Unconventional Monetary Policy and Funding Liquidity Risk*

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Abstract

This paper investigates the efficiency of various monetary policy instruments to stabilize asset prices in a liquidity crisis. We propose a macro-finance model featuring both traditional and shadow banks subject to funding risk. When banks are well capitalized, they have access to money markets and efficiently mitigate funding shocks. When aggregate bank capital is low, a vicious cycle arises between declining asset prices and funding risks. The central bank can partially counter these dynamics. Increasing the supply of reserves reduces liquidity risk in the traditional banking sector, but fails to reach the shadow banking sector. When the shadow banking sector is large, as in the US in 2008, the central bank can further stabilize asset prices by directly purchasing illiquid securities.

Keywords: Asset Pricing, Quantitative Easing, Money Markets, Shadow Banks.

JEL Classifications: E43, E44, E52, G12

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1 Introduction

In a liquidity crisis—when short-term funding markets cease to function efficiently— central banks commonly react by increasing the supply of reserves available to banks. However, during the financial crisis of 2008, a significant part of the US economy was financed by institutions outside the traditional banking sector—usually referred to as *shadow banks*—without access to these operations. This institutional feature created difficulties for the Federal Reserve Board in its role as lender-of-last-resort, and prompted expansion of the set of monetary policy instruments[1] in an attempt to ease liquidity stresses beyond the traditional banking sector (Bernanke, 2009).[2]

The recent theoretical literature on shadow banking has underscored the role of a more lenient regulatory regime for these institutions as a source of financial instability. Yet little is known of the consequences of shadow banks' lack of access to traditional central bank operations in the propagation of financial shocks and the transmission of monetary policy to asset prices.

To fill this gap, we build an asset pricing model in which heterogeneous financial intermediaries face funding shocks that lead to fire sales whenever they cannot borrow in interbank money markets. In the model, an increase in the funding risk leads to a sharp decline in asset prices as financial intermediaries require a higher return to sustain the increase in the liquidity mismatch of their balance sheets. We find that a monetary policy operation that increases the supply of reserves available to banks partly reduces funding risks in the economy, with a positive effect on asset prices. However, the existence of shadow banks, without access to central bank reserves, limits the reach of these operations. We then show that to further stabilize asset prices, the central bank still has the option of directly purchasing illiquid assets, and thereby reducing the equilibrium amount of liquidity risk in the banking sector. This new channel differs from the previous literature that links asset prices to monetary policy in a financial crisis, as it relies on diminishing the liquidity risk

[1]These instruments include new lending facilities to securities dealers and mutual funds (Term Auction Facility, the Primary Dealer Credit Facility, and the Term Securities Lending Facility), Swap Agreements with foreign central banks and the direct purchase of large amounts of mortgage-backed securities or other long-term securities usually referred to as Quantitative Easing (QE).

[2]I.e., Bernanke (2009) argues that: "providing liquidity to [bank] financial institutions does not address directly instability or declining credit availability in critical nonbank markets, such as the commercial paper market or the market for asset-backed securities, both of which normally play major roles in the extension of credit in the United States."

premium rather than redistributing wealth to banks (Brunnermeier and Sannikov, 2013, 2014) or risk to households (He and Krishnamurthy, 2013; Silva, 2015).

Our intermediary asset pricing model has two new features. First, we assume that financial intermediaries are subject to funding shocks and have to solve a liquidity management problem in the spirit of Bianchi and Bigio (2018), Drechsler, Savov, and Schnabl (2017) and Schneider and Piazzesi (2015). The consequences of these shocks for asset prices is time-varying as the economy may enter a liquidity crisis regime in which the functioning of money markets is impaired. Second, we introduce shadow banks that only differ from traditional banks by not having access to public sources of funding.[3] This element of the model contrasts with most of the prior literature on shadow banking focusing on distorted incentives for risk-taking through regulatory arbitrage (Huang, 2018; Ordoñez, 2018; Plantin, 2015) and advantage in creating liquidity (Moreira and Savov, 2017).

The first contribution of this article is to link funding risks to asset prices through the balance sheets of banks. In the model, banks engage in liquidity transformation by holding assets that are less liquid than their liabilities. After an outflow of deposits, banks have to cover the loss in fundings by either borrowing in money markets (at a negligible cost) or selling securities at a fire-sale price (at a high cost). Banks always prefer to make use of the first option but have to post securities as collateral to be able to do so. Importantly, the quantity of collateral required by lenders to secure the trade varies according to the volatility of the underlying securities. This feature endogenously creates two regimes in the economy: In normal times, banks use money markets efficiently to avoid costly fire sales, such that funding risk is low and does not impact asset prices. In a crisis, increasing volatility leads to higher margins, and some banks do not have enough collateral to acquire the funding they need in money markets. In this case, these banks have no other choice but to fire-sell some securities to settle their debt. Because banks take into account this increased fire-sale risk, securities prices have to drop when collateral is scarce. A two-way feedback loop between greater endogenous volatility and a higher need for collateral generates a large amplification of the initial drop in asset prices. This article therefore introduces a mechanism similar to the one described by Brunnermeier and Pedersen (2009) in a consumption-based asset pricing setting.

[3] This assumption is in line with the definition of shadow banks of Adrian and Ashcraft (2012): "While shadow banks conduct credit and maturity transformation similar to traditional banks, shadow banks do so without the direct and explicit public sources of liquidity and tail risk insurance via the Federal Reserve's discount window and the Federal Deposit Insurance Corporation insurance."

The model provides a tractable environment in which the central bank can counteract adverse asset prices dynamics by reducing liquidity risk in three ways. First, by increasing the supply of excess reserves to banks (liquidity injection policy), the central bank creates an ex ante buffer in banks' balance sheets to absorb funding shocks. Second, by providing access to emergency liquidity facilities (lender-of-last-resort policy), the central bank provides an ex post relief for the impact of funding shocks. Third, by buying and holding risky long-term securities (asset purchase policy), the central bank removes funding risk from the market.[4] For these three policies, the critical assumption that empowers the central bank is its ability to create reserves, which is the ultimate means of settlement in the economy. As a consequence of always being able to issue this special asset, the central bank does not face funding risk.

The second contribution is to investigate the efficiency of different monetary policy operations in various regimes (impaired and well-functioning money markets) and under different financial structures (large and small shadow banking sectors). As in the monetary policy implementation literature (Frost, 1971; Poole, 1968), we assume that central bank reserves are always accepted for interbank settlement. Hence, by holding reserves, banks can reduce their exposure to funding risk. We show how this non pecuniary benefit of holding reserves breaks Wallace's (1981) neutrality[5], such that monetary policy affects asset prices and macro variables by reducing the equilibrium level of funding risk. Its ability to create an asset that is always accepted for settlement by all banks empowers the central bank to address any surge in liquidity risk in the traditional banking sector. However, as these operations are not available to shadow banks, the positive impact of traditional monetary policy instruments on asset prices is bounded.

The third contribution of this paper is to show that in this case, the central bank can still positively affect asset prices by directly purchasing long-term illiquid assets. By doing so, the central bank reduces the funding risk component in the stochastic discount factor of financial intermediaries with a positive effect on asset prices. This mechanism differs from prior literature on the link between asset purchase policy and asset prices for two reasons.

[4]We use the term asset purchase policy rather than the more common *Quantitative Easing*, as the latter is used ambiguously to refer to both buying long term assets (on the asset side of the central bank's balance sheet) or the corresponding extension in the supply of reserves (on the liability side).

[5]Equivalent to the Ricardian equivalence result for fiscal policy, Wallace's (1981) neutrality result states that monetary policy, as a mere swap of assets, should not have any impact on an economy without liquidity frictions.

First, contrary to other asset pricing models in which an asset purchase policy has an effect, this channel does not work through a redistribution of wealth to the banking sector (Brunnermeier and Sannikov, 2013, 2014) nor a redistribution of risks to the household sector (He and Krishnamurthy, 2013; Silva, 2015). Due to the idiosyncratic nature of fire sales, the policy directly affects the quantity of risk that financial intermediaries have to bear in equilibrium—without having to transfer it to other agents. This theoretical argument is essential, as central banks are usually reluctant to create redistribution effects, which they don't view as part of their mandate. Moreover, contrary to traditional monetary policy operations, asset purchases affect the whole financial sector and not just traditional banks. This result is a consequence of the mechanism's working through a general equilibrium effect rather than a bilateral relation with the central bank.

Our analysis concludes that in the presence of a sizeable shadow banking sector and impaired money markets, traditional monetary policy operations may not be sufficient to alleviate all funding stresses. Stabilizing asset prices then requires extending lending facilities to shadow banking institutions or directly purchasing some of the illiquid securities. This conclusion concurs with the practice of central banks during the 2008-2009 crisis and formalizes the argument that the crisis pushed central banks to take responsibility as a liquidity back-up for the shadow banking sector, with potential benefits for financial stability (Mehrling, 2010).

Literature Review This work belongs to the macro-finance literature with a financial sector. Our model builds on the intermediary asset pricing models of Brunnermeier and Sannikov (2014) and He and Krishnamurthy (2013), and shares with these articles an incomplete financial market structure that leads financial intermediaries to price financial assets. As in Brunnermeier and Sannikov (2016b), our model features both inside and outside money that adapts endogenously to the demand of heterogeneous agents. The main distinction between the two articles appears in the function given to money. In their work, it is held by agents as a second-best instrument to share aggregate risk. In ours, the value of money is derived from its role as the ultimate means of settlement between banks. In Drechsler, Savov, and Schnabl (2017), banks are always fully insured against funding risks by holding enough liquidity, and monetary policy affects asset prices by varying the cost of this insurance through changes in the inflation rate. We diverge by looking at the direct effect of funding risk on risk premia and asset prices in a model in which complete insurance is not always feasible due to the existence of shadow banks. As in Silva (2015), asset purchase policy (QE) has an effect on asset prices in our model. This takes place through a change in the funding risk of banks instead of being the consequence of the redistribution of risks to agents without access to financial markets. This article is also related to the literature on asset pricing with liquidity frictions literature. As in this paper, Grossman and Vila (1992) Brunnermeier and Pedersen (2009) and Gârleanu and Pedersen (2011) study economies in which time-varying margin constraints affect asset prices with the common feature that funding liquidity is linked to asset volatility and, hence, create amplification. This article departs from these by focusing on the consequences of this link for aggregate liquidity risk and the stochastic discount factor of intermediaries rather than the collateral value of assets.

In the banking literature, Holmstrom and Tirole (1998) and Diamond and Rajan (2001, 2005) characterize optimal liquidity provision when interbank markets are affected by liquidity shocks. By focusing on money markets and having central bank reserves as an interbank settlement asset, our work also relates to Acharya and Skeie (2011), Heider, Hoerova, and Holthausen (2015) and, Allen, Carletti, and Gale (2009) who show that money markets can cease to operate when credit risk is too high. Afonso and Lagos (2015) and Bech and Monnet (2016) develop over-the-counter models of the interbank market with random matching to understand its trading dynamics. Close to this article, Bianchi and Bigio (2018), Schneider and Piazzesi (2015), Del Negro, Eggertsson, Ferrero, and Kiyotaki (2017) and Fiore, Hoerova, and Uhlig (2018) include interbank markets in macroeconomic models and study the effect of liquidity injection and lender-of-last-resort policies. We deviate from these studies by introducing a shadow banking sector and central bank asset purchases to focus on financial stability concerns within an asset pricing model.

Our paper is also linked to the literature on shadow banking: Plantin (2015), Huang (2018), and Ordoñez (2018) study the emergence of the phenomenon as a consequence of regulatory arbitrage, while Gennaioli, Shleifer, and Vishny (2013) and Luck and Schempp (2014) investigate the consequences for creditors of shadow banks that default. Our model is also close to that of Moreira and Savov (2017), as we share the view that financial fragility may arise from tightening in the collateral constraint of the shadow banking sector. Begenau and Landvoigt (2018) study the impact of tightening the capital requirement of commercial banks on the shadow banking sector. We differ by defining shadow banks as not having access to the balance sheet of the central bank to investigate the efficiency of various monetary policy instruments. Hanson, Shleifer, Stein, and Vishny (2015) share this focus on shadow bank's lack of access to public sources of liquidity and find that it leads these institutions to hold more liquid assets as compared to traditional banks.

Finally, our paper relates to the macroeconomic literature that incorporates financial frictions in Neo-Keynesian models and creates a role for unconventional monetary policy as a substitute for impaired lending (Cúrdia and Woodford, 2010; Gertler and Karadi, 2011). In particular, Cúrdia and Woodford (2011) also include both central bank reserves and direct lending to non financial companies. We mainly differ from this literature by focusing on the effect of monetary policy for financial stability and asset prices.

2 Model

The model is an infinite-horizon stochastic production economy with heterogeneous agents and financial frictions. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space that satisfies the usual conditions. Time is continuous with $t \in [0, \infty)$. The model is populated by a continuum of households, regular bankers, shadow bankers, and one central bank. Figure 1 provides a sketch of the balance sheets of these agents in equilibrium. The banking sector (shadow and regular) funds risky long-term securities holdings partly by issuing instantaneous risk-free deposits to households and partly with its own net worth. The central bank operates monetary policies through its balance sheet by holding securities, lending to banks, and issuing reserves.

2.1 Environment

Demographics Following Drechsler, Savov, and Schnabl (2017), we assume a continuous-time overlapping generation structure à la Gârleanu and Panageas (2015), in which all agents die at rate κ to avoid the economy's convergence to the trivial equilibrium in which financial intermediaries own all the wealth in the economy. New agents are born at a rate κ with a fraction η_{ss} as regular bankers, a fraction $\overline{\eta}_{ss}$ as shadow bankers, and $1 - \eta_{ss} - \overline{\eta}_{ss}$ as households. The wealth of all deceased agents is endowed equally to newly born agents. We denote variables related to shadow banks with an overline and to the central bank with an underline.

