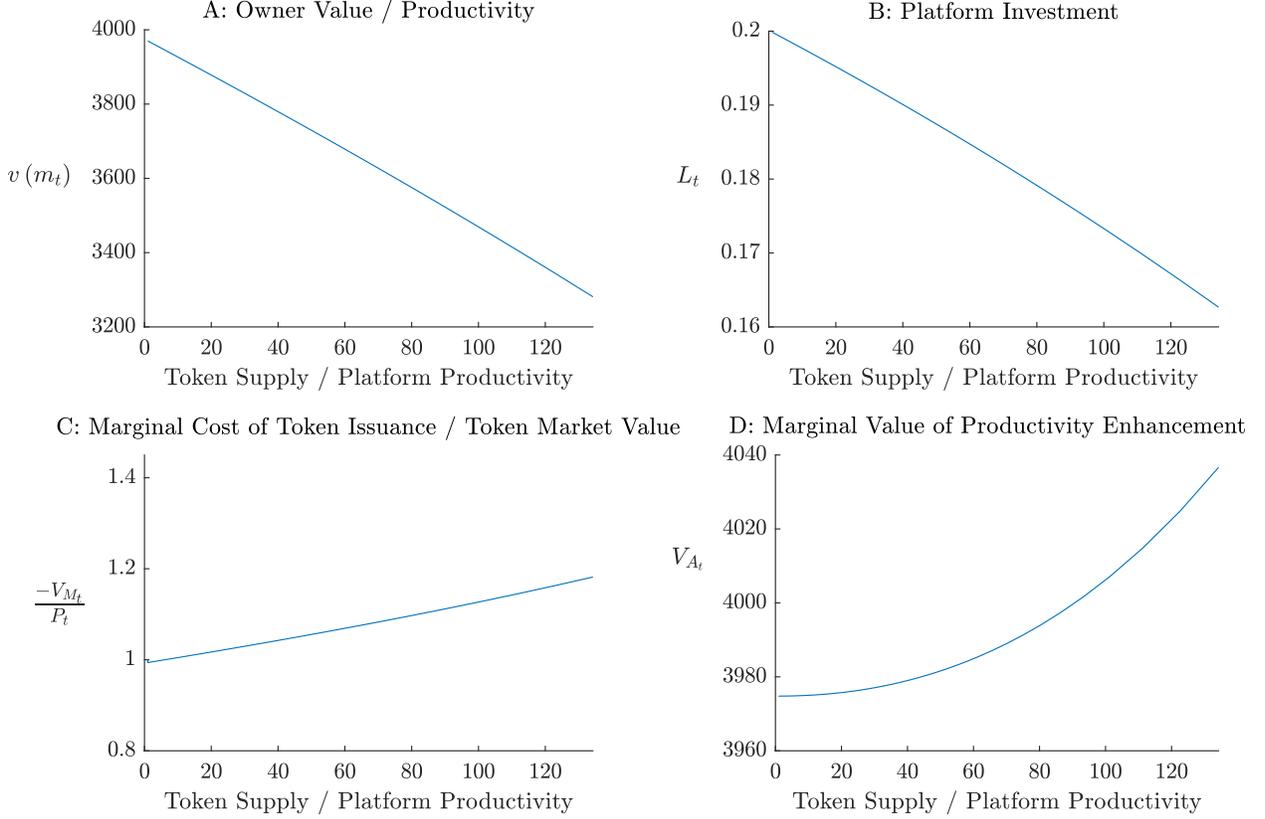
Figure 1: **Optimal Platform Investment Financed by Token Issuances**.

issuance (Panel C), even though as m_t increases, the marginal value of productivity enhancement rises (Panel D). The optimal L_t^* increases in V_{A_t}

The optimal L_t^* increases in $-V_{M_t}/P_t$, the ratio of the marginal cost of token issuance, $-V_{M_t}$, to the token market price, P_t. This ratio measures the valuation gap that exists between platform owners (insiders) and the workers and users (outsiders), i.e., the token issuance cost. When the gap is high, it is costly from the owners' perspective to gather resources using tokens. The ratio starts at one, as implied by the value-matching condition of the payout boundary. The gap widens as the token supply outpaces the growth of platform productivity, i.e., as m_t increases, and eventually, when the gap reaches $(1 + \chi)$, the platform owner's optimal buy back tokens. The increasing $-V_{M_t}/P_t$ largely contributes to the decreasing pattern of L_t^*.

