ASSET PRICES, CREDIT CONSTRAINTS, AND MACROECONOMIC FLUCTUATIONS

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Abstract. We construct and estimate a dynamic stochastic general equilibrium model with heterogenous agents and credit constraints. We argue that a financial multiplier can be effective in transforming economic shocks into the observed fluctuations in aggregate output only if the model can generate large fluctuations in asset prices. Our estimation shows that the housing demand shock in our model not only accounts for most of the observed fluctuations in the land price but also contributes to $35-45\%$ of investment fluctuations at business cycle frequencies. Such dynamic interactions between land prices and investment movements are the key to our finding of the empirically important amplification and propagation operating through endogenous credit constraints.

I. Introduction

If an investing agent is constrained by credit, movements in asset prices can in theory influence the agent's borrowing capacity and, through changes in the borrowing capacity, generate a financial multiplier that is a potentially powerful mechanism for transforming economic shocks into fluctuations in investment and output (Kiyotaki and Moore, 1997). The quantitative importance of such a financial multiplier effect, however, remains an open empirical question. To address this question, we construct

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and estimate a dynamic stochastic general equilibrium (DSGE) model with credit constraints. When fit to U.S. time series data, our model generates empirically important interactions between asset prices and investment movements. We show that such interactions provide significant amplification and propagation effects on investment and output.

The model we study consists of two agents: the representative household and the representative entrepreneur. The household consumes a homogeneous good, land services (housing) and leisure, and supplies labor and loanable funds in the competitive market. The entrepreneur consumes and produces the same homogeneous good. Production of the good requires labor, capital, and land (commercial structures) as inputs. To finance consumption, production, and investment in land and capital, the entrepreneur borrows loanable funds in the competitive market subject to a credit constraint. In particular, the borrowing capacity is constrained by a fraction of the present value of land and the accumulated capital stock. Thus, land and capital serve as both inputs for production and collateral for borrowing.[1]

The credit constraint generates a financial multiplier effect through a two-way feedback between asset prices and aggregate quantities: a decline in asset prices lowers the collateral value and contracts the entrepreneur's borrowing capacity so that investment falls; a fall in investment lowers future output, which depresses the current asset prices and reduces the collateral value. For the financial multiplier effect on output to be empirically important, therefore, an economic shock must move not only the asset price but investment as well.

Our most significant finding is that shocks to housing demand not only account for most of the fluctuations in the land price at business-cycle frequencies but also generate the empirically important interactions between land prices and investment that are necessary to give the financial multiplier effect a life of "its own." Compared to an alternative model in which we turn off the response of the entrepreneur's borrowing capacity to changes in the asset value, the dynamic responses of consumption and investment in our model can be more than three times as large. Thus, the dynamic interactions between asset prices and investment movements are essential to the amplification and propagation working through endogenous credit constraints.

[1]The land in our model can be a proxy for any fixed-supply asset or an asset that grows at a much slower rate than the capital. As Davis and Heathcote (2007) show, land grows at a very slow rate and land prices are the main driving force of housing prices observed in the U.S. We therefore interchange the terms "land" and "housing" in the paper, as does Kocherlakota (2008).

Our results yield important insights that help understand the mechanism through which a financial multiplier can be important. If an economic shock does not generate much of the asset price movements, then it will not be able to generate a strong financial multiplier effect. One such example is the neutral technology shock: the shock moves the current and future dividends as well as the loan rate in the same direction. As a result, it does not move the asset price much. These insights explain why Kocherlakota (2000) and Cordoba and Ripoll (2004) find weak multiplier effects of technology shocks in their calibrated models with credit constraints. Their findings are consistent with our estimated results: while technology shocks can be important sources of fluctuations in output as in a standard real business cycle model, they do not generate large fluctuations in asset prices so that their impact on output does not work through credit constraints.

A housing demand shock, on the other hand, induces the household to demand land and this demand drives up the land price. An increase in the land price in turn expands the entrepreneur's borrowing capacity. These two effects reinforce each other, generating a large impact on land price. As a result, an endogenous expansion or contraction of credit constraints in response to a housing demand shock provides essential room for credit cycles to function in a quantitatively significant way. Our estimates indicate that housing demand shocks alone account for $20 - 35\%$ of output fluctuations and $35 - 45\%$ of investment fluctuations at business-cycle frequencies.

II. Related Literature

The idea that borrowing constraints play a critical role in amplifying business cycles can be traced back at least to Fisher (1933). Our model builds on the recent literature that focuses on the costly contract enforcement problem (i.e., the problem of controlling over assets). Examples in this literature include Kiyotaki and Moore (1997), Kiyotaki (1998), Krishnamurthy (2003), Cordoba and Ripoll (2004), and Iacoviello (2005).[2] In this class of models, the borrowing capacity is constrained by the value of the collateral assets. We follow the literature and assume that the household (creditor) is more patient than the entrepreneur (debtor) so that a positive first-order excess return exists in steady state. Given the excess return, the borrower assigns a positive value to existing loans and borrows up to the limit. The binding credit constraint allows asset prices to interact with the debt level and therefore with investment and output. Such

[2]Open-economy extensions of this class of models include Aoki, Benigno, and Kiyotaki (2007) and Mendoza (2008), among others.

interactions generate a financial multiplier that helps amplify business cycle shocks. In this paper we provide formal evidence on the mechanism through which the financial multiplier takes effect.

Our work is related to another strand of literature that builds on the work by Townsend (1979) and Gale and Hellwig (1985) and focuses on the costly state verification problem caused by asymmetric information between creditors and debtors. Examples includes Carlstrom and Fuerst (1997), Bernanke, Gertler, and Gilchrist (1999), Cooley, Marimon, and Quadrini (2004), Gertler, Gilchrist, and Natalucci (2007), and Christiano, Motto, and Rostagno (2008). This class of models also emphasizes the interactions between asset prices and aggregate quantities. A subtle difference is that, unlike Kiyotaki and Moore (1997), loans in the model of Bernanke, Gertler, and Gilchrist (1999) are priced to take into account default risks and debtors optimally choose the amount of borrowing, taking the loan rate as given. Thus, although agency costs contribute to a positive external financing premium (i.e., the difference between the loan rate and the interest rate controlled by the government), no borrowers are constrained by credit in equilibrium and there is no steady-state excess return relative to the loan rate. This class of models generates a "financial accelerator" effect that is similar to the financial multiplier effect in Kiyotaki and Moore (1997): a decline in the asset price reduces entrepreneurs's net worth and raises the default probability and thus the external finance premium; as the cost of loans rises, entrepreneurs choose to reduce borrowing and investment and this action lowers the current asset price further.

There are few empirical studies of the empirical importance of the financial accelerator effect emphasized by Bernanke, Gertler, and Gilchrist (1999), with the notable exception of Christiano, Motto, and Rostagno (2008), who estimate an expanded version of the Bernanke, Gertler, and Gilchrist (1999) model using the time series data from the United States and the Euro Area. Christiano, Motto, and Rostagno (2008) find some financial shocks that can drive both the fluctuations in the external finance premium and investment. Their empirical work focuses on the dynamic interactions between the spread and investment. The focus of our paper is on the dynamic interactions between the asset price itself and investment. These two approaches reinforce each other, lending support to our view that the key to generating a strong financial multiplier (or accelerator) is to identify some economic shocks that move asset prices.

III. The Model

The economy is populated by two representative agents: the household and the entrepreneur. There are four types of commodities: labor, goods, land, and loanable bonds. Goods production requires labor, capital, and land as inputs. The output can be used for consumption (by both agents) and for capital investment (by the entrepreneur). The household's utility depends on consumption goods, land services (housing), and leisure; the entrepreneur's utility depends on consumption goods only.

III.1. **The representative household.** Similar to Iacoviello (2005), the household's preference is represented by the utility function

$$\mathrm{E}\sum_{t=0}^{\infty}\beta^t A_t\left\{\log(C_{ht}-\gamma_h C_{h,t-1})+\varphi_t\log L_{ht}-\psi_t N_{ht}\right\}, \tag{1}$$

where C_{ht} denotes consumption, L_{ht} denotes land holdings, and N_{ht} denotes labor hours. The parameter $\beta \in (0,1)$ is a subjective discount factor, the parameter γ_h measures the degree of habit persistence, and the term E is a mathematical expectation operator. The terms A_t, φ_t, and ψ_t are preference shocks. We assume that the intertemporal preference shock A_t follows the stochastic process

$$A_t = A_{t-1}(1+\lambda_{at}), \quad \ln\lambda_{at} = (1-\rho_a)\ln\bar{\lambda}_a + \rho_a\ln\lambda_{a,t-1} + \varepsilon_{at}, \tag{2}$$

where $\bar{\lambda}_a > 0$ is a constant, $\rho_a \in (-1,1)$ is the persistence parameter, and ε_{at} is an identically and independently distributed (i.i.d.) white noise process with mean zero and variance σ_a^2. The housing preference shock φ_t follows the stationary process

$$\ln\varphi_t = (1-\rho_\varphi)\ln\bar{\varphi} + \rho_\varphi\ln\varphi_{t-1} + \varepsilon_{\varphi t}, \tag{3}$$

where $\bar{\varphi} > 0$ is a constant, $\rho_\varphi \in (-1,1)$ measures the persistence of the shock, and $\varepsilon_{\varphi t}$ is a white noise process with mean zero and variance σ_φ^2. The labor supply shock ψ_t follows the stationary process

$$\ln\psi_t = (1-\rho_\psi)\ln\bar{\psi} + \rho_\psi\ln\psi_{t-1} + \varepsilon_{\psi t}, \tag{4}$$

where $\bar{\varphi} > 0$ is a constant, $\rho_\psi \in (-1,1)$ measures the persistence, and $\varepsilon_{\psi t})$ is a white noise process with mean zero and variance σ_ψ^2.

Denote by q_{lt} the relative price of housing (in consumption units), R_t the gross real loan rate, and w_t the real wage; denote by S_t the household's purchase in period t of the loanable bond that pays off one unit of consumption good in all states of nature in period $t+1$. In period 0, the household begins with $L_{h0} > 0$ units of housing and

$S_0 > 0$ units of the loanable bond. The flow of funds constraint for the household is given by

$$C_{ht} + q_{lt}(L_{ht} - L_{h,t-1}) + \frac{S_t}{R_t} \leq w_t N_{ht} + S_{t-1}. \tag{5}$$

The household chooses C_{ht}, $L_{h,t}$, N_{ht}, and S_t to maximize (1) subject to (5) and the borrowing constraint $S_t \geq -\bar{S}$ for some large number $\bar{S}$.

III.2. **The representative entrepreneur.** The entrepreneur has the utility function

$$\mathrm{E}\sum_{t=0}^{\infty} \beta^t \left[\log(C_{et} - \gamma_e C_{e,t-1})\right], \tag{6}$$

where C_{et} denotes the entrepreneur's consumption and γ_e is the habit persistence parameter.

The entrepreneur produces goods using capital, labor, and land as inputs. The production function is given by

$$Y_t = Z_t [L_{e,t-1}^{\phi} K_{t-1}^{1-\phi}]^{\alpha} N_{et}^{1-\alpha}, \tag{7}$$

where Y_t denotes output, K_{t-1}, N_{et}, and $L_{e,t-1}$ denote the inputs capital, labor, and land, respectively, and the parameters $\alpha \in (0,1)$ and $\phi \in (0,1)$ measure the output elasticities of these production factors. We assume that the total factor productivity Z_t is composed of a permanent component Z_t^p and a transitory component ν_t such that $Z_t = Z_t^p \nu_{zt}$, where the permanent component Z_t^p follows the stochastic process

$$Z_t^p = Z_{t-1}^p \lambda_{zt}, \quad \ln \lambda_{zt} = (1-\rho_z)\ln \bar{\lambda}_z + \rho_z \ln \lambda_{z,t-1} + \varepsilon_{zt}, \tag{8}$$

and the transitory component follows the stochastic process

$$\ln \nu_{zt} = \rho_{\nu_z} \ln \nu_{z,t-1} + \varepsilon_{\nu_z t}. \tag{9}$$

The parameter $\bar{\lambda}_z$ is the steady-state growth rate of Z_t^p; the parameters ρ_z and ρ_{ν_z} measure the degree of persistence. The innovations ε_{zt} and $\varepsilon_{\nu_z t}$ are i.i.d. white noise processes that are mutually independent with mean zero and variances given by σ_z^2 and $\sigma_{\nu_z}^2$, respectively.

The entrepreneur is endowed with K_0 units of initial capital stock. Capital accumulation follows the law of motion

$$K_t = (1-\delta)K_{t-1} + \left[1 - \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - \bar{\lambda}_I\right)^2\right] I_t, \tag{10}$$

where I_t denotes investment, $\bar{\lambda}_I$ denotes the steady-state growth rate of investment, and $\Omega > 0$ is the adjustment cost parameter.

The entrepreneur faces the flow of funds constraint

$$C_{et} + q_{lt}(L_{et} - L_{e,t-1}) + B_{t-1} = Z_t[L_{e,t-1}^{\phi} K_{t-1}^{1-\phi}]^{\alpha} N_{et}^{1-\alpha} - \frac{I_t}{Q_t} - w_t N_{et} + \frac{B_t}{R_t}, \tag{11}$$

where B_{t-1} is the amount of matured debt and B_t/R_t is the value of new debt. Following Greenwood, Hercowitz, and Krusell (1997), we interpret Q_t as the investment-specific technological change. Specifically, we assume that $Q_t = Q_t^p \nu_{qt}$, where the permanent component Q_t^p follows the stochastic process

$$Q_t^p = Q_{t-1}^p \lambda_{qt}, \quad \ln \lambda_{qt} = (1 - \rho_q) \ln \bar{\lambda}_q + \rho_q \ln \lambda_{q,t-1} + \varepsilon_{qt}, \tag{12}$$

and the transitory component μ_t follows the stochastic process

$$\ln \nu_{qt} = \rho_{\nu_q} \ln \nu_{q,t-1} + \varepsilon_{\nu_q t}. \tag{13}$$

The parameter $\bar{\lambda}_q$ is the steady-state growth rate of Q_t^p; the parameters ρ_q and ρ_{ν_q} measure the degree of persistence. The innovations ε_{qt} and $\varepsilon_{\nu_q t}$ are i.i.d. white noise processes that are mutually independent with mean zero and variances given by σ_q^2 and $\sigma_{\nu_q}^2$, respectively.

