Discussion of *HLM*², *Forward Guidance*

Jeffrey R. Campbell[1]

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[1] The views expressed are my own. They do not necessarily represent the views of the Federal Reserve Bank of Chicago, the Federal Reserve System, or its Board of Governors.

Optimal Point of Departure

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Inflation and output in New Keynesian models with a transient interest rate peg

CrossMark

Charles T. Carlstrom [a], Timothy S. Fuerst [a,b,*], Matthias Paustian [c]

[a] Federal Reserve Bank of Cleveland, OH 44101, USA
[b] Department of Economics, University of Notre Dame, Notre Dame, IN 46556, USA
[c] Board of Governors, Division of Research and Statistics, USA

Optimal Point of Departure

$$1 = \beta \left(\frac{Y_t}{Y_{t+1}} \right)^{\frac{1}{\sigma}} \frac{1 + i_t}{P_{t+1}/P_t}$$

FG in Carlstrom, Fuerst, and Paustian (2015)

Primitives

$$
\begin{aligned}
y_t &= -\sigma(i_t - \pi_{t+1} - i_t^\natural) + y_{t+1} && (1) \\
\pi_t &= \kappa y_t + \beta \pi_{t+1} && (2) \\
i_t^\natural &= \begin{cases} \varrho < 0 & \text{if } t \in \{0, \ldots, T\} \\ \rho > 0 & \text{otherwise.} \end{cases} && (3) \\
i_t &= \begin{cases} 0 & \text{if } t \in \{0, \ldots, T\} \\ \phi i_t^\natural + \pi_t & \text{otherwise} \end{cases} && (4)
\end{aligned}
$$

FG in Carlstrom, Fuerst, and Paustian (2015)

Primitives

$$
\begin{aligned}
y_t &= -\sigma(i_t - \pi_{t+1} - i_t^{\natural}) + y_{t+1} & (1)\\
\pi_t &= \kappa y_t + \beta \pi_{t+1} & (2)\\
i_t^{\natural} &= \begin{cases} \varrho < 0 & \text{if } t \in \{0, \ldots, T\} \\ \rho > 0 & \text{otherwise.} \end{cases} & (3)\\
i_t &= \begin{cases} 0 & \text{if } t \in \{0, \ldots, T\} \\ \phi i_t^{\natural} + \pi_t & \text{otherwise} \end{cases} & (4)
\end{aligned}
$$

After the Liquidity Trap

$$
\begin{aligned}
0 &= (1 + \kappa\sigma\phi)\pi_t - (1 + \kappa\sigma + \beta)\pi_{t+1} + \beta\pi_{t+2}\\
1 + \kappa\sigma\phi &> 0\\
\kappa\sigma(\phi - 1) &> 0\\
\pi_t &= c_1 \lambda_1^{-t} + c_2 \lambda_2^{-t}\\
\pi_t &= 0 \text{ always}
\end{aligned}
$$

FG in Carlstrom, Fuerst, and Paustian (2015)

Primitives

$$
\begin{aligned}
y_t &= -\sigma(i_t - \pi_{t+1} - i_t^\natural) + y_{t+1} && (1)\\
\pi_t &= \kappa y_t + \beta \pi_{t+1} && (2)\\
i_t^\natural &= \begin{cases} \varrho < 0 & \text{if } t \in \{0, \dots, T\} \\ \rho > 0 & \text{otherwise.} \end{cases} && (3)\\
i_t &= \begin{cases} 0 & \text{if } t \in \{0, \dots, T\} \\ \phi i_t^\natural + \pi_t & \text{otherwise} \end{cases} && (4)
\end{aligned}
$$

During the Liquidity Trap

$$
\begin{aligned}
y_T &= \sigma \varrho, \\
\pi_T &= \kappa y_T = \kappa \sigma \varrho \\
y_{T-1} &= y_T - \sigma \left(0 - \pi_T - i_{T-1}^\natural\right) = y_T + \sigma\left(\varrho + \kappa \sigma \varrho\right) < y_T \\
\pi_{T-1} &= \kappa y_{T-1} + \beta \pi_T < \kappa y_T + \beta \pi_T < \pi_T \\
\kappa \sigma \varrho &= \pi_t - (1 + \kappa\sigma + \beta)\pi_{t+1} + \beta \pi_{t+2}
\end{aligned}
$$

FG in Carlstrom, Fuerst, and Paustian (2015)