The optimal L_t^* increases in V_{A_t}, the marginal value of A_t, because on average, investment has a positive productivity, i.e., $\mu^H > 0$, more resources gather by token payments, L_t, means a higher expected growth of A_t. Near the buyback boundary, V_{A_t} is high because an increase

of A_t reduces m_t and the likelihood of costly buyback.

Finally, according to Equation (10), the second-order derivative of value function to productivity also affects the choice of investment, which essentially represents a precautionary motive in the choice of L_t. It is not plotted because under the current parameterization, its quantitative contribution to the dynamics of L_t is small. However, the intuition of precaution towards investment risk is still interesting. The token distribution is largely a real-option decision. While it is not completely irreversible, reversing it (i.e., buying back tokens) incurs a cost of χ.[32] The probability of incurring such costs increases as m_t approaches the buyback boundary, so the platform becomes increasingly cautious on making large investment that also brings in large exposure to investment productivity shock in Equation (2). Therefore, the platform owner may choose to invest less in order to preserve some slack, i,e., the flexibility to issue more tokens in the future.

Overall, our model reveals a rich set of trade-offs in the choice of token-compensated contributions for platform development. The model has the potential to explain various features of token distribution to open-source engineers, miners (ledger maintainers), and crowd-sourced financiers in real life.

5.4 Token Price and User Adoption

The dynamics of token price is directly linked to that of token supply. As shown in Panel A of Figure 2, the token price, P_t, declines in m_t, the productivity-adjusted token supply. From Equation (27), the diffusion of m_t is $-m_t L_t^* \sigma^H$ (through $-dA_t/A_t$), which is negative. Therefore, a positive shock in labor productivity decreases m_t by increasing A_t, moving the economy closer to the payout boundary. The token price increases in response and is procyclical with respect to the shock.

That said, in stark contrast to the 200% per annum volatility of labor productivity that we input, i.e., the fundamental volatility, σ_t^P is surprisingly small (below 0.2% in Panel B of Figure 2) due to the stabilizing effect of the endogenous cumulative token process D_t. The platform owners' incentive to pay out when m_t is low and buy back when m_t is high moderate the variation of token price through the control over token supply.

Corollary 3. *From Proposition 1, the token price is bounded in* $\left[-\left(\frac{1}{1+\chi} \right) V_{M_t}, -V_{M_t} \right]$.