In the spirit of Kiyotaki and Moore (1997), we assume that the entrepreneur face the credit constraint

$$B_t \leq \theta_t E_t[q_{l,t+1} L_{et} + q_{k,t+1} K_t], \tag{14}$$

where $q_{k,t+1}$ is the shadow price of capital in consumption units.[3] Under this credit constraint, the amount that the entrepreneur can borrow is limited by a fraction of the value of the collateral assets—land and capital. Following Kiyotaki and Moore (1997), we interpret this type of credit constraints as reflecting the problem of costly contract enforcement: if the entrepreneur fails to pay the debt, the creditor can seize the land and the accumulated capital; since it is costly to liquidate the seized land and capital stock, the creditor can recoup up to a fraction θ_t of the total value of the collateral assets. We interpret θ_t as a "collateral shock" that reflects the uncertainty in the tightness of the credit market. We assume that θ_t follows the stochastic process

$$\ln \theta_t = (1 - \rho_\theta) \ln \bar{\theta} + \rho_\theta \ln \theta_{t-1} + \varepsilon_{\theta t}, \tag{15}$$

where $\bar{\theta}$ is the steady-state value of θ_t, $\rho_\theta \in (0, 1)$ is the persistence parameter, and $\varepsilon_{\theta t}$ is an i.i.d. white noise process with mean zero and variance σ_θ^2.

The entrepreneur chooses C_{et}, N_{et}, I_t, $L_{e,t}$, K_t, and B_t to maximize (6) subject to (7) through (14).

[3]Since the price of new capital is $1/Q_t$, Tobin's q in this model is given by $q_{kt} Q_t$, which is the ratio of the value of installed capital to the price of new capital.

III.3. **Market clearing conditions and equilibrium.** In a competitive equilibrium, the markets for goods, labor, land, and loanable bonds all clear. The goods market clearing condition implies that

$$C_t + \frac{I_t}{Q_t} = Y_t, \tag{16}$$

where $C_t = C_{ht} + C_{et}$ denotes aggregate consumption. The labor market clearing condition implies that labor demand equals labor supply:

$$N_{et} = N_{ht} \equiv N_t. \tag{17}$$

The land market clearing condition implies that

$$L_{ht} + L_{et} = \bar{L}, \tag{18}$$

where $\bar{L}$ is the fixed aggregate land endowment. Finally, the bond market clearing condition implies that

$$S_t = B_t. \tag{19}$$

A competitive equilibrium consists of sequences of prices $\{w_t, q_{lt}, R_t\}_{t=0}^{\infty}$ and allocations $\{C_{ht}, C_{et}, I_t, N_t, L_{ht}, L_{et}, S_t, B_t, K_t, Y_t\}_{t=0}^{\infty}$ such that (i) taking the prices as given, the allocations solve the optimizing problems for the household and the entrepreneur and (ii) all markets clear.

III.4. **Some key properties of the equilibrium.** Following Kiyotaki and Moore (1997) and Iacoviello (2005), we assume that the household in our model are more patient than the entrepreneur.[4] The patience factor in our model is important because it leads to two related characteristics of the equilibrium: there exists a positive first-order excess return and the collateral constraint is always binding in steady state.

The first-order excess return exists because the patient household's savings depress the equilibrium loan rate and the less patient entrepreneur assigns a positive shadow value to existing loans.[5] With the excess return, the entrepreneur desires to borrow

[4]In an disaggregated environment with heterogeneous households and heterogenous entrepreneurs who all have an identical subjective discount factor, the aggregate patience factor arises if entrepreneurs face an exogenous death or exit rate as assumed by Bernanke, Gertler, and Gilchrist (1999) or if households face persistent and uninsurable idiosyncratic income risks and their precautionary savings make them appear patient in aggregation (Liu, Wang, and Zha, 2009).

[5]In Appendix B.1, we derive the first-order excess return for our model. The first-order excess return represents a key distinction between Kiyotaki and Moore (1997)'s model with costly contract enforcement and Bernanke, Gertler, and Gilchrist (1999)'s model with costly state verification. In Bernanke, Gertler, and Gilchrist (1999), the intermediary sets the loan rate to break even, taking into account the default risks; an entrepreneur optimally chooses the amount of loans, taking the loan rate

up to the credit limit to finance additional investment in land and capital. This credit constraint responds to the collateral value that is influenced by fluctuations in asset prices. It is these direct interactions between asset prices and investment movements through endogenous credit constraints that give rise to a financial multiplier, whose empirical importance we will evaluate in the following sections.

IV. Econometric Methodology

We use the Bayesian method to fit our model to quarterly U.S. time series data from 1976:Q1 through 2009:Q1. The time series that we use include the relative land price, the inverse of the quality-adjusted relative price of investment, real per capita consumption, real per capita investment (in consumption units), real per capita nonfarm and nonfinancial business debt, and per capita hours worked (as a fraction of total time endowment). Appendix A describes the details of our data.[6]

We partition the model parameters into three subsets. The first subset of parameters includes the structural parameters on which we have agnostic priors. This set of parameters, $\Psi_1 = \{\gamma_h, \gamma_e, \Omega, \bar{\lambda}_q, g_\gamma\}$, consists of the household's and entrepreneur's habit persistence parameters γ_h and γ_e, investment-adjustment cost parameter Ω, the growth rate of per capita investment $\bar{\lambda}_q$, and the growth rate of per capita output g_γ. These parameters are listed in the top panel of Table 1. We assume that the priors for γ_h and γ_e follow the beta distribution with the shape parameters given by $a = 1$ and $b = 2$. Thus, we assign positive density to $\gamma_h = \gamma_e = 0$ and let the probability density decline linearly as the value of γ_h (or γ_e) increases from 0 to 1. These hyper-parameter values imply that a lower probability (5%) bound for γ_h and γ_e is 0.0256 and an upper probability (95%) bound is 0.7761. This 90% probability interval covers most calibrated values for the habit persistence parameter used in the literature (for example, Boldrin, Christiano, and Fisher (2001) and Christiano, Eichenbaum, and Evans (2005)). The prior for the investment adjustment cost parameter Ω follows the gamma distribution with the shape parameter $a = 1$ and the "rate parameter" $b = 0.5$ so that the probability densities at both $\Omega = 0$ (no adjustment costs) and $\Omega > 0$ are

as given. The equilibrium loan rate is the same as the return on risky assets and thus the first-order excess return is always zero.

[6]The data on investment-specific technology are needed to get the sizes of standard deviations of investment technology shocks in line with those in Krusell, Ohanian, Ríos-Rull, and Violante (2000) and Fisher (2006). By using an explicit measure of investment-specific technology shocks (i.e., biased technology shocks) in our estimation, we will be able to assess the importance of biased technology shocks relative to neutral technology shocks.

positive and finite. The 90% prior probability interval for Ω ranges from 0.1 to 6, which includes most values used in the DSGE literature (e.g., Christiano, Eichenbaum, and Evans (2005), Smets and Wouters (2007), and Liu, Waggoner, and Zha (2009)). The priors for the steady-state growth rates of capital and of output follow the gamma distribution with the 90% probability interval covering the range between 0.1 and 1.5, corresponding to annual growth rates between 0.4% and 6%.

The second subset of parameters includes the structural parameters for which we can use the steady-state relations for constructing informative priors. This set of parameters, $\Psi_2 = \{\beta, \bar{\lambda}_a, \bar{\varphi}, \phi, \alpha, \theta, \delta\}$, consists of the household's discount factor β, the household's patience factor $\bar{\lambda}_a$, the housing preference parameter $\bar{\varphi}$, the leisure preference parameter ϕ, the elasticity parameter of production factors α, the average loan-to-asset ratio θ, and the capital depreciation rate δ. We construct the prior distributions for the parameters in Ψ_2 by first simulating the first set of parameters from the prior distributions and then for each simulation imposing the steady-state restrictions on both Ψ_1 and Ψ_2 such that the model matches the following moment conditions: (1) the average labor income share is 70% ($\alpha = 0.3$); (2) the average real prime loan rate is 4% per annum (Huggett, Ventura, and Yaron, 2009); (3) the capital-output ratio is on average 1.15 at the annual frequency; (4) the investment-capital ratio is on average 0.209 at the annual frequency; (5) the average land-output ratio is 0.65 at the annual frequency; (6) the average nonfarm and nonfinancial businesses' loan-asset ratio is 0.75 at the annual frequency ($\theta = 0.75$); (7) the average housing-output ratio is 1.45 at the annual frequency; and (8) the average market hours is 25% of time endowment.[7] Since the prior distributions for the parameters in Ψ_2 are of unknown form, the 90% probability bounds, reported in Table 1 (the lower panel), are generated through simulations. As shown in the table, the steady-state restrictions lead to informative probability intervals for the marginal prior distributions of the

[7]Since we have a closed-economy model with no government spending, we measure private domestic output by the sum of personal consumption expenditures and private domestic investment, where consumption is the expenditures on nondurable goods and non-housing services, and investment is the expenditures on consumer durable goods and fixed investment in equipment and software. These time series are provided by the Bureau of Economic Analysis (BEA) through Haver Analytics. Capital stock and housing stock are in annual rates. Capital includes equipment, software, and nondurables only. To compute the land-output ratio, we use the time series of the nominal value of land input and the nominal value of output in the private nonfarm and nonfinancial business sector for the period 1987-2007, both provided by the Bureau of Labor Statistics (BLS).

parameters and thus help identify the structural parameters in Ψ_2.[8] Our method for constructing the prior distributions for Ψ_2 is similar to the approach studied by Del Negro and Schorfheide (2008), which combines the Baynesian approach and the standard calibration approach for eliciting priors.

The third subset of parameters is composed of those describing the shock processes displayed in Table 2. These parameters are summarized by $\Psi_3 = \{\rho_i, \sigma_i\}$ for $i \in \{a, z, \nu_z, q, \nu_q, \varphi, \psi, \theta\}$, where ρ_i and σ_i denote the persistence parameters and the standard deviations of the eight structural shocks. We adopt agnostic priors for these parameters. Specifically, the priors for the persistent parameters follow the beta distribution with the 90% probability interval given by $[0.0256, 0.7761]$; the priors for the standard deviations follow the inverse gamma distribution with the 90% probability interval given by $[0.0001, 1.0]$. We have examined the sensitivity of our estimates by extending both the lower and the upper bounds of this interval and found that the results are not sensitive.

V. Empirical results

We now present and discuss our empirical findings. Section V.1 reports the estimated values of structural parameters, Section V.2 presents variance decompositions from the estimated model, Section V.4 discusses the importance of endogenous credit constraints, and Section V.3 presents evidence that the dynamic interactions between asset prices and investment fluctuations have played a quantitatively important role in generating aggregate fluctuations throughout our sample period.

V.1. **Parameter estimates.** Tables 1 and 2 report the posterior mode estimates of the parameters. The maximum log posterior density (MLPD) for our DSGE model is 2232.68. The MLPD for the Bayesian vector autoregression with four lags is 2145.64.[9] By the Schwarz criterion, the fit from our structural model is competitive with an atheoretical statistical model.

[8]Even with a subset of deep parameters well identified, the posterior density function is still very non-Gaussian and has many local peaks. We randomly simulate 100000 starting points and select the converged result that gives the highest posterior density. Among these starting points, many converge to the point that has the highest peak. The computing time is about 4-5 days on a cluster of 24 2.5GHz computers.

[9]We use the prior settings provided by Sims and Zha (1998). Specifically, we set the hyperparameter values as, in their notation, $\lambda_0 = 1, \lambda_1 = 0.5, \lambda_3 = 0.1, \lambda_4 = 0.1$, and $\mu_5 = \mu_6 = 1$. The MLPD is sensitive to the prior and its value can differ by 50-100 in log for different but reasonable hyperparameter values.

Our estimates suggest that the habit persistence is moderately important, with the entrepreneur's habit parameter larger than the household's (0.55 vs.0.32). The estimated value of the investment-adjustment cost parameter is only 0.18, much smaller than those obtained in the literature (for example, Christiano, Eichenbaum, and Evans (2005) obtain an estimate of Ω at around 2.5; Smets and Wouters (2007)'s reported estimate is even larger than 5.0). Since credit constraints impede investment adjustments and amplify the responses of the asset price, our model relies less on investment adjustment costs to fit the data.

The estimated growth rate of the investment-specific technology is higher than that calibrated by Greenwood, Hercowitz, and Krusell (1997) (4.8% vs. 3.2% per annum), mainly because they use a shorter sample that ends in 1990, whereas, for the subsample after the early 1990s, the United States economy experienced even more rapid declines in the quality-adjusted relative price of equipment, software, and consumer durable goods. Our estimated growth rate of per capita output is about 1.26% per annum, consistent with the average growth rate of real per capita GDP in the United States for the postwar period.

The estimated value of the patience factor is 0.005, which implies that the first-order excess return is about 1.94% per annum (see the discussion in Section III.4 and Appendix B.1). The estimated values of the other parameters in Ψ_2 are reported in the lower panel of the table.

