Commodity Prices and P-Star

by Jeffrey J. Hallman and Edward J. Bryden

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Introduction

A recent article by Hallman, Porter, and Small (1991), henceforth referred to as HPS, presented the P-Star (P*) indicator of future inflation. The HPS models exploit the stability of two long-run relationships: that between M2 and nominal output, and that between actual and potential real output. Despite paying no attention to other possible influences on inflation, such as commodity prices or interest rates, the HPS models produced better forecasts of the GNP implicit price deflator over the 1970s and 1980s than did a number of alternatives, including both univariate ARIMA models and the published forecasts of several econometric consulting firms.

Most economists believe the quantity theory relationship that underlies the P* model holds only in the long run, if at all. In the short run, standard economic theory predicts that any factors affecting aggregate supply or aggregate demand may also affect the price level. Commodity prices may give early indications of shifts in either supply or demand; if so, augmenting the P* models to include information about the recent behavior of commodity prices may be expected to improve both fit and forecasting performance, particularly over short horizons. This paper shows that these results can indeed occur.

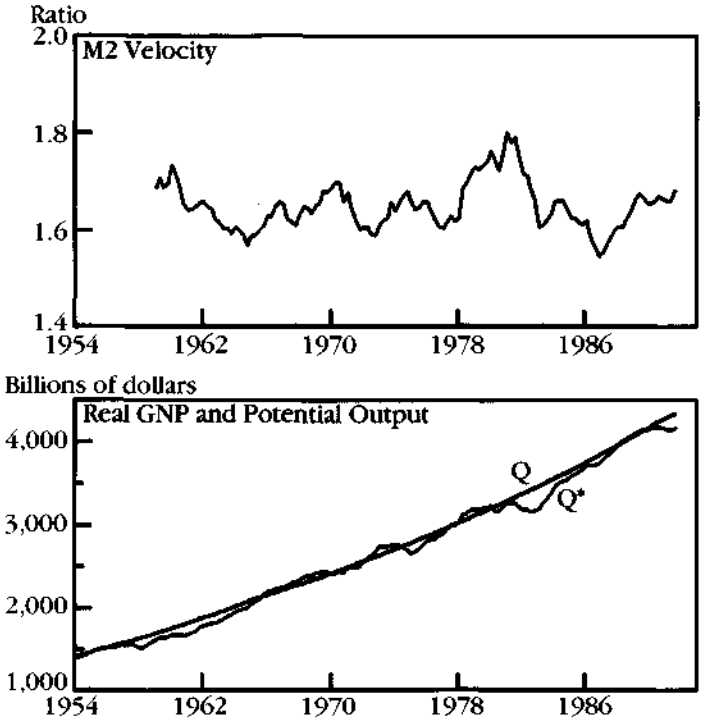
After briefly reviewing the P* idea, we show how the P* approach can be extended to incorporate commodity price data. Two notable results are obtained: First, the resulting models outperform the HPS models in fit and forecasting ability, although the improvement is not large. Second, the relative significance of the P* and commodity price terms depends on the sampling frequency of the data. The two terms are equally significant in explaining inflation at quarterly and annual frequencies, but inflation in the biennial version of the model depends exclusively on the monetary (P*) term. This accords well with the orthodox view that, while commodity market developments may significantly influence inflation in the short run, in the long run only money matters.

I. The P* Model

The HPS work was motivated by the observation that, rather than trending up or down over the past 35 years, V (the velocity of M2, defined as GNP divided by M2) has simply fluctuated around its average value of 1.65 (V*). During

FIGURE 1

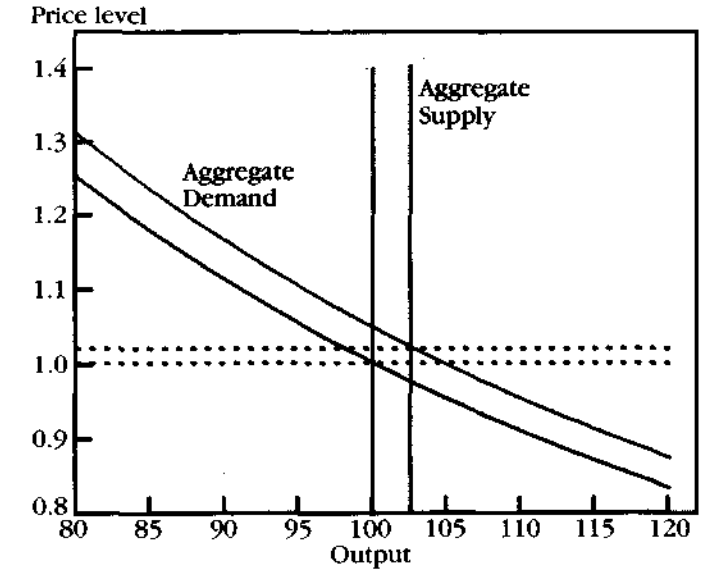
M2 Velocity, Real GNP, and Potential Output



SOURCE: Board of Governors of the Federal Reserve System.

