

# Understanding the Aggregate Effects of Credit Frictions and Uncertainty\*

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## Abstract

This paper integrates a financial accelerator mechanism à la [Bernanke et al. \(1999\)](#) and time-varying uncertainty into a Dynamic New Keynesian model. We examine the extent to which uncertainty and credit conditions interact with one another. The idea is that uncertainty aggravates the information asymmetry between lenders and borrowers, and worsens credit conditions. Already poor credit conditions amplify the effect of shocks (to both the mean and variance) on the aggregate economy. In our model, uncertainty modelled as time-varying stochastic volatility emerges from monetary policy (policy uncertainty), financial risks (micro-uncertainty), and the aggregate state of the economy (macro-uncertainty). Using a third order approximation, we find that micro-uncertainty has first order effects on economic activity through its direct impact on credit conditions. We also find that if credit conditions (as measured by the endogenous risk spread) are already poor, then additional micro-uncertainty shocks have even larger real effects. In turn, shocks to aggregate uncertainty (macro- and policy-uncertainty) have relatively small direct effects on aggregate economic activity.

*JEL Classification:* E32, E44, D8, C32

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# 1 Introduction

In the latter part of the 2000s, the U.S. economy experienced its longest recession in the post-World War II period. The unique character of this recession has sparked a renewed interest in the role of two important amplification mechanisms: credit market frictions and uncertainty. Baseline models that were popular and commonly used for policy analysis prior to the recession often feature frictionless capital markets (Christiano et al. (2005), Smets and Wouters (2007)), but even influential models with financial frictions (Bernanke et al. (1999), Christiano et al. (2010), Martínez-García (2014)) or with uncertainty (Bloom (2009)) have yet to explore the *adverse feedback loop* that can arise from increased borrowing spreads during periods of heightened uncertainty.

We introduce aggregate, policy, and financial uncertainty into a New Keynesian framework with capital accumulation financed through risky nominal loans subject to endogenous default. We argue that our framework not only explains the observed credit spreads in a model with production, but also improves upon the canonical stochastic general equilibrium business cycle model. Credit market spreads, reflecting the difference between borrowing rates and the return on savings, affect the cost of capital in our model. This, in turn, affects the real economy and leads to a strong correlation between credit spreads and aggregate quantities. The Great Recession of 2008-2009 offers a primary example of the important role that fluctuations in credit risk play in the aggregate economy. Unfortunately these developments also exposed the current need for new state-of-the-art models suitable for understanding the joint behavior of credit risk, asset prices, and the key macroeconomic aggregates in a production economy.

One interpretation of financial market imperfections stems from the information asymmetry and/or costly contract enforcement that characterizes credit markets. This gives rise to agency costs that are incorporated in financial contracts that link borrowers and lenders. The strand of research which focuses on this credit channel highlights the effect of credit frictions in propagating cyclical movements of real economic activity (Bernanke and Gertler (1989), Bernanke et al. (1996), Carlstrom and Fuerst (1997), Kiyotaki and Moore (1997), Bernanke et al. (1999)), and how those credit frictions affect monetary policy-making (Carlstrom et al. (2009), Christiano et al. (2010), Gilchrist et al. (2013), Christiano et al. (2014)).

At a general level, uncertainty is defined as the conditional volatility of a disturbance that is unforecastable from the perspective of economic agents. Research which focuses on the role of time-varying uncertainty or time-varying second-moments has also attracted much attention. In partial equilibrium settings, increases in uncertainty can depress investment, employment, and consumption if agents are subject to fixed costs or partial irreversibility (a real options effect), if agents are risk averse (a precautionary savings effect) or if financial constraints tighten in response to higher uncertainty (a financial frictions or asset pricing effect). In general equilibrium settings, many of these mechanisms continue to imply a role for time-varying uncertainty, although some of these effects may get attenuated when the general equilibrium effects are introduced.

Typically, uncertainty, which arises independently of economic and policy shocks, delays investment by changing investor sentiments and enhances the option value of waiting (see the investment under uncertainty literature—e.g., Bernanke (1983) and Pindyck (1988)). It also strengthens the precautionary saving motive of economic agents. A shock to time-varying uncertainty has been shown to have effects on consumption, output, and investment decisions (Dorofeenko et al. (2008), Alexopoulos and Cohen (2009), Bloom (2009), Fernández-Villaverde et al. (2010), Fernández-Villaverde et al. (2011), Bloom et al. (2014), Basu and

Bundick (2014), Cesa-Bianchi and Fernández-Corugedo (2014), Born and Pfeifer (2014)).<sup>1</sup>

In this paper, we examine the relationship between credit frictions and uncertainty and the role the interaction these can have in generating fluctuations in output, risk spreads, and other macroeconomic variables. We consider three types of uncertainty. *Macro-uncertainty* represents uncertainty about the evolution of the economy brought about by time-varying volatility of innovations in total factor productivity (TFP). *Micro-uncertainty* in our model is the dispersion in the distribution of the idiosyncratic technology shock of the entrepreneurs and represents the idiosyncratic uncertainty about the evolution of individual firms' productivity. This uncertainty plays a key role in the genesis of the financial frictions. Finally, *policy uncertainty* is reflected in a variance of innovations to the monetary rule that varies over time. Our primary interest is to examine how these three types of uncertainty interact with one another and the credit frictions brought about by asymmetric information and costly state verification.

Formally, we integrate a model of agency cost à la Bernanke et al. (1999) with the three types of time-varying uncertainty into an otherwise standard Dynamic New Keynesian model. Entrepreneurs seek external funds to finance the acquisition of tomorrow's stock of capital. The riskiness of the acquisition is due to an idiosyncratic technology shock that can only be observed by the entrepreneurs costlessly. Hence, lenders must resort to costly monitoring of the outcome of the risky projects in order to dissuade the entrepreneurs from misreporting their net revenues. The cost of this monitoring process, the agency cost, is a constant fraction of the value of the assets of the entrepreneur (the value of the stock of capital). This agency cost gives rise to the external finance premium required by the lenders and, therefore, raises the costs of borrowing. Uncertainty about entrepreneur productivity gets priced into the external finance premium that entrepreneurs must pay to the financial intermediaries in order to borrow. Therefore, shocks in the dispersion of idiosyncratic productivity (or micro-uncertainty) are essentially a source of exogenous fluctuations in the external finance premium. Changes in micro-uncertainty provide an additional source of shocks that offset the relatively small quantitative amplification that credit frictions appear to have for standard macro shocks (Kocherlakota (2000), Córdoba and Ripoll (2004)).

In addition, we consider the degree to which aggregate and policy uncertainty interact with credit frictions through their effect on entrepreneur net worth. First, we consider financial contracts that are written in nominal rather than real terms. This raises the possibility that uncertainty shocks that make inflation uncertain will increase the risk about the real payoff of the nominal loan contract. Second, uncertainty (both macro and policy) through their effects on the discounting of future payoffs on investment projects can affect the price of capital.<sup>2</sup> Changes in the price of capital, in turn, can affect entrepreneur net worth which also plays a role in determining the extent of the credit frictions (as measured by the external finance premium).

We model time-varying uncertainty using stochastic volatility models (as in Fernández-Villaverde (2010), Fernández-Villaverde et al. (2010), Fernández-Villaverde et al. (2011), and Born and Pfeifer (2014)). First and second order approximations are not well suited to account for time-varying uncertainty (Fernández-Villaverde et al. (2010), and Fernández-Villaverde et al. (2011)).<sup>3</sup> We solve the model using a third-order

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<sup>1</sup>Changes in uncertainty in these models provide an additional source of shocks. In Alexopoulos and Cohen (2009), Bloom (2009), and Bloom et al. (2014) this uncertainty is the time-varying variance of TFP shocks). Basu and Bundick (2014) include TFP and preference (discount factor) uncertainty shocks. Born and Pfeifer (2014) add monetary and fiscal policy uncertainty as do Fernández-Villaverde et al. (2011).

<sup>2</sup>For example, Bansal and Yaron (2004) and Bansal (2007) examine the effect of long-run uncertainty about cash flows on asset prices.

<sup>3</sup>See Schmitt-Grohé and Uribe (2004), Fernández-Villaverde (2010), Fernández-Villaverde et al. (2010), and more recently

approximation, which allows for a richer exploration of nonlinear relationships between credit frictions, uncertainty, and economic activity than a second-order approximation. We exploit this potential nonlinearity by conducting impulse response analysis that is conditional on the initial state of the economy at the time of the shock. In particular, we consider whether the effects of shocks are conditional on the degree of uncertainty and size of the credit frictions. This allows us to examine whether shocks have different qualitative effects depending on the current state of credit frictions or current degree of uncertainty. That is to say, we ask whether uncertainty, aside from being a source of shocks, amplifies the effects of possibly unrelated shocks.

We find that shocks to micro-uncertainty (or, equivalently, exogenous credit friction shocks) have first-order effects that are of similar magnitudes as shocks to the level of TFP or a traditional monetary policy shock. The response of our model economies to the micro-uncertainty shock appears to be similar to a typical financial frictions shock: There is a decline in investment and production along with a significant decline in labor use. On the other hand, TFP uncertainty shocks, on average, have effects that are orders of magnitude smaller than level TFP or micro-uncertainty shocks. Monetary policy uncertainty shocks also have effects that are substantially smaller than shocks to the level of TFP or monetary policy. However, monetary policy uncertainty has larger effects than TFP uncertainty on the dynamics of the economy. We find that the degree to which monetary policy uncertainty matters depends on the extent of nominal rigidities in the model.

We find mixed results regarding whether or not the state of credit conditions (measured by credit spreads) amplifies other shocks. Large initial spreads tend to slightly dampen the effect on output of TFP shocks while slightly amplifying that of monetary shocks. However, if spreads are already high, the effect of micro-uncertainty shocks on output is nearly 40% larger than when spreads are low. This suggests that when credit conditions are poor (high spreads), additional credit shocks make the situation disproportionately worse. On the other hand, conditioning on the amount of TFP or monetary uncertainty has virtually no qualitative effect on the responses to shocks in the model aside from the fact that shocks tend to be larger when uncertainty is higher.

While other literature has examined the impact of uncertainty on economic fluctuations, our paper differs from this previous literature in a few key respects. Like us, [Gilchrist et al. \(2013\)](#) and [Christiano et al. \(2014\)](#) include both credit frictions and uncertainty. In their models, the source of the time-varying uncertainty was changes in the distribution of entrepreneur productivity. The heterogeneity of entrepreneur productivity is the underlying source of the information asymmetry that is responsible for the credit frictions. [Christiano et al. \(2014\)](#) show that this idiosyncratic uncertainty acts as an additional source of shocks and these shocks play an important role in generating aggregate fluctuations. [Gilchrist et al. \(2013\)](#) also find that the combination of micro-uncertainty and credit frictions can be important for aggregate fluctuations and argue that this combination is quantitatively more important than the combination of micro-uncertainty and irreversibility (*option value of waiting effect*). Neither paper explores the interaction of credit frictions with other sources of uncertainty and how the various forms of uncertainty interact with one another, nor do they explore the possible non-linearity implied by the model. Furthermore, while there have been a few papers that examine how zero lower bound monetary policy interacts with aggregate uncertainty ([Basu and Bundick \(2014\)](#)), policy uncertainty ([Fernández-Villaverde et al. \(2011\)](#)), and micro-uncertainty ([Gilchrist et al. \(2013\)](#)), little other work has really explored the implications of the nonlinearity implied by both stochastic volatility and the costly-state verification credit frictions.

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[Fernández-Villaverde et al. \(2011\)](#) for a detailed discussion of the relative merits of alternative approximations.

The remainder of the paper proceeds as follows: [Section 2](#) describes our model with credit market imperfections and micro-uncertainty, macro-uncertainty, and policy uncertainty, while [Section 3](#) discusses the interaction between credit frictions and uncertainty. [Section 4](#) introduces the perturbation approach that we use to compute a third-order approximation and summarizes the parameterization strategy used for the simulations. [Section 5](#) highlights the main quantitative findings derived from our analysis of the model. [Section 6](#) provides a discussion of the main findings of this paper and the literature, and [Section 7](#) concludes. General equilibrium conditions, the zero-inflation steady state, and all listed tables and figures are provided in the [Appendix](#).<sup>4</sup>

## 2 Credit Frictions and Uncertainty

We extend the benchmark New Keynesian business cycle model with uncertainty modelled as in [Fernández-Villaverde et al. \(2011\)](#) and a financial accelerator mechanism based on the costly-state verification framework of [Townsend \(1979\)](#) and [Gale and Hellwig \(1985\)](#).<sup>5</sup> The central bank’s actions in our framework are described with a modified [Taylor \(1993\)](#)-type rule for monetary policy augmented with time-varying stochastic volatility in the monetary shock process. In this way, we capture policy uncertainty in conjunction with the financial distortions introduced by the costly monitoring of nominal financial contracts.

We distinguish here between idiosyncratic and aggregate technology shocks to productivity whereby the given financial distortion is inefficient because it prevents economic agents from fully insuring themselves against all idiosyncratic risk. Stochastic volatility on the idiosyncratic technology shocks to capital returns is introduced to model micro-uncertainty; but our model still retains aggregate productivity (TFP) shocks with a stochastic volatility component to incorporate macro-uncertainty and keep our analysis comparable with the current literature. The remainder of this section describes the building blocks of the model in more detail and further elaborates on our extensions of the benchmark New Keynesian model.

### 2.1 Households

The economy is populated by a continuum of mass one of identical and infinitely-lived households. Preferences are defined over per capita consumption,  $C_t$ , and per capita labor,  $H_t$ , based on an additively separable specification with internal habits in consumption:

$$U \equiv \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{(C_t - bC_{t-1})^{1-\chi}}{1-\chi} - \kappa \frac{H_t^{1+\xi}}{1+\xi} \right\}, \quad (1)$$

where  $0 < \beta < 1$  is the intertemporal discount factor,  $\chi \geq 0$  is the inverse of the intertemporal elasticity of substitution,  $\xi \geq 0$  is the inverse of the Frisch elasticity of labor supply,  $\kappa \geq 0$  governs the relative disutility

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<sup>4</sup>Additional technical details on the estimation and simulation of the model as well as a richer set of experiments used to evaluate the implications of the model can be found in [Balke et al. \(2017\)](#).

<sup>5</sup>Other references based on the costly-state verification framework include [Bernanke and Gertler \(1989\)](#), [Bernanke et al. \(1999\)](#), [Cohen-Cole and Martínez-García \(2010\)](#), [Martínez-García \(2014\)](#), and [Christiano et al. \(2014\)](#), among others.

of labor effort, and  $0 \leq b \leq 1$  defines the internal habit persistence on consumption.<sup>6</sup>

Households face the following nominal budget constraint:

$$P_t C_t + B_t \leq W_t H_t + I_{t-1} B_{t-1} + DIV_t. \quad (2)$$

At time  $t$ , households consume an amount  $C_t$  of the final good at price  $P_t$  and save an amount  $B_t$  through one-period deposits offered by the financial intermediaries maturing at time  $t+1$ .<sup>7</sup> Households receive a gross nominal risk-free interest rate  $I_{t-1}$  on their deposits maturing at time  $t$  ( $B_{t-1}$ ), earn income from supplying labor  $H_t$  at the prevailing competitive nominal wage rate  $W_t$ , and receive nominal dividend payments  $DIV_t$  from the profits or losses generated by the financial and nonfinancial firms they own.

Solving the households' optimization problem, we obtain the following labor supply equation and the following Euler equation for the consumption-savings decision:

$$\frac{W_t}{P_t} = \frac{\kappa H_t^\xi}{\Lambda_t}, \quad (3)$$

$$1 = \beta \mathbb{E}_t \left[ \left( \frac{\Lambda_{t+1}}{\Lambda_t} \right) \frac{P_t}{P_{t+1}} I_t \right], \quad (4)$$

where  $\Lambda_t \equiv (C_t - bC_{t-1})^{-\chi} - b\beta \mathbb{E}_t [(C_{t+1} - bC_t)^{-\chi}]$  denotes the Lagrange multiplier on the households' budget constraint expressed in units of the final good. The households' equilibrium conditions are completed with the appropriate initial and no-Ponzi transversality conditions.

## 2.2 Entrepreneurs and Financial Business Sector

There is a continuum of entrepreneurs of unit mass with identical linear preferences defined over per capita entrepreneurial consumption,  $C_t^e$ ,

$$\mathbb{E}_0 \sum_{t=0}^{\infty} (\gamma\beta)^t C_t^e, \quad (5)$$

where the parameter  $0 < \gamma < 1$  scaling the intertemporal discount factor in (5) captures the probability of each entrepreneur surviving until the next period.<sup>8</sup> The mass of entrepreneurs in each period is kept

<sup>6</sup>Whenever  $\chi = 1$  (log-utility on consumption), the specification in (1) is the one used in [Bernanke et al. \(1999\)](#). We also consider the [Jaimovich and Rebelo \(2009\)](#) preferences given by

$$U \equiv \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\left( \frac{(C_t - bC_{t-1})^{1-\chi}}{1-\chi} - \kappa \frac{H_t^{1+\xi}}{1+\xi} J_t \right)^{1-v} - 1}{1-v} \right\},$$

where the term  $J_t = \left( \frac{(C_t - bC_{t-1})^{1-\chi}}{1-\chi} \right)^\zeta J_{t-1}^{1-\zeta}$  makes these preferences non-separable in both consumption and labor. The parameter  $0 \leq \zeta \leq 1$  governs the strength of the wealth effect on labor, while  $v \geq 0$  affects the intertemporal elasticity of substitution. If  $\xi = 0$  and  $v = 0$ , then the specification corresponds to the one given in equation (1). When  $\zeta = 1$ , this reduces to the specification discussed in [King et al. \(1988\)](#). When  $\xi = 0$ , the [Greenwood et al. \(1988\)](#) utility with no wealth effect on labor supply is obtained. The results that we obtained are not significantly different with alternative specifications on preferences, and are available upon request from the authors.

<sup>7</sup>The deposits are issued by the financial intermediaries, pay the risk-free nominal interest rate prevailing at the time of issuance (which is known), and then the resources attracted through these deposits are all made available to entrepreneurs in the form of nominal loans for investment.

<sup>8</sup>This implies an expected lifetime for the entrepreneurs of  $\frac{1}{1-\gamma}$ . The assumption of finite horizons for entrepreneurs captures exogenously the phenomenon of deaths of entrepreneurial projects.

constant and equal to one by assuming full replacement of the entrepreneurial deaths.

The entrepreneurs who survive purchase raw capital from a group of *capital producers*, transform the raw capital into work capital services, and rent them to *wholesale producers* who produce wholesale goods (intermediate goods). The purchase of raw capital is financed internally and possibly externally with a loan contract between a living entrepreneur and a *financial intermediary*. The living entrepreneurs also supply labor to the wholesale producers. The entrepreneurs who *die* in period  $t$  do not purchase capital, work, or sign new contracts, but instead simply consume their accumulated net worth and disappear. The new entrepreneurs that replace them come with no net worth, but get some entrepreneurial labor income to start.

### 2.2.1 Entrepreneurs and the Agency Cost

At time  $t - 1$ , entrepreneurs purchase from capital producers the aggregate stock of capital necessary for production at time  $t$ ,  $K_t$ , at a price per unit  $Q_{t-1}$  in terms of the final good. The total nominal value of the capital acquisition,  $P_{t-1}Q_{t-1}K_t$ , is financed with a combination of the entrepreneurs' accumulated nominal net worth (internal funds),  $N_{t-1}$ , and external funding provided by the financial intermediaries (via one-period risky loans),  $L_{t-1} = P_{t-1}Q_{t-1}K_t - N_{t-1}$ . Purchased capital is subject to a purely idiosyncratic technology shock  $\omega_{t-1}$  which is i.i.d. across entrepreneurs with  $\mathbb{E}(\omega_{t-1}) = 1$  and linearly transforms capital into the actual capital services supplied by entrepreneurs to wholesale producers.