Table 2 displays the estimates of persistence parameters and standard deviations for various shock processes. The patience shock, the housing demand shock, the labor supply shock, and the collateral shock are all persistent with AR(1) coefficients above 0.9. The estimated persistence of a shock to the growth rate of neutral technology is larger than what is obtained in the literature because we include land as an input in the production function. The estimated standard deviation of the patience shock is largest (0.19) and that of the housing demand shock is second largest (0.05). The collateral shock also has a fairly sizable standard deviation (0.013). The estimated standard deviations of investment-specific technology shocks (to the level and to the growth rate) are the smallest among all shocks. This result stands in contrast to Justiniano and Primiceri (2008), who treat changes in the investment-specific technology as an unobservable and find its shock volatility much larger than that of any other shock. Our result is consistent with Krusell, Ohanian, Ríos-Rull, and Violante (2000) and Liu, Waggoner, and Zha (2009), who use direct measures of investment-specific technology.

V.2. **Variance decompositions.** Whether an exogenous economic shock has significant impact on the dynamics of asset prices and aggregate quantities does not necessarily depend on the size or the persistence of the shock itself; rather it depends on the model's internal transmission mechanism, which transforms the shocks into the fluctuations in prices and quantities. The importance of the transmission mechanism can be measured by variance decompositions of asset prices and aggregate quantities across different types of economic shocks. Tables 3 and 4 report these results at various forecast horizons.

Let us begin with analysis of the persistent patience shock, which has the largest standard deviation (almost four times the size of the second largest shock). Despite its persistence and large size, the patience shock has little impact on the dynamics of the land price, the capital price, the excess return, consumption, and debt. It has a sizable impact only on investment fluctuations (about 20%). The housing demand shock, which has the second largest standard deviation, is also very persistent. Although it has little impact on consumption, it drives most of the fluctuations in the land price and has an important impact on investment fluctuations (about 35-45%). It does not follow, however, that a shock that drives the asset price will also influence the dynamics of a real aggregate variable. The permanent shock to the investment-specific technology, for example, has negligible impact on investment itself even though it causes most of the fluctuations in the capital price. Nor does it follow that an economic shock directly influencing the borrowing capacity will have an important impact on both asset prices and investment fluctuations. The collateral shock is the case in point: the shock is very persistent and has the third largest standard deviation, but it has little impact on the dynamics of both the land price and the capital price and that its impact on output is limited (about 10%), as shown in Tables 3 and 4.

Movements in excess returns are driven by both the housing demand shock (about 65%) and the collateral shock (about 25%), as shown in Table 3. Other shocks – in particular, shocks to the growth rate and level of neutral technology – are unimportant to asset price fluctuations. Shocks to the neutral technology have considerable impact on the current and future dividends; they do not generate generate large asset-price movements, but rather influence the loan rate.

Table 4 shows that consumption fluctuations at various forecast horizons are mainly driven by the permanent neutral technology shock (about 50 to 70%) and the labor supply shock (for about 20%). Investment fluctuations are mainly driven by the housing demand shock (about 35% to 45%), the intertemporal preference (patience) shock

(about 20%), and the collateral shock (about 15%). Output fluctuations are driven not by a single shock but multiple shocks: the housing demand shock, the shock to the growth rate of neutral technology, the labor supply shock, the patience shock, and the collateral shock. The housing demand shock is most important in generating short-run fluctuation in output, while the permanent shock to the neutral technology is most important in the longer run. The shock to the level of the neutral technology is not important; both the permanent and transitory shocks to the investment-specific technology are even less important, consistent with the recent findings of Liu, Waggoner, and Zha (2009).

In the context of our heterogenous agent model, one should take caution interpreting the intertemporal shock (such as the investment specific shock) as an "intertemporal wedge." Unlike the representative agent model in Chari, Kehoe, and McGrattan (2007), a source of intertemporal distortion in our model comes naturally from the credit constraint. Although explicit intertemporal shocks do not account for a large fraction of output fluctuations, the intertemporal distortion through the credit constraint help amplify and propagate the dynamic effects of other shocks. A primary example is the housing demand shock, which accounts for about 20% to 35% of output fluctuations at business-cycle frequencies. The housing demand shock gets amplified through credit constraints as it drives movements in the asset price. Indeed, as shown in Table 3, the housing demand shock accounts for over 90% of land price fluctuations. Since land is an important collateral, changes in the land price lead to changes in the borrowing capacity and therefore investment and future output, which feeds back to changes in the current asset price and the collateral value. Our results suggest that this financial multiplier is empirically important.

V.3. **Land prices and macroeconomic fluctuations: a historical perspective.** Variance decompositions reported in Section V.2 indicate the average importance of a particular type of shock relative to other types of shocks. As Sims and Zha (2006) point out, however, variance decompositions give no indication of whether a sequence of shocks account for fluctuations in macroeconomic variables at particular points of time through the sample. As a way to quantify the historical importance of dynamic interactions between land prices and real aggregate variables, our structural model makes it an internally coherent exercise to calculate what would have happened if only housing demand shocks had occurred *throughout the history*. We turn off all the estimated structural shocks bar housing demand shocks and use our estimated model to generate the time paths of the land price, consumption, and investment by conditioning

on the estimated initial state variables and the sequence of estimated housing demand shocks. We then compare the simulation to the data.

Figure 1 displays these counterfactual histories by comparing the actual quarterly growth rates of the land price, consumption, and investment (represented by dashed lines) and the corresponding simulated paths generated by housing demand shocks alone (represented by thick solid lines). If all the other shocks had been left in place, the simulations would have matched the observed data exactly. The top panel of Figure 1 shows that the simulated path of land prices tracks the actual data almost perfectly, indicating that housing demand shocks are the dominant source of fluctuations in the observed land prices throughout the sample.

Since housing demand shocks cause most of the observed fluctuations in the land price, any dynamic interactions between land prices and business cycles, if they exist, should be driven mainly by these shocks as well. The middle panel of Figure 1 reports the simulated path of consumption. The simulation is smoother than the observed path and shows strong asymmetric effects on consumption: it tracks observed consumption in expansion periods such as late 1980s, late 1990s, and 2000s, but misses all downturns in consumption.

While housing demand shocks do not generate downturns in consumption, they generate symmetric effects on investment. As shown in the bottom panel of Figure 1, the simulated path of investment tracks many upturns as well as downturns. Although the shocks do not contribute to the movements of investment in several periods, the simulation tends to track the general pattern of the actual series. Comparing the top and bottom panels of Figure 1, one can see that the land price and investment move together for the most part of the history. These comovements, driven by housing demand shocks, are the result of the internal transmission mechanism working through endogenous credit constraints. In the next section, we show how this mechanism works using another counterfactual exercise.

V.4. **Importance of endogenous credit constraints.** In our model, the entrepreneur's credit limit is endogenous and depends on the value of collateral and therefore on asset prices. A shock can be amplified through the credit constraint if it can move asset prices, resulting in a financial multiplier effect. The strength of the amplification depends on how responsive asset prices are to the shock. Some shocks drive business cycle fluctuations, but not through the credit constraint.

One such example is the TFP shock, which, as shown by Kocherlakota (2000) and Cordoba and Ripoll (2004), generates a weak financial multiplier effect. Our analysis

suggests that TFP shocks cannot be amplified through the credit constraint because they do not influence asset prices much and therefore do not have significant effects on the borrowing capacity. Figure 2 displays the impulse responses of four macroeconomic variables following a positive, one-standard-deviation, shock to neutral technology growth for two models: our estimated model with endogenous credit limit (solid lines) and the alternative model in which the credit limit is exogenously fixed at the steady state value (dashed lines). As shown in the figure, differences between the two sets of impulse responses (solid vs. dashed lines) are negligible, as the credit limit respond little to technology shocks.

The borrowing capacity, however, is influenced considerably by two sources of economic shocks: the collateral shock that directly affects the borrowing capacity and the housing demand shock that indirectly affects the borrowing capacity by moving the land price. To assess the quantitative importance of the transmission mechanism provided by the credit constraint, we plot the impulse responses of several key variables following each of these two shocks for our model in which the credit limit responds to changes in asset prices and for the alternative model in which the credit limit does not respond to asset prices.

Figure 3 plots the impulse responses of four macroeconomic variables to a positive, one-standard-deviation, collateral shock. The amplification effect through the endogenous credit constraint is evident: compared to the alternative model with the fixed credit limit, the peak response of output in our model is more than three times as large. The responses of the land price, consumption, and investment are all amplified under the endogenous credit constraint. Figure 4 displays the distributional effects in response to the collateral shock. The shock leads to an initial drop and a subsequent persistent increase in household consumption and an initial rise and a subsequent persistent decline in entrepreneur consumption. It also leads to a shift of land holdings from the household to the entrepreneur. As the collateral shock expands the entrepreneur's borrowing capacity, the entrepreneur's demand for land increases while the loan liability reduces the entrepreneur's net worth and consumption. On the other hand, the household is induced to save more, leading to a rise in the growth rate of household consumption. Compared to the results for our model with the endogenous credit limit (solid lines), the responses of consumption and land holdings are much less pronounced when the credit limit is held fixed (dashed lines).

Similar to Figures 3 and 4, Figures 5 and 6 show that the housing demand shock also generates persistent and hump-shaped responses of macroeconomic variables and has

distributional effects between the household and the entrepreneur in our model with the endogenous credit limit (solid lines). The key difference, however, is that housing demand shock has a much larger impact on the land price than does the collateral shock (comparing the scales in Figure 3 and Figure 5). Indeed, the peak effect of a housing demand shock on the land price is greater than that of a collateral shock by an order of magnitude (0.042 vs. 0.003). As land is an important part of collateral, the housing demand shock generates large and persistent responses of investment and output through the credit constraint. Compared to the case where the credit limit is exogenously fixed (dashed lines in Figure 5), the responses of investment and output to the housing demand shock are at least three times as large (solid lines in Figure 5).

In summary, a positive housing demand shock raises the land price and thus the collateral value. As a result, the credit limit is expanded. With a larger borrowing capacity, the entrepreneur is able to increase investment in land and capital, which raises future output and marginal products of both capital and land. The rise in future marginal products raises the current asset price further, which continues to expand the credit limit. Our exercises suggest that the endogenous credit constraint provides a quantitatively important financial multiplier mechanism of housing demand shocks.

VI. Conclusion

Establishing empirical significance of the amplification and propagation channeled through endogenous credit constraints has been a challenging task (Kocherlakota, 2000). Key to this challenge is to establish an empirically important link between asset prices and business cycles. We construct and estimate a dynamic general equilibrium model with endogenous credit constraints and provide a strong empirical support for such a link. While technology shocks, as well as other shocks such as labor supply shocks, contribute to output fluctuations through usual production channels, as documented in the literature and in this paper, our new findings show that housing demand shocks paly an important role in generating real business cycles through endogenous credit constraints.

As we have shown that endogenous credit constraints can make important the amplification and prorogation mechanism through asset prices, a promising direction for future research is to build on our model and incorporate into the analysis additional features. One extension is to confront the model with capital price data (or the Tobin's q) by introducing intangible capital along the lines of Hall (2001) and McGrattan and

Prescott (2009). A challenging task, similar to that in our paper, is to identify economic shocks that can drive large fluctuations in the capital price. Another extension is to incorporate heterogenous entrepreneurs between whom a reallocation of liquidity can be essential to the effect of monetary policy (Kiyotaki and Moore, 2008).

TABLE 1. Prior distributions and posterior modes of structural parameters

	Prior					Posterior
Parameter	Distribution	a	b	Low	High	Mode
γ_h	Beta(a,b)	1.00	2.00	0.025	0.776	0.3288
γ_e	Beta(a,b)	1.00	2.00	0.025	0.776	0.5517
Ω	Gamma(a,b)	1.00	0.50	0.102	5.994	0.1810
$100(\bar{\lambda}_q - 1)$	Gamma(a,b)	1.86	3.01	0.100	1.500	1.2121
$100(g_\gamma - 1)$	Gamma(a,b)	1.86	3.01	0.100	1.500	0.3157
β	Simulated			0.9563	0.9946	0.9885
$\bar{\lambda}_a$	Simulated			0.0000	0.0509	0.0048
$\bar{\varphi}$	Simulated			0.0000	0.0697	0.0536
ϕ	Simulated			0.0655	0.0701	0.0698
δ	Simulated			0.0291	0.0485	0.0378

Note: "Low" and "High" denote the bounds of the 90% probability interval for the prior distribution.

TABLE 2. Prior Distributions and posterior modes of shock parameters

	Prior					Posterior
Parameter	Distribution	a	b	Low	High	Mode
ρ_a	Beta(a,b)	1.0000	2.0000	0.0256	0.7761	0.9246
ρ_z	Beta(a,b)	1.0000	2.0000	0.0256	0.7761	0.4374
ρ_{ν_z}	Beta(a,b)	1.0000	2.0000	0.0256	0.7761	0.0091
ρ_q	Beta(a,b)	1.0000	2.0000	0.0256	0.7761	0.6016
ρ_{ν_q}	Beta(a,b)	1.0000	2.0000	0.0256	0.7761	0.3532
ρ_φ	Beta(a,b)	1.0000	2.0000	0.0256	0.7761	0.9998
ρ_ψ	Beta(a,b)	1.0000	2.0000	0.0256	0.7761	0.9810
ρ_θ	Beta(a,b)	1.0000	2.0000	0.0256	0.7761	0.9819
σ_a	Inverse gamma(a,b)	0.3543	0.0002	0.0001	1.0000	0.1876
σ_z	Inverse gamma(a,b)	0.3543	0.0002	0.0001	1.0000	0.0046
σ_{ν_z}	Inverse gamma(a,b)	0.3543	0.0002	0.0001	1.0000	0.0042
σ_q	Inverse gamma(a,b)	0.3543	0.0002	0.0001	1.0000	0.0037
σ_{ν_q}	Inverse gamma(a,b)	0.3543	0.0002	0.0001	1.0000	0.0028
σ_φ	Inverse gamma(a,b)	0.3543	0.0002	0.0001	1.0000	0.0545
σ_ψ	Inverse gamma(a,b)	0.3543	0.0002	0.0001	1.0000	0.0065
σ_θ	Inverse gamma(a,b)	0.3543	0.0002	0.0001	1.0000	0.0129

Note: "Low" and "High" denote the bounds of the 90% probability interval for the prior distribution.

