Competition for Scarce Inputs: The Case of Airport Takeoff and Landing Slots

by Ian Gale

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Introduction

The process of deregulating airlines in the United States began in earnest with the passage of the Airline Deregulation Act of 1978. The Act set in place a timetable for removing government controls on fares and entry into routes. However, airline access to airports has not been fully deregulated. In 1968, the Federal Aviation Administration (FAA) adopted the so-called "high-density rule" to combat congestion at four airports. Specifically, limits were placed on the number of operations per hour at Kennedy, La Guardia, O'Hare, and Washington National. More and more airports are becoming crowded, given the difficulties in securing permission from local authorities to expand existing facilities or to build new ones, so this problem will persist.[1]

For many years, the right to take off and land at the four crowded airports was determined by a committee system. Multilateral negotiations took place among the incumbent carriers and prospective entrants, with the FAA stepping in if an impasse occurred. Since 1986, incumbents have had property rights called "takeoff and landing slots" or simply "slots." A slot permits the owner to make one operation (a takeoff or landing) during a specified time period. Carriers now have the right to buy and sell slots.[2] Questions remain about the optimal way to allocate existing or new slots. The first question asks why the incumbents should be given valuable property rights.[3] The second asks whether allowing carriers to buy and sell slots will lead to monopolization of airports, with resulting higher fares. Repossessing slots and then selling them back to carriers would raise revenue and would ensure that those carriers willing to pay the most would acquire the slots. This has positive welfare implications in a perfectly competitive environment, but the impact is less clear if there is the potential for wielding market power.

This *Economic Review* analyzes competition for scarce inputs such as airport takeoff and landing slots. It describes the outcome of an auction of slots between two carriers, who may

■ 1 Only one major airport has been built in the United States since 1974, that being the new Denver facility.

■ 2 For further discussion, see Grether, Isaac, and Plott (1989).

■ 3 A slot at O'Hare recently rented for $66,000 per month. See "United Wins TWA Lease," *New York Times*, March 20, 1992.

have existing slots. This allows us to evaluate the welfare implications of selling slots. It likewise describes the outcome of a merger or takeover battle in which two large incumbents seek to buy up the slots of small competitors, if not the competitors themselves.[4]

The objects for sale (the slots) are sold in a sequence of auctions. The aggregate value of the slots, and each bidder's valuation of the marginal slot, increase as slots become more concentrated in the hands of one bidder. These conditions arise naturally in market games, since control of slots can confer market power by limiting the competitiveness of one's rivals. I focus primarily on the case of two bidders who start with no slots. The final allocation is typically unequal, and many different intermediate allocations lead to the same final allocation. In addition, if bidders start with nonzero holdings of slots, a bidder who holds the entire initial allocation of slots often does not block new entry. I then discuss the extension to more than two bidders, starting with three bidders, and then with more bidders than slots.

While the focus here is on the allocation of scarce inputs, the analysis contributes to the general theory of auctions as well.[5] Most theoretical studies of auctions have considered "one-shot" auctions in which a single object is sold or several objects are sold simultaneously. In reality, related objects are often sold sequentially, either because sellers enter the market one at a time, or because it is practical for an individual seller to sell related objects as an ongoing process. (Goods auctioned sequentially include art, wine, procurement contracts, timber rights, and mineral rights.) McAfee and McMillan (1987) note that all levels of governments of western nations procure goods and services equal to 10 percent of gross national product annually, much of it by competitive bidding. Given the economic importance of these markets, further study is warranted.

A common feature of many of these settings is that the value of the object for sale, be it a contract to produce or a scarce input, depends on the other objects obtained. If the objects are identical, then this observation implies that the total value of the objects obtained is a nonlinear function of the quantity obtained, which is the case here.

I. The Model

Suppose that N identical objects called slots are offered for sale by one or more sellers.[6] Assume that $N \geq 2$ and N is an even number. The sellers could be a government agency wishing to allocate some or all of the takeoff and landing slots at an airport, or they could be carriers with small holdings of slots. (Large carriers may wish to take over the small carriers outright or they may simply wish to purchase their slots.) Two bidders compete for the slots.

I denote a typical allocation of slots by the ordered pair (x, y), where x denotes the number of slots currently held by bidder X, y denotes the number of slots currently held by bidder Y, and $0 \leq x + y \leq N$. If $x + y = N$, then (x, y) is a final allocation. At each nonfinal allocation, an auction allocates the next slot. After all slots are allocated, each bidder receives a "final payoff" that depends on the final allocation. This payoff can be thought of as the profit from production.