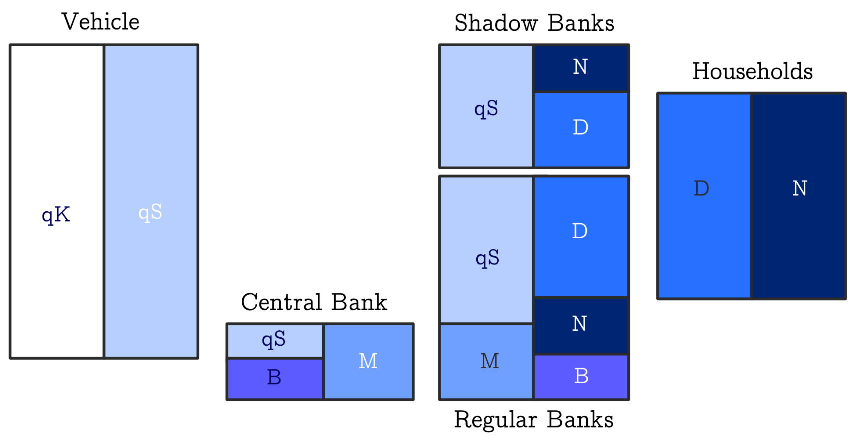

Figure 1: Balance Sheets of Agents in the Model. K represents aggregate capital, S pooled securities, q the price, N net worth, D deposits, M central bank reserves, and B long-term loans from the central bank to regular banks.

Preferences All agents have Epstein and Zin (1989) preferences with the common parameters of risk aversion γ, intertemporal elasticity of substitution ζ, and time preference ρ and implicitly taking into account the probability of death κ:

$$V_t = E_t\left[\int_t^{\infty} f_t du\right]$$

where f_t is a normalized aggregator of consumption and continuation value in each period defined as

$$f_t = \left(\frac{1-\gamma}{1-1/\zeta}\right) V_t \left[\left(\frac{c_t}{[(1-\gamma)V_t]^{1/(1-\gamma)}}\right)^{1-1/\zeta} - \rho\right].$$

We assume Epstein and Zin (1989) preferences in order to be able to separate risk aversion from the intertemporal elasticity of substitution. When $\gamma = 1/\zeta$, the felicity function converges to the constant relative risk aversion utility function.

Technology There is a positive supply of productive Lucas trees in the economy that yields output with constant returns to scale at a rate a. Trees may be destroyed with a given probability. All units of trees are pooled into an economy-wide diversified asset-backed security vehicle with total value $q_t S_t$. We write the law of motion of the stock of securities as

$$dS_t = \mu S_t dt + \sigma S_t dZ_t,$$

where $\mu > 0$ is the deterministic growth in the quantity of trees and Z_t is a standard adapted Brownian with volatility σ.

Returns As the economy only features one aggregate stochastic process dZ_t, we postulate, without loss of generality, that the stochastic law of motion of the price of a unit of securities q_t is as follows:

$$\frac{dq_t}{q_t} = \mu_t^q dt + \sigma_t^q dZ_t,$$

where μ_t^q and σ_t^q are endogenous variable that will be determined by the equilibrium. We write the flow of return on securities holdings as

$$dr_t^s = \big(\underbrace{a/q_t + \mu + \mu_t^q + \sigma\sigma_t^q}_{\mu_t^s}\big)dt + \big(\underbrace{\sigma + \sigma_t^q}_{\sigma_t^s}\big)dZ_t.$$

The drift of this process μ_t^s is composed of the dividend price ratio of holding a unit of securitized capital plus the capital gains. This formulation assumes, again without loss of generality, that the product of new investments is distributed proportionally to securities holdings. The loading factor σ_t^s consists of the sum of exogenous (the fundamental shock) and endogenous volatilities (the subsequent adjustment of asset prices).

Liquidity Management The two types of banks are subject to idiosyncratic funding shocks. Upon the arrival of a shock, a quantity $\sigma_t^d d_t$ of deposits[6] in a given bank is reshuffled to another bank. This creates a funding gap for one (the deficit bank) and a funding surplus for the other (the surplus bank). As in Drechsler, Savov, and Schnabl (2017), this sequence takes place in a short subperiod in which loans are illiquid and can only be traded at a discount fire-sale price as compared to its fundamental value.

To avoid these costly fire sales, deficit banks can use the securities on their books as

[6]Deposits are not to be interpreted in a strict sense and refer to any short-term liabilities that need to be rolled over by the creditor. As shown by Gatev and Strahan (2006), traditional bank deposits are actually among the most stable source of fundings.

collateral to borrow from surplus banks in interbank money markets. This process is subject to some frictions, and haircuts are applied to collateral such that the amount available to borrow may fall short of the funding need. In this case, shadow banks will still have to fire-sale the remaining part.

Regular banks have two additional options to mitigate this risk. First, they can hold central bank reserves—the ultimate interbank settlement asset—as a buffer against liquidity shocks. When the funding shock hits, reserves are immediately transferred from a deficit bank to a surplus one. Therefore, the size of the funding gap is reduced proportionally to reserves holdings. Second, they have access to the discount window facility. When the central bank decides to apply a haircut that is less stringent than private banks, the total amount of borrowable funds is larger for traditional banks as compared to shadow banks. We show in Appendix Section C that such a problem can be written in continuous time with the following overall transfer of wealth from a deficit to a surplus bank:

$$
\begin{aligned}
&\text{shadow banks:} \qquad && \overline{\theta}_t \equiv \lambda \max\{\sigma^d \overline{w}_t^d - \alpha_t \overline{w}_t^s, 0\}, \\
&\text{regular banks:} \qquad && \theta_t \equiv \lambda \max\{\sigma^d w_t^d - (\alpha_t + \underline{\phi}_t) w_t^s - w_t^m, 0\}.
\end{aligned}
$$

In these equations, the parameter λ is the cost of fire-sales and the variable α_t is the share of securities that can be pledged as collateral in money markets (taking into account potential haircuts) during the illiquid stage. There are two additional terms in the liquidity risk function of traditional banks. First, because these banks have access to the discount window of the central bank, they may acquire additional funds $\underline{\phi}_t$ for a given amount of collateral, provided that the central bank applies lower haircuts than private banks. Second, traditional banks can disburse their holdings of central bank reserves w_t^m without any cost and therefore avoid costly fire-sales. Hence, holding perfectly liquid reserves serves as a buffer against funding shocks. As all the losses of a deficit bank is gained by a surplus bank, the funding risk is idiosyncratic. This idiosyncratic liquidity shock is defined by the standard and adapted Brownian motion $d\widetilde{Z}_t$.[7]

[7]It is possible to represent this shock using either a Brownian motion or a Poisson shock. Both yield similar results: The Brownian motion yields simpler analytical results, while the Poisson shock is more intuitive. For the benefit of exposition, we choose the Brownian motion. We further assume that these transfers of wealth are instantaneous instead of lasting from t to $t + \Delta_d$, such that we do not have to keep track of the distribution of idiosyncratic shocks.

Central Bank To facilitate exposition, we assume that the central bank always operates with zero net worth and instantaneously redistributes any realized return (or losses) through transfers (or taxes) to private agents.[8] We scale the decision variables of the central bank by the total wealth in the economy $N_t = q_t S_t$ and write the balance sheet identity of the central bank as:

$$\underline{\nu}_t + \underline{b}_t = \underline{m}_t.$$

In this expression, $\underline{m}_t = M_t/N_t$ is the supply of reserves, $\underline{\nu}_t = q_t\underline{s}_t/N_t$ is the share of securities held by the central bank, and $\underline{b}_t = B_t/N_t$ is the total amount of loans from the central bank to traditional banks. Considering this identity, the central bank controls two of these three variables—i.e., the central bank may decide on both the size and the composition of its balance sheet. Moreover, the central bank also decides on its collateral policy at the discount window $\underline{\phi}_t$. By providing better haircuts than private markets (by setting $\underline{\phi}_t > 0$), the central bank affects the quantity of funds that traditional banks can access during the illiquid stage. We therefore define the set of monetary policy decisions as $\{\underline{m}_t, \underline{\nu}_t, \underline{\phi}_t\}$.

The distinctive role of the central bank in our economy is its capacity to issue reserves that are accepted as the ultimate means of settlement. This assumption has three implications that correspond to the three policy instruments. First, as discussed earlier, banks can hold reserves to hedge funding shocks, as reserves are perfectly liquid during the illiquid stage. Second, for a similar reason, the central bank can always lend in money markets during the illiquid stage against a better haircut than the market. Third, its reserve liabilities are always liquid, and hence the central bank does not face liquidity risk when holding illiquid assets.

Last, as in Cúrdia and Woodford (2010), we assume that the central bank may be less efficient than the private financial sector in managing securities.[9] We translate this inefficiency by assuming that securities produce less output when managed by the central bank: $\underline{a} < a$. This assumption allows us to characterize a trade-off, according to which it is not trivially always optimal for the central bank to hold all of the assets in the economy.

[8]In reality, these transfers are mediated by the fiscal authority that receives dividends from the central bank and is liable for recapitalization in case of large losses. We abstract from these concerns and assume direct transfers.

[9]Such a cost would arise in an environment in which acquiring information about the quality of the underlying capital requires special expertise that the central bank does not possess.

Overall, the balanced budget constraint for the central bank is given by the following law of motion:

$$dT_t = \underline{\nu}_t \left(\mu_t^s - \underline{\nu}_t(a - \underline{a})\right) + \underline{b}_t r_t^b - \underline{m}_t r_t^m + \underline{\nu}_t \sigma_t^s dZ_t,$$

where dT_t is the transfer of gains and losses of central bank holdings to regular and shadow banks.

2.2 Agents' Problems

Regular Banks Regular bankers face a Merton's (1969) portfolio choice problem augmented by the liquidity management component. Bankers maximize their lifetime expected recursive utility:

$$\max_{\{w_\tau^s \geq 0, w_\tau^b, w_\tau^m, w_\tau^d, c_\tau\}_{\tau=t}^{\infty}} \mathbb{E}_t \left[\int_t^\infty e^{-\rho\tau} f(c_\tau, V_\tau) d\tau \right], \tag{1}$$

subject to the law of motion of wealth:

$$\begin{aligned} dn_t = &\left(w_t^s \mu_t^s + w_t^b r_t^b + w_t^m r_t^m - w_t^d r_t^d - c_t + \mu_t^\tau \right) n_t dt + (w_t^s \sigma_t^s + \sigma_t^\tau) n_t dZ_t \\ &+ \lambda \max \left\{ \sigma^d w_t^d - (\alpha_t + \underline{\phi}_t) w_t^s - w_t^m, 0 \right\} n_t d\widetilde{Z}_t, \end{aligned} \tag{2}$$

and the balance sheet constraint:

$$w_t^s + w_t^m = 1 + w_t^d + w_t^b. \tag{3}$$

Regular bankers face a portfolio choice problem with four different assets: securities portfolio weight w_t^s, loans from the central bank with portfolio weight w_t^b, central bank reserves portfolio weight w_t^m, and deposits portfolio weight w_t^d. In equation (2), r_t^b is the interest rate on central bank loans, r_t^m the interest rate paid by the central bank on its reserves, and r_t^d the interest rate on deposits. Banks also choose their consumption rate c_t. Bankers receive a flow of transfers per unit of wealth of $d\tau_t = \mu_t^\tau dt + \sigma_t^\tau dZ_t$ from the central bank. The last term of equation (2) reflects the effect of the liquidity management problem of the regular banks on the flow of returns, as described previously.

Shadow Banks Shadow bankers face a problem similar to that of banks, except for the difference in their access to the central bank balance sheet:

$$\max_{\{\overline{w}^s_\tau \geq 0, \overline{w}^b_\tau, \overline{w}^d_\tau, \overline{c}_\tau\}_{\tau=t}^{\infty}} \mathbb{E}_t \left[\int_t^{\infty} e^{-\rho\tau} f(\overline{c}_\tau, \overline{V}_\tau) d\tau \right], \tag{4}$$

subject to the law of motion of wealth:

$$\begin{aligned} d\overline{n}_t = &\left(\overline{w}^s_t \mu^s_t - \overline{w}^d_t r^d_t - \overline{c}_t + \overline{\mu}^\tau_t \right) \overline{n}_t dt + (\overline{w}^s_t \sigma^s_t + \overline{\sigma}^\tau_t) \overline{n}_t dZ_t \\ &+ \lambda \max\{\sigma^d \overline{w}^d_t - \alpha_t \overline{w}^s_t, 0\} \overline{n}_t d\widetilde{Z}_t, \end{aligned} \tag{5}$$

and the balance sheet constraint:

$$\overline{w}^s_t = 1 + \overline{w}^d_t. \tag{6}$$

Interpretation of the variables, overlined to denote their reference shadow bankers, is the same as for regular bankers.

Households Households maximize their lifetime utility function subject to the additional assumption that they can only invest in bank deposits:

$$\max_{\{c^h_\tau\}_{\tau=t}^{\infty}} E_t \left[\int_t^{\infty} e^{-\rho\tau} f(c^h_\tau, V^h_\tau) d\tau \right], \tag{7}$$

subject to the law of motion of wealth:

$$dn^h_t = \left(r^d_t - c^h_t \right) n^h_t dt,$$

where the h index refers to households.

Equilibrium Definition

Definition 1 (Sequential Equilibrium) *Given an initial allocation of all asset variables at $t = 0$, monetary policy decisions $\{\underline{m}_t, \underline{\nu}_t, \phi_t : t \geq 0\}$, and transfer rules $\{\sigma^\tau_t, \overline{\sigma}^\tau_t, \mu^\tau_t, \overline{\mu}^\tau_t : t \geq 0\}$, a sequential equilibrium is a set of adapted stochastic processes for (i) prices $\{q_t, r^b_t, r^m_t, r^d_t : t \geq 0\}$; (ii) individual controls for regular bankers $\{c_t, w^s_t, w^m_t, w^b_t, w^d_t : t \geq 0\}$, shadow bankers $\{\overline{c}_t, \overline{w}^s_t, \overline{w}^d_t : t \geq 0\}$, and households $\{c^h_t : t \geq 0\}$; (iii) aggregate security* *stock* $\{S_t : t \geq 0\}$*; and (iv) agents' net worth* $\{n_t, \overline{n}_t, n_t^h : t \geq 0\}$ *such that:*

1. *Agents solve their respective problems, defined in equations* (1), (4), *and* (7).

