Dynamic Prudential Regulation*

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Abstract

We investigate the design of prudential bank regulation and its effects on the real and financial decisions of banks in a dynamic framework. In our model, a representative bank can dynamically alter its asset portfolio by engaging in asset substitution or risk-shifting. The regulator controls the bank's risk-shifting incentives through the threat of intervention in the bank's operations. The optimal regulatory policy, which we characterize in closed form, entails an optimal combination of an initial capital requirement, intervention to control the risk of the bank's portfolio, capital injection, and liquidation of the bank. Under the optimal risk intervention policy, the regulator intervenes when the bank's capital ratio lies inside a "band" consisting of two triggers. We calibrate the model and show that the optimal initial capital requirement is 20%, substantially higher than that suggested by the Basel II accords. Relative to a benchmark unregulated bank, regulation increases bank leverage by 15%, while lowering its credit spread. Capital injection or risk intervention alone has modest impact on the bank's social value. The optimal combination of regulatory policies, however, significantly improves social value by 4.4% and bank value by 12%. Our results suggest that prudential regulation can achieve substantial economic benefits provided it is appropriately designed. We study how optimal regulation depends on the economic and regulatory environment and find that prudential regulation should not be independent of monetary and fiscal policies.

Key Words: capital regulation, asset substitution, bank bailout, prudential regulation
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I. Introduction

The 2007–2009 financial crisis has engendered a vigorous debate on the prudential regulation of financial institutions. Despite substantial progress in regulation such as the 2010 Dodd-Frank Act, a broad consensus has yet to emerge on a number of issues that are central to the debate. How does regulation impact the various stakeholders of banks? What forms should regulatory intervention take? At what points (if at all) should regulators intervene in banks' operations to control the risk of their asset portfolios, force liquidation or recapitalization of distressed banks, or bail them out with public funds? What are the relative impacts of different components of regulation—capital regulation, intervention and control of banks' risk-taking, and capital injection—on banks' values and social welfare? How sensitive is optimal prudential regulation to the economic environment? Given the inter-temporal nature of banks' operations and regulatory intervention policies, addressing many of the above questions calls for a dynamic model that is both analytically tractable and calibratable to data. Such a model can provide qualitative insights and also potentially offer quantitative guidance on the design and impact of prudential regulation.

We take preliminary steps towards addressing some of the above issues by developing a dynamic model to analyze the design of prudential bank regulation and its effects on the real and financial decisions of banks. Our objectives are both positive and normative in that we seek to obtain insights into how existing regulatory policies affect the investment and financing decisions of banks as well as the optimal design of prudential regulation. The optimal regulatory policy, which we characterize analytically, is multi-pronged in nature in that it entails imposition of initial capital requirements, intervention to control the risk of the bank's portfolio, injection of capital after the bank becomes insolvent, and liquidation of the bank when it is no longer optimal to keep it active. Our calibration of the model shows that, relative to a benchmark unregulated bank, regulation increases bank leverage substantially, while lowering its credit spread. The optimal initial capital requirement is substantially higher than that suggested by the Basel II accords and consistent with the higher ratios advocated by recent studies (e.g. Admati, DeMarzo, Hellwig, and Pfleiderer (2011)). While capital injection and investment intervention *independently* have modest effects on social value, their optimal combination significantly improves social value by 4.4% and bank value by 12%. Overall, our results suggest that prudential regulation has substantial economic benefits provided that it is appropriately designed and the entire gamut of regulatory intervention tools are employed.

In our continuous-time model, we consider a financial institution (henceforth referred to as the "bank") that operates in a regulated environment. At date zero the bank chooses its capital structure that consists of equity and debt. For simplicity, we assume that the bank's debt structure requires it to make a fixed payment (per unit time) to debt holders. As in prior literature (e.g. Giammarino, Lewis and Sappington (1993)), there are additional deadweight costs of equity issuance relative to debt issuance. All market participants are risk-neutral and discount payoffs at

the risk-free rate.[1]

The bank can dynamically alter the distribution of its earnings by changing its investment portfolio. For simplicity, the bank can choose one of two "projects" at any date, where the term "project" is a metaphor for the composition of the bank's portfolio of investments. The bank's total earnings (before interest payments) evolve as a lognormal process under either of the projects. To focus on the distortions created by asset substitution , we assume that the first project has a higher drift and lower volatility than the second one. The bank's project choices are only observable by the regulator if it chooses to intervene in the bank's operations. We allow for a broad gamut of regulatory intervention tools. The regulator could impose an initial capital requirement for the bank, intervene in the bank's project choices, bail out an insolvent bank by injecting capital, and liquidate the bank. Regulation has potential benefits because, while the bank's management cares about shareholder value, the regulator cares about the bank's "social value" that internalizes the claims of shareholders, debtholders and tax-payers (due to possible infusion of public funds by the regulator). Further, the regulator internalizes the effects of systemic risk — the externality that a bank imposes on the economy through its financial distress.

In the hypothetical environment in which the bank is unregulated, the bank chooses its capital structure at date zero and dynamically chooses its projects to maximize shareholder value. If the bank's earnings are insufficient to meet debt payments, it can raise additional funds by issuing additional equity and/or borrowing from the central bank. In either case, it incurs additional costs due to its financial distress. The bank's capital structure balances the tradeoff between the cost advantage of debt against financial distress costs that include the excess costs of equity issuance in financial distress, additional interest costs on its borrowing from the central bank, the (endogenous) costs associated with asset substitution, and the deadweight costs of liquidation.

We solve the bank's stochastic control problem and obtain a closed-form analytical characterization of the unregulated bank's optimal behavior. The optimal strategy of the unregulated bank is characterized by three earnings thresholds: the *switching threshold* at which the bank switches between the two projects, the *liquidity threshold* at which debt payments consume all of the bank's earnings, and the *insolvency threshold* at which the bank's equity value endogenously falls to zero so that it can no longer meet debt payments. The bank optimally chooses the low-risk project whenever its earnings are higher than the switching threshold, and the high-risk project below the threshold. Because shareholders effectively hold a convex claim, it is optimal for the bank to increase risk when its earnings are sufficiently low to lower the probability of insolvency. Nevertheless, given the dynamic nature of the bank's problem, the existence of a *single* threshold at which the bank optimally switches projects is not a priori obvious; we rigorously establish this in our analysis. The bank services debt payments entirely with internal funds when its earnings are

[1] Alternatively, we can assume as in Duffie (2001, Chapter 10) that the bank's cash flows are *marketed* so that we can carry out our analysis in the risk-neutral measure under which all agents are risk-neutral and discount payoffs at the risk-free rate.

above the liquidity threshold, and issues new equity to cover debt obligations when earnings are below the liquidity threshold, but above the insolvency threshold. The bank is unable to continue servicing debt when its earnings fall to the insolvency threshold so that the bank is liquidated.

Next, we analyze the *fully regulated* environment in which the bank is completely controlled by the regulator whose objective is to maximize the bank's "social" value that incorporates the claims of shareholders, debt holders, and tax payers. To incorporate the systemic effects of the bank on the economy, we also allow the equity issuance, financial distress and liquidation costs in the regulator's objective function to differ from those of the unregulated bank. We analytically show that the optimal policy of the regulator is described by the three thresholds as in the unregulated bank's problem (although their values differ from those for the unregulated bank) as well as an additional *liquidation threshold*. When the bank's earnings fall below the insolvency threshold, the regulator takes over the bank and injects capital until either earnings fall further to the liquidation threshold or they increase so that the bank becomes solvent again. In particular, our results demonstrate that, in a dynamic environment, it could also be optimal for the regulator to permit asset substitution by increasing risk when the bank's earnings are sufficiently low. The intuition is that, because the regulator optimally injects capital when the bank's earnings fall below the insolvency threshold, the regulator effectively becomes the residual claimant and, therefore, has a "locally" convex objective function akin to that of shareholders of an unregulated bank. Consequently, it could be optimal for the regulator to increase risk to reduce the probability of liquidation and the associated liquidation costs. Nevertheless, because the regulator cares about the bank's social value, it has lower incentives to engage in asset substitution than the bank's shareholders so that the regulator's optimal switching threshold is always lower than that of the unregulated bank.

By comparing the unregulated bank's problem with that of the regulator, we derive the optimal *regulatory policy* that implements the first best policy described above. The optimal regulatory policy comprises of three components: *capital regulation, risk intervention* and *capital injection*. The regulator imposes an initial capital requirement by setting a cap on the initial leverage ratio of the bank. The risk intervention policy has a "band" structure: the regulator intervenes and forces the bank to choose the low risk project when the bank's earnings lie in between the unregulated bank's optimal switching threshold and the regulator's optimal switching threshold because it is precisely in this interval where the unregulated bank's and regulator's optimal project choices differ. Since the latter threshold is lower than the former, the regulator always intervenes nontrivially.[2]

To quantitatively analyze the impact of prudential regulation, we calibrate the model to data on U.S. banks over the period 1991 – 2008. In the aftermath of the financial crisis, it has become clear that banks were poorly regulated (if at all) during this period, but many banks were nevertheless

[2]The regulatory intervention in the model is analogous to that in the FDIC Improvement Act in 1991, which classifies banks as well capitalized ($\geq$10%), adequately capitalized ($\geq$ 8%), undercapitalized (<8%), significantly undercapitalized (<6%), and critically undercapitalized (<2%). The first two categories are not restricted in their activities, while the latter categories face increasingly severe restrictions on actions such as dividend payments, asset growth, and acquisitions, and ultimately expropriation of shareholders.

subsequently "bailed out" through injection of public funds. Consequently, the model that we calibrate to data is an "intermediate" one where the bank is free to choose its capital structure and projects, but nevertheless implicitly enjoys the protection provided by public capital injection after the bank is no longer able to raise capital to meet debt requirements. We calibrate the model to match banks' average earnings to asset ratio, risk, leverage, and yield spread.

Regulation has a substantial impact on bank leverage. Under the calibrated baseline parameters, the fully regulated bank has an optimal leverage of 80%, while the unregulated bank's leverage is 65%. The credit spread (42 bp) of the regulated bank is, however, lower than that of the unregulated bank (69 bp). The regulator's intervention in the bank's project choices significantly reduces the bank's asset risk. Further, the regulator's commitment of capital injection trims the effective cost of debt. Together these factors enable the regulated bank to issue much more debt with a lower spread than the unregulated bank. Interestingly, our results suggest that the optimal initial capital requirement, which is equal to the initial equity ratio in the fully regulated environment, is 20%, substantially higher than that suggested by the Basel II accords and consistent with the requirements being proposed by the Basel III committee. This finding also supports the proposal of Admati et al (2011) that the capital requirement be set much higher than the current regulatory levels (they recommend capital ratios between 20% and 30% in their conclusions). The optimal intervention ratio, which is the level below which the regulator intervenes in the bank's management, is 6.8%. The predicted intervention ratio is broadly consistent with the FDIC definitions of "undercapitalization" and "significant undercapitalization" ratios (8% and 6%) that trigger monitoring and restrictions on banks' activities.[3]

From an ex ante perspective, optimal regulation increases a bank's value by 12% relative to the unregulated bank, while it increases the bank's "social" value (that also incorporates the costs arising from systemic risk and deadweight social costs of capital injection) by 4.4%. Regulation has an even bigger quantitative impact ex post. The ex post bank value at the insolvency threshold increases by 87% due to capital injection by the regulator. The bank's ex post social value, which incorporates the social costs of capital injection, increases by 36% due to the injection of capital by the regulator. We also analyze the relative quantitative impacts of different regulatory tools. Risk intervention on its own improves a bank's social value by only 0.4%, while capital injection on its own reduces it by 0.7%. Together the evidence suggests that there are significant interactive effects of different regulatory tools.

The analytical characterizations of the bank's and regulator's policies greatly facilitate the exploration of various "comparative static" relationships that generate implications for the sensitivity of optimal regulation to changes in the economic environment. The optimal capital requirement decreases with the systemic cost of bank equity, because the regulator internalizes the cost of low leverage and desires higher bank borrowing. This provides a rationale for procyclical capital re-

[3]Details of the relevant rules and regulation can be found on the FDIC website at http://www.fdic.gov/regulations/laws/rules/2000-1300.html

quirements (e.g. Dewatripont, Rochet, and Tirole (2010)) as the systemic cost is likely to be higher during economic recessions. The capital requirement also depends negatively on the bank's private equity issuance cost. As higher taxes increase the private cost of equity,[4] our results imply that lower capital requirements should be imposed in a high-tax-rate regime.

We also find that the capital requirement increases with the the bank, but is virtually independent of the private distress cost. This emphasizes the point that regulatory policy should be mainly concerned with the systemic risk of the banking sector (e.g. Acharya, Cooley, Richardson, and Walter (2010)). The *expected* cost of capital injection depends *negatively* on the marginal cost of injection. This result reflects the moral hazard problem in bank bailouts (e.g., the dilemma facing the Federal Reserve when it weighed decisions on whether to save financial institutions such as Bear Stearns and Lehman Brothers). When the marginal cost of injection is smaller, the market believes that the regulator will inject more capital and, thereby, provide better insurance for creditors. As a consequence, banks are able to borrow more aggressively leading to substantial bailout costs.

Our finding that the optimal capital requirement decreases with the tax rate suggests that bank capital regulation should not be independent of fiscal policy as suggested by Admati et al. (2011). We also show that the initial capital requirement varies negatively and significantly with the risk-free rate suggesting that bank regulation should not be independent of monetary policy. This finding also suggests that uniform capital requirements across countries with differing risk-free rates could be sub-optimal.[5] In contrast, the optimal intervention ratio is quite insensitive to the risk-free rate suggesting that the trigger at which regulators intervene in banks' operations could be independent of monetary policies.

II. Literature Review

We contribute to the literature by developing a dynamic model to quantitatively investigate the design and impact of prudential regulation. We build on insights provided by a number of prior studies in the literature. One stream of the literature stresses the role of bank capital in inducing banks' monitoring efforts (Diamond (1984), Giammmarino, Lewis, and Sappington (1993), Holmström and Tirole (1997), Allen, Carletti, and Marquez (2011), Mehran and Thakor (2011), Acharya, Mehran, and Thakor (2011)). Another strand of the literature considers the banks' liquidity provision role in accepting demand deposits (Diamond and Dybvig (1983), Gorton and Winton (1995), Diamond and Rajan (2000)).

A substantial literature studies the risk-shifting incentives of banks stemming from the early studies of Merton (1977), Kareken and Wallace (1978), and Sharpe (1978).[6] While Koehn and

[4] Taxes do not substantially affect the external or systemic cost of equity because they are transfers between different agents.

[5] Acharya (2003) makes a similar point in a very different framework and for different reasons

[6] See Santos (2001, 2006) for reviews of this literature.

Santomero (1980) and Kim and Santomero (1988) show that a risk-averse banker may actually select a riskier portfolio when facing capital adequacy requirements, Rochet (1992) points out that the limited liability of the bank implies that capital regulation is still helpful in reducing risk-taking behavior. Dewatripont and Tirole (1994) propose the "representation hypothesis" that regulators monitor and supervise banks on behalf of small depositors who lack the incentives or resources to do so. Marshall and Prescott (2001, 2006) consider state-contingent regulation when the bank has moral hazard in project selection. Also related are studies of shareholder-debtholder conflicts and asset substitution for non-financial firms (Jensen and Meckling (1976), Leland (1998)).

We contribute to the above streams of the literature in a number of respects. First, we develop a continuous-time model in which the regulator has a wide gamut of intervention tools consistent with those observed in reality. Second, the bank can dynamically engage in asset substitution, and its capital structure reflects the tradeoff between the excess costs of issuing equity against financial distress costs that include the agency costs arising from asset substitution and liquidation costs. Third, we analytically characterize the optimal regulatory policy, and calibrate our model to data to obtain quantitative insights into the design and value of prudential regulation as well as the relative effects of different components of regulation on the bank's value and its social value.

A few recent studies quantitatively analyze the impact of bank capital regulation, using dynamic models with calibrated parameters. Zhu (2008) and Van den Heuvel (2009) develop dynamic models to evaluate the macroeconomic impact of captial regulation. De Nicolò, Gamba, and Lucchetta (2011) considers the impacts of capital regulation, liquidity requirements, and taxation on banks' behavior and social welfare. They abstract away from the effects of asset substitution. Analogous to the above studies, we too develop a dynamic "calibratable" model that can be used to carry out a quantitative analysis of the design and impact of prudential regulation. Because our model allows analytical solutions, we are able to obtain additional insights on the optimal prudential regulation policies and the behavior of banks in response to these policies.

III. The Model

We develop a continuous-time model of a financial institution that we refer to as a "bank" for concreteness although our model is more generally applicable to other regulated financial institutions such as insurance and securities firms. The time horizon is $[0, \infty)$. Because our focus is on shareholder-debtholder agency conflicts, we assume that the bank's management behaves in the interests of its shareholders. Hereafter, we refer to the bank's management as the "bank."

A. The Bank's Earnings and Project Choices

The bank can dynamically alter the distribution of its earnings by choosing its portfolio of projects that we hereafter refer to as its "project." As in Chapter 8 of Duffie (2001), the bank's

cash flows are *marketed.* Because all agents are value-maximizers, it follows from the fundamental theorems of asset pricing that we can, without loss of generality, work under the *risk-neutral measure* under which all agents are risk-neutral and discount payoffs at a common risk-free rate r (see also Leland (1994)).

At any date t, the bank can choose one of two possible projects, 1 or 2. In the (infinitesimal) time period $[t, t + dt]$, the bank's *total earnings* (before debt payments) C evolve as follows under the two projects:

$$dC_t = \mu_1 C_t dt + \sigma_1 C_t dB_t, \text{ if project 1 is chosen,} \tag{1}$$

$$dC_t = \mu_2 C_t dt + \sigma_2 C_t dB_t, \text{ if project 2 is chosen.} \tag{2}$$

In the above, μ_1 and μ_2 are the *risk-neutral* drifts or expected growth rates of earnings under projects 1 and 2, respectively; σ_1 and σ_2 are the volatilities, and B_t is a standard Brownian motion that we assume to be the same for the two projects (without loss of generality) to simplify the notation. Let r be the risk-free rate. We assume that

$$r > \mu_1, \mu_2 \tag{3}$$

to ensure that all asset prices are well defined. As our focus is on shareholder-debtholder agency conflicts arising from *asset substitution* or risk-shifting, we assume that

$$\mu_1 > \mu_2 \text{ and } \sigma_1 < \sigma_2. \tag{4}$$

Hence, project 1 has a higher drift and lower volatility than project 2. It is easy to see from (1), (2) and (4) that the NPV of the total future earnings from project 1 at any date is greater than that of project 2. Project 2, therefore, has higher risk and lower NPV than project 1. Our analysis can be easily generalized to accommodate the scenario where $\mu_1 > \mu_2$ and $\sigma_1 > \sigma_2$, but the tradeoffs are less interesting and relevant to the real-world regulation of institutions where the mitigation of value-destroying asset substitution plays an important role.