The specific auction format used is the second-price sealed-bid auction. In this auction, the high bid wins, and the winner pays the amount bid by the losing bidder. This format is used for ease of exposition because its outcome mimics that of the standard oral ascending-bid auction. Suppose, for example, that bidder X was willing to pay up to $100 for a slot, whereas bidder Y was willing to pay up to $80. In an oral ascending-bid auction, where the auctioneer raises the price, bidder Y should not drop out until the price hits $80 exactly. Thus, bidder X will win the auction and pay a price of (approximately) $80. If, instead, the bidders were asked to write down how much they were willing to bid, with the understanding that the bidder who submitted the higher figure would receive the slot at a price equal to the loser's figure, then the outcome should be the same. The latter scheme is the second-price sealed-bid auction.

A minimal requirement for an equilibrium of this game is that each bidder's strategy be optimal, given the strategy of the other bidder. This is the requirement that the strategies form a Nash equilibrium. A Nash equilibrium is self-enforcing in the sense that neither bidder has an incentive to change her strategy, given the strategy of the other bidder.

Unfortunately, there can be many Nash equilibria, so we must be more discerning.

■ 4 For an example, see "American Deal Will End Britt O'Hare Operation," *Chicago Tribune*, December 20, 1988.

■ 5 See McAfee and McMillan (1987) for a survey of the literature on sequential auctions.

■ 6 The model and the analysis borrow extensively from Gale and Stegeman (1993).

Suppose, for example, that bidder X values a slot at \$10 while bidder Y values it at \$8. In a second-price auction, it is a Nash equilibrium for X to bid \$10 and for Y to bid any amount strictly below \$10. Neither bidder has an incentive to change her bid. If bidder X submits a bid above Y's bid, then she receives a surplus equal to the difference between her valuation (\$10) and Y's bid. This holds regardless of X's exact bid, as long as she wins. Bidder Y has no incentive to change since he loses and receives zero surplus currently, whereas if he were to outbid X, then he would lose \$2, since he would pay \$10 for a slot that he values at \$8. Thus, any pair of bids (\$10, b_Y) with $b_Y <$ \$10 constitutes a Nash equilibrium, as does any pair of bids (b_X, \$8) with $b_X >$ \$8.

While there are many Nash equilibria, only one is "robust" in a certain sense. Once we consider the possibility that bidders might make a mistake and submit the wrong bid, then bidder X should bid \$10 and bidder Y should bid \$8. If, for example, Y bids \$9.50, expecting that X will bid \$10, but X makes a mistake and bids only \$9, then Y wins and pays \$9, yielding a net surplus of –\$1. Likewise, if X bids \$9, thinking that Y will bid \$8, but Y makes a mistake and bids \$9.50, then bidder X has lost \$.50 in surplus that would have accrued had she bid \$10. The bids b_X = \$10 and b_Y = \$8 are referred to as "dominant strategies," since they are optimal regardless of the other bidder's bid. I restrict attention to dominant strategies, which leaves each bidder with a unique bidding strategy.[7]

I study the equilibrium sequences of winners and the associated prices. (Tie bids produce multiple sequences of winners.) The bidders know the initial allocation of slots, the auction rules, and the final payoffs (the profits from production, given the final allocation of slots). I focus on the case in which both bidders start the auction with no slots, but consider other initial allocations as the analysis progresses.

A numerical example illustrates the workings of the sequential auction. Suppose that there are two slots for sale. The two firms receive a profit from production of \$2 apiece if each of them owns one slot. A firm receives a profit of \$5 if it controls both slots, in which case the other bidder receives zero. The slots are sold in a sequence of two second-price auctions. Suppose that bidder X wins the first auction. She will be willing to pay up to \$3 for the second slot, which is her marginal valuation. Bidder Y is willing to pay up to \$2, which is his marginal valuation. Bidder X will win the second auction and pay a price of \$2. Given this behavior in the second auction, both bidders are willing to bid exactly \$3 in the first auction. Therefore, the price of the first slot is \$3 and the price of the second is \$2. One bidder wins both, but both bidders receive net profits of zero.