2. *Markets for securities, central bank lending, reserves, and consumption goods clear:*

   (a) *securities:* $\int_{\mathbb{I}} w_t^s n_t^s di + \int_{\mathbb{J}} \overline{w}_t^s \overline{n}_t dj = (1 - \underline{\nu}_t) q_t S_t$

   (b) *central bank loans:* $\int_{\mathbb{I}} w_t^b n_t di = \underline{b}_t q_t S_t,$

   (c) *reserves:* $\int_{\mathbb{I}} w_t^m n_t di = \underline{m}_t q_t S_t,$

   (d) *output:* $\int_{\mathbb{I}} c_t n_t di + \int_{\mathbb{J}} \overline{c}_t \overline{n}_t dj + \int_{\mathbb{H}} c_t^h n_t^h dh = (a - \underline{\nu}(a - \underline{a})) S_t.$

2.3 Discussion of Assumptions

Households cannot hold risky securities The assumption that households cannot hold risky securities has the consequence that the stochastic discount factor of financial intermediaries is pricing the risky securities in the economy. We view this hypothesis as a parsimonious way to generate this feature for which there is strong empirical evidence (see, for instance, Adrian, Etula, and Muir, 2014, and He, Kelly, and Manela, 2017). We refer to Brunnermeier and Sannikov (2014) and He and Krishnamurthy (2013) for a microfoundation of such a constraint originating from agency frictions that force bankers to keep sufficient stakes in the bank. We could allow for banks to issue some limited outside equity to households without affecting the qualitative results of the paper, as long as this constraint is binding during times of liquidity stresses.

Shadow banks do not have access to the central bank This assumption corresponds to two existing institutional features. First, in most countries, only institutions licensed as banks (in the US, these are called depository institutions) have an account at the central bank, and hence can hold reserves. Second, access to a lender-of-last-resort facility (such as the Federal Reserve's discount window) is usually also restricted to the same set of institutions. Accordingly, the model would interpret a policy that extends access to lender-of-last-resort facilities to a larger set of institutions, such as creation of the Primary Dealer

Credit Facility or central bank swaps[10] lines, or changing some shadow banks to traditional banks.

No uncollateralized risk-free interbank loans We assume that banks can lend to each others only by issuing deposits subject to idiosyncratic funding risk or through collateralized loans during the illiquid stage. This assumption captures the fact that interbank loans were subject to increasing rollover and counterparty risk during the 2008-2009 financial crisis. As a consequence, a large part of the pre-crisis interbank loans between traditional and shadow banks were collateralized through Repurchase Agreement (repo) while non-collateralized interbank markets and credit lines collapsed during the crisis (McAndrews, 2009; Pozsar et al., 2012). This assumption is similar to the constraint on the issuance of equity to other agents as in Brunnermeier and Sannikov (2014) and generates a non-trivial allocation of aggregate and idiosyncratic risk amongst the traditional and shadow banking sectors.

Transfer rules are set to neutralize redistribution We set up the transfer rules of the central bank in order to shut down any potential redistribution of wealth as a consequence of monetary policy operations.[11] As we show below, with this rule, asset purchase policies are neutral in the absence of liquidity risk. This assumption is conservative as the results would be reinforced in the more realistic case in which central bank losses would be redistributed to households. The purpose of this assumption is to allow us to isolate the effect of liquidity risk to asset prices and monetary policy and abstract from the distributional impacts of monetary policy which have already been studied extensively (Brunnermeier and Sannikov, 2014; He and Krishnamurthy, 2013; Silva, 2015).

[10] A currency swap line is an agreement between two central banks to exchange currencies. They allow a foreign central bank to provide (US dollar) funding to its domestic banks in case of liquidity stress in (US dollar) money markets.

[11] The gains and losses of central bank holdings are distributed to regular and shadow banks such that:

$$q_t S_t dT_t = n_t d\tau_t + \overline{n}_t d\overline{\tau}_t.$$

We set up the transfer rules of the central bank to regular and shadow banks $d\tau = \mu_t^\tau dt + \sigma_t^\tau dZ_t$ and $d\overline{\tau} = \overline{\mu}_t^\tau dt + \overline{\sigma}_t^\tau dZ_t$ in order to shut down any redistribution channel of monetary policy.

2.4 Solving

Agents in the economy face a dynamic problem, by which their optimal decisions depend on the dynamics of stochastic investment opportunities composed of their SDF and securities prices. The homotheticity of Epstein-Zin preferences generates optimal strategies that are linear in the net worth of a given agent. Conveniently, this has the consequence that the distribution of net worth within each sector does not affect the equilibrium, and the state-space is reduced to two variables: the sectoral wealth of traditional and shadow banks relative to total wealth in the economy. We characterize the equilibrium as in Brunnermeier and Sannikov (2014) and Di Tella (2017) by using a recursive formulation of the problem that takes advantage of the scale invariance of the model and allows us to abstract from the level of aggregate capital stock. We guess and verify that the value function for each agent has the following power form:

$$V(n_t) = \frac{(\xi_t n_t)^{1-\gamma}}{1-\gamma}, \qquad \overline{V}(\overline{n}_t) = \frac{(\overline{\xi}_t \overline{n}_t)^{1-\gamma}}{1-\gamma}, \qquad V^h(n_t^h) = \frac{(\xi_t^h n_t^h)^{1-\gamma}}{1-\gamma},$$

for some stochastic processes $\{\xi_t, \overline{\xi}_t, \xi_t^h\}$ that capture time variations in the set of investment opportunities for a given type of agent. A unit of net worth has a higher value for a regular bank, a shadow bank, or a household in states in which ξ_t, $\overline{\xi}_t$, or ξ_t^h are, respectively, high. Without loss of generality, we postulate that the law of motion for these *wealth multipliers* follows an Ito process:

$$\frac{d\xi_t}{\xi_t} = \mu_t^{\xi} dt + \sigma_t^{\xi} dZ_t, \qquad \frac{d\overline{\xi}_t}{\overline{\xi}_t} = \overline{\mu}_t^{\xi} dt + \overline{\sigma}_t^{\xi} dZ_t, \qquad \frac{d\xi_t^h}{\xi_t^h} = \mu_t^{\xi,h} dt + \sigma_t^{\xi,h} dZ_t.$$

Recursive Formulation As a consequence of the homotheticity of preferences and linearity of technology, all agents of a same type choose the same set of control variables when stated in proportion of their net worth. Hence, we only have to track the distribution of wealth between types and not within types. The two state variables of the economy are the share of wealth in the hands of the regular and shadow banking sectors:

$$\eta_t \equiv \frac{n_t}{n_t + \overline{n}_t + n_t^h}, \qquad \overline{\eta}_t \equiv \frac{\overline{n}_t}{n_t + \overline{n}_t + n_t^h},$$

where total net worth in the economy is given by $n_t + \overline{n}_t + n^h_t = q_t S_t$. From here on, we characterize the economy as a recursive Markov equilibrium.

Definition 2 (Markov Equilibrium) *A Markov equilibrium in $(\eta_t, \overline{\eta}_t)$ is a set of functions $g_t = g(\eta_t, \overline{\eta_t})$ for (i) prices $\{q_t, r^d_t, r^m_t, r^b_t\}$; (ii) individual controls for regular bankers $\{c_t, w^s_t, w^m_t, w^b_t, w^d_t\}$, shadow bankers $\{\overline{c}_t, \overline{w}^s_t, \overline{w}^d_t\}$, and households $\{c^h_t\}$; (iii) monetary policy functions $\{\underline{m}_t, \underline{\nu}_t, \underline{\phi}_t\}$; and (iv) transfer rules $\{\sigma^\tau_t, \overline{\sigma}^\tau_t, \mu^\tau_t, \overline{\mu}^\tau_t\}$ such that:*

1. *Wealth multipliers $\{\xi_t, \overline{\xi}_t, \xi^h_t\}$ solve their respective Hamilton-Jacobi-Bellman equations.*

2. *Markets for securities, central bank loans, reserves, and consumption goods clear:*

   (a) securities: $w^s_t \eta_t + \overline{w}^s_t \overline{\eta}_t + \overline{\nu}_t = 1,$

   (b) central bank loans: $w^b_t \eta_t = \underline{b}_t,$

   (c) reserves: $w^m_t \eta_t = \underline{m}_t,$

   (e) output: $c_t \eta_t + \overline{c}_t \overline{\eta}_t + c^h_t (1 - \eta_t - \overline{\eta}_t) = (a - \underline{\nu}_t (a - \underline{a}))/q_t.$

3. *Monetary policy $\underline{m}_t$, $\underline{\nu}_t$, and $\underline{\phi}_t$ are set only as functions of the state variables.*

4. *Transfer rules σ^τ_t, $\overline{\sigma}^\tau_t$, μ^τ_t, and $\overline{\mu}^\tau_t$ are given by*

$$
\begin{aligned}
\sigma^\tau_t = \overline{\sigma}^\tau_t &= \frac{\underline{\nu}_t}{\eta_t + \overline{\eta}_t} \sigma^s_t, \\
\mu^\tau_t \eta_t &= \frac{\eta}{\eta_t + \overline{\eta}_t} \left(\mu^s_t - r^d_t - (a - \underline{a}) \right) \underline{\nu}_t + (r^b_t - r^m_t) \underline{b}_t - (r^m_t - r^d_t) \underline{\nu}_t, \\
\overline{\mu}^\tau_t \overline{\eta}_t &= \frac{\overline{\eta}_t}{\eta_t + \overline{\eta}_t} \left(\mu^s_t - r^d_t - (a - \underline{a}) \right) \underline{\nu}_t.
\end{aligned}
$$

5. *The laws of motion for the state variables η_t and $\overline{\eta}_t$ are consistent with equilibrium functions and demographics.*

First-Order Conditions The optimality conditions for control variable are derived in Appendix A by writing stationary Hamilton-Jacobi-Bellman equations. With a little bit of algebra, we can write these conditions for securities holdings as

$$\mu_t^s - r_t^d \geq \gamma(w_t^s \sigma_t^s + \sigma_t^\tau)\sigma_t^s - (1-\gamma)\sigma_t^\xi \sigma_t^s + \theta_t(\sigma^d - (\alpha_t + \underline{\phi}_t)), \tag{8}$$

$$\mu_t^s - r_t^d \geq \gamma(w_t^s \sigma_t^s + \overline{\sigma}_t^\tau)\sigma_t^s - (1-\gamma)\overline{\sigma}_t^\xi \sigma_t^s + \overline{\theta}_t(\sigma^d - \alpha_t), \tag{9}$$

Excess return from holding the risky asset (left-hand side) must be (greater than or) equal to the negative of the covariance between the return process and the stochastic discount factor (right-hand side). More precisely, excess returns compensate for taking exposure in two types of risks. The first term takes into account variations in marginal utility that originates purely from the additional wealth volatility. The second term corresponds to the compensation for correlated changes in the set of investment opportunities. Moreover, for banks, issuing short-term deposits is risky, as it creates an exposure to funding shocks. As deposits are a liability for banks, this additional risk exposure must be compensated for by a premium paid by households on deposits that shows up in the third term. For the two types of banks, this negative premium is equal to the marginal cost of the corresponding increase in liquidity risk. In other words, banks take into account that they need to raise deposits that generate liquidity risk when choosing their demand for securities. This effect is increasing in money market frictions and disappears when liquidity is so abundant that there is no liquidity risk. We derive the asset pricing equation for reserve holdings of traditional banks as:

$$r_t^b - r_t^m = \gamma\theta_t. \tag{10}$$

Central bank reserves are an asset from the perspective of banks, and holding them reduces the effect of funding shocks on wealth. Consequently, reserves also require a negative premium with respect to the risk-free central bank rate r_t^b (the marginal cost), which is equal to the marginal reduction in the impact of the funding shock (the marginal benefit). As all agents have the same recursive preferences, their optimal consumption choices are given by

$$c_t = \xi_t^{1-\zeta}, \quad (11) \qquad \overline{c}_t = \overline{\xi}_t^{1-\zeta}, \quad (12) \qquad c_t^h = (\xi_t^h)^{1-\zeta}. \quad (13)$$

Agents' consumption rates depend on their set of investment opportunities and their intertemporal elasticity of substitution parameter ζ. When $\zeta > 1$, the substitution dominates the wealth effect and agents react to an improvement in their set of investment opportunities by decreasing consumption. The reverse holds when $\zeta < 1$, and both effects cancel out when $\zeta = 1$.

3 Static Results

In this section, we study how money market frictions and monetary policy instruments affect asset prices. Since the results presented in this section hold in a simple static version of the model, we make a technical assumption to shut down the distribution of wealth as a state variables. To be more precise, we assume that the death rate is set to its infinite limit ($\kappa \to \infty$) such that $\eta_t = \eta_{ss} \equiv \eta$ and $\overline{\eta}_t = \overline{\eta}_{ss} \equiv \overline{\eta}$ and, consequently, drop the subscript t for all variables.[12] For simplicity, we also assume that $\sigma^d = 1$. We release these assumptions in the next section and show that our qualitative results are not impacted by allowing for complex feedback arising through the law of motion of state variables.

First, we solve the model without liquidity risk as a benchmark for the remaining of the discussion. We then show that an increase in money market frictions results in a drop in asset prices as higher funding liquidity risk impacts the stochastic discount factor of banks. We investigate how the different types of monetary policy may affect allocation and prices under various liquidity regimes. The tree monetary policy instruments have the potential to break Wallace's (1981) neutrality result in the presence of impaired money markets and impact asset prices. Yet, in the presence of a large shadow banking sector, liquidity injections and better discount window conditions may not be sufficient to alleviate funding risk. Asset price stabilization may then require the central bank to engage in an asset purchase policy that affects the whole banking sector.

All proofs are relegated to Appendix A.

3.1 Benchmark Without Liquidity Risk

Without funding liquidity risk—in a world in which there are no frictions in money markets, asset prices are determined by the traditional intermediary asset pricing equation.

[12] We also assume that agents value the bequest they leave such that ρ remains unaffected by the value of κ.

