Vacancies in Housing and Labor Markets[1]

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Key Words: Housing Vacancies, Job Openings JEL Codes: R23, J64

Abstract

The Great Recession of 2007–2009 has renewed a focus on the link between the housing and business cycles. In this paper, we model the housing and labor markets by means of a DMP model that treats housing and labor supply as joint decisions and highlights the interdependence of vacancies in these markets. We estimate this model at the MSA level using data on housing vacancies from the US Census Bureau's Housing Vacancy Survey (HVS) starting in 1986, on job vacancies from the Conference Board's Help-Wanted Index starting in 1951, and from the BLS's Job Opening and Labor Turnover Survey (JOLTS) since December 2000. In particular, we estimate a Beveridge curve for labor markets that includes spillovers from vacancies in the rental and homeownership housing markets, as well as a novel Beveridge curve for housing markets. We then estimate VAR models for housing and job vacancies.Results from impulse response functions show that shocks to rental vacancies and especially to homeownership vacancies have negative and significant impacts on job vacancies.

[1]We are grateful for comments received during conversations with Allen Head, Rachel Ngai and Chris Pissarides, and during presentations at Tufts, the 2014 Urban Economics Association meetings, Bethesda, MD, the ASSA/AREUEA meetings, Boston, January 3, 2015, HULM, St. Louis, April 10, 2015, and Conference in Honour of Sir Christopher Pissarides, Sciences Po, July 1, 2015. We are especially grateful to Serena Rhee for her discussion in Boston.

Version: **February 19, 2016**

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1 Introduction

The housing and business cycles are clearly tied together and this has become even more apparent since the Great Recession of 2007–2009. Figure 1 shows the national growth rate for real house prices and GDP and the unemployment rate. The house price index tracks quite well with both the real GDP growth rate and the unemployment rate; the correlation is 0.5 and -0.5, respectively. This is an indication that the national business and housing cycles are closely synchronized. In fact, Leamer (2007) has claimed that housing *is* the business cycle. He shows that, at the national level, residential investment is a much better predictor of recessions than aggregate business activity.[2] Bachmann and Cooper (2014) find, as we detail below, that housing turnover is pro-cyclical and leads the business cycle.

Starting with Oswald (1977a; b), the impact of homeownership on the labor market has received a lot of attention. He found that homeownership and unemployment rates are positively related given the relatively higher moving costs for homeowners.[3] However, as Beugnot *et al.* (2014) show theoretically, even if the unemployment and homeownership rates are positively correlated, individuals would be better off in economies where homeownership is promoted, and the costs from higher homeownership rates, if any, are principally associated with mobility costs, which are higher for homeowners. Karahan and Rhee (2014) link declines in house prices to geographical reallocation in the labor market by modeling the down payment requirement for purchasing a home as a financial friction. House price declines reduce homeowners' equity, impeding selling and moving when a local labor market is hit by a shock, thus generating a "house-lock effect" that may cause an increase in local unemployment and thus exacerbate further the local contraction. Their model accounts for 90% of the increase in dispersion of unemployment and the entire decline in net migration. Finally, Mian and Sufi (2014) and Mian, Sufi, and Trebbi (2014) have tied housing wealth to changes in the real economy.

In this paper, we link the housing and labor markets by means of a DMP model that treats housing and labor supply as joint decisions and highlights the interdependence of

[2]Ghent and Owyang (2010) show, however, that this same relationship is not as strong at the MSA level.
[3]See all Coulson and Fisher (2009) among others.

vacancies in these markets.

Vacancy rates in both the housing and labor markets emerge naturally from search models. The vacancy rate is a long-established concept in labor markets and is central to the study of frictions in those markets as established under the DMP framework. Once created, job openings may remain unfilled until suitable workers are found. Vacancies of dwelling units in housing markets are also rigorously grounded in search models (Wheaton 1990; Ngai and Sheedy 2013) and studies of housing market adjustment through search have been long-standing (Ioannides, 1975; Genovese and Han, 2012). Housing units may remain unoccupied until buyers or renters are found. But those two markets have not been studied jointly by means of consistent theoretical and empirical models.

An important feature of the housing market is the coexistence of tenure modes, renting and owning and the accordant households' choice of renting versus owner-occupancy, with both rental and homeownership vacancy rates. Our analysis highlights the interrelationship of the flows between the rental and homeownership markets and between the states of employment and unemployment. An empirical counterpart is Bachmann and Cooper (2014) who use the PSID to document the flows between renting and owning and correlations between these flows and the business cycle. Also, as we detail below, Ioannides and Kan (1996) and Ioannides and Zanella (2008) find a high correlation between moving and job changes.

A great number of recent papers have made the Beveridge curve central to business cycle analyses of labor markets.[4] Our theoretical model motivates an empirical model that includes the counterpart of the Beveridge Curve in the housing market. While vacancies in housing markets naturally correspond to vacancies in the labor market, the concept of unemployment is difficult to translate in the housing market. Our solution is motivated by the two sources of cyclical dependence of housing turnover, namely the work by Bachmann and Cooper (2014) and the evidence on the correlation between residential moves and job changes. We posit that frictions affecting renters generate an unfulfilled demand for owner occupied housing (just as unemployment is the unfulfilled demand for work). That is, some renters would

[4]See Diamond and Şahin (2014) for a discussion of the significance of shifts in the Beveridge curve and Ellsby et al. (2015) for the latest survey of the literature.

rather own, but are rationed out because they cannot get a mortgage for various reasons. A similar concept holds for owners who would rather rent. There are frictions associated with selling a home and moving. We find that the cyclical movement along this housing market Beveridge curve is in the opposite direction as that in the labor market Beveridge curve. We believe that we are the first to develop a housing market counterpart of the Beveridge curve. Our theoretical model of housing and labor market vacancies explicitly captures the interdependence of the two markets via a mechanism that we elaborate on further below. Empirically, vacancies in the housing market can shift the labor market Beveridge curve and vice versa.[5]

Our estimates are based on data obtained from: the US Census Bureau's Housing Vacancy Survey (HVS),[6] the national version of the American Housing Survey (NAHS),[7] the Help-Wanted Index from the Conference Board,[8] and BLS's Opening and Labor Turnover Survey (JOLTS).[9] We estimate models at the national level as well as for 37 US Core-Based Statistical Areas (CBSAs). Annual national-level data on housing vacancies are available starting in 1950 and MSA-level data are measured back as far as 1986. Monthly data on job vacancies begin in 1951. Following Barnichon (2010), we combine the early print index with the recent online index to construct a consistent index of job openings for 1951-2014 at both the national and CBSA level.

At the national level, we show that housing market vacancies do shift the labor market Beveridge Curve. We estimate "unfulfilled" homeownership and rental rates using a housing tenure choice equation that is estimated with multiple waves of the NAHS. We then use these predictions to estimate the housing market versions of Beveridge Curve. Our results

[5] We recognize that according to Bachmann and Cooper, total housing turnover is positively but weakly correlated with and leads the rental vacancy rate, while it is positively but weakly correlated with and lags the owner vacancy rate. However, those calculations are based on HP-filter detrended data. Detrending is of course critical for understanding the cyclical patterns but interdependence in the raw data is of interest in its own right, especially when we allow for geographic detail in the data.

[6] http://www.census.gov/housing/hvs/data/index.html

[7] http://www.census.gov/programs-surveys/ahs/data.html

[8] https://www.conference-board.org/data/

[9] http://www.bls.gov/jlt/data.htm

are consistent with the theoretical prediction that the cyclical movement along this curve is opposite to that of the labor market Beveridge curve.

Finally, using the data at the CBSA level, we estimate a VAR model of housing and labor market vacancies and use the results to calculate impulse response functions to study how shocks to either the housing or labor market will propagate themselves in the other market. The results show that shocks to the owner and rental vacancies have negative and significant impacts on job vacancies. This is consistent with the thinking that during the Great Recession of 2007-2009 the downturn in the housing market led to the subsequent decline in the labor market and the real economy.

The remainder of the paper is organized as follows. Section 2 provides a review of the recent literature that employs search models in the empirical study of housing markets. Section 3 discusses important aspects of the theoretical model, emphasizing those that capture the interdependence of the housing and labor markets with frictions. The full development of the model is relegated to Appendix A (section 10). Section 4 describes the data, section 5 presents the results, and section 6 concludes.

2 Literature Review

There have been a number of papers in the literature that employ search models in the empirical study of housing markets, though very few of them examine both the housing and labor market by means of the full complement of ideas proposed here. Both Head and Lloyd-Ellis (2012; 2014) and Rupert and Wasmer (2012) develop models of joint housing-labor search, which are complementary to one another. Head and Lloyd-Ellis (2012) focus on frictions in the housing market and the role of housing markets in generating frictions between labor markets. They do not, however, allow for frictions originating in the labor market, which they assume to be Walrasian. Our paper extends the model of Head and Lloyd-Ellis, which is therefore discussed in detail below. A notable recent study is Limnios (2014), who explores whether frictions in the rental housing market can help explain frictions in the labor market.

Rupert and Wasmer (2012) develop a theory of the relationship between unemployment and housing market frictions that focuses on the trade-off between commuting time and location decisions within a single labor market. Rupert and Wasmer show that a higher arrival rate of housing opportunities makes workers less choosy about jobs. In a variation of the model, workers receive demographic ("family") shocks, which change the valuation of the current dwelling and allows workers to sample from the existing stock of job vacancies, as opposed to just new vacancies when their jobs break up. Job separations now reflect the possibility that workers may not find an acceptable offer and thus vacancy, and the distribution of commuting distances occupied by workers is suitably adjusted [*ibid.,* Eq. (20)]. In equilibrium, the distribution of workers' commuting distances is a linear combination of the distribution function of new vacancies, weighted by the rate at which new job opportunities arrive, of the distribution function of all vacancies, weighted by the rate at which demographic shocks arrive, and of the distribution of job offers over commuting distances, conditional on their coming from acceptable commuting distances, weighted by the rate of total separations. It is thus clear that labor turnover and frictions, including demographic shocks, have profound effects on individuals' location choices. With job and housing vacancy searches being jointly indexed by commuting distance, the housing search process is subsumed into the job search. The housing market is not modelled, however, and the spatial distribution of new and existing vacancies play the role of housing supply, but demand is not rationed by housing price.

Unlike Rupert and Wasmer, Head and Lloyd-Ellis (2012) do distinguish between homeownership and renting, with Bellman equations being defined separately for employed and unemployed renters and owners and are conditional on two different city types. The housing market is intermediated by real estate firms. Head and Lloyd-Ellis rely on the steady-state equilibrium values of the Bellman equations to establish that the rent differential across the two city types is determined by unemployed renters who are assumed to move costlessly between cities, even if they do not receive a job offer. In contrast, the differential in the value of houses is determined by the marginal (unemployed) home-owner who must first receive an outside offer and then incur the (endogenous) liquidity cost of selling their house.

This result suggests that anchoring the opportunity cost of owning housing calculations on rent differentials must account for basic characteristics of labor turnover across different city types. A key friction modelled by Head and Lloyd-Ellis pertains to the illiquidity of housing for homeownership. Because homeowners accept job offers from other cities at a lower rate than do renters, a link is generated between homeownership and unemployment both at the city level and in the aggregate. Their calibration of the model in order to match aggregate US statistics on mobility, housing, and labor flows predicts that the effect of homeownership on aggregate unemployment is small. When unemployment is high, however, changes in the rate of homeownership can have economically significant effects.

In a sequence of papers, Ngai and Sheedy (2013; 2015) focus on the frictions associated with buying and selling homes. Ngai and Sheedy (2015) emphasize, in particular, the dynamic impact of the fact that the majority of housing purchase transactions involve households moving from one house to another, whereby they put their existing homes on the market and plan to buy new homes. This is motivated by households' desire to improve match quality, and consequently their decisions produce a cleansing effect on the quality distribution. Moving may be triggered by an event, like a demographic shock to a household that causes a reassessment of its housing demand. Ngai and Sheedy (2013) emphasizes sellers' decisions, namely when to put a house up for sale and when to agree to a sale. Ngai and Sheedy do not take a position on the interdependence between residential moves and job changes.

Particularly relevant for our paper are important facts reported by Bachmann and Cooper (2014) from the 1969-2009 waves of the Panel Study of Income Dynamics (PSID). They report evidence on households' propensity to move and tenure choice and how such decisions correlate with aggregate economic activity.[10] For example, 15.3% of households move at

[10] Pissarides (2013) wonders whether the recent housing market crash is an appealing explanation for the Great Recession of 2007–2009. Since homeowners are known to be less mobile than renters, the extraordinary expansion of homeownership in recent years might have contributed to the decline in residential mobility. He argues, however, that there is little evidence of "house-lock effect", namely that falling house prices and the negative equity in many houses are factors behind the fall in mobility. Pissarides argues nonetheless that composition effects due to the shift to more homeownership could still be significant. He speculates that if

each year, with roughly 55% of these moves due to renters moving to new rental dwellings, 20% by owners who change homes, 10% by owners who move to rent and 15% by renters who move to own. Only a small fraction of these moves generated net additions to the stock of owners. Bachmann and Cooper also report that whereas total housing turnover is very weakly contemporaneously correlated with the unemployment rate, it is quite strongly correlated with the aggregate growth rate of GDP (detrended by means of an HP-filter). The correlation of the unemployment rate with the owner-to-owner moving rate is substantial and negative (-0.52), with the renter-to-renter moving rate is substantial and positive (0.51) and with the renter-to-owner moving rate is non-negligible and negative ($-.32$). So moving in order to own is negatively correlated with the unemployment rate and owner-to-owner and renter-to-owner moves are positively correlated with output growth, 0.44 and 0.59, respectively. When including leads and lags, owner-to-owner moves are contemporaneous with the business cycle, renter-to-owner and renter-to-renter moves lead it, and owner-to-renter moves are acyclical. Furthermore, turnover seems to lead house prices, especially renter-to-owner moves, which also lead aggregate economic activity. The authors speculate that households start buying houses because of good news about economic activity and about the housing sector.

Bachmann and Cooper report correlations between the specific categories of housing turnover with vacancy rates. Interestingly, the correlations with the rental vacancy rate are weak but the ones with the owner vacancy rate are much stronger. The data suggest that when there are many owner-to-owner moves, which correlates strongly with higher economic activity, the owner-occupied market has a smaller number of vacancies. When there are many owner-to-renter moves, which correlates with lower economic activity, the owner-occupied market has a larger number of vacancies. Renter-to-owner moves are also negatively correlated with owner-occupied vacancies. Thus, the notion that lots of vacancies in the owner-occupied market lead to more turnover activity terminating in that market are not supported by the data. Instead, the data point to the importance of an underlying factor,

a secular decline in mobility, whatever its origin might be, is to persist, we should expect future recessions in the US to definitely impact the labor market Beveridge curve.

aggregate economic activity, that prompts households to move, and the resulting housing market adjustment is associated with higher house prices and lower vacancy rates. Because housing turnover is largely unrelated to vacancies in the rental market (*ibid.,* Table 4),which, in turn, are unrelated to economic activity (*ibid.,* Table 5), the data (plus correlations with a number of demographic characteristics that Bachmann and Cooper also report) suggest that in order to understand housing market dynamics better, deeper analyses of housing market adjustment are necessary, using both aggregate and demographic data. Bachmann and Cooper speculate that a larger number of vacancies do not seem to induce larger turnover, but rather, higher turnover activity leads to less vacancies. So overall, turnover in the U.S. housing market is procyclical and tends to lead the business cycle, as summarized by *ibid.,* Figure 7. This corroborates Leamer's claim from a disaggregated perspective.

Housing search is often associated with, as well as prompted by, job change. Specifically, using data from the PSID for 1991-1993, Ioannides and Kan (1996) report that for 1974-1983, the proportion of moves combined with job changes was 6% for household heads, while the proportion of job changes was 15% per year, and that for residential moves was 15.6%. Thus, more than 40% of the movers also changed jobs, which implies a substantial correlation between moving and job change. Furthermore, nearly two-thirds of movers did so to rent, and one-third to own. These facts agree with data from the CPS for 2004: Ioannides and Zanella (2008), Table 1, report that 17% of residential moves occur for work-related reasons and 52.7% for housing- and neighborhood- related reasons.

An important feature of the housing market is the coexistence of tenure modes, renting and owning and the accordant households' choice of renting versus owner-occupancy, with rental and homeownership vacancy rates, and interesting dynamics, as just discussed. We propose a joint model of frictional labor and housing markets and use it to motivate empirical analyses of both types of vacancy rates.

To the best of our knowledge, ours is the first paper to introduce a Beveridge curve for housing markets in a manner that is consistent with the original definition for the labor market. Peterson (2009) introduces a Beveridge curve for housing markets based on a relationship between the vacancy rate for housing and the rate of household formation, which he intends as a “long-run supply” relationship. Peterson argues that the rate of capital formation is decreasing in the housing vacancy rate because: one, the marginal cost of a new house is decreasing in the growth rate of the housing stock; and two, the probability of selling a new house is decreasing in the vacancy rate. Whereas the former assumption is counterintuitive, in view of urban congestion, the latter does agree with intuition.

3 Model

The full details of the theoretical model are developed in Appendix A. An outline of the model is as follows. The preference structure allows us to develop the Bellman equations for the conditional value functions for employed and unemployed owners and employed and unemployed renters, the specification of the supply of dwelling units, separately for the owner-occupied and rental segments of the housing market, and the determination of the value of vacant housing, which reflects critically the illiquidity of housing. The Bellman equations are solved after we have determined the relative numbers of agents in the different states by solving for housing market flows, that is the probabilities that individuals may be found as employed and unemployed owners and renters. Finally, we extend the model to allow for taste shocks leading to turnover in the homeownership market and for frictions in the rental market (neither of which are allowed by Head and Lloyd-Ellis 2012). We introduce a new concept to express the counterpart of unemployment in the housing market that allows us to introduce a Beveridge curve for housing markets.

Our starting point is the central friction that characterizes housing markets, namely renting versus owning. In a perfectly competitive economy and in the absence of uncertainty, where all assets earn the equilibrium rates of return, there should be no advantage of owning over renting for individuals with homogeneous preferences. With taste heterogeneity regarding family size and life styles, uncertain lengths of stay, heterogeneity of dwelling quality, the decentralized nature of the allocation of housing and a myriad other types of frictions (including the provision of local amenities like education), individuals must search for dwellings to rent or to own. Suppliers of rental housing services charge rents which compensate them for holding wealth in the form of rental housing stock. Suppliers of newly constructed homes for owner-occupancy (under the simplifying assumption that such dwellings comprise a distinct market from the rental housing one) must be compensated for their construction costs as well as for "inventory" costs incurred while waiting for prospective buyers. Because prospective tenants and owner occupants must search, landlords and sellers of dwelling units anticipate that their units may stay vacant until rental agreements and transactions may be completed. Furthermore, whereas the rental market may well be approximated as a Walrasian one, the owner-occupancy one typically involves a prospective buyer interacting with a potential seller over the terms of selling a substantial asset. Thus, the housing market with frictions lends itself conveniently to modeling by means of the tools known as the Diamond-Mortensen-Pissarides model, DMP for short.

The dynamic model of the housing market with frictions, due to Head and Lloyd-Ellis (2012), incorporates many of these desirable features. It models frictions in the DMP tradition, expressed in terms of housing vacancies in the homeownership market, which reflect the fact that renters must search before they may become homeowners, and that sellers of dwelling units have to hold their properties vacant until transactions take place. We extend the Head and Lloyd-Ellis model in two directions: one is to include the labor market with frictions. We do so by using the same conditional value functions for individuals as the ones defined for housing decisions to structure labor market bargaining. This leads naturally to interdependence between frictional housing and labor markets. A second direction is to allow in our model for frictions in the rental housing market. The purpose of the extended model is to characterize job vacancy rates, on the one hand, and rental and owner-occupied housing vacancy rates, on the other, in housing and labor markets at equilibrium. Working with the DMP model allows us to solve for the states in which individuals may be found in the economy, that is employed and unemployed owners and renters. This, in turn, allows us to solve for wage rates for renters and owners in terms of job, rental housing and owner vacancy rates, and finally to characterize steady state equilibrium in the extended model of frictional housing and labor markets.