While all entrepreneurs face the same capital purchasing decision problem at  $t - 1$  and make identical choices, *ex post* differences across entrepreneurs emerge due to the fact that each receives a different draw from  $\omega_{t-1}$ . Then, at time  $t$ , each entrepreneur rents  $\omega_{t-1}$  units of effective capital services to the wholesale producers. This generates a nominal per-period income stream for the entrepreneurs that comes from the earned competitive nominal rent on capital services,  $R_t^w$ , paid by the wholesale producers and from the resale value paid by the capital producers on the depreciated capital,  $\bar{Q}_t$ , expressed in units of the final good. Hence, in nominal terms, each entrepreneur earns  $\omega_{t-1} [R_t^w + P_t \bar{Q}_t (1 - \delta)]$  at time  $t$  for each unit of capital acquired at time  $t - 1$  given the depreciation rate,  $\delta$ . From here it follows that each entrepreneur's nominal return accrued on capital is simply given by  $\omega_{t-1} R_t^e$  where the aggregate nominal return on capital,  $R_t^e$ , is defined as follows:

$$\frac{R_t^e}{\Pi_t} \equiv \left[ \frac{\frac{R_t^w}{P_t} + \bar{Q}_t (1 - \delta)}{Q_{t-1}} \right], \quad (6)$$

with  $\Pi_t \equiv \frac{P_t}{P_{t-1}}$  being the inflation rate on final goods.

The idiosyncratic technology shock  $\omega_t$  is log-normally distributed—i.e.,  $\ln(\omega_t) \sim N(\mu_{\omega,t}, \sigma_{\omega,t}^2)$ . The variance,  $\sigma_{\omega,t}^2$ , reflects the dispersion of the cross-section distribution of entrepreneur productivity and, hence, the micro-uncertainty underlying the credit frictions. We allow this variance to be time-varying. Specifically,  $\sigma_{\omega,t} \equiv \sigma_{\omega} e^{\hat{\sigma}_{\omega,t}}$  where  $\hat{\sigma}_{\omega,t} \equiv \ln \sigma_{\omega,t} - \ln \sigma_{\omega}$  and

$$\hat{\sigma}_{\omega,t} = v_{\omega} \hat{\sigma}_{\omega,t-1} + \eta_{\omega} u_{\omega,t}, \quad (7)$$

where  $u_{\omega,t}$  is i.i.d.  $N(0, 1)$ . The parameter  $0 < v_{\omega} < 1$  determines the persistence of the idiosyncratic technology shock's log-volatility  $\hat{\sigma}_{\omega,t}$ . The unconditional expected volatility is given by  $\sigma_{\omega} > 0$ , while  $\eta_{\omega} \geq 0$  controls the standard deviation of the innovation to the stochastic volatility process. Furthermore, the distribution is mean-preserving to isolate the effects of pure second moment shocks (micro-uncertainty) from

the first moment or level effects of the shock.<sup>9</sup> To do this, we set the time-varying conditional mean  $\mu_{\omega,t}$  to be  $\mu_{\omega,t} = -\frac{\sigma_{\omega,t}^2}{2}$  ensuring the unconditional mean of the idiosyncratic technology shock is equal to one (i.e.,  $\mathbb{E}(\omega_t) = e^{\mu_{\omega,t} + \frac{\sigma_{\omega,t}^2}{2}} = 1$ ).<sup>10</sup>

The idiosyncratic technology shock  $\omega_{t-1}$  is costlessly observable to the entrepreneurs only after the loan terms are agreed upon, while monitoring is costly for the financial intermediaries—the financial friction in our model arises from this informational asymmetry.<sup>11</sup> Under limited liability, in case of default at time  $t$ , the financial intermediaries can only appropriate the nominal capital income generated by the defaulting entrepreneur in that period—which amounts in total to  $\omega_{t-1} [R_t^w + P_t \bar{Q}_t (1 - \delta)] K_t = \omega_{t-1} R_t^e P_{t-1} Q_{t-1} K_t$ . Financial intermediaries monitor and verify the defaulting entrepreneur’s income at a cost expressed as a fraction  $0 \leq \mu < 1$  of the nominal amount recovered—i.e., at a nominal cost of  $\mu \omega_{t-1} R_t^e P_{t-1} Q_{t-1} K_t$ .

At time  $t$ , default on a loan occurs whenever the nominal income earned by the defaulting entrepreneur after the realization of the idiosyncratic technology shock  $\omega_{t-1}$  is insufficient to cover the nominal repayment expenses on the loan, i.e., whenever

$$\omega_{t-1} R_t^e P_{t-1} Q_{t-1} K_t \leq R_t^L L_{t-1}, \quad (8)$$

where  $R_t^L$  denotes the nominal borrowing cost set by the financial intermediaries. A risky one-period loan ( $L_{t-1}$ ) is simply defined in terms of a default threshold on the idiosyncratic technology shock,  $\bar{\omega}_{t-1}$ , for which the loan repayment exactly equals the capital income accrued—i.e.,  $R_t^L L_{t-1} = \bar{\omega}_{t-1} R_t^e P_{t-1} Q_{t-1} K_t$ . In case of default ( $\omega_{t-1} < \bar{\omega}_{t-1}$ ), the financial intermediaries always choose to monitor in order to prevent the defaulting entrepreneur from misreporting the true value of  $\omega_{t-1}$  and, therefore, the nominal income that can be recovered. The entrepreneur that defaults gets nothing, while the financial intermediaries are able to recover  $(1 - \mu) \omega_{t-1} R_t^e P_{t-1} Q_{t-1} K_t$  net of monitoring costs. If the entrepreneur does not default ( $\omega_{t-1} \geq \bar{\omega}_{t-1}$ ), then he pays  $\bar{\omega}_{t-1} R_t^e P_{t-1} Q_{t-1} K_t$  back to the financial intermediaries and keeps  $(\omega_{t-1} - \bar{\omega}_{t-1}) R_t^e P_{t-1} Q_{t-1} K_t$  for himself.<sup>12</sup>

Apart from capital income net of borrowing costs, entrepreneurs get revenue also from inelastically supplying one unit of entrepreneurial labor—i.e.,  $H_t^e = 1$ —to the wholesale producers at the competitive nominal wage,  $W_t^e$ . Hence, the budget constraint of the entrepreneurs can be described in the following

<sup>9</sup>The literature has traditionally introduced stochastic volatility on log-normally distributed shocks—hence, shocks to volatility not only affect the dispersion of the shock distribution (second moment effect), but also change its conditional mean (first moment effect), potentially confounding the impact of the shock. Introducing mean-preserving volatility shocks as we do in the paper allows us to more cleanly disentangle the effect of these shocks. See the [Appendix](#) for a more detailed description of the mean-preserving uncertainty shocks.

<sup>10</sup>Given the fact that the random shock  $\omega_t$  is i.i.d., the conditional and unconditional means are equivalent in this case.

<sup>11</sup>The costly acquisition of information about these idiosyncratic shocks implies that financial contracts cannot be written down to completely diversify away these risks. We extend the work of [Bernanke and Gertler \(1989\)](#), [Bernanke et al. \(1999\)](#), [Cohen-Cole and Martínez-García \(2010\)](#), and [Martínez-García \(2014\)](#)—based on the costly-state verification set-up of [Townsend \(1979\)](#) and [Gale and Hellwig \(1985\)](#)—to express financial contracts in nominal terms. With a nominal financial contract, risks that affect inflation can also influence the allocation of capital.

<sup>12</sup>Whenever there is aggregate risk,  $R_t^e$  is not known at time  $t - 1$  when the loan is finalized. [Bernanke et al. \(1999\)](#) appeal to the assumption that entrepreneurs are risk-neutral while households are risk-averse to argue that loan contracts should require the entrepreneurs to bear all the aggregate risk to provide full insurance for the households’ savings allocated by the financial intermediaries. However, loan contracts with full insurance for the savers are not necessarily optimal in more general settings (see, e.g., [Hellwig \(2001\)](#), [Monnet and Quintin \(2005\)](#), and [Carlstrom et al. \(2016\)](#), among others). We leave the exploration of more complex risk-sharing financial arrangements for future research.



generic terms:

$$P_t C_t^e + P_t Q_t K_{t+1} \leq W_t^e H_t^e + \int_{\bar{\omega}_{t-1}}^{+\infty} [\omega_{t-1} R_t^e P_{t-1} Q_{t-1} K_t - R_t^L L_{t-1}] \phi(\omega_{t-1} | \sigma_{\omega, t-1}) d\omega_{t-1} + L_t, \quad (9)$$

where the nominal income stream from capital and labor plus the amount borrowed from the financial intermediaries ( $L_t$ ) are allocated to entrepreneurial consumption ( $C_t^e$ ) and for the acquisition of tomorrow's capital stock ( $K_{t+1}$ ). The objective of the entrepreneurs is to maximize their lifetime utility in (5) subject to the canonical sequence of budget constraints described in (9) and the entrepreneurs' balance sheet identity  $P_t Q_t K_{t+1} = L_t + N_t$ .

### 2.2.2 Optimal Loan Contract (Signed in Time Period $t$ )

There is a continuum of identical, competitive financial intermediaries of unit mass. At each time period  $t$ , financial intermediaries offer one-period, fully-insured deposits to households for saving purposes and pay a gross risk-free rate,  $I_t$  (which is known at time  $t$ ). These financial intermediaries capture all households' savings through deposits to offer one-period loans to the entrepreneurs. As explained in the [Appendix](#), the loan contracting problem reduces to optimally choosing the quantity of capital,  $K_{t+1}$ , and the threshold,  $\bar{\omega}_t$ , that maximizes the entrepreneurs' nominal return on capital net of the borrowing costs

$$P_t Q_t K_{t+1} \mathbb{E}_t [R_{t+1}^e f(\bar{\omega}_t, \sigma_{\omega, t})], \quad (10)$$

subject to a participation constraint for the lenders (financial intermediaries)

$$P_t Q_t K_{t+1} \mathbb{E}_t [R_{t+1}^e g(\bar{\omega}_t, \sigma_{\omega, t})] \geq I_t [P_t Q_t K_{t+1} - N_t], \quad (11)$$

where  $f(\bar{\omega}_t, \sigma_{\omega, t})$  and  $g(\bar{\omega}_t, \sigma_{\omega, t})$  denote the share of capital income going to the entrepreneurs and the financial intermediaries, respectively. The participation constraint simply requires financial intermediaries to be sufficiently compensated on their loans to pay back the depositors in full.

Solving the loan contract problem results in three key equilibrium conditions. First, the sharing rule between entrepreneurs and financial intermediaries resulting from the optimal loan contract implies that

$$f(\bar{\omega}_t, \sigma_{\omega, t}) + g(\bar{\omega}_t, \sigma_{\omega, t}) = 1 - \mu G(\bar{\omega}_t, \sigma_{\omega, t}), \quad (12)$$

where  $\mu G(\bar{\omega}_t, \sigma_{\omega, t})$  determines the monitoring losses associated with default.<sup>13</sup> Second, we find that

$$\frac{P_t Q_t K_{t+1}}{N_t} = 1 + \lambda(\bar{\omega}_t, \sigma_{\omega, t}) \frac{g(\bar{\omega}_t, \sigma_{\omega, t})}{f(\bar{\omega}_t, \sigma_{\omega, t})}, \quad (13)$$

where  $\lambda(\bar{\omega}_t, \sigma_{\omega, t})$  is the Lagrange multiplier on the participation constraint in (11) (which represents the shadow cost of enticing the participation of the financial intermediaries). Hence, the default threshold  $\bar{\omega}_t$  depends on the dispersion of the idiosyncratic technology shock (our measure of micro-uncertainty),  $\sigma_{\omega, t}$ ,

<sup>13</sup>The [Appendix](#) provides more details on the characterization of the functions  $f(\bar{\omega}_{t-1}, \sigma_{\omega, t-1})$ ,  $g(\bar{\omega}_{t-1}, \sigma_{\omega, t-1})$ , and  $G(\bar{\omega}_{t-1}, \sigma_{\omega, t-1})$ .

and on the asset-to-net-worth ratio of the entrepreneurs,  $\frac{P_t Q_t K_{t+1}}{N_t}$ .<sup>14</sup> Finally, expected gross returns to entrepreneurs are:

$$\mathbb{E}_t [R_{t+1}^e] = s \left( \frac{P_t Q_t K_{t+1}}{N_t}, \sigma_{\omega,t} \right) I_t, \quad (14)$$

where  $s \left( \frac{P_t Q_t K_{t+1}}{N_t}, \sigma_{\omega,t} \right)$  is the external finance premium, which is a function of the micro-uncertainty shock,  $\sigma_{\omega,t}$ , and the asset-to-net-worth ratio,  $\frac{P_t Q_t K_{t+1}}{N_t}$ , of the entrepreneurs.<sup>15</sup>

If the financial sector makes losses in equilibrium, then the participation constraint would be violated. If the financial sector makes profits, then the entrepreneurs would be better off with another loan contract that still satisfies the participation constraint of the financial intermediaries but with lower borrowing rates. Thus, the optimal external finance premium must allow financial intermediaries to recover enough income to repay their depositors in full every period from the pool of loans while breaking even (zero profits on their portfolio of loans in equilibrium). Therefore, external funding (loans) is always more expensive for entrepreneurs than internal funding (net worth) whose opportunity cost is given by the nominal risk-free rate  $I_t$  paid on deposits.

### 2.2.3 Entrepreneurs' Consumption and Net Worth under the Optimal Loan Contract

In equilibrium, whenever the optimal loan contract is implemented, the entrepreneurs' budget constraint in (9) defines an upper bound on entrepreneurial net worth  $N_t$  as follows:

$$N_t = P_t Q_t K_{t+1} - L_t \leq W_t^e H_t^e + f(\bar{\omega}_{t-1}, \sigma_{\omega,t-1}) R_t^e P_{t-1} Q_{t-1} K_t - P_t C_t^e \quad (15)$$

$$= W_t^e H_t^e + (f(\bar{\omega}_{t-1}, \sigma_{\omega,t-1}) + \lambda(\bar{\omega}_{t-1}, \sigma_{\omega,t-1}) g(\bar{\omega}_{t-1}, \sigma_{\omega,t-1})) R_t^e N_{t-1} - P_t C_t^e, \quad (16)$$

given the optimality condition in (13). Each entrepreneur dies with probability  $(1 - \gamma)$  and gets replaced by a new entrepreneur with no net worth—hence, preventing the entrepreneurs from accumulating infinite wealth and becoming self-financing. Moreover, entrepreneurs are risk-neutral and relatively more impatient than households given (5), so they choose to postpone their consumption until they die.

Hence, it follows from equation (16) that the aggregate consumption for entrepreneurs,  $C_t^e$ , can be expressed as

$$C_t^e = (1 - \gamma) (f(\bar{\omega}_{t-1}, \sigma_{\omega,t-1}) + \lambda(\bar{\omega}_{t-1}, \sigma_{\omega,t-1}) g(\bar{\omega}_{t-1}, \sigma_{\omega,t-1})) \frac{R_t^e}{\Pi_t} \left( \frac{N_{t-1}}{P_{t-1}} \right), \quad (17)$$

given that only the entrepreneurs who die consume at time  $t$ .<sup>16</sup> Furthermore, the law of motion for nominal

<sup>14</sup>The asset-to-net-worth ratio can be related to the leverage ratio since  $\frac{P_{t-1} Q_{t-1} K_t}{N_{t-1}} = 1 + \frac{L_{t-1}}{N_{t-1}}$  and  $\frac{L_t}{N_t}$  is a conventional measure of debt-to-net worth.

<sup>15</sup>The characterization of the external financing premium  $s \left( \frac{P_t Q_t K_{t+1}}{N_t}, \sigma_{\omega,t} \right)$  is discussed in further detail in the [Appendix](#).

<sup>16</sup>We interpret net worth, like capital, as accumulated output. Entrepreneurial consumption is just the fraction of that accumulated output that corresponds to the dying entrepreneurs. Hence, entrepreneurial consumption does not detract resources from current production of final goods.

entrepreneurial net worth,  $N_t$ , follows from equation (16) as

$$\begin{aligned} \frac{N_t}{P_t} &= \left\{ \gamma \left( \frac{W_t^e}{P_t} H_t^e + (f(\bar{\omega}_{t-1}, \sigma_{\omega, t-1}) + \lambda(\bar{\omega}_{t-1}, \sigma_{\omega, t-1}) g(\bar{\omega}_{t-1}, \sigma_{\omega, t-1})) \frac{R_t^e}{\Pi_t} \left( \frac{N_{t-1}}{P_{t-1}} \right) \right) \right\} + \\ &\quad \left\{ (1 - \gamma) \left( \frac{W_t^e}{P_t} H_t^e \right) \right\} + \{(1 - \gamma) 0\} \\ &= \frac{W_t^e}{P_t} H_t^e + \gamma (f(\bar{\omega}_{t-1}, \sigma_{\omega, t-1}) + \lambda(\bar{\omega}_{t-1}, \sigma_{\omega, t-1}) g(\bar{\omega}_{t-1}, \sigma_{\omega, t-1})) \frac{R_t^e}{\Pi_t} \left( \frac{N_{t-1}}{P_{t-1}} \right). \end{aligned} \quad (18)$$

This means that the net worth of the entrepreneurs above, beyond their per-period entrepreneurial labor income, is given by the income the survived entrepreneurs (with a fraction of  $\gamma$ ) earn from their capital purchases net of borrowing costs.

## 2.3 Non-Financial Business Sector

### 2.3.1 Capital Producers

There is a continuum of mass one of identical capital-producing firms. The aggregate stock of capital  $K_t$  evolves according to the following law of motion:

$$K_{t+1} \leq (1 - \delta)K_t + s_k \left( \frac{X_t}{K_t} \right) K_t, \quad (19)$$

where  $X_t$  denotes aggregate investment in terms of the final good. The production of physical capital is subject to adjustment costs as in Hayashi (1982)—and here we adopt the adjustment cost function specification proposed by Jermann (1998) and Boldrin et al. (2001), i.e.,

$$s_k \left( \frac{X_t}{K_t} \right) = \frac{s_{k1}}{1 - \frac{1}{\varphi_k}} \left( \frac{X_t}{K_t} \right)^{1 - \frac{1}{\varphi_k}} + s_{k2}, \quad (20)$$

where  $\varphi_k > 0$  regulates the degree of concavity and  $\frac{X_t}{K_t}$  denotes the investment-to-capital ratio. We impose the restrictions  $s_k(\delta) = \delta$  and  $s'_k(\delta) = 1$  to ensure that adjustment costs drop out in steady state, setting the constants  $s_{k1}$  and  $s_{k2}$  to be  $s_{k1} \equiv (\delta)^{\frac{1}{\varphi_k}}$  and  $s_{k2} \equiv - \left( \frac{1}{1 - \frac{1}{\varphi_k}} \right) \delta$ .<sup>17</sup>

At time  $t$ , entrepreneurs purchase their desired capital stock for the next period of time,  $K_{t+1}$ , from the capital producers, and sell them back the depreciated stock of existing capital  $(1 - \delta)K_t$  after the production of wholesale goods is done. Capital producers also purchase final goods in the amount of  $X_t$  at  $P_t$  to produce  $s_k \left( \frac{X_t}{K_t} \right) K_t$  units of new capital. Hence, the per-period (static) profits of the capital producers are given by  $P_t (Q_t K_{t+1} - X_t - (1 - \delta) \bar{Q}_t K_t)$ , which they aim to maximize subject to (19). Solving the capital producers' optimization, the relative price of new capital in terms of the final good (or Tobin's  $q$ )  $Q_t$  is given by:

$$Q_t = \left[ s'_k \left( \frac{X_t}{K_t} \right) \right]^{-1} = \left( \frac{X_t}{K_t} \right)^{\frac{1}{\varphi_k}} \frac{1}{\delta}, \quad (21)$$

where the parameter  $\varphi_k$  governs the elasticity of investment with respect to Tobin's  $q$ .<sup>18</sup> Note that the

<sup>17</sup>The adjustment cost function  $s_k \left( \frac{X_t}{K_t} \right)$  satisfies  $s_k(\cdot) > 0$ ,  $s'_k(\cdot) > 0$ , and  $s''_k(\cdot) < 0$ .

<sup>18</sup>Time-variation in the relative price of capital serves as an additional amplification and propagation mechanism in this framework. We follow Bernanke et al. (1999) giving ownership of the capital-producing sector to households in order to ensure that capital production decisions are not directly affecting the entrepreneurs' decision on how much capital to demand.

resale value of the depreciated stock of capital  $\bar{Q}_t$  differs from the Tobin's  $q$   $Q_t$  set by the capital producers. Imposing that these producers break even making zero profits, i.e.,

$$Q_t s_k \left( \frac{X_t}{K_t} \right) - \frac{X_t}{K_t} - (1 - \delta) (\bar{Q}_t - Q_t) = 0, \quad (22)$$

we pin down the relative resale value of capital  $\bar{Q}_t$  as a function of  $Q_t$  and the investment-to-capital ratio  $\left(\frac{X_t}{K_t}\right)$ . Note that the difference between  $Q_t$  and  $\bar{Q}_t$  is of second-order importance and omitted by Bernanke *et al.* (1999) which rely on a first-order approximation to characterize the dynamics of their model. We cannot ignore the distinction in our set-up, as we solve our model up to a higher order of approximation.