Lemma 1 (Prices without Liquidity Risk) *In the absence of money market frictions—pledgeability α is high enough such that $\theta = \overline{\theta} = 0$—equilibrium prices along the balanced growth path are given in the following set of equations:*

$$q = \frac{a}{\rho - (1-\zeta^{-1})\left(\mu - \frac{\gamma}{2(\eta+\overline{\eta})}\sigma^2\right)} \qquad \text{and} \tag{14}$$

$$r^m = r^b = r^d = \rho - \zeta^{-1}\mu + (1-\zeta^{-1})\frac{\gamma}{2(\eta+\overline{\eta})}\sigma^2. \tag{15}$$

The securities asset pricing (14) corresponds to the traditional consumption-based asset pricing equation adjusted for recursive preferences and the aggregate leverage of the banking sector $1/(\eta + \overline{\eta})$. As banks are the only agents that can take exposure to fundamental risk, precautionary motives for savings take into account that banks are levered and must bear a risk of $\gamma\sigma^2/(\eta + \overline{\eta})$ per unit of wealth. The rest of the equation is standard. The price of securities is the flow of future dividends a discounted with the stochastic discount factor of the representative banker. When the intertemporal elasticity of substitution is above one, $\zeta > 1$, the substitution effect dominates such that an increase in the drift of the capital accumulation process μ results in higher prices, while an increase in uncertainty $\sigma^2/(\eta + \overline{\eta})$ decreases asset prices. We focus on this case, as it is commonly believed to be the most relevant for macro-finance models (we refer to Bansal and Yaron 2004 for a discussion in the context of recursive preferences). For completeness, we note that when the converse holds, $\zeta < 1$, the wealth effect dominates and these relationships are inverted. As can be seen in equation (15), yields on reserves, risk-free loans, and deposits are equal. This result is intuitive as, with zero liquidity risk, banks do not value the liquidity benefits of reserves or discount the liquidity cost of deposits. This common interest rate is, therefore, simply the risk-free rate. As such, it also depends on the intertemporal elasticity of substitution. In particular, when the substitution effect dominates, an increase in uncertainty or decrease in the banking sector's relative wealth yields a reduction in the rate on deposits.

Proposition 1 (Neutrality of Monetary Policy Instruments without Liquidity Risk) *In the absence of money market frictions—pledgeability α is high enough such that $\theta = \overline{\theta} = 0$—any change in the monetary policy decision set $\{\underline{m}, \underline{\nu}, \underline{\phi}\}$ has no effect on any equilibrium variable.*

This result is straightforward for both the liquidity injection and discount window instruments because with the condition that pledgeability α is high enough such that $\theta = \overline{\theta} = 0$, all banks of the two types do not face any liquidity risk such that $\underline{m}$ and $\underline{\phi}$ do not enter into any equation. In other words, the only effect of these instruments is to lower the liquidity risk of banks. When collateral is abundant, this liquidity risk is already null and changing the amount of liquidity available to banks is inconsequential.

The logic behind the neutrality of an asset purchase policy is different. When the central bank purchases the risky securities, it does not remove the risk from the economy but rather takes it in its own balance sheet. Agents understand that they retain the exposure to the underlying risk through the future transfer (or tax) rule. This feature can be seen by first noting that market clearing conditions and the symmetry between the two types of banks, absent liquidity risk, implies that

$$w = \frac{\eta}{\eta + \overline{\eta}}(1 - \underline{\nu}), \qquad \overline{w} = \frac{\overline{\eta}}{\overline{\eta} + \overline{\eta}}(1 - \underline{\nu}).$$

After substituting for both portfolio weights and transfer rules, we can rewrite the asset pricing equations for optimal risky securities holdings as

$$\mu^s - r^d = \gamma \left(\frac{\eta}{\eta + \overline{\eta}}(1 - \underline{\nu})\sigma^s + \frac{\eta}{\eta + \overline{\eta}}\underline{\nu}\sigma^s \right) \sigma^s,$$
$$\mu^s - r^d = \gamma \left(\frac{\overline{\eta}}{\eta + \overline{\eta}}(1 - \underline{\nu})\sigma^s + \frac{\overline{\eta}}{\eta + \overline{\eta}}\underline{\nu}\sigma^s \right) \sigma^s,$$

in which central bank holdings of risky securities $\underline{\nu}$ cancels out. As agents understand the exposure that the central bank takes on their behalf, they adjust their demand for securities precisely such that the aggregate demand for securities remains unaffected. These results are a restatement of Ricardian Equivalence (Barro, 1974) for monetary policy (as in Wallace, 1981) but in terms of risk exposure rather than expected transfers. This result is also reminiscent of Black (1970), who depicts a world in which markets are so efficient in creating liquidity that the central bank loses all traction on the economy.

3.2 Money Market Frictions

In this subsection, we focus on an equilibrium with money market frictions but without monetary policy intervention. We start by characterizing the solution of the model for the case in which banks face liquidity risk.

Proposition 2 (Prices with Liquidity Risk and No Central Bank) *In an economy with positive liquidity risk in both sectors* $\theta > 0$ *and* $\overline{\theta} > 0$*, without asset purchase* $\underline{\nu} = 0$*, without reserves* $\underline{m} = 0$*, and without a discount window facility* $\underline{\phi} = 0$*, equilibrium security prices along the balanced growth path are given by*

$$q = \frac{a}{\rho - (1 - \zeta^{-1}) \left(\mu - \frac{\gamma}{2} \frac{1}{\eta + \overline{\eta}} \left(\sigma^2 + \Theta^2 \right) \right)}, \tag{16}$$

where

$$\Theta = \lambda(1 - \eta - \overline{\eta}) - \lambda\alpha.$$

When the substitution effect dominates ($\zeta > 1$), an increase in funding risks leads to a decrease in asset prices. This can be seen in the extra term Θ in equation (16) when compared to the benchmark of the previous section. This term can be interpreted as the liquidity risk in the aggregate financial sector per unit of wealth. It is proportional to the wealth of household sector $1 - \eta - \overline{\eta}$, as this is also the quantity of deposits that banks are issuing in equilibrium. The second part of this term, $-\lambda\alpha$, is the reduction of liquidity risk that is due to banks ability to access money markets in the illiquid stage. The more a given quantity of securities can generate funds in the illiquid stage (high α), the lower the liquidity risk. Overall, this idiosyncratic liquidity risk is not diversifiable by banks and is, therefore, part of their discount factor. As a consequence, higher liquidity risk yields lower equilibrium prices (when the substitution effect dominates).

Figure 2 compares the equilibrium for different levels of liquidity risk as a function of $\eta + \overline{\eta}$. For a higher level of liquidity risk due to lower pledgeability α, asset prices decrease and the net interest margin is larger.

3.3 Monetary Policy Instruments

In this subsection, we explore the pass-through of the various monetary policy instruments to asset prices. We consider, in turn, liquidity injection, discount window, and asset purchase instruments. We show how both liquidity injections and discount window instru-

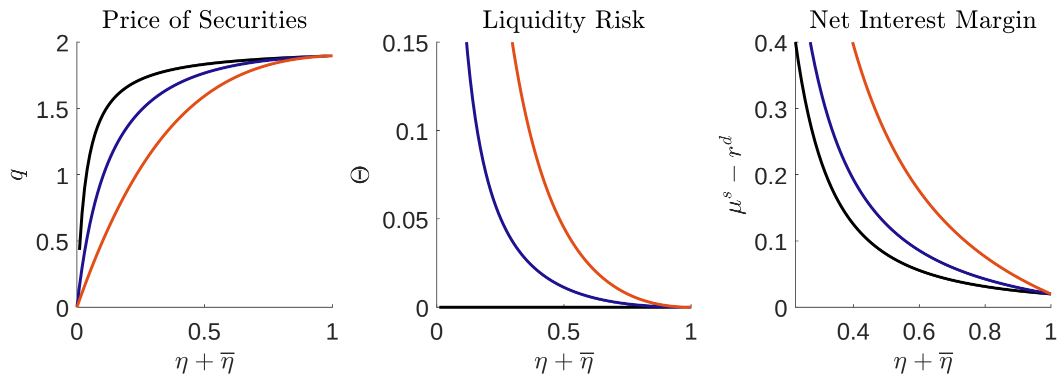

Figure 2: Liquidity Risk The figure displays how securities prices and net interest margin react to a change in money market frictions as a function of the wealth of the banking sector: benchmark with $\lambda = 0$ in black, $\lambda = 0.3$ in blue, and $\lambda = 0.6$ in red. The other parameters are set according to $a = 0.05$, $\rho = 0.03$, $\zeta = 1.1$, $\mu = 0.05$, $\gamma = 2$, σ=0.10, and $\alpha = 0.5$.

ments may positively affect asset prices but are limited, as they cannot reach the shadow banking sector. In contrast, asset purchases *get in all of the cracks* by reducing funding liquidity risk through a general equilibrium effect.

Liquidity Injections As regular banks hold reserves to hedge against funding shocks, an increase in the supply of reserves can affect asset prices when money markets are subject to frictions. We characterize the solution of the model with liquidity injection as the sole active monetary policy instrument.

Proposition 3 (Asset Prices with Positive Supply of Reserves) *In an economy without asset purchase $\underline{\nu} = 0$ and without a discount window facility $\underline{\phi} = 0$, equilibrium security prices along the balanced growth path are given by*

$$q = \frac{a}{\rho - (1 - \zeta^{-1}) \left(\mu - \frac{\gamma}{2} \frac{1}{\eta + \overline{\eta}} \left(\sigma^2 + \Theta(\underline{m})^2 + \Omega(\underline{m}) \right) \right)}, \tag{17}$$

where, if $\underline{m} < \underline{m}^{\star}$,

$$\Theta(\underline{m}) = \lambda(1 - \eta - \overline{\eta} - \underline{m}) - \lambda\alpha,$$

$$\Omega(\underline{m}) = \frac{\underline{m}^2(1-\alpha)^2\lambda^2}{\sigma^2 + (1-\alpha)^2\lambda^2}\frac{\overline{\eta}}{\eta}\sigma^2.$$

Otherwise

$$\Theta(\underline{m}) = \Theta(\underline{m}^{\star}), \qquad \Omega(\underline{m}) = \Omega(\underline{m}^{\star}),$$

and

$$\underline{m}^{\star} = (1 - \eta - \overline{\eta} - \alpha)\frac{\sigma^2 + \lambda^2(1-\alpha)^2}{\sigma^2 + \lambda^2(1-\alpha)^2 + \sigma^2\frac{\overline{\eta}}{\eta}}.$$

The supply of central bank reserves enters the asset pricing equation (17) in two ways. First, by reducing the equilibrium liquidity risk $\Theta(\underline{m})$, an increase in money supply $\underline{m}$ has a positive impact on asset prices. However, this positive relationship breaks down when the supply of reserves reaches the threshold $\underline{m}^{\star}$, which corresponds to the point at which traditional banks face zero liquidity risk and are entirely satiated with reserves. This positive effect is also dampened through the apparition of a second term $\Omega(\underline{m})$. The intuition behind this dampening effect is that as liquidity risk reduces for regular bankers, compared with shadow bankers, the former group starts to hold a larger share of the existing stock of securities. As a consequence, the distribution of fundamental risk becomes asymmetrical, which introduces an inefficiency compared with optimal fundamental risk sharing. This misallocation of fundamental risk has a negative impact on asset prices that is proportional to the size of the shadow banking sector relative to the traditional banking sector $\overline{\eta}/\eta$.

Figure 3 illustrates how the size of the shadow banking sector plays a role in determining where the liquidity satiation threshold is located. The black line represents the benchmark economy from the previous subsection without liquidity risk. Under this parametrization, asset prices do not depend on quantity of money. The blue line shows how the supply of reserves affects variables when there are only traditional banks. In this case, the central bank can always decide to inject liquidity up to the point at which banks are fully satiated

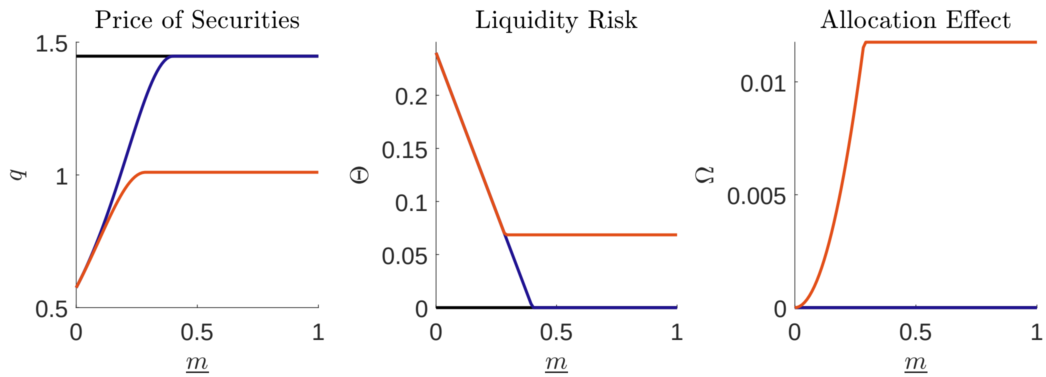

Figure 3: Liquidity Injection Policy The figure displays securities prices, stocks of liquidity risk, and the allocation effect as a function of the supply of reserves: benchmark without funding liquidity risk in black ($\lambda = 0$, $\eta = 0.05$, $\overline{\eta} = 0.05$); without a shadow banking sector in blue ($\lambda = 0.6$, $\eta = 0.10$, $\overline{\eta} = 0$); with a large shadow banking sector in red ($\lambda = 0.6$, $\eta = 0.05$, $\overline{\eta} = 0.05$). The other parameters are set according to: $a = 0.05$, $\rho = 0.03$, $\zeta = 1.1$, $\mu = 0.05$, $\gamma = 2$, $\sigma = 0.10$, $\alpha = 0.5$.

with reserves. At this point $\underline{m}^*$, there is no more liquidity risk in the economy and the price of securities q is equal to the no-liquidity-risk benchmark. When the shadow banking sector is large (red line), traditional banks may be liquidity satiated although there is still a significant amount of funding liquidity risk in the shadow banking sector and asset prices are below the no-liquidity-friction benchmark level. This effect creates an upper bound for the effect of increasing the supply of reserves to asset prices.

Discount Window By lowering haircuts at the discount window below market standards, the central bank improves the effective amount of funding that traditional banks can acquire in the illiquid stage. In doing so, it lowers the exposure to liquidity risk of traditional banks and affects equilibrium asset prices positively.

