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Stochastic Interest Rates in the Aggregate Life-Cycle/ Permanent Income Cum Rational Expectations Model **2**

Recent tests of the life cycle/permanent income cum rational expectations model have assumed either that real interest rates are constant, or that consumers know the future path of real rates. This article estimates a life cycle cum rational expectations model that allows for stochastic real interest rates. The results show that the model is strongly rejected using post-World War II U.S. data.

New Classical and New Keynesian Models of Business Cycles **20**

Both Keynesian and Classical economists have developed new models of the business cycle during the 1970s and 1980s. Both have striven to establish firm microfoundations for their theory of fluctuations, so that events may be understood in terms of the basic economic environment and the actions of individual agents. In this article, economic analyst Eric Kades presents bare-boned models of both schools, attempting to lucidly illustrate sources of business cycles. Simulations show that both models can mimic observed time series convincingly. Theoretical strengths and weaknesses are discussed, followed by a cursory examination of empirical evidence for and against each model.

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Stochastic Interest Rates in the Aggregate Life-Cycle/Permanent Income Cum Rational Expectations Model

by Kim J. Kowalewski

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There has been renewed interest in consumption behavior in the past 10 years. The origin of this interest is not so much due to deterioration in the ability of economists to predict future output and prices, although that is clearly important.

The main impetus is the challenge of the "New Classical" school. Barro (1974) argued that rational private agents do not view bond-financed increases in government spending or decreases in taxes as increases in wealth, because they know that the new bonds must be retired by additional future taxes. Rational private agents therefore will increase current saving to pay for these future taxes, no matter how far into the future they come due. This additional saving is exactly enough to purchase all of the new debt; interest rates and aggregate wealth remain unchanged. This implies that bond-financed increases in government spending have a multiplier value of 1, and that bond-financed tax cuts have a zero multiplier.

Money-financed increases in government spending also have a zero multiplier, because rational private agents view the faster growth in money as leading to a higher inflation rate in the future. This higher inflation is another "tax" that private agents will save for. That is, money-financed tax cuts have no effect on real variables, because one tax is just substituted for another.

These "New Classical" results are a direct challenge to the Keynesian and Monetarist schools, which assign higher values to these multipliers (at least in the short run), because the effects of fiscal policy actions are distinguished by how they are financed.[1]

Barro's result depends, among other things, on the assumption that private agents have the opportunity to offset these government actions. This, in turn, assumes that capital markets are perfect—that there are no transactions or other costs that drive a wedge between borrowing and lending interest rates, and that there are no informational asymmetries that are controlled with down payments,

1. See Tobin (1980) and Tobin and Buiter (1980) for good discussions of these points.

2. Muellbauer (1983) and Wickens and Molana (1984) reject the model using U.K. consumption and income data.

security interests, rationing the quantity of credit, and other non-price loan provisions. Thus, with perfect capital markets, the length of a consumer's spending horizon (that is, the time span over which a permanent increase in life-cycle wealth/permanent income is consumed) is as long as his remaining lifetime. It may be longer if, as Barro assumes, a consumer's utility function includes the utility of his direct descendants. A consumer can borrow any amount up to the current value of his net nonhuman wealth, plus the present value of all his expected future after-tax labor income, all discounted at the common rate of interest. An increase in life-cycle wealth/permanent income will be consumed over the remainder of the horizon, making the amount consumed in the short run very small.

If capital markets are imperfect, however, then the length of a consumer's planning horizon may be shortened. A consumer may not be able to borrow against all of his life-cycle wealth (or permanent income), or may do so only at a penalty rate of interest. Increases in life-cycle wealth/permanent income will be consumed over this shorter horizon, enlarging the (short-run) impact of bond-financed tax cuts or spending increases. Clearly, shorter horizons make it possible for stabilization policies to affect real variables, at least in the short run.

Thus, the recent interest in consumption behavior centers on learning the length of consumer spending horizons. The approach taken by most recent studies is to test some variant of the life-cycle/permanent income cum rational expectations (RE-LC/PI) model assuming perfect capital markets. Rejection of the RE-LC/PI model, incorporating perfect capital markets, is taken to mean that horizon lengths may not be long enough to diminish the power of stabilization policies.

Hall (1978), Flavin (1981,1985), Hayashi (1982), Muellbauer (1983), Wickens and Molana (1984), Bernanke (1982), Mankiw (1983), DeLong and Summers (1984), Boskin and Kotlikoff (1984), Kotlikoff and Pakes (1984), and Mankiw, Rotemberg, and Summers (1985) test the RE-LC/PI model with aggregate time series data, while Hall and Mishkin (1982), Bernanke (1984), and Hayashi (1985) use cross-section or panel data on individual households.

Of the studies employing micro-data, only Bernanke (1984) can reject the model. Of the studies that employ aggregate time series data, Hall (1978), Hayashi (1982), Mankiw (1983), Bernanke (1984), and Delong and Summers (1984) cannot reject the model during the post-World War II period. Kotlikoff and Pakes (1984) can reject the model, but conclude that the differences from the model are not large enough to matter in practice.[2]

These studies are not the first to be concerned with the length of consumer spending horizons. For example, Tobin (1951) argued that capital market imperfections may have accounted for the different savings behaviors of black and white Americans in the late 1940s. Houthakker (1958), in his review of Friedman's (1957) permanent income hypothesis, argued that the exclusion of capital market imperfections was the main defect of Friedman's work. Friedman (1963) argued that consumer horizon lengths were about three years.

Before rational expectations came into vogue, there were numerous tests of the life-cycle and permanent income models, beginning with Modigliani and Brumberg (1954) and Friedman (1957). The debate about the efficacy of the 1968 temporary tax increase focused on the length of consumer spending horizons see, for example, Okun (1971) and Blinder (1981). There has been considerable theoretical work done on the impact of capital market imperfections (see, for example, Tobin and Dolde [1971], Dolde [1973], Pissarides [1978], Heller and Starr [1979], Foley and Hellwig [1975], and Watkins [1975,1977]).

What is new about these recent studies is their assumption of rational expectations. Unfortunately, richness of detail seems to have been sacrified for this assumption. For exam-

ple, none of the recent models that are estimated with U.S. aggregate time series data allows for uncertain real interest rates. All of the models, except Bernanke (1982) and Mankiw (1983) assume that the real interest rate is constant. Bernanke (1982) and Mankiw (1983) allow real interest rates to vary, but assume that consumers know all future real interest rates.

It is rather curious that stochastic real interest rates have been ignored, because the real interest rate is a key variable in the life-cycle/permanent income model (and in many New Classical models). The interest rate measures the exchange rate between consuming today and saving today to consume more tomorrow. The life-cycle/permanent income model determines the utility-maximizing allocation of life-cycle wealth (permanent income) across time by balancing the marginal rate of transforming consumption today into consumption tomorrow (the interest rate) with the marginal rate of substitution (the discounted marginal utility from consuming tomorrow relative to that from consuming today). Changes in interest rates, expected or unexpected, should lead to a reallocation of consumption spending across time. Thus, an allowance for stochastic real interest rates should provide a more powerful test of the RE-LC/PI model and indirectly of the (maximum) length of the representative consumer's spending horizon.

In this article, we estimate a RE-LC/PI model that allows for uncertain future interest rates. The model is developed by Muellbauer (1983), which he estimated with United Kingdom (U.K.) data. To put Muellbauer's model into perspective, the Hall and Flavin (1981) models are also discussed and estimated. Updating the Hall and Flavin results with the 1980s data also may reveal any structural instabilities and shifts in the distribution of horizon lengths across consumers, which is a possibility ignored by all of the recent RE-LC/PI tests. Section II reviews the RE-LC/PI models, section III briefly outlines the procedures followed in estimating the three models and explains the results, and the third section concludes our study.

I. The Life-Cycle/Permanent Income Model With Rational Expectations

Tests of the RE-LC/PI model begin with Hall (1978). The consumer is assumed to maximize the expected present discounted value of current and future utility. Income is exogenous and is known in the current period, but unknown thereafter; the consumer's choice variable is the level of consumption each period. The horizon begins with the current period and ends at the (known) last period of the consumer's lifetime. There are no bequests and no capital market imperfections. Expectations are rational—functions of all information available in the current period. Real interest rates and rates of time preference are assumed to be constant. The model is:

$$(1) \qquad \max_{C_t} E_t \sum_{i=0}^{T-t} [\delta^i U(C_{t+i})]$$

subject to

$$\sum_{i=0}^{T-t} (R^i C_{t+i}) - \sum_{i=0}^{T-t} (R^i y_{t+i}) = A,,$$

where

6 is the inverse of 1, plus the pure rate of time preference, assumed constant,
R is the inverse of 1 plus the real, after-tax rate of interest r, also assumed constant, ($\delta \geq R$),
C is real life cycle consumption (not NIPA personal consumption expenditures),
y is real labor income,
A is current real nonhuman wealth,
$U(\bullet)$ is the instantaneous utility function, and
E_t is the expectations operator, conditioned on the information available at time t (variables dated t-1 and earlier).

The first order conditions for this problem are:

(2a) $E_t U'(C_{t+i}) = (R/\delta) E_t U'(C_{t+i-1})$, for $i=1$ to $T-t$; in particular, for $i=1$

(2b) $E_t U'(C_{t+1}) = (R/\delta) U'(C_t)$.

There are two things to note about (2b). First, C_t can be thought of as a sufficient statistic for C_{t+1}; that is, no variable except C_t helps predict future marginal utility of consumption $U'(C_{t+1})$. Second, with the assumption of rational expectations, marginal utility follows the regression relation:

(3) $U'(C_{t+1}) = \gamma U'(C_t) + \epsilon_{t+1}$.

The term ϵ_{t+1} represents the impact on marginal utility of all new information that becomes available in period t + 1 about the consumer's lifetime well-being. Under rational expectations, $E_t \epsilon_{t+1} = 0$ and ϵ_{t+1} is orthogonal to $U'(C_t)$. Moreover, ϵ should be white noise, that is, unpredictable using variables in the information set.

If the utility function is quadratic or "the change in marginal utility from one period to the next is small, both because the interest rate is close to the rate of time preference and because the stochastic change is small." (See Hall [1978, p. 975].) Then equation **(3)** becomes:

(4) $C_t = \gamma C_{t-1} + \epsilon_t$.

That is, life-cycle consumption follows an AR (1) process—no other variables dated t-1 or earlier affect C_t. If $\gamma = 1$, then consumption follows a random walk. It is important to notice that **(4)** is not a structural model of life cycle consumption behavior. Because it is only the first-order condition for utility maximization, it is only an implication of the life-cycle model under rational expectations. Indeed, it is only a necessary condition for this RE-LC model to be true.

Hall also shows that lifetime resources evolve as a random walk with trend. First, nonhuman wealth follows the relation:

(5) $A_t = R^{-1}(A_{t-1} + y_{t-1} - C_{t-1})$.

Second, human wealth, H_t, is the sum of current labor income and the expected present discount value of future labor income:

(6) $$H_t = \sum_{i=0}^{T-t} (R^i E_t y_{t+i}),$$

where

$$E_t y_t = y_t,$$

from which it follows that:

(7a) $H_t = R^{-1}(H_{t-1} - y_{t-1}) + \mu_t$,

where μ_t represents the present value of the changes in expectations of future income that occur between period t-1 and t:

(7b) $$\mu_t = \sum_{i=0}^{T-t} [R^i (E_t y_{t+i} - E_{t-1} y_{t+1})].$$

Again, under rational expectations, $E_{t-1}\mu_t = 0$, and μ_t should be white noise. Under certainty equivalence, $\delta_t = \alpha_t \mu_t$, where α_t is an annuity factor modified to take account of the fact that the consumer plans to make consumption grow at a proportional rate y over his remaining lifetime. Then the equation for total wealth is:

(8) $A_t + H_t = R^{-1}(1-\alpha_{t-1})(A_{t-1} + H_{t-1}) + \mu_t$.

Flavin (1981) estimates a different version of the permanent income model using the insight from (7) to eliminate the unobserved H_t. She starts by defining current consumption as the sum of permanent and transitory consumption. By equating permanent consumption with permanent income (y_t^p), she has:

(9) $C_t = y_t^p + \epsilon_{2t}$, where ϵ_{2t} is transitory consumption.

Thus, permanent income is defined to be the annuity value of the expected present discounted value of human and nonhuman wealth ($A_t + H_t$), assuming the real, after-tax rate of interest, r, is constant:

(10) $$y_t^p = r(A_t + \sum_{i=0}^{\infty} [R^{i+1} E_t y_{t+i}]).$$

3. This assumption is not unreasonable, given that her model explains short-run changes in consumption. However. in her later paper, Flavin (1985) uses annual data where it seems less likely that changes in the rate of return to capital dominate endogenous changes in wealth accumulation.