FIGURE 2

Long-Run Aggregate Supply and Demand



SOURCE: Authors' calculations.

the same period, real GNP (Q) has usually been within 5 or 6 percent of potential output (Q*). Both of these regularities can be seen in figure 1. Using lower-case letters to denote natural logarithms, HPS found that they could not reject the hypothesis that the velocity gap ($v - v^*$) and output gap ($q^* - q$) are covariance stationary.[1] A stationary series has a mean to which it tends to return infinitely often. More important, as the forecasting horizon lengthens, the optimal forecast of a stationary series tends to approach the series mean.[2]

The relation between real output, the price level as measured by the implicit price deflator (P), money, and velocity is given by the quantity equation:

$$(1) \qquad p + q = m2 + v.$$

As the velocity and output gaps are stationary, so too is the price gap ($p - p^*$), where P* is defined as M2* x V* / Q*. The reason is that the price gap is simply the sum of the velocity and output gaps. Since V* is constant, the economic interpretation of stationarity of the price gap is that, given M2 and potential output, the price level that can be supported in the long run is P*.

Historically, inflation has usually accelerated when P* has exceeded P, and slowed when the reverse was true. Letting π denote the rate of change of the implicit price deflator, HPS used a quarterly model of the form

$$(2) \qquad \Delta\pi_t = \alpha\,(p_{t-1} - p^*_{t-1}) + \sum_{j=1}^{4} \beta_j \Delta\pi_{t-j}$$

to exploit this regularity in forecasting inflation. The lagged $\Delta\pi$ terms in equation (2) reflect the fact that quarterly changes in the inflation rate are often partly reversed in subsequent quarters, perhaps due to measurement error. Lagged $\Delta\pi$ terms are unnecessary when the model is estimated using annual data. The model then becomes

$$(3) \qquad \Delta\pi_t = \alpha\,(p_{t-1} - p^*_{t-1}),$$

where the subscript t is now an annual index (measured in every fourth quarter) rather than a quarterly index as in equation (2). Estimates of the models in equations (2) and (3), and a biennial version (one observation from the fourth

1 A series y_t is covariance stationary if, for all k and j, $E(y_{t+k}) = m$, and $cov(y_{t-j}, y_t) = cov(y_{t+k-j}, y_{t+k})$.

2 Optimal here means the forecast that minimizes the expected variance of the forecast error.

TABLE 1

Estimates of the Models

	Coefficients						Regression Statistics		
	α_1	α_2	β_1	β_2	β_3	β_4	$\bar{R}^2$	Standard Error	Durbin-Watson
Frequency				**Price Gap Models**					
Quarterly[a]	−.145		−.64	−.47	−.27	−.12	.32	1.56	1.96
	(−4.2)		(−7.4)	(−4.7)	(−2.7)	(−1.3)			
Annual[b]	−.214						.38	1.19	2.36
	(−4.2)								
Biennial[c]	−.31						.70	.97	1.50
	(−5.6)								
Frequency				**Price Gap Models with Commodity Price Inflation**					
Quarterly[a]	−.107	−.039	−.70	−.55	−.34	−.15	.36	1.51	1.97
	(−3.0)	(−2.9)	(−8.1)	(−5.5)	(−3.5)	(−1.8)			
Annual[b]	−.171	−.061	−.30				.56	1.00	2.34
	(−3.5)	(−3.5)	(−2.3)						
Biennial[c]	−.37	−.015					.84	.77	2.25
	(−7.4)	(−1.8)							

a. Estimated from 1960:IIQ to 1990:IIQ; 121 observations.
b. Estimated from 1961 to 1990; 30 observations.
c. Estimated from 1962 to 1990; 15 observations.
NOTE: T-statistics are indicated in parentheses.
SOURCE: Authors' calculations.

quarter of every other year) of equation (3) are given in table 1.