Proposition 4 (Asset Prices with Discount Window) *In an economy without asset purchase $\underline{\nu} = 0$ and without liquidity injections $\underline{m} = 0$, equilibrium security prices along the balanced growth path are given by:*

$$q = \frac{a}{\rho - (1 - \zeta^{-1}) \left(\mu - \frac{\gamma}{2} \frac{1}{\eta + \overline{\eta}} \left(\sigma^2 + \Theta(\underline{\phi})^2 + \Omega(\underline{\phi}) \right) \right)} \tag{18}$$

where, if $\underline{\phi} < \underline{\phi}^{\star}$,

$$\Theta(\underline{\phi}) = \lambda(1-\eta-\overline{\eta}) - \lambda\left(\alpha + \frac{\eta\underline{\phi}}{\eta+\overline{\eta}}\right),$$

$$\Omega(\underline{\phi}) = \eta\overline{\eta}\frac{\lambda^2\phi^2}{(\eta+\overline{\eta})^2}\frac{\sigma^2 + 2\lambda^2\left(1-\left(\alpha+\frac{\overline{\eta}\underline{\phi}}{\eta+\overline{\eta}}\right)\right) - \Theta(\phi)^2}{\vartheta(\underline{\phi})}$$
$$+\lambda^2\frac{(1-\alpha)^2(1-(\alpha+\phi))^2}{\vartheta(\underline{\phi})} - \lambda^2\frac{\left(1-\left(\alpha+\frac{\eta\phi}{\eta+\overline{\eta}}\right)\right)^2\left(1-\left(\alpha+\frac{\overline{\eta}\phi}{\eta+\overline{\eta}}\right)\right)^2}{\vartheta(\underline{\phi})},$$

and

$$\vartheta(\underline{\phi}) = \sigma^2(\eta+\overline{\eta}) + (\lambda-\lambda\alpha)^2\eta + (\lambda-\lambda(\alpha+\phi)^2\overline{\eta}.$$

Otherwise,

$$\Theta(\underline{\phi}) = \Theta(\underline{\phi}^{\star}), \qquad \Omega(\underline{\phi}) = \Omega(\underline{\phi}^{\star}),$$

and $\underline{\phi}^{\star} > 0$ *is such that:*

$$\left(\sigma^2 + \lambda^2(1-\alpha)^2 - \lambda^2\underline{\phi}^{\star}\overline{\eta}\right)\left(1-\alpha-\underline{\phi}^{\star}\right) - \vartheta(\underline{\phi}^{\star}) = 0.$$

A change in the discount window policy enters the asset pricing equations in two ways. First, similar to liquidity injections, the liquidity risk term Θ is a decreasing function of the additional funds traditional banks can acquire at the discount window ϕ. This mechanism is intuitive: By providing additional funds for a given amount of collateral, the central bank decreases the total exposure of banks to liquidity risk. As only traditional banks have access to the discount window facility, the size of this effect is scaled by the share of the wealth of traditional bankers in the total banking sector $\eta/(\eta+\overline{\eta})$.

As in the case of liquidity injections, the discount window policy also affects asset prices by changing the allocation of risk. This second dimension is reflected in the term Ω, which is a non-monotonic function of ϕ. This term is more complex than for liquidity injection, because the allocation effect of the discount window instrument has implications not only for fundamental risk but also for liquidity risk, as securities have a higher collateral value

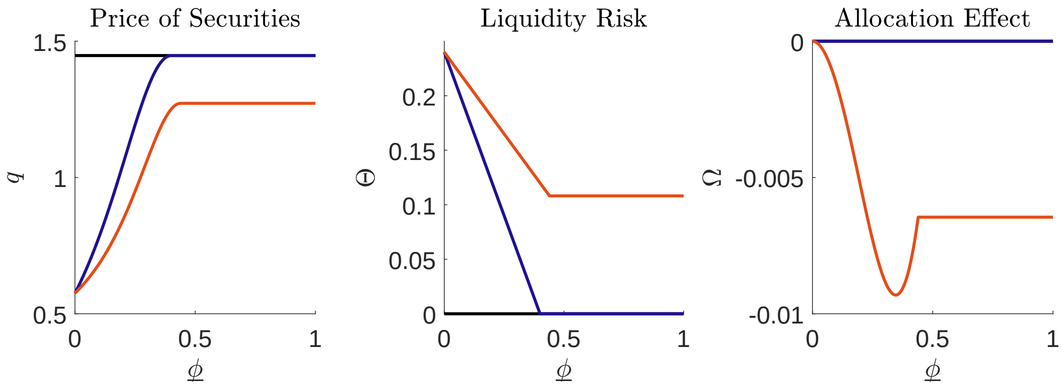

Figure 4: Discount Window Policy The figure displays securities prices, stocks of liquidity risk, and the allocation effect as a function of the discount window: benchmark without funding liquidity risk in black ($\lambda = 0$, $\eta = 0.05$, $\overline{\eta} = 0.05$); without a shadow banking sector in blue ($\lambda = 0.6$, $\eta = 0.10$, $\overline{\eta} = 0$); and with a large shadow banking sector in red ($\lambda = 0.6$, $\eta = 0.05$, $\overline{\eta} = 0.05$). The other parameters are set according to $a = 0.05$, $\rho = 0.03$, $\zeta = 1.1$, $\mu = 0.05$, $\gamma = 2$, $\sigma = 0.10$, and $\alpha = 0.5$.

when held by traditional banks. Consequently, an increase in ϕ, which leads traditional banks to hold more securities, has two opposite effects on asset prices. First, traditional banks are bearing more fundamental risk in equilibrium, which results in a larger precautionary saving motive for holding securities. Second, because traditional banks can use securities more efficiently as collateral, the aggregate liquidity risk of banks is lowered. Hence, the net outcome of this allocation effect depends on the relative strength of these two forces.

Moreover, the positive effect of lowering discount window haircuts on asset prices is bounded. Here again, the central bank may reduce the liquidity risk in the traditional banking sector but is limited in its reach to the shadow banking sector. In particular, once the threshold at which traditional banks do not face any liquidity risk $\underline{\phi}^{\star}$ is reached, any further decrease in discount window haircuts is neutral.

Figure 4 illustrates the effect of a change in discount window haircuts $\underline{\phi}$ to asset prices for different size of the shadow banking sector. The black line is the solution for the no-liquidity-risk benchmark. The red line displays a solution with liquidity risk when the shadow banking sector is large. In this case, the effect of the discount window parameter ϕ on asset prices is monotone, positive, and bounded. Once the threshold at which traditional banks no longer have liquidity risk is reached, reducing discount window haircuts is inconsequential. The central bank is, therefore, unable to push asset prices back to their benchmark level. This outcome holds despite a positive contribution of the allocation effect. The blue line represents a solution in which the shadow banking sector is nonexistent. In this case, the effect of a discount window policy is unbounded and the central bank can push asset prices up to the point at which there is no more liquidity risk in the economy and the price of securities q is equal to the no-liquidity-risk benchmark.

Asset Purchases The last type of policy we consider is the direct purchase of securities by the central bank. As the central bank does not face liquidity risk when holding securities, its purchases may positively affect asset prices by removing liquidity risk from the balance sheets of private banks.

Proposition 5 (Asset Prices with Positive Central Bank Securities Holdings) *In an economy without a discount window facility $\phi = 0$ and without central bank loans $\underline{\nu} = \underline{m}$, equilibrium securities prices along the balanced growth path are given by*

$$q = \frac{a - \Gamma(\underline{\nu})}{\rho - (1 - \zeta^{-1}) \left(\mu - \frac{\gamma}{2} \frac{1}{\eta + \overline{\eta}} \left(\sigma^2 + \Theta(\underline{\nu})^2 \right) \right)}, \tag{19}$$

where, if $\underline{\nu} < 1 - \eta - \overline{\eta}$,

$$\Theta(\underline{\nu}) = \lambda(1 - \eta - \overline{\eta} - \underline{\nu}) - \lambda\alpha(1 - \underline{\nu}).$$

Otherwise, $\Theta(\underline{\nu}) = 0$.

Central bank securities purchases also affect asset prices in two ways. First, a purchase of securities has a positive effect on asset prices by lowering the liquidity risk term Θ. When the central bank buys securities, it removes idiosyncratic liquidity risk from the balance sheets of banks. Because the central bank does not face funding liquidity risk when holding securities, these risks are extracted from the economy and, unlike fundamental risk, are not passed on to banks through future transfers. In contrast to injecting liquidity and lowering haircuts at the discount window, this mechanism is not bounded by the share of securities in the shadow banking sector.

The intuition for this result is that asset purchases remove liquidity risk through a general equilibrium mechanism by buying from the market rather than providing insurance to one particular type of agent. For the same reason, asset purchases do not create an allocation

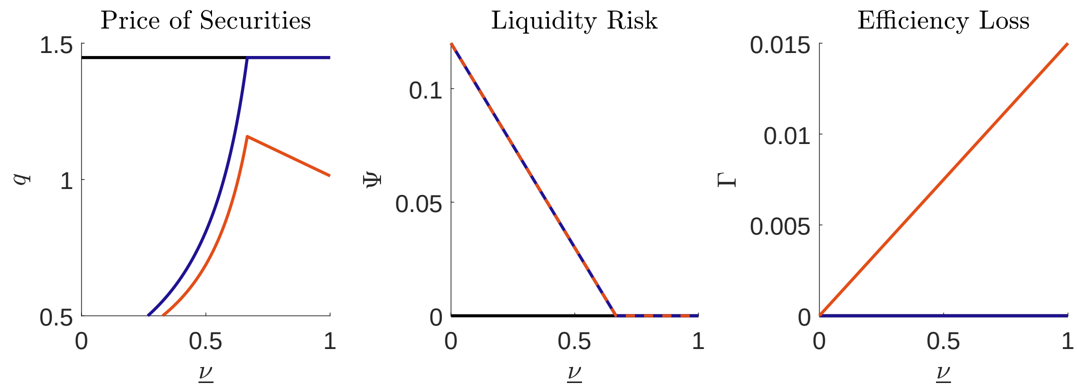

Figure 5: Asset Purchase Policy The figure displays securities prices, stocks of liquidity risk, and the convex cost of central bank management as a function of central bank's share of securities holdings: benchmark without funding liquidity risk in black ($\lambda = 0$, $\Gamma(\underline{\nu}) = 0$); without an efficiency loss in blue ($\lambda = 0.6$, $\underline{a} = a$); and with a linear efficiency loss in red ($\lambda = 0.6$, $a - \underline{a} = 0.015$). The other parameters are set according to $a = 0.05$, $\rho = 0.03$, $\zeta = 1.1$, $\mu = 0.05$, $\gamma = 2$, $\sigma = 0.10$, $\alpha = 0.5$, $\eta = 0.05$, and $\overline{\eta} = 0.05$.

effect, as the instrument does not advantage one type of bank in holding securities and holdings remain symmetrical across types.

However, asset purchases also have a countervailing negative impact on asset prices, due to the lesser ability of the central bank to manage financial assets represented by $\Gamma(\underline{\nu})$. The overall impact on securities price is a quantitative question that depends on the balance between these two forces.

Figure 5 illustrates the relationship between central bank securities holdings and asset prices for different assumptions regarding the macroeconomic cost of these holdings. The black line is the no-liquidity-risk benchmark. The blue line represents a case in which there is no difference in expertise between private banks and the central bank. In this case, the central bank is always able to push the price of securities q to its benchmark level by increasing its securities holdings. This result holds for any size of the shadow banking sector. The red line displays the solution for a positive and convex macroeconomic cost. In this instance, there is an interior maximum for the asset pricing function, after which the effect of the increase in the macroeconomic cost becomes larger than that of the decrease in liquidity risk.

4 Dynamic Results

In this section, we endogenize the frictions in the money market by explicitly modeling the haircut necessary to secure money market trades, given the volatility of assets. Then we show that the resulting collateral spiral strongly amplifies the drop in asset prices after a series of adverse shocks. Finally, we investigate, in the fully dynamic setting, how the different monetary policies may partially counteract the collateral spiral.

4.1 Numerical Procedure

We solve numerically for the global solution of the model—that is, the mapping from the pair of state variables $\{\eta_t, \overline{\eta}_t\}$ to all equilibrium variables. The numerical procedure follows the finite-difference methodology introduced by Brunnermeier and Sannikov (2016a) and extended for two state variables by d'Avernas and Vandeweyer (2017). The procedure decomposes the numerical task in two separate parts. We first solve for the wealth multiplier $\xi(\eta_t, \overline{\eta}_t)$, $\overline{\xi}(\eta_t, \overline{\eta}_t)$, and $\xi^h(\eta_t, \overline{\eta}_t)$ backward in time by using an implicit Euler method. Importantly, we evaluate the finite difference approximation of the derivative terms in the direction that preserves the numerical stability of the scheme, following Bonnans, Ottenwaelter, and Zidani (2004). Then, in between these time steps, we solve for the system of equations using a simple Newton-Raphson method to account for market clearing conditions.

4.2 Endogenous Collateral Constraint

Until this point, we have treated the proportion of available collateral α as a parameter. In reality, the quantity of secured money market loans that can be acquired with a given amount of collateral varies through time according to prevailing financial uncertainty. For instance, the need for collateral tends to shoot up during a financial crisis as traded volumes shift from uncollateralized to collateralized money markets.[13] Moreover, the haircuts on securities posted as collateral also increase in a context of high price volatility in order

[13] Kim, Martin, and Nosal (2018) document that uncollateralized interbank volumes have dropped since the crisis from about 100 billion USD to less than 5 billion USD per day.

to protect the lender in case of default.[14] To capture this link, we impose a value-at-risk constraint: The annualized probability that the collateral value becomes lower than the value of the loan must be at most p to be tolerated by lenders.[15] Thus, to borrow \$1 in collateralized money markets, the required amount of collateral χ_t satisfies:

$$\mathbb{P}\left[\chi_t \exp\left(r_t^b - (\sigma_t^s)^2/2 + \sigma_t^s(Z_{t+1} - Z_t)\right) \leq 1\right] = p, \tag{20}$$

where r_t^b is the return on risky assets under a risk-neutral measure. If a fraction κ^χ of the securities held by the bank can be used as collateral, the proportion of available collateral α_t per nominal unit of risky asset is given by

$$\alpha_t = \frac{\kappa^\chi}{\chi_t}. \tag{21}$$

Combining (20) and (21), we have that:

$$\alpha_t = \kappa^\chi \exp\left(\Phi^{-1}(p)\,\sigma_t^s + r_t^b - (\sigma_t^s)^2/2\right).$$

The effective amount of pledgeable collateral is a decreasing function of the volatility of the risky securities σ_t^s. When numerically solving our model, we use this functional forms for α_t.