TABLE 3. Variance decompositions of asset prices

Horizon	Patience	Ngrowth	Nlevel	Bgrowth	Blevel	Housing	Labor	Collateral
				Land price				
1Q	4.91	1.63	1.24	0.01	0.02	90.83	1.36	0.01
4Q	4.12	2.75	0.32	0.04	0.01	91.45	1.22	0.08
8Q	3.73	3.40	0.21	0.06	0.00	91.09	1.32	0.18
16Q	3.07	4.40	0.16	0.04	0.00	90.60	1.47	0.25
24Q	2.46	5.19	0.13	0.10	0.00	90.45	1.48	0.20
				Capital price				
1Q	9.09	0.58	24.51	46.30	1.28	12.20	3.31	2.73
4Q	4.01	0.44	7.20	75.93	0.86	7.53	1.34	2.70
8Q	1.83	0.21	3.28	88.99	0.40	3.44	0.61	1.24
16Q	0.82	0.09	1.47	95.04	0.18	1.55	0.27	0.56
24Q	0.53	0.06	0.95	96.82	0.12	1.00	0.18	0.36
				Excess Returns				
1Q	2.79	0.26	2.79	2.04	0.14	63.45	0.92	27.62
4Q	2.50	0.17	2.93	2.38	0.12	64.42	0.84	26.64
8Q	2.32	0.14	3.01	2.71	0.11	65.35	0.80	25.56
16Q	2.21	0.12	3.09	3.02	0.11	66.52	0.78	24.15
24Q	2.18	0.11	3.12	3.12	0.11	67.02	0.78	23.56

Note: Columns 2 to 9 correspond to contributions made by the intertemporal preference shock (Patience), the permanent shock to neutral technology (Ngrowth), the transitory shock to neutral technology (Nlevel), the permanent shock to biased technology (Bgrowth), the transitory shock to biased technology (Blevel), the housing demand shock (Housing), the labor supply shock (Labor), and the collateral shock (Collateral).

TABLE 4. Variance decompositions of aggregate quantities

Horizon	Patience	Ngrowth	Nlevel	Bgrowth	Blevel	Housing	Labor	Collateral
				Consumption				
1Q	5.76	49.76	9.99	0.62	0.21	3.15	28.53	1.98
4Q	2.05	68.49	1.72	0.37	0.06	1.00	25.85	0.45
8Q	1.48	66.05	1.28	0.55	0.03	5.54	23.13	1.93
16Q	3.10	63.54	1.10	0.39	0.01	9.12	19.90	2.83
24Q	3.09	66.92	0.87	1.20	0.01	7.62	18.20	2.10
				Investment				
1Q	20.33	0.77	13.47	3.19	1.81	39.89	6.99	13.56
4Q	20.30	4.82	4.75	0.85	0.37	45.04	7.07	16.82
8Q	19.46	8.53	3.60	2.89	0.27	42.48	7.59	15.18
16Q	17.54	12.40	3.09	8.25	0.24	37.49	7.97	13.02
24Q	16.11	14.33	2.83	11.88	0.22	34.50	7.78	12.35
				Output				
1Q	12.74	8.06	19.03	4.46	0.41	29.72	16.13	9.44
4Q	12.81	17.79	5.08	1.65	0.08	35.40	14.04	13.15
8Q	11.80	26.20	3.41	0.93	0.06	31.84	14.56	11.20
16Q	9.69	37.98	2.47	1.38	0.04	24.82	15.59	8.01
24Q	8.00	46.09	1.99	2.17	0.04	19.90	15.49	6.31

Note: Columns 2 to 9 correspond to contributions made by the intertemporal preference shock (Patience), the permanent shock to neutral technology (Ngrowth), the transitory shock to neutral technology (Nlevel), the permanent shock to biased technology (Bgrowth), the transitory shock to biased technology (Blevel), the housing demand shock (Housing), the labor supply shock (Labor), and the collateral shock (Collateral).

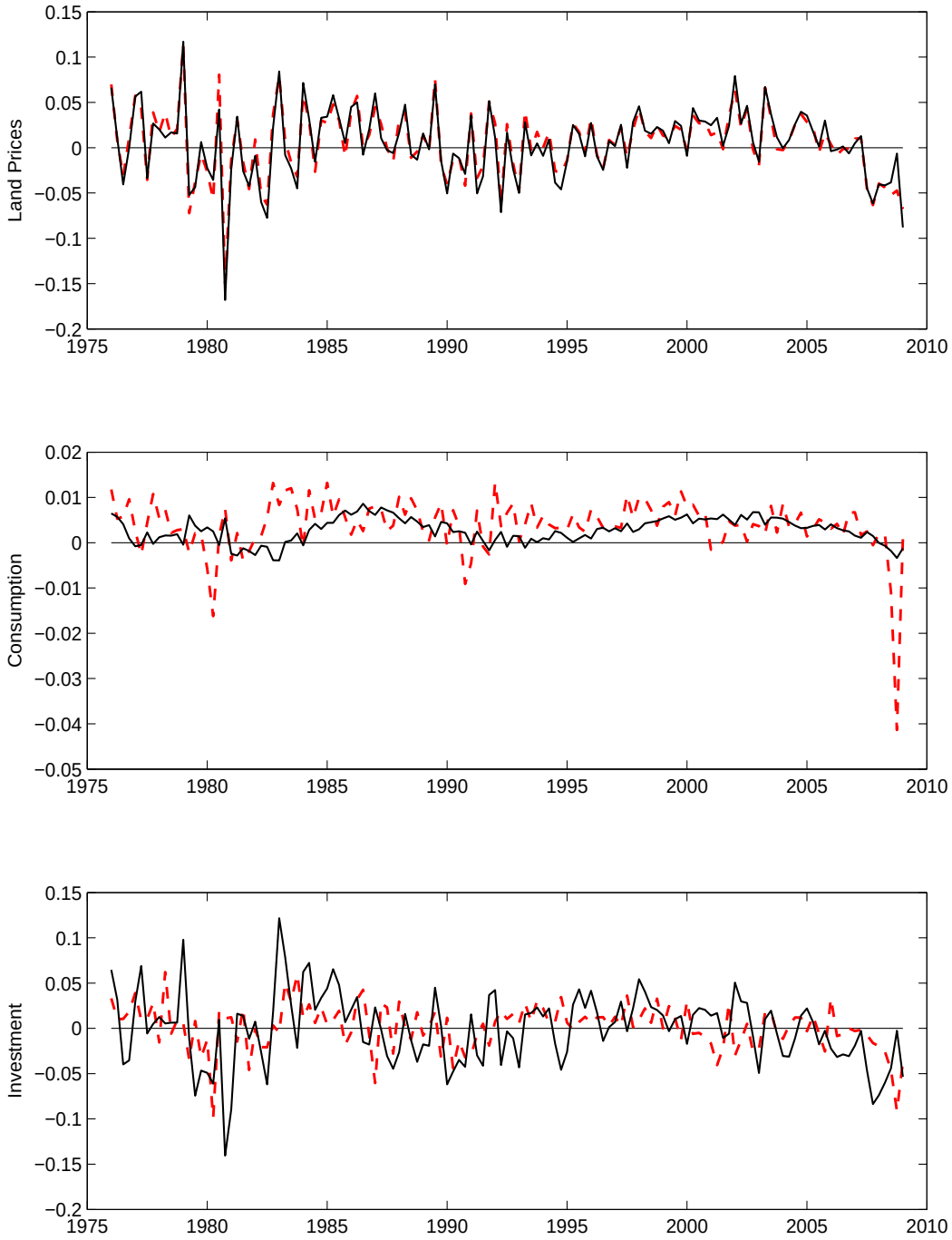

FIGURE 1. Historical paths of the land price, consumption, and investment: counterfactual versus data. Thin dashed lines represent the quarterly growth rates in the data; thick solid lines represent the simulations generated by housing demand shocks alone.

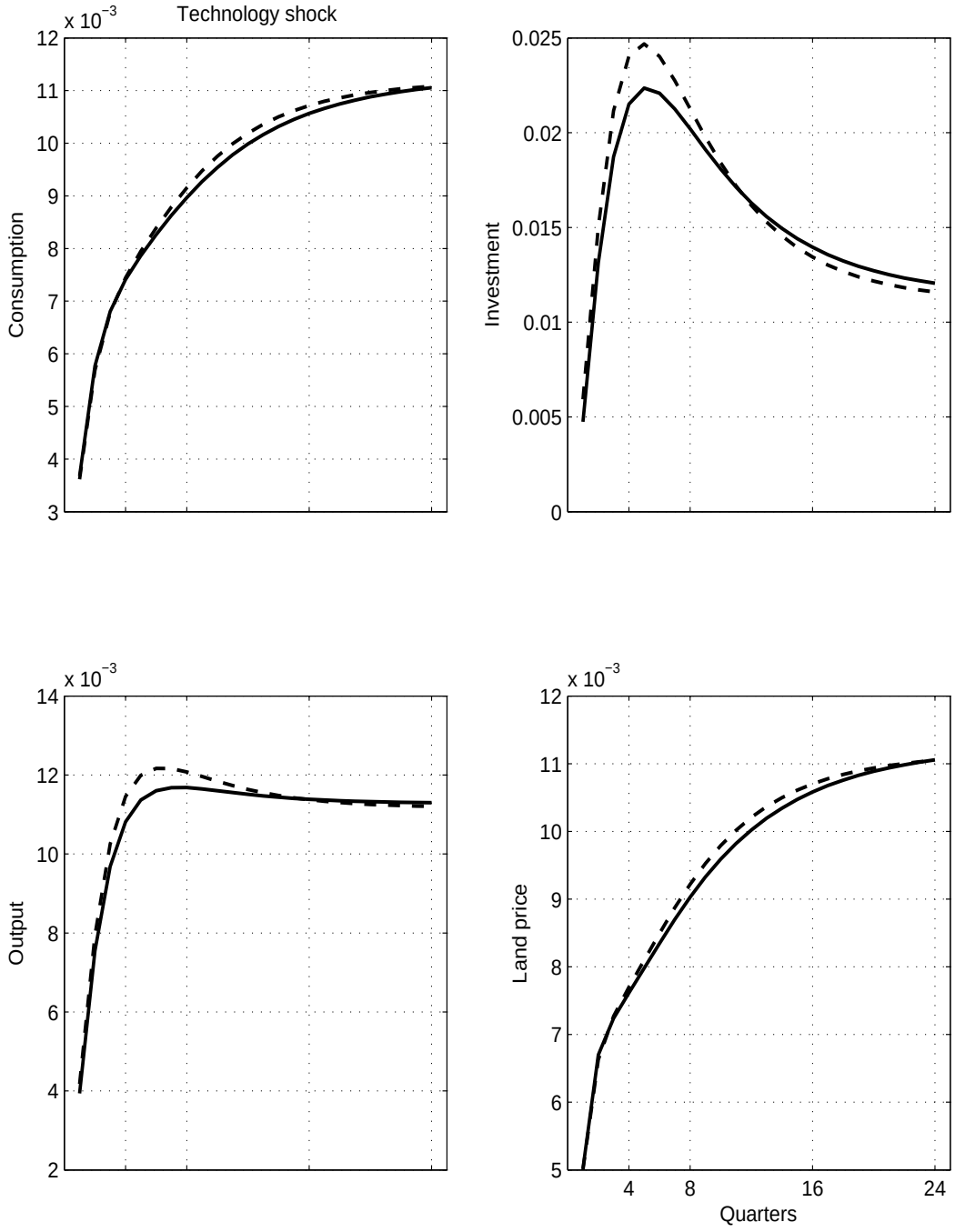

FIGURE 2. Impulse responses to a shock to neutral technology growth. Solid lines represent the model with endogenous credit limit; dashed lines represent the case with fixed credit limit.

