What You Match Does Matter: The Effects of Data on DSGE Estimation

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Abstract

This paper explores the effects of using different combinations of observables for the estimation of Dynamic Stochastic General Equilibrium (DSGE) models. I find that the estimated structural parameters and the model's outcomes are sensitive to the variables used for estimation. Depending on the set of observables the point estimate for habit formation ranges from 0.70 to 0.97. Similarly, the interest-smoothing coefficient in the Taylor rule fluctuates between 0.06 and 0.87. In terms of the model's predictions, if interest rates are excluded during estimation, the estimated structural coefficients are such that the model forecasts a strong deflation following an expansionary monetary expansion. More importantly, two ways to assess different observable sets are proposed. Based on these measures, I find that including the price of investment in the data set delivers the best results.

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1 Introduction

The search for a better understanding of the macroeconomic environment has pushed economists to formulate and analyze complex models. Such complexity makes economists rely more and more frequently on numerical solutions to their mathematical abstractions of reality. As a consequence, the policy implications of such models depend tremendously on the parameterization used to solve them. Most of this model-based policy analysis elaborates on the important contributions of Rotemberg and Woodford (1997) and Christiano, Eichenbaum, and Evans' (CEE, 2005).[1]

Although subsequent work in the spirit of CEE has adopted many of their economic assumptions (staggered price and wage contracting, costly adjustment in investment, and habit formation), there are significant departures with respect to the econometric approach and, in particular, the data used to estimate the model. CEE, for example, infer the structural parameters in their model by using nine observable variables (output, consumption, investment, interest rates, productivity, profits, inflation, money growth, and real wages). Del Negro et al. (2004), Justiniano and Primiceri (2006), Levin et al. (2005), and Smets and Wouters (2003) estimate their models using data on output, consumption, investment, labor, real wages, interest rates, and inflation. Likewise, Altig, Christiano, Eichenbaum, and Linde (2004) opt for the same seven variables plus the price of investment. Boivin and Giannoni (2006) use an even larger data set including additional measures of personal consumption and inflation. Finally, authors like Fernandez-Villaverde and Rubio-Ramirez (2007) and An and Schorfheide (2005) choose even smaller sets of observables.

At this stage it is unclear the effects that observables omission have on the estimated parameters and on the model's economic implications. Indeed, Fernandez-Villaverde and Rubio-Ramirez (2007) emphasize this lack of knowledge: " ...Unfortunately, we do not know much

[1] Recent examples of this approach include Levin et al. (2005), Schmitt-Grohe and Uribe (2004), and Eichenbaum and Fisher (2004).

about the right choice of observables and how they may affect our estimation results." An obvious question one can ask at this point is: why should we be concerned with the observables in the dataset? To answer this question, we know from regression analysis that adding more observables fictitiously improves model fitting at the cost of estimating more imprecise parameters. On the other hand, omission of relevant variables leads to biased coefficients. To make things more difficult, we also face issues such as data availability and computational costs.

In this paper I try to systematically analyze the effects of observables on the estimation of a fairly standard New-Keynesian model. To that end, I employ Bayesian methods and different combinations of observable variables. Some of the datasets typically used in the related literature are considered. My discussion of the effects of variable omission is divided in two parts. I begin analyzing the implications of different data sets on both the structural parameters' posterior distributions and the model's impulse responses. In particular, I report that the variables used for estimation influence both the median and dispersion of the estimated parameters. Next, I assess the overall effect of alternative observables based on two criteria: 1) a measure of total uncertainty associated with the estimated parameters (entropy), and 2) the mean square error (MSE) of the in-sample forecast.

In terms of the estimated parameters, my empirical exercise yields three broad results. First, the absence of observables (such as interest rates, inflation, and real wages) heavily influences the point estimates of the parameters capturing persistence in the model. For example, the point estimates for habit formation and the smoothing parameter in the Taylor rule fluctuate in the ranges $0.70 - 0.97$ and $0.06 - 0.87$, respectively.

Second, the steady state values of inflation, labor, and the growth rate of the economy are relatively robust to the exclusion of observable variables during estimation. Third, I find that the variance of the parameters' posterior distribution is significantly affected by the set of observables. Excluding labor or investment from the dataset greatly increases the uncertainty surrounding the estimated parameters.