4.3 Collateral Scarcity Spiral

Our model features a collateral spiral à la Brunnermeier and Pedersen (2009). As a series of adverse shocks hit the economy, the wealth of financial intermediaries decreases and asset prices drop. This decline in asset prices increases the endogenous volatility of the economy σ_t^q, which impacts haircuts in money markets through the value-at-risk constraint. At some point, the economy enters the collateral scarcity regime (when the funding liquidity risk

[14] See Gorton and Metrick (2012) for evidence on *haircut runs* in the repo market during the financial crisis.

[15] The value-at-risk constraint is evaluated assuming that drift μ_t^s and volatility σ_t^s are constant. That is, bankers approximate

$$\mathbb{P}\left[\chi_t \exp\left(\int_t^{t+1} (r_u^b - (\sigma_u^s)^2/2)du + \int_t^{t+1} \sigma_u^s dZ_u\right) \leq 1\right] = p$$

with equation (20). Also, for parsimony, we do not keep track of the distribution of collateral among banks.

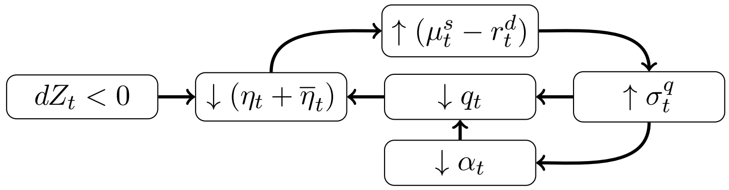

Figure 6: Schematic description of a feedback loop originating out of a negative realization of the Brownian shock when collateral is scarce.

of shadow banks, $\overline{\theta} > 0$, is positive), and the deterioration in money market conditions results in more liquidity risk, a higher drop in asset prices, and a further drop in the wealth of bankers. This link between the wealth of financial intermediaries, endogenous volatility, and haircuts creates a self-reinforcing downward spiral, illustrated in figure 6.

Proposition 6 (Amplification) *Without monetary policies, the endogenous volatility of state variables η_t and $\overline{\eta}_t$ are given by*

$$\sigma_t^{\eta}\eta_t = \frac{(\nu_t - \eta_t)\sigma}{1 - \frac{q_\eta}{q_t}(\nu_t - \eta_t) - \frac{q_{\overline{\eta}}}{q_t}(\overline{\nu}_t - \overline{\eta}_t)},$$
$$\sigma_t^{\overline{\eta}}\overline{\eta}_t = \frac{(\overline{\nu}_t - \overline{\eta}_t)\sigma}{1 - \frac{q_\eta}{q_t}(\nu_t - \eta_t) - \frac{q_{\overline{\eta}}}{q_t}(\overline{\nu}_t - \overline{\eta}_t)},$$

where

$$q_\eta = \frac{\partial q(\eta_t, \overline{\eta}_t)}{\partial \eta_t}, \qquad q_{\overline{\eta}} = \frac{\partial q(\eta_t, \overline{\eta}_t)}{\partial \overline{\eta}_t}.$$

As in Brunnermeier and Sannikov (2014), an amplification spiral arises because of a feedback loop between the lower wealth of financial intermediaries and higher endogenous volatility (see Figure 6). This can be seen from the denominator of this equation, which corresponds to the sum of two geometric series. The size of this amplification factor depends on the derivatives of the securities' price function with respect to the two state variables.

Figure 7 displays the solution of the model as a function of the total share of wealth in the hands of regular and shadow banks $\eta_t + \overline{\eta}_t$ along the diagonal line $\eta_t = \overline{\eta}_t$, when α_t is endogenously fixed to 1 and when it evolves endogenously according to the constraint (20). The drop in asset prices arises at a faster pace with the collateral spiral cycle. The

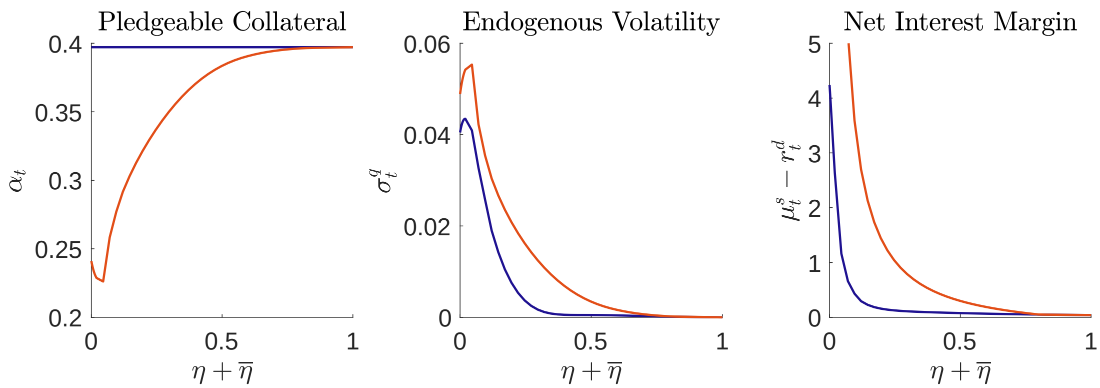

Figure 7: The figure shows the amplification mechanism when α_t is fixed to 0.4 (blue line) and α_t is endogenous (red line). The three panels display the model solution for the price of securities q_t, the endogenous volatility σ_t^q and the index of money market functioning α_t as a function of the total share of wealth in hands of regular and shadow banks $\eta_t + \overline{\eta}_t$ along the diagonal line $\eta_t = \overline{\eta}_t$. The parameters are set according to $a = 0.05$, $\rho = 0.05$, $\zeta = 1.1$, $\mu = 0.02$, $\gamma = 2$, $\sigma = 0.15$, $p = 0.1\%$, $\kappa = 0.05$, and $\kappa^\gamma = 0.5$. The fraction of newborn traditional and regular bankers η_{ss} and $\overline{\eta}_{ss}$ are set such that the stochastic steady state of the state variables η_t and $\overline{\eta}$ are both set at 0.1.

mechanism is triggered when collateral becomes scarce—α_t is so low that some banks have to fire-sell securities—and generates an increase in endogenous volatility and a drop in asset prices.

4.4 Monetary Policy in a Dynamic Setting

In this subsection we investigate, in the fully dynamic setting, how the different monetary policy instruments may partially counteract the collateral spiral. To do so, we present in Figure 8 the impulse response functions of an aggregate shock leading to a destruction of 30% of the stock of risky securities with and without policy intervention. The black line shows how net interest margin $\mu_t^s - r^d$, endogenous volatility σ_t^q, and the quantity of funding risk in the shadow banking sector $\sigma^d \overline{w}_t^d - \alpha_t \overline{w}_t^s$ evolve through time after the initial shock without any monetary policy reaction. The blue line shows the same variables when the central bank reacts to the shock by an increase in the supply of reserves from $\underline{m} = 0$ to $\underline{m} = 0.5$ (liquidity injection policy), enough to satiate traditional banks. Any further increase in money would, therefore, not change the equilibrium as reserves are neutral from this point. The red line shows how the variables evolve if the central bank decides to complement its liquidity injection policy with an asset purchase policy by increasing its

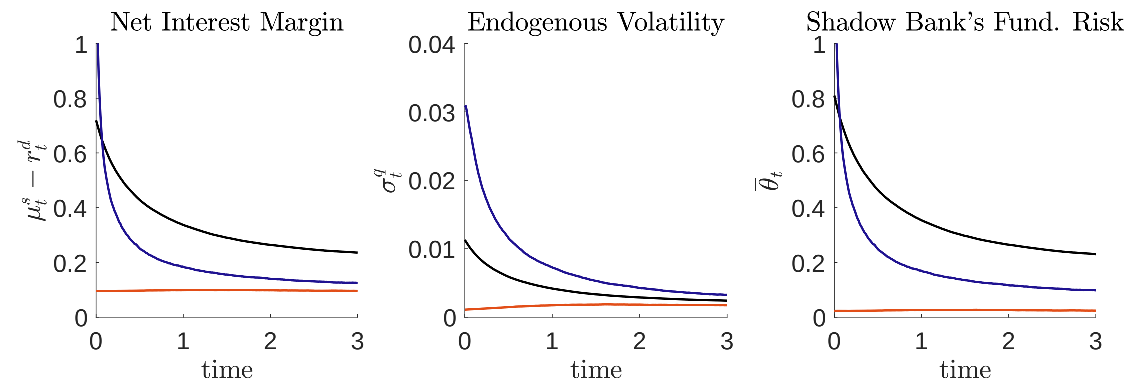

Figure 8: The figure displays the impulse response function for a 30% drop in the stock of risky securities. Starting from the stochastic steady state, we plot the average impulse response functions for net interest margin $\mu_t^s - r^d$, endogenous volatility σ_t^q, and the quantity of funding risk in the shadow banking sector $\sigma^d \overline{w}_t^d - \alpha_t \overline{w}_t^s$ after an aggregate shock to securities dZ_t that destroys 30% of the stock of securities. The black line corresponds to a no-monetary-policy benchmark. The blue line corresponds to the shock accompanied by an increase in reserves from $\underline{m} = 0$ to $\underline{m} = 0.5$ (liquidity injection policy). The red line corresponds to the same rise in reserves accompanied by an increase in central bank asset purchases from $\underline{\nu} = 0$ to $\underline{\nu} = 0.25$ (liquidity injection policy and asset purchase policy). The parameters are set according to $a = 0.05$, $\rho = 0.05$, $\zeta = 1.1$, $\mu = 0.02$, $\gamma = 2$, $\sigma = 0.15$, $p = 0.1\%$, and $\kappa^\gamma = 0.5$. The fraction of newborn traditional and regular bankers η_{ss} and $\overline{\eta}_{ss}$ are set such that the stochastic steady state of the state variables η and $\overline{\eta}$ are both set at 0.1.

holdings of securities from $\underline{\nu} = 0$ to $\underline{\nu} = 0.25$. The result derived in the static model—that asset purchase policies may have an impact on the economy when liquidity injections do not—also holds in the fully dynamic setting. In particular, the endogenous volatility of asset prices, σ_t^q, does not surge anymore following a large aggregate shock to the stock of risky securities.

5 Conclusion

In this article, we propose a framework to analyze how the presence of shadow banks may affect the transmission of conventional and unconventional monetary policy instruments to asset prices. The model allows us to study the benefits and limitations of three conceptually different types of monetary policy. Our analysis concludes that the most forceful policy mix implies, first, using the discount window and liquidity injection policies to alleviate funding stress up to the point at which traditional banks are fully satiated. If the shadow banking sector is large, this may not be sufficient to address all of the downward pressure on asset prices. In this case, the central bank can purchase long-term assets to further stabilize asset prices through a general equilibrium effect. This suggests that even when large-scale asset purchases are costly, they can be beneficial for the economy when the functioning of money markets is impaired and the shadow banking sector is large. Overall, this article highlights the importance of understanding how the development of a more decentralized and international financial system is driving central banks to extend their set of policy tools to address systemic liquidity crises originating beyond the reach of their traditional instruments.

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Appendices

A Omitted Derivations

A.1 Traditional Banks

We first write the Hamilton-Jacobi-Bellman (HJB) equation of traditional bankers' problem:

$$0 = \max_{w_t^s, w_t^b, w_t^d, w_t^m, c_t} f(c_t) + E_t\left(dV_t\right).$$

Applying Ito's lemma, we get:

$$\begin{aligned} E_t\left(dV_t\right) = {} & V_\xi \mu_t^\xi \xi_t + V_n \mu_t^n n_t + \frac{1}{2} V_{\xi\xi} (\sigma_t^\xi \xi_t)^2 + \frac{1}{2} V_{nn} \Big[(w_t^s \sigma_t^s + \sigma_t^\tau)^2 + \lambda^2 (\sigma^d w_t^d - (\alpha_t + \underline{\phi}_t) w_t^s - w_t^m)^2 \Big] n_t^2 \\ & + V_{\xi n} \sigma_t^\xi \xi_t (w_t^s \sigma_t^s + \sigma_t^\tau) n_t. \end{aligned}$$

By deriving our guess function, using the budget constraint, and substituting in the former equation, we can simplify the HJB into:

$$\begin{aligned} 0 = {} & \max_{w_t^s, w_t^d, w_t^m, c_t} f(c_t) + (\xi_t n_t)^{1-\gamma} \Bigg[\mu_t^\xi + w_t^s (\mu_t^s - r_t^d) + w_t^m (r_t^m - r_t^d) - w_t^b (r_t^b - r_t^d) + r_t^d - c_t + \mu_t^\tau \\ & - \frac{\gamma}{2} \Big((\sigma_t^\xi)^2 + (w_t^s \sigma_t^s + \sigma_t^\tau)^2 + \lambda^2 (\sigma^d (w_t^s + w_t^m - w_t^b - 1) - (\alpha_t + \underline{\phi}_t) w_t^s - w_t^m)^2 \Big) \\ & + (1-\gamma) \sigma_t^\xi \left(w_t^s \sigma_t^s + \sigma_t^\tau \right) \Bigg]. \end{aligned} \tag{22}$$

Note that the maximum function bounding the liquidity risk to being non-negative does not appear in the previous equations. We treat this kink by solving for the optimality conditions first when the maximum function is not binding and then when it is binding by simply setting $\sigma^d w_t^d - (\alpha_t + \underline{\phi}_t) w_t^s - w_t^m = 0$. We apply the maximum principle, and combine the first order conditions for the two regions in equations (8),(10) and (11). The fact that V_t is non-differentiable at the kink does not prevent the existence of a (viscosity) solution to the optimization problem.

A.2 Shadow Banks

The optimization problem of shadow banks is nested by the problem of traditional banks assuming that $w_t^m = 0$ and $\underline{\phi}_t = 0$. Solving this problem yields the first order conditions given in equations (9) and (12).

A.3 Households

Similarly, households' problem is nested when restricted to only hold risk-free deposits as a means of saving. The unique first order condition of this problem is given by equation (13).