Primitives

$$
\begin{aligned}
y_t &= -\sigma(i_t - \pi_{t+1} - i_t^{\natural}) + y_{t+1} && (1)\\
\pi_t &= \kappa y_t + \beta \pi_{t+1} && (2)\\
i_t^{\natural} &= \begin{cases} \varrho < 0 & \text{if } t \in \{0, \ldots, T\} \\ \rho > 0 & \text{otherwise.} \end{cases} && (3)\\
i_t &= \begin{cases} 0 & \text{if } t \in \{0, \ldots, T\} \\ \phi i_t^{\natural} + \pi_t & \text{otherwise} \end{cases} && (4)
\end{aligned}
$$

During the Liquidity Trap

$$
\begin{aligned}
\kappa\sigma\varrho &= \pi_t - (1 + \kappa\sigma + \beta)\pi_{t+1} + \beta\pi_{t+2}\\
\pi_t &= \iota_1 \pi_{t+1} + \kappa\sigma\varrho \frac{1 - \iota_2^{T+1-t}}{1 - \iota_2} \quad (\iota_1 > 1 > \iota_2)
\end{aligned}
$$

FG in Carlstrom, Fuerst, and Paustian (2015)

Primitives

$$
\begin{aligned}
y_t &= -\sigma(i_t - \pi_{t+1} - i_t^{\natural}) + y_{t+1} && (1)\\
\pi_t &= \kappa y_t + \beta \pi_{t+1} && (2)\\
i_t^{\natural} &= \begin{cases} \varrho < 0 & \text{if } t \in \{0, \ldots, T\} \\ \rho > 0 & \text{otherwise.} \end{cases} && (3)\\
i_t &= \begin{cases} 0 & \text{if } t \in \{0, \ldots, T\} \\ \phi i_t^{\natural} + \pi_t & \text{otherwise} \end{cases} && (4)
\end{aligned}
$$

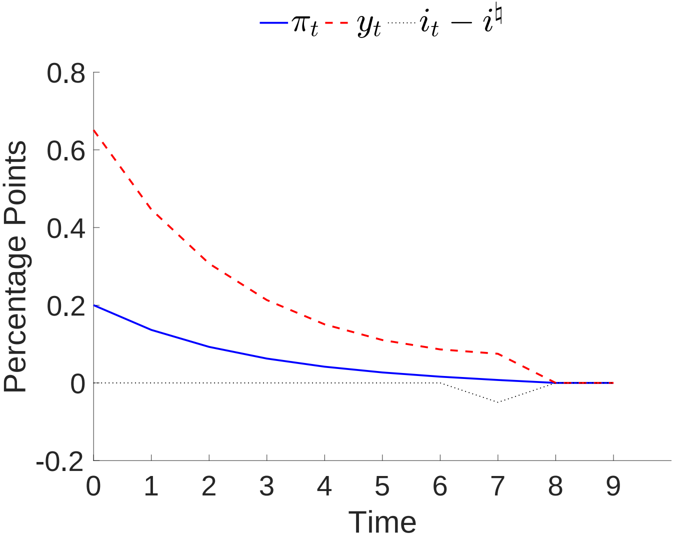
Forward Guidance

$$
\begin{aligned}
i_{T+1} &= i_{T+1}^{\natural} - \varepsilon \\
\pi_{T+1} &= \kappa \sigma \varepsilon \\
\frac{\partial \pi_t}{\partial \pi_{T+1}} &= \iota_1^{T+1-t}
\end{aligned}
$$

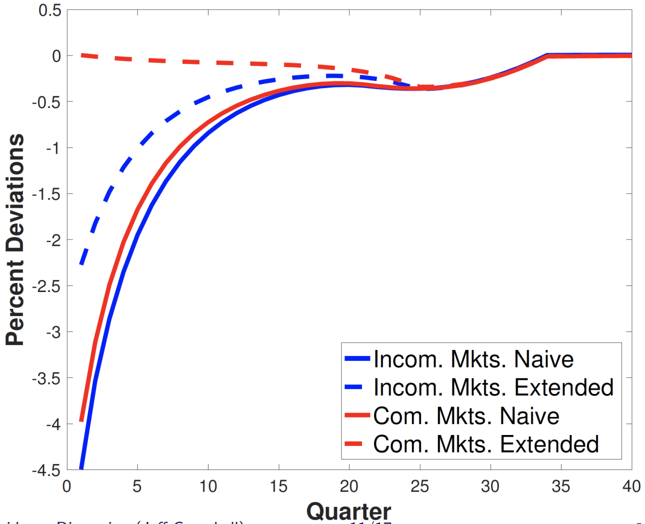
FG in Carlstrom, Fuerst, and Paustian (2015)