When I turn to the impulse responses, I notice significant differences between the predictions of the same model but estimated using alternative observables. Among all variables, I find that the impulse responses of inflation, interest rates, labor, and real wages are quite sensitive to the exclusion of observable variables during the estimation stage. Take, for example, the responses to monetary shocks; excluding interest rates from the estimation stage implies structural parameters such that the model predicts a strong deflation following a monetary expansion.

An important issue not fully discussed in the literature is the effects of the sample size on the estimated parameters. For example, papers using the price of investment stop at the fourth quarter of 2001 or earlier. This is so because a reliable measure of this price is only available up to that quarter (Fisher, 2006). When I turn to the sample size issue, I find that omitting the sample 2002:I - 2004:IV has only minor effects on the point estimates and impulse responses.

Finally, the proposed performance measures favor the data set that includes: output, consumption, investment, interest rates, total hours, inflation, real wages, and price of investment. Specifically, this set delivers simultaneously small forecast errors (low MSE) and the smallest overall parameter uncertainty (lowest entropy). Those measures also suggest that the second best option is to use the same set but to exclude the price of investment. Furthermore, I find that this choice remains the second best alternative even if we ignore the post 2001 sample.

The rest of the paper proceeds as follows. In Section 2, I discuss the main features of the model to be estimated. The econometric approach and data description are contained in Section 3. A discussion of the parameters' posterior distributions and the model's impulse responses is in Section 4. I outline the performance measures and ranking of the alternatives in Section 5. Finally, the last section provides some concluding remarks.

2 Model

My formulation builds on Altig, Christiano, Eichenbaum, and Linde (2004, henceforth ACEL), CEE, Schmitt-Grohe and Uribe (2004) and Del Negro et al. (2004). Since this type of environment has been extensively discussed in the literature, I provide a brief discussion omitting lengthly derivations.[2] The main features of the model can be summarized as follows: The economy grows along a stochastic path; prices and wages are assumed to be sticky à la Calvo; preferences display external habit formation; investment is costly; and finally, there are four sources of uncertainty: neutral and capital embodied technology shocks, preference shocks and monetary shocks.

2.1 Firms

There is a continuum of monopolistically competitive firms indexed by $j \in [0,1]$ each producing a final good out of capital services, k_j, and labor services, $L_{j,t}$. The technology function is given by

$$k_{j,t}^{\alpha}\left(S_t^L L_{j,t}\right)^{1-\alpha} - S_t^* \psi,$$

ψ makes profits equal to zero in steady state. S_t^* is the stochastic growth path of the economy (see below for its definition).[3] The neutral technology shock, S_t^L, grows at rate g_t^L which is assumed to follow the process

$$\ln g_t^L = (1-\rho_{g^L}) \ln g_{ss}^L + \rho_{g^L} \ln g_{t-1}^L + \sigma^{g_L} \varepsilon_{g^L,t},$$

where $\varepsilon_{g^L,t}$ is distributed $\mathbb{N}(0,1)$.

Firms rent capital and labor in perfectly competitive factor markets. I assume that workers must be paid in advance. As a consequence, firms must borrow the wage bill, $W_t L_{j,t}$, from a

[2] For details, the interested reader is referred to the original papers.

[3] The growth term is needed to have a well-defined steady state around which we can solve the model.

financial intermediary. The loan plus the interest rate, R_t, must be repaid at the end of the period.

Firms choose prices to maximize the present value of profits; prices are set in a Calvo fashion; that is, each period, firms optimally revise their prices with an exogenous probability $1-\xi_p$. If, instead, a firm does not re-optimize its price, then the price is updated according to the rule: $P_{j,t} = \pi_{t-1} P_{j,t-1}$, where π_{t-1} is the economy-wide inflation in the previous period. Consequently, an optimizing firm at time t sets prices according to the program

$$\max_{P_{j,t}} E_t \sum_{n=0}^{\infty} \left(\xi_p \beta\right)^n \lambda_{t+n} \left[\frac{P_{j,t} \Pi_{\tau=0}^{n-1} \pi_{t+\tau}}{P_{t+n}} y_{t+n}(j) - mc_{t+n} y_{t+n}(j) \right],$$

Here, P_t is the price index, $y_t(j)$ is the aggregate demand for good type j, mc_t is firm j's marginal cost, β is the discount factor, and λ_t is the marginal utility of consumption at time t.