Flavin shows that $E_t y^p_{t+1} = y$ fusing the insight implicit in equation (7b). Substituting (10) into (9) and using the nonhuman wealth constraint:

$$(11) \quad A_{t+1} = R^{-1} A_t + y_t - C_t.$$

Unlike equation (5), current period saving does not earn interest in equation (11). Equation (9) can be used to solve for C_{t+1} in terms of C_t:

$$(12) \quad C_{t+1} = C_t + r \sum_{i=0}^{\infty} [R^{i+1}(E_{t+1} - E_t) y_{t+i+1})] - R^{-1}\epsilon_{2t} + \epsilon_{2t+1}.$$

Flavin notes that because the coefficient of ϵ_{2t} is not -1, C_t will not evolve as a random walk unless the transitory consumption term ϵ_{2t} is zero for all t.

Equation (12) contains revisions in expectations of future real labor income. Flavin notes that "[a]s an empirical matter however, unanticipated capital gains and losses on nonhuman wealth probably constitute a significant fraction of the revisions in permanent income this model is trying to capture." (See Flavin [1981, p. 988].) She defines unanticipated capital gains as the present value of the revision in the expected earnings associated with the current nonhuman wealth position. By then assuming "... that changes in the rate of return to capital ... are quantitatively more important than the endogenous changes (in nonhuman wealth) in determining the time-series properties of the observed path of nonlabor income ...", unanticipated capital gains can be approximated as the present value of the revision in expected future nonlabor income. (See Flavin [1981, p. 988].) This permits her to use disposable personal income (YD) in place of labor income (y) in equation (12).[3]

Flavin next derives an expression for the revision in expectations of future YD by assuming that YD follows an ARMA process. She shows that the revision in the expectation of YD_{t+s} ($s>0$) between periods t and t-1 is the product of the moving average error of YD in period t (u_t) and the sth coefficient from the corresponding moving average representation for YD (B_s). Then the present discounted value of the set of revisions is:

$$(13) \quad (\sum [R_s B_s]) u_t.$$

Thus, she demonstrates that the revision in income expectations is white noise.

The ARMA model for YD plus the equation formed by substituting (13) into (12) is Flavin's permanent income consumption model. Note that (13) still contains an unobserved variable u_t. This term is included with the other error terms in estimation, making her consumption equation very similar to Hall's. The difference is that Hall's model can be viewed as a reduced form of Flavin's structural model. Flavin argues that the error terms in the two equations are correlated because her model is incomplete. The income equation error will contain additional terms because the information set probably contains variables other than past income. These omitted information set variables will also appear in the consumption equation error through (13), thus producing the correlation between the two equation errors. She dismisses this apparent specification bias by assuming that these omitted information set variables are serially uncorrelated and uncorrelated with the lagged income terms.

Hayashi (1982) also uses equation (7) to eliminate the unobserved H_t. He starts with the permanent income model in level form:

$$(14) \quad C_t = \alpha(A_t + H_t) + \epsilon_t,$$

where ϵ_t is defined as "transitory consumption"—a shock to preferences or measurement error in C_t and A_t. He notes that α, the propensity to consume, is a function of the expected real rates of return from nonhuman wealth and the subjective rate of time preference: but, like Hall and Flavin, assumes that these factors are constant over time and individuals. Using (7a) with an "overall" discount

rate $1+d$ in place of R, Hayashi eliminates H, from (14):

$$(15)\quad C_t = (1+d)C_{t-1} + \alpha[A_t - (1+d)(A_{t-1} + y_{t-1})] + v_t,$$

where $v_t = u_t - (1+d)u_{t-1} + \alpha\mu_t$. Like Flavin, Hayashi also uses a two-equation model, composed of equation (15) and a stochastic version of equation (5). He adds an error term to Hall's nonhuman wealth identity to capture unanticipated movements in asset prices and measurement errors in A_t, A_{t-1}, y_{t-1}, and C_{t-1}. Note that Hayashi's model uses labor income instead of YD and is slightly more general than either Hall's or Flavin's, because it does not assume that $1+d = R^{-1}$.

Hall, Flavin, and Hayashi test their models by adding other variables to the right-hand side of (4), the modified version of (12), and (14). It is clear that by doing so they test the *joint* hypothesis that both the life-cycle/permanent income model and the rational expectations assumption are correct. If they were interested in testing only the assumption of rational expectations, conditional upon the LC/PI model, for example, they would have compared their models with suitable transformations based on different hypotheses about expectations formation. If the joint hypothesis is correct, then no other variable in the information set except C_{t-1} will help forecast C_t. Although any set of variables could be used to test these models, income is an obvious choice, because a direct relationship between consumption and current income in these models would be strong evidence against the simple life-cycle/permanent income model assuming perfect capital markets and against Barro's neutrality hypothesis.

Recall that there is no direct structural relationship between consumption and income in these models. Current income may be correlated with current consumption, but the correlation arises only indirectly, because current income represents new information about human wealth/permanent income. Unlike Friedman (1957) and Modigliani and Brumberg (1954), who allowed for the possibility that some unexpected changes in income would not alter a consumer's estimate of his permanent income or life-cycle wealth, all unexpected income changes in the Hall, Flavin, and Hayashi models lead to revisions in permanent income or life-cycle wealth and, hence, consumption.

The models are estimated and tested with post-World War II U.S. aggregate time series data. Unfortunately, it is difficult to compare their results because they use different data and sample periods. This is partly due to the lack of reliable data on life-cycle/permanent consumption. Hall uses real, per capita PCE-nondurables and services as the consumption variable, ignoring the service flow from consumer durables because of the lack of reliable data. Flavin uses only real per capita PCE-nondurables as the consumption variable. She notes that the consumption of durable services should exhibit a lagged response to changes in permanent income due to the transactions costs of adjusting durable good stocks. The same is true of housing services, which form a large part of PCE-services. By using only PCE-nondurables, she says that she gives the benefit of the doubt to the random walk hypothesis of one-quarter adjustment.

However, this point is probably irrelevant, because Flavin detrends the consumption and income data before estimation. The strong trend in PCE-services most likely would be eliminated with detrending, allowing her to use PCE-nondurables and services as the dependent variable. Indeed, as shown below, Flavin's model rejects the RE-LC/PI model, using PCE-nondurables and services as the dependent variable. Hayashi uses real, per capita annual data constructed by Christensen and Jorgenson (1973 and updates) for the consumption variable and a modification of their labor income variable for y. The consumption data contain imputations for the service flow of consumer durables. Flavin uses real per capita YD for the income variable, and all three use this variable (or its lagged value) for testing their models.

Hall's first test consists of adding three additional lagged C terms to the right-hand side of (4) and finds them to be statistically insignificant individually and taken together. He finds the same result when one, four, and 12 lagged YD terms are added. In all cases, the coefficient on C_{t-1} is not significantly different from 1, which leads Hall to conclude that aggregate consumption is a ***random walk*** process.

However, when Hall adds four lagged stock price variables (Standard and Poor's comprehensive index of stock prices deflated by the implicit deflator for PCE-nondurables and services and divided by population), he finds that they are individually and collectively statistically significant. Hall argues that this evidence does not contradict the joint hypothesis, if it is assumed that "some part of consumption takes time to adjust to a change in permanent income. Then any variable that is correlated with permanent income in period $t-1$ will help in predicting the change in consumption in period t, since part of that change is the lagged response to the previous change in permanent income." (See Hall [1978, p. 985].) He also says that "the discovery that consumption moves in a way similar to stock prices actually supports this modification of the random walk hypothesis, since stock prices are well known to obey a random walk themselves." (See Hall [1981, p. 973].) In all tests, the Durbin-Watson statistic, which is biased downwards in these models when the autocorrelation of the errors is positive, cannot reject the hypothesis of no first-order autocorrelation. Hall thus concludes that the model cannot be rejected.

This is a rather curious inference. Hall finds a variable that contradicts the null hypothesis, and he subjectively rationalizes it! Moreover, it seems highly improbable that two truly random walks will be strongly correlated with each other. Since the two series are correlated, does this mean that the two series are not random walks, that they are random walks around a common trend, that there is a structural relationship between the two series, that the correlation is simply spurious, or that they are an artifact of aggregate time series data? Unfortunately, Hall does not report any tests of these possibilities.

Flavin adds the current and first seven lagged changes in real per capita YD to equation (12) with ΔC_t as the dependent variable. By adding these eight terms, she obtains a just-identified system. The reduced form of her model thus becomes:

$$(12a)\quad YD_t = \mu_1 + \alpha_1 YD_{t-1} + \alpha_2 YD_{t-2} + \ldots + \alpha_8 YD_{t-8} + \eta_{1t},$$

$$\Delta C_t = \mu_2 + \beta_0 (U_1 + (\alpha_1 - 1) YD_{t-1} + \alpha_2 YD + \ldots + \alpha_8 YD_{t-8}) + \beta_1 \Delta YD_{t-1} + \beta_2 \Delta YD_{t-2} + \ldots + \beta_7 \Delta YD_{t-7} + \eta_{2t},$$

where n_2, contains ϵ_2, and (13). The β's are "measures of the 'excess sensitivity' of consumption to current income, that is, sensitivity in excess of the response attributable to the new information contained in current income." (See Flavin [1981, p. 990].) Thus, a test of the joint statistical significance of the β's is a test of the RE-PI model. Over the 1949:IIIQ to 1979:IQ sample, Flavin can reject the model at a 0.5 percent significance level. The coefficient β_0 on the Δ YD, term allows her to test for a direct effect of current income on C, although her estimate of β_0 is quite large relative to those of the other Δ YD terms, its t-statistic is only 1.3, suggesting that the test "falls short of providing conclusive evidence that the permanent income-rational expectations hypothesis fails in a quantitatively significant way." (See Flavin [1981 p. 10021.)

Hayashi adds YD, to equation (14) and finds its coefficient to be of the same order of magnitude as the estimate of the discount factor, but statistically insignificant in his two-equation model. He also finds that the discount rate is statistically different from the constant real rate of return, contrary to Hall's and Flavin's assumptions. Although this is

4. It is not clear how Bernanke lets the real interest rate vary over time.

evidence in favor of the permanent income cum rational expectations hypothesis, Hayashi argues that "... the relevant measure of consumption for the liquidity-constrained households is personal consumption expenditures as defined in the National Income and Product Accounts (NIPA), which ***excludes*** service flows from consumer durables and ***includes*** expenditures on consumer durables. The foregoing test of the permanent income hypothesis seems to be in some sense unfair to the alternative hypothesis of liquidity constraints." (See Hayashi [1978, p. 908].) When he uses PCE as the dependent variable and estimates only the consumption equation (because the asset equation includes consumer durables), he finds the coefficient on current YD to be fairly large (0.892) with a t-statistic of about 20. On the basis of this result, he is persuaded to reject the permanent income cum rational expectations model. Here again is a rather curious inference. In effect, Hayashi is saying that only PCE-durables purchases can be liquidity-constrained.

Other authors have tried to relax some of the assumptions made by these writers. Bernanke (1982) and Mankiw (1983) focus on the separability issue by adding consumer durable to the life-cycle cum rational expectations model. They argue, like Flavin, that lagged stock adjustment and accelerator effects may lead to an incorrect rejection of the model. This is even true when durables are excluded from the analysis, if nondurables and durable are not separable in consumer utility functions. Moreover, as Hayashi points out, imperfections in capital markets are likely to show up in the pattern of durables purchases.

Bernanke derives a two-equation system in current period PCE-nondurables and services and next period's stock of consumer durables as the solution to the utility maximization problem. A quadratic utility function containing quadratic costs of adjusting consumer durable stocks is used. Mankiw also obtains a two-equation model, only based on the first-order conditions for utility maximization. Both show that consumption is not a random walk. In Bernanke's model, this is due to the adjustment costs, which supports Hall's assertion that adjustment costs can be consistent with the life-cycle cum rational expectations model. In Mankiw's model, consumption is not a random walk, because the real rate of interest and the relative price of durables are non-constant.

Both economists test their models with post-World War II U.S. aggregate time series data. Under the assumption of constant real interest rates, Bernanke finds that the response of consumers to an income innovation is significantly greater than predicted by the theoretical model and thus rejects the life-cycle cum rational expectations model. He claims, but unfortunately does not prove the evidence, that a similar result obtains if the real interest rate is allowed to vary.