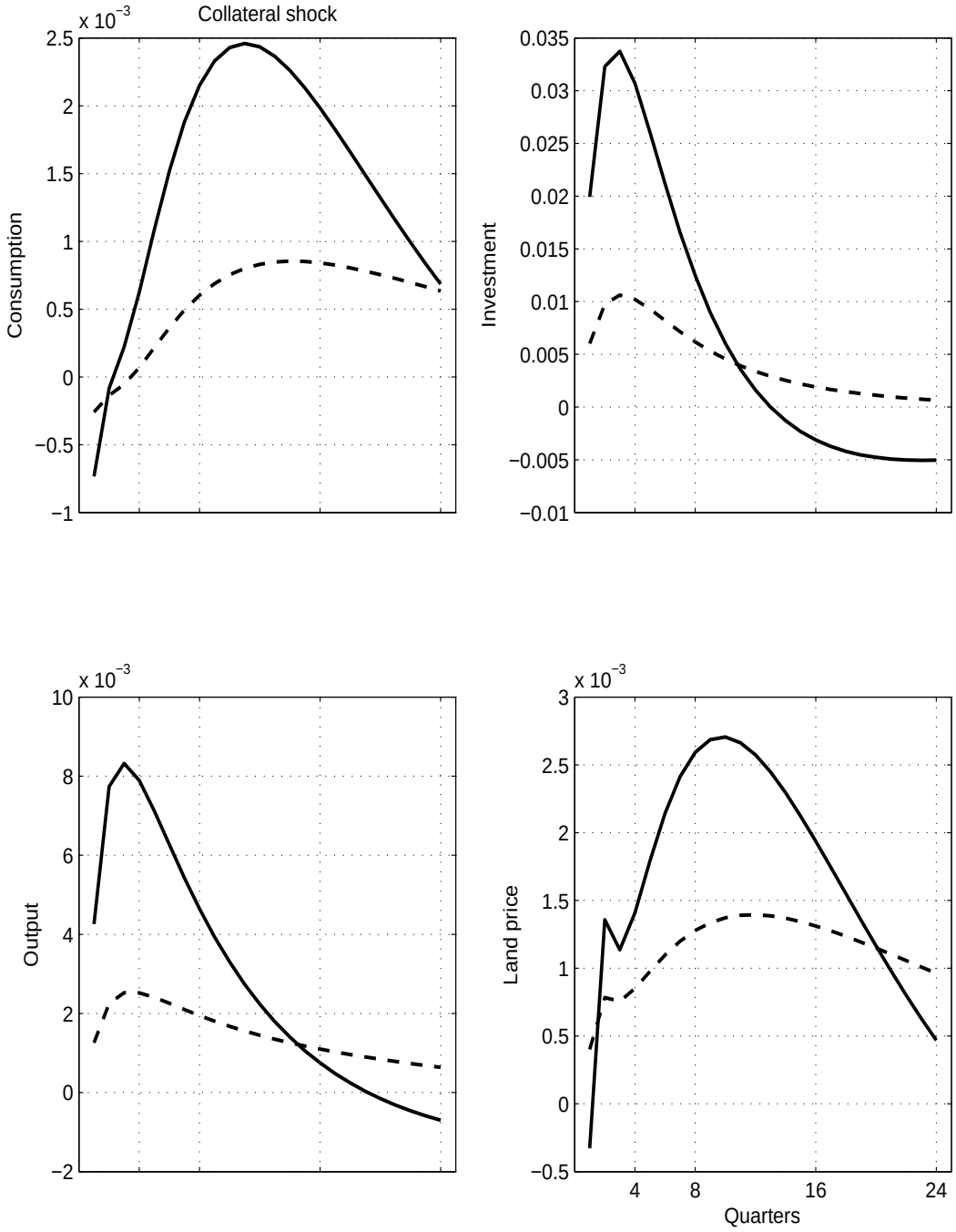

FIGURE 3. Impulse responses to a collateral shock. Solid lines represent the model with endogenous credit limit; dashed lines represent the case with fixed credit limit.

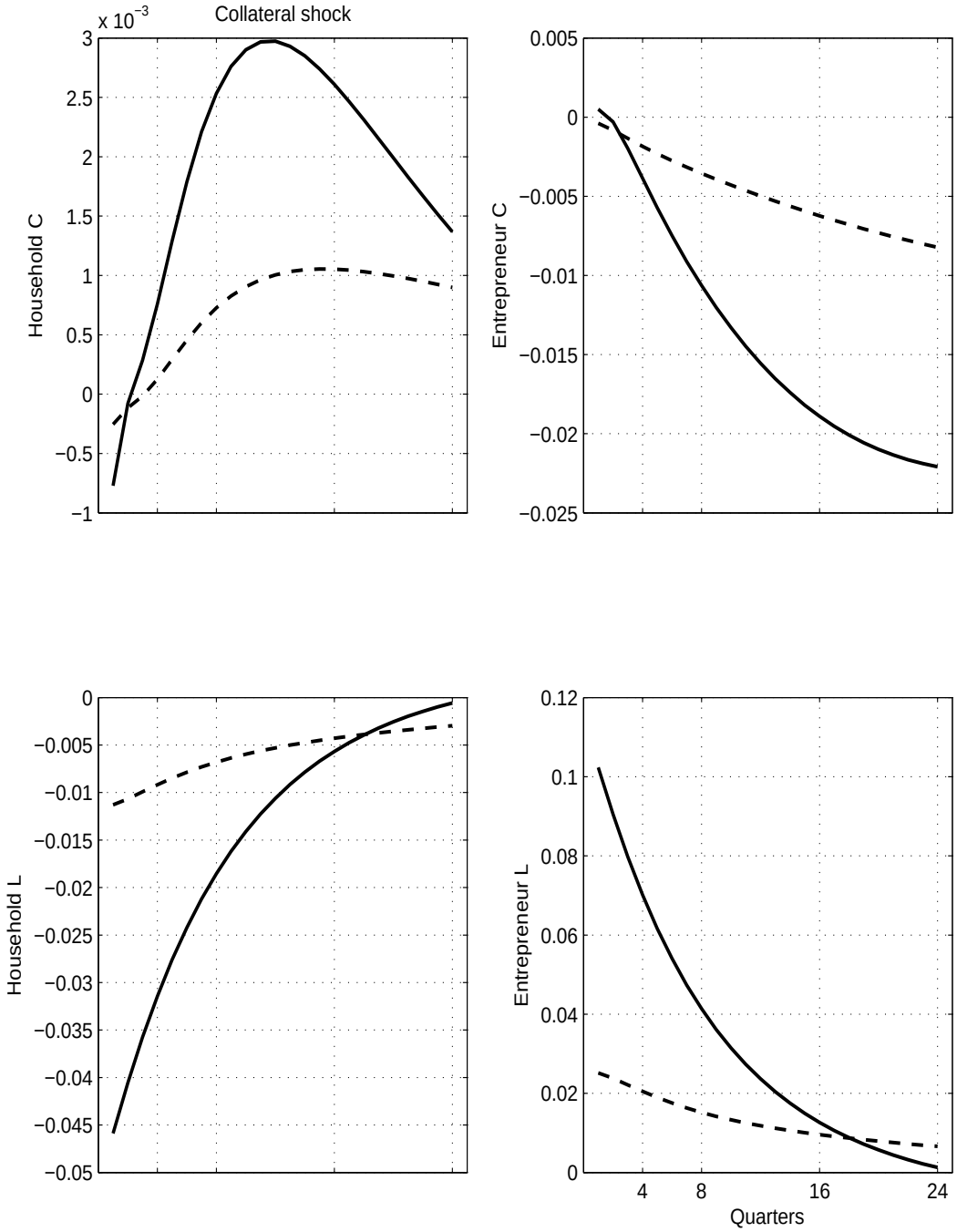

FIGURE 4. Impulse responses to a collateral shock. The letter "C" stands for "Consumption" and "L" stands for "Land." Solid lines represent the model with endogenous credit limit; dashed lines represent the case with fixed credit limit.

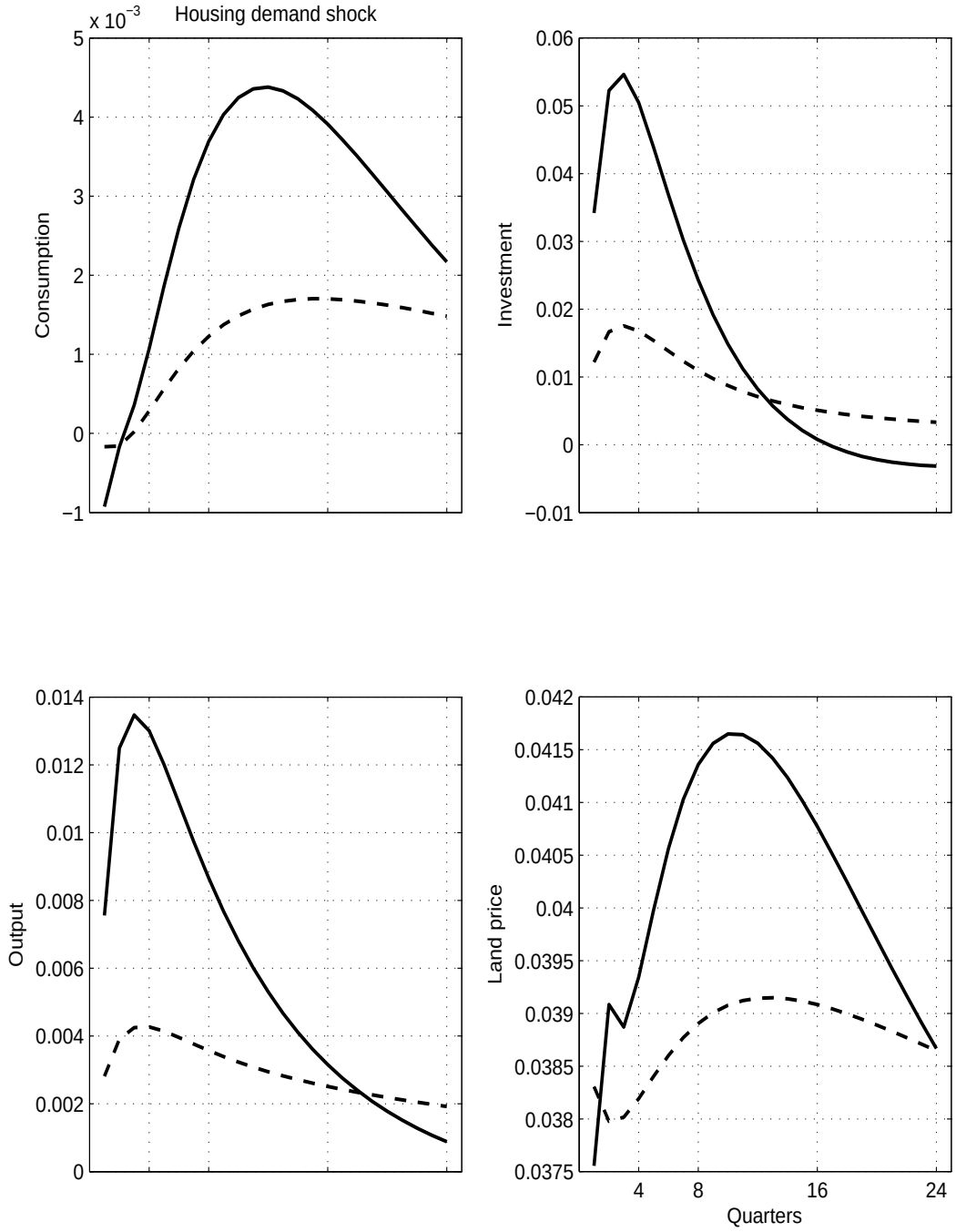

FIGURE 5. Impulse responses to a housing demand shock. Solid lines represent the model with endogenous credit limit; dashed lines represent the case with fixed credit limit.

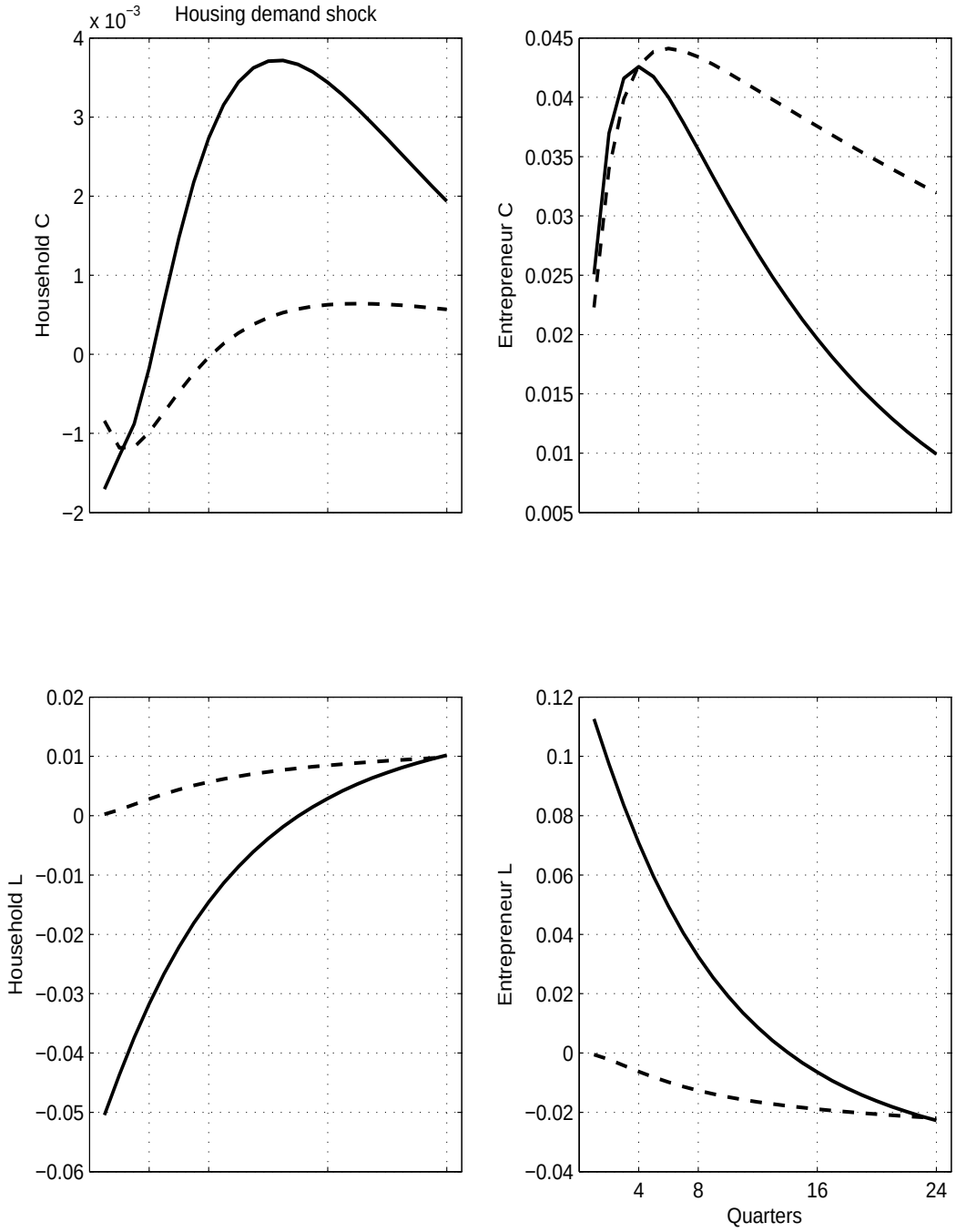

FIGURE 6. Impulse responses to a housing demand shock. The letter "C" stands for "Consumption" and "L" stands for "Land." Solid lines represent the model with endogenous credit limit; dashed lines represent the case with fixed credit limit.