This unstable root is a manifestation of the familiar local indeterminacy induced by an interest rate peg. When the difference equation is solved forward the presence of indeterminacy implies that there is a stable eigenvalue. When solved backwards, this eigenvalue is unstable. During the guidance period this unstable eigenvalue, e_1, has a fairly simple form if

The Standard Forward-Guidance Experiment



HLM^2's Forward-Guidance Experiment



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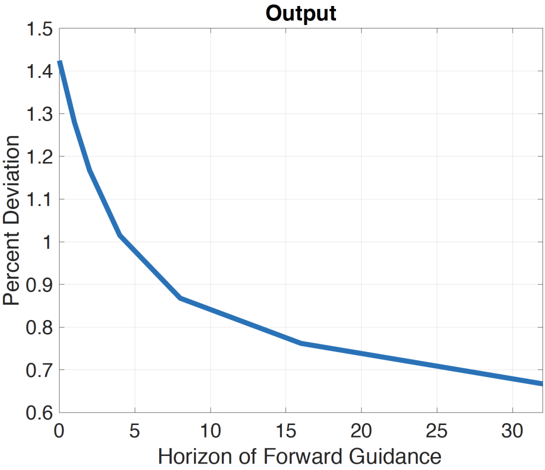
Key figure. I do not believe it.

HLM^2's Forward-Guidance Experiment



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They have flipped a root.

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Heterogeneity

- Two representative households, "Kapitalists" and "Workers"
- Workers consume hand-to-mouth.
- Capitalists own firms and trade bonds in zero net supply.

$$
\begin{aligned}
Y_t &\equiv Y_t^K + Y_t^W \\
Y_t^K &= (1-\alpha) Y_t \\
\pi_t &= \kappa y_t + \beta \pi_{t+1} \\
y_t^k &= -\frac{1}{\sigma}\left(i_t - \pi_{t+1} - i_t^\natural\right) + y_{t+1}^k \\
y_t^k &= y_t
\end{aligned}
$$

Unequal Sharing of Aggregate Risk

- Workers borrowed from Capitalists in the past in return for a perpetual *real* payment τ.

$$
\begin{aligned}
Y_t^K &= (1-\alpha)Y_t + \tau \\
y_t^k &= \frac{(1-\alpha)Y_\star}{(1-\alpha)Y_\star + \tau} y_t \\
y_t^k &= -\sigma\left(i_t - \pi_{t+1} - i_t^\natural\right) + y_{t+1}^k \\
y_t &= -\sigma\frac{(1-\alpha)Y_\star + \tau}{(1-\alpha)Y_\star}\left(i_t - \pi_{t+1} - i_t^\natural\right) + y_{t+1}
\end{aligned}
$$

Inflation Dependent Sharing of Aggregate Risk

- Workers make inflation-dependent payments to Capitalists.

$$
\begin{aligned}
Y_t^K &= (1-\alpha)Y_t + \tau\left(P_t/P_{t-1} - 1\right) \\
y_t^k &= y_t + \frac{\tau}{(1-\alpha)Y_\star}\pi_t = y_t + \theta\pi_t \\
y_t^k &= -\sigma\left(i_t - \pi_{t+1} - i_t^\natural\right) + y_{t+1}^k \\
y_t + \theta\pi_t &= -\sigma\left(i_t - \pi_{t+1} - i_t^\natural\right) + y_{t+1} + \theta\pi_{t+1} \\
-\sigma(i_t - i_t^\natural) &= \left(1 + \kappa\theta - (1 + \kappa\sigma + \beta + \kappa\theta)\, L^{-1} + \beta L^{-2}\right)\pi_t \\
&= f(L^{-1})\pi_t \\
f(0) &= 1 + \kappa\theta > 0 \\
f(1) &= -\kappa\sigma < 0
\end{aligned}
$$

Euler-equation Discounting

$$
\begin{aligned}
y_t &= -\sigma\left(i_t - \pi_{t+1} - i_t^{\natural}\right) + \delta y_t \\
\pi_t - \beta\pi_{t+1} &= -\sigma\left(i_t - \pi_{t+1} - i_t^{\natural}\right) + \delta(\pi_{t+1} - \beta\pi_{t+2}) \\
-\sigma\left(i_t - i_t^{\natural}\right) &= \left(1 - (\delta + \kappa\sigma + \beta)L^{-1} + \delta\beta L^{-2}\right)\pi_t \\
&= f(L^{-1})\pi_t \\
f(0) &= 1 > 0 \\
f(1) &= (1-\delta)(1-\beta) - \kappa\sigma
\end{aligned}
$$

How did you flip the root?