Mankiw adds disposable income growth terms to both equations in his model and finds them statistically insignificant. He thus finds no evidence against the life-cycle cum rational expectations model and argues that his model "...is a useful framework for examining the linkage between interest rates, prices, and consumer demand." (See Mankiw [1983, p. 23].) As in many past studies, he also finds that consumer durables are quite sensitive to the real rate of interest. Depending on the parameter values chosen, the short-run elasticity of the stock of consumer durables with respect to the real interest rate varies between -1.7 and -4.3. Mankiw's results also suggest that the assumption of rational expectations is unimportant because he obtains results similar to those studies that do not assume rational expectations.

Real interest rates are not handled very satisfactorily by Mankiw.[4] Consumers are assumed not to know future income, but are assumed to know future interest rates (and the relative price of durables). Thus, interest rates are allowed to vary over time in a very uninteresting way. Muellbauer (1983) and

5. In general, when real interest-rate expectations are probabilistic the coefficient on C_{t-1} depends on the joint distribution of expected real incomes and real interest rates. In both cases, the optimal forecast of current consumption requires more information than provided by C_{t-1}.

Wickens and Molana (1984) allow for random and unknown future real interest rates.

Wickens and Molana show that when the interest rate in the life-cycle cum rational expectations model is random, the first order condition for utility maximization becomes:

$$(16) \quad E_{t-1}U'(C_{t+i-1}) = \delta E_{t-1}[(1/R_{t+i})U'(C_{t+i})] \quad (i \geq 0).$$

This expression is obtained by substituting C_t out of the utility function with the period-to-period budget constraint (11) and maximizing the present discounted value of expected future utility with respect to A_t. Expectations are formed with the information set available at the end of period t-1, which includes variables dated t-1 and earlier. With the necessary assumptions, (16) can be written as:

$$(17) \quad E_{t-1}C_{t+i} = E_{t-1}\gamma_{t+i}(E_{t-1}C_{t+i-1}), \quad (i \geq 0).$$

where 2 is a function of the interest rate and the rate of time preference. Thus, as in Hall's equation (2a), the coefficient on the lagged consumption term varies with the real interest rate.[5] With the appropriate assumptions, Muellbauer obtains an expression in potentially observable variables:

$$(18) \quad \Delta lnC_t = \mu_0 + \delta_3 E_{t-1}r_{t-1} + \delta_1\sigma_{1t} + \delta_2\sigma_{2t} + \epsilon_{t+1},$$

where σ_1 and σ_2 are the innovations in period t real disposable income and the real interest rate based on information available at the end of period t-1, which includes variables dated t-1 and earlier. The Wickens and Molana model differs only slightly from this, using r_{t+1} instead of r_{t-1}, because of a minor difference in the dating of the interest rate in the cash flow constraint. Both papers use post-World War II U.K. aggregate time series data.

Also note that apart from the logarithms and the dating difference on r, Flavin's model is nested in (18). However, Muellbauer and Wickens and Molana estimate their models differently than Flavin, because the variables they use to test their consumption equations are all lagged at least one period. Recall that the Flavin model is simultaneous, because she uses AYD_t, as one of her test variables. When deriving the reduced form of her two-equation system, the equation for YD is used to substitute out the current YD term in AYD_t. The revision to permanent income due to new information provided by current YD (13) cannot be identified and thus is thrown into the error term. Because Muellbauer and Wickens and Molana only use lagged variables to test their models, the income and interest-rate innovations remain identified by the income and interest-rate equations. Thus, unlike Flavin, they can estimate the coefficients on the innovation terms.

Ignoring the interest-rate terms in Muellbauer's and Wickens and Molana's model, it is not clear that their test is more powerful than Flavin's. The presence of AYD_t in the consumption equation gives Flavin a direct test of the impact of current income on current consumption. If the RE-LC/PI model is rejected, there is some knowledge about what the correct alternative may be, or at least in what direction the search for the correct alternative might go, but she cannot test for the impact of the income innovation, an important variable of the null hypothesis. By not adding any current income terms, Muellbauer and Wickens and Molana cannot test for a direct effect of current income on current consumption, but they do have a direct test of the impact of innovations in income.

The estimation procedure used by Muellbauer and Wickens and Molana requires two steps. The first step estimates with ordinary least squares (OLS) the simple reduced forms for disposable income and the real interest rate to generate the income and interest-rate innovations and expected values. Muellbauer's $lnYD$ equation uses the first two lags of $lnYD$ and lnC_{t-1} as the information set. For his real interest-rate equation, Muellbauer argues that apart from seasonal factors, the U.K. real interest rate varies randomly about

6. It was decided not to update Hayashi's model, because it is not so easily compared with the Hall and Flavin models. The Wickens and Molana model was not updated either, because it is similar to Muellbauer's, apart from some additional terms that complicate the estimation procedure.

a constant from the 1950s until the pound sterling began to float in 1972:IIQ; it follows a random walk thereafter. Wickens and Molana say that a broader information set than one that includes only lagged values of income and real interest rates, should be used with their more general model. They use the first four lags of *ln* YD, *lnC, r, lnA,* the latter being the log of real consumer liquid assets, as the information set for both real disposable income and the real interest rate.

The second step uses the residuals for the innovation terms and fitted values for the expected value terms in OLS regressions of the consumption equations. Both papers find that their models appear to fit the U.K. data very well. Wickens and Molana do not test the joint life-cycle rational expectations hypothesis; Muellbauer does by adding the information set variables to the right-hand side of (18) and tests for their joint statistical significance. He finds the additional lagged terms to be significantly different from zero. He concludes that allowing for stochastic interest rates does not seem to be a major cause for the failure of the simple Hall model to explain U.K. consumption found earlier by Daly and Hadjimatheou (1981).

II. Updates of the Aggregate Life Cycle Cum Rational Expectations Model

We update the estimates, test the Hall (1978) and Flavin (1981) models, and present estimates of the Muellbauer model using post-World War II U.S. aggregate time series data.[6]

Updating the Hall and Flavin models serves at least four purposes. First, the updates help put the results from Muellbauer's model in perspective. The importance of allowing for stochastic interest rates is immediately clear. Second, by estimating the models through 1984, we can estimate their stability. Third, it is interesting to know how the 1980s data fit these models. Real output and prices varied over wide latitudes during the 1980s and, hence, offer macroeconometricians a rich set of high-influence data, which may help them estimate coefficients more precisely. It is likely that the 1980s data provide even stronger evidence against the RE-LC/PI model than found by Flavin.

Finally, the different models are estimated with different information sets (reduced forms) and different sample periods. It is reasonable to wonder if either the content of the information set or the estimation period has a large influence on the estimates. Our interest in these models does not lie solely in determining whether the RE-LC/PI model is accepted or rejected, although that is a very important consideration. If these models are to be useful for policymaking and forecasting, however, they should be robust to different assumptions about the underlying structure used to derive the reduced forms.

The Hall and Flavin models are updated with their original samples, specifications, and estimation techniques. To make the three models comparable, we had to make at least four decisions. The first concerns the specification of the dependent and independent variables. Hall uses per capita PCE-nondurables and services, Flavin uses the change in per capita PCE-nondurables, and Muellbauer uses the change in the logarithm of per capita (U.K.) PCE-nondurables and services. The consumption definition used in these tests is per capita PCE-nondurables and services. Although Flavin's reasons for ignoring PCE-services may be valid, most of these problems should be eliminated once the data are detrended. The change in the logarithm of consumption and the logarithm of income are used here to facilitate comparison with the Muellbauer specification. This logarithmic specification should also minimize heteroskedasticity problems. The income definition is real disposable income per capita. The log real per capita income and consumption data are detrended by their average growth trends over the 1947:IQ to 1984:IVQ period. When the same dependent

variable is used, Flavin's consumption equation is, for all practical purposes, the same as Muellbauer's with constant interest rates.

The second decision involves seasonal adjustment of the data. Muellbauer uses seasonally unadjusted data, while Hall and Flavin use seasonally adjusted data. We used seasonally adjusted data to maintain comparability with other U.S. consumption results.

A third choice concerns estimation techniques. Hall uses OLS, Flavin uses maximum-likelihood to estimate her consumption equation jointly with her income forecasting equation, and Muellbauer uses a two-step OLS procedure. The original estimation techniques used by Hall and Flavin are used to update their models with the most recent data. Maximum-likelihood is used to estimate Muellbauer's model, because the computer-generated coefficient standard errors produced by the two-step method are incorrect.[7]

A fourth choice is that of the definition of the real interest rate. Instead of using an ***ex post*** real interest rate, Muellbauer uses something like an ***ex ante*** rate—a nominal interest rate minus an ***expected*** inflation rate. He computes this real rate by subtracting from the nominal rate a fitted value from an inflation equation. This choice of real rate is rather odd, for it means that instead of using an expected real interest rate as his theory requires, he is using an expected ***expected*** real interest rate in his consumption equation. It also means that he is using a three-step estimation process, with the estimation of the inflation equation as the first step. Moreover, the inflation equation uses an information set different from that used for the income and interest-rate equations. A logical extension and correction of his model would be to specify separate forecasting equations for the nominal rate and the inflation rate, to use the same information set for all of the equations, and to use the fitted values and residuals from both equations to compute the expected real rate and its innovation. An equivalent strategy employed here is to use an *ex post* rate, as Wickens and Molana do. This requires only one forecasting equation. The ***ex post*** real three-month U.S. Treasury bill rate, (nominal rate, minus current-quarter compounded annual actual growth rate in the PCE-nondurables and services deflator) is used as the real interest rate in the estimations of Muellbauer's model shown below.

Because there is no reason to think that U.S. real interest rates have behaved as random walks during the post-World War II period, the real interest-rate equation for Muellbauer's model will have information set variables as regressors, and these will be the same as those used for the income equation—the first two lags of income, the first two lags of the real interest rate, and the first lag of consumption. This is a simple extension of Muellbauer's original information set, which consisted of the first two lags of income and the first lag of consumption.

The estimation results are shown in tables 1 to 5. The data used for the computations contain revisions through the second revised estimates for 1984:IVQ dated March 31,1985. The models in tables 1 to 3 were estimated over their original samples and over 1949:IIIQ to 1984:IVQ. For the re-estimates of Hall's model, the data were not detrended. For the re-estimates of Flavin's model, the consumption and income data were detrended using their average growth rates over the 1947:IQ to 1979:IQ period. When the two models are updated with the data through 1984:IVQ, the consumption and income data are detrended using their average growth rates over the 1947:IQ to 1984:IVQ period, and a dummy variable is added to control for the credit controls of 1980:IIQ. Detrending biases the test in favor of the random walk hypothesis, because it removes the main source of correlation from these variables. Detrending may also remove structural correlation between C and YD, again favoring the random walk hypothesis. It unfortunately leaves the trend unexplained. The dummy variable is part of the maintained

7. See Kowalewski (1985) for more detail on this point.

8. Serially correlated errors may not signal a breakdown of the model, if as Hall argues when rationalizing the statistically significant stock price index terms, consumers take more than one quarter to assimilate new information and act upon a changed expectation of life-cycle wealth.

hypothesis and is not included among the variables included in the test of the RE-LC/PI model.

The first table contains OLS estimates of Hall's model. The first equation shows the reestimates of Hall's model with only one lagged income term. The coefficients, though different from Hall's published numbers, yield the same apparent inference: the RE-LC/PI model cannot be rejected. The next equation shows the original Hall model updated through 1984:IVQ. Note that the addition of the 1980s data did not change the conclusion of the hypothesis test—the coefficient on lagged personal income is small, has the wrong sign, and is statistically insignificant. However, the Durbin h-statistic rejects the hypothesis of positive serially uncorrelated errors at better than a 5 percent significance level using a one-tailed test. Because the theory predicts that the error should be white noise, the addition of the 1980s data may be signaling a breakdown of the model.[8]

The third equation contains the change in the detrended log of per capita PCE-nondurables and services as the dependent variable and the detrended logarithm of real per capita disposable personal income as the income variable. The estimation period is 1948:IQ to 1977:IQ. Neither coefficient is large, the t-statistics are very low, and the adjusted R^2 is negative. The results change very little when the estimation period is extended through 1984:IVQ; all of the explanatory power of the right-hand side variables comes from the

Table 1 Hall Estimates

$C_t = \alpha_0 + \alpha_1 C_{t-1} + \alpha_2 Y_{t-1} + \alpha_3 \; DUM802 + \epsilon_t$

	#1	#2	#3	#4
C	*NDS/POP*	*NDS/POP*	Chg in detrended log of *NDS/POP*	Chg in detrended log of *NDS/POP*
Y	*YD72/POP*	*YD72/POP*	Detrended log of *YD72/POP*	Detrended log of *YD72/POP*
Sample	48:1Q-77:IQ	48:1Q-84:4Q	48:1Q-77:1Q	49:3Q-84:4Q
α_0	-0.0376 (-2.2620)	-0.0059 (-0.5835)	0.0007 (1.1492)	0.0005 (0.8562)
α_1	1.0811 (24.8721)	1.0081 (31.3779)		
α_2	-0.0480 (-1.6283)	-0.0008 (-0.0341)	-0.0063 (-0.4627)	-0.0060 (-0.4902)
α_3		-0.0441 (3.0626)		-0.0131 (-2.3346)
adj R^2	0.9989	0.9994	-0.0068	0.0263
Durbin h	1.358	1.7327	1.7752[a]	1.7460[a]
SER	0.0136	0.0290	0.0058	0.0056

Variables:
NDS = PCE-nondurables plus services, 1972 dollars.
YD72 = disposable personal income, 1972 dollars.
POP = non-institutionalized, civilian population.

a. Durbin-Watson statistic.