The economic intuition underlying the P* model can be illustrated using the long-run aggregate supply and long-run aggregate demand diagram in figure 2. Stationarity of the velocity and output gaps indicates the shape of the curves. The stationarity of velocity means that V is constant at V* along the long-run aggregate demand curve. But because money is also held fixed along the curve, long-run aggregate demand is represented as the hyperbola formed by the locus of points where $P \times Q = M \times V^*$. The stationarity of the output gap means that the long-run aggregate supply curve is vertical at Q*.

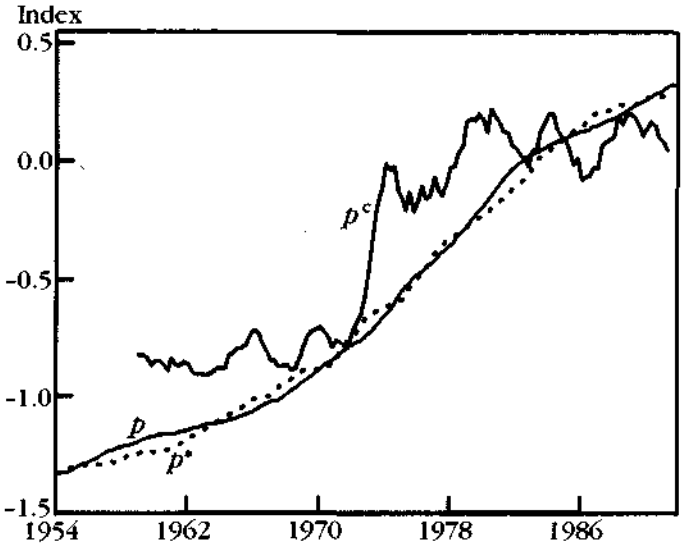
Changes in the money stock shift the long-run aggregate demand curve by proportionate amounts. A 4.5 percent increase in M2, for example, shifts the demand curve up (or equivalently, to the right) by 4.5 percent. That is, the shifted curve is the locus of (P, Q) pairs whose product is now 4.5 percent greater than before the shift. Changes in potential output are drawn as shifts in aggregate supply to the left or right. A 2.5 percent increase in Q* shifts the vertical line 2.5 percent to the right. In the example shown, the result of both shifts taken together is a 2 percent rise in prices accompanied by a 2.5 percent increase in real output.

Although the above analysis shows how money and potential output determine prices in the long run, it may have less application in the short run. The velocity and output gaps are stationary, but they are not identically zero at all times. Short-run aggregate demand may not be a hyperbola, and short-run aggregate supply may not be vertical. Furthermore, in constructing a measure of potential output, it is customary to restrict it to grow smoothly over time. The best that can be hoped for such a measure is that it will correctly capture the trend in potential output.

This catalog of short-run omissions and inadequacies is reason to hope that the P* model can be enhanced. The inclusion of variables that reflect information about the location or movements of the short-run aggregate supply and demand curves may improve the model's performance. One such candidate for inclusion is a measure of commodity prices.

FIGURE 3

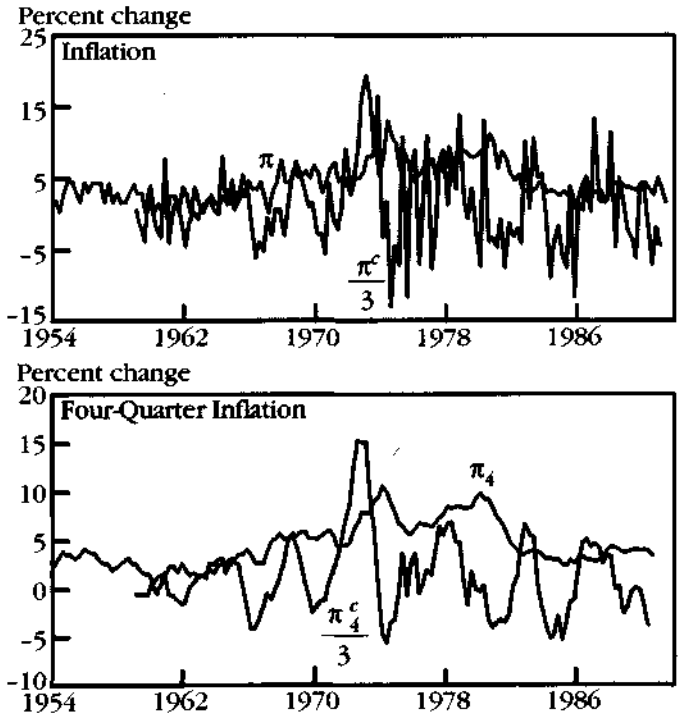
Implicit Price Deflator, P*, and the Commodity Price Index



SOURCES: Board of Governors of the Federal Reserve System, and the Commodity Research Bureau.