B Proofs

In this section, we provide the proofs to Proposition 3, Proposition 4, and Proposition 5. We find an analytical solution for the general equilibrium price by shutting down the dynamics of the state variables—that is, $\sigma^q = \sigma^\xi = \overline{\sigma}^\xi = \sigma^{\xi,h} = 0$, and $\mu^q = \overline{\mu}^\xi = \mu^{\xi,h} = 0$. We sequentially use the first order conditions of each agent together with their Hamilton-Jacobi-Belleman equations and the market clearing conditions.

One can find the price for the case without central bank policies in Proposition 2 by setting $\underline{m} = \underline{\nu} = \underline{\phi} = 0$. The price without liquidity risk in Lemma 1 can be obtained by setting $\sigma^d = 0$.

B.1 Solving the Static Model

We guess and verify the static equilibrium by setting $\sigma^d = 1$, $\sigma^q = \sigma^\xi = \overline{\sigma}^\xi = \sigma^{\xi,h} = 0$, and $\mu^q = \overline{\mu}^\xi = \mu^{\xi,h} = 0$. We start from plugging back each agent's first order conditions into their Hamilton-Jacobi-Bellman equations.

For traditional bankers:

$$
\begin{aligned}
0 = &\frac{c-\rho}{1-1/\zeta} + r^d + \gamma w^s \sigma(w^s\sigma + \sigma^\tau) + \gamma w^s \lambda^2 \left(w^d - w^m - (\alpha + \underline{\phi})w^s\right)(1-(\alpha+\underline{\phi})) \\
&- (w^s + w^m - 1 - w^d)\gamma\lambda^2 \left(w^d - w^m - (\alpha + \underline{\phi})w^s\right) - c + \mu^\tau \\
&- 1/2\gamma(w^s\sigma + \sigma^\tau)^2 - 1/2\gamma\lambda^2 \left(w^d - w^m - (\alpha + \underline{\phi})w^s\right)^2
\end{aligned}
$$

After some algebra, we have:

$$
\begin{aligned}
0 = &\frac{c-\rho}{1-1/\zeta} + r^d + \gamma/2(w^s\sigma)^2 - \gamma/2(\sigma^\tau)^2 - c + \mu^\tau \\
&+ \gamma/2\lambda^2 \left(w^d - w^m - (\alpha + \underline{\phi})w^s\right)^2 + \gamma\lambda^2 \left(w^d - w^m - (\alpha + \underline{\phi})w^s\right).
\end{aligned}
$$

For shadow bankers:

$$
0 = \frac{\overline{c}-\rho}{1-1/\zeta} + r^d + 1/2\gamma(\overline{w}^s\sigma)^2 - 1/2\gamma(\overline{\sigma}^\tau)^2 - \overline{c} + \overline{\mu}^\tau + 1/2\gamma\lambda^2 \left(\overline{w}^d - \alpha\overline{w}^s\right)^2 + \gamma\lambda^2 \left(\overline{w}^d - \alpha\overline{w}^s\right).
$$

For households:

$$
0 = \frac{c^h - \rho}{1-1/\zeta} + r^d - c^h.
$$

We solve for endogenous equilibrium portfolio choices. First, we rewrite equation (8) and (9) as

$$
r^d = \frac{a}{q} + \mu - \gamma\sigma(w^s\sigma + \sigma^\tau) - \gamma\lambda^2 \left(w^d - w^m - (\alpha + \underline{\phi})w^s\right)(1-(\alpha+\underline{\phi})).
$$

and

$$
r^d = \frac{a}{q} + \mu - \gamma\sigma(\overline{w}^s\sigma + \sigma^\tau) - \gamma\lambda^2(\overline{w}^d - \overline{w}^m - \alpha\overline{w}^s)(1-\alpha).
$$

Capital Market Clearing We then equalize the two equations:

$$
\sigma(w^s\sigma + \sigma^\tau) + \lambda^2 \left(w^d - w^m - (\alpha + \underline{\phi})w^s\right)(1-(\alpha+\underline{\phi})) = \sigma(\overline{w}^s\sigma + \overline{\sigma}^\tau) + \lambda^2 \left(\overline{w}^d - \alpha\overline{w}^s\right)(1-\alpha)
$$

After some algebra, we get:

$$
\begin{aligned}
\overline{w}^s = & w^s \frac{\sigma^2 + \lambda^2(1-(\alpha+\underline{\phi}))^2}{\sigma^2+\lambda^2(1-\alpha)^2} \\
& + \frac{\lambda^2(\underline{\nu}^s/\eta - \underline{m}/\eta)(1-(\alpha+\underline{\phi}))^2 + \lambda^2(1-\alpha)^2 - \lambda^2(1-(\alpha+\underline{\phi}))^2 + \sigma(\sigma^\tau - \overline{\sigma}^\tau)}{\sigma^2+\lambda^2(1-\alpha)^2}.
\end{aligned}
$$

Therefore, any transfer rule such that $\sigma^\tau = \overline{\sigma}^\tau$ renders asset purchases neutral in the absence of liquidity risk. For parsimony, let's define κ and κ^w such that:

$$\overline{w}^s = \kappa^w w^s + \kappa.$$

From the securities market clearing condition, we have:

$$w\eta + \overline{w\eta} + \underline{\nu} = 1,$$

which gives:

$$w^s = \frac{1-\underline{\nu}-\kappa\overline{\eta}}{\eta+\kappa^w\overline{\eta}},$$

$$\overline{w}^s = \frac{1-\underline{\nu}+\kappa/\kappa^w\eta}{\eta/\kappa^w+\overline{\eta}}.$$

Consumption Market Clearing The consumption market clearing equation is given by:

$$c\eta + \overline{c\eta} + \left(\frac{\rho}{1/\zeta} - r^d\frac{1-1/\zeta}{1/\zeta}\right)(1-\eta-\overline{\eta}) = \frac{a-\underline{\nu}(a-\underline{a})}{q}$$

HJBs We can now plug for all derived variables into the respective HJB equations and take the sum of the three of them:

$$
\begin{aligned}
0 = & \frac{a-\underline{\nu}(a-\underline{a})}{q} - \rho\zeta + (\zeta-1)r^d \\
& + (\zeta-1)\Big(1/2\gamma(w^s\sigma)^2 + 1/2\gamma\lambda^2\left(w^d - w^m - (\alpha+\underline{\phi})w^s\right)^2 + \gamma\lambda^2\left(w^d - w^m - (\alpha+\underline{\phi})w^s\right)\Big)\eta \\
& + (\zeta-1)\Big(1/2\gamma(\overline{w}^s\sigma)^2 + 1/2\gamma\lambda^2\left(\overline{w}^d - \alpha\overline{w}^s\right)^2 + \gamma\lambda^2\left(\overline{w}^d - \overline{w}^s\right)\Big)\overline{\eta} \\
& + (\zeta-1)\Big(\mu^\tau\eta + \overline{\mu}^\tau\overline{\eta} - 1/2\gamma(\overline{\sigma}^\tau)^2(\eta+\overline{\eta})\Big)
\end{aligned}
$$

Note that the transfer rules are defined as:

$$\sigma^\tau = \frac{\sigma\underline{\nu}}{\eta+\overline{\eta}},$$

$$\mu^\tau\eta = \left(r^b - r^m\right)(\underline{m}-\underline{\nu}) + \frac{\eta}{\eta+\overline{\eta}}\left(\mu^s - r^d - \frac{a-\underline{a}}{q}\right)\underline{\nu} + \left(r^d - r^m\right)\underline{\nu},$$

$$\overline{\mu}^\tau\overline{\eta} = \frac{\overline{\eta}}{\eta+\overline{\eta}}\left(\mu^s - r^d - \frac{a-\underline{a}}{q}\right)\underline{\nu}.$$

B.2 Asset Purchase Policy

We proceed to solve for the price given an asset purchase policy—that is, $\underline{\nu} = \underline{m}$ and $\underline{\phi} = 0$. Thus $w^s = \overline{w}^s = \frac{1-\underline{\nu}}{\eta+\overline{\eta}}$ and equation (22) becomes:

$$\begin{aligned}
0 =& \frac{\zeta}{\zeta-1}\frac{a-\underline{\nu}(a-\underline{a})}{q} - \frac{\zeta}{\zeta-1}\rho + \mu \\
&-\gamma\sigma^2\left(w^s + \frac{\underline{\nu}}{\eta+\overline{\eta}} - 1/2(w^s)^2(\eta+\overline{\eta}) - w^s\underline{\nu} - 1/2\gamma\frac{\underline{\nu}^2}{\eta+\overline{\eta}}\right) \\
&-\gamma\lambda^2\left(w^d - w^m - \alpha w^s\right) + \gamma\lambda^2\left(w^d - w^m - \alpha w^s\right)\alpha \\
&+\left(1/2\gamma\lambda^2\left(w^d - w^m - \alpha w^s\right)^2 + \gamma\lambda^2\left(w^d - w^m - \alpha w^s\right)\right)\eta \\
&+\left(1/2\gamma\lambda^2\left(\overline{w}^d - \alpha w^s\right)^2 + \gamma\lambda^2\left(\overline{w}^d - \alpha w^s\right)\right)\overline{\eta} \\
&+\gamma\lambda^2\left(w^d - w^m - \alpha w^s\right)(1-\alpha)\underline{\nu}
\end{aligned}$$

where $\underline{w}^s n = \underline{\nu} q S$. After some algebra, we can solve for q:

$$q = \frac{a-\underline{\nu}(a-\underline{a})}{\rho - (1-1/\zeta)\left\{\mu - \frac{1/2\gamma}{\eta+\overline{\eta}}\left[\sigma^2 + \lambda^2\left(1-\eta-\overline{\eta}-\underline{\nu}-\alpha(1-\underline{\nu})\right)^2\right]\right\}}$$

B.3 Liquidity Injection Policy

Similarly, we proceed to solve for the price given a liquidity injection policy—that is, $\underline{\nu} = 0$, $\underline{m} > 0$, and $\underline{\phi} = 0$. Thus,

$$w^s = \frac{1}{\eta + \overline{\eta}} + \frac{\overline{w}^m \lambda^2 (1-\alpha)}{\sigma^2 + \lambda^2 (1-\alpha)^2} \frac{\overline{\eta}}{\eta + \overline{\eta}},$$

$$\overline{w}^s = \frac{1}{\eta + \overline{\eta}} - \frac{\overline{w}^m \lambda^2 (1-\alpha)}{\sigma^2 + \lambda^2 (1-\alpha)^2} \frac{\eta}{\eta + \overline{\eta}}.$$

Equation (22) becomes:

$$\begin{aligned}
0 =& \frac{\zeta}{\zeta - 1} \frac{a - \underline{\nu}(a - \underline{a})}{q} - \frac{\zeta}{\zeta - 1}\rho + \mu \\
&-\gamma\sigma^2 w^s - \gamma\lambda^2 \left(w^d - \underline{w}^m - \alpha w^s\right)(1-\alpha) \\
&+\left(1/2\gamma(w^s\sigma)^2 + 1/2\gamma\lambda^2 \left(w^d - \overline{w}^m - \alpha w^s\right)^2 + \gamma\lambda^2 \left(w^d - \overline{w}^m - \alpha w^s\right)\right)\eta \\
&+\left(1/2\gamma(\overline{w}^s\sigma)^2 + 1/2\gamma\lambda^2 \left(\overline{w}^d - \alpha \overline{w}^s\right)^2 + \gamma\lambda^2 \left(\overline{w}^d - \alpha\overline{w}^s\right)\right)\overline{\eta} \\
&+\gamma\lambda^2 \left(w^d - \underline{w}^m - \alpha w^s\right)\underline{m}.
\end{aligned}$$

After some algebra, we can solve for q:

$$q = \frac{a - \underline{\nu}(a - \underline{a})}{\rho - (1 - 1/\zeta)\left\{\Phi - \frac{1/2\gamma}{\eta + \overline{\eta}}\left[\sigma^2\left(1 + \frac{\underline{m}^2\lambda^2}{\sigma^2 + \lambda^2(1-\alpha)^2}\frac{\overline{\eta}}{\eta}\right) + \lambda^2\left(1 - \eta - \overline{\eta} - \underline{m} - \alpha\right)^2\right]\right\}}.$$

These equations are valid only if $\underline{m} \leq w^d = (w^s - 1)\eta$ in the case of reserves and $0 \leq (w^s - 1)\eta$ in the case of asset purchase policy. That is,

$$\underline{m} \leq \frac{\eta}{\eta + \overline{\eta}} + \frac{\underline{w}^m \theta^2}{\sigma^2 + \theta^2} \frac{\eta\overline{\eta}}{\eta + \overline{\eta}} - \eta.$$

Equality arises if

$$\underline{m} = (1 - \eta - \overline{\eta} - \alpha)\left(\frac{\sigma^2 + \lambda^2(1-\alpha)^2}{\sigma^2 + \lambda^2(1-\alpha)^2 + \sigma^2\frac{\overline{\eta}}{\eta}}\right).$$

B.4 Lender of Last Resort Policy

Similarly, we proceed to solve for the price given a lender of last resort policy—that is, $\underline{\nu} = \underline{m} = 0$, and $\underline{\phi} > 0$. Thus,

$$w^s = \frac{\sigma^2 + \lambda^2(1-\alpha)^2 - \lambda^2\underline{\phi}\overline{\eta}}{\sigma^2(\eta+\overline{\eta}) + \lambda^2(1-\alpha)^2\eta + \lambda^2(1-\alpha-\underline{\phi})^2\overline{\eta}},$$

$$\overline{w}^s = \frac{\sigma^2 + \lambda^2(1-\alpha-\underline{\phi})^2 + \lambda^2\,(1-\alpha)\,\eta}{\sigma^2(\eta+\overline{\eta}) + \lambda^2(1-\alpha)^2\eta + (1-\alpha-\underline{\phi})^2\overline{\eta}},$$

and equation (22) becomes:

$$\begin{aligned}
0 =& \frac{\zeta}{\zeta-1}\frac{a-\underline{\nu}(a-\underline{a})}{q} - \frac{\zeta}{\zeta-1}\rho + \mu \\
&-\gamma\sigma^2 w^s - \gamma\lambda^2\left(w^d - (\alpha+\underline{\phi})w^s\right)(1-\alpha-\underline{\phi}) \\
&+\left(1/2\gamma(w^s\sigma)^2 + 1/2\gamma\left(\theta w^d - \kappa w^s\right)^2 + \gamma\lambda^2\left(w^d - (\alpha+\underline{\phi})w^s\right)\right)\eta \\
&+\left(1/2\gamma(\overline{w}^s\sigma)^2 + 1/2\gamma\lambda^2\left(\overline{w}^d - \alpha\overline{w}^s\right)^2 + \gamma\lambda^2\left(\overline{\theta}\overline{w}^d - \alpha\overline{w}^s\right)\right)\overline{\eta}.
\end{aligned}$$