NOTE:
1. The variables in equations #3 and #4 are detrended over the 1947:IQ to 1984:IVQ period.
2. The *t*-statistics are shown below the coefficient estimates.

9. Flavin (1981), proves the equivalence of these two procedures in appendix II.

10. When the consumption and income variables are detrended with their average growth rates between 1947:IQ and 1984:IVQ, the LRS for the joint test of the Δ ln YD terms becomes 13.5, which implies the rejection of the null hypothesis at about a 10 percent significance level.

dummy variable. Thus, Hall's model can find no evidence to reject the RE-LC/PI model.

The results for Flavin's model (12a) are shown in tables 2 and 3. Only the coefficients of the A YD (Δln YD) terms (the β coeffi-' cients in equation (12a) are shown because only they are relevant for the test of the RE-LC/PI model. Recall that these terms must be jointly statistically different from zero in order to reject the model. The first equation in table 2 shows the re-estimates of her original specification. Like the updates of Hall's model, these coefficients are not quantitatively the same as the original estimates; qualitatively, however, they are very similar. The coefficient on ΔYD_t, β_0, though fairly large, has a very low t-statistic; of the β's, only β_1 is significant at better than 5 percent using a one-tailed test. The likelihood ratio statistic (LRS) tests the joint significance of the ΔYD terms. Surprisingly, the RE-LC/PI model *cannot* be rejected at the original significance level.

Table 2 Flavin Re-estimates

Coef	Var	#1	#2
β_0	ΔYD_t	0.3194	0.2712
		(1.1164)	(1.4596)
β_1	ΔYD_{t-1}	0.0605	0.0650
		(1.8388)	(2.3574)
β_2	ΔYD_{t-2}	0.0079	-0.0099
		(0.2493)	(-0.3659)
β_3	ΔYD_{t-3}	-0.0662	-0.0535
		(-1.2940)	(-1.4499)
β_4	ΔYD_{t-4}	0.0415	0.0136
		(0.8088)	(0.3915)
β_5	ΔYD_{t-5}	-0.0081	-0.0082
		(-0.1410)	(-0.1908)
β_6	ΔYD_{t-6}	0.0068	0.0050
		(0.2163)	(0.1834)
β_7	ΔYD_{t-7}	0.0074	0.0169
		(0.2381)	(0.6146)
C SER		0.0103	0.0104
C D-W		2.0101	2.0521
Y SER		0.0329	0.0325
Y D-W		2.0008	2.0009
LR Statistic		11.754	17.148
Sample		**49:3Q-79:1Q**	**49:3Q-84:4Q**

NOTE: Detrending occurs from 1947:1Q to 1979:1Q.

Flavin's original likelihood ratio statistic, which is asymptotically distributed as $X^2(8)$, is 27.0, significant at better than 0.5 percent. The LRS for the test of equation (1) is only 11.8, significant at slightly better than 25.0 percent. Identical test results are obtained by estimating only the consumption-reduced-form equation with OLS and by testing for the joint significance of the lagged income terms.[9] Apparently, the results are sensitive to revisions in the data and to the use of different trend values for PCE-nondurables and YD.[10]

Equation (2) in table 2 updates Flavin's original model through 1984:IVQ. The 1947:IQ-1979:IQ trend values are used to detrend the post-1979:IQ data. Interestingly, the model can now be rejected at better than a 5.0 percent significance level; the LRS is 17.2, while the $X^2(8)$ cut-off value is 15.5 at 5.0 percent. The coefficient β_0 is now smaller, but its t-statistic is larger; the coefficient and t-statistic on $\Delta ln\, YD_{t-1}$ are also larger. Moreover, the fit of the equation is improved over the longer period; the standard errors of the two equations are smaller in the longer sample. Thus, as was expected, the 1980s data appear to tighten up coefficient standard errors and help reject the RE-LC/PI model.

Equations (3) and (4) in table 3 use the change in the logarithm of per capita real PCE-nondurables and services as the dependent variable, and the log per capita consumption and income data are detrended over the 1947:IQ to 1984:IVQ period. They compare to the Hall equations (3) and (4) in table 1. The third equation shows the unconstrained results over the 1949:IIIQ to 1979:IQ sample period. Notice that they are qualitatively similar to those of equation (1); β_0 is about 0.3 and is statistically insignificant; β_1 is large and is statistically significant. Testing the joint significance of the Aln YD terms yields a LRS of 27.1, which is significant at better than 0.5 percent, Flavin's original significance level. Note that

this result is much stronger than Flavin's original result, because the consumption variable includes PCE-services, which Flavin argued would bias the results against the RE-LC/PI model.

The fourth equation shows the estimation results over the 1949:IIIQ to 1984:IVQ sample. Qualitatively, these results are similar to those of equation (3). The LRS of the test of the lagged Δln YD terms is now 29.4, greater than the LRS over the 1949:IIIQ to 1979:IQ sample; the standard errors of the equation also are smaller in the longer sample. Again, it appears that the 1980s data provide additional stronger evidence against the RE-LC/PI model.

Tables 4 and 5 contain the estimates of Muellbauer's models. Only the coefficients on the information set, innovation, and expected interest-rate terms are shown. The dependent variable is the change in the logarithm of real per capita PCE-nondurables and services; detrending of the log real per capita consumption and income data occurs over the 1947:IQ to 1984:IVQ period. Table 4 shows the estimates of equation (18) without the interest-rate terms $E_{t-1}r_{t+1}$ and σ_2. The coefficient δ_1 on the income innovation should be positive, because positive innovations in current income should lead to upward revisions in life-cycle wealth/permanent income and, hence, in consumption. The first equation shows the results using the 1949:IIIQ to 1979:IQ sample. This equation compares to Flavin's equation (3) in table 3. The coefficient is δ_1 positive and statistically significant. Surprisingly, the RE-LC/PI model cannot be rejected by this form of Muellbauer's model, even though Flavin's model could. The LRS is only 3.8, significant at slightly less than 30 percent. Again, the results appear to be sensitive to the specification of the test.

The second equation in table 4 updates Muellbauer's model without the interest-rate terms over the 1949:IIIQ to 1984:IVQ sample. As was true of Flavin's model, Muellbauer's model without the interest-rate terms fits better with the 1980s data. Moreover, the LRS is now 14.2, significant at better than 1 percent. Again, the 1980s data lead to a convincing rejection of the RE-LC/PI model. Note that the coefficients on the information set variables are the same order both of magnitude and statistical significance in equations (1) and (2); the difference is that the model fits better with the 1980s data.

Table 5 contains the estimates of Muellbauer's model including the interest-rate terms. Recall from equation (18) that δ_3, the coefficient on the expected interest-rate term, is a positive function of the ratio of one, plus the interest rate, to one, plus the rate of time preference; hence, it should be positive. Presumably, the coefficient δ_2 on the interest-rate innovation is negative, since a higher-than-expected interest rate should cause consum-

Table 3 Flavin Estimates Using Logs

Coef	Var	#3	#4
β_0	ΔYD_t	0.2794	0.2652
		(0.8282)	(0.9903)
β_1	ΔYD_{t-1}	0.1208	0.1280
		(2.9091)	(3.3398)
β_2	ΔYD_{t-2}	0.0709	0.0597
		(1.7267)	(1.6026)
β_3	ΔYD_{t-3}	-0.0977	-0.0762
		(-1.7462)	(-1.5329)
β_4	ΔYD_{t-4}	0.0577	0.0457
		(0.6548)	(0.6887)
β_5	ΔYD_{t-5}	-0.1296	-0.1095
		(-2.7135)	(-2.5909)
β_6	ΔYD_{t-6}	0.0444	0.0459
		(1.1380)	(1.2202)
β_7	ΔYD_{t-1}	0.0162	0.0423
		(0.4045)	(1.1439)
C SER		0.0051	0.0050
C D-W		1.8636	1.9003
Y SER		0.0102	0.0098
Y D-W		1.9942	2.0022
LR Statistic		27.068	29.360
Sample		**49:3Q-79:1Q**	**49:3Q-84:4Q**

NOTE: Detrending occurs over the 1947:1Q-1984:4Q.

ers to save more in the current period. Equation (3) shows the results over the 1949:IIIQ to 1979:IQ sample. The two interest-rate coefficients appear to be small in magnitude, but this is simply a scaling difference because, interest rates are measured in percentage points. The interest-rate innovation coefficient δ_2 is statistically insignificant, while δ_3 is significant at slightly better than 10 percent. The LRS for the test of the RE-LC/PI model is 27.8, which is asymptotically distributed as $X^2(5)$, and is significant at better than 1 percent. Compared with equation (1) in table 4, the allowance for stochastic interest rates now leads to the rejection of the RE-LC/PI model. Again, the specification of the test has an important effect on the results.

Equation (4) in table 5 shows the estimates of Muellbauer's model with the interest-rate terms over the 1949:IIIQ to 1984:IVQ period. All of the coefficients are estimated more precisely, but unlike the previous results, the equation fits the longer period less well. The coefficients 62 and δ_3 now have the correct signs and about the same statistical significance as the earlier estimates. The LRS statistic for the test of the RE-LC/PI model is 26.2, rejecting the model at better than a 1 percent significance level, but it is a bit smaller than the LRS from the shorter sample period. Nevertheless, the results are qualitatively the same for both estimation periods, unlike the results of the Flavin tests.

The worse fit using the 1980s data occurs because the interest-rate equation fits less well in the longer period. This is not surprising, given that interest rates behaved so differently in the 1980s than in the earlier period.[11] Does this mean that the test is invalid because the equation generating the interest-rate expectations is wrong? This does not seem likely. Although the t-statistics on δ_2 and 63 do not provide support for the model, the LRS of the joint significance of the two interest-rate terms in equation (4) is 46.1. Thus, the interest-rate terms are undoubtedly important, even if they are poorly computed. Moreover, it is not clear how quickly interest-

Table 4 Muellbauer Estimates Without the Interest Rate

Coef	Var	#1	#2
61	*YRESID*	0.2185	0.2191
		(4.7533)	(5.3016)
β_1	$lnYD_{t-1}$	0.1207	0.1557
		(2.2775)	(3.4233)
β_2	$lnYD_{t-2}$	-0.1683	-0.1585
		(-3.4897)	(-3.5539)
β_3	lnC_{t-1}	0.0418	-0.0112
		(1.0391)	(-0.4075)
C SER		0.0053	0.0050
Y SER		0.0104	0.0104
LR Statistic		3.800	14.200
Sample		**49:3Q-79:1Q**	**49:3Q-84:4Q**

NOTE. YRESID is the current income innovation term

Table 5 Muellbauer Estimates with the Real Interest Rate

Coef	Var	#3	#4
δ_1	*YRESID*	0.2318	0.2431
		(5.2208)	(5.8538)
δ_2	*RRESID*	0.0001	-0.0001
		(0.2042)	(-0.3660)
δ_3	Er_{t-1}	0.0042	0.0026
		(1.8764)	(2.1257)
β_1	$lnYD_{t-1}$	0.0871	0.1433
		(1.6383)	(2.9831)
β_2	$lnYD_{t-2}$	-0.0000	-0.0419
		(-0.0005)	(-0.6306)
β_3	r_{t-1}	0.0006	0.0003
		(2.2494)	(1.0161)
β_4	r_{t-2}	-0.0024	-0.0017
		(-2.1887)	(-2.6248)
β_5	lnC_{t-1}	-0.0738	-0.1109
		(-1.0183)	(-1.7685)
C SER		0.0049	0.0050
Y SER		0.0111	0.0108
r SER		1.9737	2.0102
LR Statistic		27.800	26.200
Sample		**49:3Q-79:1Q**	**49:3Q-84:4Q**

NOTE: YRESID and *RRESID* are the current income and interest-rate innovations. Er_{t-1} is the expectation of last period's real interest rate based on Information available last period.