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[32]When the Synereo team has to hold multiple meetings and incur effort cost to explain to users when the team burned 33% of its cryptocurrency reserves. https://synereo.com/burn-amps/
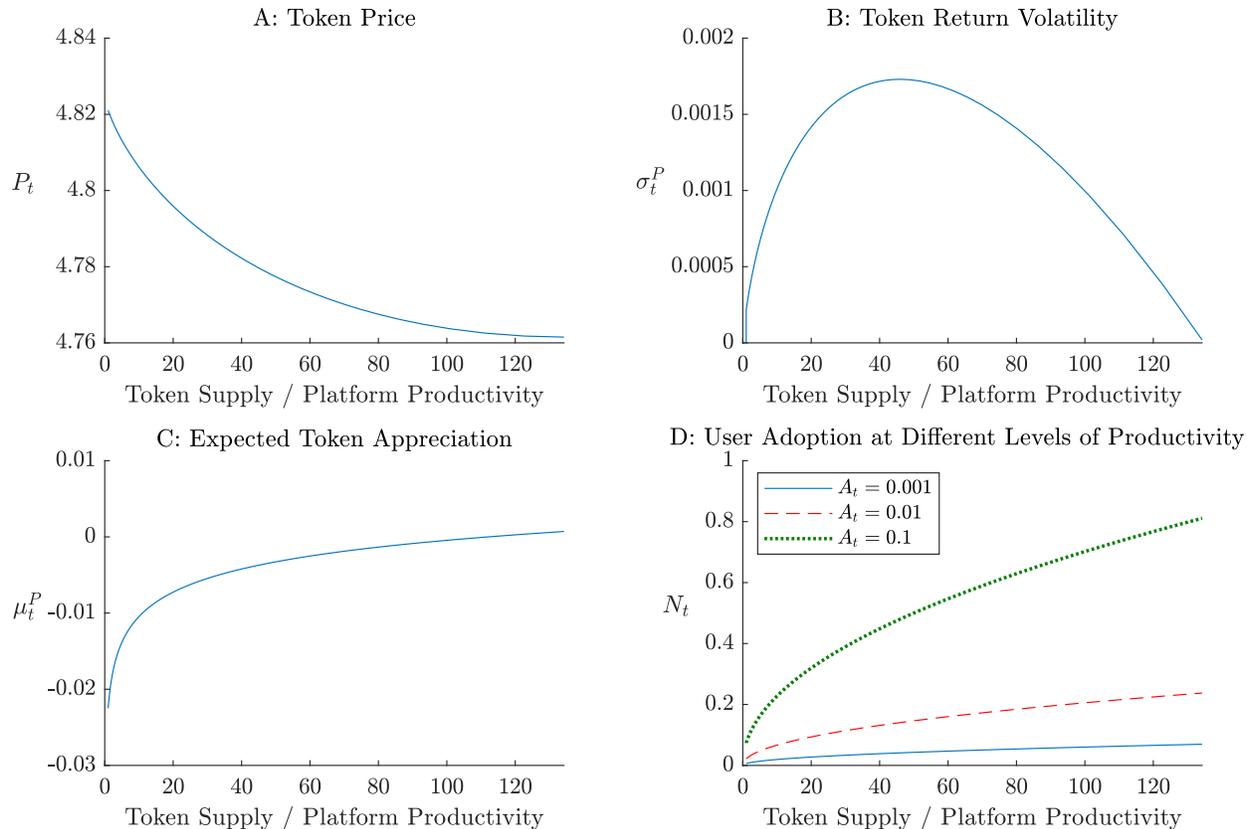
Figure 2: **Token Price Dynamics, Stablecoin, and User Adoption**.

Here the token value has two anchors. First, users need tokens for transactions. Second, when token price declines significantly below the marginal cost of having one more unit of token circulating, i.e., $P_t \leq -V_{M_t}/(1+\chi)$, the owner *optimally* chooses to exchange real resources (i.e., dollars) for tokens to reduce the supply.

Panel C of Figure 2 shows the expected token price change against m_t. When m_t is low, the expectation is negative, reflecting the likely token supply increase due to payout and labor demand (Panel A of Figure 1). The expected token price change gradually increases and eventually becomes positive because the expected token supply *change* is increasingly dominated by the declining investment (Panel A of Figure 1) and the possibility of token buyback by the platform owners.

In Proposition 2, unlike other variables in the model that only depends on m_t, the user base N_t depends on both m_t (through μ_t^P) and A_t. Panel D of Figure 2 plots the user base dynamics with different values of A_t to show that as the the risk-adjusted token appreciation, μ_t^P, increases, the user base also increases.