### 2.3.2 Wholesale Firms

There is a continuum of mass one of identical wholesale producers. Wholesale goods,  $Y_t^w$ , are produced with the following Cobb-Douglas technology:

$$Y_t^w \leq e^{a_t - a} (K_t)^\alpha (H_t^e)^\vartheta (H_t)^{1 - \alpha - \vartheta}, \quad (23)$$

combining labor from households,  $H_t$ , labor from entrepreneurs,  $H_t^e$ , and rented capital,  $K_t$ , owned by the entrepreneurs. Both capital share,  $\alpha$ , and entrepreneurial labor share,  $\vartheta$ , are elements of  $[0, 1]$ , and they give rise to the household labor share,  $(1 - \alpha - \vartheta)$ .

With persistence  $\rho_a \in (0, 1)$ , the stochastic process for aggregate productivity (TFP),  $a_t$ , is given by:

$$a_t = \mu_{a,t} + \rho_a (a_{t-1} - \mu_{a,t-1}) + \sigma_{a,t} \varepsilon_{a,t}. \quad (24)$$

Similar to the micro-uncertainty shock, a macro-uncertainty shock is defined as a shock to the stochastic volatility in the TFP,  $\sigma_{a,t} \equiv \sigma_a e^{\hat{\sigma}_{a,t}}$ , where  $\sigma_a > 0$ , and

$$\hat{\sigma}_{a,t} = v_a \hat{\sigma}_{a,t-1} + \eta_a u_{a,t}, \quad (25)$$

with  $0 < v_a < 1$  and  $\eta_a \geq 0$ . Note that  $\varepsilon_{a,t}$  and  $u_{a,t}$  are i.i.d.  $N(0, 1)$  and uncorrelated. The shock  $\varepsilon_{a,t}$  raises the productivity level (first moment shock), while  $u_{a,t}$  introduces a shock to its volatility (second moment shock).

To isolate the effects of the pure second moment shocks (macro-uncertainty) from the first moment TFP shock, we define a mean-preserving shock process by requiring the time-varying conditional mean,  $\mu_{a,t}$ , to satisfy the following recursion:  $\mu_{a,t} = -\frac{\sigma_{a,t}^2}{2} + \rho_a^2 \mu_{a,t-1}$ .<sup>19</sup> The unconditional mean of the process  $a$  can then be expressed as  $a \equiv -\frac{1}{2} \frac{\sigma_a^2}{1 - \rho_a^2}$ .

All wholesale producers operate in competitive markets and produce a homogeneous good sold only to retailers. Solving the (static) profit-maximization problem of the wholesale producers subject to the technological constraint implied by the production function in (23) results in the factors of production being

<sup>19</sup>A discussion of the mean-preserving recursion and its role in ensuring that shocks to volatility do change the dispersion—but not the mean—of the distribution can be found in the [Appendix](#).

remunerated at their marginal product,<sup>20</sup>

$$\frac{W_t}{P_t} = (1 - \alpha - \vartheta) \frac{P_t^{wr} Y_t^w}{H_t}, \quad (26)$$

$$\frac{W_t^e}{P_t} = \vartheta \frac{P_t^{wr} Y_t^w}{H_t^e}, \quad (27)$$

$$\frac{R_t^w}{P_t} = \alpha \frac{P_t^{wr} Y_t^w}{K_t}, \quad (28)$$

where labor from households and entrepreneurs is paid at competitive nominal wages,  $W_t$  and  $W_t^e$  respectively, rented capital from the entrepreneurs is compensated with a nominal rental rate,  $R_t^w$ , and the relative price of the wholesale good in terms of the final good is given by  $P_t^{wr} \equiv \frac{P_t^w}{P_t}$ .

### 2.3.3 Retailers and Final Goods Producers

There is a continuum of differentiated retail varieties  $z$  of mass one—they are indexed  $z \in [0, 1]$  and each one of them is produced by a monopolistically competitive retail firm.<sup>21</sup> All retail firms are owned by the households. The retail sector transforms homogeneous wholesale output into differentiated varieties of goods using a linear technology. Each retail variety is then bundled up by final goods producers and sold for consumption (to households and entrepreneurs) and for investment (to capital goods producers) purposes.

There is a continuum of mass one of identical final goods producers which bundle the retail varieties. The aggregate bundle of varieties  $Y_t$  defines final goods with a constant elasticity of substitution (CES) index, i.e.,  $Y_t \equiv \left[ \int_0^1 Y_t(z)^{\frac{\epsilon-1}{\epsilon}} dz \right]^{\frac{\epsilon}{\epsilon-1}}$  where  $\epsilon > 1$  is the elasticity of substitution across varieties and  $Y_t(z)$  denotes the amount of each variety  $z \in [0, 1]$ . The corresponding final goods price,  $P_t$ , is given by  $P_t = \left[ \int_0^1 P_t(z)^{1-\epsilon} dz \right]^{\frac{1}{1-\epsilon}}$ , which is a function of the prices of each variety,  $P_t(z)$ . Hence, the optimal allocation of expenditure to each variety  $z$ , i.e.,

$$Y_t(z) = \left( \frac{P_t(z)}{P_t} \right)^{-\epsilon} Y_t, \quad (29)$$

implies that retailers face a downward-sloping demand function from final goods producers.

The retail firm  $z$  then chooses price  $P_t(z)$  to maximize its expected nominal profits, i.e.,

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \lambda_t [(P_t(z) - P_t^w) Y_t(z) - s_p(P_t(z), P_{t-1}(z)) P_t Y_t], \quad (30)$$

subject to the demand function in (29) and the intertemporal discounting factor  $\lambda_t \equiv \beta^t \frac{\Lambda_t}{\Lambda_0} \frac{P_0}{P_t}$  from the households' problem. For each unit of its own variety sold, the retail firm needs to acquire a unit of the wholesale good at the competitive nominal price  $P_t^w$  to produce it. Retailers can change nominal prices every period but face a [Rotemberg \(1982\)](#) quadratic adjustment cost  $s_p(P_t(z), P_{t-1}(z))$  given by:

$$s_p(P_t(z), P_{t-1}(z)) = \frac{\varphi_p}{2} \left( \frac{P_t(z)}{P_{t-1}(z)} - 1 \right)^2, \quad \forall z \in [0, 1], \quad (31)$$

<sup>20</sup>Per-period (static) profits of the wholesale producers are expressed in nominal terms as  $P_t^w Y_t^w - R_t^w K_t - W_t H_t - W_t^e H_t^e$ . Wholesale producers make zero profits in equilibrium given the constant-returns-to-scale-technology that they use.

<sup>21</sup>Retailers' operating under imperfect competition with [Rotemberg \(1982\)](#)-style quadratic costs to nominal price adjustment introduce a time-varying mark-up in retail prices and inertia in price-setting in a tractable manner.

where  $\varphi_p \geq 0$  measures the degree of the price adjustment cost.<sup>22</sup> These costs increase in magnitude with the size of the price change and are proportional to the overall scale of economic activity given by the bundle of varieties  $Y_t$ .<sup>23</sup> All retailers face the same optimization problem and choose the same price  $P_t(z)$ , and thus, a symmetric equilibrium emerges where  $P_t(z) = P_t$  and  $Y_t(z) = Y_t$ .

By market clearing, the demand of the wholesale good from all retailers has to be equal to the total production of the wholesale firms—i.e.,  $Y_t = Y_t^w$ . Hence, we can rewrite the optimal pricing equation from the retailers' problem simply as:

$$[1 - \varphi_p (\Pi_t - 1) \Pi_t] + \varphi_p \beta \mathbb{E}_t \left[ \left( \frac{\Lambda_{t+1}}{\Lambda_t} \right) \left( (\Pi_{t+1} - 1) \Pi_{t+1} \frac{Y_{t+1}}{Y_t} \right) \right] = (1 - P_t^{wr}) \epsilon, \quad (32)$$

where  $\Pi_t \equiv \frac{P_t}{P_{t-1}}$  is the gross inflation rate, and  $P_t^{wr} \equiv \frac{P_t^w}{P_t}$  is the real marginal cost. The inverse of  $P_t^{wr}$  can be interpreted as the gross markup of each retail good over the wholesale goods.

Equilibrium in the final goods market means the production of the final good  $Y_t$  in each period is allocated to the consumption of households  $C_t$ , to investment by capital goods producers  $X_t$ , and to cover the costs associated with adjusting nominal prices in the retail sector and the costs that originate from monitoring and enforcing the optimal loan contract described earlier. This gives rise to the aggregate per-period resource constraint for final output:

$$Y_t = C_t + X_t + \underbrace{\frac{\varphi_p}{2} (\Pi_t - 1)^2 Y_t}_{\text{Loss from price adjustment costs}} + \underbrace{\mu G(\bar{\omega}_{t-1}, \sigma_{\omega, t-1}) \frac{R_t^e}{\Pi_t} Q_{t-1} K_t}_{\text{Loss from monitoring costs}}. \quad (33)$$

## 2.4 Monetary Authority

In our model, the monetary authority sets the short-term nominal interest rate,  $I_t$ , according to a standard [Taylor \(1993\)](#) monetary policy rule with inertia,

$$\frac{I_t}{I} = \left( \frac{I_{t-1}}{I} \right)^{\rho_i} \left( \left( \frac{\Pi_t}{\Pi_t^*} \right)^{\psi_\pi} \left( \frac{Y_t}{Y_{t-1}} \right)^{\psi_x} \right)^{1-\rho_i} e^{m_t - m}, \quad (34)$$

where  $I$  is the steady-state nominal interest rate,  $\Pi_t \equiv \frac{P_t}{P_{t-1}}$  is the gross inflation rate,  $\Pi_t^* = 1$  denotes the target gross inflation rate (which we set to imply zero net inflation), and  $\frac{Y_t}{Y_{t-1}}$  is the gross final output growth. The parameters  $\psi_\pi > 1$  and  $\psi_x > 0$  determine the sensitivity of the policy instrument's response to deviations of inflation from target and output growth, respectively.  $\rho_i \in [0, 1)$  sets the inertia for monetary policy.

With  $\rho_m \in (0, 1)$ , the stochastic process for the monetary policy shock,  $m_t$ , can be written as:

$$m_t = \mu_{m,t} + \rho_m (m_{t-1} - \mu_{m,t-1}) + \sigma_{m,t} \varepsilon_{m,t}. \quad (35)$$

<sup>22</sup>For the problem to be well-defined, we need to ensure that  $\frac{\varphi_p}{2} (\pi_t - 1)^2 < 1$ , i.e.,  $\pi_t \in \left( 1 - \sqrt{\frac{2}{\varphi_p}}, 1 + \sqrt{\frac{2}{\varphi_p}} \right)$ .

<sup>23</sup>[Bernanke et al. \(1999\)](#) uses the [Calvo \(1983\)](#) model to introduce price stickiness instead of the [Rotemberg \(1982\)](#) model. The two models are observationally-equivalent whenever approximated up to a first-order and around a zero inflation steady state. [Ascari and Sbordone \(2014\)](#) provide a more in-depth review of the differences that emerge between both price-setting models within the New Keynesian framework.

The stochastic volatility,  $\sigma_{m,t} \equiv \sigma_m e^{\hat{\sigma}_{m,t}}$ , is used to capture the source of policy uncertainty. Similarly to the micro- and macro-uncertainty shocks, it evolves according to an AR(1) process:

$$\hat{\sigma}_{m,t} = v_m \hat{\sigma}_{m,t-1} + \eta_m u_{m,t}, \quad (36)$$

with  $0 < v_m < 1$  and  $\eta_m \geq 0$ . The first moment policy shock  $\varepsilon_{m,t}$  and the second moment policy volatility shock  $u_{m,t}$  are i.i.d.  $N(0, 1)$  and uncorrelated. The time-varying conditional mean of the monetary policy shock,  $\mu_{m,t}$ , is adjusted to follow  $\mu_{m,t} = -\frac{\sigma_{m,t}^2}{2} + \rho_m^2 \mu_{m,t-1}$  to achieve a mean-preserving monetary policy shock. Given  $\sigma_m > 0$ , the unconditional mean of the process  $m$  can then be expressed as  $m \equiv -\frac{1}{2} \frac{\sigma_m^2}{1-\rho_m^2}$ .

### 3 Inspecting the Mechanism

Most medium-scale DSGE models such as [Christiano et al. \(2005\)](#) and [Smets and Wouters \(2007\)](#) abstract from capital market imperfections. In turn, models with financial frictions such as [Bernanke et al. \(1999\)](#) or, more recently, [Christiano et al. \(2010\)](#) and [Christiano et al. \(2014\)](#) highlight the importance of the financial accelerator's *adverse feedback loop* mechanism for the transmission of monetary policy for the propagation of shocks—but largely ignore the role of uncertainty over the business cycle and, more specifically, of uncertainty about monetary policy. We aim to explore the financial accelerator mechanism precisely as it relates to the transmission of uncertainty shocks.

For that, it might be useful to start clarifying the key aspects of the mechanism in a toy version of the model we use in the paper. Under perfect information and costless contract enforcement, the entrepreneur operates if  $\mathbb{E}_t [R_{t+1}^e] \geq I_t$  where the nominal rate  $I_t$  gives us the opportunity costs of the loanable funds obtained from the households (through financial intermediation). If  $\mathbb{E}_t [R_{t+1}^e] > I_t$ , then the entrepreneur's demand for funds would be infinite. Hence, competitive market forces will imply that  $\mathbb{E}_t [R_{t+1}^e] = I_t$ . In other words, under perfect information, asset markets would be complete and the Modigliani-Miller theorem would hold: real investment decisions in that case are independent of the financial structure in the model and, to be more precise, they are independent of whether entrepreneurs are equity or debt financed.

In the context of the [Bernanke et al. \(1999\)](#) model, private information and limited liability are the key assumptions to break away from complete asset markets and from the implications of the Modigliani-Miller theorem on the indeterminacy of the financial structure of entrepreneurs. Private information implies that only entrepreneurs can costlessly observe returns, while lenders must pay a fixed fraction of the realized return interpreted as a monitoring cost. Limited liability on the part of the entrepreneurs, in turn, introduces a lower bound (of zero) on the minimum payoff that the entrepreneurs can achieve. As a result, we end up with the following modified efficiency condition to determine the optimal choice of capital in the model as implied by (14). Combining this with the participation constraint for lenders, we obtain that

$$\frac{P_t Q_t K_{t+1}}{N_t} = \frac{1}{1 - \mathbb{E}_t \left( \frac{R_{t+1}^e}{I_t} \right) \left( \frac{\Psi(\bar{\omega}_t, \sigma_{\omega,t}) - f(\bar{\omega}_t, \sigma_{\omega,t})}{\lambda(\bar{\omega}_t, \sigma_{\omega,t})} \right)}, \quad (37)$$

where the default threshold  $\bar{\omega}_t$  is increasing in the external risk premium,  $\mathbb{E}_t \left( \frac{R_{t+1}^e}{I_t} \right)$ , based on the first-order conditions from the optimal loan contract for the threshold itself and capital (described in more detail in the [Appendix](#)).

From here it follows that

$$\frac{P_t Q_t K_{t+1}}{N_t} = \phi \left( \mathbb{E}_t \left( \frac{R_{t+1}^e}{I_t} \right), \sigma_{\omega,t} \right), \quad (38)$$

with  $\frac{\partial \phi \left( \mathbb{E}_t \left( \frac{R_{t+1}^e}{I_t} \right), \sigma_{\omega,t} \right)}{\partial \mathbb{E}_t \left( \frac{R_{t+1}^e}{I_t} \right)} > 0$ . The capital demand expressed in nominal terms can be inferred from this relationship as follows:  $P_t Q_t K_{t+1} = \phi \left( \mathbb{E}_t \left( \frac{R_{t+1}^e}{I_t} \right), \sigma_{\omega,t} \right) N_t$ , where  $\phi \left( \mathbb{E}_t \left( \frac{R_{t+1}^e}{I_t} \right), \sigma_{\omega,t} \right)$  is the optimal leverage (asset-equity ratio). Here, the optimal leverage does not depend on firm-specific factors, and this implies that we can aggregate the capital demands across entrepreneurs and think of this as an aggregate relationship where  $\frac{N_t}{P_t}$  is the aggregate real net worth and  $Q_t K_{t+1}$  is the aggregate capital demand. Our model incorporates Tobin's  $q$ ,  $Q_t$ , as an important asset-pricing channel in the determination of the demand for capital apart from the optimal leverage itself.

Inverting this relationship appropriately, we obtain the efficiency condition in (14), i.e.,

$$\mathbb{E}_t [R_{t+1}^e] = s \left( \frac{P_t Q_t K_{t+1}}{N_t}, \sigma_{\omega,t} \right) I_t, \quad (39)$$

which relates the yields to the strength of the aggregate balance sheet of the firms where  $s \left( \frac{P_t Q_t K_{t+1}}{N_t}, \sigma_{\omega,t} \right)$  is the gross interest rate spread. In equilibrium, the spread will be inversely related to the aggregate balance sheet strength of the entrepreneurs—but also to the micro-uncertainty  $\sigma_{\omega,t}$ . We can view  $\sigma_{\omega,t}$  as a measure of the dispersion of the idiosyncratic shock  $\omega_t$  and, accordingly, consider the consequences of a mean-preserving increase in the risk spread. Under some conditions, we find that the optimal leverage satisfies that

$$\frac{\partial \phi \left( \mathbb{E}_t \left( \frac{R_{t+1}^e}{I_t} \right), \sigma_{\omega,t} \right)}{\partial \sigma_{\omega,t}} < 0. \quad (40)$$

In other words, increasing the idiosyncratic risk reduces capital demand by *tightening the margins* and reducing the optimal leverage ratio required.

This is the heart of the mechanism that we explore quantitatively in the remainder of this paper and the heart of our paper's contribution. [Christiano et al. \(2014\)](#), among others, have recognized the role that *risk shocks* or shocks to the spread can play in accounting for business cycles. Our theory builds on the existing general equilibrium models with financial market imperfections to provide a rationale for (financial) *risk shocks* based on the idea of uncertainty about idiosyncratic shocks—that cannot be fully insured against. Other forms of aggregate uncertainty—including monetary policy uncertainty, in particular—would only have an additional impact through capital demand on investment and, therefore, over the business cycle to the extent that they feed through asset values (Tobin's  $q$ ) and their effects on them, or to the extent that they influence the decisions of households through a real options effect (from fixed costs or partial irreversibility), or through a precautionary savings effect (from risk aversion). Our theory and our subsequent quantitative work suggest that asset market incompleteness is critical not just to pin down the financial structure of entrepreneurs but also to explain why idiosyncratic shocks would matter.

Moreover, our paper also makes another important point that—to our knowledge—has not yet received much attention in the literature. What our inspection of the mechanism reveals is that the strength of the balance sheet effects operating through the optimal leverage depends inversely on the uncertainty attached



to the idiosyncratic shocks. Therefore, and this is what is significant, the endogenous propagation of other shocks through the investment channel described here can be potentially different depending on whether idiosyncratic shocks are more or less uncertain. In other words, the financial accelerator’s propagation of shocks is conditional on the amount of microeconomic uncertainty. We find that whenever micro-uncertainty  $\sigma_{\omega,t}$  is high, the optimal leverage is low and—accordingly—the risk spreads are low as well. In that environment, even conventional monetary policy shocks (as well as any other level shocks) would have a more muted impact on the capital demand resulting in smaller amplification effects on investment and economic activity arising from the financial accelerator channel. However, this also implies a lower level of capital. That is to say, we find that greater uncertainty about the idiosyncratic shocks would result in precautionary lower levels of the leverage but would also result in less capital accumulation. We therefore highlight the importance of uncertainty (micro-uncertainty in particular) as a shifter in the propagation dynamics for other first-moment and even second-moment shocks.

## 4 Solving the Model

The traditional log-linearizing approach is not suitable here because, with a first-order approximation of the solution, stochastic volatility would not enter into the decision rules in an interesting way.<sup>24</sup> In order to obtain an independent effect of the stochastic volatility terms, we use a third-order perturbation solution to approximate the general rational expectations solution (see [Schmitt-Grohé and Uribe \(2004\)](#) and [Aruoba et al. \(2006\)](#)).