The bank's earnings as well as the project parameters $\mu_1, \mu_2, \sigma_1, \sigma_2$ are observable by all market participants. The bank's actual project choices are, however, only observable by the regulator if it intervenes in the bank's operations. We show later that the regulatory intervention policy can be designed so that the bank's dynamic project choices are optimal from the standpoint of the regulator. In other words, the "first best" can be achieved through a suitable intervention strategy so that there is no deadweight loss due to the fact that the regulator can only observe the bank's project choices when it intervenes in the bank's management. For future reference, we denote the information filtration generated by the earnings process C by $\mathcal{F}_t$.

B. Capital Structure

The bank chooses its capital structure at date 0 that comprises of equity and debt. For simplicity, we assume that the bank's debt structure requires it to make a constant payment of θ per unit time to debtholders. For example, if the bank's debt comprises of deposits, θ is the withdrawal rate of depositors per unit time.[7] The bank's capital structure is determined by the parameter θ.

As in several prior studies (e.g. Giammarino et al. (1993)), there are additional deadweight costs of equity issuance is costly relative to debt issuance. These costs include direct costs such as the tax disadvantage of equity relative to debt as well as *shadow* costs arising from various sources such as the comparative advantage that banks possess in raising capital through deposits rather than equity, investors' demand for liquid demand deposits, etc. Some of these costs are more naturally interpreted as "flow" costs, while others are easier to interpret as "lump sum" costs that are incurred whenever equity is issued. It would greatly complicate the notation and exposition to separately model "lump sum" and "flow" costs, and it is also hard to independently calibrate the corresponding parameters in our quantitative analysis. Consequently, we choose to model the excess costs of equity as "flow" costs throughout. It should be borne in mind, however, that we can alternately model these as "lump sum" costs or a combination of "flow" and "lump sum" costs. The alternate modeling choices do not affect the main insights of the study.

More precisely, in any period $[t, t + dt]$, as long as the bank is solvent, the cash flow to debt holders is θdt and the cash flow to equity holders is

$$\begin{aligned} \text{Net cash flow to equity holders} &= (C_t - \theta)dt - \lambda_1(C_t - \theta)dt \text{ if } C_t \geq \theta \quad (5) \\ &= (C_t - \theta)dt - \lambda_2(\theta - C_t)dt \text{ if } C_t < \theta, \quad (6) \end{aligned}$$

where

$$\lambda_1, \lambda_2 > 0 \quad (7)$$

In (5), λ_1 ais the excess cost of equity modeled as a flow cost. In (6), λ_2 is the proportional cost associated with the bank's financial distress when its earnings are insufficient to meet debt payments. We allow for the bank to raise funds in financial distress by issuing equity and/or borrowing from the central bank. To keep the model parsimonious, we assume that either channel is equally costly for the bank so that we can assume henceforth that the bank issues additional equity in financial distress. Inequality (7) incorporates the realistic possibility that the bank incurs additional financial distress costs.

The bank services debt payments entirely as long as it is solvent. If the bank's earnings are insufficient to meet debt payments, it can issue additional equity. We also allow for the regulator to inject capital to service debt payments. In our framework, it is actually optimal for the bank to

[7] More generally, a bank's debt structure includes demand deposits as well as longer-term debt obligations. The explicit incorporation of the different components of a bank's debt structure is well beyond the scope of this study.

service debt entirely by issuing equity (if necessary) as long as the value of equity is positive. After the equity value falls to zero, the regulator services debt payments by injecting capital until it is no longer optimal to continue to do so.

The bank chooses to service debt by issuing equity until an $\mathcal{F}_t-$stopping time τ_B when it becomes insolvent (recall that $\mathcal{F}_t$ is the information filtration generated by the bank's earnings process.) For $t > \tau_B$, the regulator takes control of the bank (in effect, it becomes the owner of the bank), injects capital, and continues to do so until a stopping time $\tau_L \geq \tau_B$. Hence, the bank services debt for $t < \tau_B$, and the regulator services debt by injecting capital for $t \in (\tau_B, \tau_L)$. Note that, in the region (τ_B, τ_L), the bank's equity value is negative, that is, the regulator effectively holds a negative equity stake in the firm. The bank's assets are liquidated at $t = \tau_L$ when the regulator decides to stop injecting capital, and debtholders receive the resulting liquidation payoff.

There are deadweight liquidation costs that we model by assuming that debtholders receive a proportion $1 - \alpha$; $\alpha \in (0, 1)$ of the *unlevered bank value* at the liquidation threshold. More precisely, it follows immediately from (1) that the value of the (hypothetically) unlevered bank at the liquidation threshold is

$$V_{\tau_L}^{unlevered} = \frac{(1 - \lambda_1)C_{\tau_L}}{r - \mu_1}. \tag{8}$$

The payoff to debtholders is

$$D_{\tau_L} = (1 - \alpha)\frac{(1 - \lambda_1)C_{\tau_L}}{r - \mu_1} \tag{9}$$

In reality, bank debt comprises of demand deposits that are protected by deposit insurance as well as longer-term debt obligations that are not insured. Our framework actually incorporates the possibility of deposit insurance because we allow the regulator to inject capital after the bank goes bankrupt. In our setting, the regulator is essentially the "social planner" who also embodies the role of the deposit insurer. As our subsequent analysis shows, from a normative standpoint, it is optimal for the regulator to inject capital and service the bank's debt until its earnings hit the liquidation threshold τ_L. Consequently, the main normative implications of our study are not affected by the explicit incorporation of deposit insurance. Further, insofar as a bank incurs financial distress costs (for example, through the loss of business, reputation, etc) regardless of whether a portion of its debt is insured, and there are limits to the amount of debt that can be insured (which is true in reality), our positive implications are not significantly affected either.

C. Regulation

The regulator observes the bank's earnings and can choose to intervene in the bank's management. The regulator's intervention policy could involve controlling the bank's risk as well as injecting capital to keep the bank solvent. As discussed in the previous section, the regulator injects capital in the region (τ_B, τ_L).

We denote the bank's strategy by an $\mathcal{F}_t-$adapted process

$$\Pi = (P, \tau_B). \tag{10}$$

In the above, $P_t \in \{1, 2\}$ is the bank's project choice at date t, and τ_B is the $\mathcal{F}_t-$stopping time at which the bank stops issuing equity to service debt and declares insolvency .

We denote the regulator's intervention policy by an $\mathcal{F}_t-$adapted process

$$\Pi^{reg} \equiv (P^{reg}, \tau_L). \tag{11}$$

In the above, $P_t^{reg} \in \{1, 2\}$ is the regulator's project choice at date t and τ_L is the $\mathcal{F}_t-$stopping time at which the regulator stops injecting capital and liquidates the bank. If the regulator forces the bank to choose a particular project at date t, then $P_t = P_t^{reg}$. Note that the above formulation is quite general and encompasses the possibilities that the regulator chooses not to intervene in the bank's operations at a particular date t and control its project/risk choice.

The proportional deadweight social cost of capital injection by the regulator is λ_2^{social}, which could differ from the cost of capital injection by the bank in financial distress. In other words, if the regulator injects capital k_t, the social cost is $\lambda_2^{social} k_t$. The wedge between λ_2^{social} and λ_2 reflects the effects of *systemic risk*. The regulator internalizes the effects of the externality that the bank imposes on the economy because of its financial distress.

D. Equity, Debt, Bank and Social Values

Let Π and Π^{reg} denote strategies for the bank and the regulator, respectively, as defined in the previous section. Suppose that the bank's debt level is θ. At any date t, the value of the bank's debt is

$$D_t(\theta, \Pi, \Pi^{reg}) = E_t \int_t^{\tau_L} e^{-r(u-t)} \theta du + E_t[e^{-r(\tau_L - t)} D_{\tau_L}], \tag{12}$$

where E_t denotes the conditional expectation at date t. The debt payoff D_{τ_L} upon liquidation is given by (9).

By (6), the bank's equity value at date t is

$$S_t(\theta, \Pi, \Pi^{reg}) = E_t \int_t^{\tau_B} e^{-r(u-t)} \left[\begin{array}{c} 1_{C_u(\Pi;\Pi^{reg}) \geq \theta}(1 - \lambda_1)(C_u(\Pi, \Pi^{reg}) - \theta) + \\ 1_{C_u(\Pi;\Pi^{reg}) < \theta}(1 + \lambda_2)(C_u(\Pi, \Pi^{reg}) - \theta) \end{array} \right] du. \tag{13}$$

The upper limit of the integral above is the stopping time $\tau_B \leq \tau_L$ at which the bank becomes insolvent. In the integrand, we explicitly indicate the fact that the earnings process C_u depends on the bank's and the regulator's strategies.

The bank's total value at date t is the sum of its equity and debt values, and is given by

$$\begin{aligned} F_t(\theta,\Pi,\Pi^{reg}) &= D_t(\theta,\Pi,\Pi^{reg}) + S_t(\theta,\Pi,\Pi^{reg}) \\ &= E_t \int_t^{\tau_B} e^{-r(u-t)} \left\{ \left[\begin{array}{c} 1_{C_u(\Pi,\Pi^{reg})\geq\theta}\left((1-\lambda_1)C_u(\Pi,\Pi^{reg})+\lambda_1\theta\right)+ \\ 1_{C_u(\Pi,\Pi^{reg})<\theta}\left((1+\lambda_2)C_u(\Pi,\Pi^{reg})-\lambda_2\theta\right) \end{array} \right] \right\} ds \\ & E_t \int_{\tau_B}^{\tau_L} \theta ds + E_t[e^{-r(\tau_L-t)} D_{\tau_L}]. \end{aligned} \tag{14}$$

Finally, we define the bank's *social* value at date t, which also includes the net cash flows to the regulator when it takes control of the bank during the interval $[\tau_B, \tau_L]$ and injects capital.

$$\begin{aligned} F_t^{social}(\theta,\Pi,\Pi^{reg}) &= E_t \int_t^{\tau_L} e^{-r(u-t)} \left\{ \left[\begin{array}{c} 1_{C_u(\Pi,\Pi^{reg})\geq\theta}\left((1-\lambda_1^{social})C_u(\Pi,\Pi^{reg})+\lambda_1^{social}\theta\right)+ \\ 1_{C_u(\Pi,\Pi^{reg})<\theta}\left((1+\lambda_2^{social})C_u(\Pi,\Pi^{reg})-\lambda_2^{social}\theta\right) \end{array} \right] \right\} dsIt \\ & +E_t[e^{-r(\tau_L-t)} D_{\tau_L}^{social}], \end{aligned} \tag{15}$$

where

$$D_{\tau_L}^{social} = (1-\alpha^{social})\frac{(1-\lambda_1^{social})C_{\tau_L}}{r-\mu_1}. \tag{16}$$

The bank's social value (15) differs from its total value (14) in a number of important dimensions. First, the excess cost of equity in "good" states from the regulator's standpoint, λ_1^{social}, could differ in general from λ_1, reflecting the fact that a portion of the excess cost of equity, such as the tax disadvantage of equity, are simply wealth transfers that do not affect the bank's social value (see Duffie (2001, Chapter 8) and Admati et al. (2011)). On the other hand, other costs of equity issuance such as those affecting a bank's role as an intermediary in issuing deposits are deadweight costs that are not simply transfers. The regulator, therefore, acknowledges the important role that banks play as financial intermediaries and, in particular, providers of liquidity through instruments such as demand deposits. Second, the financial distress cost λ_2^{social} from the regulator differs, in general, from λ_2 because the regulator incorporates the effects of systemic risk — the externality that the bank imposes on the economy through its financial distress. For similar reasons, the proportional liquidation cost α^{social} as seen by the regulator also differs from the proportional liquidation cost, α, incurred by the bank.

We first analyze the benchmark scenario where the bank is completely unregulated, that is, the regulator's intervention policy is the "null" policy of no intervention at all. Hence, the bank's equity, debt and total values are only functions of the debt level θ and the bank's project choices Π. Further, because the regulator injects no capital at all, the liquidation time τ_L coincides with the insolvency time τ_B. Given a debt level θ , the bank chooses its optimal strategy—its dynamic project choices and the insolvency time—to maximizes its equity value, that is, the bank's optimal

strategy solves

$$\Pi^*(\theta) = \arg\max_{\Pi} S_0(\theta, \Pi). \tag{17}$$

In the above, we explicitly indicate the fact that the bank's optimal strategy depends on the bank's debt level. Note that the regulator's intervention strategy does not appear at all as an argument because it is the " null" policy of no intervention. The bank's capital structure, which is determined by the debt level θ maximizes its total value at date zero, that is, the bank's optimal capital structure solves

$$\theta^* = \arg\max_{\theta} F_0(\theta, \Pi^*(\theta)) \tag{18}$$

Next, we consider the "full regulation" or "first best" scenario in which the regulator completely controls the bank and chooses its capital structure as well as its projects to maximize the bank's social value. Given a debt level θ, the regulator's optimal strategy—its project choices and liquidation time—solves

$$\Pi^{*reg}(\theta) = \arg\max_{\Pi} F_0^{social}(\theta, \Pi^{reg}). \tag{19}$$

The regulator chooses the bank's capital structure to maximize the bank's social value, that is,

$$\theta^{*reg} = \arg\max_{\theta} F_0^{social}(\theta, \Pi^{*reg}(\theta)) \tag{20}$$

To analyze the effects of different regulatory intervention tools—capital regulation, intervention in the bank's project choices, and capital injection—we also explore a number of "intermediate" scenarios in which one or more of these tools are "turned off." To avoid complicating the exposition here, we discuss these scenarios when we analyze them later.

IV. The Unregulated Bank

In the hypothetical scenario where the bank is completely unregulated, the regulatory intervention policy is the "null" policy where the regulator does not intervene at all in the bank's operations as discussed in Section D. The stopping time τ_B, where the bank's equity value falls to zero (it becomes insolvent), and the stopping time τ_L, where the bank's assets are liquidated, coincide. The bank's dynamic project choices, the insolvency time, and capital structure solve (17) and (18).

We first derive the bank's optimal dynamic project and insolvency time choices given a capital structure described by the debt level θ. We then derive the bank's capital structure choice. We first analytically characterize the bank's equity and debt values as well as the insolvency threshold for a *given* switching policy described by a trigger C_S.

Proposition 1 *[Equity and Debt Values for Given Switching Policy] Suppose the debt level is θ and the bank adopts a policy where it chooses project 1 if $C_t \geq C_S$ and project 2 for $C_t < C_S$.*
(i) Suppose that $C_S \geq \theta$. The bank's equity value at any date t is a function of the current earnings

level C_t and is given by

$$S_t = S(C_t) = \begin{cases} \frac{(1-\lambda_1)C_t}{r-\mu_1} - \frac{(1-\lambda_1)\theta}{r} + A_1(C_S,\theta)C_t^{\gamma_1^-}, & \text{if } C_t \geq C_S, \\ \frac{(1-\lambda_1)C_t}{r-\mu_2} - \frac{(1-\lambda_1)\theta}{r} + A_2(C_S,\theta)C_t^{\gamma_2^-} + A_3(C_S,\theta)C_t^{\gamma_2^+} & \text{if } C_S > C_t \geq \theta \\ \frac{(1+\lambda_2)C_t}{r-\mu_2} - \frac{(1+\lambda_2)\theta}{r} + A_4(C_S,\theta)C_t^{\gamma_2^-} + A_5(C_S,\theta)C_t^{\gamma_2^+}, & \text{if } \theta > C_t \geq C_B(C_S,\theta), \end{cases} \tag{21}$$

Furthermore, the equity value satisfies the following value matching and smooth pasting conditions at the insolvency threshold $C_B(C_S,\theta)$

$$S(C_B(C_S,\theta)) = S'(C_B(C_S,\theta))) = 0. \tag{22}$$

(ii) Suppose that $C_S < \theta$. The bank's equity value at any date t is given by

$$S_t = S(C_t) = \begin{cases} \frac{(1-\lambda_1)C_t}{r-\mu_1} - \frac{(1-\lambda_1)\theta}{r} + A_1(C_S,\theta)C_t^{\gamma_1^-}, & \text{if } C_t \geq \theta, \\ \frac{(1+\lambda_2)C_t}{r-\mu_1} - \frac{(1+\lambda_2)\theta}{r} + A_2(C_S,\theta)C_t^{\gamma_1^-} + A_3(C_S,\theta)C_t^{\gamma_1^+} & \text{if } \theta > C_t \geq C_S \\ \frac{(1+\lambda_2)C_t}{r-\mu_2} - \frac{(1+\lambda_2)\theta}{r} + A_4(C_S,\theta)C_t^{\gamma_2^-} + A_5(C_S,\theta)C_t^{\gamma_2^+}, & \text{if } C_S > C_t \geq C_B(C_S,\theta), \end{cases} \tag{23}$$

Further, the equity value satisfies the value matching and smooth pasting conditions (22).
(iii) The bank's debt value at any date t in both cases above is

$$D_t = D(C_t) = \begin{cases} \frac{\theta}{r} + B_1(C_S,\theta)C_t^{\gamma_1^-}, & \text{if } C_t \geq C_S, \\ \frac{\theta}{r} + B_2(C_S,\theta)C_t^{\gamma_2^-} + B_3(C_S,\theta)C_t^{\gamma_2^+}, & \text{if } C_S > C_t \geq C_B(C_S,\theta). \\ D(C_B(C_S,\theta)) = \frac{(1-\alpha)(1-\lambda_1)C_B(C_S,\theta)}{r-\mu_1} \end{cases} \tag{24}$$

The coefficients $A_i(C_S,\theta); i = 1,...,5$; $B_j(C_S,\theta); j = 1,...,3$, and the insolvency level $C_B(C_S,\theta)$ are determined by the conditions that the equity and debt values are differentiable for all $C_t > C_B(C_S,\theta)$. In the above, $\gamma_i^+, \gamma_i^-; i = 1,2$ are the positive and negative roots, respectively, of the equation

$$\frac{\sigma_i^2}{2}\gamma^2 + (\mu_i - \frac{\sigma_i^2}{2})\gamma - r = 0; i = 1,2. \tag{25}$$

In the expressions above, we explicitly indicate the fact that the coefficients $A_i(.), B_i(.)$, and the insolvency level $C_B(.)$ depend on the switching trigger C_S and the debt level θ. Note that the coefficients $A_i(.)$ differ in the two scenarios $C_S \geq \theta$ and $C_S < \theta$. We avoid complicating the notation by explicitly indicating the differing functional forms in the two cases. The asset values and the insolvency level can be analytically characterized in closed form.