We introduce the following notation followed by the basic ingredients of the model:

- the population consists of identical individuals and is given, $N : N(t) = N(0)e^{\nu t}$;
- the housing stocks, endogenous: total and per person owner-occupancy stock, $H, h = \frac{H}{N}$; total and per person rental stock, $R, r = \frac{R}{N}$.
- Allocation of agents into four different states, endogenous: $N^{WR}, N^{UR}, N^{WH}, N^{UH}$; alternatively expressed as the probabilities that agents may be found in different states, $n^{WR}, n^{UR}, n^{WH}, n^{UH}$, where

$$n^{WR} + n^{UR} + n^{WH} + n^{UH} = 1. \tag{1}$$

- The stocks of vacant units in the owner and rental segments of the housing market, endogenous: υ^H and υ^R.

  The owner vacancy rate is:

$$\text{vown} = \frac{\upsilon^H}{H} = \frac{H - N^{WH} - N^{UH}}{H} = 1 - \frac{1}{h}\left(n^{WH} + n^{UH}\right); \tag{2}$$

  The rental vacancy rate,

$$\text{vrent} = \frac{\upsilon^R}{R} = \frac{R - N^{WR} - N^{UR}}{R} = 1 - \frac{1}{r}(n^{WR} + n^{UR}). \tag{3}$$

- The unemployment rate, u; the stock of unemployed is uN;
- The employment rate, μ; the stock of employed is μN;
- the job vacancy rate, υ; the stock of job vacancies is υN. The unemployment and job vacancy rates are both endogenous.
- Labor market tightness, θ, is the ratio of the job vacancy rate to the unemployment rate, $\theta = \frac{\upsilon}{u}$.
- γ, the contact rate, the rate per unit of time at which prospective buyers arrive per vacant owner-occupancy unit, is defined as the product of the arrival rate of vacant dwelling units per prospective buyer, λ, times the homeownership market tightness, measured by the ratio of the number of prospective homeowners (in this case, all renters), to the number of vacant units in the homeownership market, $\frac{n^{WR}+n^{UR}}{h-n^{WH}-n^{UH}}$:

$$\gamma = \lambda \frac{n^{WR} + n^{UR}}{h - n^{WH} - n^{UH}}. \tag{4}$$

Unlike in the canonical DMP model, the matching rate λ is exogenous, but the contact rate for units with prospective buyers is endogenous.

- δ, the exogenous rate at which jobs break up.
- ν, the exogenous rate of growth of the population
- The value of vacant rental housing is equal to the rental housing supply costs, which increase with rental housing per person.
- The value of vacant owner-occupied housing is equal to the owner-occupied housing supply costs, which increase with owner-occupied housing stock per person.

Expressing the flows of individuals across the four states yields three equations, which together with Eq. (1) comprise the flow equations that may be used to solve for the four states, $n^{WR}, n^{UR}, n^{WH}, n^{UH}$ in terms of μ, the employment rate, an endogenous variable that is expressed as an increasing function of labor market tightness, θ, $\mu(\theta), \mu' > 0$, and the job matching function in the labor market. The states also depend on a number of quantities, that is, γ, the contact rate between vacant dwelling units and prospective buyers, δ, the rate at which jobs break up, and ν, the rate of growth of the population. The associated homeownership rate at the steady state is equal to $\frac{\lambda}{\lambda+\nu}$, and thus the homeownership market tightness is equal to $\frac{\nu}{(\lambda+\nu)h-\lambda}$, a decreasing function of the owner-occupied housing stock per person. The associated rate of unemployment is equal to $\frac{\delta+\nu}{\delta+\nu+\mu}$.

We develop, in detail in Appendix A the Bellman equations for renters and owners and use their solutions to characterize the determination of wages and job vacancy rates. The model predicts different wage rates for owners and renters. When individuals transition from renting to owning, their bargaining position in the labor market changes. This emanates from the logic of firms' bargaining with workers over the division of the surplus, where both parties are aware that renters' lifetime utility reflects the prospect that they may be become owners. Similarly, our extension of the Head and Lloyd-Ellis model that accounts for owner-to-renter transition makes owners' lifetime utility reflect the prospect that they may, at some point, be become renters.

If owners remain owners forever, the *wage curve for owners*, that is the DMP counterpart of the labor supply curve for owners, expresses the wage rate as a convex combination of unemployment compensation and the benefit to the firm from making a hire, which is equal to the marginal revenue plus the saving in hiring costs to the firm. The latter is an increasing function of the labor market tightness for owners.

The *wage curve for renters*, that is the DMP counterpart to the labor supply curve for renters, is made up of two components. One is the remuneration per unit of labor from accepting a job under the terms of employment for renters, which is expressed as a convex combination of the annuity benefit to the firm associated with filling a job (that is, the marginal revenue from the increase in employment plus the saving in hiring costs from the hire) and of unemployment compensation. The second component, a negative one, adjusts for the spillover effect due to the fact that renting is associated with the prospect of becoming a homeowner. It is for this reason that the wage curve for renters also depends on the labor market tightness for owners. That is, other things being equal, the prospect from becoming an owner serves as indirect benefit from renting. It also depends on the housing market tightness in the homeownership market, as the latter determines the matching rate of renters with prospective sellers and therefore the prospect of the transition from renting to owning.

The demand by firms for labor is expressed through the *job creation condition*, for either renters or owners, who are perfect substitutes in production, in effect the DMP model counterpart for the demand for labor. The job creation condition equates the wage rate plus the capitalized value of the firm's hiring costs, which are foregone once a person is hired, to the marginal revenue of an additional worker.

Working with the job creation conditions and the wage curves as a system of simultaneous equations allows us to solve for wage rates and labor market tightness for owners and renters, as functions housing market tightness conditions. These solutions imply job vacancy and unemployment rates for owners and renters, which also depend on housing market tightness.

The equilibrium rent readily follows from the rental housing supply equation, because the proportion of renters is equal to $\frac{\nu}{\lambda+\nu}$, and in the Head and LLoyd-Ellis model, the rental housing market is not frictional. Once wage rates and labor market tightness have been solved for renters and owners, they are expressed as functions of housing market tightness. So are the conditional value functions, and thus via them the value of vacant housing for owner occupancy. Therefore, the supply equation for owner-occupied housing determines the equilibrium value of the stock of owner occupied units. In other words, equilibrium is determined from asset equilibrium. The rental vacancy rate is, of course, equal to 0, if the rental housing market is frictionless. The solution for the owner-occupied housing vacancy rate follows once the owner-occupied housing stock is determined. Thus, rental and owner vacancy rates are jointly determined with job vacancy and unemployment rates. The solutions establish the presence of spillovers from the housing market to the labor market and vice versa.

Such spillovers are not present in the original Head and Lloyd-Ellis model. They emanate in our model from the assumption of a frictional labor market. We go beyond Head and Lloyd-Ellis in two other directions, too. First, we confer symmetry to the two segments of the housing market by introducing a frictional rental housing market. Newly produced rental housing stock is assumed to enter the market as vacant, and its value must earn the equilibrium rate of return. Consistently with the owner occupancy market, its supply price depends on the occupied plus the vacant rental housing stock as a proportion of the total population. The value of vacant rental housing from the demand side is in turn determined by Bellman equations for occupied and vacant housing units. These equations depend on the tightness in rental and owner-occupied housing markets. Rental housing units vacated by renters who become owners must be matched with new prospective renters. Therefore, equilibrium conditions in the two segments of the housing market become interdependent, although the dwelling units are distinct and individuals are flowing through them as housing tenure changes. The equilibrium quantities of the two types of housing stock are jointly determined, and so are the respective vacancy rates. Thus, given an exogenous population, which may be growing, the housing market adjusts to accommodate individual housing needs, while both segments of the housing market and the labor market are frictional and characterized by non-zero unemployment, rental, and owner vacancy rates.

A second direction in which we extend the model of frictions in the housing market is by allowing for the impact of imperfections in the form of agents' being unable to fulfill their desired choices. That is, owners may be mismatched; given their circumstances some owners would rather rent. Similarly for renters: given their circumstances some renters would rather own. Both types of mismatch express the joint impact of financial and mobility frictions.

Let the numbers of mismatched individuals be $N_{u,own}$ and $N_{u,rent}$, unfulfilled owners and renters, respectively. Let the respective shares of mismatched renters who would rather own and owners who would rather rent, be denoted by msm^R and msm^H, respectively:

$$\mathrm{msm}^R = \frac{N_{u,rent}}{N^{WR} + N^{UR}}, \; \mathrm{msm}^H = \frac{N_{u,own}}{N^{WH} + N^{UH}}. \tag{5}$$

This notation is purely to facilitate using the mathematical derivations obtained for the benchmark model. The introduction of renter mismatch, $0 < msm^R < 1$, but not of owners, $msm^H = 0$, does not affect the flow equations, which continue to hold with the modification that instead of λ, the rate at which prospective buyers find dwelling units, we now have $\lambda^R = \lambda \mathrm{msm}^R$. The introduction of owner mismatch, $0 < msm^H < 1$, does affect the flow equations quite extensively, as we detail in Appendix A. The rate at which prospective owners find rental dwelling units may now be written as $\lambda^H = \lambda \mathrm{msm}^H$.

Allowing for owner mismatch in the form of unfulfilled renters, msm^H, constitutes a more significant modification of the model. That is, renting is no longer a transient state, with renters seeking to become owners at the first opportunity. There are now transitions of owners, unemployed and employed, into renters. While the model continues to be tractable, the homeownership rate in the long run is now less than one, if population growth is zero, which removes an awkward feature of our version of the Head and Lloyd-Ellis model.

3.1 Labor and Housing Market Beveridge Curves

The Beveridge Curve for labor markets, the job vacancy rate as a function of the unemployment rate, is a well-established and a widely researched concept. Its derivation is straightforward.[11] At the steady state with a constant labor force we have that:

$$u = \frac{\delta}{\delta + \mu(\frac{v}{u})}.$$

where δ is of job destruction and μ is the employment rate. Since μ is increasing in labor market tightness, $\theta = \frac{v}{u}$, it is easy to show that the unemployment rate is decreasing in the job vacancy rate.

The similarities between housing and labor markets leads us to develop a Beveridge Curve for housing. Analogous to vacancies in the labor market, which is unsatisfied demand for workers by firms, there correspond vacancies in the housing market, which is unsatisfied demand for occupants by sellers . Analogous to unemployed individuals, which is unsatisfied demand for employment by individuals, there are unsatisfied renters who wish to own, and owners who wish to rent. They are prevented from doing so by frictions. Our development of Beveridge Curves for housing markets is adapted to the institutional features of housing markets, where there are owners and renters and conforms to the notion of the Beveridge curve as an accounting relationship in the steady state.

We work first with the homeownership market; the vacancy rate, vown, is given by (**??**). We next express it in terms of a concept that serves as the unemployment counterpart in housing markets. We allow for mismatch among renters giving rise to unsatisfied homeownership demand, the solutions for n^{WH} and n^{UH} depend on λmsm^R instead of just λ and thus on the incidence of mismatch. Working with the solution (see Appendix A, (41)), and assuming that $\text{msm}^H = 0$, we have that the equilibrium homeownership rate is:

$$n^{WH} + n^{UH} = \frac{\lambda \text{msm}^R}{\lambda \text{msm}^R + \nu}. \tag{6}$$

The equilibrium homeownership rate decreases with the probability of mismatch. That is, an increase, due to mismatch of renters, in the number of individuals searching to buy homes reduces the homeownership rate.[12]

In developing a Beveridge curve for the homeownership market, we propose the concept of the unfulfilled homeownership rate as the counterpart of the unemployment rate and

[11]Pissarides (1986) is the first joint empirical model of unemployment and job vacancies.

[12]In view of the generalization of the matching model in footnote 3 above, the rate at which buyers contact dwelling units, λ, may be written in terms of the matching function $\Gamma(.,.)$, and the ratio of potential buyers

normalize it appropriately. We start with the definition of

$$\text{uhr} = \frac{N_{u,rent}}{N_{u,rent} + N^{WH} + N^{UH}},$$

where $N_{u,rent}$ is defined as the number of renters who prefer to own, a quantity that we impute based on a tenure choice estimation, and $N^{WH} + N^{UH}$ all self-reported owners. This quantity may be at most equal to the rental rate, and therefore normalizing it by the rental rate yields the relative unfulfilled homeownership rate,

$$\text{ur}^H = \frac{\text{uhr}}{n^{WR} + n^{UR}}. \tag{7}$$

This serves as our analog of the unemployment rate for the ownership market. It ranges between 0 and 1, if all renters wish to become owners, which Head and Lloyd-Ellis assume.

By manipulating the definitions we may express uhr in terms of the n's and then solving the flow equations and using (2) yields the analog of the Beveridge curve for the ownership market:

$$\text{vown} = 1 - \frac{1}{h} + \frac{1}{h}\frac{\nu}{\mu}\frac{1}{\text{ur}^H}. \tag{8}$$

Thus, the Beveridge curve for the ownership market is a decreasing function of ur^H, the respective ownership "unemployment rate," which agrees with the Beveridge curve for labor markets.[13] In this expression, the owner-occupied housing stock per capita, h, is endogenous, which may cause the Beveridge curve to shift and tilt by the cyclical variation in h.

Turning to the rental market, we propose the concept of the unfulfilled rental rate as the analog of the unemployment rate for the rental market. We start with the definition of the auxiliary quantity

$$\text{urr} = \frac{N_{u,own}}{N_{u,own} + N^{WR} + N^{UR}},$$

where $N_{u,own}$ is defined as the number of owners who prefer to rent, a quantity that we impute based on the same tenure choice estimation as the one used for renters, and $N^{WR} + N^{UR}$ all

to vacant units, ϕ. That is:

$$\lambda = \Gamma(1, \phi^{-1}).$$

[13]The expression in (58) is modified if $\text{msm}^H \neq 0$, but its property with respect to ur^H is not affected.

self-reported renters. This quantity may be at most equal to the ownership rate, if all owners wish to be renters. Normalizing it by the ownership rate yields the relative unfulfilled rental rate,

$$\mathrm{ur}^R = \frac{\mathrm{urr}}{n^{WH} + n^{UH}}. \tag{9}$$

This serves as our analog of the unemployment rate for the rental market. ur^R ranges between 0, the assumption made by Head and Lloyd-Ellis, and 1, which would mean that all owners wish to become renters.

By manipulating the definitions we have an expression for urr in terms of the n's. From the solution of the flow equations we have expressions for the n's in terms of parameters, including the imputed shares of mismatched renters and owners, $\mathrm{msm}^R, \mathrm{msm}^H$. Working from (3) we obtain an expression for the Beveridge Curve for rental housing markets:

$$\mathrm{vrent} = 1 - \frac{1}{r} + \frac{1}{r}(n^{WH} + n^{UH}) = 1 - \frac{1}{r} + \frac{1}{r}\frac{\mathrm{urr}}{\mathrm{ur}^R}. \tag{10}$$

In general, both msm^R and msm^H enter the expression for urr. Since renting and owning are interdependent, in the most general case, it is not surprising that the vacancy rates share parameters. As with the vacancy rate in the homeownership market, the rental vacancy rate depends on r, the rental housing stock per person, which is endogenous and varies procyclically, thus shifting and tilting the rental Beveridge curve.

Not surprisingly, the wage curve for renters does depend on housing market magnitudes: renters become homeowners at the first opportunity, and thereafter stay as homeowners. Forward-looking agents anticipate this prospect. We note that the result implies a spill-over effect from the housing market. An increase in θ^H, the labor market tightness in the labor market for homeowners, increases the employment rate for homeowners and shifts upwards the wage curve for renters. A decrease in housing market tightness, γ, associated for example with an increase in housing per person, shifts downwards the wage curve in this labor market. This causes, *cet. par.,* a decrease in the labor market tightness for renters, θ^R, which increases unemployment and decreases vacancies. This, in turn, shifts the Beveridge curve upwards, exactly as it was observed during the downturn associated with the Great Recession of 2007-2009 in the US. Therein lies the power of the Beveridge curve tool: it allows us to track structural shifts in the overall economy as well as at sectoral or regional levels.

Before we move on with the empirics, a remark is in order. At first sight, it might be puzzling that the wage rates for owners and renters are different, since individuals are identical with respect to their productivity. Yet differences in wages between renters and owners follows directly from the model. The economic interpretation rests on the fact that firms anticipate different turnover patterns between renters and owners, due to different mobility costs.

There are, of course, numerous ways in which the model can be extended, in addition to developing fully the case of turnover by owners and its implications for wage determination, unemployment and labor and housing vacancy rates. A particularly interesting feature that is worth exploring is to allow for correlation between residential moves and job changes. As discussed in section 2 above, more than one-third of moves are also associated with job change.

4 Data

Annual data at the national level on homeownership and rental vacancies is available from the Census Bureau starting in 1950. Data on housing vacancies at the MSA level come from the Census Bureau's Housing Vacancy Survey (HVS). The HVS is a regular part of the Current Population Survey (CPS). Units that are found to be vacant or were otherwise not interviewed are included in the HVS.[14] These data are available from 1986-present on

[14]The definition of a vacant housing unit as given by the Census Bureau is "A housing unit is vacant if no one is living in it at the time of the interview, unless its occupants are only temporarily absent. In addition, a vacant unit may be one which is entirely occupied by persons who have a usual residence elsewhere. New units not yet occupied are classified as vacant housing units if construction has reached a point where all exterior windows and doors are installed and final usable floors are in place. Vacant units are excluded if they are exposed to the elements, that is, if the roof, walls, windows, or doors no longer protect the interior from the elements, or if there is positive evidence (such as a sign on the house or block) that the unit is to be demolished or is condemned. Also excluded are quarters being used entirely for nonresidential purposes,

an annual basis for the largest 75 MSAs (though there are less than 75 MSAs in the early years). These data are somewhat problematic since MSA definitions change over time.

Data on monthly job vacancies starting in 1951 come from the Help-Wanted Index for the fifty largest MSAs; these are an aggregate of ads carried by the press that is provided by the Conference Board.[15] However, it is known that since the mid- to late-1990s, this "print"-based measure of vacancy posting has become increasingly unrepresentative as advertising over the internet has become more prevalent. Figure 2 plots the National print Help-Wanted Index starting in 1977 (note that it coincides with the composite index until 1994). One can see the drop off around 2000. Barnichon (2010) builds a vacancy posting index that captures the behavior of total"print" and "online"-help-wanted advertising, by combining the print with the online Help-Wanted Index published by the Conference Board since 2005. Figure 2 includes our version of the combined index. It closely replicates Barnichon's index which goes through 2009 and extends it through June 2014. The details of our computations are given in Appendix C (12).

Figure 3 plots the job vacancy data (the composite Help-Wanted Index divided by the size of the labor force) along with homeownership and rental vacancy rates for 1956-2014. The correlation coefficients are given below the figure. There is a reasonably strong negative correlation between job vacancies and both rental and homeownership vacancy rates. This is explained by the following observations: (1) there tend to be more job vacancies when the labor market is "hot," as workers can be more selective, and hence firms find it harder to hire; (2) there are fewer rental or homeownership vacancies when the housing market is hot, as renters are motivated to enter the homeownership market (though there is more churning), and (3) the labor and housing markets tend to be hot at the same time (see Figure 1). We believe the latter fact has not been noticed before. A potential causal mechanism is

such as a store or an office, or quarters used for the storage of business supplies or inventory, machinery, or agricultural products. Vacant sleeping rooms in lodging houses, transient accommodations, barracks, and other quarters not defined as housing units are not included in the statistics in this report. A vacant unit for rent consists of "units offered for rent and those offered both for rent and sale."