### 4.1 Pruned Third-Order Approximation

The set of equilibrium conditions that characterizes the solution of the model can be written in a compact way as

$$\mathbb{E}_t f(y_{t+1}, y_t, x_{t+1}, x_t, \epsilon_{t+1}, \epsilon_t) = 0, \quad (41)$$

where  $\mathbb{E}_t$  denotes the mathematical expectations operator conditional on information available at time  $t$ ,  $y_t$  is a vector of  $n_y$  control variables,  $x_t$  is a vector of  $n_x$  state variables, and the vector  $\epsilon_t$  contains all  $n_\epsilon$  structural shocks. The solution to the system given in equation (41) can be written in terms of the following two equations:

$$y_t = g(x_t, \sigma), \quad (42)$$

$$x_{t+1} = h(x_t, \sigma) + \sigma \eta \epsilon_{t+1}, \quad (43)$$

where  $\eta$  is an  $n_x \times n_\epsilon$  variance-covariance matrix of structural shocks and  $\sigma$  is the perturbation parameter that scales the variance-covariance matrix.<sup>25</sup> We are seeking a higher-order approximation to functions  $g(\cdot)$  and  $h(\cdot)$  around the deterministic steady state where  $x_t = x$  and  $\sigma = 0$ . To be more precise, we seek a third-order approximation. As in [Fernández-Villaverde \(2010\)](#), [Fernández-Villaverde et al. \(2010\)](#), [Fernández-Villaverde et al. \(2011\)](#), we note that the first, second, and third partial derivatives of  $g(\cdot)$  and

<sup>24</sup>It is worth noting that when considering mean-preserving-spread uncertainty shocks, the time-varying variance shows up in the first-order approximation as changes in the mean.

<sup>25</sup>The additive linear structure of the shocks is not restrictive as we augment the state vector to include the shock terms.

$h(\cdot)$  with respect to the components of  $x_t$  as well as the perturbation parameter  $\sigma$  are used to compute the third-order approximation.

Following [Andreasen et al. \(2013\)](#), we examine a pruned third-order approximation that eliminates approximation terms of a higher order than three from the impulse responses and other dynamic analysis as these higher order terms can introduce dynamic instability. From [Andreasen et al. \(2013\)](#), the pruned third-order approximation has the form:

$$y_t^{rd} = g_x \left( x_t^f + x_t^s + x_t^{rd} \right) + \frac{1}{2} G_{xx} \left( \left( x_t^f \otimes x_t^f \right) + 2 \left( x_t^f \otimes x_t^s \right) \right) + \frac{1}{6} G_{xxx} \left( x_t^f \otimes x_t^f \otimes x_t^f \right) + \dots \quad (44)$$

$$\frac{1}{2} g_{\sigma\sigma} \sigma^2 + \frac{3}{6} g_{\sigma\sigma x} \sigma^2 x_t^f + \frac{1}{6} g_{\sigma\sigma\sigma} \sigma^3$$

$$x_{t+1}^f = h_x x_t^f + \sigma \eta \epsilon_{t+1}, \quad (45)$$

$$x_{t+1}^s = h_x x_t^s + \frac{1}{2} H_{xx} \left( x_t^f \otimes x_t^f \right) + \frac{1}{2} h_{\sigma\sigma} \sigma^2, \quad (46)$$

$$x_{t+1}^{rd} = h_x x_t^{rd} + H_{xx} \left( x_t^f \otimes x_t^s \right) + \frac{1}{6} H_{xxx} \left( x_t^f \otimes x_t^f \otimes x_t^f \right) + \frac{3}{6} h_{\sigma\sigma x} \sigma^2 x_t^f + \frac{1}{6} h_{\sigma\sigma\sigma} \sigma^3, \quad (47)$$

where  $y_t^{rd}$  are the pruned third-order approximations for the control variables,  $x_t^f$  are state variables based on the first-order approximation,  $x_t^s$  are the additional second-order approximation terms for state variables, and  $x_t^{rd}$  are the additional third-order terms for the state variables. Note that if the first order approximation is stationary, then so are the pruned second-order and third-order approximations.

The first-order derivatives are:  $g_x$  ( $n_y \times n_x$  matrix) and  $h_x$  ( $n_x \times n_x$  matrix). The second-order derivatives are:  $G_{xx}$  ( $n_y \times n_x^2$  matrix),  $H_{xx}$  ( $n_x \times n_x^2$  matrix),  $g_{\sigma\sigma}$  ( $n_y \times 1$  vector), and  $h_{\sigma\sigma}$  ( $n_x \times 1$  vector). The third-order derivatives are:  $G_{xxx}$  ( $n_y \times n_x^3$  matrix),  $H_{xxx}$  ( $n_x \times n_x^3$  matrix),  $g_{\sigma\sigma x}$  ( $n_y \times n_x$  matrix),  $h_{\sigma\sigma x}$  ( $n_y \times n_x$  matrix),  $g_{\sigma\sigma\sigma}$  ( $n_y \times 1$  vector), and  $h_{\sigma\sigma\sigma}$  ( $n_x \times 1$  vector). We use Dynare to find the second- and third-order perturbation solutions and extract the matrices relevant for the pruned third-order approximation.<sup>26</sup>

## 4.2 Fitting the Model to the Data

[Table 1](#) summarizes the model parameters used in our simulations. Since we adopt the set-up of the financial accelerator model, our model parameterization is roughly similar to the existing literature based on the seminal work of [Bernanke et al. \(1999\)](#), except where otherwise noted. We take the estimates of the aggregate productivity (TFP) and the monetary policy stochastic volatility parameters from [Born and Pfeifer \(2014\)](#) who obtain them directly from U.S. data on TFP and the short-term interest rates.

We set the values of nine key parameters ( $\kappa$ ,  $b$ ,  $\varphi_k$ ,  $\varphi_p$ ,  $\gamma$ ,  $\mu$ ,  $\sigma_\omega$ ,  $v_\omega$ , and  $\eta_\omega$ ) so as to match simulated moments from the model to values that are consistent with key empirical regularities found in U.S. data and our model parameter normalizations. [Table 2](#) lists these moments, their data sources, and their empirical values. The parameter values for  $\kappa$ ,  $b$ ,  $\varphi_k$ ,  $\varphi_p$ ,  $\gamma$ ,  $\mu$ ,  $\sigma_\omega$ ,  $v_\omega$ , and  $\eta_\omega$  are chosen to minimize the squared distance between the key moments implied by the model and those of the data. The moments were:

1. the mean of the spread ( $400 \times \ln \left( \frac{R_{t+1}^L}{R_{t+1}^E} \right)$ ),
2. the mean of the leverage ratio ( $100 \times \frac{N_t}{P_t Q_t K_{t+1}}$ ),

<sup>26</sup>Note that the stochastic volatilities in our model would be included in the state vector  $x_t$  and are not directly related to the perturbation parameter  $\sigma$ .

3. the mean default probability ( $100 \times \Phi_t^{default}$ ),
4. the mean hours ( $400 \times \ln(H_t)$ ),
5. the variance of the spread,
6. the ratio of the variance of investment ( $400 \times \ln(X_t)$ ) to the variance of output ( $400 \times \ln(Y_t)$ ),
7. the first-order autocorrelation of the spread,
8. the first-order autocorrelation of nondurable consumption ( $400 \times \ln(C_t)$ ),
9. the first-order autocorrelation of inflation ( $400 \times \ln(\Pi_t)$ ).

For the mean default probability ( $100 \times \Phi_t^{default}$ ) and mean hours ( $400 \times \ln(H_t)$ ), no sample data was used; the *target* moments were normalized to 0.75 (Carlstrom and Fuerst (1997) and Bernanke et al. (1999)) and zero, respectively.

Specifically, we

$$\min_{\kappa, b, \varphi_k, \varphi_p, \gamma, \mu, \sigma_\omega, \nu_\omega, \eta_\omega} M' M$$

where

$$M = \begin{pmatrix} \frac{\sum_{t=1}^T \widehat{spread}_t}{T} \\ \frac{\sum_{t=1}^T \widehat{leverage}_t}{T} \\ 0.75 - \mathbb{E}_{\text{model}}(\Phi^{default}) \\ \mathbb{E}_{\text{model}}(\ln(H)) \\ \sum_{t=1}^T \left[ \frac{\widehat{spread}_t^2 - \text{VAR}_{\text{model}}(\ln(\frac{R^L}{R^e}))}{T} \right] \\ \sum_{t=1}^T \left[ \frac{\widehat{spread}_t \times \widehat{spread}_{t-1} - \rho_{\text{model}}(\ln(\frac{R^L}{R^e})) \times \widehat{spread}_t^2}{T} \right] \\ \sum_{t=1}^T \left[ \frac{\widehat{\ln(X_t)}^2 - \frac{\text{VAR}_{\text{model}}(\ln(X)) \times \widehat{\ln(Y_t)}^2}{\text{VAR}_{\text{model}}(\ln(Y))}}{T} \right] \\ \sum_{t=1}^T \left[ \frac{\widehat{\ln(\Pi_t)} \times \widehat{\ln(\Pi_{t-1})} - \rho_{\text{model}}(\ln(\Pi)) \times \widehat{\ln(\Pi_t)}^2}{T} \right] \\ \sum_{t=1}^T \left[ \frac{\widehat{sprdev}_t \times \widehat{sprdev}_{t-1} - \rho_{\text{model}}(\ln(\frac{R^L}{R^e})) \times (\widehat{sprdev}_t)^2}{T} \right] \end{pmatrix},$$

and  $\widehat{spread}_t = \left[ \ln\left(\frac{R_{t+1}^L}{R_{t+1}^e}\right) - \mathbb{E}_{\text{model}}\left(\ln\left(\frac{R^L}{R^e}\right)\right) \right]$ ,  $\widehat{leverage}_t = \left[ \frac{N_t}{P_t Q_t K_{t+1}} - \mathbb{E}_{\text{model}}\left(\frac{N}{P Q K}\right) \right]$ ,  $\widehat{\ln(X_t)} = \ln(X_t) - \mu_{\ln X}$ ,  $\widehat{\ln(Y_t)} = \ln(Y_t) - \mu_{\ln Y}$ ,  $\widehat{\ln(C_t)} = \ln(C_t) - \mu_{\ln C}$ , and  $\widehat{\ln(\Pi_t)} = \ln(\Pi_t) - \mu_{\ln \Pi}$  with  $\mu_Z$  being the sample mean of the variable  $Z_t$ .  $\mathbb{E}_{\text{model}}(\cdot)$  is the analytical unconditional mean implied by the pruned third-order approximation while  $\text{VAR}_{\text{model}}(\cdot)$  and  $\rho_{\text{model}}(\cdot)$  are the simulated unconditional variances and first-order autocorrelations implied as the pruned third-order approximation, respectively.

Simulated variances and autocorrelations are based on 20,000 simulated values of the model. For each parameter value evaluated, the same random number seed was used to generate the simulated samples. Note that our problem is essentially a simulated method of moments estimation except that for two of the moment conditions (the default probability ( $\Phi_t^{default}$ ) and mean hours ( $\ln(H_t)$ )) there is no sampling variation.<sup>27</sup>

<sup>27</sup>In practice, we add a tiny bit of sampling noise to these two moment conditions for computational convenience so that we use these moments along with the rest of the moment conditions in the same computer subroutine.

It turns out that our estimated parameters for  $\gamma$ ,  $\mu$ , and  $\sigma_\omega$  are quite similar to those in [Bernanke et al. \(1999\)](#) while the parameters for  $v_\omega$  and  $\eta_\omega$  are not very far from those used by [Christiano et al. \(2014\)](#). Similarly, the parameters for  $b$ ,  $\varphi_k$ , and  $\varphi_p$  are well within the ranges typically seen in the literature.<sup>28</sup> Furthermore, we find that:

- Rather than take the values for  $\gamma$ ,  $\mu$ , and  $\sigma_\omega$  from [Bernanke et al. \(1999\)](#), we estimate these parameters that are key to determining the terms of the financial contract together with the parameters governing the time-varying dynamics of the dispersion in entrepreneur productivity ( $v_\omega$  and  $\eta_\omega$ ). In our exercise, the dispersion of the idiosyncratic shock is time-varying unlike in the [Bernanke et al. \(1999\)](#) framework, making it all the more relevant that we pin down these parameters to be consistent with the features observed in the data. In this task, we note that the means implied by the model for the risk spread ( $\ln\left(\frac{R_{t+1}^L}{R_{t+1}^E}\right)$ ), the leverage ratio ( $\frac{N_t}{P_t Q_t K_{t+1}}$ ), and the default probability ( $\Phi_t^{default}$ ) as well as the variance and first-order autocorrelation of the risk spread ( $\widehat{spread}_t$ ) are largely responsible for determining the values of  $\gamma$ ,  $\mu$ ,  $\sigma_\omega$ ,  $v_\omega$ , and  $\eta_\omega$ .
- We use the variance of investment ( $X_t$ ) relative to the variance of output ( $Y_t$ ) as well as the autocorrelations for inflation ( $\Pi_t$ ) and consumption ( $C_t$ ) to help determine the values of  $b$ ,  $\varphi_k$ , and  $\varphi_p$ . In this sense, we aim to select a parameter  $\varphi_p$  that would result in nominal rigidities whose impact on inflation is not too persistent—as suggested by the data.
- Finally, one observes that the value of  $\kappa$  is largely determined by normalizing average log household hours ( $H_t$ ) to zero.

## 5 Main Quantitative Findings

### 5.1 Summary of Business Cycle Statistics

[Table 3](#), [Table 4](#), and [Table 5](#) display some standard statistics to assess the business cycle properties implied by the benchmark model as well as various modelling alternatives in which we shut down different features of the benchmark model each time. In particular, we examine alternative specifications where all stochastic variances are shut down, where each stochastic volatility is shut down singly, where financial frictions are shut down, where nominal price rigidities are shut down, and the case where risk aversion (intertemporal elasticity of substitution) is set to be high (low) ( $\chi = 7$ ).

[Table 3](#) displays the variance of output and the variances of alternative macro variables relative to output for the data as well as for the different alternative model specifications that we consider. [Table 4](#) shows the autocorrelation of key variables for various model specifications. [Table 5](#) displays the correlations of key variables with output and the risk spread for the benchmark as well as the competing model specifications. Note that most of these variances and covariances were not used to estimate the model parameters.

Comparing the benchmark model ( $M1$ ) with the model without stochastic volatilities ( $M2$ ) in [Table 3](#), one observes that the stochastic volatilities are an important contributor to the overall variance of output implied by the model (contributing to nearly 40 percent of the variance of output). Of the various sources

<sup>28</sup>References for  $b$  include [Christiano et al. \(2005\)](#), [Christiano et al. \(2010\)](#), and [Christiano et al. \(2014\)](#); for  $\varphi_k$  references include [Bernanke et al. \(1999\)](#), [Christiano et al. \(2010\)](#), and [Justiniano et al. \(2011\)](#); and for  $\varphi_p$  references include [Carlstrom et al. \(2009\)](#) and [Ascari and Sbordone \(2014\)](#).

of time-varying uncertainty that are incorporated in our benchmark model, it seems that monetary policy uncertainty (*M5*) has the largest impact on output volatility. In terms of relative variability, shutting down the stochastic volatilities does not have a dramatic impact on the variances of other variables of interest relative to the variance of output. We interpret this as suggesting that monetary policy uncertainty—but more generally all forms of uncertainty including micro-uncertainty and TFP uncertainty—have a significant contribution to overall volatility but only a modest effect on the relative volatility observed across the key macro variables of interest.

While shutting down financial frictions (*M6*) does not have a large impact on output variance, it does lower (counterfactually) the relative variance of leverage and the risk spread. A model with high risk aversion (and low intertemporal elasticity of substitution) (*M8*) dramatically lowers the relative variance of consumption to output. In turn, we find that introducing stochastic volatility and even the presence of financial frictions and nominal rigidities has little effect on the autocorrelation implications of the model (see [Table 4](#)), with the exception being that inflation persistence is substantially lower when there are no nominal frictions. We argue, therefore, that persistence among the variables of interest is largely unaffected by key features of the model such as the stochastic volatilities or the presence of frictions (financial and in the form of nominal rigidities). We find that these same features and frictions play a major role in driving the overall macro volatility of the model with financial frictions in particular opening an important conduit for propagation through the funding of capital.

From [Table 5](#), one observes that the model gets many of the cross-correlations of output with other real macro variables largely right (the most notable exception being the correlation between output and the real wage). Comparing columns *M1* through *M5* suggests that including stochastic volatility does not change the correlations of output with other real macro variables all that much, and it does not change the cross-correlations of output and the risk spread in a substantial way either.

We observe that the standard agency-cost model without stochastic volatility of [Bernanke et al. \(1999\)](#) results in procyclical movements in the external finance premium (and default probabilities).<sup>29</sup> That is, endogenous movements in credit spreads are not strongly countercyclical. Including time-varying micro-uncertainty results in a more plausible countercyclical external finance premium, but even so the model correlations are smaller (in absolute value) than those found in the data. This suggests that the standard model with credit frictions, even one that includes time-varying micro-uncertainty, misses a substantial amount of the interaction between the risk spread and economic activity seen in the data.

## 5.2 Impulse Response Analysis

To assess the model’s implications for how the economy responds to shocks in the exogenous driving processes, we would like to conduct something similar to the standard impulse response analysis typically done with linear models. Given the nonlinear nature of the model solution that we pursue here, we use instead the generalized impulse response approach of [Koop et al. \(1996\)](#) and calculate how the conditional expectation

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<sup>29</sup>The [Bernanke et al. \(1999\)](#) framework implies that asset price movements serve to reinforce financial imperfections. It is the absence of this mechanism that causes [Gomes et al. \(2003\)](#) to favor this model over the [Carlstrom and Fuerst \(1997\)](#) framework. [Faia and Monacelli \(2007\)](#) provide additional theoretical analysis on the implications of the [Bernanke et al. \(1999\)](#) and [Carlstrom and Fuerst \(1997\)](#) frameworks on the countercyclicality of the external finance premium.

of the endogenous variables would change as a result of a shock. Specifically, we examine:

$$IRF(k, \epsilon_t, x_{t-1}) = \mathbb{E} [y_{t+k}^{rd} | \epsilon_t, x_{t-1}] - \mathbb{E} [y_{t+k}^{rd} | x_{t-1}], \quad (48)$$

where we use the pruned third-order approximation so the state vector can be expressed as  $x_{t-1} = (x_{t-1}^f, x_{t-1}^s, x_{t-1}^{rd})'$ .<sup>30</sup> Note that unlike standard impulse responses for a linear (or first-order approximation) model, the impulse responses in (48) do not, in general, scale up with the size of the shock,  $\epsilon_t$ . Furthermore, the impulse responses depend on the initial condition of the state variables. Given the large number of state variables that we often handle in a medium-size DSGE model, there would be in fact many possible initial conditions to evaluate.

Typically, researchers will take the initial condition to be a particular realization, say the deterministic steady state or the unconditional mean of the state variables. Unfortunately, while these responses are relatively easy to compute, one is not sure how to evaluate how likely it is that the economy would be at that particular initial state. Our approach is to use the information not only in the unconditional mean, but also in the unconditional distribution implied by the model to help choose initial conditions for the impulse response analysis that are economically relevant for our analysis.<sup>31</sup>

### 5.2.1 Unconditional (or Average) Impulse Responses

As part of our analysis, we would like to get a sense of the *average* effect of shocks over all possible initial conditions. In other words, we are interested in the expected (or average) impulse response given the unconditional distribution implied by the model, i.e.,

$$IRF_{average}(k, \epsilon_t) = \int IRF(k, \epsilon_t, x) p(x) dx, \quad (49)$$

where  $p(x)$  is the unconditional density of the state vector  $x$  implied by the model,  $k$  is the endogenous variable of interest, and  $\epsilon_t$  refers to the shock innovations that we are evaluating. To obtain an estimate of the unconditional distribution of the endogenous variables implied by the model, we simulate the pruned model, starting at the unconditional mean for 300 time periods. We do this 20,000 times to get an estimate of the unconditional distribution. We then draw a sample of initial conditions (500 draws), calculate the change in conditional expectations for each initial condition, and then average the responses.<sup>32</sup>

Figure 1 displays the average impulse responses for the various shocks in the model. In general, shocks that affect the conditional means of TFP and monetary policy directly (TFP and interest rate shocks) have substantially larger effects than shocks that affect their volatilities. In the benchmark model, on average, shocks to aggregate uncertainty (whether to TFP or monetary policy), are truly not of first-order importance. Aggregate uncertainty shocks have some precautionary saving effects (causing consumption to fall, hours and

<sup>30</sup>We use the analytical solution for the impulse response functions based on [Andreasen et al. \(2013\)](#) to calculate the change in conditional expectations.