Appendix A. Data Description

All data are either taken directly from the Haver Analytics Database or constructed by Patrick Higgins at the Federal Reserve Bank of Atlanta. The construction methods are described below.

The model estimation is based on six U.S. aggregate variables: the relative price of land (q_{lt}^{Data}), the inverse of the relative price of investment (Q_t^{Data}), real per capita consumption (C_t^{Data}), real per capita investment in consumption units (I_t^{Data}), real per capita nonfinancial business debt (B_t^{Data}), and per capita hours (L_t^{Data}).

These series are derived as follows.

- $q_{lt}^{\text{Data}} = \frac{\text{LiqLAND_PI_OFHEO}}{\text{PriceNonDurPlusServExHous}}$;
- $Q_t^{\text{Data}} = \frac{\text{PriceNonDurPlusServExHous}}{\text{GordonPriceCDplusES}}$;
- $C_t^{\text{Data}} = \frac{\text{(NomConsNHSplusND)/PriceNonDurPlusServExHous}}{\text{LNNReviseQtr}}$;
- $I_t^{\text{Data}} = \frac{\text{(CD@USECON + FNE@USECON)/PriceNonDurPlusServExHous}}{\text{LNNReviseQtr}}$;
- $B_t^{\text{Data}} = \frac{\text{(PL10TCR5@FFUNDS + PL11CRE5@FFUNDS)/PriceNonDurPlusServExHous}}{\text{LNNReviseQtr}}$;
- $L_t^{\text{Data}} = \frac{\text{LXNFH@USECON}}{\text{LNNReviseQtr}}$.

The original data, the constructed data, and their sources are described as follows.

LNNReviseQtr: civilian noninstitutional population with ages 16 years and over (NSA, Thous) by eliminating breaks in population from 10-year censuses and post 2000 American Community Surveys using "error of closure" method. This fairly simple method is used by the Census Bureau to get a smooth monthly population series to reduce the unusual influence of drastic demographic changes. The detailed explanation can be found in http://www.census.gov/popest/archives/methodology/intercensal_nat_meth.html. Source: BLS.

PriceNonDurPlusServExHous: the consumption deflator. The Tornqvist procedure is used to construct this deflator as a weighted aggregate index from nondurables consumption and services consumption excluding housing services. Source: BEA.

LiqLAND_PI_OFHEO: the liquidity-adjusted price index for residential land from Davis and Heathcote (2007) (http://www.marginalq.com/morris/landdata.html). The adjustment methods of Quart and Quigley (1989, 1991) are used to be consistent with the volatility measure provided by Lin and Liu (2008).

GordonPriceCDplusES: the quality-adjusted price index for consumer durable goods, equipment investment, and software investment. This index is a weighted

one from a number of individual price series within this category. For each individual price series from 1947 to 1983, we use Gordon (1990)'s quality-adjusted price index. Following Cummins and Violante (2002), we estimate an econometric model of Gordon's price series as a function of a time trend and a few macroeconomic indicators in the National Income and Product Account (NIPA), including the current and lagged values of the corresponding NIPA price series; the estimated coefficients are then used to extrapolate the quality-adjusted price index for each individual price series for the sample from 1984 to 2008. These constructed price series are annual. We use Denton (1971)'s method to interpolate these annual series on a quarterly frequency. We then use the Tornquist procedure to construct the quality-adjusted price index from the interpolated individual quarterly price series. Source: BEA.

NomConsNHSplusND: nominal personal consumption expenditures: non-housing services and nondurable goods. Source: BEA.

CD@USECON: nominal personal consumption expenditures: durable goods. Source: BEA.

FNE@USECON: nominal private nonresidential investment: equipment & software. Source: BEA.

PL10TCR5@FFUNDS: nonfarm nonfinancial corporation business liabilities: credit market debt. Source: BEA.

PL11CRE5@FFUNDS: nonfarm nonfinancial noncorporate business liabilities: credit market instruments. Source: BEA.

LXNFH@USECON: nonfarm business sector: hours of all persons (1992=100). Source: BLS.

Appendix B. Detailed derivations

B.1. **The excess returns.** In this section, we provide an intuitive derivation of the first-order excess returns in the presence of binding credit constraints.

The representative entrepreneur has two types of assets: land and capital. Each asset can be intuitively thought of as a Lucas tree bearing fruits and growing at a gross rate of g_γ. The entrepreneur can trade a portion of the tree in the market, and the return on this tree depends on the price of a unit of the tree as well as the marginal product (fruit) of the remaining tree. In steady state, it should be g_γ/β. To see if this intuition works in the model when the entrepreneur faces the borrowing constraint, we

first derive the expected return on each of these assets. We begin with the return on land.

Suppose the entrepreneur purchases one unit of land at the price q_{lt} in period t. Since she can pledge a fraction θ_t of the present value of the land as a collateral, the net out-of-pocket payment (i.e., the down payment) to purchase the land is given by

$$u_t \equiv q_{lt} - \theta_t \mathrm{E}_t \frac{q_{l,t+1}}{R_t}, \tag{A1}$$

where R_t is the loan rate. The land is used for period $t+1$ production and yields $\phi\alpha Y_{t+1}/L_{et}$ units of extra output. In addition, the entrepreneur can keep the remaining value of the land in period $t+1$ after repaying the debt so that the total payoff from the land is $\phi\alpha Y_{t+1}/L_{et} + q_{l,t+1} - \theta_t \mathrm{E}_t q_{l,t+1}$. The return on the land from period t to $t+1$ is thus given by

$$R_{l,t+1} = \frac{\phi\alpha Y_{t+1}/L_{et} + q_{l,t+1} - \theta_t \mathrm{E}_t q_{l,t+1}}{q_{lt} - \theta_t \mathrm{E}_t \frac{q_{l,t+1}}{R_t}}. \tag{A2}$$

We can similarly derive the return on capital, which is given by

$$R_{k,t+1} = \frac{\phi\alpha Y_{t+1}/K_t + q_{k,t+1}(1-\delta) - \theta_t \mathrm{E}_t q_{k,t+1}}{q_{kt} - \theta_t \mathrm{E}_t \frac{q_{k,t+1}}{R_t}}. \tag{A3}$$

To see how these returns relate to the entrepreneur's optimal decisions, we denote by μ_{et} the Lagrangian multiplier for the flow of funds constraint (11), μ_{kt} the multiplier for the capital accumulation equation (10), and μ_{bt} the multiplier for the credit constraint (14). With these notations, the shadow price of capital in consumption units is given by

$$q_{kt} = \frac{\mu_{kt}}{\mu_{et}},$$

and the marginal utility of income, μ_{et}, is equal to the marginal utility of consumption:

$$\mu_{et} = \frac{1}{C_{et} - \gamma_e C_{e,t-1}} - \mathrm{E}_t \frac{\beta\gamma_e}{C_{e,t+1} - \gamma_e C_{et}}.$$

The optimal decision on the entrepreneur's borrowing can be described by

$$\frac{1}{R_t} = \beta \mathrm{E}_t \frac{\mu_{e,t+1}}{\mu_{et}} + \frac{\mu_{bt}}{\mu_{et}}. \tag{A4}$$

The above Euler equation implies that the credit constraint is binding (i.e., $\mu_{bt} > 0$) if and only if the interest rate is lower than the entrepreneur's intertemporal marginal rate of substitution. The entrepreneur's optimal decisions on land and capital can be

described by the following two Euler equations:

$$q_{lt} = \beta \mathrm{E}_t \frac{\mu_{e,t+1}}{\mu_{et}} \left[\alpha\phi \frac{Y_{t+1}}{L_{et}} + q_{l,t+1} \right] + \frac{\mu_{bt}}{\mu_{et}} \theta_t \mathrm{E}_t q_{l,t+1}, \tag{A5}$$

$$q_{kt} = \beta \mathrm{E}_t \frac{\mu_{e,t+1}}{\mu_{et}} \left[\alpha(1-\phi) \frac{Y_{t+1}}{K_t} + q_{k,t+1}(1-\delta) \right] + \frac{\mu_{bt}}{\mu_{et}} \theta_t \mathrm{E}_t q_{k,t+1}. \tag{A6}$$

Using (A4), we can rewrite Equations (A5) and (A6) as

$$1 = \beta \mathrm{E}_t \frac{\mu_{e,t+1}}{\mu_{et}} R_{j,t+1}, \quad j \in \{l, k\}. \tag{A7}$$

Since consumption grows at the rate g_λ in equilibrium and the utility function is of logarithmic form, (A7) implies that $R_j = g_\lambda/\beta$.

On the other hand, the loan rate R_t is determined by the household's intertemporal Euler equation:

$$\frac{1}{R_t} = \beta \mathrm{E}_t \frac{\mu_{h,t+1}}{\mu_{ht}}, \tag{A8}$$

where μ_{ht} is the Lagrangian multiplier for the flow of funds constraint (5). It represents the marginal utility of income and is equal to the marginal utility of consumption:

$$\mu_{ht} = A_t \left[\frac{1}{C_{ht} - \gamma_h C_{h,t-1}} - \mathrm{E}_t \frac{\beta\gamma_h}{C_{h,t+1} - \gamma_h C_{ht}} (1 + \lambda_{a,t+1}) \right].$$

It follows from (A8) that in steady state, $R = \frac{g_\gamma}{\beta(1+\bar{\lambda}_a)}$, where $\bar{\lambda}_a > 0$ measures the extent to which the household is more patient than the entrepreneur. The steady state excess return is then given by

$$R_j^e \equiv R_j - R = \frac{g_\gamma}{\beta} \frac{\bar{\lambda}_a}{1 + \bar{\lambda}_a}, \quad j \in \{l, k\}. \tag{A9}$$

Clearly, the steady-state excess return is positive if and only if the patience factor, $\bar{\lambda}_a$, is positive.

To see how a positive first-order excess return is related to the entrepreneur's credit constraint, one can derive from (A4) the following steady state relationship:

$$\frac{\beta\bar{\lambda}_a}{g_\gamma} = \frac{\tilde{\mu}_b}{\tilde{\mu}_e}.$$

Thus, the credit constraint is binding (i.e., $\tilde{\mu}_b > 0$) if and only if the household is more patient than the entrepreneur (i.e., $\bar{\lambda}_a > 0$).

This result carries over to the dynamics of excess returns. Denote by $R_{j,t+1}^e \equiv R_{j,t+1} - R_t$ the excess return for asset $j \in \{l, k\}$. By combining the bond Euler equation (A4) and the asset-pricing equation (A7), we obtain

$$\beta \mathrm{E}_t \frac{\mu_{e,t+1}}{\mu_{et}} R_{j,t+1}^e = \frac{\mu_{bt}}{\mu_{et}} R_t, \quad j \in \{l, k\}. \tag{A10}$$

As in the standard asset-pricing model, the mean excess return depends on the asset's riskiness measured by the covariance between the return and the marginal utility of consumption. Unlike the standard model, however, the excess return in our model contains a first-order term that is positive if and only if the borrowing constraint is binding (i.e., $\mu_{bt} > 0$).

B.2. **Euler equations.** Denote by μ_{ht} the Lagrangian multiplier for the flow of funds constraint (5). The first-order conditions for the household's optimizing problem are given by

$$\mu_{ht} = A_t \left[\frac{1}{C_{ht} - \gamma_h C_{h,t-1}} - \mathrm{E}_t \frac{\beta \gamma_h}{C_{h,t+1} - \gamma_h C_{ht}} (1 + \lambda_{a,t+1}) \right], \tag{A11}$$

$$w_t = \frac{A_t}{\mu_{ht}} \psi_t, \tag{A12}$$

$$q_{lt} = \beta \mathrm{E}_t \frac{\mu_{h,t+1}}{\mu_{ht}} q_{l,t+1} + \frac{A_t \varphi_t}{\mu_{ht} L_{ht}}, \tag{A13}$$

$$\frac{1}{R_t} = \beta \mathrm{E}_t \frac{\mu_{h,t+1}}{\mu_{ht}}. \tag{A14}$$

Equation (A11) equates the marginal utility of income and of consumption; equation (A12) equates the real wage and the marginal rate of substitution (MRS) between leisure and income; equation (A13) equates the current relative price of land to the marginal benefit of purchasing an extra unit of land, which consists of the current utility benefits (i.e., the MRS between housing and consumption) and the land's discounted future resale value; and equation (A14) is the standard Euler equation for the loanable bond.