[15] Pissarides (1986) for Britain, and Blanchard and Diamond (1989) for the US have used the Help-Wanted Index in studying labor market adjustment.

that as vacancies increase in the housing market, job vacancies decrease since this opens up more residential locations and allows workers to make better job matches (given the joint residential location and job matching decision process).

We have the same job vacancy data at the MSA level for 40 MSAs. We have applied a similar procedure to create a combined Help-Wanted Index (HWI) for each of the MSAs.[16] Details about the construction of our MSA-level combined HWI are given in the Appendix. Summary statistics for the composite HWI for 1986 - present are given in Table 1.

Additional data on monthly job vacancies starting in December 2000 are available from the Bureau of Labor Statistics in the Job Openings and Labor Turnover Survey (JOLTS). These data are only provided at the level of the four Census regions for total nonfarm employment as well as aggregated by a number of industrial categories.

We use the National version of the American Housing Survey (NAHS) to estimate renter's unfulfilled desire to be home owners (and vice versa). The NAHS is an unbalanced panel of more than 50,000 housing units that are interviewed every two years and contains detailed information on dwelling units and their occupants through time, including the current owner's evaluation of the unit's market value. We use the NAHS for survey years 1985-2013. Summary statistics for all the variables used in this calculation are given in Table 1.

5 Empirics

5.1 Beveridge Curve Regressions: labor market

Recall that the Beveridge curve plots job vacancies versus the unemployment rate. Movement along the Beveridge curve indicates positions in the business cycle: higher unemployment

[16] We have data for 51 MSAs for the HW online index (HWOL) and 49 MSAs for the print index. But Austin, Buffalo, Honolulu, Las Vegas, Orlando, Portland, Providence, San Jose, Tucson, and Virginia Beach are included in the online data but not in the print data, whereas Albany, Allentown, Dayton, Knoxville, Omaha, Syracuse, Toledo, and Tulsa are included in the print data but not in the online data. Also, Houston is missing the print index for 1996.9 to 2003.7 so it is excluded.

and lower vacancies in periods of recession, and lower unemployment and higher vacancies in periods of expansion. Shifts in the Beveridge curve can arise for a variety of reasons: changes in the efficiency of the job matching process, skill mismatch, changes in the labor force participation rate, and others, such as economic and policy uncertainty. See Diamond and Sahin (2014) and Pissarides (2011) for details on shifts in the US and UK Beveridge curves.

Figure 4 plots the National Beveridge Curve for 1951-2014. The job vacancy rate, vjobs, is the composite help wanted index divided by the labor force. The data are split into five episodes that are determined by apparent shifts in the Beveridge curve. Figure 5 plots the Beveridge curve for the most recent episode using the monthly JOLTS data. The curve begins in the upper left corner in January 2001 in a period of low unemployment and a high job vacancy rate. It then moves south east and ends up in the southeast corner at the end of 2009 in a weak period of high unemployment and low job vacancies. There appears to be an outward shift in the Beveridge curve over the next six months followed by a north-west movement to a stronger economic position in 2014.

Our theory establishes spillovers between the wage curves for owners and renters, where the latter depends on conditions in the housing market.[17] The prediction for the full effect requires that we solve jointly the two wage-job creation curve systems for the labor market for renters and owners.

Denote the unemployment rate in MSA i and time t as $\text{unempl}_{i,t}$, the job vacancy rate as $\text{vjobs}_{i,t}$, the homeownership vacancy rate as $\text{vown}_{i,t}$, and the rental vacancy rate as $\text{vrent}_{i,t}$. Then the augmented Beveridge curve is specified as

$$\text{vjobs}_{i,t} = \alpha_{0i} + \alpha_1 \text{unempl}_{i,t}^{-1} + \alpha_2 \text{vown}_{i,t} + \alpha_3 \text{vrent}_{i,t} + \epsilon_{i,t}. \tag{11}$$

where vjobs isthe national composite help wanted index for 1951-2014 divided by the size

[17]This is evident in the last term on the r.h.s. of Eq. (75), via γ, the rate at which new dwelling units sold by construction firms are matched with potential buyers, and $\mu(\theta^H)$, the employment rate in the labor market for homeowners. A larger γ causes a downward shift of the wage curve for renters, thus increasing the respective labor market tightness and causing a downward movement along the corresponding Beveridge curve.

of the labor force. The results for 1959–2014 are given in Table 2. Columns (1)and (2) present results for the model that includes the inverse of the unemployment rate and the homeownership and rental vacancy rates as regressors. OLS results are given in column (1). The elasticity of the jobs vacancy rate with respect to the unemployment rate to -0.53. Both housing vacancy rates are significant. Surprisingly the corresponding coefficient estimates are opposite in sign. The coefficient for the rental vacancy rate has a smaller $p-$value and an elasticity near -1. This is in line with the negative relationship between labor and housing market vacancy rates that we hypothesized earlier. Given the interdependence of the vacancy rates in the housing and labor markets, the former are likely to be endogenous in this model. We instrument for the homeownership and rental vacancy rates using new housing permits and starts for 1 unit and 2or more unit structures. These instruments pass the over-identification and weak instrument tests but the test for the exogeneity of the homeownership and rental vacancy rates is not rejected. Not surprisingly, the IV results (in column (2) of Table 2) are very similar to the OLS results. We next include indicators of the shifts in the Beverage Curve (see Figure 4). OLS and IV results are given in columns (3) and (4) of Table 2. Not surprisingly, their addition significantly improves the fit of the model. Also not a surprise is the fact that this lowers the significance and magnitude of the homeownership and rental vacancy rate coefficients since it is these shits that the housing vacancy variables help explain.

5.2 Beveridge Curve Regressions: housing market

Our theory aims at a symmetric treatment of the labor and housing markets over and above the presence of spillovers. In particular, we develop a housing market Beveridge curve. Whereas the vacancy rate concept applies equally well to the housing and labor market, at this point, no obvious counterpart of unemployment in the housing market has been proposed. In the labor market, the unemployment rate measures the extent of the unfulfilled desire of labor market participants to work. We posit a counterpart concept in terms of an unfulfilled desire on the part of renters to become homeowners. Renters are prevented from owning homes due to the inability to get a mortgage because of down payment constraints, poor credit, or because of discrimination in the mortgage credit market.

We proceed by estimating a housing tenure choice equation and then using these results to predict the probability of owning. We define renters with a probability of homeownership greater than or equal to 0.5 as unfulfilled owners. The number of this group is denoted as $N_{u,rent,t}$ and the number of owners as $N_{own,t}$. The unfulfilled homeownership rate is defined in (7) above.

5.3 Unfulfilled Rental and Homeownership rates: Estimation via Tenure Choice

In order to calculate the unfulfilled homeownership and rental rate variables, $\text{uhr}_t and \text{urr}_t$, defined in (7) and (9) above, we estimate a tenure choice equation by probit, where the probability of owning is given by $\Phi(\mathcal{X}_{i,m,t}\hat{\alpha})$, where $\Phi(\cdot)$ denotes the cumulative normal distribution, and the vector $\mathcal{X}_{i,m,t}$ includes all characteristics used so far. Details of this estimation are given in Appendix B (section 11) below.

We define renter i, $\text{own}_{i,m,t} = 0$, as an unfulfilled renter,

$$\text{u-rent}_{i,m,t} = 1, \text{if } \Phi(\mathcal{X}_{i,m,t}\hat{\alpha}) \geq 0.5; \text{and } \ \text{own}_{i,m,t} = 0. \tag{12}$$

It follows that according to (7) the unfulfilled homeownership rate is given by:

$$\text{uhr}_t = 100 \times \frac{\frac{N_{u-rent,}}{N_{u-rent,t}+N_{own,t}}}{\frac{N_{rent,t}}{N_{rent,t}+N_{own,t}}},$$

where $N_{u-rent,t} = \sum_i \ \text{u-rent}_{i,m,t}$, and $N_{rent,t}, N_{own,t}$ all self-reported renters, owners.

Working in a like manner according to (9), we define the unfulfilled rental rate as:

$$\text{urr}_t = 100 \times \frac{\frac{N_{u,own,t}}{N_{u,own,t}+N_{rent,t}}}{\frac{N_{own,t}}{N_{rent,t}+N_{own,t}}},$$

where $N_{u,own,t} = \sum_i \ \text{u,own}_{i,m,t}$, and $N_{rent,t}, N_{own,t}$ all self-reported renters, owners. Note that the estimation of equ. (12) is sufficient, in view of the binary nature of the tenure choice here. That is, $\text{u,own}_{i,m,t} = 1 - \ \text{u,rent}_{i,m,t}$.

We estimate the share of unfulfilled homeowners and renters using the NAHS. The mean of the number of unfulfilled homeowners, those who wish to be renters, as a share the number of all homeowners over the sample period is equal to 13%. Correspondingly, the mean number of unfulfilled renters, those who wish to be owners, as a share the number of all renters over the sample period is equal to 44%. These estimates are of course sensitive to the choice of the 50% cutoff point above.

The US housing Beveridge curves for 1985-2013 are plotted in Figure 6 with the homeownership and rental vacancy rates on the left and right axis, respectively. Each curve appears to have shifted outward in the latter half of the period. Note that both curves follow fairly similar patterns, and they are roughly distorted "negative" logistic. The homeownership Beveridge Curve is located in the southeast corner in 2005, indicating a "hot" market. It then moves to the northwest corner in 2009, indicating a cold market. There is then a movement back towards the southeast, indicating that this market has rebounded.

While visually, prices in the rental market appear to move in step with those in the housing market (see Figure 7), the correlation between the real price growth in the two markets is only 0.36 over the 1975 to 2015 period. Differences clearly emerge starting in the early 2000s. The rental market did not suffer the huge decline in prices that occurred in the housing market starting around 2005. Another difference between the two markets is that the vacancy rate is significantly higher in the rental market compared to the housing market; 7.4 versus 1.6 between 1956 and 2014. Still, lie the Beveridge Curve in the housing market, the Beveridge Curve in the rental market is in northwest corner in 2009 and then there was a strong movement towards the southeast, indicating market recovery (see Figure 6). We believe that this is the first time that Beveridge curves for the housing market have been drawn. [18]

[18]Peterson (2009) defines a long-run "Beveridge Curve" in the housing market as the rate of household formation as a decreasing function of the vacancy rate for housing, which he finds to be true for the owner-occupied market, the rental market, and the total market for housing irrespective of homeownership status. He sees this as a long-run supply condition that he explains by assuming that, one, the cost to produce new housing is decreasing in the growth rate of the housing stock, and two, the likelihood of selling a new house is decreasing in the vacancy rate. The first condition clashes with a long-held stylized fact of urban

Next, we specify and estimate the augmented housing Beveridge curves for the homeownership and rental markets

$$vx_{i,t} = \beta_0 + \beta_1 \mathrm{uxr}_{i,t}^{-1} + \beta_2 \mathrm{vjobs}_{i,t} + \varepsilon_{i,t}, \tag{13}$$

where $x = h(\mathrm{own}), r(\mathrm{rent})$. Results for the Beveridge curves that include the labor market vacancy rate are given in Columns (1) and (3) of Table 3. Of course, this is really only illustrative at this point since we only have 15 observations. The coefficient estimates for $\mathrm{uhr}_{i,t}^{-1}$ are positive but not significant in both cases but and the elasticities are -0.39 and -1.49. for the homeownership and rental models, respectively. The coefficient estimate for $\mathrm{vjobs}_{i,t}$ is negative and significant (at 10% or better) in both cases and the elasticities are identical. The results including the shifts in the homeownership and rental market Beveridge Curves are included in columns (2) and (4) in Table 3. Not surprisingly, the coefficient estimates for $\mathrm{vjobs}_{i,t}$ are no longer significant since this variable is included to account for shifts in the housing market Beveridge Curves.

5.4 VAR Models and Impulse Response Functions for Owner, Rental, and Job Vacancy Rates

We next specify and estimate VAR models of job and housing homeownership and rental vacancy rates using the CBSA-level data. The purpose is to establish the interrelationship between the two markets and then to calculate how shocks in one market propagate themselves in the other market using an impulse response function. These models are extensions of the augmented labor and housing market Beveridge curves that include lags of the explanatory and dependent variables, the CBSA house price index as an additional control variable, and CBSA fixed effects. We have data for 37 CBSAs for 1991–2012.

First, we check for unit roots in each of the time series (using xtunitroot in Stata). All variables, including the unemployment rate (and its inverse) and the owner-occupied house price index are found to have a unit root. So we run the VAR regressions in first differences.

congestion; the second one is, however, consistent with the search model.

Next, we test for Granger Causality. These three regressions for owner, rental, and job vacancy rates include two lags of these variables, fixed effects, and time dummies. Whether we run these tests in levels or in first differences, the only (Granger) causality runs from owner and renter vacancy rates to job vacancy rates.

The VAR equations are reduced form (there are no contemporaneous variables included as explanatory variables). That is:

$$\Delta \text{ vx}_{i,t} = \alpha_{0,x} + \sum_{j=1,2} \alpha_{1,j,x} \Delta \text{vown}_{i,t-j} + \sum_{j=1,2} \alpha_{2,j,x} \Delta \text{vrent}_{i,t-j}$$

$$+ \sum_{j=1,2} \alpha_{3,j,x} \Delta \text{vjobs}_{i,t-j} + \sum_{j=1,2} \alpha_{4,j,x} \Delta \mathbf{X}_{i,t-j} + u_{t,x} + v_{i,x} + \varepsilon_{it,x}, \tag{14}$$

where vx = own, rent, job vacancy rates, that is, o, r, j, $\mathbf{X}_{i,t-j}$ is a vector containing the inverse of the unemployment rate and the house price index. First, we estimate these three equations (14) and get coefficient estimates with two lags included. The results are given in Table 4 below.

After estimating these three equations, we calculate responses to shocks to vjobs, vown, and vrent. We do so by adding a one standard deviation increase in (values given in Table 1) and following the changes in Δ vx$_{i,t}$, $x = o, r, j$ over time. This produces three sets of impulse response functions (IRFs); with shocks to the first-differences in owner, rental, and job vacancy rates. Note that this means that the ordering of the variables in not necessary. IRFs for the three cases are given below; Figure 8. These are cumulative in the levels of vown, vrent, vjobs. These results reinforce the Granger causality findings. The responses to the owner and rental vacancy rates due to a shock to job vacancies are small and not significantly different from zero. The responses to the owner vacancy rates due to a shock to rental vacancies are small and not significantly different from zero (and vice versa). But the shocks to the owner and rental vacancies do have negative and significant impacts on job vacancies. In the case of the shock to owner vacancies, there is a long-term negative and significant impact of about -0.04 in the job vacancies variable. In the case of the shock to rental vacancies, there is a negative and significant impact for the first few periods but the long-term impact of about -0.15 is not significant. The RMSE from the VAR equation for job vacancies is 0.27 so the ratio of the long-term impact from the shock to the owner vacancy rate is 0.15, a reasonably large impact.

These results reinforce the thinking on the Great Recession where it was the downturn in the housing market that resulted in the subsequent decline in the real economy.

6 Conclusions

This paper explores the interdependence between the housing and labor markets by means of a DMP-type model. The model gives rise naturally to equilibrium vacancy rates in housing and labor markets. The labor market model with frictions produces as an outcome the Beveridge Curve and we develop a Beveridge curve for housing; one each for the homeownership and rental markets. We propose a housing market counterpart for the concept of unemployment; the unfulfilled homeownership and rental rates. Movement along the housing Beveridge Curves is opposite that of the housing market Beveridge Curve. That is, there is a movement to the south east as the housing market (and economy) improves. Our model predicts negative spillovers from the housing market to the labor market and vice versa. In the case of the labor market, the mechanism is that an increase in house vacancies increasing the matching efficiency as it easier for workers to move to new jobs given that it is easier to find new housing. This result implies that the increase in vacancies in the housing market results in an inwards shift in the Beveridge Curve in the labor market. Despite this inward shift, the labor market Beveridge Curve shifted outward during the recent recession.

We estimate the model using data at the MSA level on housing vacancies from the US Census Bureau's Housing Vacancy Survey starting in 1986 and data on job vacancies from the Help-Wanted Index that starts in 1951 and the online version that starts in 2005. We first estimate a Beveridge curve for labor markets that includes rental and homeownership vacancies as explanatory variables as predicted by our model. We find that rental vacancies have a significant negative impact on vacancies in the housing market whereas the impact of homeownership vacancies is not significant. This makes sense if most households that change jobs and residences move into rental housing (at least initially). Next we estimate Beveridge curves for the homeownership and rental markets. We use the 1985-2013 waves of the National AHS to calculate our equivalent measure of "unemployment" in the housing market; the unfulfilled desire to own and to rent. This results in 15 observations so the results are really illustrative. Still, we find that labor market vacancies have a negative and significant impact on both homeownership and rental vacancies with an elasticity of around -0.4 in both cases. Again, the mechanism is that given that many residential moves are joint with job decisions, more job vacancies mean that households are able to better match their housing needs given the greater availability of job openings.

The results from the VAR models for labor market and housing vacancies are used to study how shocks to either the housing or labor markets will propagate themselves in the other market. We find evidence that spillovers from the rental and housing markets affect labor market vacancies and not vice versa. This is consistent with the belief that the Great Recession of 2007-2009 started in the housing market and spilled over into the real economy.