The following proposition shows that the optimal dynamic project choice policy of the bank has the structure assumed in Proposition 1, that is, there is a single time-independent trigger at which the bank switches between the two projects.

Theorem 2 *[Optimal Switching Policy] Given an initial choice of debt level θ, there exists a unique cash flow threshold $C_S^*(\theta)$ such that the bank optimally chooses project 1 when $C_t \geq C_S^*(\theta)$ and project 2 when $C_S^*(\theta) > C_t > C_B^*(\theta)$. $C_B^*(\theta)$ is the insolvency trigger that is determined in Proposition 1 when the switching trigger is $C_S^*(\theta)$.*

By the discussion in Section A, in the absence of any frictions, the bank would always choose project 1 because it has a higher NPV than project 2. In the presence of shareholder-debtholder agency conflicts, however, the bank's shareholders have the incentive to engage in asset substitution, thereby diverting cash flows from debtholders. Asset substitution potentially occurs when the bank's earnings are below a threshold so that the bank is sufficiently leveraged. Note that, depending on the drifts and volatilities of the two projects, it may be optimal for the bank to always choose project 1, that is, not engage in risk-shifting at all. This scenario corresponds to the case where the switching threshold $C_S^*(\theta)$ coincides with the insolvency threshold $C_B^*(\theta)$.

As we show in the proof of Theorem 2, the switching point $C_S^*(\theta)$ is determined by the "super-contact" condition

$$\left.\frac{\partial^2 S}{\partial C^2}\right|_{C=C_S^*(\theta)+} = \left.\frac{\partial^2 S}{\partial C^2}\right|_{C=C_S^*(\theta)-}. \tag{26}$$

The above condition implies that the equity value must be *twice differentiable* throughout. The optimal switching trigger can be analytically characterized in closed form as we show in the proof.

Given the debt structure θ, the initial leverage ratio of the bank is $Lev_0 = D(C_0;\theta)/F(C_0;\theta)$, where we also indicate the dependence of the debt value and bank value on the debt level θ. The debt value and bank value are determined by Proposition 1 with the switching trigger set to the optimal switching trigger $C_S^*(\theta)$. We define the bank's leverage ratio at the switching point $C_S^*(\theta)$ as the *switching leverage*. The switching leverage is given by

$$\textit{Switching Leverage} = D(C_S^*(\theta);\theta)/F(C_S^*(\theta);\theta). \tag{27}$$

As described by Theorem 2, the bank has an incentive to engage in asset substitution by selecting the riskier, lower NPV project 2 once its leverage reaches the switching leverage. Therefore, the switching leverage is of importance to regulators who would like to limit the agency costs of asset substitution. The following proposition provides a simple and useful characterization of the switching point and leverage for a given debt level θ.

Proposition 3 *1) The default boundary $C_B^*(\theta)$ and switching trigger $C_S^*(\theta)$ are linear functions of θ, that is,*

$$C_S^*(\theta) = c_S\theta;\ C_B^*(\theta) = c_B\theta, \quad . \tag{28}$$

2) The switching leverage is a constant that does not depend on the initial choice of θ.

Intuitively, the banker chooses the switching point in proportion to the initial debt level. When

the debt level is high, agency costs are larger, and the banker would choose to switch to the risky project at a higher threshold. The switching leverage, however, is not sensitive to the initial choice of debt level, rather, it is sensitive to the relative characteristics (drifts and volatilities) of the two projects. The result of Proposition 3 is useful because it provides a justification for some aspects of the Basel accords where intervention by regulators is triggered when a bank's leverage reaches a pre-specified threshold *regardless of its initial leverage level.*

The unregulated bank's optimal capital structure choice is determined by solving (18). The switching and insolvency triggers at the optimal capital structure, θ^{unreg} (the superscript indicates that this is the scenario with no regulation) are, therefore, given by

$$C_S^{\text{unreg}} = C_S^*(\theta^{\text{unreg}}) = c_S\theta^{\text{unreg}};\ C_B^{\text{unreg}} = C_B^*(\theta^{\text{unreg}}) = c_B\theta^{\text{unreg}}, \quad . \tag{29}$$

where the constants c_S and c_B are defined in Proposition 3. Since the optimal capital structure cannot be analytically characterized in closed form, we compute it numerically in our subsequent analysis of the calibrated model.

V. The Fully Regulated Bank

In the scenario in which the regulator completely controls the bank's operations, the regulatory intervention policy is the policy of "total intervention" that is the polar opposite of the "null" intervention policy considered in the previous section. As we discussed in Section D, the bank's capital structure and project choices maximize the regulator's objective, which is the bank's social value. By comparing the results of this section with those of the previous one, we derive the optimal intervention policy of the regulator.

From the regulator's standpoint, the bank's capital structure, project choices, the insolvency time, and the liquidation time should maximize the bank's social value, that is, they should solve

$$(\theta^{\text{reg}}, \Pi^{\text{reg}}) = \arg\max_{(\theta,\Pi)} F_0^{social}(\theta, \Pi). \tag{30}$$

In the above, the superscript, "reg" indicates the fact that this is the scenario where the regulator completely controls the bank's operations.

As in the previous section, we first derive the regulator's dynamic project choices for a given capital structure, and then derive the optimal capital structure. It turns out that the regulator's optimal project choice policy has a similar structure to that of the bank in the previous section. There exists a threshold level of the bank's earnings such that it is optimal for the regulator to choose project 1 above the threshold and project 2 below the threshold. The regulator's optimal "switching threshold" differs, in general, from the bank's switching threshold derived in the previous section. The following proposition analytically characterizes the bank's social value for a given switching policy described by a trigger C_S. We omit its proof because it follows using exactly the same

arguments used to prove Proposition 1.

Proposition 4 *[Equity, Debt and social Values for Given Switching Policy] Suppose the debt level is θ and the regulator adopts a policy where it chooses project 1 if $C_t \geq C_S$ and project 2 for $C_t < C_S$.*
(i) Suppose that $C_S \geq \theta$. The bank's social value at any date t is a function of the current earnings level C_t and is given by

$$F_t^{social} = F^{social}(C_t) = \begin{cases} \frac{(1-\lambda_1^{social})C_t}{r-\mu_1} + \frac{\lambda_1^{social}\theta}{r} + X_1(C_S,\theta)C_t^{\gamma_1^-}, & \text{if } C_t \geq C_S, \\ \frac{(1-\lambda_1^{social})C_t}{r-\mu_2} + \frac{\lambda_1^{social}\theta}{r} + X_2(C_S,\theta)C_t^{\gamma_2^-} + X_3(C_S,\theta)C_t^{\gamma_2^+} & \text{if } C_S > C_t \geq \theta \\ \frac{(1+\lambda_2^{social})C_t}{r-\mu_2} - \frac{\lambda_2^{social}\theta}{r} + X_4(C_S,\theta)C_t^{\gamma_2^-} + X_5(C_S,\theta)C_t^{\gamma_2^+}, & \text{if } \theta > C_t \geq C_L(C_S,\theta), \\ \overbrace{F^{social}(C_L(C_S,\theta)) = \frac{(1-\alpha^{social})(1-\lambda_1^{social})C_L(C_S,\theta)}{r-\mu_1}}^{\text{value matching}} & \\ \overbrace{F^{social\prime}(C_L(C_S,\theta)) = \frac{(1-\alpha^{social})(1-\lambda_1^{social})}{r-\mu_1}}^{\text{smooth pasting}} & \end{cases} \tag{31}$$

(ii) Suppose that $C_S < \theta$. The bank's social value at any date t is given by

$$F_t^{social} = F^{social}(C_t) = \begin{cases} \frac{(1-\lambda_1^{social})C_t}{r-\mu_1} + \frac{\lambda_1^{social}\theta}{r} + X_1(C_S,\theta)C_t^{\gamma_1^-}, & \text{if } C_t \geq \theta, \\ \frac{(1+\lambda_2^{social})C_t}{r-\mu_2} - \frac{\lambda_2^{social}\theta}{r} + X_2(C_S,\theta)C_t^{\gamma_1^-} + X_3(C_S,\theta)C_t^{\gamma_1^+} & \text{if } \theta > C_t \geq C_S \\ \frac{(1+\lambda_2^{social})C_t}{r-\mu_2} - \frac{\lambda_2^{social}\theta}{r} + X_4(C_S,\theta)C_t^{\gamma_2^-} + X_5(C_S,\theta)C_t^{\gamma_2^+}, & \text{if } C_S > C_t \geq C_L(C_S,\theta), \\ \overbrace{F^{social}(C_L(C_S,\theta)) = \frac{(1-\alpha^{social})(1-\lambda_1^{social})C_L(C_S,\theta)}{r-\mu_1}}^{\text{value matching}} & \\ \overbrace{F^{social\prime}(C_L(C_S,\theta)) = \frac{(1-\alpha^{social})(1-\lambda_1^{social})}{r-\mu_1}}^{\text{smooth pasting}} & \end{cases} \tag{32}$$

The coefficients $X_i(C_S,\theta); i = 1,...,5$ in the above are determined by the conditions that the social value is differentiable throughout. The liquidation level $C_L(C_S,\theta)$ is determined by the "value matching" and "smooth pasting" conditions for the social value at the liquidation level.
(iii) The banks's equity values at any date t in the two cases above are given by (21) and (23) where the switching trigger is C_S. The endogenous insolvency threshold at which the equity value falls to zero is $C_B(C_S,\theta)$.

(iv) The bank's debt value at any date t is

$$D_t = D(C_t) = \begin{cases} \frac{\theta}{r} + B_1(C_S,\theta)C_t^{\gamma_1^-}, & \text{if } C_t \geq C_S, \\ \frac{\theta}{r} + B_2(C_S,\theta)C_t^{\gamma_2^-} + B_3(C_S,\theta)C_t^{\gamma_2^+}, & \text{if } C_S > C_t \geq C_L(C_S,\theta). \\ D(C_L(C_S,\theta)) = \frac{(1-\alpha^{social})(1-\lambda_1^{social})C_L(C_S,\theta)}{r-\mu_1} \end{cases} \tag{33}$$

As in the case of Proposition 1, the expressions for the social value and the equity value differ depending on whether the switching trigger is above or below the debt level θ. Comparing the two propositions, we note that the insolvency trigger is now replaced by the liqudation trigger $C_L(C_S,\theta)$.

The following proposition shows that the optimal dynamic project choice policy of the regulator has a structure similar to that of the bank, that is, there is a single time-independent trigger at which the regulator switches between the two projects.

Theorem 5 *[Regulator's Optimal Switching Policy] Given an initial choice of debt level θ, there exists a unique cash flow threshold $C_S^{*reg}(\theta) \leq \theta$ such that the regulator optimally chooses project 1 when $C_t \geq C_S^{*reg}(\theta)$ and project 2 when $C_S^{*reg}(\theta) > C_t > C_L^{*reg}(\theta)$. $C_L^{*reg}(\theta)$ is the liquidation trigger that is determined in Proposition 1 when the switching trigger is $C_S^{*reg}(\theta)$.*

The above theorem shows that, contrary to received wisdom in anecdotal discussions of prudential regulation, it could actually be optimal for the regulator too to engage in asset substitution if the bank's earnings are below a threshold. The theorem, however, shows that the regulator's optimal switching trigger is never greater than the debt level, θ. If the regulator finds it optimal to engage in asset substitution, therefore, it always does so when the bank is in financial distress, which is not the case for the unregulated bank as shown by Theorem 2 and its proof. It is worth mentioning that the regulator's optimal switching trigger could actually lie below the insolvency threshold at which the bank's equity value falls to zero, that is, it could lie in the "capital injection" region.

The intuition for why the regulator could also find it optimal to engage in asset substitution is as follows. First, when it injects capital, the regulator effectively holds a (negative) *equity stake* in the bank. The regulator's objective function in this region is, therefore, akin to that of shareholders of the unregulated bank in the previous section. Consequently, when the bank's earnings fall sufficiently far, it could be optimal for the regulator to increase risk to lower the probability of liquidation of the bank and, thereby, the associated liquidation costs. Second, by increasing risk, the regulator also delays liquidation and, thereby, lowers the expected costs of capital injection after insolvency.

Because the regulator cares about the bank's social value rather than just its equity value, it is intuitive that it has lower incentives to engage in asset substitution than the bank's shareholders. The following proposition formalizes this intuition.

Proposition 6 *[Comparison of Bank's and Regulator's Optimal Switching Policies] The optimal switching trigger for the regulator is less than or equal to the optimal switching trigger for the unregulated bank, that is,*

$$C_S^{*reg}(\theta) \leq C_S^*(\theta). \tag{34}$$

Analogous to Proposition 3, the following proposition shows that the switching, insolvency, and liquidation thresholds are linear in the debt level θ, and the switching leverage is constant. We omit the proof because it follows along the lines of the proof of Proposition 3.

Proposition 7 *1) The insolvency threshold $C_B^{*reg}(\theta)$, the switching threshold $C_S^{*reg}(\theta)$, and the liquidation threshold $C_L^{*reg}(\theta)$ are all proportional to the debt level θ,*

$$C_B^{*reg}(\theta) = c_B^{reg}\theta, \quad C_S^{*reg}(\theta) = c_S^{reg}\theta, \quad C_L^{*reg}(\theta) = c_L^{reg}\theta. \tag{35}$$

where $c_B^{reg}, c_S^{reg}, c_L^{reg}$ are constants independent of θ.
2) The switching leverage is a constant that does not depend on the initial choice of θ.

The regulator chooses the bank's capital structure to maximize the social value, that is, the optimal debt level θ^{reg} solves

$$\theta^{\text{reg}} = \arg\max_{\theta} F_0^{social}(\theta), \tag{36}$$

where the bank's social value, $F_0^{social}(\theta)$, is given by Proposition 4 with the switching trigger set to $\widetilde{C}_S^*(\theta)$. The switching, insolvency and liquidation triggers at the optimal capital structure are, therefore, given by

$$C_S^{\text{reg}} = C_S^{*reg}(\theta^{\text{reg}}) = c_S^{reg}\theta^{\text{reg}};\ C_B^{\text{reg}} = C_B^{*reg}(\theta^{\text{reg}}) = c_B^{reg}\theta^*, C_L^{\text{reg}} = C_L^{*reg}(\theta^{\text{reg}}) = c_L^{reg}\theta^{\text{reg}}. \tag{37}$$

VI. Optimal Regulatory Policy

We now use the results of the previous two sections to describe the optimal intervention policy of the regulator.

First, by (36), it is optimal for the regulator to force the bank to choose a debt level θ^{reg}. For future reference, we define the *initial capital requirement* to be $1 - D(\theta^{\text{reg}})/F(\theta^{\text{reg}})$, i.e., one minus the initial leverage ratio or the equity capital ratio.

Second, by Theorem 2, given the debt level θ^{reg}, the bank chooses a project choice policy where the switching trigger is $C_S^*(\theta^{\text{reg}})$. However, by Proposition 5, from the standpoint of the regulator, it is optimal to switch projects at C_S^{reg}. By Proposition 6, $C_S^{\text{reg}} < C_S^*(\theta^{\text{reg}})$. In other words, the regulator has lower incentives to shift risk than the bank's shareholders. To align the bank's project choices with that of the regulator, the regulator intervenes when the bank's earnings cross the threshold $C_S^*(\theta^{\text{reg}})$ and forces the bank to choose the lower risk project 1. We define the *intervention ratio* to be $1 - Lev_{C_S^*(\theta^{\text{reg}})}$, or the capital ratio of the bank at the switching leverage.

Third, by Proposition 5, it is optimal for the regulator to choose the higher risk project 2 when earnings fall below the threshold C_S^{reg}. Consequently, it is optimal for the regulator to stop intervening in the bank's operations and allow it to shift risk in this region. Hence, the regulator follows a "band policy," that is, it intervenes in the bank's project choice in the region $(C_S^{\text{reg}}, C_S^*(\theta^{\text{reg}}))$.

Fourth, by (37), the insolvency level at which the equity value falls to zero is C_B^{reg} and the liquidation level is C_L^{reg}. The regulator, therefore, allows the bank to issue equity to service debt payments as long as the earnings exceed C_B^{reg}. When earnings fall below C_B^{reg}, but exceed C_L^{reg}, it is optimal for the regulator to inject capital to keep the bank afloat.

Fifth, if earnings hit the level C_L^{reg}, it is optimal for the regulator to liquidate the bank's assets and pay off debtholders with the liquidation proceeds.

Figure 1 plots sample paths for a fully regulated bank to illustrate the above optimal regulatory policy. The bank begins with an earnings level above the debt level θ^{reg} and chooses the low-risk project 1. When earnings hit the *liquidity threshold* θ^{reg}, the bank begins to issue external equity in order keep servicing debt. When earnings further declines to *the bank's switching threshold* $C_S^*(\theta^{\text{reg}})$, despite the bank's incentive to switch to the high-risk project 2, the regulator steps in and forces the bank to continue with the low-risk project 1.[8] When the bank's earnings reaches the *insolvency threshold* C_B^{reg}, the bank's equity value falls to zero and the regulator bails out the bank by starting to inject capital into the bank. The regulator switches the bank's project to the high-risk project 2 when earnings reaches *the regulator's switching threshold* C_S^{reg}, and liquidates the bank if earnings further deteriorates to the *liquidation threshold* C_L^{reg}. The figure shows that the regulator's intervenes in the bank's project choices when earnings lie between the intervention band $[C_S^{\text{reg}}, C_S^*(\theta^{\text{reg}})]$. In another sample path plotted in the figure, the bank's earnings improve and the bank stops issuing equity when earnings rise to the liquidity threshold θ^{reg}.

VII. Quantitative Analysis

To obtain a reasonable set of baseline parameter values for our quantitative analysis of the effects of regulation, we calibrate the model to data on U.S. banks over the period 1991–2008. It has become evident, over most of this time period, U.S. banks were only weakly regulated, if at all. Yet, many distressed banks were bailed out with public funds after the crisis. Consequently, the model that we calibrate to the data is an "intermediate" one where the bank chooses its capital structure and projects, but enjoys the "safety net" provided by capital injection after the bank's equity value falls to zero.