11. The standard error of the consumption equation also increased, but this is probably due to the poorer fit of the interest-rate equation through the cross-equation constraints.

rate forecasting models were adjusted in the 1980s. Given the lag in the learning process, the number of quarters for which the interest-rate equation may be wrong is probably smaller than 20. Even if the interest-rate equation is wrong, it is not necessarily irrational. Finally, the fit of the model did not worsen so much that this is likely to be the sole reason that the RE-LC/PI model is rejected.

III. What Has Been Learned?

The estimation results provide ample evidence to reject this form of the RE-LC/PI model during the postwar period, especially when the 1980s data are included. Even though Hall's specification cannot reject the model, minor generalizations of Flavin and Muellbauer can, and Muellbauer's specification including uncertain interest rates can reject the model with or without the 1980s data. It would appear that an important assumption for Barro's neutrality hypothesis does not hold.

Unfortunately, this rejection of the RE-LC/PI model does not offer an explicit alternative as a replacement. As mentioned earlier, these tests cannot distinguish the assumption of rational expectations from that of the life-cycle/permanent income model. All that can be inferred from these tests is that the joint hypothesis can be rejected. Flavin (1985) attempts to determine whether the rejection of the RE-LC/PI model is due to the assumption of perfect capital markets or to that of the permanent income model. She uses her original model augmented with an equation for the unemployment rate, which is a proxy for the number of liquidity-constrained consumers. However, there are many problems using such a crude variable for such a complex hypothesis; her tests undoubtedly have little power.

Nor do these tests provide many clues about the exact length of consumer spending horizons, or how the distribution of horizon lengths changes as interest rates, the distribution of income, or the supply of consumer credit changes.

That the distribution of consumer horizon lengths may vary over time is suggested by the increased significance of the likelihood ratio tests when the 1980s data are included. The early 1980s were apparently a time when the distribution of horizons lengths was skewed toward the shorter end, increasing the correlation of aggregate consumption to current disposable income. Additional evidence about changes in the distribution of consumer spending horizons is provided by Kowalewski (1982), who studies the time series behavior of aggregate personal bankruptcy filings in the United States. Personal bankruptcy filings are countercyclical, increasing in recessions and falling in recoveries. For a variety of reasons discussed in the article, it is likely that just before they file for bankruptcy, personal bankrupts have about the shortest spending horizons of all consumers.

Thus, increases in the number of personal bankruptcy filings might indicate a shift in the distribution of consumer spending horizons towards shorter lengths. In a regression explaining per capita personal bankruptcy filings, transitory income had a much larger impact than permanent income, suggesting that liquidity is very important for these financially distressed consumers. The composition of consumer portfolios was also significantly related to the behavior of personal bankruptcy filings. Unfortunately, this evidence is only about one tail of the distribution. It is clear that much work remains to be done before the time series behavior of aggregate consumption is understood.

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New Classical and New Keynesian Models of Business Cycles

by Eric Kades

"Not the least misfortune in a prominent falsehood is the fact that tradition is apt to repeat it for truth."

HOSEA BALLOU

The alleged demise of classical economics was greatly exaggerated in the Keynesian era after World War II. The supposed death blow was the seeming inability of the purely competitive model to explain the vagaries of the business cycle. But in the last two decades, a number of articles have demonstrated that fluctuations with many of the central characteristics of observed business cycles can arise in "classical" market-clearing models.' Market-clearing notions are among the strongest in economics, and the ***New Classical*** ability to explain business cycles has breathed new life into the equilibrium approach and many of its provocative conclusions. The existence of business cycles is no longer a reason to ring the death knell for classical models.

Keynesian models have never had trouble explaining business cycles. Observed movements of output and prices have shaped Keynesian thinking first and foremost, and their models have always admitted these facts. Perhaps because of this preoccupation with empirical regularities, general equilibrium microfoundations for Keynesian economics failed to arise quickly. Much of the New Classical rebellion against Keynesian orthodoxy in the late 1960s and 1970s was understandably inspired by this lack of a strong choice-theoretic basis for the neoclassic synthesis (the IS-LM and Phillips curve model). Economic theorists of all schools have become less and less willing to accept models not derived from explicit maximizing behavior in a general equilibrium setting.

Such shortcomings led to premature eulogies for Keynesian theories; New Classical economists found the inflation of the late 1960s and the stagflation of the 1970s evidence of the failure of Keynesian ideas and policies. But the theoretical deficiencies have, in large part, been remedied, and indeed, the ***New Keynesian*** tradition employs more precision and adherence to general equilibrium rigor than the New Classical.[2]

Both New Keynesians and New Classical theorists either implicitly or explicitly are searching for the central cause or causes of business fluctuations. Economists of both

1. See for example, Lucas (1972), (1979); Prescott and Kydland (1980); or Long and Plosser (1983).

2. *For a New Keynesian example, see Benassy* (1976); *Malinvaud* (1977); *Bohm* (1978); *or Grandmont* (1982). *For a representative New Classical example, compare existence proofs in Dreze* (1975) *or in van den Heuvel with Lucas* (1979) *or in Long and Plosser* (1983). *Although the label "New Keynesian" is not uncontroversial, I feel its use is warranted. First, Keynes states clearly in the General Theory that his model generalizes on the classical perspective. This is a central point of this paper, in reference to present-day theories. Second. market failures are at the root of Keynes's model. New Keynesian theory merely formalizes insights due in large part to the General Theory. Finally, the modern authors who developed this approach (Benassy, Younes, Grandmont, and Dreze) refer to their models as "Keynesian," "neo-Keynesian," etc. Thus the use of "New Keynesian" is historically accurate.*

schools agree that many factors are involved, but find the ***rough*** equivalence of cycles (in covariances; not in frequencies and amplitudes) striking and believe the essence of the issue can be illustrated in relatively simple models. When examining Classical and Keynesian models of the business cycle, one is weighing the evidence and deciding which fundamental insight best agrees with the data.

For Keynesians, the central cause of the business cycle has always been market failure. The formal definition of ***market clearing*** equilibrium, that prices adjust to the attributes of agents so that trades balance under desired behavior, is employed by all theorists today. This rigorous definition of market clearing did not arise until the 1950s (Debreu and Arrow [1954]), and it was not for another decade that Clower (1965) clarified the Keynesian idea of ***market failure.*** This idea was then formalized and rigorously established as valid in a general equilibrium framework (Benassy [1975] and Dreze [1975]), by the ***New Keynesians*** in the early 1970s. The basic notion of market failure is that quantities adjust faster than prices. Prices then do not clear markets, and the entire market-clearing house of cards collapses. Such Keynesian models are compatible with rational expectations and full information. And they do not rely on "strange" utility or production functions; indeed we will see that, at present, disequilibrium models are more robust than equilibrium models as to the specification of these fundamentals.

One way to highlight the difference between the Keynesian and Classical perspectives is to describe their view of the existing market mechanism. Keynesians view this market structure as an ***endowment*** that, at least over moderate horizons, agents must take as given, much like their endowments of various goods, such as labor, time, assets, etc. Conversely, Classical theorists view the market structure as much more fluid; any possibility for gains from trade between agents (taking into consideration search and transactions costs) can and will be exercised. This is reflected in a price mechanism that works rapidly and effectively. In Keynesian models, the imperfect market structure causes business cycles. For Classical theory, fluctuations must arise from other sources.

The essence of New Classical business cycles lies in agents' intertemporal substitution of consumption and labor in response to technology (supply) or other shocks. Agents desire to smooth their consumption paths and, to achieve this end, substitute between present and future consumption, present and future leisure and, intratemporally, between labor and leisure. Combined with very simple technology shocks, such a model can mimic observed business cycles.

Both schools of thought, then, have constructed models that "explain" business cycles in that they reproduce the basic empirical regularities of observed fluctuations. Transacted quantities of all goods exhibit high positive correlation over time, and quantity movements tend to persist in the same direction for many periods. Further, both generate pro-cyclical real wages. These are the most basic features of observed business cycles.

How are economists to choose which model better explains economic fluctuations? Are the two theories observationally equivalent so that it is impossible to determine which truly describes the real economy? This question is important, since Keynesian models call for activist policy to smooth business cycles, while in Classical models these fluctuations are desired paths for the economy.

We will demonstrate that the New Classical (NC) model is a special case of the New Keynesian (NK) model. Thus, the NC model can be distinguished by the restrictions it places on the more general theory. In the decision-theoretic foundations of statistical scientific inquiry, we can state precisely that there will be less risk in working with the NK model, since it places less ***a priori*** restrictions on parameters. And although testing hypotheses on these

highly abstract models is controversial (due to lack of desired statistics, problems of aggregation, and other problems with available data), existing empirical evidence casts doubt on the *a priori* restrictions of the NC model.

We illustrate these points by presenting simple, but essentially complete, NC and NK models of the economy and business cycles that illustrate the central forces behind fluctuations in each. We then discuss theoretical and statistical arguments for and against each model. The models examined are intentionally bare-boned; they assume perfect information, rational expectations, and model only labor and goods markets. No assets exist; money is solely a unit of account.

I. Equilibrium Model

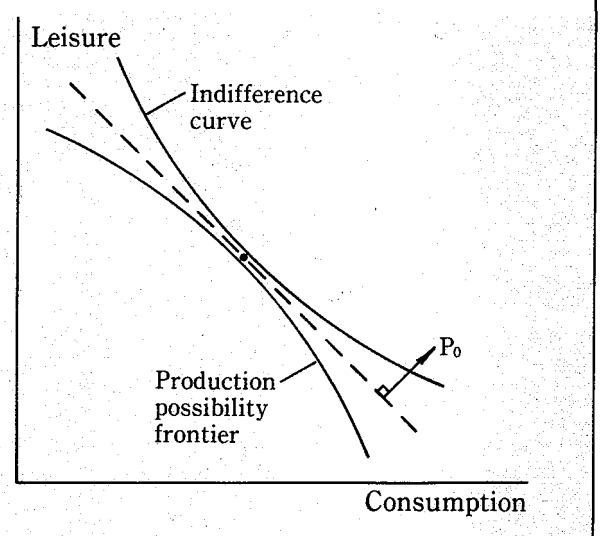
We choose Long and Plosser's (1983) equilibrium model of business cycles for its simplicity; it captures the essence of the New Classical explanation of economic fluctuations. Unlike earlier models, such as Lucas', this formulation requires no monetary authority along with asymmetric information to fool agents and jolt the economy into fluctuations.[3] For clarity, we assume perfect information and rational expectations. Business cycles arise from technology shocks and intertemporal labor, leisure, and consumption substitutions in response to these surprises.

In equilibrium models, the market works instantaneously at every date. Prices, although theoretically exogenous to households and firms, are actually precisely determined by the attributes of these agents. Imagine a representative firm and household with very well-behaved production and utility functions. For this Robinson Crusoe and Friday economy, we have equilibrium at the tangency of the indifference curves and production frontier in leisure-commodity space. (See figure 1.)

The key point is the equating of prices and marginal tradeoffs. Consumers equate the wage with the marginal utility of leisure; firms equalize wages and labor's marginal product. Although there are technical complications in extending the equilibrium model to a world with many periods, economically this approach reduces to applying these marginal equalities over time, while correcting for interest rates and agents' time preference. These marginal conditions are *equivalent to* the traditional notion of efficiency in economics (Pareto optimality); full markets insure that all gains from trade are achieved and that exogenous (government) policy measures cannot improve on this outcome.

Equilibrium prices at time t, then, are determined precisely by the fundamental nature of agents: endowments and utility (or profit) functions. These basic parameters are completely summarized by excess demand (Z). Excess demands of the agents at time t (Z_t) must determine prices *continuously* for market-clearing equilibrium to hold.[4] This may seem obscure, but it is important in understanding the nature of New Classical price adjustments. The idea can be lucidly illustrated by the basic functions involved.

Fig. 1 Equilibrium model One-Period Solution



3. Lucas (1972, 1979) requires monetary policy measures along with asymmetric information to constantly confound agents to produce cycles. Given the appearance of business cycles under an extremely wide range of monetary policy regimes, in all modern economies, and for hundreds of years, Lucas' model cannot be considered a general explanation of fluctuations.

4. In a discrete time model, prices are set at intervals frequent enough so that excess demands do not change within a period.

5. In a representative agent model, with only one consumer, the interest rate is determined by his or her rate of time preference, i.e. $p = 1/(1+r)$. This would simplify the equations in (5), since 9 and (1+r) would cancel out in the denominators of (b) and (c). They have been included in (5) to explicitly show the role of r and p in intertemporal optimization.