2.2 Households

The economy is populated by a continuum of households indexed by i. Every period households must decide how much to consume, work, and invest. In addition, they must choose the amount of money to be sent to a financial intermediary. I assume agents in the economy have access to complete markets; such assumption is needed to eliminate wealth differentials arising from wage heterogeneity (CEE, and Erceg, Henderson, and Levin, 2000). Households maximize the expected present discounted value of utility

$$E_0^i \sum_{t=0}^{\infty} \beta^t \left[S_t^{Uc} \log(C_{i,t} - bC_{t-1}) - \Phi \frac{L_{i,t}^{1+1/\gamma}}{1+1/\gamma} + \psi_m \left(\frac{M_{i,t}}{S_t^* P_t} \right)^{1-\zeta_m} \right] \tag{1}$$

subject to

$$P_t C_{i,t} + \frac{P_t}{S_t^K} \left(I_{i,t} + a(x_t) K_{i,t} \right) + \mathcal{M}_{i,t} = R_t(\mathcal{M}_{i,t-1} - M_{i,t} + T_t) + R_t^K x_t K_{i,t} + W_{i,t} L_{i,t} + A_{i,t},$$

and

$$K_{i,t+1} = (1-\delta)K_{i,t} + I_{i,t}\left(1 - \Gamma(\frac{I_{i,t}}{I_{i,t-1}})\right),$$

Here, E_t^i is the time t expectation operator conditional on the information set of household i; S^{Uc} is a preference shock that follows the process $\log S_t^{Uc} = \rho_{Uc} \log S_{t-1}^{Uc} + \sigma^{Uc}\varepsilon_{U_c,t}$ with $\varepsilon_{U_c,t}$ distributed $\mathbb{N}(0,1)$; preferences display external habit formation, measured by $b \in (0,1)$; and Γ is a function reflecting the costs associated to adjusting the investment portfolio. This function is assumed to be increasing and convex satisfying $\Gamma = \Gamma' = 0$ and $\kappa \equiv \Gamma'' > 0$ in steady state. $\mathcal{M}_{i,t-1}$ is household i's beginning of period t stock of money, whereas T_t is a lump-sum transfer by the government. Households send the amount $\mathcal{M}_{i,t-1} - M_{i,t} + T_t$ to a financial intermediary where it earns the interest rate, R_t. The stochastic trend, $S_t^* = S_t^L \left(S_t^K\right)^{\alpha/(1-\alpha)}$, in the money term is required to have a well-defined steady state. The term S_t^K is an investment specific shock whose growth rate obeys

$$\log g_t^K = (1-\rho_{g^K}) \log g_{ss}^K + \rho_{g^K} \log g_{t-1}^K + \sigma^{g^K} \varepsilon_{g^K,t},$$

where $\varepsilon_{g^K,t}$ is distributed $\mathbb{N}(0,1)$.

As in ACEL, CEE, and Schmitt-Grohe and Uribe (2004), I assume that physical capital can be used at different intensities. Furthermore, using the capital with intensity x_t entails a cost $a(x_t)$, which satisfies $a(1) = 0; a"(1) > 0; a'(1) > 0$. For future reference, define $\varkappa_a = a"(1)$.

The term $A_{i,t}$ captures net payments from complete markets plus profits from producers. The individual consumption good is assumed to be a composite made of differentiated goods indexed by j according to the aggregator

$$C_{i,t} = \left(\int_0^1 c_t(i,j)^{\frac{\zeta-1}{\zeta}} dj\right)^{\frac{\zeta}{\zeta-1}}, \quad 1 \leq \zeta < \infty,$$

where $c(i,j)$ is the demand of household i for good type j. With this type of composite good,

the demand for goods of type j is given by

$$c(i,j) = \left(\frac{P_{j,t}}{P_t}\right)^{-\zeta} C_{i,t}.$$

Here, The nominal price index is $P_t = \left(\int_0^1 P_{j,t}^{1-\zeta} dj\right)^{\frac{1}{1-\zeta}}$. Similarly, I assume that individual investment obeys $I_{i,t} = \left(\int_0^1 I_t(i,j)^{\frac{\zeta-1}{\zeta}} dj\right)^{\frac{\zeta}{\zeta-1}}$. As with consumption, $I(i,j)$ denotes household i's demand for investment good of type j.