After some algebra, we can solve for q:

$$q = \frac{a}{\rho - (1-\zeta^{-1})\left(\mu - \frac{\gamma}{2}\frac{1}{\eta+\overline{\eta}}\left(\sigma^2 + \Theta(\underline{\phi})^2 + \Omega(\underline{\phi})\right)\right)}$$

where, if $\underline{\phi} < \underline{\phi}^\star$,

$$\Theta(\underline{\phi}) = \lambda(1-\eta-\overline{\eta}) - \lambda\left(\alpha + \frac{\eta\underline{\phi}}{\eta+\overline{\eta}}\right),$$

$$\begin{aligned}
\Omega(\underline{\phi}) =& \eta\overline{\eta}\frac{\lambda^2\phi^2}{(\eta+\overline{\eta})^2}\frac{\sigma^2 + 2\lambda^2\left(1-\left(\alpha+\frac{\overline{\eta}\underline{\phi}}{\eta+\overline{\eta}}\right)\right) - \Theta(\phi)^2}{\vartheta(\underline{\phi})} \\
&+\lambda^2\frac{(1-\alpha)^2(1-(\alpha+\phi))^2}{\vartheta(\underline{\phi})} - \lambda^2\frac{\left(1-\left(\alpha+\frac{\eta\phi}{\eta+\overline{\eta}}\right)\right)^2\left(1-\left(\alpha+\frac{\overline{\eta}\phi}{\eta+\overline{\eta}}\right)\right)^2}{\vartheta(\underline{\phi})},
\end{aligned}$$

and

$$\vartheta(\underline{\phi}) = \sigma^2(\eta + \overline{\eta}) + (\lambda - \lambda\alpha)^2\eta + (\lambda - \lambda(\alpha + \phi)^2\overline{\eta}.$$

Otherwise,

$$\Theta(\underline{\phi}) = \Theta(\underline{\phi}^\star), \qquad\qquad \Omega(\underline{\phi}) = \Omega(\underline{\phi}^\star),$$

and $\underline{\phi}^\star > 0$ is such that:

$$\Big(\sigma^2 + \lambda^2(1-\alpha)^2 - \lambda^2\underline{\phi}^\star\overline{\eta}\Big)\Big(1 - \alpha - \underline{\phi}^\star\Big) - \vartheta(\underline{\phi}^\star) = 0.$$

C Micro-Foundations for Funding Liquidity Risk

We describe the liquidity management problem of banks as a discrete-time problem with two subperiods. In the first subperiod, banks can freely adjust their portfolio. In the second subperiod, access to money markets is limited by a collateral constraint and illiquid assets can only be traded at a fire-sale price. We then show that it can be approximated in continuous-time as in equations (2) and (5). This micro-foundation can be understood as building on a combination of Bianchi and Bigio (2018) (allowing for fire-sale of securities) and He and Xiong (2012) (allowing for the borrowing in money markets and the disbursement of reserves).

Timing Time is discrete with an infite horizon. Each period is divided into two stages: the liquid stage ℓ and the illiquid stage i. In the liquid stage, there is no liquidity frictions and portfolios can be adjusted at market prices without any cost. At the end of the liquid stage, the macroeconomic shock on risky securities realizes, output is consumed and interest rates are paid. At the beginning of the illiquid stage, deposits are randomly reshuffled from some banks—the deficit banks—to other banks—the surplus banks. To settle the debt created by this shock to deposits, a deficit bank can either borrow in money markets subject to a collateral constraint or use its reserves. If, after having done so, some debt remains, the deficit banks has no choice but to fire-sell some of its securities at a high cost. After the end of the illiquid stage, the economy enters into a new liquid stage for the next period.

The Liquid Stage In the liquid stage, all banks can trade assets without frictions. Holding risky securities s_t exposes banks to *aggregate* risk realizing in the liquid stage. We write the return received from holding securities during the liquid stage as:

$$r_t^s = \mu_t^s s_t \Delta t + \sigma_t^s s_t \varepsilon_t^\ell \sqrt{\Delta t}$$

where ε_t^ℓ is binomial stochastic variable distributed with even probabilities:

$$\varepsilon_t^\ell = \begin{cases} +1 & \text{with } p = 1/2, \\ -1 & \text{with } p = 1/2. \end{cases}$$

The law of motion for the wealth of banks in the liquid stage can therefore be written as:

$$\Delta^\ell n_t = \left(\mu_t^s q_t s_t + r_t^m m_t - r_t^b b_t - r_t^d d_t - c_t n_t + \mu_t^\tau n_t\right)\Delta t + (\sigma_t^s s_t + \sigma_t^\tau)\epsilon_t^\ell \sqrt{\Delta t}$$

where the definition of the variables corresponds to the ones given in the article (i.e., m_t corresponds to $w_t^m n_t$: the total amount of reserves held by a given bank at time t). Note that, as in the continuous-time definition, σ_t^τ are the transfers set by the government conditional on the realization of the macroeconomic shock ϵ_t^ℓ while μ_t^τ are the unconditional transfers.

The Illiquid Stage Each individual banks is subject to an *idiosyncratic* deposit shock:

$$\Delta^i d_t = \sigma_t^d d_t \varepsilon_t^i \sqrt{\Delta t}$$

where ε_t^i is a binomial stochastic variable distributed with even probabilities:

$$\varepsilon_t^i = \begin{cases} +1 & \text{with } p = 1/2, \\ -1 & \text{with } p = 1/2. \end{cases}$$

The balance sheet constraint of the bank imposes that the flow of deposits is matched with an equivalent flow of securities s_t, acquired money market loans b_t, and/or central bank reserves m_t. That is,

$$\Delta^i d_t + \Delta^i b_t = \Delta^i s_t + \Delta^i m_t.$$

In words, after a negative shock to deposits, either reserves have to be disbursed, an interbank loan needs to be contracted, or risky securities needs to be disbursed.

The flows of assets $\Delta^i s_t$, $\Delta^i b_t$, and $\Delta^i m_t$ are chosen by deficit banks in order to minimize the net cost of transactions. To simplify the model, we assume that the quantities exchanged during the illiquid period are determined by the deficit banks and that the cost of trading securities in the illiquid stage are fixed exogenously[16] and transferred to the

[16] We do not provide a micro-foundation for the cost of fire-sale but we refer to the large literature in which it arises either as a consequence of shift in bargaining power under a strong selling pressure (see Brunnermeier and Pedersen, 2005; Duffie and Strulovici, 2012; Duffie, Gârleanu, and Pedersen, 2005, 2007) or asymmetry of information (see Malherbe, 2014; Wang, 1993). The intuition is that using reserves or other liquid money market assets will have a negligible cost as compared to having to sell risky securities.

surplus bank. We capture this cost with the parameter λ. Because the policy functions are linear in the agents' wealth, the distribution of these flows do not impact the recursive competitive equilibrium.

We can then write net impact of the deposit shock on an individual bank's wealth as:

$$\Delta^i n_t = \lambda \Delta^i s_t.$$

Substituting for the balance sheet constraint, we have:

$$\Delta^i n_t = \lambda \left(\Delta^i d_t + \Delta^i b_t - \Delta^i m_t \right). \tag{23}$$

Moreover, money market loans b_t can only be contracted during the illiquid period up to a threshold base on eligible collateral:

$$\Delta^i b_t \leq \alpha_t s_t \sqrt{\Delta t}. \tag{24}$$

In the illiquid stage, a deficit bank can only increase its amount of interbank borrowing up to a proportion α_t of its securities holdings. This proportion may be lower than 1 to reflect that not all securities can be pledged as collateral and are subject to haircuts (i.e. over-collateralization). To match our definition of the stochastic shock to deposits and be able to converge to a Brownian shock in the continuous time approximation, we assume that the this amount is proportional to $\sqrt{\Delta t}$. We also have to add the following constraint to make sure that securities holdings and reserves cannot be disbursed by the deficit bank more than existing amounts outstanding:

$$0 \geq \Delta^i m_t \geq -m_t \sqrt{\Delta t}, \tag{25}$$

$$0 \geq \Delta^i s_t \geq -s_t \sqrt{\Delta t}. \tag{26}$$

The optimisation problem of deficit banks in the illiquid stage simply amounts to the

The model could be easily extended to assume that trading others assets are also costly during the illiquid stage.

static[17] minimization of their losses under the liquidity constraints and liquidity costs:

$$\min_{\Delta^i m_t, \Delta^i b_t, \Delta^i s_t} \Delta^i n_t$$

where Δn_t^i is given by (23) with $\Delta^i d_t = -\sigma^d \sqrt{\Delta t}$ and subject to the liquidity frictions (24), (25) and (26).

We first consider the case where liquid assets are not sufficient for a deficit bank to cover its funding needs as given by the condition: $\sigma^d d_t > \alpha_t s_t - m_t$. As using risky securities s_t is the most costly asset, deficit banks always first use reserves m_t and money market borrowings b_t and only then resort to selling securities in order to settle remaining debts. Hence, the optimal portfolio adjustments are given by:

$$\begin{aligned}
\Delta^i m_t &= -m_t \sqrt{\Delta t}, \\
\Delta^i b_t &= \alpha_t s_t \sqrt{\Delta t}, \\
\Delta^i s_t &= \Delta^i d_t - \Delta_t^i b_t + \Delta^i m_t.
\end{aligned}$$

Intuitively, in order to avoid having to fire-sale illiquid securities at a cost λ, deficit banks mobilize as much as they can from their other (more liquid) asset holdings. Note that, all losses from a deficit bank is gained by a surplus bank. We can therefore write the law of motion of bank's wealth when there is not enough liquidity to avoid all fire-sales of securities as:

$$\Delta^i n_t = -\lambda \left(\sigma^d d_t - m_t - \alpha_t s_t \right) \sqrt{\Delta t}. \tag{27}$$

Let's now consider the case where liquidity is sufficient to cover a negative funding shock: $\sigma^d d_t \leq \alpha_t s_t - m_t$. In this case, the deficit bank does not have to fire-sell any securities. As we model the usage of both reserves and money markets as costless, this case yields the absence of any fire-sale risk for banks abd the law of motion for the wealth of banks is given by:

$$\Delta^i n_t = 0.$$

[17] The problem is static as banks are able to fully readjust their balance sheet at the beginning of the next period.

Continuous-time approximation We can combine the law of motion of both stages to get:

$$\begin{aligned}\Delta n_t &= \Delta^\ell n_t + \Delta^i n_t \\ &= \Big(\mu_t^s q_t s_t + r_t^m m_t + r_t^f f_t - r_t^d d_t - c_t n_t + \mu_t^\tau n_t\Big)\Delta t + \Big(\sigma_t^s s_t + \sigma_t^\tau n_t\Big)\varepsilon_t^\ell \sqrt{\Delta t} \\ &+ \lambda \max\big\{\sigma^d d_t - m_t - \alpha_t s_t, 0\big\}\varepsilon_t^i \sqrt{\Delta t}.\end{aligned}$$

Finally, the limit when Δt tends to 0 is given by:

$$\begin{aligned}dn_t = &\Big(\mu_t^s q_t s_t + r_t^m m_t + r_t^f f_t - r_t^d d_t - c_t n_t + \mu_t^\tau n_t\Big)dt + \Big(\sigma_t^s s_t + \sigma_t^\tau n_t\Big)dZ_t \\ &+ \lambda \max\big\{\sigma^d d_t - m_t - \alpha_t s_t, 0\big\}d\widetilde{Z}_t.\end{aligned}$$

where Z_t is an aggregate Brownian motion and $\widetilde{Z}_t$ is an idiosyncratic Brownian motion.

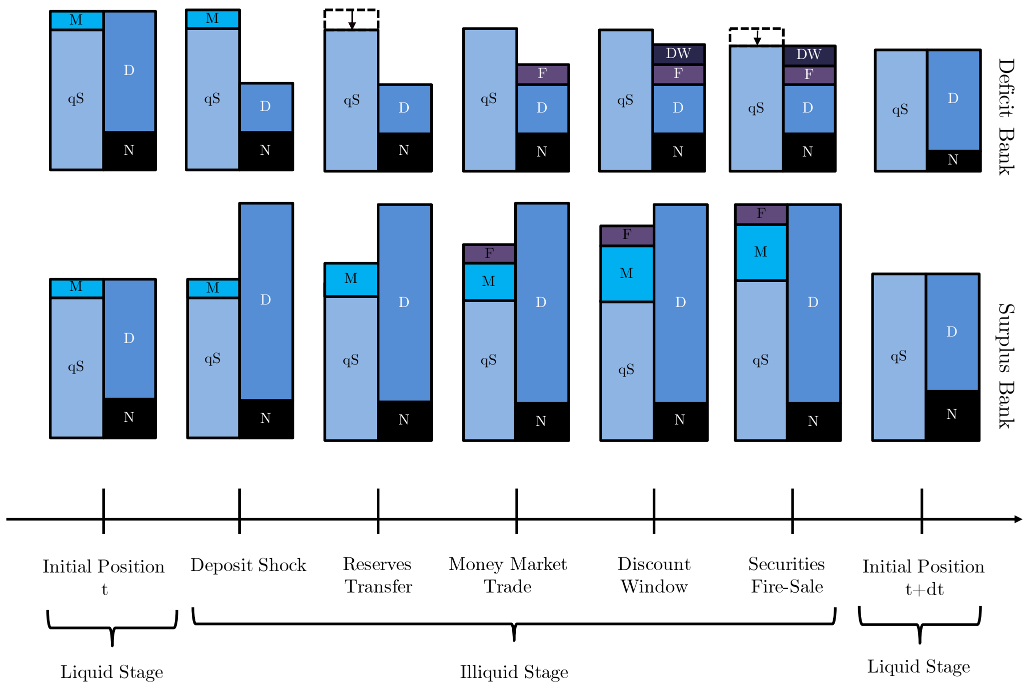

Figure 9: Sketch of Balance-Sheet Adjustments in the Discrete-Time Model