Without loss of generality, we normalize the bank's initial earnings, C_0—the state variable in

[8] For illustration purposes, the figure plots a particular case where $\theta^{\text{reg}} > C_S^*(\theta^{\text{reg}})$, while in general it is possible that $\theta^{\text{reg}} \leq C_S^*(\theta^{\text{reg}})$.

the model—to 1. The set of key parameters of the model that need to be calibrated is

$$\Gamma \equiv \left\{ r, \alpha, \alpha^{social}, \lambda_1, \lambda_1^{social}, \lambda_2, \lambda_2^{social}, \mu_1, \mu_2, \sigma_1, \sigma_2 \right\}$$

We set the risk-free rate r to 0.035 to match the average short-term U.S. treasury rate over the period 1991 – 2008.[9] For our baseline set of parameter values, we set $\alpha = \alpha^{social}$. The liquidation cost $\alpha = 0.20$ is in the ranges estimated by Andrade and Kaplan (1998) and Davydenko, Strebulaev, and Zhao (2011). Tax shields on debt interest payments represent one of the sources of equity issuance costs faced by individual banks. However, from a social standpoint, tax shields are merely wealth transfers, which would suggest that λ_1^{social} should be less than λ_1. However, the bank's personal costs of equity issuance may not completely reflect the "general equilibrium" effects of the redistribution of wealth, and the relative composition of securities issued by the bank among heterogeneous investors some of whom prefer information-insensitive debt-like securities, while others prefer risky equity-like securities. These effect could cause λ_1^{social} to even exceed λ_1. Since it is difficult (if not impossible) to calibrate λ_1^{social}, we set it equal to λ_1 in our baseline analysis, and subsequently explore the effects of varying its value. We set $\lambda_2^{social} = \lambda_2$. In reality, the social costs of financial distress are likely to be much higher than the bank's personal cost because of the effects of systemic risk that the bank does not internalize. Our baseline analysis, therefore, provides a conservative estimate or lower bound on the effects of regulation. Later, we explore the effects of a wedge between social and personal costs of financial distress by allowing λ_2^{social} to vary from its baseline value.

To calibrate the project return and risk characteristics, we use Compustat data for U.S. banks over the sample time period . In the model, the bank chooses the low risk project except possibly in financial distress. Accordingly, we set the risk, σ_1, of the low-risk project to 0.126 to match the asset risk of the average bank in the sample.[10] We set the risk of the high-risk project σ_2 to 0.148, which corresponds to the 75th percentile of the asset risk of banks in the sample. In the absence of default risk, the bank's value is

$$V_0 = \frac{(1-\lambda_1)C_0}{r-\mu_1} + \frac{\lambda_1 \theta}{r}, \tag{38}$$

where θ is the interest payment. The ratio of the average bank's earnings to assets and interest payments to assets are 3.8% and 2.8%, respectively. Setting C_0/V_0 to 3.8% and θ/V_0 to 2.8% in (38), respectively, we obtain

[9] We consider this time period because the Basel I Accord was published in 1988 and enforced by G-10 countries in 1992. The risk-free rates are downloaded from Kenneth French's website.

[10] We calculate the asset risk by the formula $\sigma_A = \sqrt{(\theta_D \sigma_D)^2 + (\theta_E \sigma_E)^2}$, where θ_D and σ_D are the fraction and risk of bank debt, and θ_E and σ_E are the fraction and risk of equity. We assume that the risk of deposit debt is zero. The debt risk σ_D is estimated to be 0.08, the same as the long-term treasury bond, and the equity risk is estimated by monthly stock returns of U.S banks over the period 1991–2010.

$$\mu_1 = r\left(1 - \frac{(1-\lambda_1)0.038}{r - 0.028\lambda_1}\right) \tag{39}$$

To avoid introducing additional arbitrary parameters in our analysis, we assume that the two projects have the same market price of risk. It follows from the fundamental theorems of asset pricing (see Duffie (2001)) that the risk-neutral drifts and volatilities of the projects are related as follows:

$$\mu_1 = r - \pi\sigma_1 \tag{40}$$

$$\mu_2 = r - \pi\sigma_2, \tag{41}$$

where π is the market price of risk of the two projects. From (41), we have

$$\frac{r-\mu_1}{\sigma_1} = \frac{r-\mu_2}{\sigma_2} \tag{42}$$

We then calibrate the drifts of the projects μ_1, μ_2, and the equity cost parameters λ_1 and λ_2 subject to the constraints (39) and (42). We calibrate these parameters by matching the average leverage and credit spread of banks. The average leverage ratio of banks in the sample is 87%. Krishnan, Ritchken, and Thomson (2006) find that the the credit spread of 10-year bonds issued by average highly rated (A- or better) banks is 106 bp. We choose μ_1, μ_2, λ_1, andλ_2 so that the equations (39) and (42) are satisfied, and the optimal initial leverage ratio as well as the credit spread match the empirical values (87% and 106 bp) as closely as possible. We thereby obtain $\mu_1 = 0.0\%, \mu_2 = -0.6\%, \lambda_1 = 0.305$, and $\lambda_2 = 0.539$. The following table shows the calibrated parameters.

	Parameter	Calibrated Value
Risk-free rate	r	0.035
Drift of low-risk project	μ_1	0.000
Volatility of low-risk project	σ_1	0.126
Drift of high-risk project	μ_2	-0.006
Volatility of high-risk project	σ_2	0.148
Liquidation Cost	α,α^{social}	0.20
Excess Cost of Equity	λ_1,λ_1^{social}	0.305
Excess Cost of Equity in Distress	λ_2,λ_2^{social}	0.539

A. Impact of Regulation

In our first set of numerical experiments, we compare the fully regulated bank with the fully unregulated bank. The parameters of the model take their baseline values described in the previous

sub-section. We compute the optimal leverage ratios and risk-shifting policies of the unregulated and fully regulated bank, respectively, as described in Sections IV and V, respectively. Table I displays the results.

First, we note that the optimal leverage of the regulated bank (80%) is much higher than that of the unregulated bank (65%), which is due to two factors. First, the regulator's intervention in the bank's project choices substantially reduces the risk of the bank. Second, the promise of capital injection by the regulator reduces the effective cost of debt. These factors enable the regulated bank to borrow much more than an unregulated bank. By the discussion in Section VI, the optimal leverage of the regulated bank implies an initial capital requirement of 20% imposed by the regulator, which is much higher than those recommended by the Basel accords, and consistent with the much higher capital ratios proposed by recent studies such as Admati et al (2011).

Second, the higher initial debt level of the regulated bank implies that the bank would want to switch to the risky project earlier, that is, at a higher threshold (0.753) than in the unregulated case (0.581). This intuition follows from Proposition 3. However, the regulator prevents asset substitution from happening at such an early point by intervening in the bank's management at the switching threshold of the bank, or at the intervention ratio of 6.8%. Interestingly, consistent with Theorem 5, the regulator also finds it optimal to engage in risk-shifting when the bank's earnings are sufficiently low (0.335). In the baseline model, the risk-switching threshold of the regulator is less than the insolvency/bailout threshold (0.579). In other words, the regulator shifts risk when its equity stake in the bank is negative.

Third, despite having a higher leverage, the regulated bank borrows at a lower spread (42 bp) than the unregulated bank (69 bp). Regulation lowers credit risk despite increasing leverage by mitigating the asset substitution problem and by injecting capital after the bank's equity value falls to zero. Both spreads are, however, very low, reflecting the fact that banks' assets have much lower volatility than those of a typical non-financial company.

Fourth, the ex ante benefit of optimal regulation is economically significant and equals 4.4% of the bank value. Because there are additional deadweight social costs of capital injection, the bank's value (equity value plus debt value) is greater than the social (or social) value of the bank. Optimal regulation boosts the bank's value by 12.4%, implying huge benefits for the equity and debt holders of the bank. The benefits to the bank's claimholders are, however, partially offset by a significant external social cost of capital injection, equaling 8% of bank value.

Fifth, regulation has an even greater impact on the bank's value *ex post* at the insolvency boundary. The ex post debt value at insolvency increases from the case with no capital injection to the case where the regulator injects capital by 86.9%.[11] Even after excluding the external social cost of injection, the net benefit of the bank bailout is 36.3% of the value of the unbailed bank.

The significant ex ante and ex post benefits of intervention by the regulator in the bank's project choices as well as capital injection when the bank is insolvent suggest that both are, indeed,

[11] Note that at the insolvency boundary or later, the bank value is equal to the debt value of the bank.

valuable regulatory tools *provided* they are optimally used.

B. Impact of Individual Regulatory Policies

We further explore the effects of different regulatory tools by analyzing environments in which the regulator has less than the full set of regulatory tools considered in the previous section. By considering individual regulatory policies, we isolate the effects of different components of bank regulation. Table II compares the solutions in several environments where the regulator only employ certain subsets of regulatory tools.

Capital Injection

We first study the scenario in which the regulator can promise to make capital injections but has no control over the bank's project choices or its leverage. Column 1 of Table II reports the solution to the regulator's problem in this enviornment. In this setting, the regulator injects capital after equity value drops to zero, providing implicit insurance to debt holders. This allows the bank to choose a much higher initial leverage (87%) than in the unregulated environment (65%). Capital injection alone actually reduces the bank's social value by -0.7%. The ex ante costs of capital injection arises from the fact that the bank is able to engage in asset substitution without intervention by the regulator. Unconstrained asset substitution by the bank not only decreases social value, but also leads the regulator to inject less capital because the bank is more risky and less worthy of salvation.

Intervention in Project Choices

We now examine the environment in which the regulator can restrict the bank's project choices through, but is not able to inject capital into the bank. This scenario seems particularly relevant given the vigorous academic and political debate on the rationales for capital injection.

Column 3 of Table II reports the optimal policies in such an environment, and compares them with those in the unregulated environment. Because the regulator does not incur external capital injection costs in this setup, the bank value is equal to the social value. Therefore, the optimal initial leverage for the bank is the same as that for the regulator. This optimal leverage is 66.4%, slightly higher than that in the unregulated case (65%), because of the reduced asset substitution problem. Ex ante, capital regulation in this setup improves firm/social value by only 0.4%.

Comparing Column 2 and 3 in Table II , we see that capital injection should not be used alone as a regulatory tool, and capital regulation that aims at reducing asset substitution is more effective. The combination of these policies (Column 5 of Table II), however, yields a much greater ex ante benefit (4.4%) than the benefit of capital regulation (0.4%) without injection. Therefore, our results suggest that there is a large impact of the interaction between capital regulation and

capital injection policies, when they are both optimally chosen.

Regulation without Initial Capital Requirement

We next consider the environment where the regulator optimally intervenes in the bank's project choices, injects capital, and liquidates the bank, but does not impose an initial capital requirement. Studying such an environment allows us to understand the role of the initial capital requirement when other mechanisms such as investment intervention and capital injection are in place.

Column 4 of TableII compares the policies in such an environment with those in the unregulated environment. When the initial leverage is unrestricted, the optimal leverage chosen by the bank is 100% and the bank value increases by 33% compared to the unregulated case. Meanwhile, the social value of the bank decreases by 16.5% relative to the unregulated bank. The intuition behind these results is that because investment intervention and capital injection by the regulator protect debtholders, the bank capitalizes on these protection devices by increasing its debt as much as possible, thereby substantially increasing the expected social costs of capital injection. These results illustrate the necessity of imposing a minimum initial capital requirement, which is analogous to the capital requirement specified in Basel accords, especially when other regulatory tools that aim at reducing bank risk are in place.

C. Impact of Changes in Initial Capital Requirement

We now analyze the scenario where the regulator imposes an initial capital requirement that may be suboptimal, but administers other regulatory policies optimally. Figure 2 presents the impact of the initial capital requirement in such an environment. Recall that the optimal leverage is equal to one minus the initial capital requirement. When initial capital requirement varies, the intervention ratio stays constant (Proposition 7shows that the switching leverage and thus the intervention ratio is independent of initial capital structure). The ex ante social benefit of regulation decreases substantially when the initial capital requirement is relaxed. For example, when the initial capital requirement is equal to 8%, i.e., the Basel II requirement, the ex ante social benefit decreases by 2.4% of bank value. Our result thus suggests that adopting the optimal capital requirement, which is much higher than those required by the Basel II accords, can improve social welfare substantially.

D. Comparative Statics

We now examine how variations in the values of key underlying parameters affect the optimal regulatory policies and the associated benefits. The closed-form solution of the model greatly simplifies the computation of these "comparative static" relationships. Figure 3 shows the impact of changes in the parameters on optimal regulation.

The Systemic and Internal Costs

Panels A and B of Figure 3 display the effects of the social cost of equity issuance λ_1^{social} and the bank's internal cost λ_1. The initial capital requirement decreases with and is very sensitive to the social cost λ_1^{social}. When λ_1^{social} is high, the regulator has a strong preference for the bank to preform its welfare-improving functions, such as providing demand deposits, and therefore, imposes a less strict capital requirement on the bank. As a consequence, at times when the functions of banks are crucial, for example, during an economic depression or credit crunch, the regulator should optimally reduce the capital requirement of banks. Our model thus provides a rationale for procyclical capital requirement proposed by many critiques of the Basel II requirement (see e.g. Drumond (2009) and Dewatripont, Rochet, and Tirole (2010)).

The initial capital requirement also depends negatively on the bank's personal cost of equity λ_1 (to a lesser extent than λ_1^{social}), for a very different reason. The (unreported) optimal regulatory initial debt level is virtually independent of λ_1, which implies that the optimal regulatory initial leverage is higher (and capital requirement is lower) for higher λ_1 since the bank's equity value depends negatively on λ_1. The intervention ratio is relatively insensitive to changes in λ_1^{social} or λ_1. The intuition is that the incentives for risk-shifting, which determine the optimal switching trigger and, therefore, the capital adequacy ratio, mainly depend on the project characteristics, and not on the other parameters of the model. The ex ante social benefits of regulation increase sharply as λ_1^{social} increases due to the higher initial leverage which allows the bank to perform its functions better. In contrast, the social benefits are not sensitive to changes in λ_1 as the regulator does not incorporate the costs that are only internal to the bank.

Here we can relate our findings to tax policy and capital regulation. When the government imposes a higher corporate tax rate on banks, it makes the bank's internal cost of equity λ_1 higher, but should not affect the social cost λ_1^{social} as the tax policy is simply a wealth transfer.[12] Therefore, our comparative statics suggest that in a high-tax-rate regime, capital requirements should be less stringent, because higher tax rate decreases banks' equity value while does not affect debt value substantially. Importantly, our results suggest that bank capital regulation should not be independent of fiscal policy as suggested by Admati et al. (2011).

Panels C and D show the comparative statics for the systemic cost λ_2^{social} and the bank's personal distress cost λ_2. In contrast to the result with λ_1^{social}, when λ_2^{social} increases, the optimal initial capital requirement increases due to two factors. First, the higher systemic cost of distress leads to lower optimal initial leverage. Second, a higher λ_2^{social} implies a higher social cost of injection and thus the regulator imposes a lower initial leverage to control the expected costs of capital injection. Interestingly, the expected cost of injection (the gap between the benefits to the bank value and the social value) depends *negatively* on the marginal social cost of injection λ_2^{social}.

[12] This is a simplified analysis because tax policy may have social benefits or costs associated with redistribution of wealth, which is beyond the scope of this paper.

This reflects the classical moral hazard problem in bank bailouts, and echoes the dilemma facing the Federal Reserve when it weighed decisions on saving financial institutions such as Bear Stearns, Lehman Brothers, and AIG. When the marginal cost λ_2^{social} is smaller, the market anticipates the regulator to inject more capital into the bank. Therefore, the bank debt is expected to be better insured, which enables the banker to borrow more aggressively, leading to substantial bailout costs for the regulator. Since the regulator is concerned only with the systemic cost of distress, the optimal regulatory policies and social benefits are virtually independent of the bank's internal distress cost λ_2.

Panels E and F show the effects of the social and personal costs of liquidation α^{social} and α. Surprisingly, the optimal initial capital requirement changes little with α^{social} or α, contrary to the usual intuition that leverage should decrease with default costs. The reason is that the regulator is able to inject capital, delay liquidation, and provide significant insurance to the debt holders, offsetting the negative impact on debt value and optimal leverage of high liquidation cost. Consistent with the above intuition, for higher liquidation cost α^{social}, the ex ante benefits of regulation is larger.

The Risk-Free Rate

Panel G of Figure 3 shows the impact of varying the risk-free interest rate r. The optimal initial capital requirement decreases with the risk-free rate. This implies that capital regulation should not be independent of monetary policy, and keeping a constant capital requirement as recommended by the Basel II accords could be sub-optimal. From the regulator's point of view, a looser monetary policy should be combined with a tighter capital requirement. Our findings also suggest that it could be sub-optimal to impose the same capital requirements in different countries with different monetary policies.[13]

In contrast with the effects of the risk-free rate on the initial capital requirement, the intervention ratio is relatively insensitive to the risk-free rate, for the same reason discussed above. The social benefit of regulation increases slightly with the risk-free rate, possibly due to the fact that the benefits of reducing asset substitution and capital injection are higher when the initial leverage of the bank is higher, i.e., when initial capital requirement is lower.

Project Drifts and Volatilities

Panel H of Figure 3 shows the effects of varying the drift, μ_1, of the low-risk project. The intervention ratio declines sharply when the drift μ_1 increases. The intuition is that, when the drift of the low-risk project increases, the bank's incentives for risk-shifting decline. The optimal initial capital requirement, however, increases with μ_1, because a higher drift or earnings growth

[13] Acharya (2003) makes a similar point using a very different framework and for different reasons.

rate implies a lower debt capacity of the bank. The social benefit of regulation decreases with μ_1 due to the leverage effect.

Panel I shows the effects of varying the volatility of the low-risk project σ_1. When σ_1 increases, the bank's average risk increases. Consequently, the optimal leverage declines, and the initial capital requirement increases. Perhaps surprisingly, the intervention ratio declines with σ_1. As σ_1 increases, the incentives for choosing the high-risk project decline because the wedge between the volatilities of the two projects declines. Consequently, the optimal intervention ratio is lower. In other words, even though the average risk of the bank is higher, the capital ratio at which the regulator intervenes is actually *lower*.

Finally, Panels J and K present the comparative statics for the drift, μ_2, and volatility, σ_2, of the high-risk project. The intervention ratio is very sensitive to these parameters. The intuition is straightforward. An increase in the drift and/or the volatility of the high-risk project *aggravate* the asset substitution problem that leads to higher optimal capital requirement. The initial capital requirment, however, is largely stable with μ_2 and σ_2, which reflects the fact that the optimal leverage is primarily driven by the characteristics of the low-risk project because this is the project that the bank chooses in "normal" circumstances.