<sup>31</sup>[Balke et al. \(2017\)](#) provide a very detailed exploration of the model under the most relevant initial conditions. We also report multiple other results aimed at exploring the potential asymmetries of the model and the effects of the size of the shocks on the propagation that complement the results included in the paper. That detailed exploration cannot be included in the current draft due to length constraints, naturally.

<sup>32</sup>[Balke et al. \(2017\)](#) includes a figure that compares the average (over the unconditional distribution of initial conditions) impulse responses with the impulse responses with an initial condition equal to the unconditional means. The two sets of responses are qualitatively similar, but the *average* response to interest rate shocks is larger in magnitude than the corresponding response starting at the unconditional mean.

output to rise) but these effects are relatively small.

The effects of aggregate uncertainty shocks on the risk spread as well as on the price of capital and nominal interest rate are also negligible. This result seems to be consistent with the literature. In both [Born and Pfeifer \(2014\)](#) and [Fernández-Villaverde et al. \(2011\)](#), most of the individual stochastic volatility shocks had small effects. In fact, when examining the importance of stochastic volatility shocks, both papers display the response to a simultaneous two standard deviation shock to all sources of uncertainty aiming to represent events of heightened uncertainty (albeit events that occur very infrequently).<sup>33</sup>

In contrast, in our baseline model, micro-uncertainty shocks have sizeable first-order effects. A mean-preserving spread shock to the distribution of entrepreneur productivity has effects of similar magnitude as shocks to TFP and monetary policy. The effect of a micro-uncertainty shock is strongly negative on output, investment, hours, and the price of capital while strongly positive on the risk spread.

In the agency-cost model that underpins the financial accelerator in our model, credit markets are incomplete and idiosyncratic technology shocks cannot be fully insured due to information asymmetries between borrowers and lenders. Hence, greater micro-uncertainty results in a higher required external finance premium because it makes lending to entrepreneurs more risky and leads to a higher default probability as well. The high cost of borrowing discourages investment, pushes up the price of capital, and encourages entrepreneurs to free up more internal funds. In response to the falling investment, households increase consumption and cut down labor input. As a result, output shrinks.

To see how different features of the model affect the *unconditional* impulse responses, [Figure 2](#) plots the average impulse responses of output, the spread, and the interest rate for alternative models. We observe that:

- In one of our experiments, we increase the curvature of the utility function by increasing  $\chi$  from one to seven in the baseline model, making households both more risk averse and less willing to substitute consumption (net of habits) over time. This increases the responses of output, spreads, and interest rates to TFP uncertainty and monetary policy uncertainty shocks. This is perhaps not too surprising as greater risk aversion increases the precautionary saving in the face of an increase in uncertainty—hence, the greater increase in output. Nonetheless, the effects of aggregate uncertainty shocks (related to TFP or monetary policy) are still rather small. For other first-order moment shocks, higher values of  $\chi$  typically dampen the effects of the shocks.
- If prices were perfectly flexible ( $\varphi_p = 0$ ), the effect of the aggregate uncertainty shocks, already small in the benchmark model, virtually disappears. This suggests that price stickiness does magnify the effects of aggregate uncertainty shocks. In contrast, price flexibility lessens the effect of micro-uncertainty shocks while increasing the effect of TFP shocks on output.
- Comparing the baseline model with a model where the financial frictions are not present ( $\mu = 0$ ), we see that the presence of financial frictions does not qualitatively change the average response of the variables to first-order moment shocks. The stochastic volatility shocks related to TFP and monetary policy appear to have modest effects anyway, and the removal of the financial accelerator channel for propagation does not appear to alter this finding of our model all that much. The lone exception in

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<sup>33</sup>Below, we also consider two standard deviation shocks and show that they essentially scale up the effects of the one standard deviation shocks.

all of this is, of course, the micro-uncertainty shocks which have no effects when there are no financial frictions.

- Recall that the micro-uncertainty shocks represent a mean-preserving increase in the dispersion of the distribution of entrepreneurial idiosyncratic productivity. Although economic agents cannot insure themselves against aggregate shocks (systematic risk), a complete asset market would allow for perfect risk-sharing and therefore full insurance against idiosyncratic technology shocks. The no-financial-friction model approximates such a complete asset markets allocation by making monitoring costless and, hence, only uninsurable aggregate shocks (such as TFP uncertainty and monetary policy uncertainty) have an effect. Insurable idiosyncratic technology shocks (such as micro-uncertainty) do not.
- As for the other stochastic volatility shocks, the responses are larger in some cases when the financial frictions are present (TFP uncertainty), but slightly smaller in other cases (TFP shocks, monetary policy shocks, and monetary uncertainty shocks).

As noted above, in general, shocks in nonlinear models do not scale up as in the linear case nor do they imply symmetric responses to positive and negative shocks. [Figure 3](#) displays the response of output to  $\pm$  one and two standard deviation shocks. While the pruned third-order approximation allows for asymmetry, for the baseline model considered here, on average, the responses are close to being symmetric; the responses to negative and positive shocks are virtually mirror images of one another. Furthermore, the responses to two standard deviation shocks appear to be quite similar to scaled up versions of the responses to one standard deviation shocks. Note that these results are obtained from the *unconditional* or average IRFs. This suggests that for the baseline model, on average, the response of output to shocks behaves symmetrically, similar to how we observe it in the linear model.<sup>34</sup>

### 5.2.2 Conditional Impulse Responses

Thus far, we have examined the *average* effect of shocks on key variables in our model. However, as pointed out above, in a nonlinear model the effect of shocks depends on the initial state of the economy at the time of the shock. It is interesting to discern how and to what extent the response to a shock differs depending on the state of the economy. For example, do shocks have different effects when credit frictions are large as opposed to when they are small?<sup>35</sup> To get at this notion of conditional responses, we consider impulse responses defined by:

$$IRF_{y=y_0}(k, \epsilon_t) = \int IRF(k, \epsilon_t, x) p(x|y = y_0) dx, \quad (50)$$

where  $p(x|y = y_0)$  is the conditional density of the vector of states  $x$  implied by the model when variable  $y$  is initially at  $y_0$ . That is, given that a variable  $y$  is initially at  $y_0$ , we average over the possible states consistent with this initial condition.

As we are interested in the interaction of credit conditions and shocks, we first consider impulse response analysis in which the initial state corresponds to states where the level of the risk spread is either high or

<sup>34</sup>This is not to say that a first-order approximation will yield the same quantitative responses as the pruned third-order approximation. For instance, in the first-order case, the response of output to macro uncertainty shocks (related to TFP or monetary policy) is essentially zero regardless of the degree of risk aversion we are willing to assume.

<sup>35</sup>Balke (2000) in the context of a threshold vector autoregression examines whether the effects of shocks depend on current credit conditions.



low. With credit frictions in the [Bernanke et al. \(1999\)](#) model, the expected costs of monitoring defaulting entrepreneurs are priced in the spreads that lenders charge on their loans. In this sense, spreads reflect the extent to which credit frictions and entrepreneur risk are present and, thus, instances where the spread is high are thought to coincide with instances in which credit frictions are large. Specifically, we take *high spread* initial states to be states where the spread is roughly equal to the 95<sup>th</sup> percentile of its unconditional distribution and *low spread* initial states to be states where the spread is at the 5<sup>th</sup> percentile.<sup>36</sup>

Our key results can be summarized as follows:

- [Figure 4](#) displays the response of output to the five structural shocks conditional on the spread being either high (approximately 95<sup>th</sup> percentile) or low (approximately 5<sup>th</sup> percentile). The effect of TFP shocks turns out to be slightly smaller if the spread is initially high. That is, if credit frictions are already large, the expansionary effect of a TFP shock is muted somewhat. For a shock to monetary policy, the opposite is true. If the spread is initially high, the effect of a contractionary monetary shock is larger. Similarly, the contractionary effect on output of an increase in micro-uncertainty when the spread is initially high is about 40% larger than when the spread is initially low. This suggests that if current credit conditions are poor (high spreads), then the effect of supply-side shocks (TFP) tends to get dampened while the effect of financing shocks (monetary policy and micro-uncertainty) gets magnified. In turn, the effect of both TFP uncertainty and monetary policy uncertainty shocks does not depend on the current state of credit conditions—the responses are virtually identical and very small regardless of whether the spread was initially high or low.
- [Figure 5](#) displays the response of the spread itself to various shocks conditional on whether the spread was initially high or low. The positive response of the spread to TFP, monetary policy, and micro-uncertainty shocks is even larger when the spread is already high. This is particularly the case for micro-uncertainty shocks—the effect on the risk spread of a positive micro-uncertainty shock is nearly twice as large when the spread is initially high as opposed to when the spread is initially small.

What leads to high or low levels of the risk spread? The unconditional impulse responses displayed in [Figure 1](#) suggest that high or low spreads are likely the result of cumulative shocks to micro-uncertainty. To get a more precise sense of which shocks contribute more to high or low risk spreads, [Figure 6](#) displays scatter plots implied by the unconditional distribution of the baseline model. The variables plotted are the risk spread against the five exogenous state variables: the two measures of aggregate uncertainty ( $\hat{\sigma}_{a,t}$  and  $\hat{\sigma}_{m,t}$ ), the measure of micro-uncertainty ( $\hat{\sigma}_{\omega,t}$ ), the level of TFP ( $a_t$ ), and the monetary shock ( $m_t$ ). As is clear from the scatter plot, there is a strong relationship between micro-uncertainty and the risk spread. The correlation coefficient between  $\hat{\sigma}_{\omega,t}$  and the spread is 0.84. Note that this relationship also appears to be slightly nonlinear. None of the other exogenous state variables appears to have a strong relationship with the risk spread.<sup>37</sup> This tends to confirm that spread fluctuations largely arise from exogenous micro-uncertainty shocks as indicated before and are not necessarily the endogenous response to other exogenous shocks.

We also consider whether impulse responses are conditional on the level of uncertainty itself. For micro-uncertainty, the responses of output to shocks conditional on the level of micro-uncertainty ( $\hat{\sigma}_{\omega,t}$ ) are very

<sup>36</sup>For impulse responses conditional on variable  $y$  being at its  $k$ -th percentile,  $y(k^{th})$ , we average the impulse responses for initial conditions corresponding to realizations from the unconditional distribution where variable  $y \in [y(k - .003)^{th}, y(k + .003)^{th}]$ . Given our 20,000 draws from the unconditional distribution, this amounted to 121 alternative initial conditions.

<sup>37</sup>The correlation coefficients between the spread and  $\{\hat{a}_t, \hat{m}_t, \hat{\sigma}_{a,t}, \hat{\sigma}_{m,t}\}$  are  $\{0.07, 0.00, 0.00, 0.00\}$ , respectively.

similar to the output responses conditional on the spread. Given the relatively strong relationship between realizations of  $\hat{\sigma}_{\omega,t}$  and realizations of the spread discussed previously, this is not too surprising. Figure 7 displays the responses of output to the various shocks conditional on TFP uncertainty and monetary uncertainty as well. The response of output to TFP shocks is dramatically larger when TFP uncertainty rises—primarily because a high level of TFP uncertainty increases the size of TFP shocks. Similarly, a high level of monetary policy uncertainty substantially enhances the effect of monetary policy shocks on output by increasing their size. Aside from these two effects, the response of output to the other shocks does not depend very much on the state of TFP uncertainty or monetary policy uncertainty.

## 6 Discussion

We obtain relatively small effects from fluctuations in aggregate uncertainty (modeled as stochastic volatility in TFP and monetary policy). Aside from implying that macro shocks are on average larger in magnitude when aggregate uncertainty is high, this type of uncertainty is of second-order importance in the baseline model. Increasing risk aversion increases the effects of these uncertainty shocks, but they are still relatively small compared to shocks that affect the level of TFP or monetary policy directly.

Why are the macro- and policy-uncertainty effects relatively small? It is because the primary effect of aggregate uncertainty in our model appears to work through the precautionary saving motive which is generally modest. Even though we have adjustment costs for changing the prices and the capital stock, the *option value of waiting* when making decisions that are costly to undo still doesn't appear to have large quantitative effects.

Another way to think about why uncertainty has relatively small effects is there is simply not a lot of *curvature* in the model. The standard neoclassical medium-size DSGE model—and even the New Keynesian variant that we consider here—is not that far from being log-linear, and the addition of quadratic adjustment costs for price changes and capital does not add much more nonlinear structure. As the time-varying stochastic volatility shows up in the higher order (3<sup>rd</sup> and above) approximation terms, the lack of substantial nonlinearities in the neighborhood of the deterministic steady state means these higher-order terms are relatively small. Perhaps, additional adjustment costs such as the costs of changing labor input might provide greater avenues for uncertainty to matter. Alternatively, adding additional sources of uncertainty might increase the quantitative influence of aggregate uncertainty. However, given the results of Born and Pfeifer (2014), other forms of uncertainty (such as on taxes) are not likely to have large effects either.

In contrast, we find that micro-uncertainty is important both qualitatively and quantitatively in our analysis—we would say of first-order importance. It is not the micro-uncertainty in and of itself that matters, but the fact that this form of uncertainty exacerbates the credit frictions that arise because of asymmetric information. In our benchmark model, taking away the credit frictions kills the financial accelerator channel and removes any role that micro-uncertainty can play on economic activity. If there are other frictions that are related to micro-uncertainty (e.g., firm hiring decisions or firm-specific adjustment costs), then increases in micro-uncertainty could have an additional effect independent of credit frictions.

While micro-uncertainty and credit frictions interact with one another, the interactions between aggregate uncertainty variables and credit frictions are relatively small. The effects of aggregate uncertainty shocks do not depend on the current credit conditions, nor does the effect of a credit friction shock (micro-uncertainty)

depend significantly on current aggregate uncertainty (neither related to TFP nor to monetary policy). In fact, as suggested above, spread fluctuations appear to be driven primarily by exogenous shocks in micro-uncertainty (Figure 6). All the macro (TFP and monetary policy) shocks including to the level and to uncertainty have relatively small effects on the spread.

What accounts for the minimal interaction between credit frictions and the macro shocks? The only way for macroeconomic conditions to affect the spread is through their effect on the leverage ratio ( $\frac{N_t/P_t}{Q_t K_{t+1}}$ ). It turns out that while macro shocks can have a large effect on individual components of the leverage ratio, these affect the numerator and denominator of the leverage ratio roughly in the same proportion (through their effect on the price of capital). As a result, the leverage ratio moves very little which in turn implies a small response by the spread. Consequently, only changes in the exogenous micro-uncertainty have substantial effects on the spread and the extent of credit frictions.

## 7 Concluding remarks

In this paper, we examined the interaction between aggregate uncertainty and credit frictions through the lens of a New Keynesian DSGE model with stochastic volatility and credit frictions arising from asymmetric information (as in Bernanke et al. (1999)). We use a pruned third order approximation to solve the model, calculate various business cycle moments implied by the model, and conduct impulse response analysis. We find that the interaction between aggregate uncertainty and the extent of the credit frictions (as measured by the risk spread) is relatively small. While micro-uncertainty (or equivalently, exogenous credit friction shocks) have *first-order* effects that are of comparable magnitudes as shocks to the level of TFP or monetary policy shocks, macro-uncertainty and policy uncertainty shocks have relatively small effects. Our nonlinear impulse response analysis suggests that the effect of an increase in micro-uncertainty tends to be larger as the existing risk spread is higher (indicating a worsening credit condition). On the other hand, conditioning on the amount of TFP or monetary uncertainty has virtually no qualitative impact on the responses to shocks in the model aside from the fact that shocks tend to be larger when uncertainty is high.

In our model (and in the Bernanke et al. (1999) model as well) the leverage ratio and, hence, the risk spread move very little in response to most shocks. The exception is micro-uncertainty shocks which have a direct effect on the spread independent of the leverage ratio. Perhaps, incorporating longer-term contracts might increase the effect of aggregate uncertainty, particularly if contracts are set in nominal terms. Furthermore, in the model, net worth is largely affected by the price of capital ( $Q_t$ ); how large fluctuations in the price of capital depends, in turn, largely on the adjustment costs of changing capital. Adding a stronger asset price channel might generate greater fluctuations in net worth and larger fluctuations in the risk spread.

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# Appendix

## A Optimal Loan Contract

**On the Aggregate Sharing of Nominal Income from Capital.** At time  $t$ , the nominal income from capital net of borrowing costs at time  $t + 1$  anticipated by the entrepreneurs for each possible state of aggregate risk—where aggregate risk arises from productivity (TFP) shocks, monetary shocks, and from the stochastic volatility on the aggregate productivity (TFP), monetary, and idiosyncratic technology shocks—can be calculated as follows:

$$\begin{aligned} & \int_{\bar{\omega}_t}^{+\infty} [\omega_t R_{t+1}^e P_t Q_t K_{t+1} - R_{t+1}^L L_t] \phi(\omega_t | \sigma_{\omega,t}) d\omega_t \\ &= R_{t+1}^e P_t Q_t K_{t+1} \left[ \int_{\bar{\omega}_t}^{+\infty} (\omega_t - \bar{\omega}_t) \phi(\omega_t | \sigma_{\omega,t}) d\omega_t \right] = R_{t+1}^e P_t Q_t K_{t+1} f(\bar{\omega}_t, \sigma_{\omega,t}), \end{aligned} \quad (51)$$

where

$$f(\bar{\omega}_t, \sigma_{\omega,t}) \equiv \int_{\bar{\omega}_t}^{+\infty} \omega_t \phi(\omega_t | \sigma_{\omega,t}) d\omega_t - \bar{\omega}_t (1 - \Phi(\bar{\omega}_t | \sigma_{\omega,t})). \quad (52)$$

By the law of large numbers,  $f(\bar{\omega}_t, \sigma_{\omega,t})$  can be interpreted as the fraction of the expected nominal income from capital obtained by the entrepreneurs.

Similarly, the nominal income from capital net of monitoring costs at time  $t + 1$  anticipated by the financial intermediaries at each possible state of aggregate risk is equal to:

$$\begin{aligned} & (1 - \mu) \int_0^{\bar{\omega}_t} [\omega_t R_{t+1}^e P_t Q_t K_{t+1}] \phi(\omega_t | \sigma_{\omega,t}) d\omega_t + \int_{\bar{\omega}_t}^{+\infty} [R_{t+1}^L L_t] \phi(\omega_t | \sigma_{\omega,t}) d\omega_t \\ &= R_{t+1}^e P_t Q_t K_{t+1} \left[ (1 - \mu) \int_0^{\bar{\omega}_t} \omega_t \phi(\omega_t | \sigma_{\omega,t}) d\omega_t + \bar{\omega}_t \int_{\bar{\omega}_t}^{+\infty} \phi(\omega_t | \sigma_{\omega,t}) d\omega_t \right] \\ &= R_{t+1}^e P_t Q_t K_{t+1} g(\bar{\omega}_t, \sigma_{\omega,t}), \end{aligned} \quad (53)$$

where

$$g(\bar{\omega}_t, \sigma_{\omega,t}) \equiv (1 - \mu) \int_0^{\bar{\omega}_t} \omega_t \phi(\omega_t | \sigma_{\omega,t}) d\omega_t + \bar{\omega}_t (1 - \Phi(\bar{\omega}_t | \sigma_{\omega,t})). \quad (54)$$

By the law of large numbers,  $g(\bar{\omega}_t, \sigma_{\omega,t})$  can be interpreted as the fraction of the expected nominal capital income that accrues to the financial intermediaries.

Based on these definitions, the sharing rule on nominal income from capital implied by the loan contract satisfies that:

$$f(\bar{\omega}_t, \sigma_{\omega,t}) + g(\bar{\omega}_t, \sigma_{\omega,t}) = 1 - \mu G(\bar{\omega}_t, \sigma_{\omega,t}), \quad (55)$$

where  $\mu G(\bar{\omega}_t, \sigma_{\omega,t})$  characterizes the losses due to monitoring of defaulting entrepreneurs, i.e.,

$$\mu G(\bar{\omega}_t, \sigma_{\omega,t}) \equiv \mu \int_0^{\bar{\omega}_t} \omega_t \phi(\omega_t | \sigma_{\omega,t}) d\omega_t. \quad (56)$$

As can be inferred from equation (55), the nominal income from capital split between entrepreneurs ( $f(\bar{\omega}_t, \sigma_{\omega,t})$ )



and financial intermediaries ( $g(\bar{\omega}_t, \sigma_{\omega,t})$ ) does not add up to one. The fraction,  $\mu G(\bar{\omega}_t, \sigma_{\omega,t})$ , is lost due to defaults and the costs of monitoring them. The income shares for the entrepreneurs and the financial intermediaries as well as the costly monitoring losses depend on the default threshold,  $\bar{\omega}_t$ , but also on the realization of the micro-uncertainty shock,  $\sigma_{\omega,t}$ .