7 References

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8 Figures

1. US Real House Price Growth and GDP growth rates, and Unemployment Rates: 1976:1–2015:3.

2. National Help Wanted Index, 1977:1-2014:6: Conference Board Print Index. Composite Index, JOLTS National (rescaled).

3. Annual Rental, Homeowner and Job Vacancy Rates: 1956-2014.

4. U.S. Beveridge Curve, Help Wanted Index: 1951-2014.

5. US Beveridge Curve, Labor Markets, JOLTS Data: 2000:12–2014:3.

6. U.S. Housing Beveridge Curves: 1997-2013.

7. National Real House and Rental Market Growth Rates, 1975:1–2015:2.

8. Impulse Response Functions

Figure 1

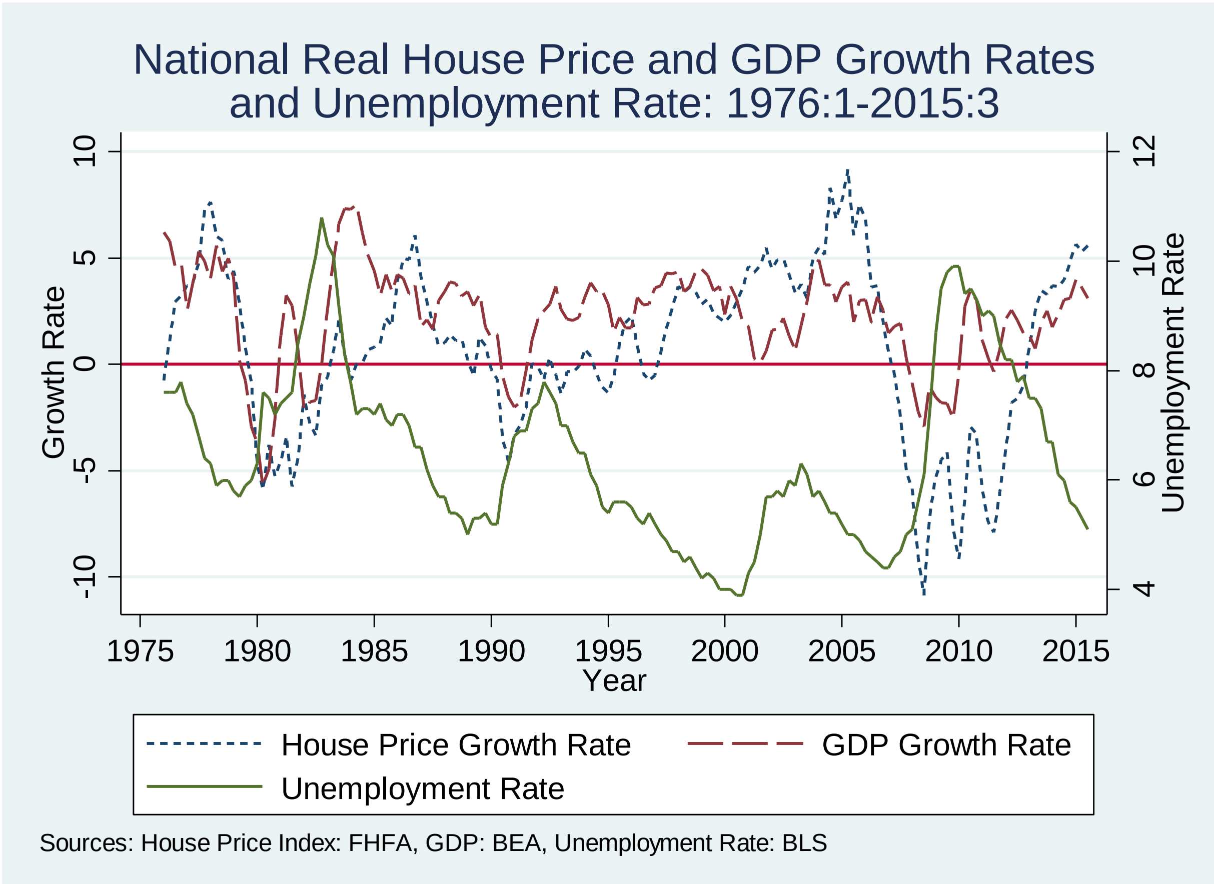

National Real House Price and GDP Growth Rates
and Unemployment Rate: 1976:1-2015:3
Growth Rate
Unemployment Rate
Year
10
5
0
-5
-10
12
10
8
6
4
1975
1980
1985
1990
1995
2000
2005
2010
2015
House Price Growth Rate
GDP Growth Rate
Unemployment Rate
Sources: House Price Index: FHFA, GDP: BEA, Unemployment Rate: BLS

Figure 2

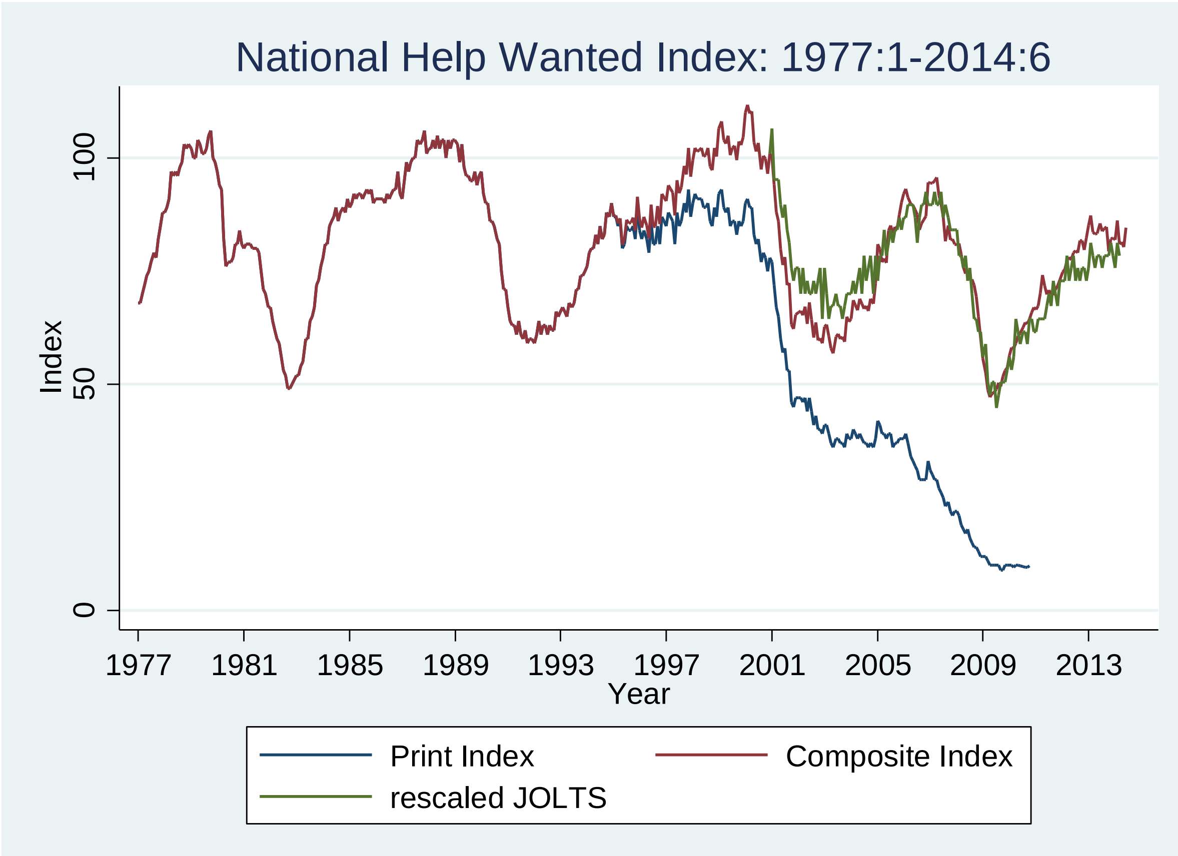
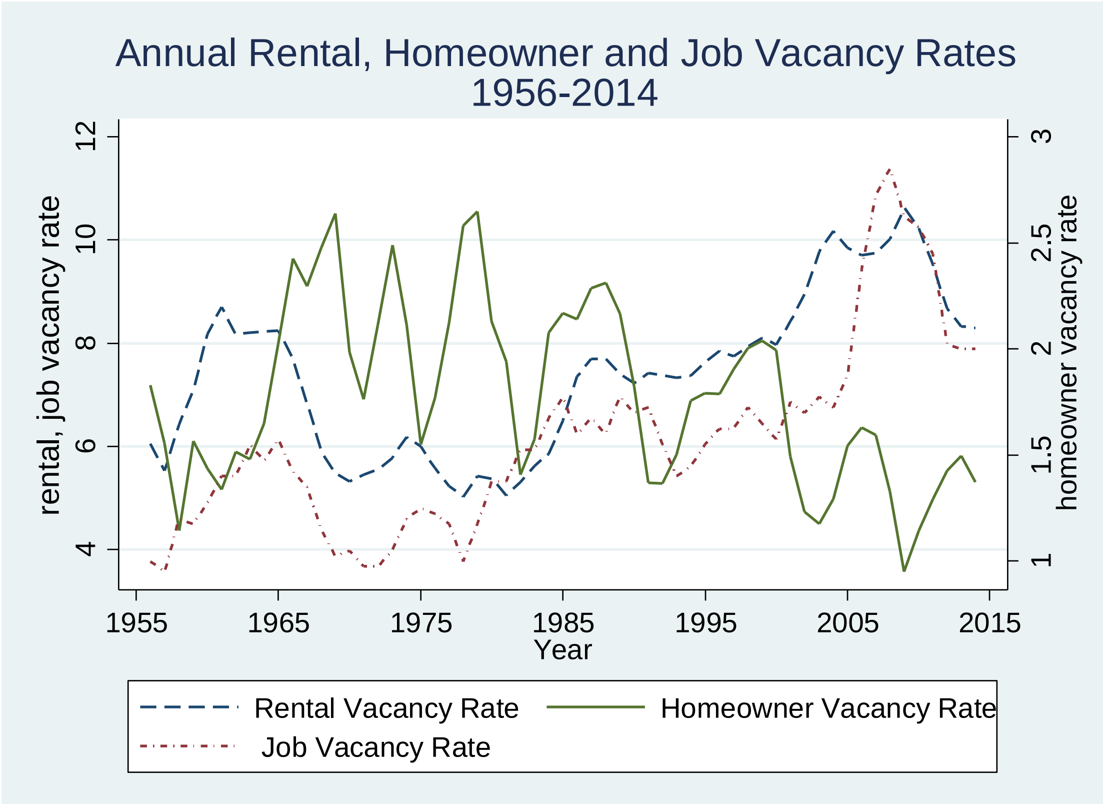

National Help Wanted Index: 1977:1-2014:6
Index
0
50
100
1977
1981
1985
1989
1993
1997
2001
2005
2009
2013
Year
Print Index
Composite Index
rescaled JOLTS

Figure 3



Correlations

	Rental Vacancy	Home Vacancy	Job Vacancy
Home vacancy	0.805		
Job Vacancy	-0.591	-0.495	
Unemployment Rate	0.003	0.294	-0.468

Figure 4

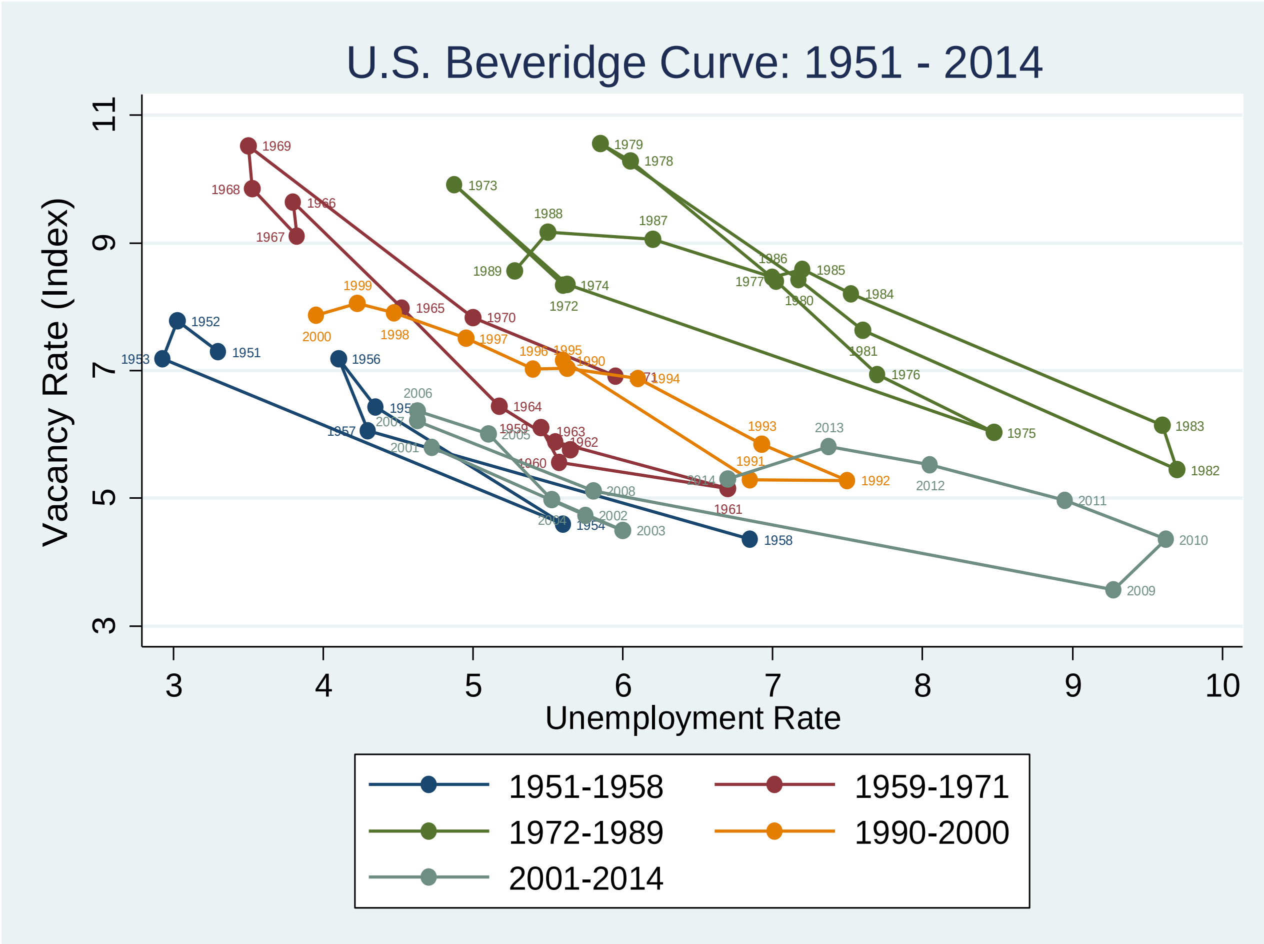
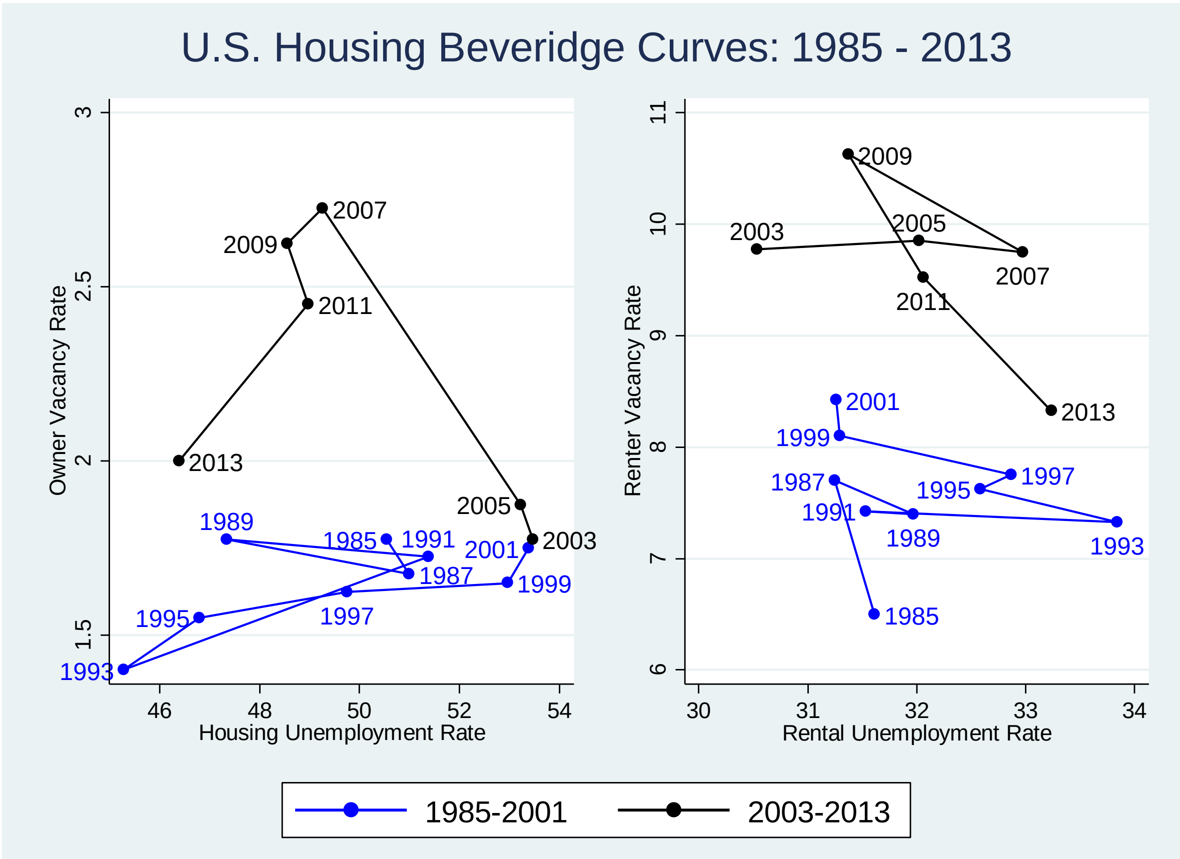

U.S. Beveridge Curve: 1951 - 2014
Vacancy Rate (Index)
Unemployment Rate
1951-1958
1959-1971
1972-1989
1990-2000
2001-2014

Figure 5

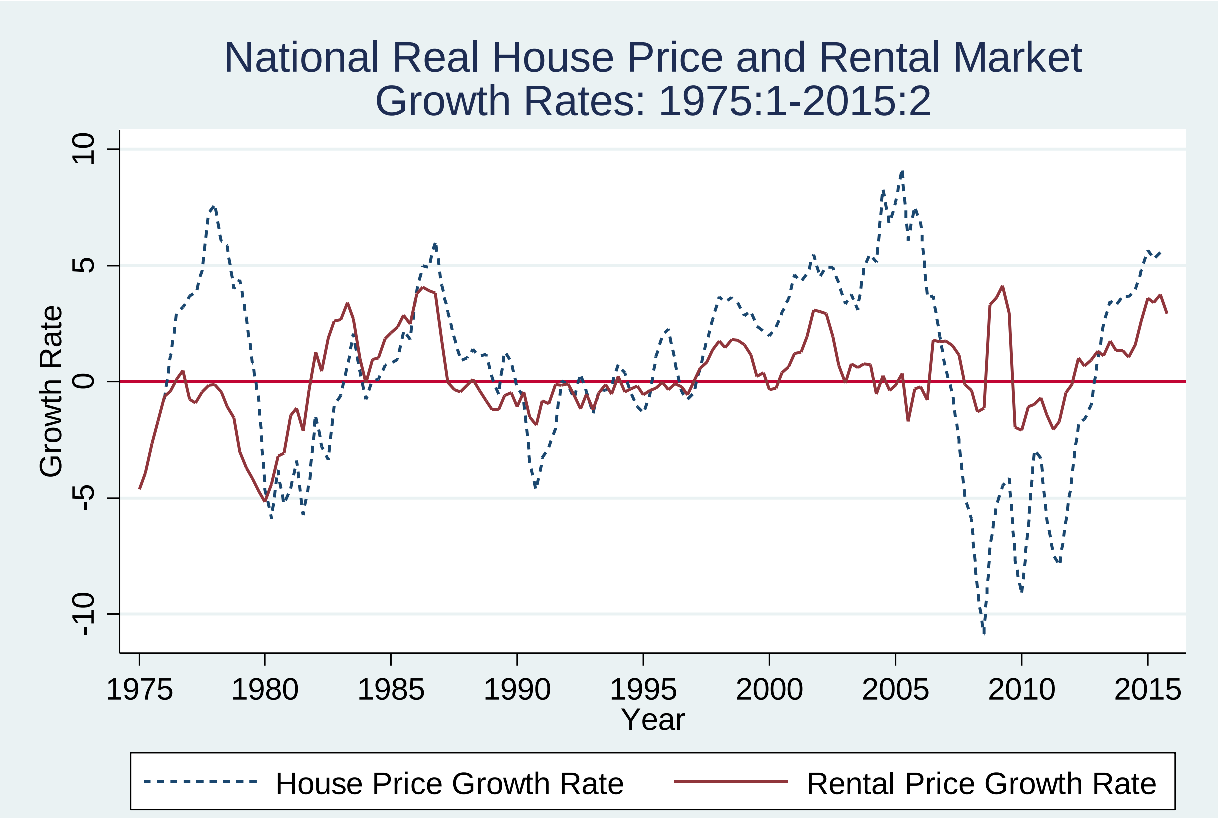

U.S. Beveridge Curve: 2000:12 - 20014:3
Vacancy Rate
Unemployment Rate
4
3.5
3
2.5
2
1.5
4
5
6
7
8
9
10
11
2001.1
2014.2
2011.9
2010.4
2009.2
2009.10
2009.7