VIII. Conclusion

We develop a continuous-time model in which a bank can dynamically engage in asset substitution or risk-shifting. The bank's capital structure reflects the tradeoff between the excess costs of issuing equity against the agency costs of asset substitution and liquidation costs of debt. The regulator can impose initial capital requirement, intervene in the bank's risk choices, inject capital, and force liquidation of the bank. Under the optimal regulatory policy, the regulator intervenes in the bank's risk choices when the bank's capital ratio hits a switching threshold (the *intervention ratio*) to mitigate the social costs of asset substitution behavior of the bank's shareholders. However, when the bank's earnings are sufficiently low, the regulator may find it optimal to substitute assets and make more risky investment as the bank does. The optimal investment intervention policy has a "band" structure where the regulator intervenes in the bank's operations if and only if the bank's capital ratio lies in an interval. By calibrating the model to data, we find that the optimal initial capital requirement is much higher than those suggested by the Basel II accords. Prudential regulation can achieve substantial social benefits when it is appropriately designed. The optimal combination of investment intervention and capital injection polices significantly improve social value over either policy alone.

Appendix: Proofs

Proof of Proposition 1

By (13),

$$S_t = E_t \int_t^{\tau_B} e^{-r(u-t)} \left[1_{C_u \geq \theta}(1-\lambda_1)(C_u - \theta) + 1_{C_u < \theta}(1+\lambda_2)(C_u - \theta)\right] du. \tag{43}$$

From the above, the equity value must satisfy the following flow equation (expressed in infinitesimal form for convenience)

$$S_t = \left(1_{C_t \geq \theta}(1-\lambda_1)(C_t - \theta) + 1_{C_t < \theta}(1+\lambda_2)(C_t - \theta)\right) dt + E_t\left[e^{-rdt} S_{t+dt}\right]. \tag{44}$$

Subtracting the L.H.S. from the R.H.S., dividing throughout by dt and eliminating terms of $o(dt)$, we get

$$E_t[\frac{S_{t+dt} - S_t}{dt} - rS_t + (1_{C_t \geq \theta}(1-\lambda_1)(C_t - \theta) + 1_{C_t < \theta}(1+\lambda_2)(C_t - \theta))] = 0.$$

Applying Ito's lemma to the above, it follows from (1) and (2) that the equity value $S(C)$ must satisfy the following system of ODEs

$$\begin{aligned}
\frac{1}{2}\sigma_1^2 C^2 \frac{d^2 S}{dC^2} + \mu_1 C \frac{dS}{dC} - rS + (1_{C \geq \theta}(1-\lambda_1)(C-\theta) + 1_{C<\theta}(1+\lambda_2)(C-\theta)) &= 0 \text{ for } C > C_S \\
\frac{1}{2}\sigma_2^2 C^2 \frac{d^2 S}{dC^2} + \mu_2 C \frac{dS}{dC} - rS + (1_{C \geq \theta}(1-\lambda_1)(C-\theta) + 1_{C<\theta}(1+\lambda_2)(C-\theta)) &= 0 \text{ for } C < C_S.
\end{aligned} \tag{45}$$

(i) Suppose that $C_S > \theta$. The solution of the above system of ODEs is as follows.

$$\begin{aligned}
S(C) &= A_1 C^{\gamma_1^-} + A_1' C^{\gamma_1^+} + \frac{(1-\lambda_1)C}{r-\mu_1} - \frac{(1-\lambda_1)\theta}{r} \quad \text{for } C > C_S \\
&= A_2 C^{\gamma_2^-} + A_3 C^{\gamma_2^+} + \frac{(1-\lambda_1)C}{r-\mu_2} - \frac{(1-\lambda_1)\theta}{r} \quad \text{for } C_S > C > \theta \\
&= A_4 C^{\gamma_2^-} + A_5 C^{\gamma_2^+} + \frac{(1+\lambda_2)C}{r-\mu_2} - \frac{(1+\lambda_2)\theta}{r} \quad \text{for } C > C_B,
\end{aligned} \tag{46}$$

where γ_i^+, γ_i^- are the positive and negative root, respectively of (25). (It is easy to show that (25) has one positive and one negative root.) In the above, we have suppressed the dependence of the coefficients $A_1, A_1', A_2, A_3, A_4, A_5$ on C_S and θ.

Because $S(C) \sim \frac{(1-\lambda_1)C}{r-\mu_1}$ as $C \longrightarrow \infty$, we must have $A_1' = 0$. Further, because the equity value is zero at insolvency , and insolvency is optimally chosen by the bank to maximize its equity value, it follows from well-known arguments (e.g. see Leland (1994)) that the equity value must satisfy the value matching and smooth pasting conditions (22), which ensure that the equity value is differentiable at the insolvency threshold C_B. The coefficients $A_1, ..., A_5$ and the insolvency threshold C_B are determined by the conditions that the equity value must be differentiable at the switching threshold C_S, the debt level θ and the insolvency threshold C_B, that is, we have 6 unknowns and

6 equations.

(ii) We can use similar arguments to those used in (i) to show that the equity value must be given by (23). The only difference is that $C_S < \theta$.

(iii) Using arguments similar to those used for the equity value, the debt value satisfies the following system of ODEs:

$$\begin{aligned} \frac{1}{2}\sigma_1^2 C^2 \frac{d^2 D}{dC^2} + \mu_1 C \frac{dD}{dC} - rD + \theta &= 0 \text{ for } C > C_S \\ \frac{1}{2}\sigma_2^2 C^2 \frac{d^2 D}{dC^2} + \mu_2 C \frac{dD}{dC} - rD + \theta &= 0 \text{ for } C < C_S. \end{aligned} \tag{47}$$

The general solution to the above system is

$$\begin{aligned} D(C) &= B_1 C^{\gamma_1^-} + B_1^{'} C^{\gamma_1^+} + \frac{\theta}{r} \quad \text{for } C > C_S \\ &= B_2 C^{\gamma_2^-} + B_3 C^{\gamma_2^+} + \frac{\theta}{r} \quad \text{for } C < C_S. \end{aligned}$$

Since $D(C) \sim \frac{\theta}{r}$ as $C \longrightarrow \infty$, $B_1^{'} = 0$. By (9), we must have

$$D(C_B) = \frac{(1-\alpha)(1-\lambda_1)C_B}{r - \mu_1}. \tag{48}$$

The coefficients B_1, B_2, B_3 are determined by(48) and the conditions that the debt value is differentiable at C_S, that is, we have 3 unknowns and 3 equations. Q.E.D.

<u>Proof of Theorem 2</u>

We begin by stating the relevant dynamic programming verification theorem for our analysis.

Proposition 8 *[Dynamic Programming Verification Theorem] Let $S_q(C)$ denote the equity value when the current earnings level is C if the bank follows a switching policy where it chooses project 1 when its earnings exceed q and project 2 when the earnings are below q. Suppose that $S_q(C)$ satisfies the following Hamilton-Jacobi-Bellman (HJB) equation:*

$$\max_{i \in \{1,2\}} L_i S_q + (1_{C \geq \theta}(1-\lambda_1)(C-\theta) + 1_{C<\theta}(1+\lambda_2)(C-\theta)) = 0;$$
$$S_q(C_B) = S_q^{'}(C_B) = 0, \tag{49}$$

where

$$L_i S_q = \frac{1}{2}\sigma_i^2 C^2 \frac{d^2 S_q}{dC^2} + \mu_i C \frac{dS_q}{dC} - rS_q; i \in \{1,2\} \tag{50}$$

Then $S_q(C)$ is the optimal equity value function among all possible dynamic project choice policies (including non-stationary policies) and q is the optimal switching trigger.

Since the above follows from the general verification theorem for dynamic programming, we omit its proof for brevity and refer the reader to Fleming and Soner (1992).

We use the following lemma frequently in the proof.

Lemma 9 *We have*

$$\begin{aligned} \gamma_i^+ &> 1 \text{ for } i \in \{1,2\} && (51)\\ \gamma_1^- &< \gamma_2^- && (52)\\ r - \mu_i\gamma_i^- &> 0 \text{ for } i \in \{1,2\} && (53)\\ r - \mu_i\gamma_i^+ &> 0 \text{ for } i \in \{1,2\} && (54) \end{aligned}$$

Proof. By (3), and since γ_i^+, γ_i^- are the roots of (25), we have $0 > \mu_i - r = \frac{1}{2}\sigma_i^2(1)^2 + (\mu_i - \frac{1}{2}\sigma_i^2)(1) - r = \frac{1}{2}\sigma_i^2(1-\gamma_i^+)(1-\gamma_i^-)$. Since $\gamma_i^- < 0$, we must have $1 < \gamma_i^+$.

Next, we note that, because $\mu_2 < \mu_1$ and $\sigma_1 < \sigma_2$,

$$\mu_2 - \frac{1}{2}\sigma_2^2 < \mu_1 - \frac{1}{2}\sigma_1^2$$

Because $\gamma_1^- < 0$, it follows that

$$\frac{1}{2}\sigma_2^2(\gamma_1^-)^2 + (\mu_2 - \frac{1}{2}\sigma_2^2)(\gamma_1^-) - r > \frac{1}{2}\sigma_1^2(\gamma_1^-)^2 + (\mu_1 - \frac{1}{2}\sigma_1^2)(\gamma_1^-) - r = 0,$$

Because γ_2^+, γ_2^- are the roots of $\frac{1}{2}\sigma_2^2(x)^2 + (\mu_2 - \frac{1}{2}\sigma_2^2)(x) - r = 0$, we have

$$\frac{1}{2}\sigma_2^2(\gamma_1^-)^2 + (\mu_2 - \frac{1}{2}\sigma_2^2)(\gamma_1^-) - r = \frac{1}{2}\sigma_2^2(\gamma_1^- - \gamma_2^-)(\gamma_1^- - \gamma_2^+) > 0.$$

Since $\gamma_1^- < 0 < \gamma_2^+$, it follows from the last inequality above that we must have $\gamma_1^- < \gamma_2^-$.

To prove the third inequality, we proceed as follows.

$$0 = \frac{1}{2}\sigma_i^2(\gamma_i^-)^2 + (\mu_i - \frac{1}{2}\sigma_i^2)(\gamma_1^-) - r = \frac{1}{2}\sigma_i^2\left((\gamma_i^-)^2 - \gamma_i^-\right) + \mu_i(\gamma_1^-) - r$$

Because $\gamma_1^- < 0$, $\frac{1}{2}\sigma_i^2\left((\gamma_i^-)^2 - \gamma_i^-\right) > 0$. Consequently, it follows from the above that (53) must hold.

To prove the fourth inequality, observe that

$$0 = \frac{1}{2}\sigma_i^2(\gamma_i^+)^2 + (\mu_i - \frac{1}{2}\sigma_i^2)(\gamma_1^+) - r = \frac{1}{2}\sigma_i^2\left((\gamma_i^+)^2 - \gamma_i^+\right) + \mu_i(\gamma_1^+) - r$$

By (51), $\frac{1}{2}\sigma_i^2\left((\gamma_i^+)^2 - \gamma_i^+\right) > 0$ so that inequality (54) follows from the above. Q.E.D.

We now proceed with the proof of the main theorem. Because there are different cases to consider, we prove the theorem by stating and proving propositions that deal with each case. The following proposition establishes a *necessary and sufficient* condition for the bank to optimally choose project 1 always, that is, engage in no risk-shifting.

Proposition 10 *[No Asset Substitution] Suppose that*

$$L_2S_0 + (1+\lambda_2)(C-\theta)|_{C=C_B(0)+} \leq 0, \tag{55}$$

where S_0 is the equity value function when the bank always chooses project 1 and $C_B(0)$ is the corresponding endogenous insolvency level. Then it is optimal for the bank to always choose project 1, that is, no asset substitution is optimal.

Proof. By Proposition 7, it suffices to show that

$$L_2 S_0 + (1_{C\geq\theta}(1-\lambda_1)(C-\theta) + 1_{C<\theta}(1+\lambda_2)(C-\theta)) \leq 0 \text{ for all } C \geq C_B(0). \tag{56}$$

By Proposition 1,

$$\begin{aligned} S_0(C) &= A_1 C^{\gamma_1^-} + \frac{(1-\lambda_1)C}{r-\mu_1} - \frac{(1-\lambda_1)\theta}{r} \text{ for } C \geq \theta \\ &= A_2 C^{\gamma_1^-} + A_3 C^{\gamma_1^+} + \frac{(1+\lambda_2)C}{r-\mu_1} - \frac{(1+\lambda_2)\theta}{r} \text{ for } C_B(0) < C < \theta. \end{aligned} \tag{57}$$

First, we note that

$$A_1 > 0 \tag{58}$$

because the equity value function must be greater than $\frac{(1-\lambda_1)C}{r-\mu_1} - \frac{(1-\lambda_1)\theta}{r}$, which is the equity value function in the hypothetical scenario where shareholders are not protected by limited liability.

Matching the value and derivative of the function S_0 at $C = \theta$, we have

$$\begin{aligned} A_1\theta^{\gamma_1^-} + \frac{(1-\lambda_1)\theta}{r-\mu_1} - \frac{(1-\lambda_1)\theta}{r} &= A_2\theta^{\gamma_1^-} + A_3\theta^{\gamma_1^+} + \frac{(1+\lambda_2)\theta}{r-\mu_1} - \frac{(1+\lambda_2)\theta}{r}, \\ \gamma_1^- A_1\theta^{\gamma_1^-} + \frac{(1-\lambda_1)\theta}{r-\mu_1} &= \gamma_1^- A_2\theta^{\gamma_1^-} + \gamma_1^+ A_3\theta^{\gamma_1^+} + \frac{(1+\lambda_2)\theta}{r-\mu_1}. \end{aligned} \tag{59}$$

From the above, we obtain

$$(\gamma_1^+ - \gamma_1^-)A_3\theta^{\gamma_1^+} = (\lambda_1+\lambda_2)\theta\left[\frac{\gamma_1^- - 1}{r-\mu_1} - \frac{\gamma_1^-}{r}\right] = (\lambda_1+\lambda_2)\theta\frac{\mu_1\gamma_1^- - r}{r(r-\mu_1)}.$$

By (53), and the fact that $\gamma_1^+ > \gamma_1^-$, the above implies that

$$A_3 < 0. \tag{60}$$

Again, from (59), we obtain

$$(\gamma_1^+ - \gamma_1^-)A_1\theta^{\gamma_1^-} + (\lambda_1+\lambda_2)\theta\left[\frac{r-\mu_1\gamma_1^+}{r(r-\mu_1)}\right] = (\gamma_1^+ - \gamma_1^-)A_2\theta^{\gamma_1^-}.$$

By (54) and (58), the above implies that

$$A_2 > 0. \tag{61}$$

Next, we note that, for $C < \theta$,

$$\begin{aligned} L_2 S_0 + 1_{C<\theta}(1+\lambda_2)(C-\theta) &= A_2 \left[\frac{1}{2}\sigma_2^2(\gamma_1^-)^2 + (\mu_2 - \frac{1}{2}\sigma_2^2)\gamma_1^- - r\right] C^{\gamma_1^-} \\ &+ A_3 \left[\frac{1}{2}\sigma_2^2(\gamma_1^+)^2 + (\mu_2 - \frac{1}{2}\sigma_2^2)\gamma_1^+ - r\right] C^{\gamma_1^+} + \frac{(\mu_2-\mu_1)C}{r-\mu_1} \\ &= A_2 \frac{1}{2}\sigma_2^2(\gamma_1^- - \gamma_2^-)(\gamma_1^- - \gamma_2^+)C^{\gamma_1^-} + A_3 \frac{1}{2}\sigma_2^2(\gamma_1^+ - \gamma_2^-)(\gamma_1^+ - \gamma_2^+)C^{\gamma_1^+} \\ &+ \frac{(1+\lambda_2)(\mu_2-\mu_1)C}{r-\mu_1}, \end{aligned} \tag{62}$$

where the last equality follows from the fact that γ_2^+ and γ_2^- are the roots of (25) for $i = 2$. Since $\gamma_1^- < \gamma_2^- < \gamma_2^+$ by (52), it follows from (61) that

$$A_2 \frac{1}{2}\sigma_2^2(\gamma_1^- - \gamma_2^-)(\gamma_1^- - \gamma_2^+)C^{\gamma_1^-} > 0. \tag{63}$$

We need to consider two cases.

Case 1: $\gamma_1^+ > \gamma_2^+$

By (60),

$$A_3 \frac{1}{2}\sigma_2^2(\gamma_1^+ - \gamma_2^-)(\gamma_1^+ - \gamma_2^+)C^{\gamma_1^+} \leq 0. \tag{64}$$

Since $\mu_2 < \mu_1$, $\frac{(1+\lambda_2)(\mu_2-\mu_1)C}{r-\mu_1} < 0$ and decreases with C. It then follows from (62), (63), and (64) that $L_2 S_0 + 1_{C<\theta}(1+\lambda_2)(C-\theta)$ decreases with C. Since $L_2 S_0 + (1+\lambda_2)(C-\theta)|_{C=C_B(0)+} \leq 0$ by (55), we see that

$$L_2 S_0 + (1+\lambda_2)(C-\theta) \leq 0 \text{ for all } \theta > C \geq C_B(0).$$

Case 2: $\gamma_1^+ \leq \gamma_2^+$

In this case,

$$A_3 \frac{1}{2}\sigma_2^2(\gamma_1^+ - \gamma_2^-)(\gamma_1^+ - \gamma_2^+)C^{\gamma_1^+} \geq 0. \tag{65}$$

By (62), (63), (65), and (51), the function $L_2 S_0 + 1_{C<\theta}(1+\lambda_2)(C-\theta)$ tends to ∞ as $C \longrightarrow 0$ and as $C \longrightarrow \infty$ and has one unique global minimum that must be negative by (55). We will show that the function is decreasing at $C = \theta$ that, by condition (55) would imply that it is negative in the region $\theta > C \geq C_B(0)$.