FIGURE 4

Inflation Rates for the Implicit Price Deflator and the Commodity Price Index



SOURCES: Board of Governors of the Federal Reserve System, and the Commodity Research Bureau.

II. Commodity Prices and the Deflator

Figure 3 plots the logarithms of P, P*, and the Commodity Research Bureau's (CRB) spot market price index for 22 commodities, labeled p^c in the figure. While both p and p^c have increased over the years, a simple long-run relationship between their levels is not apparent. Rather, there has been a decline in $(p^c - p)$, the log of real commodities prices.

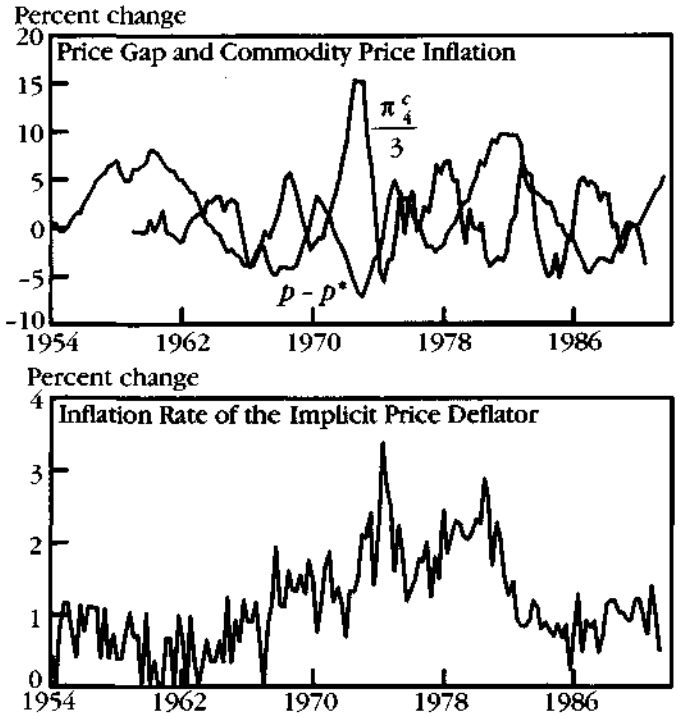
There is no reason to expect a simple relationship to hold between the levels of a commodity price index and the implicit price deflator. The deflator is meant to measure the price of all of the economy's outputs, while most of the 22 CRB commodities are primarily used as inputs to various production processes. Even if firms price on a pure markup basis, output prices can diverge from commodity prices if the relative prices of other, noncommodity inputs change. One can easily imagine that continuing technological progress in agriculture, for example, will result in a downward trend in the price of wheat relative to the price of labor. Trends in the real prices of commodities are simply trends in the prices of commodities relative to the prices of other goods, so we should not be surprised to find them in the data.

While it is easy to see how a trend in $(p^c - p)$ might arise from such forces as technological progress, it is much harder to imagine a scenario in which the inflation rates π and π^c would trend apart indefinitely. Not only would this imply a trend in the real price of commodities, but it would require a trend in the trend, resulting in explosive real commodity prices. That kind of relative behavior is not evident in the data, so there is an implied long-run relationship between the inflation rates of prices in general and commodity prices.

The top panel of figure 4 plots quarterly inflation rates for the GNP deflator and the commodity price index. The commodity inflation index is divided by three to put it on the same scale in the plot. The relationship between the two indices is clearer in the lower panel, where inflation rates are calculated over four quarters, resulting in a smoother plot. Notice that in keeping with the downward trend in real commodity prices evident in figure 3, the arithmetic mean of the commodity inflation index is lower than the arithmetic mean of GNP inflation. Finally, figure 5 shows that both $(p - p^*)$ and π_4^c (the change in p^c over four quarters) foreshadow subsequent changes in the inflation rate.

FIGURE 5

Inflation Predictions by the Price Gap and by Commodity Price Changes



SOURCES: Board of Governors of the Federal Reserve System, and the Commodity Research Bureau.

TABLE 2

Significance of the Error-Correction Terms

	T-statistics	
Frequency	$(p - p^*)$	$(\pi - \pi^c)$
Quarterly	–3.0	–2.9
Annual	–3.5	–3.5
Biennial	–7.4	–1.8

SOURCE: Authors' calculations.