2.3 Wage Setting

Following Erceg, Henderson, and Levin (2000), I assume that each household is a monopolistic supplier of a differentiated labor service, $L_{i,t}$. Households sell these labor services to a competitive firm that aggregates labor and sell it to final firms. The technology used by the aggregator is

$$\widetilde{L}_t = \left[\int_0^1 L_{i,t}^{\frac{\zeta_w-1}{\zeta_w}} dj\right]^{\frac{\zeta_w}{\zeta_w-1}}, \quad 1 \leq \zeta_w < \infty.$$

It is straightforward to show that the relation between the labor aggregate and the wage aggregate, W_t, is given by

$$L_{i,t} = \left[\frac{W_t}{W_{i,t}}\right]^{\zeta_w} \widetilde{L}_t.$$

To induce wage sluggishness, I assume that households set their wages in Calvo fashion. In particular, with exogenous probability ξ_w a household does not re-optimize wages each period. If this is the case, wages are set according to the rule of thumb $W_{i,t} = \pi_{t-1} W_{i,t-1}$. Similar to the firms, households set wages according to the program

$$\max_{W_{i,t}} E_t \sum_{n=0}^{\infty} (\xi_w \beta)^n \left[-\Phi \frac{L_{i,t+n}^{1+1/\gamma}}{1+1/\gamma} + \lambda_{t+n} \frac{W_{i,t} \Pi_{\tau=0}^{n-1} \pi_{t+\tau}}{W_{t+n}} \frac{W_{t+n}}{P_{t+n}} L_{i,t+n}\right].$$

I do not index the marginal utility of consumption, λ, by i to reflect our assumption of complete markets.

2.4 Government

The monetary authority sets the short-term interest rate according to a Taylor rule. In particular, the central bank smoothes interest rates and responds to deviations of actual inflation from steady state inflation, π, and deviations of output from its trend level, $(Y/S^*)_t$.

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_r} \left[\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi} \left(\frac{Y_t/S_t^*}{(Y/S^*)_t}\right)^{\phi_y}\right]^{1-\rho_r} \exp(\sigma_m \varepsilon_{m,t}). \tag{2}$$

The term $\varepsilon_{m,t}$ is a random shock to the systematic component of monetary policy and is assumed to be standard normal; σ_m is the size of the monetary shock. Other authors have implemented similar Taylor rules, e.g. Del Negro et al. (2004) and Justiniano and Primiceri (2006). As in the related literature (ACEL, Christiano et al., 2005, Levin et al., 2005), I assume that the government has access to lump-sum taxes and implements a Ricardian fiscal policy.

2.5 Financial Intermediaries

Financial intermediaries receive money from two sources: households' deposits and transfers from the government $\int (\mathcal{M}_{i,t-1} - M_{i,t})di + T_t$. All this money is lent to final good firms so they can pay workers at the beginning of each period. Consequently, the clearing condition in the loan market is $\int (W_t L_{j,t})\, dj = T_t + \int (\mathcal{M}_{i,t-1} - M_{i,t})di$.

3 Estimation

I estimate the model using different sets of observables. Since the model allows for growth over time, I follow the standard practice of re-scaling the growing variables by appropriate factors so that the new model is stationary. The resulting model is then solved using a linear

approximation about the steady state. For the baseline formulation, I include seven variables: output, consumption, investment, real wage, labor, nominal interest rates, and inflation. Several recent DSGE papers have used those variables for estimation purposes; hence, they seem a natural starting point.[4] Next, I successively suppress one observable variable at a time and re-estimate the model. The resulting median estimates and 95% probability intervals are reported in Table 3. To provide a comprehensive analysis, I also consider three additional cases. First, as in ACEL and Levin et al. (2005) I include the price of investment as an extra observable variable. Since a reliable measure for this variable is only available for a smaller subsample, I then estimate the original seven-variable specification using the reduced sample (see Data section for details). The third specification involves the estimation of the model using the following six observable variables: output, investment, real wages, inflation, interest rates, and the price of investment.[5] A total of 11 alternatives are considered.

While the list of alternatives I consider is far from exhaustive, it provides concrete evidence that the set of observables does indeed have an effect not only on the estimated parameters but also on the model's predictions.