Now suppose that owning one slot is worth \$3 instead, all else equal. After the first auction, the winner has a marginal valuation of \$2, whereas the loser has a marginal valuation of \$3. The loser of the first auction will win the second and pay a price of \$2 for that slot. Thus, neither bidder will be willing to pay more than \$2 to win the first slot, since by losing he or she will win the second slot. Similarly, each bidder will be willing to pay up to \$2 for the first slot. Since winning the first auction at any price strictly less than \$2 is better than winning the second auction at a price of \$2, it follows that both bidders will bid \$2 for the first slot. This process generalizes to N slots. The equilibrium bids are determined by backwards recursion, as outlined below.

At any point during the sequence of auctions, the current allocation of slots is an ordered pair (x, y), where bidder X owns x slots and bidder Y owns y slots. The allocation (x, y) is followed by $(x + 1, y)$ or $(x, y + 1)$, depending on who wins at (x, y). For each number of slots, $x = 0, 1, \ldots, N$ and each bidder $i = X, Y$; $f_i(x)$ denotes the final payoff given to bidder i if the final allocation is $(x, N - x)$. Since the slots are assets, f_X is increasing in x and f_Y is decreasing. Each bidder i seeks to maximize the difference between her final payoff $f_i(x)$ and her total expenditure for slots purchased in auctions.

Let $V_i(x, y)$, $i = X, Y$, denote bidder i's valuation of allocation (x, y). In other words, $V_i(x, y)$ is bidder i's equilibrium payoff in the game commencing at (x, y). The valuation of a final allocation is the final payoff:

$$(1) \qquad V_i(x, N - x) \equiv f_i(x),$$

$$x = 0, 1, 2, \ldots, N; \quad i = X, Y.$$

The valuation of an intermediate allocation (x, y) depends on the valuations of its sequels $(x + 1, y)$ and $(x, y + 1)$. There is a unique valuation of (x, y) if each bidder adopts the strategy in auction (x, y) and in all subsequent auctions of bidding the increment to value resulting from obtaining the next slot. (As noted

7 Strictly speaking, I eliminate, iteratively, all strategies that are dominated by some other strategy.

above, any auction in which the resulting bids are unequal has many Nash equilibria, but the possibility of mistakes justifies the assumption that bidders will play dominant strategies.)

Consider first the penultimate allocations (x, y), where $0 \leq x$, $0 \leq y$, and $x + y = N - 1$. The value of that next slot to bidder X, say, is the difference between her final payoff when she wins and when she loses: $V_X(x+1, y) - V_X(x, y+1)$. It is a dominant strategy for bidder i to bid $B_i(x, y)$ in auction (x, y), where

$$\text{(2a)} \quad B_X(x, y) \equiv V_X(x+1, y) - V_X(x, y+1);$$

$$\text{(2b)} \quad B_Y(x, y) \equiv V_Y(x, y+1) - V_Y(x+1, y).$$

If $B_X(x, y) > B_Y(x, y)$, then bidder X wins the next slot and pays bidder Y's bid. If $B_X(x, y) < B_Y(x, y)$, then bidder Y wins the next slot. If $B_X(x, y) = B_Y(x, y)$, then the tie-breaking rule determines the winner.

The valuations at each penultimate allocation can now be calculated, given these equilibrium bids. This allows bids to be determined one auction earlier, at the antepenultimate allocations, and so on. Bids at any earlier allocation also take the form of (2). The bidders' valuations of each allocation can be calculated recursively. The price paid in auction (x, y) is min $[B_X(x, y), B_Y(x, y)]$. The tie-breaking rule, which I leave unspecified for now, cannot affect bids or valuations in any given auction.

The equilibrium of the auction game comprises the bidding functions $B_X(x, y)$ and $B_Y(x, y)$, for all (x, y). Henceforth, I assume that the bidders are symmetric with respect to their final payoffs. The outcomes will typically not be symmetric, however.

II. Competition for Scarce Inputs

I now put some additional structure on the model. First, symmetry of bidders is imposed in (3a). Second, aggregate final payoffs, and each bidder's valuation of the marginal slot, are assumed to rise as the final concentration of slots increases (that is, as the allocation of slots between bidders becomes more unequal). These conditions arise naturally for scarce inputs because higher concentration typically leads to higher output prices, increasing both aggregate profits and the value of marginal output. Assumptions (3b) and (3c) mean that each bidder's valuation of the marginal slot rises as the final concentration of slots increases, while assumption (3d) means that aggregate final payoffs increase with the final concentration of slots. (Note that [3c] says that bidder Y's final payoff is increasing and concave in his own number of slots.) Finally, assumptions (3e) and (3f) are invoked for concreteness. The example discussed below satisfies these conditions, but the results of Theorem 1 do not require them.