Discussion: stablecoins. Practitioners have proposed several candidate designs for stablecoin, a powerful alternative to fiat money or even digital payments through the commercial banking system (e.g., Duffie, 2019). A popular approach is to mimic open market operation done by central banks. When token price is low, the platform issues token bonds to buy back tokens. Token bonds promise to pay the principal with interest in the future, but all payments are in tokens. The problem with this design is that an inter-temporal substitution between current and future tokens tilts the schedule of token supply over time, but it does not introduce any real resource to support token price, nor does it provide any incentive to economic agents to devote such resources. A champion of this design, the Basis stablecoin project that attracted $133 million of venture capital in April 2017, has closed down all operations, citing US securities regulation as the reason for its decision.[33]

An alternative design is collateralization, backing token value with real resources such as dollar (e.g., Tether, Circle, Gemini, JPM coin, and Paxos), oil reserves (e.g., Venezuelas El Petro, OilCoin, and PetroDollars). A derivative of such design is to further tranche the claims on real resources, so tokens as means of payment are the most senior tranche, which is less information-sensitive and thus has a stable secondary-market value. Such designs are often subject to frauds and market manipulations (e.g., Griffin and Shams, 2018).

The way the buyback policy stabilizes token value differs from the existing proposals. First, we introduce the value of platform ownership — the continuation or franchise value — that derives value from token dividends that are optimally chosen instead of a prescheduled bond coupons do. Then, given the economic cost of buying back tokens, χ, we characterize the incentive for the platform owners to voluntarily buy back tokens using the numeraire goods (real resources). The parameter χ may change, and can even be a function of macroeconomic state variables to better represent the cost of token buyback.

Note that it is in the interest of platform owners such as the founders to buy back tokens, which provides an incentive-compatible support for token price. Later we introduce commitment using smart contracting which can further help creating stablecoins.

6 Blockchain as a Commitment Device

The rise of tokens as means of payment on digital platforms is a recent phenomenon with many applications inspired by the success of Bitcoin, Ethereum, and other blockchain-based

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[33]https://icoexaminer.com/ico-news/133-million-basis-stablecoin-project-ceases-and-desists-citing-regulatory-concerns/

startups. So far, our analysis has been focusing on the case of discretionary token supply policy of the platform – depending on the state of the world, the platform can freely adjust the token issuance to finance platform development or to pay (or be bought back by) the owners. The blockchain technology allows consensus protocols and token supply rules to be immune to adjustments after the launching of platform. Next, we study platforms' commitments to predetermined rules of token supply. Our analysis helps understand why tokens become a viable payment solution after the blockchain technology becomes available and what are the welfare gains for platform owners and users from blockchain-enabled commitment.[34]

Specifically, we consider $F(L_t, A_t)/P_t dt = \mu^M M_t dt$, which implies a constant growth of token supply in the interior region ($dD_t = 0$) to finance the enhancement of platform productivity:

$$dM_t = F(L_t, A_t)/P_t dt = \mu^M M_t dt, \tag{37}$$

which is popular among the blockchain applications (e.g., Ethereum) as a way to create scarcity and avoid inflation of tokens. This rule of token supply implies that the resources a platform gathers, L_t, is fixed given the current productivity A_t, token price P_t, and token supply M_t. In our numerical solution, we use $\mu^M = 10$, which is the average level of token growth in the baseline model.

We still allow the platform owners to receive token dividends and buy back tokens, but with L_t fixed, the owners' only control variable is dD_t and the HJB equation directly specifies a differential equation for the value function:

$$rV(M_t, A_t) dt = \max_{dD_t} P_t dD_t \left[\mathbb{I}_{\{dD_t \geq 0\}} + (1+\chi) \mathbb{I}_{\{dD_t < 0\}} \right] + V_{M_t} \left[\mu^M M_t dt + dD_t \right] \tag{38}$$

$$+ V_{A_t} A_t L_t \mu^H dt + \frac{1}{2} V_{A_t A_t} A_t^2 L_t^2 (\sigma^H)^2 dt.$$

Comparing it with Equation (9), the token paid for L_t is replaced by $\mu^M M_t dt$. Under the parametric choices in Section 5.1 and in the interior where $dD_t = 0$, we have

$$rv(m_t) = v'(m_t) m_t \mu^M + \left[v(m_t) - v'(m_t) m_t \right] L_t \mu^H + \frac{1}{2} v''(m_t) m_t^2 L_t^2 (\sigma^H)^2, \tag{39}$$