Figure 6



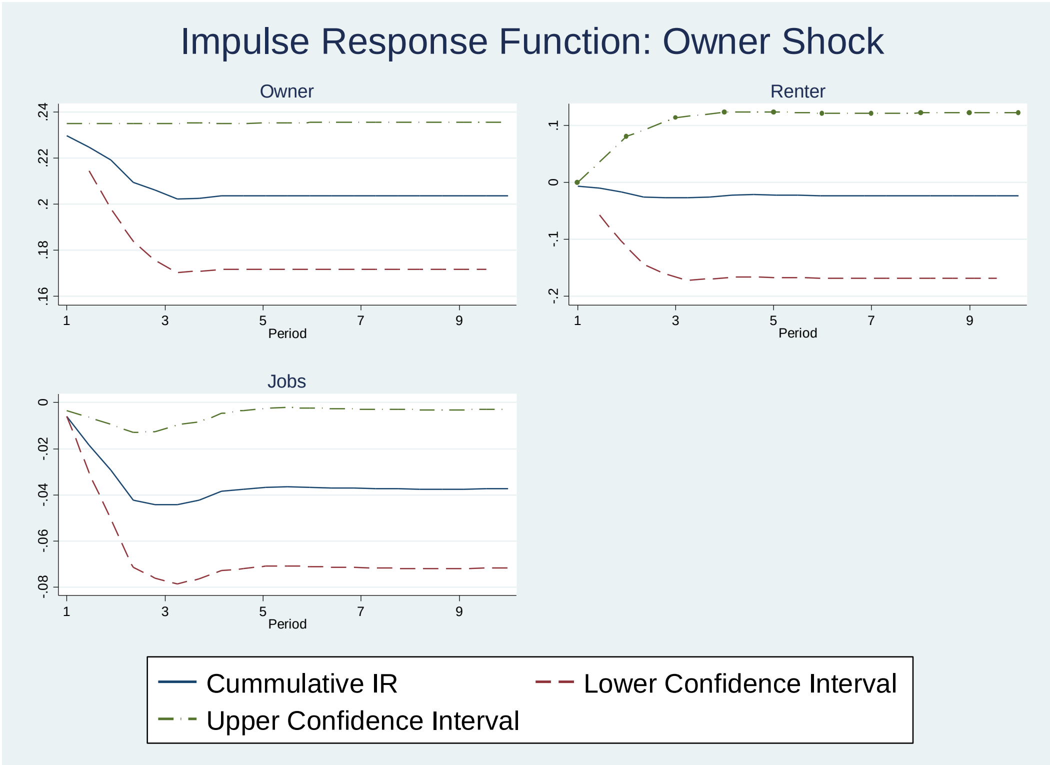
U.S. Housing Beveridge Curves: 1985 - 2013
Owner Vacancy Rate
3
2.5
2
1.5
2007
2009
2011
2013
2005
2003
1989
1985
1991
2001
1987
1999
1995
1997
1993
46
48
50
52
54
Housing Unemployment Rate
Renter Vacancy Rate
11
10
9
8
7
6
2009
2003
2005
2007
2011
2013
2001
1999
1997
1987
1995
1991
1989
1993
1985
30
31
32
33
34
Rental Unemployment Rate
1985-2001
2003-2013

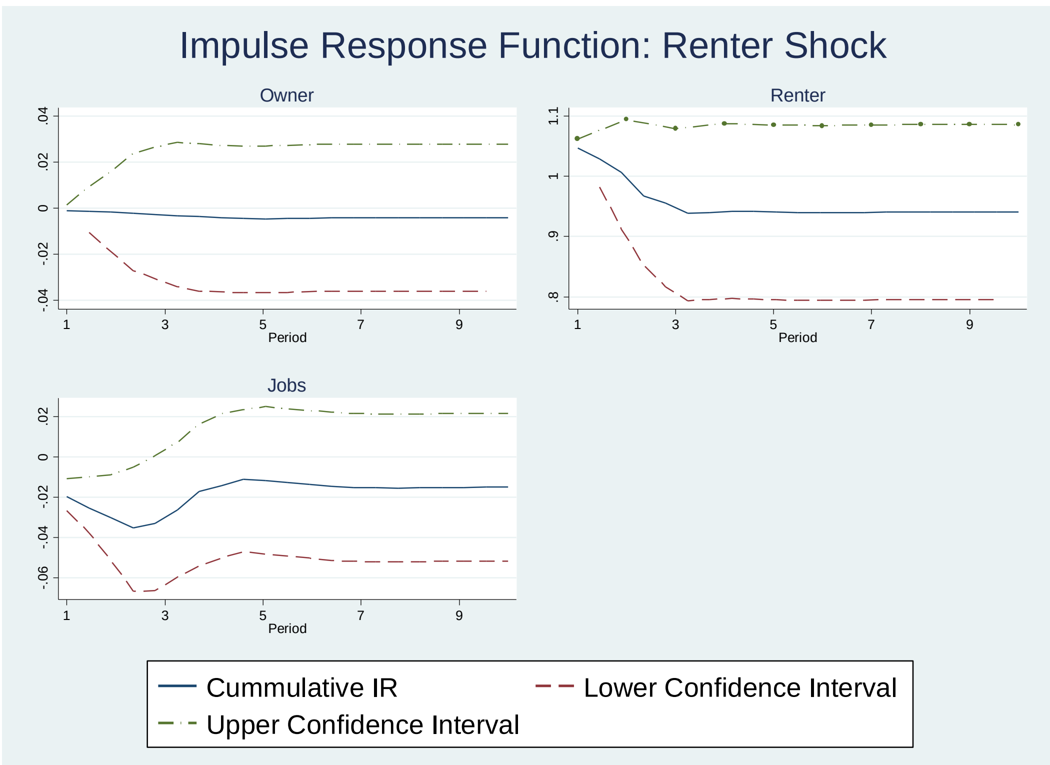
Figure 7

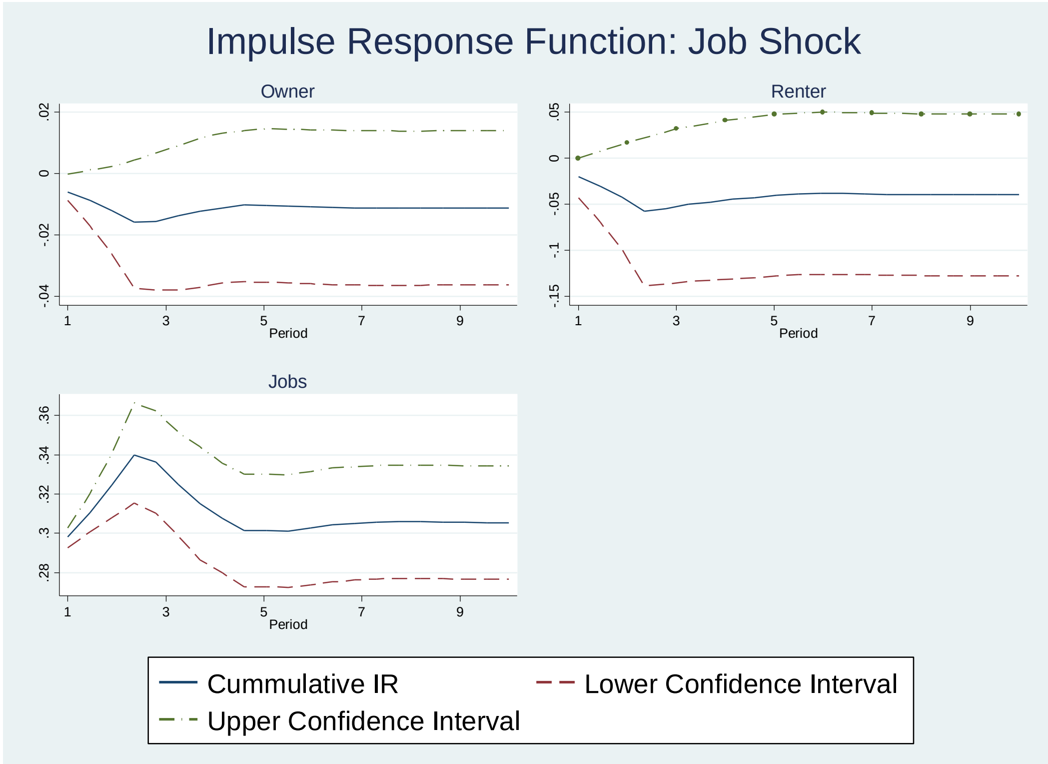


Sources: House Price Index: FHFA, Rental Price Index: FRED

Figure 8: Impulse Response Functions






Impulse Response Function: Job Shock
Owner
Renter
Jobs
Period
Cummulative IR
Lower Confidence Interval
Upper Confidence Interval

9 Tables

1. Summary statistics
2. Beveridge Curve 1950 –2014: Dependent Variable is Job Vacancy Rate
3. Housing Beveridge Curve Results: 1997-2011
4. VAR Regressions for Vacancy Rates: Homeowner, Rental, and Job, CBSA Level

Table 1: Summary Statistics

Variable	Nobs	Mean	Std. Dev.	Minimum	Maximum
HVS Sample					
Single Family House Price Index	1423	144.44	48.26	67.31	333.53
Single Family Housing Permits (in hundreds)	1423	85.76	90.35	0.52	615.58
Owner Occupied Vacancy Rate (HVS)	1423	1.67	1.00	0.10	6.30
Natural Vacancy Rate (HVS)	1423	8.56	4.21	2.50	21.10
Employment (1,000s)	1351	1006.96	1174.33	16.79	7737.40
Fair Market Rent	1423	715.03	231.19	370.16	1791.65
Population (1,000s)	1423	2439.26	2847.47	105.18	19069.80
Per Capita Income (1,000s)	1150	34.31	8.65	15.59	80.14
Age Adjusted Ownership Rate	1423	56.32	6.97	37.75	78.30
Unemployment Rate	1421	5.62	2.21	1.56	15.87
Unemployment Compensation (millions)	1423	396.46	793.45	2.31	11456.67
Wages (1,000s)	1150	39.08	9.75	19.73	94.75
ACS Sample					
Single Family House Price Index	2465	181.32	37.07	105.03	362.87
Single Family Housing Permits (in hundreds)	2465	20.19	45.64	0.10	615.58
Owner Occupied Vacancy Rate (HVS)	2465	2.34	1.31	0.10	11.90
Employment (1,000s)	2112	283.90	653.85	14.79	7737.40
Fair Market Rent	2465	726.51	191.21	356.17	1730.00
Population (1,000s)	2465	716.26	1596.95	70.26	19069.80
Per Capita Income (1,000s)	2458	35.29	6.79	17.29	80.14
Age Adjusted Ownership Rate	2465	61.05	6.22	38.90	78.30
Unemployment Rate	2463	6.82	2.99	2.07	29.67
Unemployment Compensation (millions)	2458	180.48	567.83	0.59	11456.67
Wages (1,000s)	2458	38.91	6.81	24.44	94.75
Wages – Construction (1,000s)	2443	34.81	8.79	10.48	67.48
Monthly Composite Help Wanted Index					
HWI	13680	98.40	67.27	11.35	628.13
JOLTS Data					
Jobs Vacancy Rate – National	160	2.65	0.41	1.6	3.8
Jobs Vacancy Rate – Northeast	160	2.52	0.36	1.7	4
Jobs Vacancy Rate – Midwest	160	2.39	0.39	1.4	3.8
Jobs Vacancy Rate – South	160	2.82	0.49	1.7	3.9
Jobs Vacancy Rate – West	160	2.77	0.5	1.6	4.3

Table 2: Beveridge Curve Results: 1956-2014
Dependent Variable is Job Vacancy Rate

	OLS	IV	OLS	IV
Variables	(1)	(2)	(3)	(4)
$Unempl^{-1}$	22.600***	22.416***	28.453***	28.560***
	(2.389)	(2.969)	(2.541)	(2.510)
Owner Vacancy	0.967**	0.852	0.639	0.995
	(0.384)	(0.748)	(0.436)	(0.633)
Rental Vacancy	-0.946***	-1.060***	-0.396***	-0.675***
	(0.118)	(0.241)	(0.123)	(0.238)
1 if 1972-1989			2.011***	1.661***
			(0.309)	(0.437)
1 if 1990-2000			0.152	0.170
			(0.223)	(0.246)
1 if 2001-2014			-0.475	-0.152
			(0.429)	(0.606)
Constant	8.619***	9.688***	3.450***	4.977***
	(0.860)	(1.121)	(1.034)	(1.882)
	Elasticities			
Unemployment	-0.52	-0.02	-0.66	-0.66
Owner Vacancy	0.22	0.19	0.14	0.23
Rental Vacancy	-1.00	-1.12	-0.42	-0.72
	IV Test Statistics			
Over ID: p-value		0.42		0.86
Endogeneity: p-value		0.21		0.26
1st Stage F stat:				
Owner Vacancy		23.16		23.16
Rental Vacancy		17.73		17.73
Observations	56	56	56	56
R-squared	0.709	0.693	0.844	0.830

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1
Instruments: 1 unit permits, 2 or more unit permits, 1 unit starts, 2or more unit starts

Table 3: Housing Beveridge Curve Results: 1985-2013

	Dependent Variable			
	Owner Vacancy Rate		Rental Vacancy Rate	
Variables	(1)	(2)	(3)	(4)
Unfulfilled Ownership	14.22	23.99		
	(90.972)	(71.479)		
Unfulfilled Rental			243.16	275.73
			(270.877)	(175.364)
Job Vacancy Index	-0.12*	0.01	-0.52***	-0.17
	(0.061)	(0.064)	(0.146)	(0.126)
1 if 2003-2015		0.60**		1.70***
		(0.207)		(0.404)
Constant	2.36	1.12	4.19	0.21
	(1.876)	(1.533)	(8.560)	(5.616)
	Elasticities			
Unfulfilled Ownership	-0.39	-0.65		
Unfulfilled Rental			-1.49	-1.69
Job Vacancy Index	-0.43	0.03	-0.43	-0.14
Observations	15	15	15	15
R-squared	0.233	0.567	0.538	0.823

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 4 - VAR Regression Results

	Dependent Variable: Vacancy Rates (if First-Differences)		
	Ownership	Rental	Jobs
Vacancies in First Diff			
Ownership_{t-1}	-0.077	-0.090	-0.167
	(0.048)	(0.219)	(0.055)**
Ownership_{t-2}	-0.077	-0.093	0.002
	(0.048)	(0.220)	(0.056)
Rental_{t-1}	-0.002	-0.066	-0.049
	(0.010)	(0.048)	(0.012)**
Rental_{t-2}	-0.004	-0.069	0.034
	(0.011)	(0.048)	(0.012)**
Jobs_{t-1}	-0.067	-0.236	0.323
	(0.033)*	(0.149)	(0.038)**
Jobs_{t-2}	0.026	0.103	-0.210
	(0.031)	(0.143)	(0.036)**
Other Variables in First-Diff			
Unpl^{-1}_{t-1}	-0.661	-2.080	0.351
	(0.459)	(2.091)	(0.528)
Unpl^{-1}_{t-2}	0.019	0.140	-0.244
	(0.443)	(2.019)	(0.510)
$\text{House Price Index}_{t-1}$	-0.002	-0.012	-0.006
	(0.001)	(0.006)*	(0.002)**
$\text{House Price Index}_{t-2}$	0.000	0.001	0.004
	(0.001)	(0.006)	(0.001)**
Constant	0.002	0.026	0.053
	(0.044)	(0.198)	(0.050)
$\hat{\sigma}_{\varepsilon}$	0.233	1.062	0.269
R-squared	0.16	0.37	0.41
Observations	740	740	740

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

10 Appendix A: A Model of Housing Labor Market Vacancies as Joint Outcomes

This appendix details our extension of the theory of housing markets with frictions of Head and Lloyd-Ellis (2012). Our extension involves, primarily, the following components: one, incorporating a frictional labor market along the lines of Pissarides (1985; 2000); two, allowing for a frictional housing market. In addition, we develop a novel concept of unemployment for rental and ownership housing markets, which in turn makes possible to derive Beveridge curves for housing markets. These extensions are critical in helping us structure the empirical investigations reported in the main body of the paper.

10.1 Preferences

Let W^j, U^j, denote expected lifetime utility, conditional on being employed, unemployed, for a renter, homeowner, $j = R, H$ respectively, which is expressed in real terms, and under the assumption of unrestricted borrowing or lending at a fixed rate of interest, ρ. These are generated by utility flow, denoted by π^j, and defined in terms of preference over consumption c , labor supply, l, and housing consumption, z, in a typical period:

$$\pi^j(c, l, z) = c^j - l + z^j, j = R, H. \tag{15}$$

We assume that a person is either employed, supplying one unit of labor, in area i earning w_i, or unemployed, receiving $b_i < w_i$. We suppress the location-specific subscript i, unless it is strictly necessary for clarity.

We allow for the definition of utility flow (15) to account for housing consumption to depend on housing unit costs that differ for renters and owners and to depend on local conditions. Let: non-housing consumption be the numeraire, with its price set equal to 1, and let κ be rent per unit of rental housing. Ignoring commuting costs, the quantity of housing consumed by renters in a particular area is given by rent expenditure divided by κ. Let p_h be the annual user cost of owner-occupied housing. The latter is defined in standard fashion [Poterba (1986); Henderson and Ioannides (1987)], given a housing price index, p, as the annualized user cost of housing per unit of housing value. A dwelling unit of value V generates an annualized user cost of $p_h V$.[19] The respective quantity of housing consumed, that is, housing services, is given by $\frac{p_h V}{p}$. Suppose that there are no taxes, nor maintenance, depreciation, and appreciation, and an individual borrows at the real rate of interest ρ to finance living in a house of value V. She would thus incur annual housing costs equal to the opportunity cost of housing of value V, ρV. Equivalently, since housing is durable, services from an actual housing stock V emanate at the real rate of interest, ρ, and are given by $\rho\frac{V}{p}$. More generally, the user cost of housing reflects the implications of the tax treatment of housing as well as its durability.[20]

Under the assumption of perfect capital markets, with individuals' being able to borrow against their expected future income and to save at rate ρ, the Bellman equations for $W^j, U^j, j =$ H(omeowner), R(enter), may be defined as follows. Spending in a particular period is equal to the wage (or unemployment compensation) rate plus dissaving. This plus spending on non-housing consumption equals the wage rate (or unemployment compensation, as appropriate) plus dissaving. Following Head et al. (2014), Eq. (3), we let the flow

[19]This definition maintains consistency between the rental and the homeownership sectors. However, this could be modified so as to be based on transactions values instead of the vacant unit value. Also property tax rates, maintenance and depreciation rates as well as housing price appreciation rates may be area-specific.

[20]Following Poterba (1986) and Henderson and Ioannides (1987) the user cost of housing reflects mortgage payments at a rate of interest ι, times the portion of the value of a dwelling unit that is financed, $1 - \text{equ}$, and adjusted for the tax deductability of mortgage interest associated with the portion of the value of owner-occupied housing that is leveraged, by multiplying by 1 minus the marginal US income tax rate, $1-\tau$. Property taxes, denoted by rate τ_p here, are also deductible for US income tax purposes. In addition, allowing for maintenance and depreciation, at rates maint and depr, respectively, and deducting the rate of expected housing price appreciation, appr^e, yield the annual user cost of housing as:

$$p_h = \left[(1-\tau)[\iota(1-\text{equ}) + \tau_p] + \text{depr} + \text{maint} - \text{appr}^e\right].$$

This definition maintains consistency between the rental and the homeownership sectors. However, this could be modified so as to be based on transactions values instead of the vacant unit value. Also property taxes rates, maintenance and depreciation rates as well as housing price appreciation rates may be area-specific. In the absence of taxation, maintenance and capital gains, and with $\iota = \rho$, the above definition yields $p_h = \rho$ as a special case of the user cost concept.

of utility be linear in non-housing and housing consumption. For a homeowner, ignoring the disutility of work and allowing for institutional considerations entering through the definition of p_h, utility per period may be written as:

$$\pi^H(w) = w - \rho V + \text{dissaving} + \frac{p_h V}{p}, \tag{16}$$

where $-\rho V$ denotes the opportunity cost (dissaving) associated with holding (durable) housing stock of value V. For a renter:

$$\pi^R(w) = w - \text{rent expenditure} + \text{dissaving} + \frac{\text{rent expenditure}}{\kappa}. \tag{17}$$

For an unemployed individual, b takes the place of w in both sides of the above expressions.