(3a) $f_X(x) = f_Y(N - x)$ for all x.

(3b) $f_X(x)$ is strictly increasing and convex, for $x \geq N/2$.

(3c) $f_Y(x)$ is strictly decreasing and concave, for $x \geq N/2$.

(3d) $f(x) \equiv f_X(x) + f_Y(x)$ is strictly increasing, for $x \geq N/2$.

(3e) $f_X(x) = \alpha_X + \beta_X x + \delta_X x^2/2$, and

(3f) $f_Y(x) = \alpha_Y + \beta_Y x - \delta_Y x^2/2$,

where $\delta_Y > \delta_X > 0$.

Assumptions (3a) – (3d) place additional restrictions on the parameters, which I leave unspecified for now.

Winning a particular slot affects a bidder directly by increasing her current holdings of slots, but it also affects the competition for the remaining slots. Two countervailing effects determine the number of slots won by, say, bidder X. The first effect argues for one bidder to acquire all slots, since that outcome maximizes the sum of the final payoffs. This effect is countered by the fact that the more slots bidder X wins, the higher is the marginal valuation of another slot to bidder Y (presuming that $y < x$), and the fiercer is the competition for the marginal slot. If bidder Y ends up with very few slots, then bidder X is a near-monopolist. Bidder Y benefits from X's relative passivity in the production game, so marginal capacity is most valuable to bidder Y when he has very little. (Empirically, airlines that dominate airports tend to have the highest fares. See Borenstein [1992].) This trade-off leads to an equilibrium allocation of slots that is typically neither symmetric nor monopolistic.

A heuristic argument provides some intuition for the determinants of the equilibrium allocation of slots. I proceed by conjecturing the form

of the equilibrium. I then show that it is optimal in each auction for bidders to bid in the way prescribed. The heuristic argument for determining the equilibrium allocation is now given.

Consider an allocation (x, y) such that, in equilibrium, bidder X wins any auction $(x', y') \geq (x, y)$ such that $y' \leq y + 2$. In other words, if bidder X has at least x slots and bidder Y has y, $y + 1$, or $y + 2$ slots, then bidder X will win all remaining slots in equilibrium. If a deviation at (x, y) causes bidder Y to win one more slot, then any additional slot that bidder Y wins will, in equilibrium, be his last slot. Therefore, following the deviation, bidder Y bids the value of that last slot, $B_Y(N - y - 2, y + 1)$, in every remaining auction. If the deviation did not occur, bidder Y would bid $B_Y(N - y - 1, y)$ in every remaining auction. Since bidder X wins all remaining slots, even if a deviation causes her to lose one of those slots, the marginal contribution to her final payoff of winning auction (x, y) is equal to the marginal value of winning the last auction, $B_X(N - y - 1, y)$.

By winning auction (x, y), bidder X increases the price that she must pay upon winning the remaining $N - x - y - 1$ auctions by $\Delta B_Y(N - y - 2, y + 1) \equiv B_Y(N - y - 1, y) - B_Y(N - y - 2, y + 1)$. Therefore, bidder X wins auction (x, y) if and only if $B_X(N - y - 1, y) \geq (N - x - y - 1)\Delta B_Y(N - y - 2, y + 1) + B_Y(N - y - 1, y)$. Recalling that $f(x) = f_X(x) + f_Y(x)$, the condition can be rewritten:

$$(4) \quad f(N - y) - f(N - y - 1) \geq (N - x - y - 1)\Delta B_Y(N - y - 2, y + 1).$$

In other words, bidder X wins auction (x, y) only if the increment to aggregate final payoffs (the left-hand side) exceeds the increment to total prices paid (the right-hand side).

Define μ to be the smallest integer such that $(y + 2, y)$ satisfies (4) for $y = \mu, \mu + 1, N/2 - 2$. The equilibrium allocation will be $(N - \mu, \mu)$, which I call the "modal allocation," or its symmetric counterpart.

Theorem 1. When (3) holds, either bidder X wins $N - \mu$ slots and bidder Y wins μ, or the reverse holds.