**The Contracting Problem.** With the information available at time  $t$ , the entrepreneurs' expected nominal income from capital net of borrowing costs implied by equation (51) is:

$$P_t Q_t K_{t+1} \mathbb{E}_t [R_{t+1}^e] f(\bar{\omega}_t, \sigma_{\omega,t}). \quad (57)$$

Similarly, financial intermediaries' expected nominal income from capital net of monitoring costs given by equation (53) is:

$$P_t Q_t K_{t+1} \mathbb{E}_t [R_{t+1}^e] g(\bar{\omega}_t, \sigma_{\omega,t}). \quad (58)$$

The formal contracting problem, at time  $t$ , reduces to choosing the quantity of capital,  $K_{t+1}$ , and the default threshold,  $\bar{\omega}_t$ , that maximize the entrepreneurs' expected nominal income from capital net of borrowing costs given by (57) subject to the following participation constraint for the financial intermediaries, i.e.,

$$P_t Q_t K_{t+1} \mathbb{E}_t [R_{t+1}^e] [1 - f(\bar{\omega}_t, \sigma_{\omega,t}) - \mu G(\bar{\omega}_t, \sigma_{\omega,t})] \geq I_t L_t = I_t [P_t Q_t K_{t+1} - N_t], \quad (59)$$

where the left-hand side combines the expected nominal income from capital of the financial intermediaries in (58) with the sharing rule equation in (55), and the equality on the right-hand side follows from the balance sheet equation of the entrepreneurs (i.e., from  $P_t Q_t K_{t+1} = N_t + L_t$ ). All financial intermediaries share equally in the pool of loans to entrepreneurs. If lenders participate in this loan contract, they always supply enough loans,  $L_t$ , as long as they achieve a rate of return on their loan portfolio greater than or equal to the nominal interest rate,  $I_t$ . In other words, we do not explicitly consider here the possibility of credit rationing.

It follows from the first-order condition with respect to  $\bar{\omega}_t$  that:

$$-f_{\omega}(\bar{\omega}_t, \sigma_{\omega,t}) + \lambda(\bar{\omega}_t, \sigma_{\omega,t}) [f_{\omega}(\bar{\omega}_t, \sigma_{\omega,t}) + \mu G_{\omega}(\bar{\omega}_t, \sigma_{\omega,t})] = 0, \quad (60)$$

where  $\lambda(\bar{\omega}_t, \sigma_{\omega,t})$  is the Lagrange multiplier on the financial intermediaries' participation constraint in (59). Here, we define  $f_{\omega}(\bar{\omega}_t, \sigma_{\omega,t}) \equiv \frac{\partial f(\bar{\omega}_t, \sigma_{\omega,t})}{\partial \bar{\omega}_t}$  and  $G_{\omega}(\bar{\omega}_t, \sigma_{\omega,t}) \equiv \frac{\partial G(\bar{\omega}_t, \sigma_{\omega,t})}{\partial \bar{\omega}_t}$ . By virtue of this optimality condition, we say that the shadow cost of enticing the participation of the financial intermediaries in this loan contract is given by:

$$\lambda(\bar{\omega}_t, \sigma_{\omega,t}) = \frac{f_{\omega}(\bar{\omega}_t, \sigma_{\omega,t})}{f_{\omega}(\bar{\omega}_t, \sigma_{\omega,t}) + \mu G_{\omega}(\bar{\omega}_t, \sigma_{\omega,t})}. \quad (61)$$

This, in turn, implies that the participation constraint must always be binding since the Lagrange multiplier is non-zero. The binding participation constraint can be re-written as:

$$\frac{P_t Q_t K_{t+1}}{N_t} \mathbb{E}_t \left( \frac{R_{t+1}^e}{I_t} \right) (1 - f(\bar{\omega}_t, \sigma_{\omega,t}) - \mu G(\bar{\omega}_t, \sigma_{\omega,t})) = \left[ \frac{P_t Q_t K_{t+1}}{N_t} - 1 \right], \quad (62)$$

or, more compactly, as

$$\frac{P_t Q_t K_{t+1}}{N_t} = \frac{1}{1 - \mathbb{E}_t \left( \frac{R_{t+1}^e}{I_t} \right) \left( \frac{\Psi(\bar{\omega}_t, \sigma_{\omega, t}) - f(\bar{\omega}_t, \sigma_{\omega, t})}{\lambda(\bar{\omega}_t, \sigma_{\omega, t})} \right)}, \quad (63)$$

where  $\Psi(\bar{\omega}_t, \sigma_{\omega, t})$  is

$$\begin{aligned} \Psi(\bar{\omega}_t, \sigma_{\omega, t}) &\equiv f(\bar{\omega}_t, \sigma_{\omega, t}) + \lambda(\bar{\omega}_t, \sigma_{\omega, t}) (1 - f(\bar{\omega}_t, \sigma_{\omega, t}) - \mu G(\bar{\omega}_t, \sigma_{\omega, t})) \\ &= f(\bar{\omega}_t, \sigma_{\omega, t}) + \lambda(\bar{\omega}_t, \sigma_{\omega, t}) g(\bar{\omega}_t, \sigma_{\omega, t}), \end{aligned} \quad (64)$$

given the nominal income split rule in (55).

The contracting problem also requires the following first-order condition with respect to capital,  $K_{t+1}$ , to hold:

$$\mathbb{E}_t \left( \frac{R_{t+1}^e}{I_t} \right) \Psi(\bar{\omega}_t, \sigma_{\omega, t}) - \lambda(\bar{\omega}_t, \sigma_{\omega, t}) = 0, \quad (65)$$

where we implicitly are conjecturing that  $\bar{\omega}_t$  is conditioned only on variables known at time  $t$  (to be more precise, we conjecture that  $\bar{\omega}_t$  is conditioned on  $\left( \frac{P_t Q_t K_{t+1}}{N_t}, \sigma_{\omega, t} \right)$ ).<sup>38</sup> Simply re-arranging this expression gives us that,

$$\mathbb{E}_t \left( \frac{R_{t+1}^e}{I_t} \right) = \frac{\lambda(\bar{\omega}_t, \sigma_{\omega, t})}{\Psi(\bar{\omega}_t, \sigma_{\omega, t})}. \quad (66)$$

This optimality condition determines the excess returns per unit of capital above the nominal interest rate that are required to make the loan contract worthwhile to both entrepreneurs and financial intermediaries.

If we combine equations (63) and (66), then it follows that:

$$\begin{aligned} \frac{P_t Q_t K_{t+1}}{N_t} &= \frac{1}{1 - \frac{\lambda(\bar{\omega}_t, \sigma_{\omega, t})}{\Psi(\bar{\omega}_t, \sigma_{\omega, t})} \left( \frac{\Psi(\bar{\omega}_t, \sigma_{\omega, t}) - f(\bar{\omega}_t, \sigma_{\omega, t})}{\lambda(\bar{\omega}_t, \sigma_{\omega, t})} \right)} \\ &= \frac{1}{1 - \left( \frac{\Psi(\bar{\omega}_t, \sigma_{\omega, t}) - f(\bar{\omega}_t, \sigma_{\omega, t})}{\Psi(\bar{\omega}_t, \sigma_{\omega, t})} \right)} = \frac{\Psi(\bar{\omega}_t, \sigma_{\omega, t})}{f(\bar{\omega}_t, \sigma_{\omega, t})}, \end{aligned} \quad (67)$$

which validates our conjecture that the default threshold  $\bar{\omega}_t$  is a function of observables implying that  $\bar{\omega}_t \equiv \bar{\omega} \left( \frac{P_t Q_t K_{t+1}}{N_t}, \sigma_{\omega, t} \right)$ . Given the relationships derived in (66) and (67), a formulation of the external finance premium arises as follows:

$$\mathbb{E}_t \left[ \frac{R_{t+1}^e}{I_t} \right] = s \left( \frac{P_t Q_t K_{t+1}}{N_t}, \sigma_{\omega, t} \right), \quad (68)$$

or simply

$$\mathbb{E}_t [R_{t+1}^e] = s \left( \frac{P_t Q_t K_{t+1}}{N_t}, \sigma_{\omega, t} \right) I_t, \quad (69)$$

given that  $I_t$  is known at time  $t$  and can be taken out of the expectation. This characterization of the external finance premium expands the [Bernanke et al. \(1999\)](#) financial accelerator framework by modelling nominal contracts explicitly and by linking the premium itself to the micro-uncertainty shocks,  $\sigma_{\omega, t}$ .

<sup>38</sup>The micro-uncertainty shock and the assets-to-net-worth ratio are known by all agents at time  $t$  when the loan contract is signed.

**Loan Contract Terms Under Log-Normality.** The idiosyncratic technology shock  $\omega_t$  is log-normally distributed, i.e.,  $\ln(\omega_t) \sim N(\mu_{\omega,t}, \sigma_{\omega,t}^2)$ , where the volatility part is given by  $\sigma_{\omega,t} = \sigma_{\omega} e^{\hat{\sigma}_{\omega,t}}$  and  $\hat{\sigma}_{\omega,t} \equiv \ln \sigma_{\omega,t} - \ln \sigma_{\omega}$ . The density function of  $\omega_t$  is

$$\phi(\omega_t | \mu_{\omega,t}, \sigma_{\omega,t}) = \frac{1}{\omega_t \sigma_{\omega,t} \sqrt{2\pi}} e^{-\frac{(\ln \omega_t - \mu_{\omega,t})^2}{2\sigma_{\omega,t}^2}}, \quad \omega_t > 0, \quad (70)$$

and its cumulative distribution function is

$$\Phi(\omega_t | \mu_{\omega,t}, \sigma_{\omega,t}) = \Pr(\omega_t \leq \omega | \mu_{\omega,t}, \sigma_{\omega,t}^2) = \frac{1}{2} \left[ 1 + \operatorname{erf} \left( \frac{\ln \omega_t - \mu_{\omega,t}}{\sigma_{\omega,t} \sqrt{2}} \right) \right], \quad (71)$$

$$\operatorname{erf} \left( \frac{\ln \omega_t - \mu_{\omega,t}}{\sigma_{\omega,t} \sqrt{2}} \right) \equiv \frac{2}{\sqrt{\pi}} \int_0^{\frac{\ln \omega_t - \mu_{\omega,t}}{\sigma_{\omega,t} \sqrt{2}}} e^{-x^2} dx, \quad (72)$$

where  $\operatorname{erf}(\cdot)$  denotes the Gaussian error function.

Given the log-normal assumption in (60) – (62), we obtain that:<sup>39</sup>

$$\begin{aligned} \int_{\bar{\omega}_t}^{+\infty} \omega_t \phi(\omega_t | \sigma_{\omega,t}) d\omega_t &= \int_{\bar{\omega}_t}^{+\infty} \frac{1}{\sigma_{\omega,t} \sqrt{2\pi}} e^{-\frac{(\ln \omega_t - \mu_{\omega,t})^2}{2\sigma_{\omega,t}^2}} d\omega_t = \left( -\frac{1}{2} e^{\mu_{\omega,t} + \frac{\sigma_{\omega,t}^2}{2}} \operatorname{erf} \left( \frac{\mu_{\omega,t} + \sigma_{\omega,t}^2 - \ln \omega_t}{\sigma_{\omega,t} \sqrt{2}} \right) \right) \Big|_{\bar{\omega}_t}^{+\infty} \\ &= \left( -\frac{1}{2} e^{\mu_{\omega,t} + \frac{\sigma_{\omega,t}^2}{2}} \operatorname{erf}(-\infty) \right) - \left( -\frac{1}{2} e^{\mu_{\omega,t} + \frac{\sigma_{\omega,t}^2}{2}} \operatorname{erf} \left( \frac{\mu_{\omega,t} + \sigma_{\omega,t}^2 - \ln \bar{\omega}_t}{\sigma_{\omega,t} \sqrt{2}} \right) \right) \\ &= \frac{1}{2} e^{\mu_{\omega,t} + \frac{\sigma_{\omega,t}^2}{2}} \left[ 1 + \operatorname{erf} \left( \frac{\mu_{\omega,t} + \sigma_{\omega,t}^2 - \ln \bar{\omega}_t}{\sigma_{\omega,t} \sqrt{2}} \right) \right], \end{aligned} \quad (73)$$

and

$$\Phi(\bar{\omega}_t | \mu_{\omega,t}, \sigma_{\omega,t}) = \frac{1}{2} \left[ 1 + \operatorname{erf} \left( \frac{\ln \bar{\omega}_t - \mu_{\omega,t}}{\sigma_{\omega,t} \sqrt{2}} \right) \right]. \quad (74)$$

Hence, it is possible to characterize the share  $f(\bar{\omega}_t, \sigma_{\omega,t})$  in (52) as:

$$f(\bar{\omega}_t, \sigma_{\omega,t}) = \frac{1}{2} e^{\mu_{\omega,t} + \frac{\sigma_{\omega,t}^2}{2}} \left[ 1 + \operatorname{erf} \left( \frac{\mu_{\omega,t} + \sigma_{\omega,t}^2 - \ln \bar{\omega}_t}{\sigma_{\omega,t} \sqrt{2}} \right) \right] - \bar{\omega}_t (1 - \Phi(\bar{\omega}_t | \sigma_{\omega,t})) \quad (75)$$

in terms of the error function  $\operatorname{erf}(\cdot)$ .

Similarly, given the log-normal assumption in (60) – (62), the function  $G(\bar{\omega}_t, \sigma_{\omega,t})$  can be re-expressed as:

$$\begin{aligned} G(\bar{\omega}_t, \sigma_{\omega,t}) &\equiv \int_0^{\bar{\omega}_t} \omega_t \phi(\omega_t | \sigma_{\omega,t}) d\omega_t \\ &= \int_0^{+\infty} \omega_t \phi(\omega_t | \sigma_{\omega,t}) d\omega_t - \int_{\bar{\omega}_t}^{+\infty} \omega_t \phi(\omega_t | \sigma_{\omega,t}) d\omega_t, \end{aligned} \quad (76)$$

<sup>39</sup>The properties of the error function  $\operatorname{erf}(\cdot)$  imply that  $\lim_{x \rightarrow -\infty} \operatorname{erf}(x) = -1$ .

where  $\mathbb{E}(\omega_t) = \int_0^{+\infty} \omega_t \phi(\omega_t | \sigma_{\omega,t}) d\omega_t = e^{\mu_{\omega,t} + \frac{\sigma_{\omega,t}^2}{2}}$ . Hence, the functional form for  $G(\bar{\omega}_t, \sigma_{\omega,t})$  becomes:

$$\begin{aligned} G(\bar{\omega}_t, \sigma_{\omega,t}) &= 1 - \int_{\bar{\omega}_t}^{+\infty} \omega_t \phi(\omega_t | \sigma_{\omega,t}) d\omega_t = 1 - \int_{\bar{\omega}_t}^{+\infty} \frac{1}{\sigma_{\omega,t} \sqrt{2\pi}} e^{-\frac{(\ln \omega_t - \mu_{\omega,t})^2}{2\sigma_{\omega,t}^2}} d\omega_t \\ &= 1 - \frac{1}{2} e^{\mu_{\omega,t} + \frac{\sigma_{\omega,t}^2}{2}} \left[ 1 + \operatorname{erf} \left( \frac{\mu_{\omega,t} + \sigma_{\omega,t}^2 - \ln \bar{\omega}_t}{\sigma_{\omega,t} \sqrt{2}} \right) \right], \end{aligned} \quad (77)$$

in terms of the error function  $\operatorname{erf}(\cdot)$ . The functional forms for  $f(\bar{\omega}_t, \sigma_{\omega,t})$  and for  $G(\bar{\omega}_t, \sigma_{\omega,t})$  together with equation (55) completely characterize the split of the nominal income from capital between entrepreneurs and financial intermediaries, while also accounting for the losses due to monitoring. Here, we assume a mean-preserving distribution that ensures the unconditional mean is equal to one in every period (i.e.,  $\mathbb{E}(\omega_t) = 1$ ) by setting  $\mu_{\omega,t} = -\frac{\sigma_{\omega,t}^2}{2}$ .

Given the functional form  $f(\bar{\omega}_t, \sigma_{\omega,t})$  derived in (74)–(75), we can compute the derivative of  $f(\bar{\omega}_t, \sigma_{\omega,t})$  with respect to  $\bar{\omega}_t$  as:

$$\begin{aligned} f(\bar{\omega}_t, \sigma_{\omega,t}) &= \frac{1}{2} e^{\mu_{\omega,t} + \frac{\sigma_{\omega,t}^2}{2}} \left[ 1 + \operatorname{erf} \left( \frac{\mu_{\omega,t} + \sigma_{\omega,t}^2 - \ln \bar{\omega}_t}{\sigma_{\omega,t} \sqrt{2}} \right) \right] - \bar{\omega}_t \left( 1 - \frac{1}{2} \left[ 1 + \operatorname{erf} \left( \frac{\ln \bar{\omega}_t - \mu_{\omega,t}}{\sigma_{\omega,t} \sqrt{2}} \right) \right] \right), \\ f_{\omega}(\bar{\omega}_t, \sigma_{\omega,t}) &\equiv \frac{\partial f(\bar{\omega}_t, \sigma_{\omega,t})}{\partial \bar{\omega}_t} = -\frac{1}{2} \left[ 1 + \operatorname{erf} \left( \frac{\mu_{\omega,t} - \ln(\bar{\omega}_t)}{\sigma_{\omega,t} \sqrt{2}} \right) \right]. \end{aligned} \quad (78)$$

Similarly, we can compute the derivative of  $G(\bar{\omega}_t, \sigma_{\omega,t})$  with respect to  $\bar{\omega}_t$  as:

$$\begin{aligned} G(\bar{\omega}_t, \sigma_{\omega,t}) &= 1 - \frac{1}{2} e^{\mu_{\omega,t} + \frac{\sigma_{\omega,t}^2}{2}} \left[ 1 + \operatorname{erf} \left( \frac{\mu_{\omega,t} + \sigma_{\omega,t}^2 - \ln \bar{\omega}_t}{\sigma_{\omega,t} \sqrt{2}} \right) \right], \\ G_{\omega}(\bar{\omega}_t, \sigma_{\omega,t}) &\equiv \frac{\partial G(\bar{\omega}_t, \sigma_{\omega,t})}{\partial \bar{\omega}_t} = 0.398942 \left( \frac{\bar{\omega}_t^{\frac{\mu_{\omega,t}}{\sigma_{\omega,t}^2}}}{\sigma_{\omega,t}} \right) e^{-\left( \frac{(\mu_{\omega,t})^2 + (\ln(\bar{\omega}_t))^2}{2\sigma_{\omega,t}^2} \right)}. \end{aligned} \quad (79)$$

Finally, the Lagrange multiplier  $\lambda(\bar{\omega}_t, \sigma_{\omega,t}) = \frac{f_{\omega}(\bar{\omega}_t, \sigma_{\omega,t})}{f_{\omega}(\bar{\omega}_t, \sigma_{\omega,t}) + \mu G_{\omega}(\bar{\omega}_t, \sigma_{\omega,t})}$  can be expressed as

$$\lambda(\bar{\omega}_t, \sigma_{\omega,t}) = \frac{-\frac{1}{2} \left[ 1 + \operatorname{erf} \left( \frac{\mu_{\omega,t} - \ln(\bar{\omega}_t)}{\sigma_{\omega,t} \sqrt{2}} \right) \right]}{-\frac{1}{2} \left[ 1 + \operatorname{erf} \left( \frac{\mu_{\omega,t} - \ln(\bar{\omega}_t)}{\sigma_{\omega,t} \sqrt{2}} \right) \right] + \mu \left( 0.398942 \left( \frac{\bar{\omega}_t^{\frac{\mu_{\omega,t}}{\sigma_{\omega,t}^2}}}{\sigma_{\omega,t}} \right) e^{-\left( \frac{(\mu_{\omega,t})^2 + (\ln(\bar{\omega}_t))^2}{2\sigma_{\omega,t}^2} \right)} \right)}. \quad (80)$$

## B Mean-Preserving Spreads

In our framework, all shock processes with stochastic volatility—idiosyncratic technology, aggregate productivity (TFP) and monetary policy shocks, i.e.,  $z_t \in \{\ln(\omega_t), a_t, m_t\}$ —can be cast in the following canonical

form:

$$z_t = \mu_{z,t} + \rho_z (z_{t-1} - \mu_{z,t-1}) + \sigma_z e^{\widehat{\sigma}_{z,t}} \varepsilon_{z,t}, \quad (81)$$

$$\mu_{z,t} = -\frac{(\sigma_z e^{\widehat{\sigma}_{z,t}})^2}{2} + (\rho_z)^2 \mu_{z,t-1}, \quad (82)$$

$$\widehat{\sigma}_{z,t} = \nu_z \widehat{\sigma}_{z,t-1} + \eta_z u_{z,t}, \quad (83)$$

where  $\widehat{\sigma}_{z,t} = \ln \sigma_{z,t} - \ln \sigma_z$  and  $\sigma_{z,t} \equiv \sigma_z e^{\widehat{\sigma}_{z,t}}$ . The innovation terms  $\varepsilon_{z,t}$  and  $u_{z,t}$  are i.i.d.  $N(0,1)$  and uncorrelated. Differences between conditional and unconditional moments of the distribution can arise under this canonical form—hence, we must clarify that in this paper the notion of a mean-preserving spread that we adopt is conditional on the history of the volatility shocks.