We see the implications of this formulation, in the simplest possible case at the steady state, with renters and owners maintaining their housing tenure status for ever. Let δ denote the job destruction rate and μ the job finding rate (which will be specified in section 10.3 below as a function of labor market tightness). The Bellman equations for the conditional value functions are, first for owners:

$$\rho W^H = \pi^H(w) + \delta[U^H - W^H]; \tag{18}$$

$$\rho U^H = \pi^H(b) + \mu[W^H - U^H]; \tag{19}$$

and correspondingly for renters:

$$\rho W^R = \pi^R(w) + \delta[U^R - W^R]; \tag{20}$$

$$\rho U^R = \pi^R(b) + \mu[W^R - U^R]; \tag{21}$$

where the flow utilities are specified in (16)-(17) above, except that the term dissaving is of course dropped when we integrate from the flow to the stock (the value functions). From now on, we will use $\pi^j, j = H, R$ without the term dissaving.[21] We solve below for the

[21] In contrast to Head and Lloyd-Ellis, our definition of $\pi^H(w)$ in (16) above makes it dependent on V, in general, which is endogenous. We will ignore this endogeneity from now on, when we derive the equilibrium value of V below. However, V cancels out of the expressions for the $\pi^j, j = H, R$ if we assume that $p = 1$, and $p_h = \rho$.

expressions for the conditional value functions, under more general conditions. We also use the resulting solutions to motivate a housing tenure choice estimation, which we detail in section 11 below and use to estimate the probabilities of unfulfilled renters and owners in section 5.3.

The associated steady-state unemployment rate is given by: $\frac{\delta}{\delta+\mu}$. The job destruction rate is typically assumed to be exogenous. It may vary across MSAs because of differences in their industrial compositions. The job finding rate is typically specified in terms of the the job matching process and labor market tightness, to which we come further below. It can reflect individual characteristics, which is relevant at the empirical stage. Housing spells of homeowners are initially assumed to last forever, if job market events and housing tenure events are independent. At such a steady state, we could assume that housing units for renters and owners are perfect substitutes.

10.2 Frictions in Housing Markets

Both housing and labor markets are subject to frictions. The individual (or household, the two terms will be used interchangeably) is subject to the risk of job loss: jobs break up at a Poisson rate δ, and the unemployed individual finds a job at a rate μ, per unit of time. Dwelling units, either of owner-occupancy or renting may be occupied or vacant. Frictions are present in matching of dwelling units and individuals via search, which leads to the determination of vacancy rates for homeownership and rental housing markets. Suppose first, as Head and Lloyd-Ellis (2012) do, that rental units may be found instantaneously and thus frictionlessly, while units for owner-occupancy involve a matching process, frictions. Consequently, the values of vacant units as assets may differ from the transaction prices at which they change owners.

Specifically, let γ denote the rate at which new dwelling units sold by construction firms match with prospective homeowners. Head and Lloyd-Ellis specify γ as the product of the rate at which prospective homeowners match with dwelling units, λ, times the ratio of prospective homeowners to vacant units on the market, ϕ:

$$\gamma = \lambda\phi. \tag{22}$$

Clearly, ϕ and thus γ may vary across areas, and we may introduce a subscript i, when it is necessary for clarity. This definition may be generalized by specifying, in the standard Pissarides fashion, a matching function for individuals and vacant dwelling units.[22] It may also be generalized to account for the time it takes owner-occupied houses to be transferred from one household to another, when turnover in owner-occupied units is allowed; see section 10.2.7 below. The fact that matching in housing markets involves frictions makes housing to some extent illiquid; its value when vacant depends on the speed with which a buyer can be found for dwelling units on the market. The model highlights this effect. We specify further below ϕ, the ratio of buyers to vacant units in the market. In section 10.2.5 below, we allow for frictions in the rental housing market as well.

The population consists of N individuals whose number is assumed to grow at a rate ν. Individuals may be found in one of four different states, employed and unemployed homeowners, N^{WH} and N^{UH}, and employed and unemployed renters, N^{WR} and N^{UR}, respectively. and sum up to the total population:

$$N^{WR} + N^{UR} + N^{WH} + N^{UH} = N. \tag{23}$$

It is more convenient to work with the relative numbers of agents. By using lower case n's, i.e. $n^{WH} = \frac{N^{WH}}{N}$, (23) becomes:

$$n^{WH} + n^{UH} + n^{WR} + n^{UR} = 1. \tag{24}$$

[22] Let $I_{b,t}, I_{s,t}$, denote the stock of buyers searching for houses and the stock of sellers searching for buyers, respectively. Let the matching process be specified in the standard fashion in terms of the Poisson rate of contacts generated, denoted by $\Gamma_t = \Gamma\left(I_{b,t}, I_{s,t}\right)$. So, in general, the rate of arrivals of contacts to the typical dwelling unit in MSA i is: $\gamma = \frac{1}{I_{s,t}}\Gamma\left(I_{b,t}, I_{s,t}\right)$, which under the assumption of constant returns to scale, this may be written as:

$$\gamma = \Gamma(\phi, 1) = \Gamma\left(\frac{I_{b,t}}{I_{s,t}}, 1\right).$$

This differs from the Head and Lloyd-Ellis assumption, (22) above, only because of the nonlinearity, but is consistent with the assumptions typically made about matching models. Parameter λ is subsumed in this formulation.

Let R, H denote the total housing stock, rental and homeownership, respectively. In view of (22), we may write the homeownership market tightness, γ^H, in terms of the matching parameter, λ, and ϕ, is the ratio of the number of potential buyers (employed and unemployed renters) to that of vacant units, the latter being equal to the stock of dwelling units for owner-occupancy that is not owned. That is:

$$\gamma^H = \lambda \frac{N^{WR} + N^{UR}}{H - N^{WH} - N^{UH}}. \tag{25}$$

10.2.1 Housing Supply: The Rental Housing Market

Following Glaeser *et al.* (2014), and Head and Lloyd-Ellis, *op. cit.*, we assume free entry in the housing construction-real estate business and specify a supply equation for rental housing units: the present value of rents equals the asset value of their construction costs, that is:

$$\frac{\kappa}{\rho} = c_0 + c^R \frac{R}{N},$$

where c_0 denotes fixed construction cost, and $c^R \frac{R}{N}$ variable costs that depend linearly on the rental housing stock, R, relative to population, N. We assume initially that the entire stock of rental units are occupied as soon as they are produced, that is, the rental housing market is not subject to frictions. Since all rental units are occupied, $R = N^{WR} + N^{UR}$, the above equation may be rewritten instead in terms of n^{WR} and n^{UR} :

$$\frac{\kappa}{\rho} = c_0 + c^R \left(n^{WR} + n^{UR} \right). \tag{26}$$

Housing is assumed to live for ever, and the rental price κ may be assumed to include maintenance costs. Under the assumption of free entry, we need not worry about the owners of rental housing stock. We modify this equation further below in section 10.2.5 in order to introduce frictions in the rental housing market, too.

10.2.2 Housing Supply: Homeownership Market

The value of units for owner-occupancy when vacant must compensate their producers:

$$V = c_0 + c^H \frac{H}{N}, \tag{27}$$

where $c^H \frac{H}{N}$ denotes variable costs, that depend linearly on the housing stock for homeownership, H, relative to the population, in order to express the cost of land due to congestion. We could also allow for costly conversion of dwelling units from one mode of tenure to another.

We note that the supply equations (26)-(27) link "prices," that is rents and values of vacant units, to their respective stocks relative to the numbers of individuals. Next, we relate supply equations to demand conditions.

10.2.3 The Value of Vacant Housing in the Owner-Occupied Market

The value of vacant dwellings in the ownership market must, at asset equilibrium, reflect the fact that it may be purchased by either employed or unemployed renters, whose willingness to pay may be different. The return per unit of time to holding an asset of value V is equal to the probability per unit of time that it may be sold either to an employed renter, at price P^W, or an unemployed renter, at price P^U, whichever of the two bids is higher:

$$\rho V = \gamma \mathcal{E} \max_j \left\{ P^j - V \right\}, j = W, U. \tag{28}$$

The expectation on the rhs of (28), the arbitrage equation for V, may be written out by recognizing that a unit will either be purchased by an employed renter if $P^W > P^U$, an event that occurs with probability equal to the proportion of those employed among all renters, $\alpha = \frac{N^{WR}}{R}$; correspondingly, the event $P^W < P^U$ occurs with probability $1 - \alpha = \frac{N^{UR}}{R}$.

Consistent with the literature of markets with frictions, a seller and a buyer of a dwelling unit when they come into contact engage in Nash bargaining and split the surplus from the transaction, with a share σ of V going to the seller and $1 - \sigma$ of $W^H - W^R$ to the buyer. So, the prices paid by employed and unemployed households satisfy:

$$P^W = \sigma V + (1-\sigma) \left[W^H - W^R \right]; P^U = \sigma V + (1-\sigma) \left[U^H - U^R \right]. \tag{29}$$

Solving for V from (28) and (29) we have:

$$V = \frac{(1-\sigma)\gamma}{\rho + (1-\sigma)\gamma} \left[\alpha [W^H - W^R] + (1-\alpha)[U^H - U^R] \right]. \tag{30}$$

Recall that we assume here that once renters purchase dwelling units and become homeowners they remain so forever. Their conditional value functions are given by (18)–(19)

above. Renters, on the other hand, are faced with opportunities, at a rate γ, to purchase dwelling units and become homeowners. Thus, the respective Bellman equations, the counterparts of (20)–(21), for the conditional value functions become:

$$\rho W^R = \pi^R(w) + \delta[U^R - W^R] + \gamma \left[W^H - P^W - W^R\right]; \tag{31}$$

$$\rho U^R = \pi^R(b) + \mu[W^R - U^R] + \gamma \left[U^H - P^U - U^R\right]. \tag{32}$$

We may modify the model to allow for interdependence between employment and housing tenure mode transitions, but for the moment, such transitions are assumed to be independent. However, conditions in the housing market have a profound effect on the conditional value functions.

Next, we use (30) in order to express the transaction prices, P^W, P^R, in terms of the conditional value functions, W^H, W^R, and U^H, U^R. We substitute back into the Bellman equations, (18-19) for owners, and (31-32) for renters, solve for the conditional value functions, namely for W^H, W^R, U^H, U^R, as functions of the real wage rate and unemployment compensation, on one hand, and of labor market and housing market tightness, on the other. Labor market tightness enters the job finding rate for owners and renters, as we discuss in more detail in section 10.3 below.

10.2.4 Housing Market Flows and Conditional Value Functions

Given wages and other magnitudes including the value of vacant units for sale from (30), which are determined from the model of labor markets with frictions, we may solve the model. The transaction prices for owner-occupied units, P^W, P^R, may be expressed in terms of the four conditional value functions, W^H, W^R, U^H, U^R. There are seven unknowns, namely:

- the total stock for owner-occupancy, H; in per capita: $h = \frac{H}{N}$;
- the rental stock, R; in per capita: $r = \frac{R}{N}$;
- the rent, κ;
- and the four numbers of agents in different states, $N^{WR}, N^{UR}, N^{WH}, N^{UH}$.

By solving equ. (18), (19), (31) and (32) for the conditional value functions, we can express the value of a vacant home, V, in terms of the four unknown numbers of agents in different states, , $N^{WR}, N^{UR}, N^{WH}, N^{UH}$, the unknown housing rental price, κ, and the wage rate.

It is more convenient to think of the model in a steady state, with the number of individuals growing at an exogenous rate ν. Along the steady state, all absolute quantities grow at the same rate leaving relative numbers of agents constant. This leads to four flow relationships in terms of relative numbers of agents: First, all N individuals distribute themselves over the four states, so that their respective relative numbers sum up to 1: (24) holds.

Second, the change in the number of employed renters in a given city i, $\frac{dN^{WR}}{dt}$ equals the number of unemployed renters who become employed, μN^{UR}, minus the measure of employed renters whose jobs are destroyed, δN^{WR}, and minus those who cease to be renters because they become owners, λN^{WR}. That is:

$$\frac{dN^{WR}}{dt} = \mu N^{UR} - (\delta + \lambda) N^{WR}.$$

Dividing by N and imposing the condition that for a steady state, $\frac{dN^{WR}}{dt} = \nu N^{WR}$, and rewriting, yields:

$$(\nu + \delta + \lambda) n^{WR} - \mu n^{UR} = 0. \tag{33}$$

Third, working in like manner, the change in the relative number of unemployed homeowners, $\frac{dN^{UH}}{dt} = \nu N^{UH}$, is equal to minus those unemployed homeowners who find jobs, μn^{UH}, plus the number of those employed homeowners who lose their jobs, plus the number of unemployed renters who become homeowners, λn^{UR}. Rewriting, we have:

$$(\mu + \nu) n^{UH} - \delta n^{WH} - \lambda n^{UR} = 0. \tag{34}$$

Fourth, the increase in the number of employed homeowners, νn^{WH}, is equal to the number of unemployed homeowners getting jobs, μn^{UH}, plus the number of employed renters who become homeowners, λn^{WR}, minus those employed homeowners who become unemployed. Rewriting, we have:

$$\nu n^{WH} + \delta n^{WH} - \lambda n^{WR} - \mu n^{UH} = 0. \tag{35}$$

By rewriting the above equations in matrix form, we have:

$$\begin{bmatrix} \delta+\lambda+\nu & -\mu & 0 & 0 \\ 0 & -\lambda & -\delta & \mu+\nu \\ -\lambda & 0 & \delta+\nu & -\mu \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} n^{WR} \\ n^{UR} \\ n^{WH} \\ n^{UH} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}. \tag{36}$$

The matrix on the l.h.s. of (36) depends on the parameters $(\delta, \lambda, \mu, \nu)$ only. This yields the solution:

$$n^{WR} = \frac{\mu\nu}{(\lambda+\nu)(\lambda+\nu+\delta+\mu)}, n^{UR} = \frac{\nu(\lambda+\nu+\delta)}{(\lambda+\nu)(\lambda+\nu+\delta+\mu)}; \tag{37}$$

$$n^{WH} = \frac{\lambda\mu(\lambda+\nu+\delta)+\lambda\mu(\mu+\nu)}{(\lambda+\nu)(\delta+\mu+\nu)(\lambda+\nu+\delta+\mu)}, n^{UH} = \frac{\lambda\delta(\lambda+\nu+\delta+\mu)+\lambda\nu(\delta+\lambda+\nu)}{(\lambda+\nu)(\delta+\mu+\nu)(\lambda+\nu+\delta+\mu)}. \tag{38}$$

With these results, the share of employed renters, $\alpha = \frac{n^{WR}}{n^{WR}+n^{UR}}$, and the unit matching rate introduced in (25) are given by:

$$\alpha = \frac{\mu}{\lambda+\nu+\delta+\mu}; \tag{39}$$

$$\gamma^H = \lambda \frac{\frac{\nu}{\lambda+\nu}}{h - \frac{\lambda}{\lambda+\nu}}, \tag{40}$$

where $h = \frac{H}{N}$, is the owner-occupied stock per capita. In equilibrium, the denominator above must be positive. The model implies an equilibrium homeownership rate,

$$\mathrm{hr} = n^{WH} + n^{UH} = \frac{\lambda}{\lambda+\nu}. \tag{41}$$

The equilibrium rental rate is thus equal to $\frac{\nu}{\lambda+\nu}$.

We note that these rates depend critically on the rate of growth of the population. In contrast, the equilibrium unemployment rate does not depend on population growth. Clearly, in order to arrive at a more general model, we need to specify churning within the housing market. We go some way further in this direction below in sections 10.2.5 and 10.2.7. In section 10.2.5, we also relax the assumption that all renters seek to become homeowners, which boosts the equilibrium rental rate, and we rework the system of equations of (36) accordingly.

The conditional value functions for homeowners may be obtained by solving (18–19). Thus, we have:

$$W^H = \frac{1}{\rho(\delta+\mu+\rho)}\left[(\rho+\mu)\pi^H(w^H)+\delta\pi^H(b)\right]. \tag{42}$$

$$U^H = \frac{1}{\rho(\delta+\mu+\rho)}\left[\mu\pi^H(w^H)+(\delta+\rho)\pi^H(b)\right]. \tag{43}$$

These expressions allow for the possibility that bargaining between firms and workers may lead to wage rates that are different between renters and owners, w^H, w^R, respectively, from now on.

We can solve for the conditional value functions for renters, (31-32), after we have expressed the transaction prices for vacant units in terms of the conditional value functions. Recall (29), which gives give transaction prices via Nash bargaining. From (29) and (30), we have instead of (31), (32), respectively:

$$\rho W^R = \pi^R(w) + \delta[U^R - W^R]$$
$$+\gamma^H\frac{\sigma\rho+\sigma(1-\sigma)\gamma^H(1-\alpha_1)}{\rho+(1-\sigma)\gamma^H}\left[W^H-W^R\right]-\gamma^H\frac{\sigma(1-\sigma)\gamma^H(1-\alpha_1)}{\rho+(1-\sigma)\gamma^H}\left[U^H-U^R\right]; \tag{44}$$

$$\rho U^R = \pi^R(b) + \mu[W^R - U^R]$$
$$+\gamma^H\frac{\sigma\rho+\sigma(1-\sigma)\gamma^H\alpha_1}{\rho+(1-\sigma)\gamma^H}\left[U^H-U^R\right]-\gamma^H\frac{\sigma(1-\sigma)\gamma^H\alpha_1}{\rho+(1-\sigma)\gamma^H}\left[W^H-W^R\right]. \tag{45}$$

These equations in concise matrix notation become:

$$\begin{bmatrix} \rho+\delta+\sigma\gamma^H\frac{\rho+(1-\sigma)\gamma^H(1-\alpha_i)}{\rho+(1-\sigma)\gamma^H} & -\left(\delta+\frac{\sigma(1-\sigma)(\gamma^H)^2(1-\alpha_i)}{\rho+(1-\sigma)\gamma^H}\right) \\ -\left(\mu+\frac{\sigma(1-\sigma)(\gamma^H)^2\alpha_i}{\rho+(1-\sigma)\gamma^H}\right) & \rho+\mu+\sigma\gamma^H\frac{\rho+(1-\sigma)\gamma^H\alpha_i}{\rho+(1-\sigma)\gamma^H} \end{bmatrix}\begin{bmatrix} W^R \\ U^R \end{bmatrix}$$
$$=\begin{bmatrix} \pi^R(w^R)+\frac{\gamma^H\sigma\rho W^H}{\rho+(1-\sigma)\gamma^H}+\frac{(\gamma^H)^2\sigma(1-\sigma)(1-\alpha_i)}{\rho+(1-\sigma)\gamma^H}\left[W^H-U^H\right] \\ \pi^R(b)+\frac{\gamma^H\sigma\rho U^H}{\rho+(1-\sigma)\gamma^H}-\frac{(\gamma^H)^2\sigma(1-\sigma)\alpha_i}{\rho+(1-\sigma)\gamma^H}\left[W^H-U^H\right] \end{bmatrix} \tag{46}$$

While a full solution of (46) is straightforward and would allow us to compute agents' expected lifetime utility for the purpose of welfare analysis, it is helpful to solve for $W^R - U^R$, which is a crucial input to the bargaining problem between firms and unemployed renters.

$$W^R(w^R;b)-U^R(w^R;b)=\frac{1}{D}\frac{\rho(\rho+\gamma^H)}{\rho+(1-\sigma)\gamma^H}\left(w^R-b+\gamma^H\sigma\rho\left[W^H(w^H;b)-U^H(w^H;b)\right]\right), \tag{47}$$

where D denotes the determinant of the matrix on the LHS of (46). Naturally, parameters and all variables reflecting housing market conditions through α and γ^H that enter the matrix in (46).