The equilibria of the original game and the game starting from any intermediate allocation have a regular form. From here on, I assume that ties are won by bidder X if the allocation is even. This tie-breaking rule ensures that bidder X always has at least as many slots as bidder Y, so the equilibrium allocation in this case is $(N - \mu, \mu)$. If $x = y$, then there will be a tie in the next auction, which buyer X wins by the tie-breaking rule. Buyer X also wins the next auction to maintain her lead. Thereafter, different patterns are possible. If I assume that bidder Y wins all ties when he is behind, then the bidders alternate victories until buyer X commences her final string of victories, and the modal allocation is reached. If $x > y$, then buyer Y wins a string of auctions first, but not enough to catch up. Either buyer Y catches up to within one slot, and then a pattern of alternation persists until the modal allocation is reached, or else he never catches up to within one, in which case buyer X wins all auctions after Y's string of victories. The point at which buyer X commences her string of victories is determined by (4).

The theorem is illustrated by the following numerical example. Suppose that each slot represents one unit of capacity in a subsequent production game, where the inverse demand is $P = 24 - Q$ and average cost is zero, up to capacity. If bidder X has x slots, then she is able to produce up to x units of output. For quantities strictly less than x, marginal and average cost both equal zero, but for quantities strictly above x, marginal and average cost are infinite.[8]

Suppose that the firms behave like Cournot duopolists in the production game. In a Cournot duopoly, each firm chooses the optimal quantity to produce, given the level produced by its competitor. In the absence of capacity constraints, each duopolist produces eight units of output in the Cournot equilibrium. To see this, note that if bidder X produces q_X units of output, then bidder Y's profit from producing q units is $(24 - q_X - q)\,q$, a concave function that takes its maximum at $q = (24 - q_X)/2$. Likewise, if Y produces q_Y, bidder X's optimal strategy is to produce $q = (24 - q_Y)/2$. If both bidders are unconstrained, then the unique Cournot equilibrium has $q_X = q_Y = 8$.

Assume, however, that exactly $N = 16$ slots are available, and $x > 8$. Concavity of the profit function ensures that the best responses for the two bidders are

$$(5a) \quad q_X = \min\{(24 - q_Y)/2, x\}$$

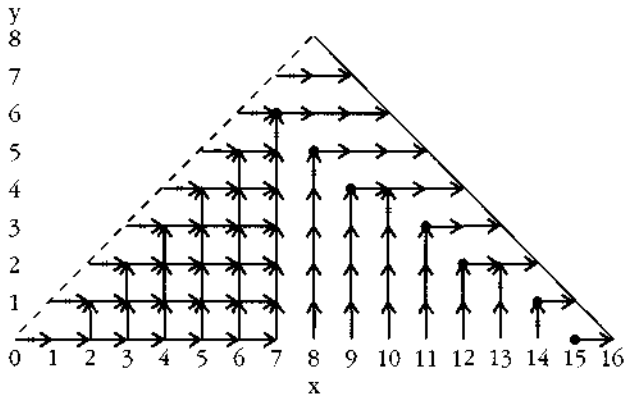
$$(5b) \quad q_Y = \min\{(24 - q_X)/2, 16 - x\}.$$

There are now four possibilities, depending on whether the bidders are constrained. The bidders cannot both be unconstrained, since that would require that bidder Y produce eight units, which exceeds his capacity. It is also immediate

8 In reality, the capacity constraint is not absolute, since more seats can be put on an airplane, but the qualitative properties need not change when this possibility is permitted

FIGURE 1

Possible Sequences of Equilibrium Victories



SOURCE: Author's calculations.

that the bidders cannot both be constrained. If Y produced $16 - x$ units, X would wish to produce only $[24 - (16 - x)]/2 = 4 + (x/2)$, whereas he has available x units of capacity. Because $x > 8$, $4 + (x/2) < x$. Now suppose that X is constrained and Y is unconstrained. If $q_X = x$, $q_Y = \min\{(24 - x)/2, 16 - x\} = 16 - x$, which is a contradiction. The only remaining possibility is that bidder X is unconstrained and bidder Y is constrained. It is straightforward to confirm that $q_X = 4 + (x/2)$ and $q_Y = 16 - x$ are the unique equilibrium outputs. Total output is $20 - (x/2)$. The price of output is $(8 + x)/2$, so the final payoffs (that is, the profits in the Cournot production game) are

(6a) $f_X(x) = (8 + x)^2/4$,

(6b) $f_Y(x) = (8 + x)(16 - x)/2$.