- **Data**

In the baseline formulation, I estimate the model using seven U.S. variables: the growth rates of output, consumption, investment, and real wages ($\Delta \ln Y_t, \Delta \ln C_t, \Delta \ln I_t,\ \Delta \ln (w/P)_t$), and the levels of labor, nominal interest rates, and inflation ($\ln L_t, i_t, \pi_t$). The data is that used in Justiniano and Primiceri (2006); it is extracted from Haver Analytics database and spans from 1954:III up to 2004:IV. They build the series as follows: Real GDP results from dividing nominal GDP by population (16 years and older) and the GDP deflator. Real consumption is the sum of personal consumption of non-durables and services. Real investment consists of personal consumption expenditures of durables and gross private domestic investment. To obtain per

[4] Examples include Del Negro, Schorfheide, Smets and Wouters (2004), and Justiniano and Primiceri (2006).
[5] This choice is based on the recent work by Fernandez-Villaverde and Rubio-Ramirez (2007):

capita measures, both real consumption and real investment are divided by population. The log of hours of all persons in the non-farm business sector divided by population corresponds to labor in the paper. Real wages result from dividing nominal wage per hour in the non-farm business sector by the GDP deflator. Interest rates correspond to the effective Federal Funds Rate while inflation is the quarterly log difference of the GDP deflator. Finally, when the price of investment is included in the estimation I use Fisher's (2006) investment series spanning from 1954:III to 2001:IV.

- **Bayesian Inference**

Following Schorfheide (2000), Smets and Wouters (2003), and Del Negro et al. (2004), I estimate the linearized version of the model using Bayesian methods. In particular, the posterior distribution of the structural parameters is characterized using a Markov chain Monte Carlo (MCMC) approach (for details of this algorithm see the appendix and the excellent surveys of An and Schorfheide, 2007, and Geweke, 1999). Since there are seven observable variables and only four structural shocks, I avoid stochastic singularity by following Sargent (1989) in including measurement errors to the state space representation used to estimate the model.[6] These errors are assumed to be iid and distributed $\mathbb{N}(0, \sigma)$. The scale of these errors can vary across the measurement equations. The results in the next sections are based on a Markov chain of 100,000 draws after discarding 20,000 replications from a burn in phase. The convergence properties of the chains are discussed in the appendix.

- **Priors**

A subset of the parameter space was fixed: $\alpha = 0.36$, $\delta = 0.025$, $\zeta_w = 21$, $\zeta_m = 10.58$, $\psi_m = 0.055$. The parameters for money demand correspond to those estimated in Christiano et al. (2005). Since I choose to estimate labor in steady state, the parameter Φ is endogenously

[6] Since we observe the exact values of interest rates, measurement errors were not included in the equation corresponding to interest rates in the state space representation.

determined. The prior distributions for the remaining parameters are reported in Table 1. These priors are loose and consistent with those typically used in the literature (see Del Negro et al., 2004, Levin et al., 2005, Justiniano and Primiceri, 2006, and Fernandez-Villaverde and Rubio-Ramirez, 2007). For future reference, let Ξ denote the set of all parameters to be estimated.

4 Results

4.1 Posterior Distributions

Table 2 reports the median estimates for the elements of the set Ξ under the baseline formulation (estimation based on seven observables). Numbers in parenthesis correspond to the 2.5th and 97.5th percentiles computed with the draws of the posterior simulator (a 95% probability interval). The shocks, σ_i, are expressed in percentage points. The absence of the price of investment as an observable variable implies that the two trends in the model, S^L and S^K, are not separately identified in the baseline scenario. Therefore, I set the steady state growth rate of the investment-specific shock to one, $g^K = 1$. The last six entries in the third panel of Table 2 correspond to the scales of the measurement errors.

Broadly speaking, the estimates are in line with the results previously found in the literature (Christiano, Eichenbaum, and Evans, 2005, and Smets and Wouters, 2004). For example, prices and wages are re-optimized on average every 6.6 and 3 quarters, respectively. The model displays significant habit formation, around 0.93, and some adjustment costs in investment, around 0.75. These values, however, are consistent with the recent evidence reported in Fernandez-Villaverde and Rubio-Ramirez (2007). The estimated Taylor rule implies that the central bank actively responds to inflation and smoothes lagged interest rates with coefficients similar to those found in Justiniano and Primiceri (2006). Finally, the Frisch elasticity, γ, is well within the values used in related studies. Christiano et al. (2005), for example, set this elasticity to 1.