The shock  $z_t \in \{\ln(\omega_t), a_t, m_t\}$  is specified in logs as a stochastic (Gaussian) process, but appears in the model equilibrium conditions in levels (as  $e^{z_t}$ ). Under the log-normality assumption, an increase in  $\sigma_{z,t}$  increases not only the variance of the shock (the dispersion for  $e^{z_t}$ ) but also the expected mean value of  $e^{z_t}$ . Since we are interested in mean-preserving spreads that arise solely because of shifts in the dispersion of the distribution and not from indirect effects coming through the mean, we introduce a recursive correction given in (82) that reverses the conditional mean-effect of volatility on the time-varying conditional mean of the shock process  $\mu_{z,t}$ .

To show that this recursive correction is conditional mean-preserving, note first that  $z_t$  and  $\mu_{z,t}$  can be expanded backwards as follows:

$$z_t = \mu_{z,t} + \sum_{i=0}^{\infty} (\rho_z)^i \sigma_{z,t-i} \varepsilon_{z,t-i}, \quad (84)$$

$$\mu_{z,t} = -\sum_{i=0}^{\infty} (\rho_z^2)^i \frac{\sigma_{z,t-i}^2}{2}. \quad (85)$$

Hence, when we compute the mean of the process  $e^{z_t}$  conditional on the history of the volatility shocks, we obtain the following expression under the time-varying conditional mean  $(\mu_{z,t})$  recursion given in (82):

$$\begin{aligned} & \mathbb{E}[e^{z_t} \mid \sigma_{z,t-i}, i = 0, \dots, \infty] \\ &= \mathbb{E}\left[ e^{\mu_{z,t} + \sum_{i=0}^{\infty} (\rho_z)^i \sigma_{z,t-i} \varepsilon_{z,t-i}} \mid \sigma_{z,t-i}, i = 0, \dots, \infty \right] \\ &= e^{\left(-\sum_{i=0}^{\infty} (\rho_z^2)^i \frac{\sigma_{z,t-i}^2}{2}\right)} \mathbb{E}\left[ e^{\left(\sum_{i=0}^{\infty} (\rho_z)^i \sigma_{z,t-i} \varepsilon_{z,t-i}\right)} \mid \sigma_{z,t-i}, i = 0, \dots, \infty \right] \\ &= e^{\left(-\sum_{i=0}^{\infty} (\rho_z^2)^i \frac{\sigma_{z,t-i}^2}{2}\right)} e^{\mathbb{E}\left(\sum_{i=0}^{\infty} (\rho_z)^i \sigma_{z,t-i} \varepsilon_{z,t-i} \mid \sigma_{z,t-i}, i=0, \dots, \infty\right) + \frac{1}{2} \mathbb{V}\left(\sum_{i=0}^{\infty} (\rho_z)^i \sigma_{z,t-i} \varepsilon_{z,t-i} \mid \sigma_{z,t-i}, i=0, \dots, \infty\right)} \\ &= e^{\left(-\sum_{i=0}^{\infty} (\rho_z^2)^i \frac{\sigma_{z,t-i}^2}{2}\right)} e^{\left(\sum_{i=0}^{\infty} (\rho_z^2)^i \frac{\sigma_{z,t-i}^2}{2}\right)} = 1, \end{aligned} \quad (86)$$

since  $\varepsilon_{z,t}$  are i.i.d.  $N(0,1)$  innovations. As a result, this shows that the recursive correction proposed in (82) ensures that the conditional mean of the shock  $z_t$  in levels is not affected by a change in the second moment  $\sigma_{z,t}$ . This is the sense in which the specification of the stochastic volatility shocks is said to be mean-preserving in our model.

**The Deterministic Steady State.** The standard way we characterize the deterministic steady state is to: (a) assume that  $\varepsilon_{z,t}$  and  $u_{z,t}$  are replaced by their unconditional means (i.e., replaced by  $\mathbb{E}(\varepsilon_{z,t}) = \mathbb{E}(u_{z,t}) = 0$ ), and (b) drop the time subscript in the corresponding dynamic equations in the canonical form. Hence, we get the following set of equations for the deterministic steady state:

$$z = \mu_z = -\frac{1}{2} \frac{\sigma_z^2}{1 - \rho_z^2}, \quad \widehat{\sigma}_z = 0, \quad \mu_z = -\frac{1}{2} \frac{\sigma_z^2}{1 - \rho_z^2}. \quad (87)$$

This describes the steady state for all shocks  $z$  under our mean-preserving correction which we must derive explicitly to solve the model as it appears in the set of equilibrium conditions that characterizes the solution. This is the case because we introduce aggregate productivity (TFP) and monetary policy shocks in deviations from their respective deterministic steady state values to ensure that the steady state around which we make our approximation is not distorted.

## C Tables and Figures

Table 1. Parameters Used in the Model Simulations

<i>Preference and Technological Parameters</i>	Parameter	Value	Parameterization Source
Households' Intertemporal Discount Factor	$0 < \beta < 1$	0.99	Bernanke et al. (1999)
Households' Inverse of the Intertemporal Elasticity of Substitution	$\chi \geq 0$	1	Bernanke et al. (1999), Christiano et al. (2014)
Households' Inverse of the Frisch Elasticity of Labor Supply	$\xi \geq 0$	$\frac{1}{3}$	Bernanke et al. (1999)
Households' Scaling Parameter on Labor Disutility	$\kappa \geq 0$	0.73752	Simulated Method of Moments
Households' Habit Parameter	$0 \leq b \leq 1$	0.73766	Simulated Method of Moments
Elasticity of Substitution Across Varieties	$\epsilon > 1$	10	Basu (1996)
Capital Share	$0 \leq \alpha \leq 1$	0.35	Bernanke et al. (1999)
Entrepreneurial Labor Share	$0 \leq \theta \leq 1$	0.01	Bernanke et al. (1999)
Depreciation Rate	$0 < \delta \leq 1$	0.025	Bernanke et al. (1999)
<i>Adjustment Cost Parameters</i>			
Capital Adjustment Cost	$\varphi_k > 0$	3.36850	Simulated Method of Moments
Rotemberg (1982) Price Adjustment Cost	$\varphi_p \geq 0$	121.73	Simulated Method of Moments
<i>Agency Cost Parameters</i>			
Monitoring Cost	$0 \leq \mu < 1$	0.14462	Simulated Method of Moments
Survival Rate of Entrepreneurs	$0 < \gamma < 1$	0.97840	Simulated Method of Moments
<i>Taylor Rule Parameters</i>			
Interest Rate Smoothing	$0 \leq \rho_i < 1$	0.836	Born and Pfeifer (2014)
Sensitivity to Inflation Deviations from Target	$\psi_\pi > 1$	1.777	Born and Pfeifer (2014)
Sensitivity to Output Gap	$\psi_x > 0$	0.319	Born and Pfeifer (2014)
<i>Exogenous Shock Parameters</i>			
Unconditional Std. Dev. of Idiosyncratic Risk Shock	$\sigma_\omega > 0$	0.29996	Simulated Method of Moments
Persistence of the Stochastic Volatility of Idiosyncratic Risk Shock	$0 < v_\omega < 1$	0.96568	Simulated Method of Moments
Std. Dev. of the Stochastic Volatility of Idiosyncratic Risk Shock	$\eta_\omega \geq 0$	0.025394	Simulated Method of Moments
TFP Shock Persistence	$0 < \rho_a < 1$	0.814	Born and Pfeifer (2014)
TFP Shock Unconditional Standard Deviation	$\sigma_a > 0$	0.0054	Born and Pfeifer (2014)
Persistence of the Stochastic Volatility on TFP	$0 < v_a < 1$	0.632	Born and Pfeifer (2014)
Std. Dev. of the Stochastic Volatility on TFP	$\eta_a \geq 0$	0.312	Born and Pfeifer (2014)
Monetary Shock Persistence	$0 < \rho_m < 1$	0.367	Born and Pfeifer (2014)
Monetary Shock Unconditional Standard Deviation	$\sigma_m > 0$	0.0014	Born and Pfeifer (2014)
Persistence of the Stochastic Volatility of Monetary Shock	$0 < v_m < 1$	0.921	Born and Pfeifer (2014)
Std. Dev. of the Stochastic Volatility of Monetary Shock	$\eta_m \geq 0$	0.363	Born and Pfeifer (2014)

**Table 2. Moments Used to Set Values of  $\kappa$ ,  $b$ ,  $\varphi_k$ ,  $\varphi_p$ ,  $\gamma$ ,  $\mu$ ,  $\sigma_\omega$ ,  $v_\omega$ , and  $\eta_\omega$**

Variable	Moment	Value	Data Source
1. Mean risk spread	$\mathbb{E} \left( \ln \left( \frac{R_{t+1}^L}{R_{t+1}^c} \right) \right)$	2.29	Annualized spread between commercial paper and T-bill rate
2. Mean leverage rate	$\mathbb{E} \left( \frac{N_t}{P_t Q_t K_{t+1}} \right)$	52.14	(Net worth/Total assets)x100 (Non-financial corporate business)
3. Mean quarterly default probability	$\mathbb{E} \left( \Phi_t^{default} \right)$	0.75	<a href="#">Bernanke et al. (1999)</a>
4. Mean log hours	$\mathbb{E} (\ln (H_t))$	0	Normalization
5. Variance of risk spread	$\text{VAR} \left( \ln \left( \frac{R_{t+1}^L}{R_{t+1}^c} \right) \right)$	0.52	Annualized spread between commercial paper and T-bill rate
6. Var(investment)/Var(output)	$\frac{\text{VAR}(\ln(X_t))}{\text{VAR}(\ln(Y_t))}$	17.85	NIPA fixed investment plus consumer durables
7. Autocorrelation of the spread	$\rho \left( \ln \left( \frac{R_{t+1}^L}{R_{t+1}^c} \right) \right)$	0.90	Annualized spread between commercial paper and T-bill rates
8. Autocorrelation of consumption	$\rho (\ln (C_t))$	0.90	NIPA non-durable consumption
9. Autocorrelation of inflation	$\rho (\ln (\Pi_t))$	0.39	NIPA GDP deflator

Note:  $\mathbb{E}(\cdot)$  denotes unconditional mean,  $\text{VAR}(\cdot)$  denotes unconditional variance, and  $\rho(\cdot)$  denotes the first-order autocorrelation.



**Table 3. Simulated and Empirical Business Cycle Volatilities for Various Models**

	<b>Data</b>	<b>M1</b>	<b>M2</b>	<b>M3</b>	<b>M4</b>	<b>M5</b>	<b>M6</b>	<b>M7</b>	<b>M8</b>
$\sigma(z_t)$ (%)	4.62	4.45	2.57	4.29	4.38	2.95	4.62	2.72	3.69
$y_t$	0.64	0.38	0.44	0.38	0.36	0.44	0.35	0.52	0.09
$x_t$	4.23	4.2	3.77	3.95	4.24	4.31	3.57	4.13	4.98
$h_t$	1.66	1.61	1.66	1.61	1.6	1.66	1.56	0.55	1.79
$w_t - p_t$	0.96	1.54	1.51	1.6	1.56	1.35	1.5	1.19	1.94
$\pi_t$	0.14	0.42	0.42	0.44	0.43	0.38	0.41	1.79	0.49
$i_t$	0.23	0.12	0.12	0.12	0.12	0.12	0.1	0.26	0.17
Leverage	0.58	0.32	0.06	0.05	0.32	0.48	0.01	0.52	0.38
Spread	0.16	0.15	0.02	0.02	0.15	0.23	0	0.25	0.19

Note: The endogenous variables  $z_t$  included are output ( $y_t$ ), household consumption ( $c_t$ ), investment ( $x_t$ ), household's hours worked ( $h_t$ ), real wages ( $w_t - p_t$ ), inflation ( $\pi_t$ ), and nominal interest rates ( $i_t$ ). Lowercase letters denote variables that are logged, expressed in annualized percentages, and filtered with a one-sided HP-filter. Leverage refers to the ratio  $N_t P_t Q_t K_{t+1}$  expressed in logs, annualized, and filtered with a one-sided HP-filter, while Spread refers to the external finance premium  $\lambda_{t+1} / \Psi_{t+1}$  expressed in logs, annualized, and filtered with a one-sided HP-filter. The table shows the standard deviation  $\sigma(z_t)$  and the standard deviation relative to output  $\sigma(z_t) / \sigma(y_t)$ . We report the results for the following variants of the model: M1 = benchmark model, M2 = without all stochastic volatilities, M3 = without micro stochastic volatility only, M4 = without TFP stochastic volatility only, M5 = without monetary stochastic volatility, M6 = without financial frictions, M7 = without nominal rigidities, and M8 = with high risk aversion.

Table 4. Simulated and Empirical Business Cycle Persistence for Various Models

	Data	M1	M2	M3	M4	M5	M6	M7	M8
$\rho(z_t, z_{t-1})$									
$y_t$	0.91	0.63	0.68	0.64	0.62	0.67	0.63	0.73	0.57
$c_t$	0.9	0.9	0.91	0.89	0.89	0.91	0.89	0.91	0.91
$x_t$	0.93	0.58	0.58	0.55	0.58	0.64	0.56	0.69	0.58
$h_t$	0.96	0.56	0.55	0.57	0.56	0.54	0.56	0.54	0.54
$w_t - p_t$	0.73	0.56	0.59	0.56	0.56	0.6	0.56	0.66	0.57
$\pi_t$	0.39	0.52	0.51	0.52	0.52	0.51	0.52	0	0.51
$i_t$	0.94	0.75	0.78	0.75	0.75	0.79	0.76	0.48	0.78
Leverage	0.96	0.99	0.97	1	0.99	0.99	1	0.99	0.99
Spread	0.89	0.88	0.96	0.96	0.88	0.88	0.93	0.88	0.89

Note: The endogenous variables  $z_t$  included are output ( $y_t$ ), household consumption ( $c_t$ ), investment ( $x_t$ ), household's hours worked ( $h_t$ ), real wages ( $w_t - p_t$ ), inflation ( $\pi_t$ ), and nominal interest rates ( $i_t$ ). Lowercase letters denote variables that are logged, expressed in annualized percentages, and filtered with a one-sided HP-filter. Leverage refers to the ratio  $N_t/P_t Q_t K_{t+1}$  expressed in logs, annualized, and filtered with a one-sided HP-filter, while Spread refers to the external finance premium  $\lambda_{t+1}/\Psi_{t+1}$  expressed in logs, annualized, and filtered with a one-sided HP-filter. The table shows the first-order autocorrelation  $\rho(z_t, z_{t-1})$ . We report the results for the following variants of the model: M1 = benchmark model, M2 = without all stochastic volatilities, M3 = without micro stochastic volatility only, M4 = without TFP stochastic volatility only, M5 = without monetary stochastic volatility, M6 = without financial frictions, M7 = without nominal rigidities, and M8 = with high risk aversion.

Table 5. Simulated and Empirical Business Cycle Cyclicity for Various Models

Data	M1	M2	M3	M4	M5	M6	M7	M8
$\rho(z_t, y_t)$								
$y_t$	1	1	1	1	1	1	1	1
$c_t$	0.9	0.7	0.8	0.69	0.67	0.75	0.73	0.49
$x_t$	0.93	0.94	0.95	0.94	0.92	0.97	0.89	0.97
$h_t$	0.88	0.79	0.55	0.83	0.54	0.8	-0.1	0.73
$w_t - p_t$	0.04	0.94	0.96	0.94	0.89	0.97	0.89	0.93
$\pi_t$	0.35	0.83	0.69	0.86	0.62	0.84	-0.12	0.85
$i_t$	0.65	-0.78	-0.94	-0.77	-0.62	-0.93	-0.55	-0.69
Leverage	-0.02	0.06	-0.03	0.06	0.11	-0.06	0.11	0.08
Spread	-0.51	-0.07	0.06	-0.07	-0.12	0.01	-0.11	-0.07
$\rho(z_t, \text{Spread})$								
$y_t$	-0.51	-0.07	0.06	-0.07	-0.12	0	0.01	-0.11
$c_t$	-0.51	-0.04	0.09	-0.05	-0.06	0	0.00	-0.03
$x_t$	-0.42	-0.10	0.03	-0.10	-0.15	0	0.01	-0.16
$h_t$	-0.44	-0.03	0.14	-0.03	-0.05	0	0.01	-0.15
$w_t - p_t$	-0.04	-0.01	0.07	-0.01	-0.04	0	0.01	-0.02
$\pi_t$	-0.18	0.00	0.12	0.00	-0.01	0	0.01	-0.00
$i_t$	-0.53	-0.07	-0.02	-0.07	-0.09	0	-0.00	-0.06
Leverage	0.02	0.18	-0.99	0.18	0.18	0	-0.10	0.19
Spread	1	1	1	1	1	0	1	1

Note: The endogenous variables  $z_t$  included are output ( $y_t$ ), household consumption ( $c_t$ ), investment ( $x_t$ ), household's hours worked ( $h_t$ ), real wages ( $w_t - p_t$ ), inflation ( $\pi_t$ ), and nominal interest rates ( $i_t$ ). Lowercase letters denote variables that are logged, expressed in annualized percentages, and filtered with a one-sided HP-filter. Leverage refers to the ratio  $N_t/P_t Q_t K_{t+1}$  expressed in percentages and filtered with a one-sided HP-filter, while Spread refers to the external finance premium  $\lambda_{t+1}/\Psi_{t+1}$  expressed in logs, annualized, and filtered with a one-sided HP-filter. The table shows the contemporaneous correlation with output  $\rho(z_t, y_t)$ . We report the results for the following variants of the model: M1 = benchmark model, M2 = without all stochastic volatilities, M3 = without micro stochastic volatility only, M4 = without TFP stochastic volatility only, M5 = without monetary stochastic volatility, M6 = without financial frictions, M7 = without nominal rigidities, and M8 = with high risk aversion.