Observe that, for $C > \theta$

$$\frac{d}{dC}[L_1 S_0 + (1-\lambda_1)(C-\theta)] = \frac{1}{2}\sigma_1^2 C^2 S_0^{'''} + \sigma_1^2 C S_0^{''} + \mu_1 S_0^{'} + \mu_1 C S_0^{''} - r S_0^{'} + (1-\lambda_1) = 0.$$

For $C < \theta$

$$\frac{d}{dC}[L_1 S_0 + (1+\lambda_2)(C-\theta)] = \frac{1}{2}\sigma_1^2 C^2 S_0^{'''} + \sigma_1^2 C S_0^{''} + \mu_1 S_0^{'} + \mu_1 C S_0^{''} - r S_0^{'} + (1+\lambda_2) = 0.$$

Since S_0 is twice differentiable at $C = \theta$, we have

$$\frac{1}{2}\sigma_1^2 C^2 S_0'''(\theta+) - \frac{1}{2}\sigma_1^2 C^2 S_0'''(\theta-) = \lambda_1 + \lambda_2 > 0.$$

As $\sigma_1 < \sigma_2$,

$$\frac{1}{2}\sigma_2^2 C^2 S_0'''(\theta+) - \frac{1}{2}\sigma_2^2 C^2 S_0'''(\theta-) > \lambda_1 + \lambda_2. \tag{66}$$

Now note that, by (66),

$$\begin{aligned} & \frac{d}{dC}[L_2 S_0 + (1-\lambda_1)(C-\theta)]|_{C=\theta+} - \frac{d}{dC}[L_2 S_0 + (1+\lambda_2)(C-\theta)]|_{C=\theta-} \\ = \; & \frac{1}{2}\sigma_2^2 C^2 S_0'''(\theta+) - \frac{1}{2}\sigma_2^2 C^2 S_0'''(\theta-) - (\lambda_1 + \lambda_2) > 0. \end{aligned} \tag{67}$$

By (57), for $C > \theta$,

$$L_2 S_0 + (1-\lambda_1)(C-\theta) = A_1 \frac{1}{2}\sigma_2^2 (\gamma_1^- - \gamma_2^-)(\gamma_1^- - \gamma_2^+) C^{\gamma_1^-} + \frac{(1-\lambda_1)(\mu_2 - \mu_1)C}{r - \mu_1}.$$

By (52), (58), and because $\mu_2 < \mu_1$, the R.H.S. above decreases with C and, in particular, is decreasing at $C = \theta +$. By (67), therefore, $\frac{d}{dC}[L_2 S_0 + (1+\lambda_2)(C-\theta)]|_{C=\theta-} < 0$. Consequently,

$$L_2 S_0 + (1+\lambda_2)(C-\theta) \leq 0 \text{ for all } \theta > C \geq C_B(\theta).$$

It remains to show that

$$L_2 S_0 + (1-\lambda_1)(C-\theta) < 0 \text{ for all } C > \theta.$$

Because S_0 is twice differentiable at $C = \theta$, it follows from the above that

$$L_2 S_0 + (1-\lambda_1)(C-\theta)|_{C=\theta+} < 0$$

We have already shown that $L_2 S_0 + (1-\lambda_1)(C-\theta)$ is decreasing for $C > \theta$. It follows from the above that it must be negative for $C > \theta$.

By Proposition 8, therefore, choosing project 1 throughout is optimal for the bank. Q.E.D.

The following proposition establishes a *necessary and sufficient* condition for the optimal switching trigger to exceed the debt level.

Proposition 11 *[Optimal Switching Trigger Greater than Debt Level] Suppose that*

$$L_1 S_\theta|_{C=\theta-} < 0, \tag{68}$$

where S_θ is the equity value function corresponding to the policy where the manager chooses project 2 for $C < \theta$ and project 1 for $C \geq \theta$. There exists q^* with $\theta < q^* < \infty$ such that the policy of switching projects at q^* is optimal for the bank and S_{q^*} is the corresponding equity value function.

Proof. We split the proof into several steps.

Step 1. Let S_∞ be the policy of always choosing project 2. It follows using arguments similar

to those used to prove Proposition 1 that

$$S_\infty(C) = A_\infty C^{\gamma_2^-} + \frac{(1-\lambda_1)C}{r-\mu_2} - \frac{\theta}{r} \text{ for } C > \theta. \tag{69}$$

Further, because equity holders are protected by limited liability, $S_\infty(C) > \frac{(1-\lambda_1)C}{r-\mu_2} - \frac{\theta}{r}$ so that $A_\infty > 0$. Next,

$$L_1 S_\infty(C) + (1-\lambda_1)(C-\theta) = \frac{1}{2}\sigma_1^2 A_\infty(\gamma_2^- - \gamma_1^-)(\gamma_2^- - \gamma_1^+)C^{\gamma_2^-} + \frac{(1-\lambda_1)C(\mu_1-\mu_2)}{r-\mu_2}$$

Because, $\mu_1 > \mu_2$ and $\gamma_2^- < 0$,

$$L_1 S_\infty(C) + (1-\lambda_1)(C-\theta) \longrightarrow \infty \text{ as } C \longrightarrow \infty. \tag{70}$$

Step 2. The function

$$\Gamma(q) = L_1 S_q + (1_{C\geq\theta}(1-\lambda_1)(C-\theta) + 1_{C<\theta}(1+\lambda_2)(C-\theta))\,|_{C=q-}$$

is a continuous function of q. By (70), $\Gamma(q) \longrightarrow \infty$ as $q \longrightarrow \infty$. By (68), there exists $q^* \in (\theta, \infty)$ such that

$$L_1 S_{q^*} + (1-\lambda_1)(C-\theta)|_{q^*-} = 0. \tag{71}$$

Since S_{q^*} is the equity value function corresponding to the policy of choosing project 2 for $C < q^*$ and project 1 for $C > q^*$,

$$\begin{aligned} L_1 S_{q^*} + (1-\lambda_1)(C-\theta)|_{q^*+} &= 0, & (72)\\ L_2 S_{q^*} + (1-\lambda_1)(C-\theta)|_{q^*-} &= 0. & (73) \end{aligned}$$

Subtracting (71) from (72), and using the fact that S_{q^*} is differentiable at q^* , we see that

$$\frac{d^2 S_{q^*}}{dC^2}|_{q=q^*+} = \frac{d^2 S_{q^*}}{dC^2}|_{q=q^*-}. \tag{74}$$

By (73), (73), and the fact that S_{q^*} is differentiable at q^*, see that

$$L_2 S_{q^*} + (1-\lambda_1)(C-\theta)|_{q^*+} = 0. \tag{75}$$

Step 3. We show that q^* is the optimal switching trigger. By Proposition 8, we need to show that

$$\begin{aligned} L_2 S_{q^*} + (1-\lambda_1)(C-\theta) &\leq 0 \text{ for } C > q^* \\ L_1 S_{q^*} + (1-\lambda_1)(C-\theta) &\leq 0 \text{ for } q^* \geq C > \theta \\ L_1 S_{q^*} + (1+\lambda_2)(C-\theta) &\leq 0 \text{ for } \theta \geq C > C_B(q^*). & (76) \end{aligned}$$

By (21), for $C > q^*$,

$$L_2 S_{q^*} + (1-\lambda_1)(C-\theta) = \frac{1}{2}\sigma_2^2 A_1 C^{\gamma_1^-}(\gamma_1^- - \gamma_2^-)(\gamma_1^- - \gamma_2^+) + \frac{(1-\lambda_1)(\mu_2-\mu_1)C}{r-\mu_1}.$$

Since $\mu_2 < \mu_1$, the second term on the R.H.S. above is negative. By (52), we must have $A_1 > 0$ for (72) to hold. In this case, however, the R.H.S. of the above is a strictly decreasing function of C. The first condition in (76) then follows from (72).

Step 4. We now show that the second condition in (76) holds. Since S_{q^*} is differentiable at q^*,

$$\begin{aligned} A_1 (q^*)^{\gamma_1^-} + \frac{(1-\lambda_1)q^*}{r-\mu_1} - \frac{(1-\lambda_1)\theta}{r} &= A_2 (q^*)^{\gamma_2^-} + A_3 (q^*)^{\gamma_2^+} + \frac{(1-\lambda_1)q^*}{r-\mu_2} - \frac{(1-\lambda_1)\theta}{r}, \\ \gamma_1^- A_1 (q^*)^{\gamma_1^-} + \frac{(1-\lambda_1)\theta}{r-\mu_1} &= \gamma_1^- A_2 (q^*)^{\gamma_2^-} + \gamma_1^+ A_3 (q^*)^{\gamma_2^+} + \frac{(1-\lambda_1)q^*}{r-\mu_2}. \end{aligned} \tag{77}$$

After some algebra, we obtain

$$\left(\gamma_2^+ - \gamma_2^-\right) A_1 (q^*)^{\gamma_1^-} + \frac{(\gamma_2^+ - 1)(1-\lambda_1)C(\mu_1-\mu_2)}{(r-\mu_1)(r-\mu_2)} = \left(\gamma_2^+ - \gamma_2^-\right) A_2 (q^*)^{\gamma_2^-} .$$

Since $\gamma_2^+ - 1 > 0$ by (51) and $\mu_1 > \mu_2$, it follows from our earlier results that $A_1 > 0$ that

$$A_2 > 0. \tag{78}$$

For $\theta < C < q^*$,

$$\begin{aligned} L_1 S_{q^*} + (1-\lambda_1)(C-\theta) &= \overbrace{A_2 \frac{1}{2}\sigma_1^2(\gamma_2^- - \gamma_1^-)(\gamma_2^- - \gamma_1^+)C^{\gamma_2^-}}^{<0} + A_3 \frac{1}{2}\sigma_2^2(\gamma_2^+ - \gamma_1^-)(\gamma_2^+ - \gamma_1^+)C^{\gamma_1^+} \\ &\quad + \frac{(1-\lambda_1)C(\mu_1-\mu_2)}{r-\mu_2} \end{aligned}$$

The first term on the R.H.S. above is negative by (52) and (78). The third term is positive because $\mu_1 > \mu_2$. There are two cases to consider.

Case 1. Suppose that the second term on the R.H.S. above is positive.

It is then easy to see that the entire expression on the R.H.S. is increasing. It follows from (71) that the second condition in (76) holds.

Case 2. Suppose the second term on the on the R.H.S. above is negative. In this case, it follows from (51) that the expression tends to $-\infty$ as $C \longrightarrow 0$, to $-\infty$ as $C \longrightarrow \infty$, and has a unique local (and global) maximum. If we show that $L_1 S_{q^*} + (1-\lambda_1)(C-\theta)$ is increasing to the left of q^*, it will follow that it is negative for $\theta < C < q^*$. We proceed as follows.

By the arguments in *Step 3,*

$$\frac{d}{dC}\left[L_2 S_{q^*} + (1-\lambda_1)(C-\theta)\right]|_{C=q^*+} < 0 \tag{79}$$

Since $L_2 S_{q^*} + (1-\lambda_1)(C-\theta) = 0$ for $\theta < C < q^*$,

$$\frac{d}{dC}\left[L_2 S_{q^*} + (1-\lambda_1)(C-\theta)\right]|_{C=q^*-} = 0. \tag{80}$$

Subtracting (80) from (79), evaluating the derivatives, and using the fact that S_{q^*} is twice

differentiable at q^* by (74), we conclude that

$$\frac{d^3}{dC^3}\left(S_{q^*}\right)|_{C=q^*+} - \frac{d^3}{dC^3}\left(S_{q^*}\right)|_{C=q^*-} < 0. \tag{81}$$

Next, we note that, because $L_1 S_{q^*} + (1-\lambda_1)(C-\theta) = 0$ for $C > q^*$,

$$\frac{d}{dC}\left[L_1 S_{q^*} + (1-\lambda_1)(C-\theta)\right]|_{C=q^*+} = 0. \tag{82}$$

From the above, (81), and the twice differentiability of S_{q^*} at q^*, we see that

$$\frac{d}{dC}\left[L_1 S_{q^*} + (1-\lambda_1)(C-\theta)\right]|_{C=q^*-} > 0,$$

which is what we wanted to prove. It follows from (71) that $L_1 S_{q^*} + (1-\lambda_1)(C-\theta)$ is negative and increasing for $\theta < C < q^*$. This establishes the second condition in (76).

Step 5. We now show that the third condition in (76) holds.

Since S_{q^*} is differentiable at θ,

$$\begin{aligned} A_2\theta^{\gamma_2^-} + A_3\theta^{\gamma_2^+} + \frac{(1-\lambda_1)\theta}{r-\mu_2} - \frac{(1-\lambda_1)\theta}{r} &= A_4\theta^{\gamma_2^-} + A_5\theta^{\gamma_2^+} + \frac{(1+\lambda_2)\theta}{r-\mu_2} - \frac{(1+\lambda_2)\theta}{r}, \\ \gamma_2^- A_2\theta^{\gamma_2^-} + \gamma_2^+ A_3\theta^{\gamma_2^+} + \frac{(1-\lambda_1)\theta}{r-\mu_2} &= \gamma_2^- A_4\theta^{\gamma_2^-} + \gamma_2^+ A_5\theta^{\gamma_2^+} + \frac{(1+\lambda_2)\theta}{r-\mu_2}. \end{aligned} \tag{83}$$

After some algebra, we can show that

$$\left(\gamma_2^+ - \gamma_2^-\right) A_2\theta^{\gamma_2^-} + (\lambda_1+\lambda_2)\theta\left[\frac{r-\gamma_2^+\mu_2}{r(r-\mu_2)}\right] = \left(\gamma_2^+ - \gamma_2^-\right) A_4\theta^{\gamma_2^-}.$$

The first term on the L.H.S. above is positive by (78). The second term is also positive by (54). Consequently,

$$A_4 > 0. \tag{84}$$

For $C < \theta$,

$$\begin{aligned} L_1 S_{q^*} + (1+\lambda_2)(C-\theta) &= \overbrace{A_4\frac{1}{2}\sigma_1^2(\gamma_2^- - \gamma_1^-)(\gamma_2^- - \gamma_1^+)C^{\gamma_2^-}}^{<0} + A_5\frac{1}{2}\sigma_2^2(\gamma_2^+ - \gamma_1^-)(\gamma_2^+ - \gamma_1^+)C^{\gamma_1^+} \\ &\quad + \frac{(1+\lambda_2)C(\mu_1-\mu_2)}{r-\mu_2} \end{aligned}$$

The first term on the R.H.S. above is negative by (52) and (84). The third term is positive because $\mu_1 > \mu_2$. There are again two cases to consider.

Case 1. Suppose that the second term on the R.H.S. above is positive.

It is then easy to see that the entire expression on the R.H.S. is increasing. By the arguments in Step 4,

$$L_1 S_{q^*} + (1+\lambda_2)(C-\theta)|_{C=\theta+} < 0.$$

Since S_{q^*} is twice differentiable at $C = \theta$,

$$L_1 S_{q^*} + (1 + \lambda_2)(C - \theta)|_{C=\theta-} < 0. \tag{85}$$

Since $L_1 S_{q^*} + (1 + \lambda_2)(C - \theta)$ is increasing for $C < \theta$, it is negative for $C < \theta$.

Case 2. Suppose the second term on the on the R.H.S. above is negative. In this case, it follows from (51) that the expression tends to $-\infty$ as $C \longrightarrow 0$, to $-\infty$ as $C \longrightarrow \infty$, and has a unique local (and global) maximum. If we show that $L_1 S_{q^*} + (1 - \lambda_1)(C - \theta)$ is increasing to the left of θ, it will follow that it is negative for $C < \theta$. We proceed as follows.

First, note that, because

$$\begin{aligned} L_2 S_{q^*} + (1 - \lambda_1)(C - \theta) &= 0 \text{ for } C > \theta, \\ L_2 S_{q^*} + (1 + \lambda_2)(C - \theta) &= 0 \text{ for } C < \theta, \end{aligned}$$

$$\frac{d}{dC} L_2 S_{q^*}|_{C=\theta+} - \frac{d}{dC} L_2 S_{q^*}|_{C=\theta-} = \lambda_1 + \lambda_2 > 0.$$

Since S_{q^*} is twice differentiable at θ,

$$\frac{d}{dC} L_2 S_{q^*}|_{C=\theta+} - \frac{d}{dC} L_2 S_{q^*}|_{C=\theta-} = \frac{1}{2}\sigma_2^2 \left[\frac{d^3}{dC^3} \left(S_{q^*}\right)|_{C=\theta+} - \frac{d^3}{dC^3} \left(S_{q^*}\right)|_{C=\theta-} \right] > 0$$

Since $\sigma_2 > \sigma_1$

$$\begin{aligned} \frac{1}{2}\sigma_2^2 \left[\frac{d^3}{dC^3} \left(S_{q^*}\right)|_{C=\theta+} - \frac{d^3}{dC^3} \left(S_{q^*}\right)|_{C=\theta-} \right] &> \frac{1}{2}\sigma_1^2 \left[\frac{d^3}{dC^3} \left(S_{q^*}\right)|_{C=\theta+} - \frac{d^3}{dC^3} \left(S_{q^*}\right)|_{C=\theta-} \right] \\ &= \frac{d}{dC} L_1 S_{q^*}|_{C=\theta+} - \frac{d}{dC} L_1 S_{q^*}|_{C=\theta-} \end{aligned}$$

It follows from the above that

$$\frac{d}{dC} L_1 S_{q^*}|_{C=\theta+} - \frac{d}{dC} L_1 S_{q^*}|_{C=\theta-} - (\lambda_1 + \lambda_2) < 0$$

By the results of Step 4, $\frac{d}{dC}\left[L_1 S_{q^*} + (1 - \lambda_1)(C - \theta)\right]|_{C=\theta+} > 0$. It follows from the above that

$$\frac{d}{dC}\left[L_1 S_{q^*} + (1 + \lambda_2)(C - \theta)\right]|_{C=\theta-} > 0,$$

which is exactly what we wanted to prove. It follows from (85) that $L_1 S_{q^*} + (1 + \lambda_2)(C - \theta) < 0$ for $C < \theta$.

In conclusion, we have shown that all three conditions of (76) hold and, moreover, the inequalities are strict. Consequently, by Proposition 8, the policy of switching projects at q^* is the unique optimal policy for the bank. Q.E.D.

The following proposition completes the remaining step in the proof of the theorem by establishing conditions under which the optimal switching trigger is less than the debt level.

Proposition 12 *[Optimal Switching Trigger Less than Debt Level] Suppose that*

$$L_1 S_\theta|_{C=\theta-} \geq 0, L_2 S_0 + (1+\lambda_2)(C-\theta)|_{C=C_B(0)+} > 0. \tag{86}$$

There exists $q^* \leq \theta$ such that the policy of switching projects at q^* is optimal for the bank and S_{q^*} is the corresponding equity value function.

Proof. We again split the proof up into several steps.

Step 1. Suppose first that $L_1 S_\theta|_{C=\theta-} > 0$. By (86), It follows as a special case of Proposition 8 that, within the sub-class of policies characterized by a single switching trigger that is less than θ, the policy of always choosing project 1 as well as the policy of switching at $C = \theta$ are both sub-optimal. By the continuity of the bank's objective function in the switching trigger, it follows that there exists $q^* < \theta$ such that the policy of switching projects at q^* is optimal within the restricted sub-class of policies characterized by a single switching trigger. We will show that the policy of switching projects at q^* is, in fact, *globally optimal* among all possible dynamic project choice policies.