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[34]We cannot over-emphasize the possibility of commitment brought forth by blockchains because commitment lies at the heart of many economic issues. For example, firms' inability to commit to future funding choices has profound consequences for understanding capital structure dynamics, as Admati, DeMarzo, Hellwig, and Pfleiderer (2018) demonstrates. In fact, in the AFA presidential address in 2019, DeMarzo highlights that commitment issues are first-order in understanding capital structure dynamics and collateralization presents one form of commitment (Demarzo, 2019).
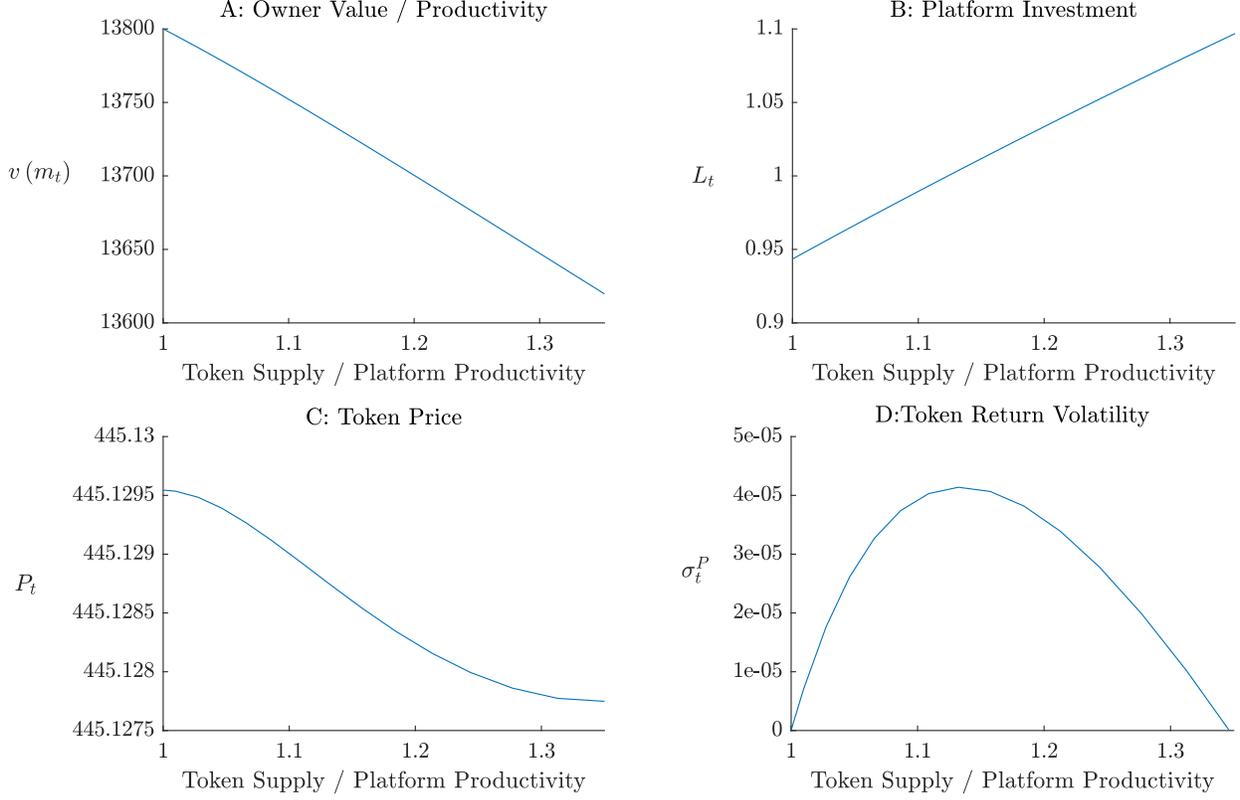
Figure 3: **Constant Growth of Token Supply for Platform Development**.

where L_t is given by

$$L_t + \frac{\theta}{2}L_t^2 = \mu^M P_t m_t. \tag{40}$$

The boundary conditions are the same as those of the baseline model. Because the left-hand side increases when $L_t > -1/\theta$ and $L_t \geq 0$, so platform investment increases in P_t and m_t. Intuitively, when token is more valuable, the platform gathers more decentralized contributions; when the token supply is high, the amount of newly issued tokens is also high per unit of time, leading to a larger payment for L_t.