Figure 1. Unconditional impulse responses for the baseline model

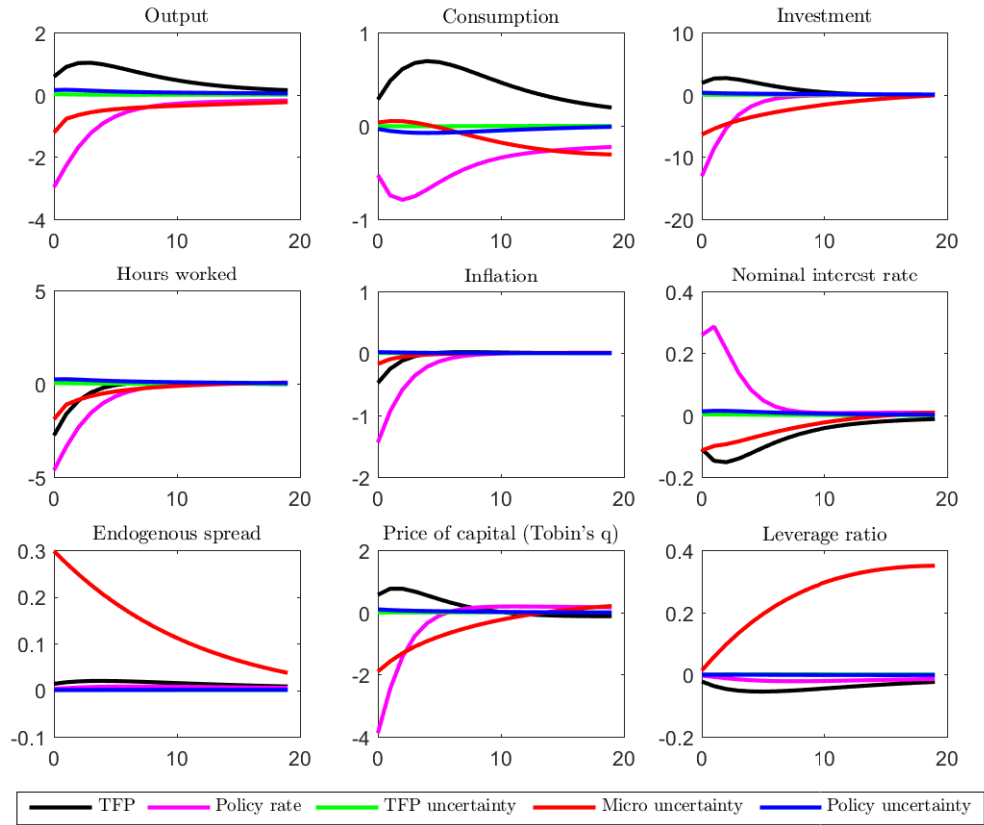
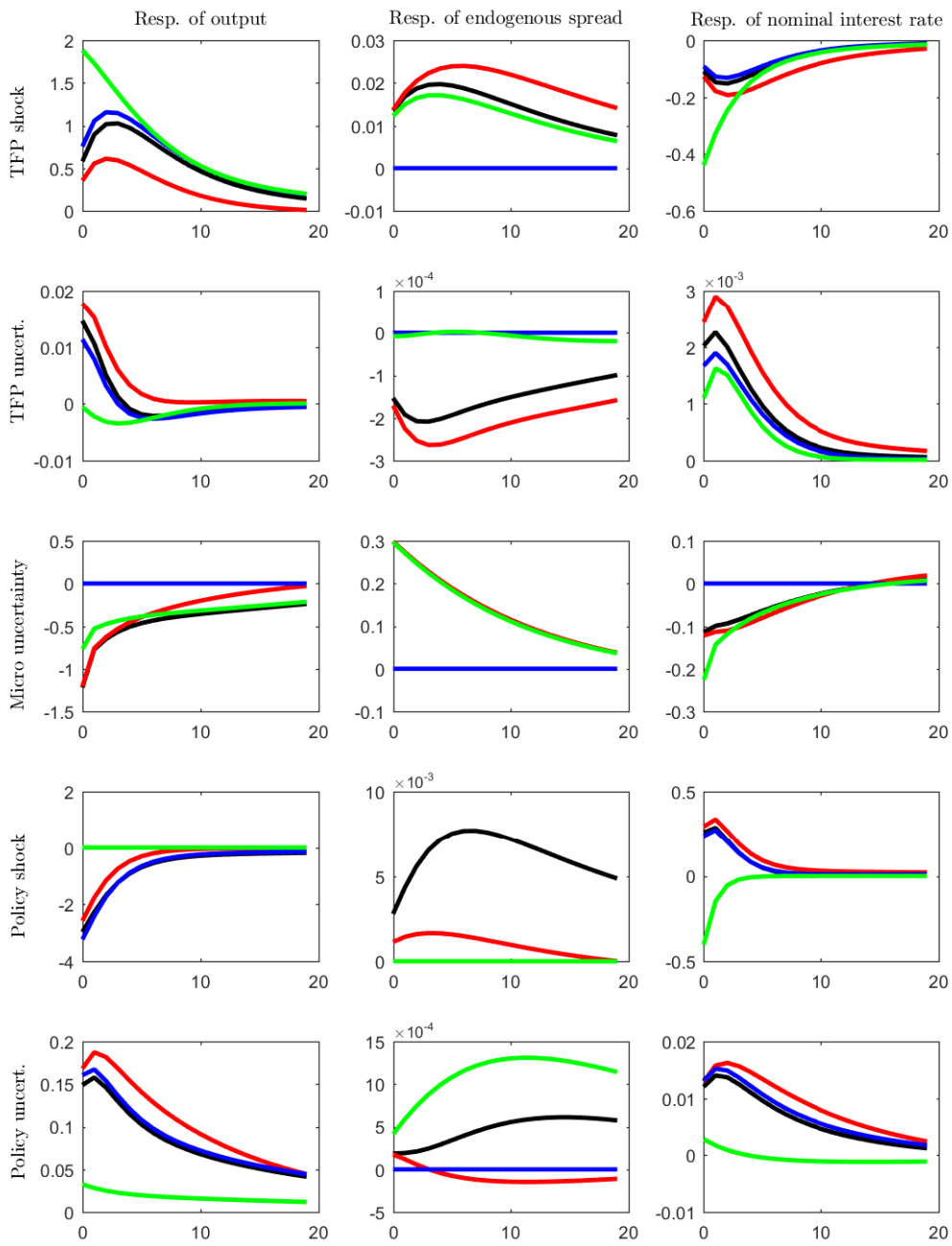


Figure 2. Unconditional impulse responses for alternative models



— Baseline — High risk aversion ( $\chi = 7$ ) — No credit frictions ( $\mu = 0$ ) — Flexible prices ( $\phi_p = 0$ )

Figure 3. Response of output to alternative shock sizes and directions

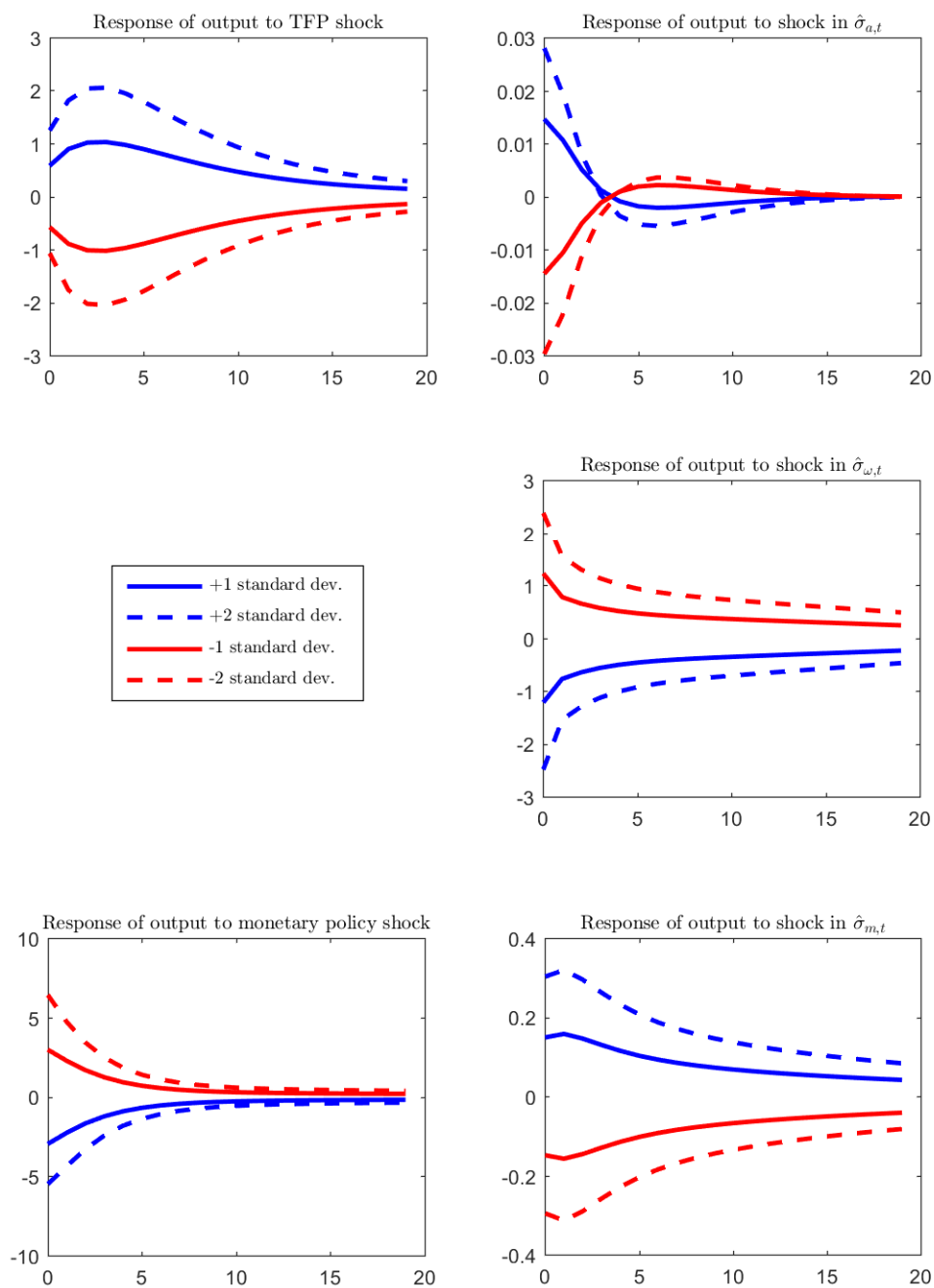


Figure 4. Response of output to shocks conditional on the value of the spread

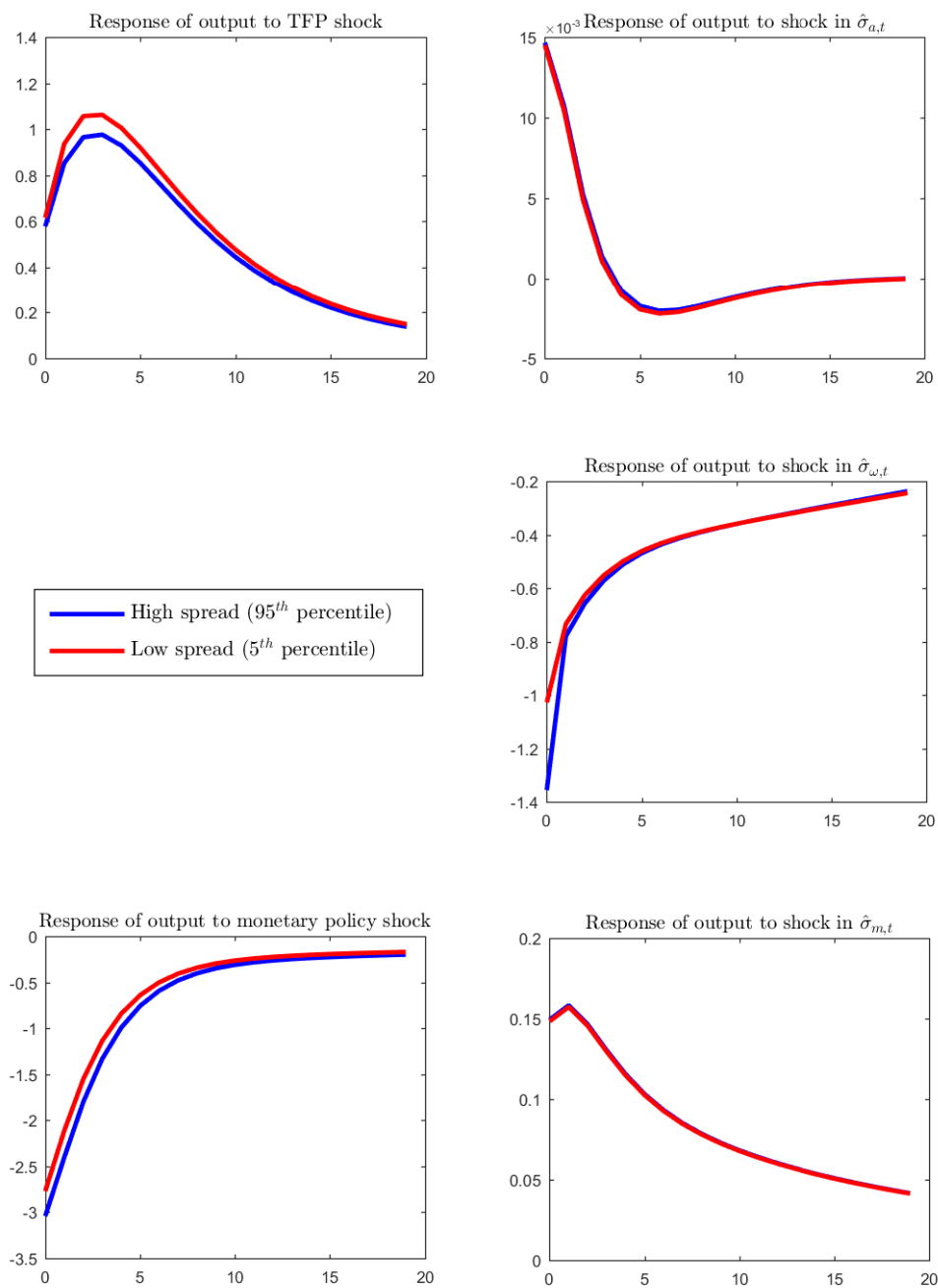


Figure 5. Response of spread to shocks conditional on the value of the spread

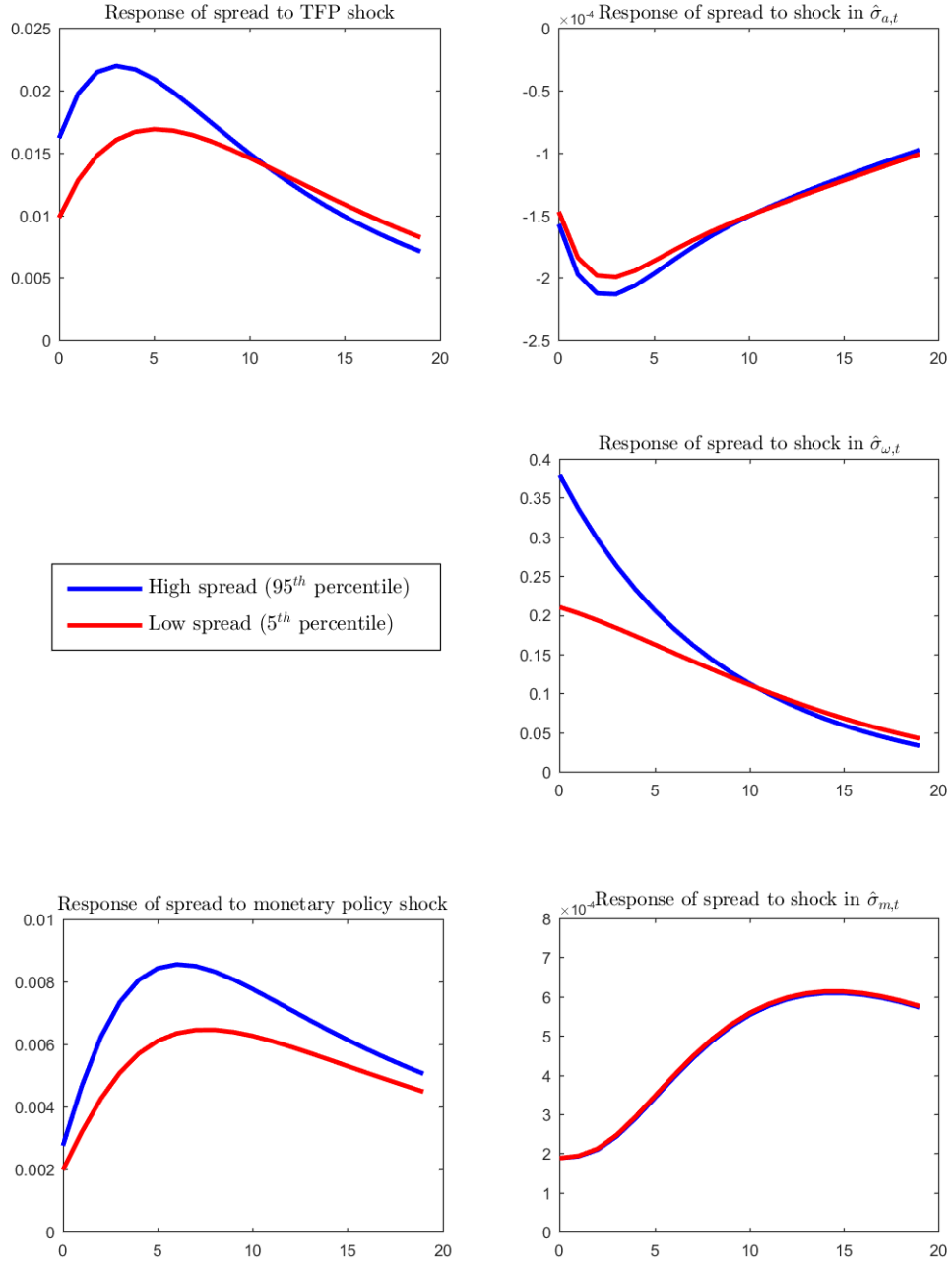




Figure 6. Scatterplot of the unconditional distribution for the baseline model

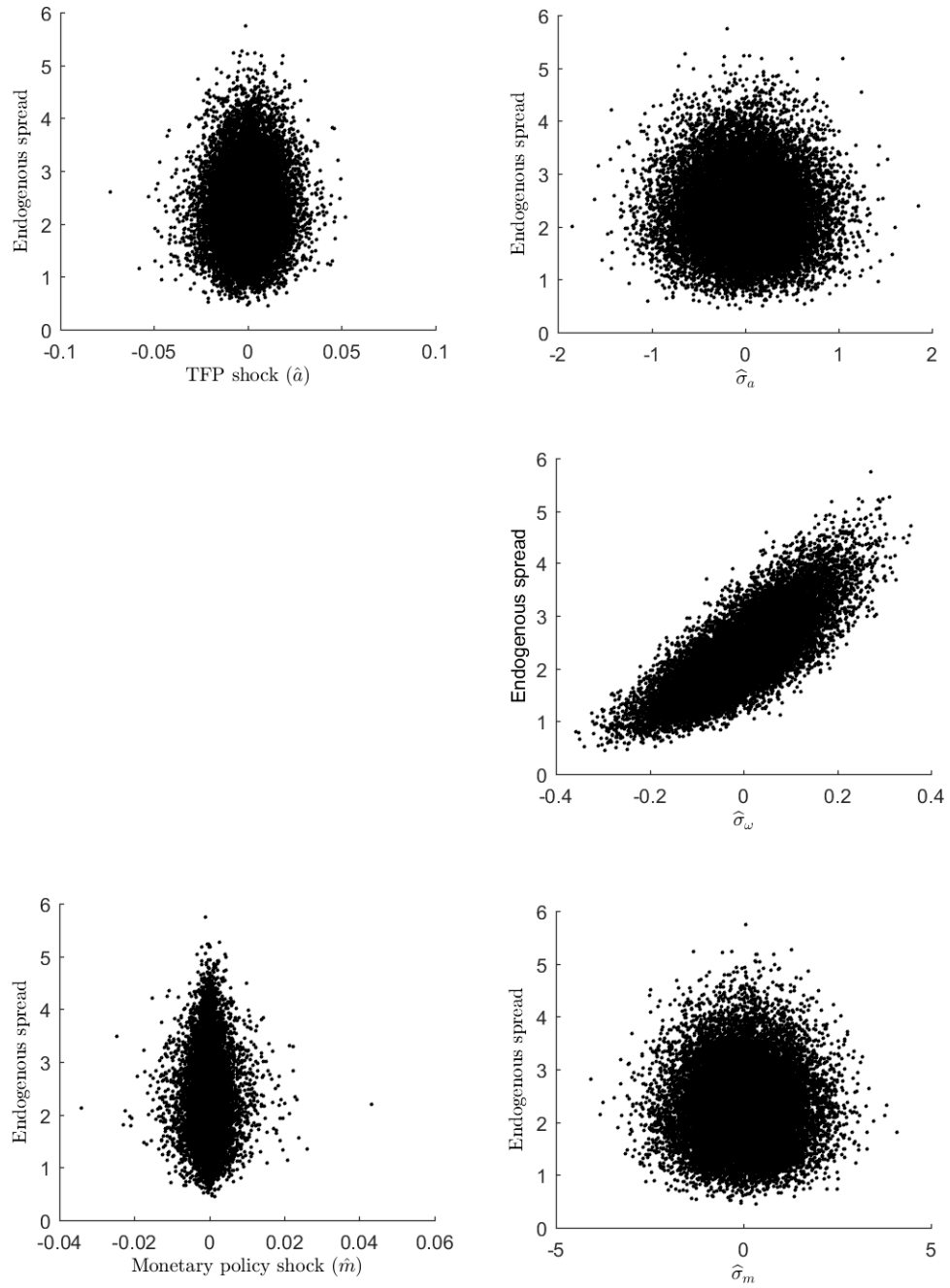


Figure 7. Response of output to shocks conditional on the values of macro- and policy-uncertainty