To recapitulate, the conditional value functions (W^H, U^H, W^R, U^R) have been solved in terms of the wage rates, w^H, w^R, the unit matching rate, γ^H, which in view of (40) depends on h, and the labor market tightness that enters via the employment rate, μ. From (26) and in view of (37), the rent κ is determined as a function of the share of renters $n^{WR} + n^{UR} = \frac{\nu}{\lambda+\nu}$, and thus is determined and is exogenous. Finally, from (30), V may be expressed, via the conditional value functions, in terms of the wage rates, w^H, w^R, labor market tightness (via the job finding rate μ) and the unit matching rate $\gamma^H = \lambda \frac{\frac{\lambda}{\lambda+\nu}}{h-\frac{\nu}{\lambda+\nu}}$. These derivations when used in (27) yield an equation for the relative stock of owner-occupied units, h. Finally, the equilibrium is fully determined once the wage rates are set. We turn to this further below, which requires looking at the labor market with frictions, after we introduce frictions in the rental housing market, too. Head and Lloyd-Ellis assume frictionless rental housing and labor markets.

10.2.5 Allowing for Frictions in Rental Markets

The model so far treats the rental housing market as frictionless. However, rental housing units may be vacant, and in fact data on rental market vacancies are also available. The purpose of this extension of the Head and Lloyd-Ellis model is to introduce rental market vacancies, which helps structure the use of such data in our estimation. To the variables denoting the relative numbers of individuals in different labor market states, $(n^{WR}, n^{UR}, n^{WH}, n^{UH})$, and the housing stocks for owner-occupancy and renting, H and R, we need to add the stocks of vacant units, υ^H and υ^R, respectively. We extend the model of housing market frictions by also allowing for mismatch of owners, that is, given their circumstances some owners would rather be renting. Similarly, for renters, that is, given their circumstances some renters would rather be owning. Both types of mismatch express financial and mobility frictions. Let the numbers of mismatched individuals be $N_{u,own}$ and $N_{u,rent}$, respectively — unfulfilled owners and renters, respectively. Let the respective shares of mismatched renters, who would rather own, and owners, who would rather rent, be denoted by msm^R and msm^H, respectively, be defined as follows:

$$\text{msm}^R = \frac{N_{u,rent}}{N^{WR}+N^{UR}}, \ \text{msm}^H = \frac{N_{u,own}}{N^{WH}+N^{UH}}, \tag{48}$$

If mismatched renters are introduced, $msm^R \neq 0$, but not mismatched owners, $msm^H = 0$, the first three equations in (36) continue to hold with the modification that instead of λ, the rate at which prospective buyers find dwelling units, we now have λmsm^R.

By definition, the rental housing stock may be occupied by employed or unemployed renters or be vacant. It thus satisfies

$$r = n^{WR} + n^{UR} + \frac{\upsilon^R}{R} r. \tag{49}$$

Accordingly, we rewrite (26), the supply equation for rental housing stock, for the expected value of a vacant rental unit, V^R,

$$V^R = c_0 + c^R \left(n^{WR} + n^{UR} + \frac{\upsilon^R}{R} r \right). \tag{50}$$

The matching model for rental housing units, to be developed shortly, allows us to obtain an expression for the "demand" for rental housing units.

Introducing owner mismatch in the form of unfulfilled renters, $msm^H \neq 0$, requires a greater modification of the model. That is, there are now transitions of owners, unemployed and employed, into renters. This needs to be accounted for in the Bellman equations and in eq. (36) that determine the equilibrium distribution of agents across states. The number of employed and of unemployed renters must account for inflow of employed and unemployed owners who are mismatched.

The algebra is straightforward but tedious. Specifically, it involves two steps. First, in eq. (33)–(35) λ is replaced by $\lambda^R \equiv \bar{\lambda}^R(1 - \text{msm}^R)$, where $\bar{\lambda}^R$ denotes the rate at which renters make contacts with dwelling units for sale. Second, the term $\lambda^H n^{WH}$, where $\lambda^H \equiv \bar{\lambda}^H \text{msm}^H$, is added to the lhs of (33), the term $\lambda^H n^{UH}$ is added to the lhs of (34), and the term $-\lambda^H n^{WH}$ is added to the rhs of (35), where $\bar{\lambda}^R$ denotes the rate at which owners make contacts with dwelling units for renting. The resulting counterpart of eq. (36) is more complicated but still straightforward to solve.[23] The extended model has the advantage that owning is no longer an absorbing state and a nonzero probability of renting is possible even if $\nu = 0$, which removes a drawback of the previous model.

These definitions allow us to complete the determination of the values of rental housing units, V_v^R, V_o^R. At equilibrium, vacant and occupied rental units must earn the market return, that is:

$$\rho V_v^R = \text{maint} + \gamma^R \left[V_o^R - V_v^R \right]; \tag{52}$$

$$\rho V_o^R = \kappa + \lambda \frac{\text{msm}^R (N^{WR} + N^{UR})}{H - N^{WH} - N^{UH}} \left[V_v^R - V_o^R \right], \tag{53}$$

where the matching rates with dwelling units of prospective renters and of prospective owners, γ^R and γ^H, are now defined as:

$$\gamma^R = \lambda \frac{\text{msm}^H (N^{WH} + N^{UH})}{R - N^{WR} - N^{UR}}; \; \gamma^H = \lambda \frac{\text{msm}^R (N^{WR} + N^{UR})}{H - N^{WH} - N^{UH}}. \tag{54}$$

By solving this system of linear equations (52)– (53) in terms of (V_v^R, V_o^R) we obtain an expression for the expected value of a vacant rental unit, V^R, expressing demand. That is:

$$V^R = \frac{\upsilon^R}{R} V_v^R + \left(1 - \frac{\upsilon^R}{R} \right) V_o^R. \tag{55}$$

The system of equations (49), (50) and (55) determines the remaining endogenous variables, the vacancy rate and the rental capita stock per capita, $(\frac{\upsilon^R}{R}, \frac{R}{N})$. Here we take κ, the housing rent as given.[24] Since $\text{msm}^H, \text{msm}^R$, maint are given, the solutions for $(n^{WR}, n^{UR}, n^{WH}, n^{UH})$ obtained above allow us to solve explicitly for the vacancy rate in rental housing markets, $\frac{\upsilon^R}{R}$.

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[23]System (36) becomes:

$$\begin{bmatrix} \delta + \lambda^R + \nu & -\mu & \lambda^H & 0 \\ 0 & -\lambda^R & -\delta & \mu + \nu + \lambda^H \\ -\lambda^R & 0 & \delta + \nu - \lambda^H & -\mu \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} n^{WR} \\ n^{UR} \\ n^{WH} \\ n^{UH} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}. \tag{51}$$

[24]Rental housing transactions also involve landlords and prospective tenants coming into contact. For symmetry with our treatment of the ownership market, we could specify the determination of the rent, κ, by means of bargaining between landlords and prospective tenants. But the agreements typically involve relatively shorts spells of stay, and it is thus appropriate to assume that κ is determined competitively.

10.2.6 Housing Beveridge Curves

The Beveridge Curve for labor markets is a well-established and a widely researched concept. See Pissarides (1985; 1986) and Blanchard and Diamond (1989). The intuitive similarities between housing and labor markets behoove us to exploit analogies in order to obtain a Beveridge Curve for housing. Analogous to vacancies in labor markets, which is unsatisfied demand for workers by firms, there correspond prospective buyers and prospective renters in housing markets, which is unsatisfied demand by individuals for housing. Analogous to unemployed individuals, which is unsatisfied demand for employment by individuals, there are unsatisfied renters who wish to own, and owners who wish to rent. There are prevented from doing so by frictions. Our development of Beveridge Curves for housing markets is adapted to the institutional features of housing markets, where there are owners and renters and adheres to the notion of Beveridge curve as an accounting relationship at the steady state.

We work first with the homeownership market; the vacancy rate, vown, is given by (**??**). We next express it in terms of a concept that serves as the unemployment counterpart in housing markets. We allow for mismatch among renters giving rise to unsatisfied homeownership demand, the solutions for n^{WH} and n^{UH} depend on $\lambda \mathrm{msm}^R$ instead of just λ and thus on the incidence of mismatch. Working with the solution (see Appendix A, (41)), and assuming that $\mathrm{msm}^H = 0$, we have that the equilibrium homeownership rate is:

$$n^{WH} + n^{UH} = \frac{\lambda \mathrm{msm}^R}{\lambda \mathrm{msm}^R + \nu}. \tag{56}$$

The equilibrium homeownership rate decreases with the probability of mismatch. That is, an increase, due to mismatch of renters, in the number of individuals searching to buy homes reduces the homeownership rate.[25]

[25] In view of the generalization of the matching model in footnote 3 above, the rate at which buyers contact dwelling units, λ, may be written in terms of the matching function $\Gamma(.,.)$, and the ratio of potential buyers to vacant units, ϕ. That is:

$$\lambda = \Gamma(1, \phi^{-1}).$$

In developing a Beveridge curve for the homeownership market, we propose the concept of the unfulfilled homeownership rate as the counterpart of the unemployment rate and normalize it appropriately. We start with the definition of

$$\mathrm{uhr} = \frac{N_{u,rent}}{N_{u,rent} + N^{WH} + N^{UH}},$$

where $N_{u,rent}$ is defined as the number of renters who prefer to own, a quantity that we impute based on a tenure choice estimation, and $N^{WH} + N^{UH}$ all self-reported owners. This quantity may be at most equal to the rental rate, and therefore normalizing it by the rental rate yields the relative unfulfilled homeownership rate,

$$\mathrm{ur}^H = \frac{\mathrm{uhr}}{n^{WR} + n^{UR}}. \tag{57}$$

This serves as our analog of the unemployment rate for the ownership market. It ranges between 0 and 1, if all renters wish to become owners, which Head and Lloyd-Ellis assume.

By manipulating the definitions we may express uhr in terms of the n's. That is:

$$\frac{\mathrm{uhr}}{1 - \mathrm{uhr}} = \mathrm{msm}^R \frac{n^{WR} + n^{UR}}{n^{WH} + n^{UH}}.$$

Solving the flow equations and using (2) yields the analog of the Beveridge curve for the ownership market:

$$\mathrm{vown} = 1 - \frac{1}{h} + \frac{1}{h}\frac{\nu}{\mu}\frac{1}{\mathrm{ur}^H}. \tag{58}$$

Thus, the Beveridge curve for the ownership market is a decreasing function of ur^H, the respective ownership "unemployment rate," which agrees with the Beveridge curve for labor markets.[26] In this expression, the owner-occupied housing stock per capita, h, is endogenous, which may cause the Beveridge curve to shift and tilt by the cyclical variation in h.

Turning to the rental market, we propose the concept of the unfulfilled rental rate as the analog of the unemployment rate for the rental market. We start with the definition of the auxiliary quantity

$$\mathrm{urr} = \frac{N_{u,own}}{N_{u,own} + N^{WR} + N^{UR}},$$

[26]The expression in (58) is modified if $\mathrm{msm}^H \neq 0$, but its property with respect to ur^H is not affected.

where $N_{u,own}$ is defined as the number of owners who prefer to rent, a quantity that we impute based on the same tenure choice estimation as the one used for renters, and $N^{WR} + N^{UR}$ all self-reported renters. This quantity may be at most equal to the ownership rate, if all owners wish to be renters. Normalizing it by the ownership rate yields the relative unfulfilled rental rate,

$$\mathrm{ur}^R = \frac{\mathrm{urr}}{n^{WH} + n^{UH}}. \tag{59}$$

This serves as our analog of the unemployment rate for the rental market. ur^R ranges between 0, the assumption made by Head and Lloyd-Ellis, and 1, which would mean that all owners wish to become renters.

By manipulating the definitions we have an expression for urr in terms of the n's. By manipulating the definitions we may express uhr in terms of the n's. That is:

$$\frac{\mathrm{urr}}{1 - \mathrm{urr}} = \mathrm{msm}^H \frac{n^{WH} + n^{UH}}{n^{WR} + n^{UR}}.$$

From the solution of the flow equations we have expressions for the n's in terms of parameters, including the imputed shares of mismatched renters and owners, $\mathrm{msm}^R, \mathrm{msm}^H$. Working from (3) we obtain an expression for the Beveridge Curve for rental housing markets:

$$\mathrm{vrent} = 1 - \frac{1}{r} + \frac{1}{r}(n^{WH} + n^{UH}) = 1 - \frac{1}{r} + \frac{1}{r}\frac{\mathrm{urr}}{\mathrm{ur}^R}. \tag{60}$$

In general, both msm^R and msm^H enter the expression for urr. Since renting and owning are interdependent, in the most general case, it is not surprising that the vacancy rates share parameters. As with the vacancy rate in the homeownership market, the rental vacancy rate depends on r, the rental housing stock per person, which is endogenous and varies procyclically, thus shifting and tilting the rental Beveridge curve.

10.2.7 Residential Moving and Intra-city Turnover

In the model so far, owners stay for ever in the houses they buy. Yet, it is common experience that owners move, as their circumstances change, by selling their homes to buy other homes or to rent. We generate turnover by homeowners by adopting a modeling trick due to Wheaton (1990), also used by Head and Lloyd-Ellis and Ngai and Sheedy, namely that homeowners suffer a taste, or demographic shock. That is, we assume with probability equal to msm[27] home-owners experience housing taste shocks. Upon experiencing a shock, the service flow a home-owner receives from their current home falls permanently to $\pi^H - \epsilon$, *cet. par.,* while the service flow potentially available to them from other houses remains equal to π^H. Analytically, this is analogous to the exogenous job destruction rate that so simplifies the DMP model. This creates a mismatch, and all such *mismatched* owners immediately become potential buyers, search for a new house and match with vacant houses via the same technology as renters. Once they find a new house, they immediately sell their old house to a real estate firm at the market price for vacant units. The real estate firm sells the new house at a price that reflects the usual bargaining outcome:

$$P_i^{WH} = (1-\sigma)\left[W^H - \tilde{W}^H\right] + \sigma V_i, P_i^{UH} = (1-\sigma)\left[U^H - \tilde{U}^H\right] + \sigma V_i,$$

where $\tilde{W}^H, \tilde{U}^H$ denote the conditional value functions for mismatched owners. We need to express this new set of possibilities by rewriting the Bellman equations for home-owners as follows:

$$\rho W^H = \pi^H(w^H) + \delta(1-\mathrm{msm})[U^H - W^H] + \mathrm{msm}\ \delta_m \left[W^H - \tilde{W}^H\right];$$

$$\rho U^H = \pi^H(b) + \mu(1-\mathrm{msm})[W^H - U^H] + \mathrm{msm}\ \delta_m \left[U^H - \tilde{U}^H\right],$$

and for the mismatched owners, the conditional value functions satisfy:

$$\rho \tilde{W}^H = \pi^H(w^H) - \epsilon + \delta[\tilde{U}^H - \tilde{W}^H] + \gamma_i \left[W^H - P_i^{WH} + V_i - \tilde{W}^H\right];$$

$$\rho \tilde{U}^H = \pi^R(b) - \epsilon + \mu[\tilde{W}^H - \tilde{U}^H] + \gamma_i \left[W^H - P_i^{UH} + V_i - \tilde{U}^H\right].$$

To these new set of possibilities, there correspond steady state unemployment and home-ownership rates.

10.3 The Labor Market with Frictions

The model so far has taken as given the wage rate and the employment rate. The treatment that follows completes the analysis by employing the same preference structure to examine

[27] Note, there is no superscript, to distinguish from msm^H, msm^R.

symmetrically the labor market under frictions. Since housing market magnitudes enter the analysis, it follows that housing market outcomes show up as determinants of wages and the unemployment rate. That is, we embed the above model of individuals into a DMP model, by following the pared down approach of Pissarides (1985), as presented in Pissarides (2000), the canonical equilibrium model of search unemployment.

10.3.1 Labor market flows

Consider a labor market in a steady state with a fixed number of labor force participants, L who are either employed or unemployed. Time is continuous and agents have infinite time horizons. Recalling the basic details, jobs are destroyed at the exogenous rate δ; all employed workers thus lose their jobs and enter unemployment at the same rate. Unemployed workers enter employment at the rate μ which is endogenously determined, as we see shortly below. Frictions in the labor market are modeled by a matching function of the form

$$M = \mathcal{M}(uN, \upsilon N), \tag{61}$$

where uN is the number of unemployed workers, a stock, and υN the number of job vacancies, also a stock. The matching function is taken as increasing in both arguments, concave and exhibiting constant returns to scale.

Unemployed workers find jobs at the rate

$$\mu = \frac{\mathcal{M}(uN, \upsilon N)}{uN} = \mu(\theta),$$

where $\theta \equiv \frac{\upsilon}{u}$ is labor market tightness. It follows that firms fill vacancies at the rate

$$q = \frac{\mathcal{M}(uN, \upsilon N)}{\upsilon N} = \mathcal{M}\left(\left(\frac{\upsilon}{u}\right)^{-1}, 1\right) = q\left(\frac{\upsilon}{u}\right) = q(\theta). \tag{62}$$

It follows[28] that:

$$\mu'(\theta) > 0, \; q'(\theta) < 0.$$

[28]By definition:

$$\mu(\theta) = \theta q(\theta).$$

Differentiating with respect to θ we have:

$$\mu'(\theta) = q(\theta) + \theta q'(\theta).$$

The intuition is straightforward: the tighter the labor market, the easier it is for workers to find a job, and the more difficult for firms to fill a vacancy. A steady state in the labor market requires that the unemployment rate is unchanging over time. This occurs when the inflow from employment into unemployment, $\delta(1-u)N$, equals the outflow from unemployment to employment, $\mu(\theta)uN$. The steady-state unemployment rate[29] is thus given as:

$$u = \frac{\delta + \nu}{\delta + \mu(\theta) + \nu}. \tag{64}$$

Since $\mu(\theta)$ is increasing in its argument, this equation also implies a negative relationship, at the *steady state*, between unemployment and vacancies known as the Beveridge curve, typically depicted on an unemployment rate – vacancy rate space, (u, v) space. In an important sense, this is a mechanical accounting relationship, the consequences of flow balance, and it is this feature that we sought to emulate in section 10.2.6 above in defining Beveridge curves for housing markets.

A deterioration of matching efficiency, i.e., a decline in job finding given a certain level of

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From the definition of q the second term above becomes:

$$\theta q'(\theta) = -\theta \frac{\partial \mathcal{M}}{\partial (uN)} \left(\frac{v}{u}\right)^{-2} = -\frac{\partial \mathcal{M}}{\partial (uN)} (\frac{v}{u})^{-1} < 0.$$

Greater labor market tightness reduces the rate at which firms fill their vacancies. Therefore,

$$\mu'(\theta) = \mathcal{M}\left(\theta^{-1}, 1\right) - \theta^{-1} \frac{\partial \mathcal{M}}{\partial (uN)} > 0.$$

The inequality follows from the concavity of $\mathcal{M}(\cdot,\cdot)$ by a simple geometric argument, provided that $\lim_{ul \to 0} \mathcal{M}(uN, vN) \to \infty$.

[29]The full dynamic equation for the unemployment rate readily follows: At any point in time, $(1-u)N$ people are employed. Of these, per unit of time, $(1-u)N\delta dt$ people lose their jobs and enter unemployment. Again, during time dt, $uN\mu(\theta)$ are finding jobs, thus reducing the ranks of the unemployed and νdtN people enter the economy and become unemployed. Consequently,

$$d(un) = Ndu + udN = (1-u)N\delta dt - uN\mu(\theta)\, dt + \nu N dt.$$

Using the fact that $\frac{dN}{N} = \nu dt$ and rewriting this as a differential equation we have:

$$N\dot{u} + u\nu N = (1-u)N\delta - uN\mu(\theta) + \nu N. \tag{63}$$

The Beveridge curve follows if we impose the condition that the unemployment rate remains constant, equilibrium unemployment $\dot{u} = 0$.

tightness, involves an outward shift of the Beveridge curve in the (u, v) space. An increase in the job destruction rate, possibly induced by faster sectoral reallocation of jobs, is also associated with an outward shift of the Beveridge curve. The Beveridge curve, computed using U.S. monthly data on unemployment and vacancies, is regularly reported by BLS and is based on its Job Openings and Labor Turnover Survey (JOLTS) [`www.bls.gov/ljt`]. The movements in the unemployment rate, u, measured here as unemployment divided by the labor force, and v, the job openings (vacancy) rate, measured here as openings divided by employment plus openings. Monthly observations are used to track the business cycle. See Figures 4 and 5 below.