The excess demands are constructed by the hypothetical process of calculating excess demands (i.e., quantity desired minus endowment level) at all possible price vectors. So we have the function:

$$(1) \qquad Z_t = Z_t(p).$$

This function is assumed to have a unique root, p_t^*, which gives an equilibrium. But this equilibrium price vector p is determined by the excess demands. That is:

$$(2) \qquad p_t^* = Z_t(p_t^*).$$

Immediately we see that p_t^* is defined by a function of itself. To avoid any time paradox in the determination of p_t^* and the root of Z, these quantities must be determined simultaneously — instantaneous market clearing — at every date.

Long and Plosser do not develop their price dynamics in a full general equilibrium model; they limit most of their study to a simple example. We will carry the analysis of the general case further, since it lucidly illustrates some of the central issues in equilibrium cycles.

Consumers have an unchanging utility function:

$$(3) \qquad u(C_t, L_t),$$

where L, represents labor and C, consumption in period t. Instead of a single-period maximization problem, the consumer in this model must solve the multi-period problem:

$$(4) \qquad max\ U = \sum_{t=1}^{\infty} \beta^t u(C_t, L_t),$$

subject to labor constraints in each period. β is the discount factor of the representative agent. Although solving this dynamic maximization problem in general is not possible, if the utility function is strictly concave and all markets are perfect so that there are no kinks in the budget set at any date, then there can be no corner solution. In this case, the following first-order conditions must hold:

$$(5a) \qquad \frac{W_t}{P_t} = \frac{\partial u/\partial L_t}{\partial u/\partial C_t},$$

$$(5b) \qquad \frac{P_{t+1}}{\beta(1+r_t)P_t} = \frac{\partial u/\partial C_{t+1}}{\partial u/\partial C_t},$$

$$(5c) \qquad \frac{W_{t+1}}{\beta(1+r_t)W_t} = \frac{\partial u/\partial L_{t+1}}{\partial u/\partial L_t},$$

where r is the interest rate, W is the nominal wage, and P is the nominal price of the consumption good.[5]

These are the extensions of the marginal conditions to a dynamic setting. Equation (a) intratemporally requires the real wage to equal the marginal utility of leisure; (b) equates trade-offs of consumption over successive periods via the rate of time preference β times the price ratio across periods; and (c) requires that the labor/leisure decisions equal the interperiod wage ratio multiplied by the time preference rate. Even though no assets exist in the model, such intertemporal trades are feasible because of the rich market structure of the NC model. For example, there exists a contingent futures market for the consumption good in period t+s if a negative technology shock occurs; contracts on this market can be purchased with labor services in any period between t and $t+s$. Of course, the price on such a market varies over time according to the tastes and technology of the agents. The important point is that the markets do exist, and so all trades are possible. It is not yet clear that this model will produce business cycles. Indeed, since the economy is assumed to have a unique and stable equilibrium, w*, the phase diagram (see figure 2) for this model in the real wage w=W/P seems to indicate that cycles will not occur:

As mentioned above, Long and Plosser do not attempt to show that cycling occurs in the

unrestricted case. NC models have not, in general, been shown to produce cycles. To derive concrete results, they specify a utility function that embodies the intertemporal substitutions necessary for NC business cycles. Long and Plosser continue their argument with specific utility and production functions:

$$(6) \quad u(C_t, L_t) = \Theta_0 \ln L_t + \sum_{i=1}^{n} \Theta_i \ln C_{it}$$

$$Y_{i,t+1} = \lambda_{i,t+1} L_{it}^{b} \Pi X_{ijt}^{a}$$

The standard logarithmic utility function has elasticities Θ_i which are constrained to be non-negative, ruling out inferior goods. Presumably, if a Θ_i is zero, the good has some use in production. Otherwise, it is superfluous in the economy, since Long and Plosser assume free disposal. The Cobb-Douglas production function is unusual only in the appearance of $\lambda_{i,t+1}$, the stochastic shock to the production of good i in period t. The subscript t+1 refers to the date of completed production; that is, when it is ready for consumption. X is the vector of goods used as productive inputs.

Fig. 2 Phase Diagram for Equilibrium Model

Then, in each period, the consumer maximizes expected utility according to:

$$(7) \quad E(U|S_t) = E\{ \sum \beta^{s-t} [\Theta_0 \ln A_t + \sum \Theta_i \ln c_{it} | S_t]\},$$

where $S_t = (Y_t, A_{i,t+1})$. We require maximization under the expectations operator E, since A is stochastic. A shock in the technology for producing a good will obviously change present consumption. We expect co-movement of most goods, since they are all normal—changes in income call for marginal increments or decrements of each in the equilibrium consumption basket. Because leisure is also a normal good, some of the gain or loss due to the windfall will be taken in increased or decreased work hours. Co-movement is the first empirical regularity of the business cycle captured by the NC model.

Intuitively, persistence arises as consumers spread unexpected income changes over time as well as over all goods within a period. Savings or dissavings due to windfall gains or losses (from shocks) are used to increase or decrease income in many future periods. So, even without serially correlated shocks, we will have persistence in fluctuations.

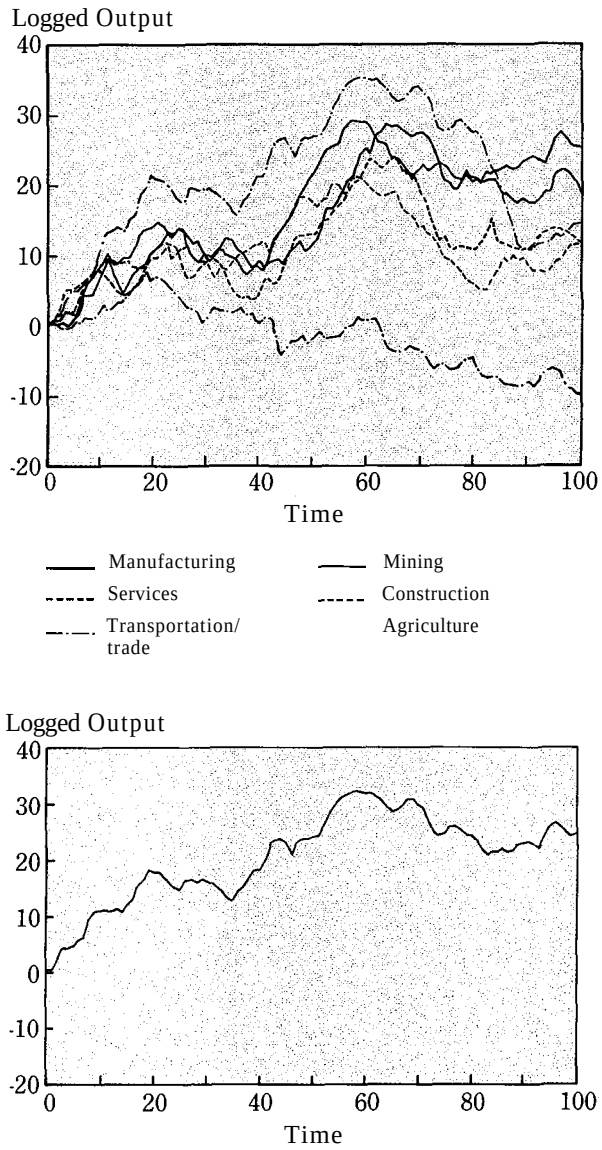
The utility function has been the source of co-movement and persistence in quantity fluctuations discussed to this point. Production technology is another source of these business cycle characteristics. Since most goods are inputs in the production of some other goods, a technology windfall (disaster) not only increases (decreases) present and future consumption of that good, but also, since all goods are superior, some of the windfall (disaster) is used to produce more (less) of all other goods requiring it as an input. Again, it is all part of the smoothing over time, as well as among commodities of any unexpected change in income. This leads to cycles, even in response to serially uncorrelated technology shocks.

The real wage can easily move pro-cyclically in this model. If tastes are fixed, then any increase in output (which requires more labor input) drives up the real wage required to induce workers to provide necessary labor services. As long as any decrease in labor's marginal product doesn't dominate this effect, the real wage will rise. Since increased output is associated with a windfall of some good that can serve as capital (increasing productivity), a pro-cyclical real wage seems likely. Although observed, it is beyond the scope of simple non-monetary models like this to pro-

duce co-movement among price for many or all goods.

In an elaborate simulation for a six-sector model of their economy (Long and Plosser [1983], p. 65) observed the paths for sectoral and aggregate output shown in figure 3.

Fig. 3 Simulated Output of Equilibrium Model



SOURCE: Long and Plosser (1983, p. 65).

Co-movement of different goods appears clearly. The long swings in the time series indicates higher degrees of autocorrelation than 1, suggesting persistence.

Long and Plosser's equilibrium model, then, can generate three of the central aspects of observed business cycles: quantities in almost all industries move roughly together, output variations tend to persist for many periods, and the real wage moves pro-cyclically. These characteristics arise from intertemporal tradeoffs to smooth consumption along with shocks to the production function.

Long and Plosser emphasize that these are not the only factors in the business cycle, but claim their model is a "useful benchmark" for evaluating other models. We take this to mean what we said at the outset: they are positing the central driving force of business cycles. They also point out that, in their model, business cycles are preferred paths; any policy attempting to smooth these fluctutations will be at best Pareto-equivalent with the free market outcome and may well be Pareto-dominated.

II. Disequilibrium Model

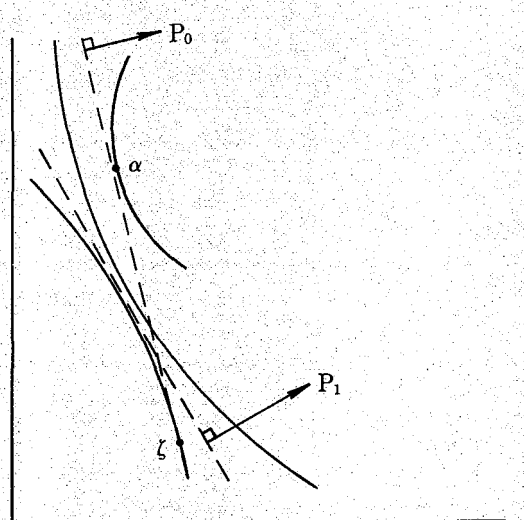
The only departure of disequilibrium models from the purely competitive Arrow-Debreu framework is the supposition that quantities may adjust faster than prices. Taken to the limit, this leads to fixprice models ***in each period*** with quantity rationing to balance trades. The market system that the economy is ***endowed with*** may not permit the perfectly fluid media for trade that exists in classical models. Some mutually desirable trades may simply not be possible under the constraint of an imperfect market structure. Dreze and Benassy (1975) laid the static foundations for this model; dynamic extensions have been numerous (see Bohm [1977] and Kades [1985]).

Here, we outline a simple version of a dynamic fixprice model (as outlined by Grandmont [1982], this is called a temporary equilibrium framework) and show how business cycles arise in it.

In contrast to figure 1 above, a Robinson Crusoe and Friday disequilibrium economy can be illustrated, as in figure 4.

The price vector p_o is exogenous ***within each period,*** so that the unique (under our same assumptions of very well-behaved utility and production functions) Pareto optimal Walrasian price vector p outcome almost never obtains. Under p_o, the consumer will wish to trade to point φ and the producer to point ζ, so it is not an equilibrium. Instead, we will have a new type of equilibrium, a fixprice equilibrium. We explicitly demonstrate this new type of equilibrium below. In this outcome, in general, one or both agents fail to obtain desired quantities at given prices and so are rationed. Note that, if the exogenous price vector is p, we have an equilibrium model. Here we clearly see that in the static world disequilibrium models are more general than equilibrium models; they allow for both equilibrium and rationing outcomes. NC models maintain that agents will always be able to find or create markets that will yield ***market-clearing*** prices.

Fig. 4 Basic Structure of Disequilibrium



Since the marginal rates of substitution in consumption and the marginal rates of transformation in production are not equated to prices, this economy lacks the Pareto optimality of the NC model. Within such a framework, it is likely that government policies could improve the welfare of all agents.

We will more fully specify a dynamic disequilibrium model following Malinvaud (1977) and Kades (1985a). Cycles occur in a more general model here than with Long and Plosser; there is no need to adopt specific utility and production functions.

We use L, C, P, W, and w, as in the equilibrium model. Consumers are described by a utility function U that is constrained only to be quasi-concave. Our representative consumer's sole endowment consists of time that may be "spent" on either labor or leisure. A simple concave stochastic production function, $F(L,) + \xi_b$ describes the activity of the firm. Consumers maximize utility, and firms maximize profits.

Instead of assuming that the very special Walrasian price vector is found, the fixprice approach imagines that the price vector is truly parametric ***at a given trading date*** and will be Walrasian only by accident. Between dates, the price vector moves according to the so-called law of supply and demand; excess demand for a good in period t (and possibly in previous periods) tends to pull prices up, while excess supply causes prices to fall. This does not restore the auctioneer and the instantaneous achievement of the equilibrium price vector. It more modestly posits that market forces work in the right direction and possibly with lags. Thus, there are other forces beyond current excess demands $Z(p)$ (and specifically, its root) that may enter into the function determining prices.