Note that these final payoffs are consistent with (3a) – (3f).

I now return to the auction. The different possible sequences of equilibrium victories are noted in figure 1. Arrows indicate which allocations can follow a given allocation. For example, starting from the allocation (2,0), where bidder X currently has two slots and bidder Y currently has none, the allocations (3,0) and (2,1) can both be reached. This means that the two bidders submit the same bid at (2,0), and the tie-breaking rule determines whether X wins and (3,0) is reached or whether Y wins and (2,1) is reached.

The modal allocation is (10,6) here. Straightforward calculations confirm that (4) amounts to

(7) $(2y + 1)/4 \geq (13 - 2y)$,

or $y \geq 5.1$. Thus, $\mu = 6$. Price drops occur between auctions separated by double cross marks. On any path from the origin, the price is 10.5 in the first auction, 6.25 in the next 12 auctions, and 5.5 in the last three auctions. The equilibrium paths in the example have several other properties.

1. While the leader may ultimately win considerably more slots than the follower, the leader strictly outbids the follower only when his lead is reduced to one or after the follower has already won all of her slots. A consequence is that there is an equilibrium path in which bidder X wins the first two slots, then bidder Y wins one, and then they alternate victories until the allocation $(\mu + 1, \mu)$ is reached. At that point, bidder X wins all remaining slots.

2. The trade-off that determines the number of slots that the leader wants to win, along with the preceding observation, ensures that many intermediate allocations lead to the same final allocation. Any initial allocation (x, y) at which $0 \leq y \leq x \leq \mu + 1$ and $(x, y) \neq (\mu + 1, \mu + 1)$ leads to the same final allocation. In the example, the modal allocation is (10,6), and any initial allocation (x, y) with $0 \leq y \leq 7$, excepting only (7, 7), leads to the modal allocation. Thus, many different histories lead to the same equilibrium allocation of slots. This implies that a sizable initial advantage need not be maintained.

3. If the initial allocation is (0,0), then the price drops either once or twice. It drops immediately after the leader is established (which equalizes net profits). It also declines immediately after the follower wins his last slot if a deviation causing him to lose that slot would have caused him to lose all subsequent slots as well. Since the follower loses some surplus in that case, he bids more than he would bid if a loss would be compensated by his winning another auction subsequently. The existence of at most three distinct prices depends on the precise functional form assumed.

4. An incumbent monopolist typically does not block entry if new slots become available. For instance, suppose that the initial allocation of slots is (12,0), which effectively makes firm X an unconstrained monopolist. Calculations show that if four new slots become available, then the prospective entrant, firm Y, wins the first two slots before firm X wins the last two.

The reason for this is simply that it is often too costly to preempt an entrant completely, since the incumbent monopolist would leave the capacity idle whereas the entrant would find the capacity very valuable because the incumbent would still be a near-monopolist after entry.

III. More Bidders

In many markets, there are exactly two competitors of consequence. This is particularly true at many airports and in many airline markets, so the assumption of two bidders is realistic in such contexts. I now consider the impact of having more than two bidders. The extension is qualitatively different because the value of winning a slot typically depends on which rival would otherwise win it.[9] Hence, a bidder often has no dominant strategy, so I focus on the equilibria in which each bidder bids the value of the next slot to her, assuming that the rival making the highest bid would otherwise win the auction.

Multiple equilibria are common when productive capacity is sold, because keeping capacity away from constrained bidders is a public good that the unconstrained bidders may individually prefer not to provide. Thus, at an airport with two dominant incumbents, if new capacity becomes available, each incumbent may prefer that the other buy the new capacity and keep the entrant out. The following example illustrates the public good aspect in a case where all bidders start with no capacity.

Suppose that three firms bid for three slots. Each slot represents one unit of capacity, where inverse demand is $P = 8(4 - Q)$ and production is again costless up to the capacity constraint. In the post-auction production game, the firms again act as Cournot competitors. The final allocation (3,0,0) returns final payoffs of 32, zero, and zero, respectively. The allocation (2,1,0) returns payments of 18, 12, and zero. The allocation (1,1,1) returns a payment of 8 to each bidder. The payments from the other final allocations are determined symmetrically.

Equilibria are computed through backwards recursion. (The price paid in the second-price sealed-bid auction is the larger of the two losing bids here.) Consider the allocation (2,0,0). The last slot is worth 14 to the leader and bidder X, and 12 to the others. Therefore, the leader pays 12 for the last slot. The payoff to entering this "subgame" (the game that takes place when the initial allocation is (2,0,0)) is 32 – 12 = 20 for the leader and zero for the followers.