During the Great Recession, a marked outward shift in the Beveridge curve has been observed. Earlier recessions were also associated with such shifts, though not as pronounced.[30] The reasons for this shift are not yet fully understood. However, it is clear that the curve is turning around, exactly as predicted by Pissarides' theory. We come to that shortly below. This feature of the observed Beveridge curve has consequences for the housing market, and it is one of the aims of the present paper to explore it fully.

10.3.2 The Behavior of Workers at Bargaining

Our benchmark model assumes exogenous search effort. Workers' decisions can influence unemployment only through their impact on wage setting. Workers care about their expected present values of incomes and recognize that these values depend on labor market transition rates as well as wages while employed and unemployment benefits while unemployed.

We work with the value functions developed before. Recall $U^j, j = H, R$ denote the expected present value of income of an unemployed worker and $W^j, j = H, R$, the corresponding present value of an employed worker. Equ. (42-43) for owners, and Equ. (46), for renters, are solved for the quantities, namely for W^H, W^R, U^H, U^R, as functions of the real wage rate, unemployment compensation, and labor market and housing market tightness. The solutions for the conditional value functions enter the bargaining model. Here we are

[30]For recent discussions of shifts in the Beveridge curve for labor markets, see Elsby *et al.* (2014) and Diamond and Sahin (2014).

faced with a modeling choice, namely whether or not to make wage determination conditional on whether the individual is a renter or an owner. That is, in modeling bargaining between prospective workers and employers whether we assume that the benefit from employment for the worker is either $W^H - U^H$ or $W^R - U^R$. If we assume that bargaining is conditional of housing tenure status, then the resulting labor market tightness will also be similarly conditional. The spirit of bargaining theory in the context of search models suggests that the tenure status ought to affect bargaining, because the utility of the threat point does. Of course, there is no reason that otherwise equivalent labor should be discriminated in employment decisions, unless of course they have different reservation wages. If we were to posit that the benefit from employment, for the purpose of bargaining, WS, is the expected surplus, it is expressed as follows:

$$\mathrm{WS} = \frac{n^{UR}}{n^{UR} + n^{UH}}[W^R - U^R] + \frac{n^{UH}}{n^{UR} + n^{UH}}[W^H - U^H]. \tag{65}$$

Still, the logic of the bargaining model requires that the transactions price be dependent on the buyer's being employed or unemployed, as expressed by (29) above, so in wage bargaining the same logic requires that we treat firms' bargaining with workers separately for homeowners and renters.[31]

10.3.3 The Behavior of Firms at Bargaining

Jobs are created by firms that decide to open new positions. Job creation involves some costs and firms care about the expected present value of profits, net of hiring costs. The unit value of a firm's output is p_g. Assume, as is standard in this literature, that firms are small, in the sense that each firm has only one job that is either vacant or occupied by a worker. There is a flow cost, $p_g c$, associated with a vacancy, per unit of time. Note it is defined in terms of the value of the output. Let V_u denote that expected present value of having a vacancy unfilled and V_f the corresponding value of having a job occupied, a vacancy filled, by a worker. A job vacancy is an asset from which the firm expects to make money. A job

[31] A way to get around this awkward step would be to assume competitive search, as modeled by Diaz and (2013) and Moen (1997).

vacancy is filled at the rate $q(\theta)$, whereas an occupied job is destroyed at the rate δ. The value functions associated with a vacancy and a filled job satisfy, respectively, the following equations:

$$\rho V_u = -p_g c + q(\theta)(V_f - V_u). \tag{66}$$

$$\rho V_f = p_g - w + \delta(V_u - V_f), \tag{67}$$

where p_g denotes the price of the good produced, which for consistency with the earlier part of the paper can be set equal to 1, as the good is the numeraire. The l.h.s. of (66) is the opportunity cost per unit of time of a vacancy. Its r.h.s. is the expected return, when costs are incurred per unit of time, $p_g c$, plus the expected capital gain from a job vacancy's getting filled, $q(\theta)(V_f - V_u)$. Similarly, the l.h.s. of (67) is the opportunity cost per unit of time of a filled vacancy. Its r.h.s. is the expected return, which consist of output minus the wage rate, profit per unit of time, $p_g - w$, plus the expected capital gain from a job's becoming vacant, $\delta(V_u - V_f)$.

Hiring by firms is done indirectly by firms' opening vacancies. Firms open vacancies as long as it is profitable to do so. As firms open up vacancies, the value of a vacancy decreases. At the free entry equilibrium, $V_u = 0$.

So, solving (66) in terms of V_u and setting $V_u = 0$, yields an equation for V_f :

$$V_f = \frac{p_g c}{q(\theta)}. \tag{68}$$

Solving (67) for V_f and setting $V_u = 0$, yields a second equation for V_f, $V_f = \frac{p_g - w}{\rho + \delta}$. From these two equations for V_f we have:

$$w = p_g - (\rho + \delta)\frac{p_g c}{q(\theta)}. \tag{69}$$

Here, p_g is the value of the marginal product of labor. Once filled, each job produces a unit of output per unit of time. It is equal to the wage rate plus the capitalized value of the firm's hiring cost. A vacancy once created is expected to last for $q(\theta)^{-1}$ periods of time, generating costs $\frac{pc}{q(\theta)}$. Each vacancy is created with probability δ per unit of time and the hiring cost incurs an interest cost at a rate ρ. So, So, the capitalized value of the firm's hiring cost is given by $(\rho+\delta)\frac{p_g c}{q(\theta)}$. From this relationship, w as a function of θ in (θ, w) space is downwardsloping.

Equ. (69) is referred to as the *job creation* condition. It plays the role of the demand for labor in the standard model of a labor market without frictions, where the quantity of labor is represented by labor market tightness, $\theta = \frac{v}{u}$, the ratio of the vacancy rate to the unemployment rate. Note that in equilibrium, from Equ. (69), that given p_g, w, the incentives to create vacancies are reduced by a higher real interest rate, a higher job destruction rate and a higher vacancy cost. Vacancy creation is encouraged by improved matching efficiency that exogenously increases the rate at which the firm meets job searchers.

10.4 Wage bargaining

Since the labor market is characterized by frictions and bilateral meetings, the standard wage determination mechanism is not appropriate. The main approach that has been used by the markets with frictions literature assumes that there is bargaining between the employer and the worker. So suppose that wages are set through individual worker-firm bargaining. The logic of our model requires that we distinguish between homeowners and renters in their bargaining with employers.[32]

10.4.1 Homeowners' bargaining and labor market equilibrium

The expected capital gain for an unemployed homeowner from becoming employed is equal to $W^H - U^H$. A firm, on the other hand, gives up $V_u = 0$, in order to gain V_f. Following generalized Nash bargaining, the wage rate is determined so as to split the total surplus,

$$\text{Total Surplus}^H = W^H - U^H + V_f - V_u, \tag{70}$$

[32]Our approach to both housing and labor markets is based on the original formulation of labor markets with frictions due to Pissarides (1985). It can be extended by means of competitive search models, along the lines of Diaz and Jerez (2013), which is applied to housing markets, and Moen (1997), which aims at job market applications.

in order to

$$\max_{w^H} : \left(W^H - U^H\right)^{1-\sigma_L} (V_f - V_u)^{\sigma_L}, \tag{71}$$

where $1 - \sigma_L$ is a measure of the worker's relative bargaining power. With free entry of vacancies, $V_u = 0$, and thus: $V_f = \frac{p-w}{\rho+\delta}$. Note that the threat points in the Nash bargaining are taken to be what the worker and the firm would receive upon separation from each other. As Hall and Milgrom (2008) note, the job-seeker then returns to the market and the employer waits for another applicant. A consequence is that the bargained wage is a weighted average of the applicant's productivity on the job and the value of unemployment. That latter value, in turn, depends in large part on the wages offered by other jobs.[33]

The first-order condition for the maximization of the total surplus is:

$$W^H - U^H = (1 - \sigma_L)\left[W^H - U^H + V_f\right],$$

which yields $\sigma_L(W^H - U^H) = (1 - \sigma_L)V_f = (1 - \sigma_L)\frac{p-w^H}{\rho+\delta}$. Following Pissarides (2000), we do not work with the reduced forms for W^H and U^H, to express $W^H - U^H$ and instead follow the logic of the Nash equilibrium and solve for W^H from (18), substitute into the l.h.s. of the above equation and solve for w^H to get:

$$w^H = (1 - \sigma_L)p_g + \sigma_L \rho U^H.$$

Next, we solve for U^H from (19) using the alternative expression for $W^H - U^H$, obtained from $\sigma_L(W^H - U^H) = (1 - \sigma_L)V_f = (1 - \sigma_L)\frac{p_g c}{q(\theta)}$. Solving for ρU^H we have: $\rho U^H = b + \theta p_g c \frac{1-\sigma_L}{\sigma_L}$. This yields:

$$w^H = \sigma_L b + (1 - \sigma_L)p_g(1 + c\theta^H), \tag{72}$$

[33] Some researchers have made alternative assumptions about the threat points. Hall and Milgrom (2008) assume that the threat point is to delay and postpone bargaining and agreement instead of threatening to walk out of the deal, as Pissarides does. "The bargainers have a joint surplus, arising from search friction, that glues them together." Hall and Milgrom (2008). They assume that the threats are to extend bargaining rather than to terminate it. The result is to loosen the tight connection between wages and outside conditions of the Mortensen–Pissarides model. When the labor market is hit with productivity shocks, the Hall–Milgrom bargaining model delivers greater variation in employer surplus, employer recruiting efforts, and employment than does the Nash bargaining model.

the wage curve for owners. Not surprisingly, it does not depend on housing market conditions: once individuals become homeowners, they stay as homeowners. Of course, this would change if we were to modify the model and allow for turnover for homeowners as well, as in Section (10.2.7) above.

The two equations, the wage curve and the job creation condition, (72) and (73), the latter being the same as (69) but expressed in terms of (w^H, θ^H) for clarity,

$$w^H = p_g - (\rho + \delta)\frac{p_g c}{q(\theta^H)}, \tag{73}$$

play the role of the labor supply and demand curves, respectively. Solving them jointly determines the wage rate and labor market tightness in the labor market for homeowners, (θ^H, w^H). It might sound odd that labor market conditions depend on housing tenure status, but the logic of the model is that bargaining is adapted to all parties' specific circumstances. One way to rationalize this is to consider that owners would likely have smaller separation rate δ that renters. Thus, firms are prepared to invest more in employment relationships with owners.

10.4.2 Renters' bargaining and labor market equilibrium

Working in like manner, we formulate the bargaining problem for renters in order to obtain the wage curve for renters. Because in the model renting is a transitional state, as individuals look forward to becoming owners, the wage curve reflects conditions both for renters and owners. The bargaining model is defined as maximizing $\max_{w^R} : \left(W^R - U^R\right)^{1-\sigma_L} (V_f - V_u)^{\sigma_L}$, subject to a total surplus condition, like (70), yields:

$$\sigma_L(W^R - U^R) = (1 - \sigma_L)V_f = (1 - \sigma_L)\frac{p_g - w^R}{\rho + \delta}. \tag{74}$$

Following Pissarides (2000), we do not work with the reduced forms for W^R, U^R to write for $W^R - U^R$ from (47). Instead we follow the logic of Nash equilibrium and solve for W^R from (44):

$$\left[\rho + \delta + \gamma\frac{\sigma\rho + \sigma(1-\sigma)\gamma(1-\alpha_1)}{\rho + (1-\sigma)\gamma}\right] W^R = \pi^R(w) + \left[\delta + \gamma\frac{\sigma(1-\sigma)\gamma_i(1-\alpha_1)}{\rho + (1-\sigma)\gamma_i}\right] U^R$$

$$+\gamma\frac{\sigma\rho+\sigma(1-\sigma)\gamma_i(1-\alpha_1)}{\rho+(1-\sigma)\gamma}W^H-\gamma\frac{\sigma(1-\sigma)\gamma(1-\alpha_1)}{\rho+(1-\sigma)\gamma}U^H.$$

By substituting into (74), we solve the resulting equation for w^R, in terms of (W^H, U^H) and U^R. Next, we solve from (45) for U^R, in terms of (W^H, U^H) and by using the alternative version of the maximization condition for the surplus, $\sigma_L(W^R - U^R) = (1-\sigma_L)\frac{pc}{q(\theta)}$. Finally, we use the solutions (42-43) in the resulting expression for the wage curve.

A short cut (that uses the reduced form solution and is not fully accurate) yields the following wage curve for renters:

$$w^R+\frac{\gamma\rho}{\delta+\rho+\mu(\theta^H)}w^H=b\left(1+\frac{\gamma\rho}{\delta+\rho+\mu(\theta^H)}\right)+\frac{\sigma_L}{1-\sigma_L}p_g\left[(\rho+\delta+\gamma\sigma)\frac{c}{q(\theta^R)}+c\theta^R\right]. \tag{75}$$

Since $q(\theta^R)$ is decreasing in θ^R, the wage curve, which plays the role of the supply curve here, is increasing in θ^R. Equation (75) and the job creation condition for renters,

$$w^R = p_g - (\rho+\delta)\frac{p_g c}{q(\theta^R)}, \tag{76}$$

when solved as a simultaneous system determine (θ^R, w^R) labor market tightness and the wage rate for renters.

11 Appendix B: Tenure Choice Estimation

To obtain uhr, we estimate the following equation for the propensity for household head i in MSA m, and year t to be a homeowner:

$$\text{own}^*_{i,m,t} = \alpha_0 + \alpha_1 \frac{\text{index}^{\text{value}}_{mt}}{\text{index}^{\text{rent}_{mt}}}$$

$$+\alpha_2 \text{income}^p_{imt} + \alpha_2 \text{income}^T_{imt} + \mathbf{X}_{imt}\alpha_4 + \epsilon_{it}, \tag{77}$$

where the discrete indicator $\text{own}_{i,m,t}$ is defined as $\text{own}_{i,m,t} = 1$, if

$$\epsilon_{it} \geq -\left(\alpha_0 + \alpha_1 \frac{\text{index}^{\text{value}}_{imt}}{\text{index}^{\text{rent}_{imt}}} + \alpha_2 \text{income}^p_{imt} + \alpha_2 \text{income}^T_{imt} + \mathbf{X}_{imt}\alpha_4\right), \tag{78}$$

and renting otherwise:

$$\text{own}_{i,m,t} = 0, \text{otherwise}$$

where $\text{index}^{\text{value}}_{imt}$, $\text{index}^{\text{rent}}_{imt}$ are rental and house value indices (to be explained below).[34] The value to rent ratio is included in the housing tenure equation to capture the relative cost of owning versus renting. The variables income^p_{imt}, income^T_{imt} are permanent and transitory annual household income. Due to mortgage market imperfections, they have different impacts. In the absence of suitable data, income^p_{imt} proxies for wealth. The vector $\mathbf{X}_{i,m,t}$ includes socioeconomic characteristics, like individual education, gender, race, age, and household size.

We generate the auxiliary variables $\text{index}^{\text{rent}}_{mt}$, $\text{index}^{\text{value}}_{mt}$ from the following hedonic equations, for renters and owners, respectively:

$$\ln(\text{rent}_{imt}) = \alpha_{0,m} + \alpha_1 \mathbf{Y}_{1,i,m,t} + \epsilon_{1,i,t}, i = \text{renter}, \tag{79}$$

[34] One could think of this estimation approach as a reduced form corresponding to a structural form, like in Henderson and Ioannides (1986). That is, the tenure choice probability is evaluated in terms of a comparison of indirect utility values, including shocks, associated with renting and owning. Individuals, however, may not attain their optimal mode of tenure, because of rationing associated with financial frictions and the like.

where rent_{imt} is reported monthly rent paid, and

$$\ln(\text{price}_{imt}) = \beta_{0,m} + \beta_1 \mathbf{Y}_{1,i,m,t} + \beta_2 \mathbf{Y}_{2,i,m,t} + \epsilon_{2,i,t}, i = \text{owner}, \tag{80}$$

where price_{imt} is the respondent's estimate of the property's market price, $\mathbf{Y}_{1,i,m,t}$ denotes a vector of dwelling unit characteristics, and $\mathbf{Y}_{2,i,m,t}$ property tax and lot size. The intercepts of the above hedonic equations vary by MSA, m. Then the rent and value indices are calculated as follows:

$$\text{index}_{mt}^{\text{value}} = 100 \times \exp[\beta_{0,m}]; \tag{81}$$

$$\text{index}_{mt}^{\text{rent}} = 100 \times \exp[\alpha_{0,m}] \tag{82}$$

The predicted values of the permanent and transitory components of household incomes, $\text{income}_{imt}^{p}, \text{income}_{imt}^{T}$ are obtained as the predicted value and residual, respectively, from the following equation:

$$\ln\left(\text{income}_{imt}\right) = \gamma_{0,m} + \gamma_1 \mathbf{Z}_{i,m,t} + \epsilon_{2.m,t} \tag{83}$$

where income_{imt} denotes reported household income and $\mathbf{Z}_{i,m,t}$ denotes a vector that includes functions of education, age, race, and gender.

12 Appendix C: Generating the Composite Help Wanted Index

The method we use to construct the full National help wanted index up through 2014 is similar to Barnichon (2010) but not as complicated. It consists of the following 4 steps.

Step 1. 1951-1994: online help-wanted (HWOL) index, O_t does not exist, $O_t = 0$. As in Barnichon, we use the HWI print index through 1994; $H_t = P_t$ where H_t and P_t are the composite and print Help-Wanted advertising indices, respectively.

Step 2. 1995-2005:5: $O_t > 0$, but not observed. Step 2 is also the same as in Barnichon. To get the composite index, we inflate the print index by the estimated print share: $H_t = P_t/\hat{s}_t^p$. But the procedure wee use to estimate is the simpler version in Barnichon. That is, we fit a quartic polynomial to P_t over 1951-2010:10 (the last month that we have the National HWI print index), and estimate as the ratio of the polynomial's value at time t to the polynomial's value in 1994:12. Figure A1 below reproduces Figure 2 in Barnichon. One can see that the print share based on the polynomial trend fits the Sp-JOLTs print share very well. What is key here is that, unlike in Barnichon, the Sp-JOLTs print share DOES exhibit a constant rate of decline and does NOT appear to follow an $S-$curve. Hence, we use the polynomial trend in the above calculation to estimate and not the more complicated method used by Barnichon.

Step 3. 2005:6-2010:10: both O_t and P_t are observed. Same as in Barnichon: H_t is constructed from

$$\frac{dH_t}{H_{t-1}} = s^p_{t-1}\frac{dH_t}{P_{t-1}} + (1 - s^p_{t-1})\frac{dO_t}{O_{t-1}}$$

where O_t is the online help-wanted advertising index.

What we have from the online data is the total number of ads (seasonally adjusted and not seasonally adjusted) and the total number of new ads (seasonally adjusted and not seasonally adjusted). We use the seasonally adjusted total number of ads to construct O_t.

Step 4. 2010:11-2014:6: Only O_t is observed. We construct H_t from from $d\ln H_t = d\ln O_t$. That is, we assume that $s^p_t = 0$ starting in 2010:11 (the estimated value from the polynomial trend is 0.008). The composite index, the print index and the rescaled JOLTS index are plotted on Figure 5.

We can use the same procedure at the MSA level. The one complication is that the last date that the print index is observed varies across MSAs and can be less than 2010.10; the earliest date for this is June 2005. Between this date and 2010:11 (call this Step 3.1), we use the inflated value of O_t to construct H_t from $d\ln H_t = \ln \frac{O_t}{1-\hat{s}^p_t} - \ln \frac{O_{t-1}}{1-\hat{s}^p_{t-1}}$.