It is already easy to illustrate that, dynamically, NC models are a special case of NK

models. The most general form of the price equation is:

(8) $p_t = A(\chi)$,

where χ is the vector of all conceivable state variables in the world. The object of theory is to pare down the size of the vector χ as far as possible *without* ignoring anything of importance. NC models reduce the dimension of χ to only contemporaneous excess demands Z_t:

(9) $p_t = f(Z_t)$.

They further require that f(0) obtain at every date. NK models allow for a broader range of variables to enter, such as lagged excess demands or even lagged prices. Here are some examples:

(10) $p_t = g[\lambda Z_t + (1-\lambda) Z_{t-1}]$
$p_t = h[\lambda f(Z_t) + (1-\lambda) p_{t-1}]$

Further, the Z_t's are allowed to take non-zero values.

If the New Classical special case held in reality, proper econometric estimation of equation (12) would find that A was statistically indistinguishable from 1. And only this singular result could yield direct evidence that New Keynesian theories were over-parametrized.

Returning to the outline of the model, there is no reason to believe that Walrasian supplies and demands will balance at an arbitrary price vector in a disequilibrium world. More structure must be imposed here to define demands and to determine actual transactions. The most basic requirement imposed in fixprice models is voluntary trade: no agent is ever forced to trade (supply or demand) more of a good than he desires—what his preferences dictate. Since markets do not clear and we disallow forced transactions, agents will have to be rationed in quantities at the given price vector to balance trades. This model requires a new definition of "equilibrium."

Fixprice equilibrium means the maximization of quantity-constrained utility and profit functions with trades balancing. Disequilibrium Benassy (1975) demands, which we will refer to (following the ideas of Clower) as *effective* demands, are derived from considering all constraints except the constraint in the individual market where demand is being formed.

Fig. 5 Firm's Demand (Production Function)

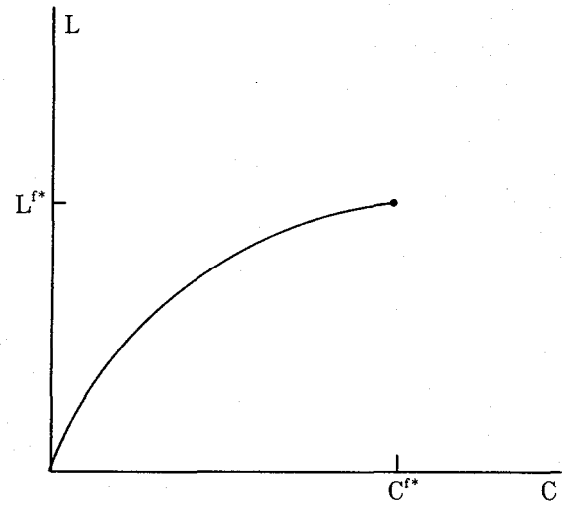
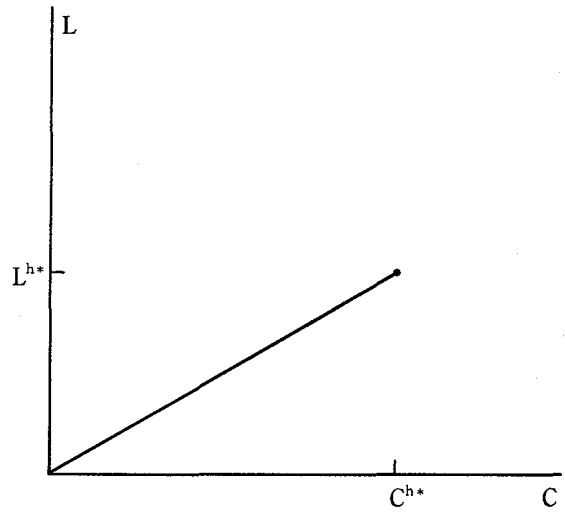


Fig. 6 Household's Demand (Consumption Expansion Line)



We denote them with a + superscript; they are defined from the maximization problems:

(11) Households:

$L^{h+} = MAX\ u(L,C,w)$ subject to $WL < pC$

$C^{h+} = MAX\ u(L,C,w)$ subject to $WL < pC$

Firms:

$L^{f+} = MAX\ r(L,C,w)$ subject to $C < F(L)$

$C^{f+} = MAX\ r(L,C,w)$ subject to $C < F(L)$,

where C and L are perceived constraints on other markets, and r is the profits function.

Benassy showed that, when solved, these demands yield balanced trades while simultaneously determining perceived constraints. The perceived constraints are the minimum of the effective demands when the system of simultaneous demands is solved. Thus agents' maximizing decisions under these constraints balance in the aggregate, yielding a fixprice equilibrium with rationing. The rationing mechanism is usually assumed to be stochastic. Formally, however, this point needn't be addressed in representative agent models.

We develop some graphs to represent this model. (See figures 5 and 6.) We will be using graphs to show the behavior of the household and firm in the trade space (L,C). The firm simply obeys "efficient production" in this model and always produces somewhere along the production function $C = F(L)$. However, the firm will never produce beyond its Walrasian point (L^{f*}, C^{f*}) under the given wage and price (the exogenous parameter x) since, beyond this point, the exogenous wage exceeds labor's marginal product. The shape stems from our assumptions on the production function.

The household obeys "efficient consumption"; it consumes along a line going through the origin (no work, no pay) whose slope is dictated by the real wage rate.

The household will never work beyond its notional quantities (L^{h*}, C^{h*}) since, beyond this point, the marginal utility of the good falls below the marginal utility of leisure.

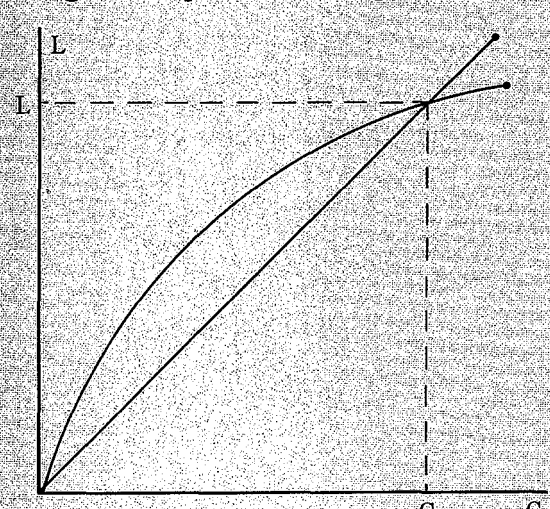
To determine the fixprice equilibrium, we combine the two curves. (See figure 7.)

Beyond the possibility of a Walrasian equilibrium (WE) when notional points coincide, there are two possible outcomes to this model. If consumers are rationed in selling labor and firms in selling the consumption good, then we have general excess supply. This has been labeled a Keynesian equilibrium (KE). If general excess demand prevails, we have an inflationary equilibrium (IE).

Thus, disequilibrium Benassy demands give rise to a much broader range of market outcomes than Walrasian models, where $Z_i = 0$ in all markets. Even at an arbitrary price vector, Walras' Law holds for New Classical demands: excess demand in one market is, by definition of budget constraints, balanced by excess supply in another market. General excess supply or demand cannot arise even hypothetically in an equilibrium model. Clower correctly stressed that the key to disequilibrium models must be to establish a rigorous framework within which Walras' Law did not hold. This is one way to describe the main accomplishment of New Keynesian theorists.

The dynamics of our disequilibrium model are very simple, since there is only one state

Fig. 7 Fixprice Static Equilibrium



variable, the real wage w. In the state space R^1, we have a unique value of w, w*, that gives a Walrasian equilibrium. But the movement of the real wage in KE and IE regions (on either side of the Walrasian equilibrium) is, at first inspection, undetermined. In the case of KE, labor is in excess supply in terms of effective demands, so the nominal wage should fall. But the commodity is also in excess supply, and so its price also should drop. Qualitatively, it seems difficult to determine the direction of real wage movements. The same holds for IE, where we have general excess (effective) demand.

Elsewhere (Kades 1985b), it has been shown that it is likely that steady states exist in both the KE region and the IE region. How does this occur? In Keynesian steady states, the nominal price of both labor and the good fall at the same rate in the price (vector-valued) function. Then the real wage rate is unchanging, and since it is the only state variable in this simple model, a steady state obtains. A symmetric case explains a steady state in the IE region.

Further, all Keynesian steady states of the model are stable (Kades 1985b); in a one-dimensional model, this implies uniqueness. Since lagged demands are generally included, the WE will almost never be an equilibrium (i.e., it is a measure zero event). Inflationary steady states may be either stable or unstable. Figure 8 presents a typical phase diagram for this system.

This system can easily give rise to cycles in the presence of exogenous shocks to the production function. The system can move further and further into either the KE region (a recession) or the IE region (boom). It can move either towards a stable or away from an unstable node until any type of shock moves the system to the other side, changing the cycle. The unstable IE effectively marks the border between the two regimes. White noise shocks can produce outcomes much like those observed in real economies. Figure 9 shows a simulation of this model similar to Long and Plosser's for aggregate output only. Further, this model fully captures the observed co-movement of prices and quantities. By making C a vector, it is easy to show that different quantities move together in the model.

So the fixprice/disequilibrium paradigm explains the most fundamental aspects of observed business cycles, and does so without recourse to special utility and production functions. The only reason for such fluctuations in the model is the general inability of the market mechanism to always find the market-clearing price vector. This economy is endowed with a cumbersome market structure that may or may not accurately reflect reality.

Fig. 8 Phase Diagram for Disequilibrium Model

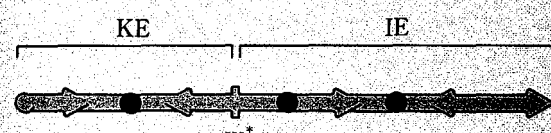
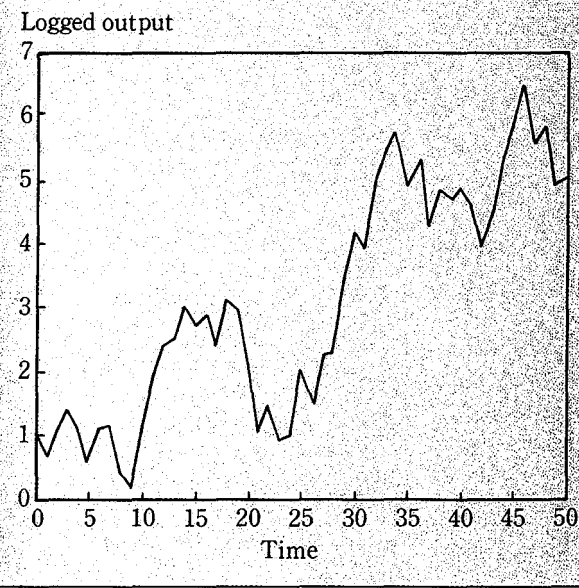


Fig. 9 Simulation of Disequilibrium Model



III. The Evidence

It is difficult to directly test hypotheses on whether or not all markets clear. But we can heuristically and formally examine evidence and arguments on a number of issues and measure the degree to which equilibrium and disequilibrium business cycle models agree with observation and rigorous thought.

The Great Depression stands as perhaps the most memorable single twentieth-century cyclical swing. The ability of a business cycle theory to plausibly explain this experience is important in establishing its credibility. Therefore, we first discuss the extent to which both models can explain this event.

Pigou and other Classical theorists in the 1930s blamed the Great Depression on an excessive reservation wage rate demanded by laborers. Thus for them, recessions were caused by a market imperfection in labor markets. In a sense, this view stands closer to disequilibrium paradigms, although the classical notion of market failure differs substantially from the New Keynesian view discussed above. For many early Keynesians, this was also seen as the cause of the Great Depression; they disagreed with Classical theorists only on the effectiveness of expansionary policies.

Today's New Classicals must argue that recessions occur when *low* wages are expected; workers then find leisure less costly in terms of wages foregone and bide their time until renumeration rates improve. But can the Great Depression best be explained as a multiyear withdrawal from labor markets by most Americans because they expected an eventual wage rise? The other explanations that New Classical theory can offer seem no more credible. One is that the utility function of most laborers called for a "... spontaneous outburst of demand for leisure ..." from 1929-1939. Another possibility is that a large negative shock to production technology was responsible, but then the problem becomes specifying the source of this shock.