Denote by μ_{et} the Lagrangian multiplier for the flow of funds constraint (11), μ_{kt} the multiplier for the capital accumulation equation (10), and μ_{bt} the multiplier for the borrowing constraint (14). With these notations, the shadow price of capital in consumption units is given by

$$q_{kt} = \frac{\mu_{kt}}{\mu_{et}}. \tag{A15}$$

The first-order conditions for the entrepreneur's optimizing problem are given by

$$\mu_{et} = \frac{1}{C_{et} - \gamma_e C_{e,t-1}} - \mathrm{E}_t \frac{\beta \gamma_e}{C_{e,t+1} - \gamma_e C_{et}}, \tag{A16}$$

$$w_t = (1 - \alpha) Y_t / N_{et}, \tag{A17}$$

$$\frac{1}{Q_t} = q_{kt} \left[1 - \frac{\Omega}{2} \left(\frac{I_t}{I_{t-1}} - \bar{\lambda}_I \right)^2 - \Omega \left(\frac{I_t}{I_{t-1}} - \bar{\lambda}_I \right) \frac{I_t}{I_{t-1}} \right]$$

$$+\beta\Omega \mathrm{E}_t \frac{\mu_{e,t+1}}{\mu_{et}} q_{k,t+1} \left(\frac{I_{t+1}}{I_t} - \bar{\lambda}_I \right) \left(\frac{I_{t+1}}{I_t} \right)^2, \tag{A18}$$

$$q_{kt} = \beta \mathrm{E}_t \frac{\mu_{e,t+1}}{\mu_{et}} \left[\alpha(1-\phi) \frac{Y_{t+1}}{K_t} + q_{k,t+1}(1-\delta) \right] + \frac{\mu_{bt}}{\mu_{et}} \theta_t \mathrm{E}_t q_{k,t+1}, \tag{A19}$$

$$q_{lt} = \beta \mathrm{E}_t \frac{\mu_{e,t+1}}{\mu_{et}} \left[\alpha\phi \frac{Y_{t+1}}{L_{et}} + q_{l,t+1} \right] + \frac{\mu_{bt}}{\mu_{et}} \theta_t \mathrm{E}_t q_{l,t+1}, \tag{A20}$$

$$\frac{1}{R_t} = \beta \mathrm{E}_t \frac{\mu_{e,t+1}}{\mu_{et}} + \frac{\mu_{bt}}{\mu_{et}}. \tag{A21}$$

Equation (A16) equates the marginal utility of income to the marginal utility of consumption since consumption is the numéraire; equation (A17) is the labor demand equation which equates the real wage to the marginal product of labor; equation (A18) is the investment Euler equation, which equates the cost of purchasing an additional unit of investment good and the benefit of having an extra unit of new capital, where the benefit includes the shadow value of the installed capital net of adjustment costs and the present value of the saved future adjustment costs; equation (A19) is the capital Euler equation, which equates the shadow price of capital to the present value of future marginal product of capital and the resale value of the un-depreciated capital, plus the value of capital as a collateral asset for borrowing; equation (A20) is the land Euler equation, which equates the price of the land to the present value of the future marginal product of land and the resale value, plus the value of land as a collateral asset for borrowing; equation (A21) is the bond Euler equation for the entrepreneur, which reveals that the borrowing constraint is binding (i.e., $\mu_{bt} > 0$) if and only if the interest rate is lower than the entrepreneur's intertemporal marginal rate of substitution.

B.3. **Stationary equilibrium.** We are interested in studying the fluctuations around the balanced growth path. For this purpose, we focus on a stationary equilibrium by appropriately transforming the growing variables. Specifically, we make the following transformations of the variables

$$\tilde{Y}_t \equiv \frac{Y_t}{\Gamma_t}, \quad \tilde{C}_{ht} \equiv \frac{C_{ht}}{\Gamma_t}, \quad \tilde{C}_{et} \equiv \frac{C_{et}}{\Gamma_t}, \quad \tilde{I}_t \equiv \frac{I_t}{Q_t \Gamma_t}, \quad \tilde{K}_t \equiv \frac{K_t}{Q_t \Gamma_t}, \quad \tilde{B}_t \equiv \frac{B_t}{\Gamma_t},$$
$$\tilde{w}_t \equiv \frac{w_t}{\Gamma_t}, \quad \tilde{\mu}_{ht} \equiv \frac{\mu_{ht}\Gamma_t}{A_t}, \quad \tilde{\mu}_{et} \equiv \mu_{et}\Gamma_t, \quad \tilde{\mu}_{bt} \equiv \mu_{bt}\Gamma_t, \quad \tilde{q}_{lt} \equiv \frac{q_{lt}}{\Gamma_t}, \quad \tilde{q}_{kt} \equiv q_{kt} Q_t, \tag{A22}$$

where $\Gamma_t \equiv [Z_t Q_t^{(1-\phi)\alpha}]^{\frac{1}{1-(1-\phi)\alpha}}$. In Appendix B, we describe the stationary equilibrium and derive the log-linearized equilibrium conditions around the steady state for solving the model. To solve the log-linearized equilibrium system requires the input of several key steady-state values. These include the shadow value of the loanable funds $\frac{\tilde{\mu}_b}{\tilde{\mu}_e}$, the

ratio of commercial real estate to aggregate output $\frac{\tilde{q}_l L_e}{\tilde{Y}}$, the ratio of residential land to commercial real estate $\frac{L_h}{L_e}$, the ratio of loanable funds to output $\frac{\tilde{B}}{\tilde{Y}}$, the capital-output ratio $\frac{\tilde{K}}{\tilde{Y}}$, and the "big ratios" $\frac{\tilde{C}_h}{\tilde{Y}}$, $\frac{\tilde{C}_e}{\tilde{Y}}$, and $\frac{\tilde{I}}{\tilde{Y}}$. The model implies a set of restrictions between these steady-state ratios and the parameters, and we will use these restrictions along with the first moments of selected time series in the data to sharpen our priors and to help identify a subset of the parameters in our estimation.

Denote by $g_{\gamma t} \equiv \frac{\Gamma_t}{\Gamma_{t-1}}$ and $g_{qt} \equiv \frac{Q_t}{Q_{t-1}}$ the growth rates for the exogenous variables Γ_t and Q_t. Denote by g_γ the steady-state value of $g_{\gamma t}$ and $\lambda_k \equiv g_\gamma \bar{\lambda}_q$ the steady-state growth rate of capital stock. On the balanced growth path, investment grows at the same rate as does capital, so we have $\bar{\lambda}_I = \lambda_k$.

The stationary equilibrium is the solution to the following system of equations:

$$\tilde{\mu}_{ht} = \frac{1}{\tilde{C}_{ht} - \gamma_h \tilde{C}_{h,t-1}\Gamma_{t-1}/\Gamma_t} - \mathrm{E}_t \frac{\beta\gamma_h}{\tilde{C}_{h,t+1}\Gamma_{t+1}/\Gamma_t - \gamma_h \tilde{C}_{ht}}(1+\lambda_{a,t+1}), \tag{A23}$$

$$\tilde{w}_t = \frac{\psi_t}{\tilde{\mu}_{ht}}, \tag{A24}$$

$$\tilde{q}_{lt} = \beta \mathrm{E}_t \frac{\tilde{\mu}_{h,t+1}}{\tilde{\mu}_{ht}}(1+\lambda_{a,t+1})\tilde{q}_{l,t+1} + \frac{\varphi_t}{\tilde{\mu}_{ht} L_{ht}}, \tag{A25}$$

$$\frac{1}{R_t} = \beta \mathrm{E}_t \frac{\tilde{\mu}_{h,t+1}}{\tilde{\mu}_{ht}} \frac{\Gamma_t}{\Gamma_{t+1}}(1+\lambda_{a,t+1}). \tag{A26}$$

$$\tilde{\mu}_{et} = \frac{1}{\tilde{C}_{et} - \gamma_e \tilde{C}_{e,t-1}\Gamma_{t-1}/\Gamma_t} - \mathrm{E}_t \frac{\beta\gamma_e}{\tilde{C}_{e,t+1}\Gamma_{t+1}/\Gamma_t - \gamma_e \tilde{C}_{et}}, \tag{A27}$$

$$\tilde{w}_t = (1-\alpha)\tilde{Y}_t/N_t, \tag{A28}$$

$$\begin{aligned} 1 = {} & \tilde{q}_{kt}\left[1 - \frac{\Omega}{2}\left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}}\frac{Q_t\Gamma_t}{Q_{t-1}\Gamma_{t-1}} - \bar{\lambda}_I\right)^2 - \Omega\left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}}\frac{Q_t\Gamma_t}{Q_{t-1}\Gamma_{t-1}} - \bar{\lambda}_I\right)\frac{\tilde{I}_t}{\tilde{I}_{t-1}}\frac{Q_t\Gamma_t}{Q_{t-1}\Gamma_{t-1}}\right] \\ & + \beta\Omega \mathrm{E}_t \frac{\tilde{\mu}_{e,t+1}}{\tilde{\mu}_{et}}\frac{Q_t\Gamma_t}{Q_{t+1}\Gamma_{t+1}}\tilde{q}_{k,t+1}\left(\frac{\tilde{I}_{t+1}}{\tilde{I}_t}\frac{Q_{t+1}\Gamma_{t+1}}{Q_t\Gamma_t} - \bar{\lambda}_I\right)\left(\frac{\tilde{I}_{t+1}}{\tilde{I}_t}\frac{Q_{t+1}\Gamma_{t+1}}{Q_t\Gamma_t}\right)^2, \end{aligned} \tag{A29}$$

$$\tilde{q}_{kt} = \beta \mathrm{E}_t \frac{\tilde{\mu}_{e,t+1}}{\tilde{\mu}_{et}}\left[\alpha(1-\phi)\frac{\tilde{Y}_{t+1}}{\tilde{K}_t} + \tilde{q}_{k,t+1}\frac{Q_t\Gamma_t}{Q_{t+1}\Gamma_{t+1}}(1-\delta)\right] + \frac{\tilde{\mu}_{bt}}{\tilde{\mu}_{et}}\theta_t \mathrm{E}_t \tilde{q}_{k,t+1}\frac{Q_t}{Q_{t+1}}, \tag{A30}$$

$$\tilde{q}_{lt} = \beta \mathrm{E}_t \frac{\tilde{\mu}_{e,t+1}}{\tilde{\mu}_{et}}\left[\alpha\phi\frac{\tilde{Y}_{t+1}}{L_{et}} + \tilde{q}_{l,t+1}\right] + \frac{\tilde{\mu}_{bt}}{\tilde{\mu}_{et}}\theta_t \mathrm{E}_t \tilde{q}_{l,t+1}\frac{\Gamma_{t+1}}{\Gamma_t}, \tag{A31}$$

$$\frac{1}{R_t} = \beta \mathrm{E}_t \frac{\tilde{\mu}_{e,t+1}}{\tilde{\mu}_{et}}\frac{\Gamma_t}{\Gamma_{t+1}} + \frac{\tilde{\mu}_{bt}}{\tilde{\mu}_{et}}. \tag{A32}$$

$$\tilde{Y}_t = \left(\frac{Z_t Q_t}{Z_{t-1}Q_{t-1}}\right)^{-\frac{(1-\phi)\alpha}{1-(1-\phi)\alpha}} [L_{e,t-1}^{\phi}\tilde{K}_{t-1}^{1-\phi}]^{\alpha} N_t^{1-\alpha}, \tag{A33}$$

$$\tilde{K}_t = (1-\delta)\tilde{K}_{t-1}\frac{Q_{t-1}\Gamma_{t-1}}{Q_t\Gamma_t} + \left[1 - \frac{\Omega}{2}\left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}}\frac{Q_t\Gamma_t}{Q_{t-1}\Gamma_{t-1}} - \bar{\lambda}_I\right)^2\right]\tilde{I}_t, \tag{A34}$$

$$\tilde{Y}_t = \tilde{C}_{ht} + \tilde{C}_{et} + \tilde{I}_t, \tag{A35}$$

$$\bar{L} = L_{ht} + L_{et}, \tag{A36}$$

$$\alpha\tilde{Y}_t = \tilde{C}_{et} + \tilde{I}_t + \tilde{q}_{lt}(L_{et} - L_{e,t-1}) + \tilde{B}_{t-1}\frac{\Gamma_{t-1}}{\Gamma_t} - \frac{\tilde{B}_t}{R_t}, \tag{A37}$$

$$\tilde{B}_t = \theta_t \mathrm{E}_t\left[\tilde{q}_{l,t+1}\frac{\Gamma_{t+1}}{\Gamma_t}L_{et} + \tilde{q}_{k,t+1}\tilde{K}_t\frac{Q_t}{Q_{t+1}}\right]. \tag{A38}$$

We solve these 16 equations for 16 variables summarized in the vector

$$[\tilde{\mu}_{ht}, \tilde{w}_t, \tilde{q}_{lt}, R_t, \tilde{\mu}_{et}, N_t, \tilde{I}_t, \tilde{Y}_t, \tilde{C}_{ht}, \tilde{C}_{et}, \tilde{q}_{kt}, L_{et}, L_{ht}, \tilde{K}_t, \tilde{B}_t, \tilde{\mu}_{bt}]'.$$

B.4. **Steady state.** To get the steady-state value for $\frac{\tilde{\mu}_b}{\tilde{\mu}_e}$, we use the stationary bond Euler equations (A26) for the household and (A32) (described in the Appendix) to obtain

$$\frac{1}{R} = \frac{\beta(1+\bar{\lambda}_a)}{g_\gamma}, \quad \frac{\tilde{\mu}_b}{\tilde{\mu}_e} = \frac{\beta\bar{\lambda}_a}{g_\gamma}. \tag{A39}$$

Since $\bar{\lambda}_a > 0$, we have $\tilde{\mu}_b > 0$ and the borrowing constraint is binding in the steady-state equilibrium.