Step 2. We now show that

$$L_1 S_{q^*} + (1+\lambda_2)(C-\theta)|_{q^*-} = L_2 S_{q^*} + (1+\lambda_2)(C-\theta)|_{q^*+} = 0. \tag{87}$$

Suppose to the contrary that $L_1 S_{q^*} + (1+\lambda_2)(C-\theta)|_{q^*-} < 0$. We can use Ito's lemma to show that there exists $q^{**} > q^*$ such that the value of the policy of switching projects at q^{**} is greater than the corresponding value for switching projects at q^*. Suppose, on the other hand, that $L_1 S_{q^*} + (1+\lambda_2)(C-\theta)|_{q^*-} > 0$. In this case, we can use Ito's lemma to show that there exists $q^{**} < q^*$ such that the value of the policy of switching projects at q^{**} is greater than the corresponding value for switching projects at q^*. In either case, the policy of switching projects at q^* is suboptimal within the sub-class of policies characterized by a single switching trigger, which is a contradiction. Hence, $L_1 S_{q^*} + (1+\lambda_2)(C-\theta)|_{q^*-} = 0$. Since the bank chooses project 1 for $C > q^*$, $L_1 S_{q^*} + (1+\lambda_2)(C-\theta)|_{q^*+} = 0$. It then immediately follows that $L_2 S_{q^*} + (1+\lambda_2)(C-\theta)|_{q^*+} = 0$. Moreover, the super contact condition (74) holds at q^*, that is, the value function is twice differentiable at q^*.

Step 3. By Proposition 8, we need to show that

$$\begin{aligned} L_2 S_{q^*} + (1-\lambda_1)(C-\theta) &\leq 0 \text{ for } C > \theta \\ L_2 S_{q^*} + (1+\lambda_2)(C-\theta) &\leq 0 \text{ for } \theta \geq C > q^* \\ L_1 S_{q^*} + (1+\lambda_2)(C-\theta) &\leq 0 \text{ for } q^* \geq C > C_B(q^*). \end{aligned} \tag{88}$$

By (23),

$$L_2 S_{q^*} + (1-\lambda_1)(C-\theta) = \frac{1}{2}\sigma_2^2 A_1 C^{\gamma_1^-}(\gamma_1^- - \gamma_2^-)(\gamma_1^- - \gamma_2^+) + \frac{(1-\lambda_1)(\mu_2-\mu_1)C}{r-\mu_1} \tag{89}$$

By the arguments in Step 1, S_{q^*} is strictly greater than the equity value from the policy of always choosing project 1. The latter, in turn, exceeds $\frac{(1-\lambda_1)C}{r-\mu_1}$ because equity is protected by limited liability. Consequently,

$$A_1 > 0. \tag{90}$$

It follows from (52) and the fact that $\mu_1 > \mu_2$ that both terms on the R.H.S. of (89) are

decreasing. Hence, $L_2 S_{q^*} + (1-\lambda_1)(C-\theta)$ decreases for $C > \theta$.

Next, observe that, for $C > \theta$

$$\frac{d}{dC}[L_1 S_{q^*} + (1-\lambda_1)(C-\theta)] = \frac{1}{2}\sigma_1^2 C^2 S_{q^*}^{'''} + \sigma_1^2 C S_{q^*}^{''} + \mu_1 S_{q^*}^{'} + \mu_1 C S_{q^*}^{''} - r S_{q^*}^{'} + (1-\lambda_1) = 0.$$

For $q^* < C < \theta$

$$\frac{d}{dC}[L_1 S_{q^*} + (1+\lambda_2)(C-\theta)] = \frac{1}{2}\sigma_1^2 C^2 S_{q^*}^{'''} + \sigma_1^2 C S_{q^*}^{''} + \mu_1 S_{q^*}^{'} + \mu_1 C S_{q^*}^{''} - r S_{q^*}^{'} + (1+\lambda_2) = 0.$$

Since S_{q^*} is twice differentiable at $C = \theta$, we have

$$\frac{1}{2}\sigma_1^2 C^2 S_{q^*}^{'''}(\theta+) - \frac{1}{2}\sigma_1^2 C^2 S_{q^*}^{'''}(\theta-) = \lambda_1 + \lambda_2 > 0.$$

As $\sigma_1 < \sigma_2$,

$$\frac{1}{2}\sigma_2^2 C^2 S_{q^*}^{'''}(\theta+) - \frac{1}{2}\sigma_2^2 C^2 S_{q^*}^{'''}(\theta-) > \lambda_1 + \lambda_2. \tag{91}$$

Now note that, by (66),

$$\begin{aligned} & \frac{d}{dC}[L_2 S_{q^*} + (1-\lambda_1)(C-\theta)]|_{C=\theta+} - \frac{d}{dC}[L_2 S_{q^*} + (1+\lambda_2)(C-\theta)]|_{C=\theta-} \\ = \; & \frac{1}{2}\sigma_2^2 C^2 S_{q^*}^{'''}(\theta+) - \frac{1}{2}\sigma_2^2 C^2 S_{q^*}^{'''}(\theta-) > 0. \end{aligned} \tag{92}$$

We have already shown earlier that $\frac{d}{dC}[L_2 S_{q^*} + (1-\lambda_1)(C-\theta)]|_{C=\theta+} < 0$. Consequently, $\frac{d}{dC}[L_2 S_{q^*} + (1+\lambda_2)(C-\theta)]|_{C=\theta-} < 0$.

Therefore, we have shown that $L_2 S_{q^*} + 1_{C<\theta}(1+\lambda_2)(C-\theta) + 1_{C\geq\theta}(1+\lambda_2)(C-\theta)$ is strictly decreasing for all $C > q^*$. It then follows from (87) that it must be negative for all $C > q^*$. Hence, we have established the first two conditions in (88). It remains to establish the third condition.

Step 4. By our previous arguments,

$$\frac{d}{dC}[L_2 S_{q^*} + (1+\lambda_2)(C-\theta)]|_{C=q^*+} < 0. \tag{93}$$

Since the bank chooses project 2 for $C < q^*$,

$$\frac{d}{dC}[L_2 S_{q^*} + (1+\lambda_2)(C-\theta)]|_{C=q^*-} = 0. \tag{94}$$

Subtracting (94) from (93), we get

$$S_{q^*}^{'''}(q^*+) - S_{q^*}^{'''}(q^*-) > 0. \tag{95}$$

Next, because the bank chooses project 1 for $C > q^*$,

$$\frac{d}{dC}[L_1 S_{q^*} + (1+\lambda_2)(C-\theta)]|_{C=q^*+} = 0. \tag{96}$$

By (95) and (96),

$$\frac{d}{dC}[L_1 S_{q^*} + (1+\lambda_2)(C-\theta)]|_{C=q^*-} > 0.$$

We can use arguments similar to those we have used previously in the proof to show that the above condition implies that $L_1 S_{q^*} + (1+\lambda_2)(C-\theta)$ is strictly increasing for $C < q^*$. It then follows from (87) that it is strictly negative for $C < q^*$, which establishes the third condition in (88).

Step 5. Suppose first that $L_1 S_\theta|_{C=\theta-} = 0$. We can use arguments very similar to those used above to show that the policy of switching projects at θ is optimal. We omit the arguments for brevity. Q.E.D.

The main theorem follows from Propositions 10, 11 and 12. Q.E.D.

Proof of Proposition 3

By Proposition 4, the equity, debt and bank values for a given switching policy satisfy the following ODE:

$$\frac{1}{2}\sigma_i^2 C^2 \frac{d^2Y}{dC^2} + \mu_i C \frac{dY}{dC} - rY + y(C,\theta) = 0 \text{ when project } i \in \{1,2\} \text{ is chosen,}$$

where $y(C,\theta)$ is the corresponding payout flow to equity, debt, or the bank. Note that $y(C,\theta)$ is homogeneous of degree 1 in C and θ, that is,

$$y(\xi C, \xi\theta) = \xi y(C,\theta).$$

Further, the boundary conditions that determine the solution to the corresponding equity, debt and firm value functions are also homogeneous of degree 1 in C and θ. Consequently, the value function must itself be homogeneous of degree 1 in C and θ. It immediately follows from the proof of Theorem 2 that the optimal switching threshold and insolvency threshold must both scale linearly with θ as stated in the proposition. The constancy of the switching leverage follows from the fact that the optimal switching trigger is proportional to θ, and the debt value and bank value are homogeneous of degree 1 in current cash flow level and the debt level. Q.E.D.

Proof of Theorem 5

As in the proof of Theorem 2, we establish the theorem by stating and proving intermediate propositions. The following proposition states the relevant dynamic programming verification theorem.

Proposition 13 *[Verification Theorem for Regulator's Problem] Let $F_q^{social}(C)$ denote the social value when the current earnings level is C if the regulator follows a switching policy where it switches projects at the earnings level q. Suppose that $F_q^{social}(C)$ satisfies the following HJB equation:*

$$\max_{i\in\{1,2\}} L_i F_q^{social}(C) + \left[1_{C\geq\theta}\left\{(1-\lambda_1^{social})C + \lambda_1^{social}\theta\right\} + 1_{C<\theta}\left\{(1+\lambda_2^{social})C - \lambda_2^{social}\theta\right\}\right] = 0;$$

$$F_q^{social}(C_L) = F_q^{social'}(C_L) = 0, \tag{97}$$

Then $F_q^{social}(C)$ is the optimal social value function among all possible dynamic project choice policies (including non-stationary policies) and q is the optimal switching trigger.

The following proposition establishes the necessary and sufficient condition for the regulator to optimally choose project 1 throughout, that is, engage in no risk-shifting.

Proposition 14 *[No Asset Substitution for Regulator] Suppose that*

$$L_2 F_0^{social} + (1+\lambda_2^{social})C - \lambda_2^{social}\theta|_{C=C_L(0)+} \leq 0, \tag{98}$$

where F_0^{social} is the social value function when the regulator always chooses project 1 and $C_L(0)$ is the corresponding endogenous liquidation level. Then it is optimal for the regulator to always choose project 1, that is, no asset substitution is optimal.

Proof. By (32), the social value when the regulator always chooses project 1 is given by

$$\begin{aligned} F_0^{social}(C) &= X_1 C^{\gamma_1^-} + \frac{(1-\lambda_1^{social})C}{r-\mu_1} + \frac{\lambda_1^{social}\theta}{r} \text{ for } C \geq \theta \\ &= X_2 C^{\gamma_1^-} + X_3 C^{\gamma_1^+} + \frac{(1+\lambda_2^{social})C}{r-\mu_1} - \frac{\lambda_2^{social}\theta}{r} \text{ for } C < \theta. \end{aligned} \tag{99}$$

The social value function in the hypothetical scenario where the bank's assets are never liquidated and/or debt is completely risk-free is $\frac{(1-\lambda_1^{social})C}{r-\mu_1} + \frac{\lambda_1^{social}\theta}{r}$. Since $F_0^{social}(C)$ must clearly be less, we must have

$$X_1 < 0. \tag{100}$$

Next, we note that, for $C > \theta$,

$$L_2 F_0^{social} + (1-\lambda_1^{social})C + \lambda_1^{social}\theta = \frac{1}{2}\sigma_2^2 X_1(\gamma_1^- - \gamma_2^-)(\gamma_1^- - \gamma_2^+)C^{\gamma_1^-} + \frac{(1-\lambda_1^{social})C(\mu_2-\mu_1)}{r-\mu_1}.$$

Because $\gamma_1^- < \gamma_2^-$ by (52), it follows from (100) that the first term on the R.H.S. above is strictly negative. The second term is also strictly negative because $\mu_2 < \mu_1$. Consequently, $L_2 F_0^{social} + (1-\lambda_1^{social})C + \lambda_1^{social}\theta < 0$, which establishes the verification condition for $C > \theta$.

Since the social value function is differentiable at θ,

$$\begin{aligned} X_1\theta^{\gamma_1^-} + \frac{(1-\lambda_1^{social})\theta}{r-\mu_1} + \frac{\lambda_1^{social}\theta}{r} &= X_2\theta^{\gamma_1^-} + X_3\theta^{\gamma_1^+} + \frac{(1+\lambda_2^{social})\theta}{r-\mu_1} - \frac{\lambda_2^{social}\theta}{r} \\ \gamma_1^- X_1\theta^{\gamma_1^-} + \frac{(1-\lambda_1^{social})\theta}{r-\mu_1} &= \gamma_1^- X_2\theta^{\gamma_1^-} + \gamma_1^+ X_3\theta^{\gamma_1^+} + \frac{(1+\lambda_2^{social})\theta}{r-\mu_1} \end{aligned}$$

From the above, we obtain

$$(\gamma_1^+ - \gamma_1^-)X_3\theta^{\gamma_1^+} = (\lambda_1^{social} + \lambda_2^{social})\theta\left[\frac{\gamma_1^- - 1}{r-\mu_1} - \frac{\gamma_1^-}{r}\right] = (\lambda_1^{social} + \lambda_2^{social})\theta\frac{\mu_1\gamma_1^- - r}{r(r-\mu_1)}.$$

By (53), and the fact that $\gamma_1^+ > \gamma_1^-$, the above implies that

$$X_3 < 0. \tag{101}$$

Next, note that, for $C < \theta$,

$$L_2 F_0^{social} + (1+\lambda_2^{social})C - \lambda_2^{social}\theta = \frac{1}{2}\sigma_2^2 X_2(\gamma_1^- - \gamma_2^-)(\gamma_1^- - \gamma_2^+)C^{\gamma_1^-} + \frac{1}{2}\sigma_2^2 X_3(\gamma_1^+ - \gamma_2^-)(\gamma_1^+ - \gamma_2^+)C^{\gamma_1^+} + \frac{(1-\lambda_1^{social})C(\mu_2-\mu_1)}{r-\mu_1}. \quad (102)$$

Because F_0^{social} is twice differentiable at $C = \theta$, it follows from our earlier result that $L_2 F_0^{social} + (1-\lambda_1^{social})C + \lambda_1^{social}\theta < 0$ for $C > 0$ that

$$L_2 F_0^{social} + (1+\lambda_2^{social})C - \lambda_2^{social}\theta|_{C=\theta-} = L_2 F_0^{social} + (1-\lambda_1^{social})C + \lambda_1^{social}\theta|_{C=\theta+} < 0 \quad (103)$$

We need to consider two cases.

Case 1: $\gamma_1^+ \geq \gamma_2^+$.

In this case, the second term on the right hand side of (102) is non-positive by (101). The third term is negative because $\mu_2 < \mu_1$. If $X_2 \leq 0$, then the first term is non-positive by (52) so that the entire expression on the R.H.S. of (102) is negative for $C < \theta$. If $X_2 > 0$, then the expression on the R.H.S. of (102) is decreasing with C. In this case, condition (98) implies that it is again negative for $C_L(0) < C < \theta$.

Case 2: $\gamma_1^+ < \gamma_2^+$

In this case, the second term on the right hand side of (102) is positive by (101). If $X_2 < 0$, then the first term is positive by (52). The function on the R.H.S. of (102) tends to ∞ as $C \longrightarrow 0$ and as $C \longrightarrow \infty$ and has a unique local (and global) minimum that is strictly negative by (98). Condition (98) and (103) together imply that $L_2 F_0^{social} + (1+\lambda_2^{social})C - \lambda_2^{social}\theta$ must, in fact, be negative for $C_L(0) < C < \theta$.

In summary, we have shown that F_0^{social} satisfies the HJB equation. Moreover, $L_2 F_q^{social}(C) + \left[1_{C\geq\theta}\left\{(1-\lambda_1^{social})C + \lambda_1^{social}\theta\right\} + 1_{C<\theta}\left\{(1+\lambda_2^{social})C - \lambda_2^{social}\theta\right\}\right] < 0$ for $C_L(0) < C < \infty$, which implies that choosing project 1 always is the unique optimal policy for the regulator. Q.E.D.

The following proposition shows that, if condition (98) does not hold, the regulator optimally switches projects at a trigger that is below the debt level.

Proposition 15 *[Optimal Switching Trigger] Suppose that*

$$L_2 F_0^{social} + (1+\lambda_2^{social})C - \lambda_2^{social}\theta|_{C=C_L(0)+} > 0. \quad (104)$$

There exists $q^* < \theta$ such that it is optimal for the regulator to choose project 2 for $C < q^*$, and project 1 for $C \geq q^*$.

Proof. We split the proof into several steps.

Step 1. Consider the policy where the switching trigger is equal to θ. By (31), the social value function for $C > \theta$ has the form

$$F_\theta^{social}(C) = X_1 C^{\gamma_1^-} + \frac{(1-\lambda_1^{social})C}{r-\mu_1} + \frac{\lambda_1^{social}\theta}{r}.$$

By the same argument used in the proof of Proposition 14,

$$X_1 < 0. \quad (105)$$

Consequently, for $C > \theta$,

$$L_2 F_\theta^{social} + (1-\lambda_1^{social})C + \lambda_1^{social}\theta = \frac{1}{2}\sigma_2^2 X_1(\gamma_1^- - \gamma_2^-)(\gamma_1^- - \gamma_2^+)C^{\gamma_1^-} + \frac{(1-\lambda_1^{social})C(\mu_2-\mu_1)}{r-\mu_1} < 0, \tag{106}$$

so that

$$L_2 F_\theta^{social} + (1-\lambda_1^{social})C + \lambda_1^{social}\theta|_{C=\theta+} < 0. \tag{107}$$

We know that, because project 2 is chosen for $C < \theta$, and project 1 for $C > \theta$,

$$\begin{aligned} L_2 F_\theta^{social} + (1+\lambda_2^{social})C - \lambda_2^{social}\theta|_{C=\theta-} &= 0, \\ L_1 F_\theta^{social} + (1-\lambda_1)C + \lambda_1\theta|_{C=\theta+} &= 0. \end{aligned}$$

From the above equations and (107), we obtain

$$L_1 F_\theta^{social} + (1+\lambda_2^{social})C - \lambda_2^{social}\theta|_{C=\theta-} > 0. \tag{108}$$

By (104) and (108), the policy of always choosing project 1, and the policy of switching projects at θ are both sub-optimal within the restricted class of "single switching trigger" policies where the switching trigger is less than or equal to θ. By the continuity of the regulator's objective function, there exists $q^* < \theta$ such that the policy of switching projects at q^* is optimal within the restricted class of "single switching trigger" policies. We will show that the policy is, in fact, *globally optimal* as in the proof of Proposition 12.

Step 2. By arguments similar to those used in Step 2 of the proof of Proposition 12, we must have

$$L_2 F_{q^*}^{social} + (1+\lambda_2^{social})C - \lambda_2^{social}\theta|_{C=q^*+} = 0, \tag{109}$$

$$L_1 F_{q^*}^{social} + (1+\lambda_2^{social})C - \lambda_2^{social}\theta|_{C=q^*-} = 0. \tag{110}$$

Further, the above imply that $F_{q^*}^{social}$ is twice differentiable at q^*.