New Keynesian explanations of the Great Depression are likewise unconvincing. Ironically, the most prominent possibility is due to Milton Friedman, a theorist not usually associated with Keynesian ideas. Friedman and Schwartz (1961) argued that a major cause of the Great Depression was the decline in the money supply from 1929-1933. In a slightly modified version of our New Keynesian model with money (Malinvaud 1977), it can be shown that low money-growth rates (or a *fortior* money declines) are associated with Keynesian recessionary outcomes. But far-reaching questions have been raised about this evidence (Temin 1975) and it is not clear which way causation runs between money and output. Further, as argued in footnote 3, cycle theories based on monetary phenomenon are less robust than real theories since cycles have occurred under a wide range of monetary systems. Perhaps monetary factors contributed to the *severity* of the Great Depression, but their role must be explicitly tied into a general model of cycles to provide a satisfactory story. Like New Classical theories, New Keynesian explanations may point to some particularly violent shock as the root cause of the Great Depression, but then the difficulty becomes uncovering and explaining the shock. No convincing explanation has been presented.

Although some economists find merit in these heuristic arguments, they are based on vague notions and "stylized facts," and lack precision. In a formal econometric study, Mankiw, Rotemberg, and Summers (1985) test the first order conditions in equation (5) for a utility function more general than Long and Plosser's. That is, they test the first-order conditions of consumers' maximization in the NC model. Although not sufficient, the first-order conditions are still *necessary* for any interior solution; if they are rejected, then the model can be rejected. There are, of course, difficult questions of aggregation in treating national data as if it is created by a representative consumer. No consensus on a solution to this issue exists, and this methodology is, at present, the *de-facto* standard for empirical work.

Mankiw, Rotemberg, and Summers find that the data (NIPA) reject the hypotheses, that these maximizations are carried out by consumers. None of the three over-identifying restrictions in equation (5) placed by equilibrium models is supported by the data. Further, the rejections occur for almost all permutations of the specifications of the hypothesis tests: separable or non-separable utility, annual, or quarterly data. Indeed, many of the restrictions actually force the shape of the utility function to be convex, in which case a maxima would occur at a corner and the Classical tangency conditions illustrated in figure 1 could not hold. When the utility function is concave, either leisure or "consumption" (NIPA) becomes an inferior good—which like convexity casts serious doubt on the model. Simultaneous estimation of all three restrictions in (5) is similarly rejected and produces either a convex utility function or inferiority of either leisure or consumption.

This rejection can be interpreted in two ways. Mankiw, Rotemberg, and Summers argue that the data show that markets (both labor and capital) fail to clear. There is another possibility: the structure of the utility function may be such that intertemporal substitution effects are very weak. In this case, a radically different utility function must be specified to dovetail with observation. At any rate, either explanation leads us to question Long and Plosser's equilibrium paradigm of-business cycles. It seems that either markets fail to clear, or that substantial intertemporal elasticities of substitution do not exist; both interpretations of the evidence reject this NC explanation of business cycles.

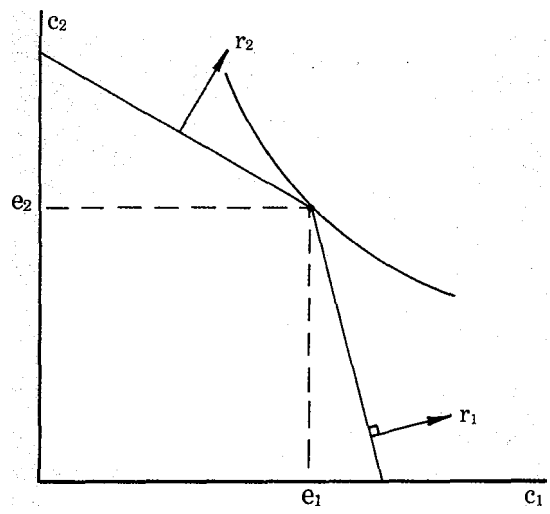
The disequilibrium model cannot be rejected by any such hypotheses concerning the structure of the utility function; it requires only that the utility function be quasi-concave. Beyond this, the disequilibrium model is robust to the form of the utility function.

Apart from rejecting the restricted form of the utility function needed to generate equilibrium business cycles, there is also strong economic evidence that key markets do not clear. Specifically, we shall discuss evidence that capital (lending) markets fail to clear.

Recall from the first-order conditions in the equilibrium model (5) that the interest rate appears in consumers' decisions just as in any other price. Equilibrium models require that agents can buy ***or*** sell as much of a good as they want at a ***uniform*** price, subject only to their endowment constraint. This constraint prevents any kinks from existing in the agents' budget sets so that, with a concave utility function, no corner solutions to maximization problems exist.

Keynesians (Old and New) have long argued that consumers, in reality, face liquidity constraints: either they cannot borrow at all against future income or they must pay an interest rate greater than the rate they receive for lending funds (even accounting for risk premia). Figure 10 shows that if agents lend at one price, but borrow at another, they are likely to solve maximization problems at ***corners*** of their budget set. Here, the equality of prices and intrapersonal utility trade-offs breaks down, and the economy may no longer be efficient.

Fig. 10 Corner Solution for Liquidity-Constrained Consumer



6. It is interesting to note that asset markets are almost always assumed to more closely approximate the competitive ideal than other markets. If the data show that these markets fail to clear, then it seems dubious to assume that labor and goods markets clear.

Agents are endowed with $e=(e_1, e_2)$ of a good in periods one and two respectively. The interest rate for borrowing in period one is r_1, while the lending rate is less, r_2. With a concave utility map, it is then immediately apparent that a corner solution can occur.

Strong evidence exists that such liquidity constraints have been binding for significant numbers of American consumers. Fumio Hayashi (1985), modifying an idea originally appearing in Kowalewski and Smith (1979), uses cross-sectional data and divides consumers into high-and low-savings groups. He assumes that high-savings households are unlikely to be liquidity-constrained, so they may be used as a control group to be compared to other (potentially liquidity-constrained) households. By estimating consumption behavior for each group separately, and then by comparing the two parameter sets, Hayashi finds a significant difference that can be explained by the existence of liquidity constraints. Although there are other explanations for the result, they require the rejection of either the permanent income hypothesis or of market clearing. Since both market clearing and the permanent income hypothesis embody the New Classical idea of the ***markets'abilities*** to smooth consumption over time, this interpretation too, casts doubt on the equilibrium business cycle model. Flavin (1981) and Kowalewski (1985) provide time series evidence that liquidity constraints have persistently shaped agents' budget sets in the postwar American economy.

On the other hand, the disequilibrium model is robust to either interpretation of Hayashi's results. If liquidity constraints do exist, they are an instance of the imperfect markets of New Keynesian theory.[6] If we view the results as a rejection of all utility functions that give rise to permanent-income consumption paths, we already know that the NK model is not subject to this criticism.

In discussing the compatibility of both models with observed business cycles, we have examined only three central patterns: the co-movement of different quantities, the persistence of trends, and the positive correlation between quantities and the real wage. But there are other empirical regularities in business cycles that both models should similarly mimic if they are to be adequate representations of the central force in business cycles. Although they were raised by Arthur Okun (1980) in objection to Lucas's equilibrium model (Lucas 1972), they also point to shortcomings in Long and Plosser's model and in NC models in general.

First, many secondary aspects of labor markets (beyond pro-cyclical wages) are at odds with the NC model. Productivity may or may not be pro-cyclical in the NC model. It depends on the size of the technology shocks and on the intensity of the disutility of labor. But observed productivity is strongly pro-cyclical. In non-market clearing models, this phenomenon is explained by implicit contract theory, where workers are insured against unemployment by their employers in return for a lower wage. When demand slackens, there are no layoffs; with the same amount of labor and less production, productivity must decline. As demand improves, the same work force is called on to produce more; hence, productivity increases. Implicit contract theory comprises one market imperfection that could be the fundamental source of fixed prices (wages) in the short run. The market clears by a non-price mechanism. No institutional factor (that is, exogenous parameter) explaining market-clearing could produce pro-cyclical productivity in Long and Plosser's model. Similarly, quits induced by pro-cyclical factors, counter-cyclical layoffs (moreover, the existence of layoffs, which involve rationing the sale of labor), and wage increases in recessions seem inexplicable in the present NC model.

Although Long and Plosser examine and discuss only the consumer side of their model, firms as well as the household may seek to

smooth over time their objective—profits. One rationale for such behavior is that since households own the firms, smoothing profits is simply one part of smoothing income. This motivation is superfluous in an NC model, since in market-clearing models economic profits have, by definition, a zero expected value in each period (under the usual assumption of constant returns to scale). However, we observe very large pro-cyclical fluctuations in profits. New Classical theorists must explain why the value of entrepreneurial talent and risking capital fluctuate so sharply with the business cycle to lend credibility to their paradigm. Conversely, pro-cyclical profits exist under implicit contracts in a New Keynesian framework, since wage costs are constant while productivity varies with business cycles. However, this criticism must be tempered by remembering the substantial controversies in defining and, moreover, in measuring economic profits.

Finally, Fisher (1984) has raised a methodological objection to the equilibrium paradigm. In response to any shock, these models require that prices adjust so rapidly—almost instantly—that agents never face disequilibrium prices. However, even in the world of physics, adjustment to a new shock takes time, and a mechanical system must move out of one equilibrium before a new rest state is attained. Even in the case of a unique equilibrium, New Classical dynamic behavior violates the usual properties of differential equations in avoiding disequilibrium, but we can imagine the shock (ξ_t) and the real wage (w_t) move in tandem precisely to produce continuous market clearing. (See figure 11.) If NC models contain a dynamic structure as rich as the New Keynesian (to avoid the necessity of specifying a restricted class of utility functions to produce cycles), then the hypothesis of continuous market clearing cannot be maintained. (See figure 12.)

The system must move ***through*** the unstable disequilibrium γ and cannot give continuous market clearing. It is not clear why New Classical theorists feel that economic adjustments can be approximated by instantaneous movements from one equilibrium to another. They must provide a plausible mechanism for this behavior before it can be used convincingly. Disequilibrium dynamics call for the economy to adjust along paths more in line with established notions about change over time.

Fig. 11 Continuous Market Clearing with a Unique Stable Equilibria

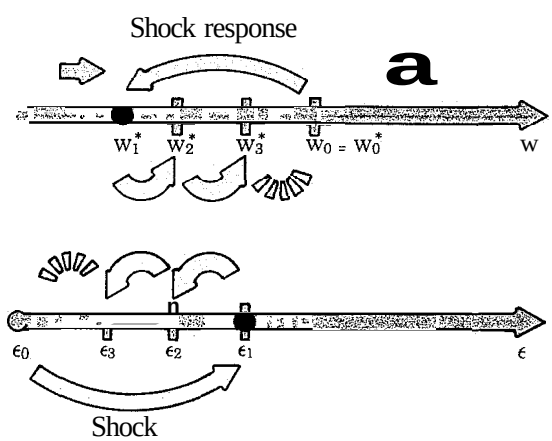
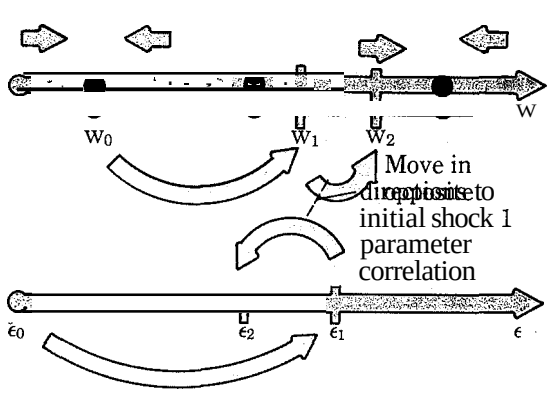


Fig. 12 Failure of Continuous Market Clearing with Multiple Equilibria



IV. Conclusion

Both equilibrium and disequilibrium theories can construct model economies that mimic the basic behavior of real business cycles: strong co-movement among quantities of different goods, persistence of quantity movements in the same direction for many periods, and pro-cyclical real wages. But existing NC models cannot explain other aspects of observed business cycles, such as pro-cyclical productivity or other observed characteristics of the labor market. Further, evidence exists that capital markets do not clear. Finally, the data reject the New Classical utility function exhibiting strong intertemporal substitutions. Without such a utility function, the model has not been shown to produce persistent output cycles.

The NK model is robust to most of these criticisms. It requires no specific utility function to generate cycles, it fits the observed regularities of business cycles more fully, and it employs a more general model of price movements over time. However, like the NC model, it provides no convincing explanation of the Great Depression. Since equilibrium models:

(1) Comprise a subset of disequilibrium models;

(2) employ identifying restrictions that are not empirically validated; and

(3) require nonstandard dynamical adjustments; it appears that, at present, despite this shortcoming, New Keynesian theories provide a better paradigm of the business cycle.

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