Consider next the allocation (1,1,0). The last slot is worth 10 to a leader if it would otherwise go to the follower, bidder Z, but it is worth only 6 if it would otherwise go to the other leader. It is worth 8 to the follower. If tie bids are broken fairly, then there are three equilibria of interest. The follower bids 8 in all three equilibria. In one equilibrium, one leader bids 10 and the other 6; in another, their roles are reversed; in the last, they both bid 8. In the first two equilibria, the payoff to entering this subgame is 10 for the leader who wins, 12 for the other leader, and zero for the follower. Both leaders are willing to bid enough to shut out the follower, but each prefers that the other do it. If we assume that the leaders randomly coordinate on one of the two equilibria, the average payoff to entering this subgame is thus 11 for each leader and zero for the follower.

At the allocation (1,0,0), the next slot is worth (32 – 12) – 11 = 9 to the leader and 11 to each follower. (If the leader wins, he goes on to win the third slot at a price of 12.) One of the followers wins, and the payoffs to entering this subgame are 11 for the leader and zero for the followers. At the initial allocation (0,0,0), each firm bids 11.

Summarizing, the assumption that firms randomly coordinate among equilibria at the time of the auction generates an equilibrium allocation that is a permutation of (2,1,0). The final payoffs to the firms are 18, 12, and zero, and the equilibrium prices are $p_1 = 11$, $p_2 = 11$, and $p_3 = 8$.

It is possible for three firms to earn higher profits if they coordinate on an equilibrium before the auctions begin. In particular, suppose that they coordinate on the equilibrium in which firm X wins auctions (1,1,0) and (1,0,1) and firm Y wins auction (0,1,1). Then the leader's payoffs in these subgames are either 10 or 12, instead of 11. In auction (1,0,0) the equilibrium price is 12, and in auctions (0,1,0) and (0,0,1) the equilibrium price is 10. The payoffs in these three subgames are now 10 for the leader and zero for the followers, except that firm Z's payoff in subgame (0,1,0) is 2 instead of zero, and its payoff in subgame (0,0,1) is 12 instead of 10. If tie bids are broken fairly, then firm Z wins the initial auction (0,0,0) with a bid of 11 and pays a price of 10. Since the payoff to winning is 12, firm Z earns profits of 2, unlike the zero profits earned without advance

9 This is reminiscent of the situation in baseball pennant races, where contending teams attempt to trade for a player whose team is out of the race. How much a team is willing to pay depends on whether the other team vying for that player's services is in the same division.

coordination. The price sequence is $p_1 = 10$, $p_2 = 10$, $p_3 = 8$. Profits appear because advance coordination creates an asymmetry that gives firm Z an advantage from the start. This holds because firm Z is never put in the position of purchasing capacity that will not be fully utilized, since the other leader is assumed to win at (1,0,1) and at (0,1,1). In those cases, the other leader produces only 1.5 units even though it purchases 2.

Finally, consider the possibility of there being more bidders than slots available. In Krishna (1993), an incumbent monopolist and potential entrants bid for new capacity. The monopolist wins only the last unit of new capacity, assuming that marginal costs are constant, market demand is concave, and entrants always produce to capacity. The presence of potential entrants, each of whom will produce up to capacity, means that the bid from an entrant is always the equilibrium price of output should the entrant win. With two or three bidders, all bidders may have some slots, in which case they all internalize the impact of increased (aggregate) production on the value of their current holdings. This makes the potential entrants less competitive in the small numbers case, so an incumbent monopolist will win more slots.

IV. Concluding Remarks

This paper has provided a framework for analyzing competition for scarce inputs such as airport takeoff and landing slots. The analysis describes the outcome of an auction of slots between two carriers, who may have existing slots, and it also depicts the outcome of a merger or takeover wave. The equilibrium allocation of slots is typically asymmetric, even though firms are ex ante identical. It is not typically monopolistic, however, since the more concentrated the allocation of slots becomes, the higher is the price that the leader must pay for the marginal slot.

Many different histories, and many different allocations of slots, lead to the same equilibrium allocation of slots, implying that an initial advantage need not be maintained. Thus, the concern with monopolization may be misplaced. Future work will consider risk aversion, capital constraints, and matching of slots between airports.

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