Step 3. Using arguments similar to those used in the proof of Proposition 14, we can show that, condition (109) implies that $L_2 F_{q^*}^{social} + (1+\lambda_2^{social})C - \lambda_2^{social}\theta < 0$ for $q^* < C < \theta$.

Step 4. It remains to show that $L_1 F_{q^*}^{social} + (1+\lambda_2^{social})C - \lambda_2^{social}\theta < 0$ for $C_L(q^*) < C < q^*$. By arguments similar to those used in Step 4 of the proof of Proposition 12, we can show that $L_1 F_{q^*}^{social} + (1+\lambda_2^{social})C - \lambda_2^{social}\theta$ is increasing for $C < q^*$. Condition (110), therefore, implies that it is negative for $C < q^*$.

Hence, the function $F_{q^*}^{social}$ satisfies all the conditions of the verification theorem. Moreover, the results that

$$\begin{aligned} L_2 F_{q^*}^{social}(C) + \left[1_{C\geq\theta}\left\{(1-\lambda_1^{social})C + \lambda_1^{social}\theta\right\} + 1_{C<\theta}\left\{(1+\lambda_2^{social})C - \lambda_2^{social}\theta\right\}\right] &< 0 \text{ for } C > q^*, \\ L_1 F_{q^*}^{social}(C) + \left[1_{C\geq\theta}\left\{(1-\lambda_1^{social})C + \lambda_1^{social}\theta\right\} + 1_{C<\theta}\left\{(1+\lambda_2^{social})C - \lambda_2^{social}\theta\right\}\right] &< 0 \text{ for } C < q^*, \end{aligned}$$

together imply that switching projects at q^* is the unique optimal policy for the regulator. Q.E.D.

Proof of Proposition 6

To simplify the notation, let q^* and $\widetilde{q}^*$ denote the optimal switching triggers in the bank's and regulator's problems, respectively. We need to show that $\widetilde{q}^* \leq q^*$.

Step 1. Consider a switching policy described by a trigger q, where $q \geq C_B(q)$, where $C_B(q)$ is the endogenous insolvency threshold corresponding to the switching policy. Define the function

$$G_q = F_q^{social} - S_q \tag{111}$$

By (13) and (15), G satisfies the following system of ODEs:

$$\begin{aligned} L_1 G_q + \theta &= 0 \text{ for } C > q \\ L_2 G_q + \theta &= 0 \text{ for } C_B(q) < C < q \\ L_2 G_q + (1-\lambda_1)C + \lambda_1 \theta &= 0 \text{ for } C_L(q) < C < C_B(q), \end{aligned} \tag{112}$$

where $C_L(q)$ is the endogenous liquidation threshold corresponding to the switching policy characterized by the trigger q. By arguments similar to those used in the proof of Proposition 1, G has the following form for $C > q$:

$$G_q(C) = Y_1 C^{\gamma_1^-} + \frac{\theta}{r}. \tag{113}$$

Since $\frac{\theta}{r}$ is the value of $G_q(C)$ in the hypothetical scenario where debt is completely risk-free and liquidation never occurs, we must have

$$Y_1 < 0. \tag{114}$$

Next, we note that

$$L_2(G_q) + \theta = \frac{1}{2} Y_1 \sigma_2^2 (\gamma_1^- - \gamma_2^-)(\gamma_1^- - \gamma_2^+) C^{\gamma_1^-} < 0 \tag{115}$$

because $Y_1 < 0$ and $\gamma_1^- < \gamma_2^-$ by (52).

Step 2. Suppose that $q^* = C_B(q^*)$, which implies that project 1 is always chosen. By (55),

$$L_2 S_{q^*} + (1+\lambda_2)(C-\theta)|_{C=C_B(q^*)+} \leq 0. \tag{116}$$

An examination of the proof of Theorem 5 reveals that it suffices to show that

$$L_2 F_{q^*}^{social} + (1+\lambda_2)C - \lambda_2 \theta|_{C=C_B(q^*)+} \leq 0$$

to ensure that the regulator also optimally chooses project 1 for all $C \geq C_B(q^*)$, that is, the regulator's optimal switching trigger (if it exists) is less than $C_B(q^*)$. By (111) and (115),

$$\begin{aligned} L_2 F_{q^*}^{social} + (1+\lambda_2)C - \lambda_2 \theta &= L_2 S_{q^*} + (1+\lambda_2)(C-\theta) + \\ L_2(G_{q^*}) + \theta &< 0, \end{aligned}$$

where the last inequality above follows from (116) and (115).

Step 3. Suppose that $q^* > C_B(q^*)$. Since the regulator's optimal switching trigger is less than θ by Theorem 5, it suffices to consider the case where $q^* < \theta$.

By the proof of Theorem 2,

$$L_2 S_{q^*} + (1+\lambda_2)(C-\theta)|_{C=q^*+} = 0 \tag{117}$$

By the proof of Theorem 5, it suffices to show that

$$L_2 F_{q^*}^{social} + (1+\lambda_2)C - \lambda_2\theta|_{C=q^*+} \leq 0.$$

The above, however, immediately follows from (115) and (117). Q.E.D.

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Table I: The Impact of Optimal Prudential Regulation

This table reports the solutions to the case of the unregulated bank and the case where the regulator optimally implements the full set of regulatory tools.

	Unregulated Bank	Full Regulation
Debt Level and Thresholds		
Initial Coupon Level	0.635	0.822
Switching Boundary (Bank)	0.581	0.753
Switching Boundary (Regulator)		0.335
Insolvency Boundary	0.432	0.579
Liquidation Boundary		0.297
Leverage and Regulatory Ratios		
Initial Leverage	64.9%	79.8%
Initial Capital Requirement		20.2%
Switching/Intervention Leverage	90.2%	93.2%
Intervention Ratio		6.8%
Switching Leverage (Regulator)		145.7%
Spread (bp)	68.6	42.2
Ex Ante Values		
Bank Value	23.346	26.251
Social Value	23.346	24.365
Ex Ante Benefit (Bank Value, %)		12.4%
Ex Ante Benefit (Social Value, %)		4.4%
Ex Post Values at Insolvency		
Ex Post Benefit (Bank Value, %)		86.9%
Ex Post Benefit (Social Value, %)		36.3%

Table II: The Impact of Individual Components of Prudential Regulation

This table reports the key regulatory ratios and ex ante benefits in several cases where different sets of regulatory tools are available. Column 1 reports the case of the unregulated bank. Column 2 reports the case where the regulator injects capital when the bank is insolvent and optimally liquidates the bank, but does not intervenes in project choices or impose initial capital requirement. Column 3 reports the case where the regulator's only action is intervention in the bank's project choices. Column 4 reports the case where the regulator employs all regulatory tools (investment intervention, capital injection, and liquidation) except the initial capital requirement. Column 5 reports the case where the regulator optimally employ all regulatory tools.

	Unregulated Bank	Capital Injection Only	Investment Intervention Only	Full Regulation with No Initial Capital Requirement	Full Regulation
Leverage and Regulatory Ratios					
Initial Leverage	64.9%	87%	66.4%	100%	79.8%
Initial Capital Requirement					20.2%
Intervention Ratio		8.0%	9.8%	6.8%	6.8%
Ex Ante Values					
Firm Value	23.346	25.226	23.442	31.133	26.251
Social Value	23.346	23.177	23.442	19.496	24.365
Ex Ante Benefit (Bank Value, %)		8.1%	0.4%	33.4%	12.4%
Ex Ante Benefit (Social Value, %)		-0.7%	0.4%	-16.5%	4.4%

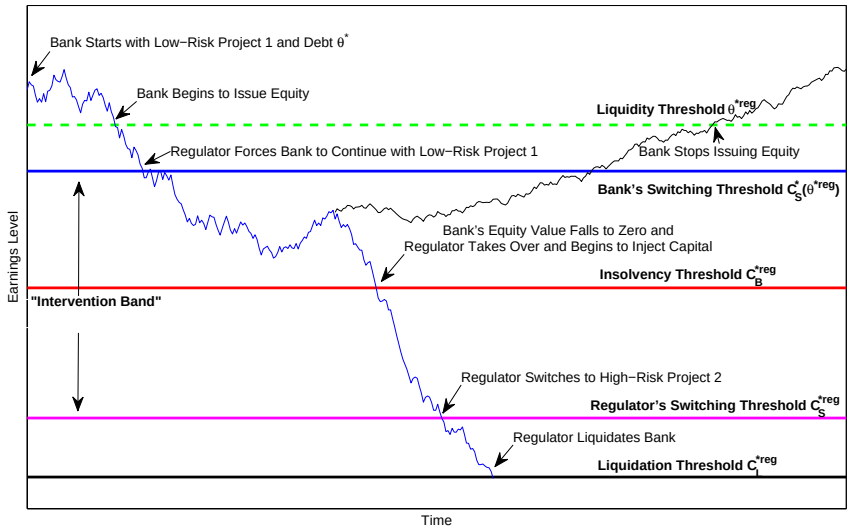

Figure 1: **Optimal Prudential Regulation: An Illustration** This figure plots sample earnings paths and project choices of a fully regulated bank.

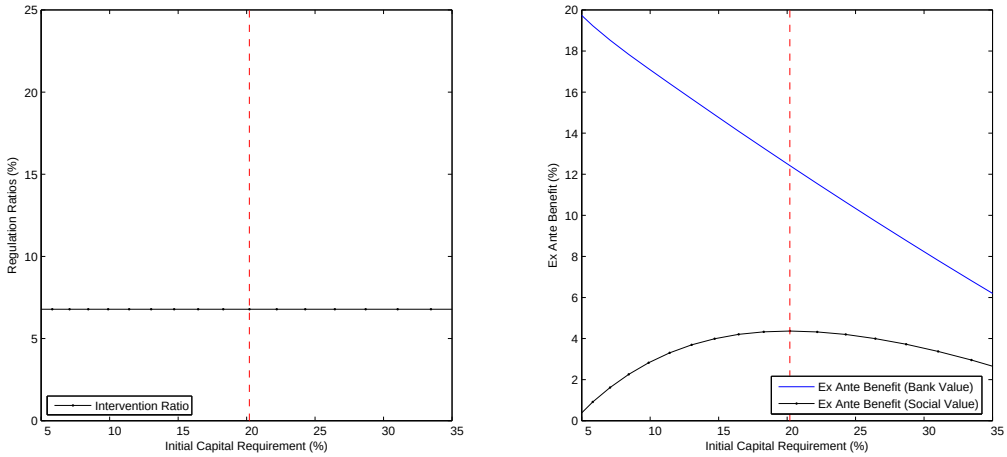

Figure 2: **Impact of Changes in Initial Capital Requirement** This figure plots the changes in optimal regulatory policy and the associated benefits when the initial capital requirement deviates from the optimal level. The model parameters are the baseline parameters. The red dashed lines correspond to the case of optimal regulation.

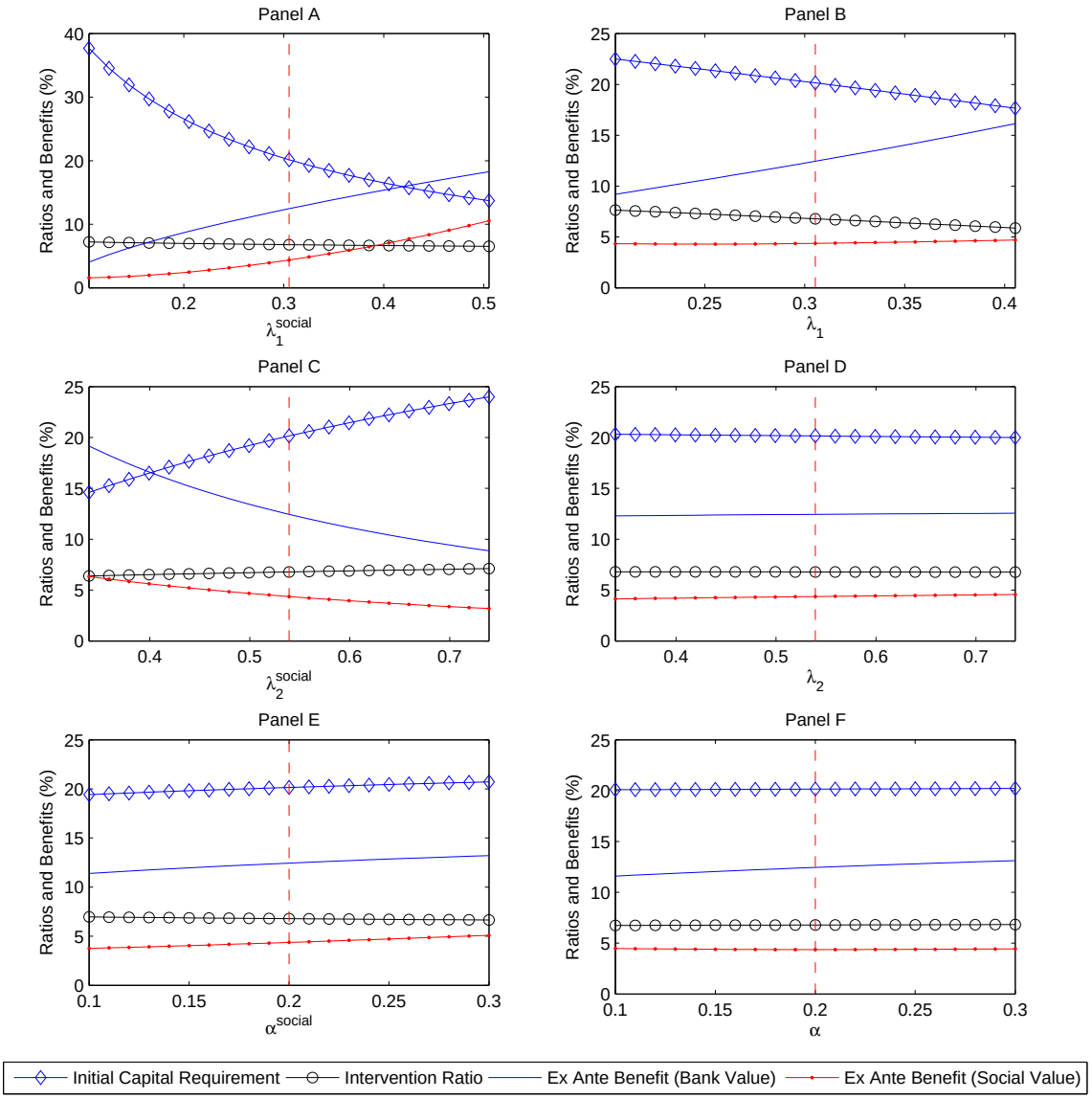

Figure 3: **Optimal Prudential Regulation: Comparative Statics** This figure reports the optimal regulatory ratios and the benefits to the bank and social welfare of optimal regulation under different values of the model's parameters. The red dashed lines correspond to the baseline case.

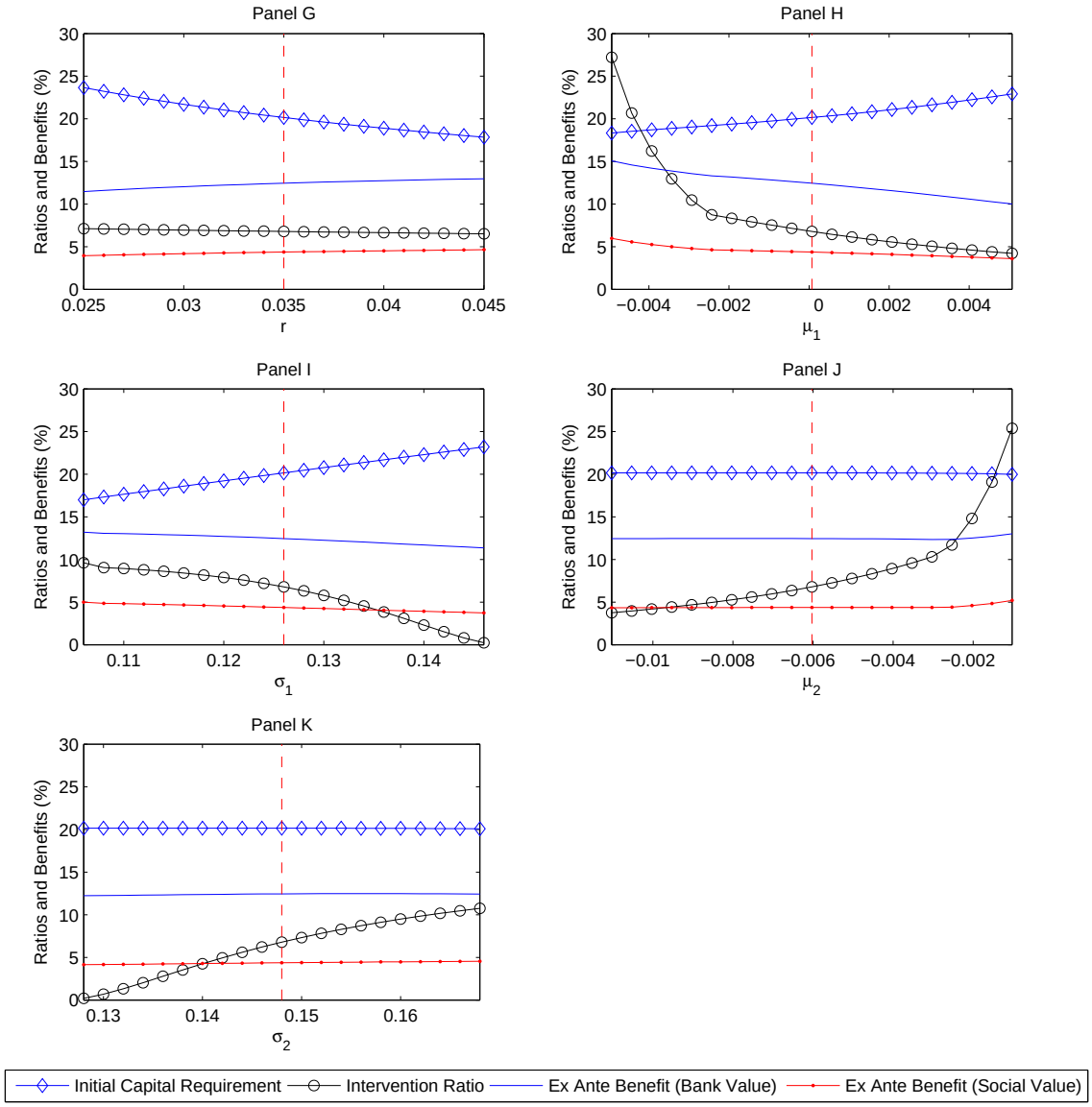

Figure 3: **Optimal Prudential Regulation: Comparative Statics (Continued)**