III. P* with Commodity Prices

The figures argue for a specification of the dynamic relationship between p and p^c that preserves a long-run relationship in which the commodity inflation rate is less than GNP deflator inflation. Just as the term $(p_{t-1} - p^*_{t-1})$ in equations (2) and (3) enforces a long-run relationship between p and p^*, the introduction of a term $(\pi_{4,t-1} - \pi^c_{4,t-1} - m)$ with a negative coefficient will enforce a similar relationship between the inflation rates for P and P^c. Here, m denotes the mean difference of the two inflation rates, so that the real commodity inflation rate $(\pi_4 - \pi^c_4)$ enters as a deviation about its mean. The resulting equation for the quarterly model is

$$(4) \quad \Delta\pi_t = \alpha_1 (p_{t-1} - p^*_{t-1}) + \alpha_2(\pi_{4,t-1} - \pi^c_{4,t-1} - m) + \sum_{j=1}^{4} \beta_j \Delta\pi_{t-j},$$

and the implied equilibrium has both $p = p^*$ and a trend of $-m$ in the real price of commodities. The annual and biennial versions of this model take the form

$$(5) \quad \Delta\pi_t = \alpha_1 (p_{t-1} - p^*_{t-1}) + \alpha_2(\Delta_4 p_{t-1} - \Delta_4 p^c_{t-1} - m) + \beta_1 \Delta\pi_{t-1}.$$

The biennial version does not require the lagged $\Delta\pi$ term. Estimates for all three versions may be found in table 1.

In all of the estimated models, both α coefficients are significantly less than zero, and the equations using the price gap model with commodity price inflation fit better than those that do not use the commodity price term.

Table 2 shows an interesting comparison of the error-correction t-statistics taken from table 1. The quarterly model predicts inflation over the coming quarter, the annual model over the coming year, and the biennial model over two years. At the lowest frequency, the explanatory power of the commodity inflation term is minor; nearly all of the explanatory power of the model comes from the price gap term. This result confirms that while supply shocks reflected in $(\pi - \pi^c)$ affect inflation in the short run, over longer horizons money (reflected in $[p - p^*]$) is more important. If both velocity and real output eventually return to their long-run values, supply shocks not accommodated

TABLE 3

Year-Ahead Forecasting Performance

		Quarterly ($p - p^*$)		Annual ($p - p^*$)		Annual ($p - p^*$) and ($\pi - \pi^c$)	
Year	Actual	Predicted	Error	Predicted	Error	Predicted	Error
1970	5.17	5.41	−0.24	5.52	−0.35	7.14	−1.97
1971	6.09	4.94	1.15	4.99	1.10	5.39	0.70
1972	4.42	6.39	−1.97	6.38	−1.96	6.49	−2.08
1973	8.25	5.42	2.82	5.05	3.19	7.45	0.79
1974	9.96	8.88	1.09	8.94	1.02	8.10	−1.86
1975	8.35	9.69	−1.34	9.25	−0.91	9.14	−0.80
1976	5.74	7.97	−2.23	7.75	−2.01	7.81	−2.07
1977	6.82	6.73	0.09	6.08	0.75	7.52	−0.70
1978	7.98	7.30	0.69	7.47	0.51	6.98	1.00
1979	8.87	7.82	1.05	7.95	0.92	8.57	0.30
1980	9.88	7.49	2.39	7.90	1.98	8.10	1.78
1981	8.76	8.49	0.27	8.17	0.59	8.00	0.76
1982	5.06	6.92	−1.85	6.73	−1.67	6.27	−1.20
1983	3.64	3.70	−0.06	3.02	0.62	4.28	−0.64
1984	3.42	3.29	0.13	2.94	0.48	5.13	−1.71
1985	2.94	3.31	−0.38	3.12	−0.18	3.11	−0.17
1986	2.58	3.44	−0.86	3.26	−0.68	2.60	−0.02
1987	2.96	3.79	−0.83	3.75	−0.80	3.69	−0.74
1988	4.14	3.46	0.68	3.76	0.37	4.44	−0.30
1989	3.73	4.39	−0.66	4.59	−0.86	4.68	−0.96
1990	3.98	3.74	0.25	3.80	0.19	3.57	0.41
1991		3.03		3.80		3.57	
Mean forecast error			0.01		0.11		−0.27
Mean absolute forecast error			1.001		1.006		0.999
Root mean squared forecast error			1.28		1.24		1.15

NOTE: Entries represent the annual growth rate of the GNP implicit price deflator over fourth-quarter to fourth-quarter periods ending in the indicated year.
SOURCE: Authors' calculations.

by the monetary authority will have no long-run effect on the overall price level. In the short run, they can influence the rate at which P converges to P*, but the fit of the biennial equation indicates that the effect dissipates within two years.