Proposition 5 (**Predetermined Token Growth**). *Under the commitment to constant growth of token supply for productivity enhancement, platform investment given by Equation (40) increases in $P_t m_t$. The owner receives token dividends ($dD_t^* > 0$) only if $P_t \geq -V_{M_t}$, and buys back and burns tokens out of circulation ($dD_t^* < 0$) only if $-V_{M_t} \geq P_t(1+\chi)$. Token price is determined by Equation (19) as in the baseline model.*

Figure 3 plots the key variables from the solution under the commitment of token growth. Comparing Panel A of Figure 3 with Panel A of Figure 1, we can see that commitment

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significantly increases the platform owners' value. By comparing Panel B in the two figures, we see that such an increase mainly comes from an about five times larger level of investment. Therefore, a key impact commitment brings is the mitigation of under-investment.

Commitment adds value because the financing costs χ creates a conflict of interest between the platform owner and users, which manifests itself when we examine the usage value and the investment value of tokens. To the owner, the value of tokens is $-V_{M_t}$, while to the users, it is P_t. The wedge between $-V_{M_t}$ and P_t widens as the productivity-adjusted token supply, m_t, increases and reaches χ at the buyback boundary. Therefore, as shown in Equation (10), the financing cost translates into a dynamic cost of token issuance to contributors that discourages investment. Intuitively, under-investment occurs because the platform owner fully bears this dynamic token issuance cost, $-V_{M_t}/P_t > 1$, yet the resulting productivity enhancement benefits both the owners (by decreasing m_t and thus increasing $v(m_t)$) and the users (via the token usage value — a larger flow of convenience yield). Moreover, at the buyback boundary, the platform owner bears χ while the token buyback benefits both the owner (also by decreasing m_t and increasing $v(m_t)$) and the users (through the token investment value — the resulting token appreciation).

Commitment mitigates the under-investment problem. With commitment, the constant growth of token supply in the interior region quarantines investment from the influence of financing cost. As a result, investment depends directly on the state variable m_t as shown in Equation (40). When m_t increases, the declining token price (Panel C of Figure 3) drives down L_t but the increasing token supply drives up L_t. In Panel B of Figure 3, the latter force dominates, so the large m_t is, the higher platform investment, exhibiting a pattern of investment opposite to that in the baseline model (Panel B of Figure 1).

Commitment also creates more frequent token buyback. The range of m_t in Figure 3 is reduced by one hundred times in comparison with that in Figure 1 and 2. In other words, the platform owner is more willing to buy back and burn tokens at a much lower threshold level of productivity-adjusted token supply. As m_t increases, the ratio of owner's token value, $-V_{M_t}$, to users' token value, P_t, increases and reaches $1+\chi$ at the buyback boundary. A tighter range of m_t suggests that the ratio rises faster in m_t under commitment. When investment increases in m_t, an increasing amount of tokens are issued at the expense of platform owner to boost productivity that benefits users. Therefore, as m_t increases, the divergence of owner's and users' interest, $-V_{M_t}/P_t$, becomes increasingly large, reaching $1+\chi$ fast and justifying token buyback at a low level of m_t.

Even though the platform owner ends up paying more frequently the financing cost χ

under commitment, the owner's value is still higher than the baseline model of full discretion because the higher level of platform investment translates into a higher token value through users expectation of productivity growth. The value added from commitment is analogous to that in other settings of corporate finance. For example, firms' ability to commit to future capital-structure choices improves the firm value (Demarzo, 2019).