To get the ratio of commercial real estate to output, we use the land Euler equation (A31) for the entrepreneur, the definition of $\tilde{\mu}_e$ in (A27), and the solution for $\frac{\tilde{\mu}_b}{\tilde{\mu}_e}$ in (A39). In particular, we have

$$\frac{\tilde{q}_l L_e}{\tilde{Y}} = \frac{\beta\alpha\phi}{1-\beta-\beta\bar{\lambda}_a\bar{\theta}}. \tag{A40}$$

To get the investment-output ratio, we first solve for the investment-capital ratio by using the law of motion for capital stock in (A34) and then solve for the capital-output ratio using the capital Euler equation (A30). Specifically, we have

$$\frac{\tilde{I}}{\tilde{K}} = 1 - \frac{1-\delta}{\lambda_k}, \tag{A41}$$

$$\frac{\tilde{K}}{\tilde{Y}} = \left[1 - \frac{\beta}{\lambda_k}(\bar{\lambda}_a\bar{\theta} + 1 - \delta)\right]^{-1}\beta\alpha(1-\phi), \tag{A42}$$

where we have used the steady-state condition that $\tilde{q}_k = 1$, as implied by the investment Euler equation (A29). The investment-output ratio is then given by

$$\frac{\tilde{I}}{\tilde{Y}} = \frac{\tilde{I}}{\tilde{K}}\frac{\tilde{K}}{\tilde{Y}} = \frac{\beta\alpha(1-\phi)[\lambda_k - (1-\delta)]}{\lambda_k - \beta(\bar{\lambda}_a\bar{\theta} + 1 - \delta)}. \tag{A43}$$

Given the solution for the ratios $\frac{\tilde{q}_l L_e}{\tilde{Y}}$ and $\frac{\tilde{K}}{\tilde{Y}}$ in (A40) and (A42), the binding borrowing constraint (A38) implies that

$$\frac{\tilde{B}}{\tilde{Y}} = \bar{\theta} g_\gamma \frac{\tilde{q}_l L_e}{\tilde{Y}} + \frac{\bar{\theta}}{\bar{\lambda}_q} \frac{\tilde{K}}{\tilde{Y}}. \tag{A44}$$

The entrepreneur's flow of funds constraint (A37) implies that

$$\frac{\tilde{C}_e}{\tilde{Y}} = \alpha - \frac{\tilde{I}}{\tilde{Y}} - \frac{1 - \beta(1 + \bar{\lambda}_a)}{g_\gamma} \frac{\tilde{B}}{\tilde{Y}}. \tag{A45}$$

The aggregate resource constraint (A35) then implies that

$$\frac{\tilde{C}_h}{\tilde{Y}} = 1 - \frac{\tilde{C}_e}{\tilde{Y}} - \frac{\tilde{I}}{\tilde{Y}}. \tag{A46}$$

To solve for $\frac{L_h}{L_e}$, we first use the household's land Euler equation (i.e., the housing demand equation) (A25) and the definition for the marginal utility (A23) to obtain

$$\frac{\tilde{q}_l L_h}{\tilde{C}_h} = \frac{\bar{\varphi}(g_\gamma - \gamma_h)}{g_\gamma (1 - g_\gamma/R)(1 - \gamma_h/R)}, \tag{A47}$$

where the steady-state loan rate is given by (A39).

Taking the ratio between (A47) and (A40) results in the solution

$$\frac{L_h}{L_e} = \frac{\bar{\varphi}(g_\gamma - \gamma_h)(1 - \beta - \beta\bar{\lambda}_a\bar{\theta})}{\beta\alpha\phi g_\gamma (1 - g_\gamma/R)(1 - \gamma_h/R)} \frac{\tilde{C}_h}{\tilde{Y}}. \tag{A48}$$

Finally, we can solve for the steady-state hours by combining the labor supply equation (A24) and the labor demand equation (A28) to get

$$N = \frac{(1 - \alpha) g_\gamma (1 - \gamma_h/R)}{\bar{\psi}(g_\gamma - \gamma_h)} \frac{\tilde{Y}}{\tilde{C}_h}. \tag{A49}$$

B.5. **Log-linearized equilibrium system.** Upon obtaining the steady-state equilibrium, we log-linearize the equilibrium conditions (A23) through (A38) around the steady state. We define the constants $\Omega_h \equiv (g_\gamma - \beta(1 + \bar{\lambda}_a)\gamma_h)(g_\gamma - \gamma_h)$ and $\Omega_e \equiv$

$(g_\gamma - \beta\gamma_e)(g_\gamma - \gamma_e)$. The log-linearized equilibrium conditions are given by

$$\begin{aligned}
\Omega_h \hat{\mu}_{ht} &= -[g_\gamma^2 + \gamma_h^2 \beta(1+\bar{\lambda}_a)]\hat{C}_{ht} + g_\gamma \gamma_h (\hat{C}_{h,t-1} - \hat{g}_{\gamma t}) \\
&\quad -\beta \bar{\lambda}_a \gamma_h (g_\gamma - \gamma_h) \mathrm{E}_t \hat{\lambda}_{a,t+1} + \beta(1+\bar{\lambda}_a) g_\gamma \gamma_h \mathrm{E}_t (\hat{C}_{h,t+1} + \hat{g}_{\gamma,t+1}), && \text{(A50)} \\
\hat{w}_t + \hat{\mu}_{ht} &= \hat{\psi}_t, && \text{(A51)} \\
\hat{q}_{lt} + \hat{\mu}_{ht} &= \beta(1+\bar{\lambda}_a)\mathrm{E}_t\left[\hat{\mu}_{h,t+1} + \hat{q}_{l,t+1}\right] \\
&\quad +[1-\beta(1+\bar{\lambda}_a)](\hat{\varphi}_t - \hat{L}_{ht}) + \beta \bar{\lambda}_a \mathrm{E}_t \hat{\lambda}_{a,t+1}, && \text{(A52)} \\
\hat{\mu}_{ht} - \hat{R}_t &= \mathrm{E}_t\left[\hat{\mu}_{h,t+1} + \frac{\bar{\lambda}_a}{1+\bar{\lambda}_a}\hat{\lambda}_{a,t+1} - \hat{g}_{\gamma,t+1}\right], && \text{(A53)} \\
\Omega_e \hat{\mu}_{et} &= -(g_\gamma^2 + \beta\gamma_e^2)\hat{C}_{e,t} + g_\gamma \gamma_e (\hat{C}_{e,t-1} - \hat{g}_{\gamma t}) + \beta g_\gamma \gamma_e \mathrm{E}_t (\hat{C}_{e,t+1} + \hat{g}_{\gamma,t+1}), && \text{(A54)} \\
\hat{w}_t &= \hat{Y}_t - \hat{N}_t, && \text{(A55)} \\
\hat{q}_{kt} &= (1+\beta)\Omega\lambda_k^2 \hat{I}_t - \Omega\lambda_k^2 \hat{I}_{t-1} + \Omega\lambda_k^2(\hat{g}_{\gamma t} + \hat{g}_{qt}) \\
&\quad -\beta\Omega\lambda_k^2 \mathrm{E}_t[\hat{I}_{t+1} + \hat{g}_{\gamma,t+1} + \hat{g}_{q,t+1}], && \text{(A56)} \\
\hat{q}_{kt} + \hat{\mu}_{et} &= \frac{\tilde{\mu}_b}{\tilde{\mu}_e}\frac{\bar{\theta}}{\bar{\lambda}_q}(\hat{\mu}_{bt} + \hat{\theta}_t) + \frac{\beta(1-\delta)}{\lambda_k}\mathrm{E}_t(\hat{q}_{k,t+1} - \hat{g}_{q,t+1} - \hat{g}_{\gamma,t+1}) + \left(1 - \frac{\tilde{\mu}_b}{\tilde{\mu}_e}\frac{\bar{\theta}}{\bar{\lambda}_q}\right)\mathrm{E}_t \hat{\mu}_{e,t+1} \\
&\quad +\frac{\tilde{\mu}_b}{\tilde{\mu}_e}\frac{\bar{\theta}}{\bar{\lambda}_q}\mathrm{E}_t(\hat{q}_{k,t+1} - \hat{g}_{q,t+1}) + \beta\alpha(1-\phi)\frac{\tilde{Y}}{\tilde{K}}\mathrm{E}_t(\hat{Y}_{t+1} - \hat{K}_t), && \text{(A57)} \\
\hat{q}_{lt} + \hat{\mu}_{et} &= \frac{\tilde{\mu}_b}{\tilde{\mu}_e} g_\gamma \bar{\theta}(\hat{\theta}_t + \hat{\mu}_{bt}) + \left(1 - \frac{\tilde{\mu}_b}{\tilde{\mu}_e} g_\gamma \theta\right)\mathrm{E}_t \hat{\mu}_{e,t+1} + \frac{\tilde{\mu}_b}{\tilde{\mu}_e} g_\gamma \bar{\theta} \mathrm{E}_t(\hat{q}_{l,t+1} + \hat{g}_{\gamma,t+1}) \\
&\quad +\beta \mathrm{E}_t \hat{q}_{l,t+1} + (1 - \beta - \beta\bar{\lambda}_a\bar{\theta})\mathrm{E}_t[\hat{Y}_{t+1} - \hat{L}_{et}], && \text{(A58)} \\
\hat{\mu}_{et} - \hat{R}_t &= \frac{1}{1+\bar{\lambda}_a}\left[\mathrm{E}_t(\hat{\mu}_{e,t+1} - \hat{g}_{\gamma,t+1}) + \bar{\lambda}_a \hat{\mu}_{bt}\right], && \text{(A59)} \\
\hat{Y}_t &= \alpha\phi \hat{L}_{e,t-1} + \alpha(1-\phi)\hat{K}_{t-1} + (1-\alpha)\hat{N}_t - \frac{(1-\phi)\alpha}{1-(1-\phi)\alpha}[\hat{g}_{zt} + \hat{g}_{qt}], && \text{(A60)} \\
\hat{K}_t &= \frac{1-\delta}{\lambda_k}[\hat{K}_{t-1} - \hat{g}_{\gamma t} - \hat{g}_{qt}] + \left(1 - \frac{1-\delta}{\lambda_k}\right)\hat{I}_t, && \text{(A61)} \\
\hat{Y}_t &= \frac{\tilde{C}_h}{\tilde{Y}}\hat{C}_{ht} + \frac{C_e}{\tilde{Y}}\hat{C}_{e,t} + \frac{\tilde{I}}{\tilde{Y}}\hat{I}_t, && \text{(A62)} \\
0 &= \frac{L_h}{\bar{L}}\hat{L}_{ht} + \frac{L_e}{\bar{L}}\hat{L}_{et}, && \text{(A63)} \\
\alpha\hat{Y}_t &= \frac{\tilde{C}_e}{\tilde{Y}}\hat{C}_{e,t} + \frac{\tilde{I}}{\tilde{Y}}\hat{I}_t + \frac{\tilde{q}_l L_e}{\tilde{Y}}(\hat{L}_{et} - \hat{L}_{e,t-1}) \\
&\quad +\frac{1}{g_\gamma}\frac{\tilde{B}}{\tilde{Y}}(\hat{B}_{t-1} - \hat{g}_{\gamma t}) - \frac{1}{R}\frac{\tilde{B}}{\tilde{Y}}(\hat{B}_t - \hat{R}_t), && \text{(A64)} \\
\hat{B}_t &= \hat{\theta}_t + g_\gamma \bar{\theta}\frac{\tilde{q}_l L_e}{\tilde{B}}\mathrm{E}_t(\hat{q}_{l,t+1} + \hat{L}_{et} + \hat{g}_{\gamma,t+1})
\end{aligned}$$

$$+\left(1-g_\gamma\bar{\theta}\frac{\tilde{q}_l L_e}{\tilde{B}}\right)\mathrm{E}_t(\hat{q}_{k,t+1}+\hat{K}_t-\hat{g}_{q,t+1}). \tag{A65}$$

The terms $\hat{g}_{zt}$, $\hat{g}_{qt}$, and $\hat{g}_{\gamma t}$ are given by

$$\hat{g}_{zt} = \hat{\lambda}_{zt}+\hat{\nu}_{zt}-\hat{v}_{z,t-1}, \tag{A66}$$

$$\hat{g}_{qt} = \hat{\lambda}_{qt}+\hat{\nu}_{qt}-\hat{v}_{q,t-1}, \tag{A67}$$

$$\hat{g}_{\gamma t} = \frac{1}{(1-(1-\phi)\alpha)}\hat{g}_{zt}+\frac{(1-\phi)\alpha}{(1-(1-\phi)\alpha)}\hat{g}_{qt}. \tag{A68}$$

The technology shocks follow the processes

$$\hat{\lambda}_{zt} = \rho_z\hat{\lambda}_{z,t-1}+\hat{\varepsilon}_{zt}, \tag{A69}$$

$$\hat{\nu}_{zt} = \rho_{\nu_z}\hat{\nu}_{z,t-1}+\hat{\varepsilon}_{\nu_z t}, \tag{A70}$$

$$\hat{\lambda}_{qt} = \rho_q\hat{\lambda}_{q,t-1}+\hat{\varepsilon}_{qt}, \tag{A71}$$

$$\hat{\nu}_{qt} = \rho_{\nu_q}\hat{\nu}_{q,t-1}+\hat{\varepsilon}_{\nu_q t}. \tag{A72}$$

$$\tag{A73}$$

There preference shocks follow the processes

$$\hat{\lambda}_{at} = \rho_a\hat{\lambda}_{a,t-1}+\hat{\varepsilon}_{at}, \tag{A74}$$

$$\hat{\varphi}_t = \rho_\varphi\hat{\varphi}_{t-1}+\hat{\varepsilon}_{\varphi t}, \tag{A75}$$

$$\hat{\psi}_t = \rho_\psi\hat{\psi}_{t-1}+\hat{\varepsilon}_{\psi t}. \tag{A76}$$

The liquidity shock follows the process

$$\hat{\theta}_t = \rho_\theta\hat{\theta}_{t-1}+\hat{\varepsilon}_{\theta t}. \tag{A77}$$

We solve the 19 equations (A50) through (A68) for the 19 unknowns in the vector

$$x_t = [\hat{\mu}_{ht}, \hat{w}_t, \hat{q}_{lt}, \hat{R}_t, \hat{\mu}_{et}, \hat{\mu}_{bt}, \hat{N}_t, \hat{I}_t, \hat{Y}_t, \hat{C}_{ht}, \hat{C}_{et}, \hat{q}_{kt}, \hat{L}_{ht}, \hat{L}_{et}, \hat{K}_t, \hat{B}_t, \hat{g}_{\gamma t}, \hat{g}_{zt}, \hat{g}_{qt}]'.$$

The state variables consist of the predetermined variables and the exogenous forcing processes summarized in the vector

$$s_t = [\hat{C}_{h,t-1}, \hat{C}_{e,t-1}, \hat{I}_{t-1}, \hat{L}_{e,t-1}, \hat{K}_{t-1}, \hat{B}_{t-1}, \hat{\lambda}_{zt}, \hat{\nu}_t, \hat{\lambda}_{qt}, \hat{\mu}_t, \hat{\lambda}_{at}, \hat{\varphi}_t, \hat{\psi}_t, \hat{\theta}_t]'$$

We use Chris Sims's gensys algorithm to solve the model.

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