Table 3 compares the out-of-sample forecasting performance of some of these models. The first three columns show the performance of the model in equation (1), while the remaining columns are for the annual models in equations (3) and (5). The forecasts reported in the table are not truly out of sample for two reasons. First, the estimates of v^* and q^* used in constructing the p^* series are actually based on the full sample of observations. However, all of the models use this p^* series, so none of them has an unfair advantage. This "cheating" would be important if we were comparing the performance of P* models with other models, as was done in HPS.[3] Second, the year-ahead forecasts made from the quarterly model cheat in the sense that we assume the values taken by p^* over the intervening three quarters were known. This is not true of the year-ahead forecasts for the annual models making up the rest of the table, since all of their right-hand-side variables are lagged at least once.

The root mean squared forecast error for the annual price gap model (equation [3]) is slightly smaller than for the quarterly version, showing that the short-run dynamics modeled by the lagged $\Delta\pi$ terms in the quarterly model are not

■ 3 In their paper, HPS did try estimates of q^* and v^* based only on information that would have been available to a forecaster operating in real time and found that it made little difference to the forecasting performance of their models.

TABLE 4

Encompassing Tests

Independent Variable (right-hand side)	Dependent Variable (left-hand side)	T-statistic
$e^q - e^{p^*}$	e^q	1.08
	e^{p^*}	0.01
$e^q - e^{cp}$	e^q	2.23
	e^{cp}	1.18
$e^{p^*} - e^{cp}$	e^{p^*}	2.24
	e^{cp}	1.63

SOURCE: Authors' calculations.

important for forecasting a year ahead. The results for the annual models (equations [3] and [5]) show that adding the commodity price term yields smaller forecast errors. Another way to compare competing forecasts is to ask whether the forecast error from a given model can be explained (encompassed) by the forecast of another model, as elucidated in Chong and Hendry (1986). To make this comparison, let e_t^1 and e_t^2 represent the forecast errors made by models 1 and 2. Then examine the t-statistics for the coefficients α^1 and α^2 in the regressions

$$e_t^1 = \alpha^1(e_t^1 - e_t^2), \quad e_t^2 = \alpha^2(e_t^2 - e_t^1).$$

If α^1 is significantly different from zero, model 2 encompasses model 1, as it contains useful information (for forecasting purposes) that is not in model 1. If α^1 is significant but α^2 is not, the encompassing is one-way; that is, model 1 is encompassed (by model 2) but is not encompassing. Model 2 is then clearly better on statistical grounds. The statistics in table 4 show that the annual model (equation [5]) with commodity prices holds just such a relationship with the quarterly and annual price gap models, while neither of the latter encompasses the other.

Although the comparisons in tables 3 and 4 show that commodity price information can improve the forecasting performance of the price gap model, the t-statistic comparison in table 2 indicates that the improvement will be less at longer horizons. Commodity prices themselves are notoriously difficult to forecast (as implied by efficient markets theory), so it is probably impractical to make multistep forecasts from models like equations (4) and (5) to predict at longer horizons.

For 1991, the quarterly and annual price gap models predict inflation rates of 3.1 and 3.8 percent, respectively. Augmenting the annual model with the commodity price term yields a somewhat lower prediction of 3.6 percent. The low prediction of the quarterly model reflects the unusually slow money growth in the first three quarters of this year. If we assume that P, P*, and P^c all grew at a 2 percent annual rate for the fourth quarter of 1991, the inflation rate for all of 1991 would be 3.2 percent. Both annual models (equations [3] and [5]) would then forecast 1992 inflation at 2.1 percent.

IV. Conclusion

The P* approach to forecasting inflation exploits the long-run tendencies of output to return to potential and velocity to return to its mean. However, other factors may also influence the inflation rate over shorter horizons. Incorporating such influences into the HPS model can be accomplished simply by including additional terms that measure the lagged discrepancy between the actual inflation rate and the rate that would be predicted based on the relationship between inflation and the new factor.

The commodity inflation data used here are only one of many possible augmentations. Inflation as measured by a wholesale price index, for example, would be one reasonable addition; another might be an expectations measure derived from the yield curve. So long as these additional factors are related to the inflation rate, rather than to the price level, the latter will continue to be determined only by money and potential output. If potential output is taken to be exogenous, then ultimately, only money matters.

References

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