Stablecoins under commitment. Many blockchain applications aim for creating tokens with stable value so that tokens may perform the roles of both means of payment and unit of account. From a theoretical perspective, there are various merits to a unit of account whose value is stable (e.g., Doepke and Schneider, 2017). A rigid token supply rule is often considered as a contributing factor to price volatility because volatile demands directly translate into price fluctuations. However, as shown in Panel D of 3, the annual volatility of token return, dP_t/P_t, is less than one basis point in spite of the 200% annual volatility of productivity shock. The commitment to constant token growth thus brings further stability on top of the volatility reduction that we see in the baseline model (Figure 2).[35]

The intuition is straightforward. Because commitment increases the sensitivity of $-V_{M_t}/P_t$ to m_t, buyback happens at a low threshold level of productivity-adjusted token supply. As shown in the baseline model, token buyback reduces the token volatility. When the owner buys back tokens more frequently under commitment, volatility is reduced accordingly.

The commitment to constant token growth brought by blockchain technology not only achieves stability of token price but also simplifies the protocol design. Moreover, since the platform investment increases in the token market capitalization (Equation (40)), the expected growth of platform productivity is relatively easy to calculate. In other words, under commitment, there exists an one-to-one mapping between token market capitalization and the enhancement of platform productivity, suggesting a simple way to rank token-based platforms by their quality.

More broadly, our findings highlight how blockchains can potentially provide an alternative commitment device and in a corporate finance context, facilitates rethinking many key issues such as capital structure. For one, it helps if a firm can commit to future financing choices or if a regulator can commit to capital requirements and leverage limits. That

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[35]If the end goal is price stability, one can even trivially achieve perfect stability by committing to have $M_t = A_t$ always, because then the driver for price dynamics, m_t, becomes a constant. Here our analysis is motivated by the tokens with constant growth rate of supply, for example, Ether.

said, what blockchains and smart contracts allow us to commit to is not unrestricted. We have analyzed a plausible form of commitment here but the commitment power from the technology in general remains a rich area for future research.

7 Conclusion

We develop a dynamic model of platform economy, where tokens are issued and used as means of payment among users, contributors, and founding entrepreneurs. Users demand tokens for their platform transactions. Dispersed contributors such as distributed record-keepers, open-source developers, and crowdfunders make on-demand contributions to improve the platform in exchange for token compensation. Entrepreneurs maximize their overall seigniorage by managing token-supply dynamics, subject to the conditions that users break even inter-temporally and labor markets are competitive.

We characterize the dynamic token allocation strategy and its implications on user base dynamics, endogenous platform growth, the dynamics of token price. A key mechanism is the divergence between insiders' (entrepreneurs') token valuation and that of outsiders (users and workers) – when the valuation wedge falls to zero, the platform owners optimally issuing token dividends to themselves; when it rises to an endogenously determined threshold, they optimally burn tokens out of circulation to stabilize token value. By serving as a commitment device, blockchains enable rule-based token supply schedule. This distinguishing feature can help mitigate under-investment and boost platform franchise value, user adoption, and token price. It also improves upon the novel dynamic token-supply mechanism for creating stablecoins.

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Appendix A - A Model of Platform Transaction Surplus

In this section, we provide a theoretical foundation for our specification of transaction surplus on the platform. Time, $t \in \mathbb{R}_+$, is continuous. The quality of the blockchain platform, A_t, evolves stochastically according to a general diffusion process determined in the equilibrium

$$dA_t = \mu(A_t)dt + \sigma(A_t)dZ_t.$$

The economy is populated with a unit measure of infinitely lived risk-neutral agents who has a discount rate $r > 0$. Agents have investment opportunities that occur at Poisson arrival times, $\{T_n\}_{n=1}^{+\infty}$, with time-varying and agent-specific intensity, $\lambda_{i,t}$. At a given Poisson time, T_n, agent i is endowed with a technology, $\omega_i F(\cdot)$, that transforms labor into goods, and is matched with another agent who can supply the required labor input. Agent-specific productivity is captured by ω_i. To simplify the exposition, we assume that the labor supply has a constant marginal cost of one, and the supplier breaks even, so the full trade surplus accrues to agent i.

Agent i's labor demand, denoted by h, is not restricted by the real balance of token holdings, $P_t k_{i,T_n-}$, where k_{i,T_n-} denotes the units of tokens carried to T_n. Since the focus of this paper is not on financial constraints, we allow the agent to borrow dollars (an instantaneous loan) at zero cost, so h may exceed agent i's wealth at the moment. Once the production is complete, the loan is repaid immediately by the goods produced.

The lumpy payment for labor incurs a transaction cost that is proportional to the total payment value, δh ($\delta > 0$), but using tokens as means of payment save the transaction cost by $U(P_t k_{i,T_n-})$ ($U' > 0$, $U'' < 0$) because agent i does not need to exchange dollars for tokens, the required means of payment on the platform. This transaction cost can be interpreted as the cost of traditional bank transfer service, legal costs of contracting. Native tokens in many cases allow transaction parties unknown and untrusted to still complete a value transfer at distance, or to contract on simple terms, thanks to the blockchain ledger and smarting contracting functionalities. For these reasons, platforms such as Kin require all on-platform transactions to be mediated using Kin tokens.

Agent i maximizes the investment profit, which is a jump in wealth,

$$\max_h \omega_i F(h) - h - (\delta h - U(P_t k_{i,T_n-})), \tag{41}$$

where the last term is the transaction cost. The optimal labor demand, h^*, is given by

$$\omega_i F'(h^*) = 1 + \delta, \tag{42}$$

so that the marginal value of production is equal to the marginal cost of labor plus the

transaction cost, δ. We can substitute the constant h^* into the investment profit to have

$$\omega_i F\left(h^*\right) - \left(1 + \delta\right) h^* + U\left(P_t k_{i,T_n-}\right). \tag{43}$$

We assume that ω_i is sufficiently high so $h^* \geq P_t k_{i,T_n-}$. The conversion between the local currency (token) and other assets can be costly, especially when a lumpy transaction is required within a short period of time. By holding tokens, agents save such costs.

Therefore, at time t, agent i has an expected gain of $\lambda_{i,t} U\left(P_t k_{i,t}\right) dt$ by holdings $k_{i,t}$ units of tokens for dt. To obtain a tighter analytical characterization of the equilibrium, we specify $\lambda_{i,t} = \left(N_t^\gamma A_t e^{u_i}\right)^\alpha$ ($\alpha \in (0,1)$). A larger community (N_t) makes it easier to find transaction counterparties. A higher platform quality (A_t) makes matching more efficient. And u_i captures agent-specific transaction needs. We specify $U\left(P_t k_{i,t}\right) = \chi\left(P_t k_{i,t}\right)^{1-\alpha}$, so the expected transaction costs saved are

$$\lambda_{i,t} U\left(P_t k_{i,t}\right) = \left(N_t^\gamma A_t e^{u_i}\right)^\alpha \left(P_t k_{i,t}\right)^{1-\alpha} \chi dt. \tag{44}$$

In the model we normalize $\chi = 1$ because its scaling effect can be subsumed by the level of A_t.

We may reinterpret h as goods or services other than labor, and the investment profit as a burst of consumption or utility value from transactions. The features our micro-foundation captures are two-fold: (i) the arrival of transaction opportunities depends on the user base, the platform quality, and agent-specific factors; (ii) holding tokens on the tokenized platform save transaction costs for lumpy payments. In essence, we model the flow utility of token holdings as a form of convenience yield, as emphasized by John Cochrane.[36]

We have many applications of native tokens as means of payment on platforms. In the case of Kin, entrepreneurs obtain information from consumer surveys that helps improve product quality. Consumers are rewarded by the native currency, Kin tokens. In our model, we do not differentiate buyers and sellers among users because in reality, the one entrepreneur are often consumers of other sellers' products in the platform market place. They can simultaneously take on different roles (investor, user, developer or operator), and actively participate in a sharing economy.

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[36] Please refer to https://johnhcochrane.blogspot.com/2017/11/bitcoin-and-bubbles.html