Next I report the effects that the removal or addition of observables has on the structural

parameters' point estimates and their confidence intervals. To save space, I only report a subset of the estimated coefficients (see Table 3).[7,8] First, I provide a general discussion of the results and then elaborate on the individual impact of each observable variable.

- **General Results**

Table 3 displays the medians and 95% probability intervals from the estimation of the model with different observable variables. For example, the second row presents the results when labor is omitted from the estimation. Similarly, the row labeled "Price Investment" reports the estimated parameters when the observables in the baseline scenario plus the price of investment are used for inference. A quick look at Table 3 reveals several interesting results. First, note that except for two cases the parameter for sticky prices, ξ_p, is consistently estimated around 0.85, a value that implies that price contracts are reviewed every 6.6 quarters.[9] Moreover, this parameter is tightly estimated; note the small 95% probability interval. The parameters capturing the steady state of inflation, π, the discount factor, β, the growth rate of neutral technology, g_L, and labor, L, enjoy similar consistency.

Second, the exclusion of observables has a sizable effect on parameters generating smoothness in the model. Specifically, habit formation, b, sticky wages, ξ_w, and costly investment, κ, are quite sensitive to the data; the first parameter ranges from 0.70 to 0.97 while the second one is in the interval 0.43 to 0.79.

Third, all coefficients in the Taylor rule vary with the set of observables. For example, ρ, the interest rate smoothing parameter, can be as low as 0.06 (no output) and as high as 0.87 (no wages). The response to output fluctuations, ϕ_y, is estimated in the range 0.02 - 0.77. In addition, the reaction to inflation, ϕ_π, shows small but still important deviations from the baseline estimation. More importantly, the presence of interest rates in the measurement

[7] I omit the persistence coefficients for the structural shocks and the scales of the measurement errors. They are available upon request.

[8] Each time an observable equation was removed, I also dropped its measurement error from the estimation process.

[9] The two exceptions happened when either labor or real wages were not included as observable variables.

equations is crucial to pin down the Taylor rule; note the significant changes reported in the sixth row of Table 3.

Fourth, the structural shocks in the economy also present considerable fluctuations relative to the baseline estimates. Take, for example, the size of the neutral technology shock; it fluctuates in the range 0.18 - 0.91. Changes in the structural shocks are important as they affect the second order properties of the model and impulse responses. Further discussion on this issue is provided in section 4.2.Finally, the Frisch elasticity of labor supply, γ, presents some variability but less extreme than with the smoothing parameters.

Next, I provide an analysis of the impact of different observable sets. Understanding the effects that omitting each observable variable has on each single parameter is a daunting task. In fact, each case can be easily the subject of a separate paper. Here, I pursue a more modest approach and concentrate only on a reduced number of relevant results.

- **No Labor**

In the absence of labor nothing pins down its steady state in the model. Therefore, I set this value to the mean of labor in the sample, which is 52.85. Interestingly, the absence of labor has a small impact on the point estimates. Two important exceptions are the Calvo probabilities for prices and the adjustment cost in investment. According to the new estimates, price contracts are reviewed on average every 2.5 quarters.

- **No Consumption**

Surprisingly, excluding consumption from the estimation has practically no effect on the estimated parameters. Except for the standard deviation of the preference shock, σ_{U_c}, all the remaining parameters are no more than 5% away from the baseline estimates. To understand the increase in the size of the preference shock, note this shock is directly attached to consumption through the Euler equation

$$\lambda_t = S_t^{U_c} \frac{1}{C_t - bC_{t-1}}, \tag{3}$$

where λ is the Lagrangian multiplier associated to the budget constraint. The absence of consumption as an observable variable gives extra freedom to the preference shock. In particular, its size can go up to provide more variability in the model, as now consumption volatility is no longer a concern.

- **No Real Wage**

CEE argue that wage sluggishness is needed for two reasons. First, it provides the persistent behavior of real wages. Second, sticky wages help to avoid abrupt changes in real costs; such changes are undesirable as they induce volatility in inflation. In the absence of real wages as an observable, wage stickiness only plays its second role; as a consequence, the model seems to require a low wage persistence, $\xi_w = 0.44$, combined with high price sluggishness, $\xi_p = 0.93$.

There is an important impact on the coefficients of the Taylor rule. Note that interest rate-smoothing, ρ_R, increases while the central bank's reaction to inflation, ϕ_π, declines by an appreciable margin. High price stickiness induces smooth inflation reducing the need for the central bank to tame inflation. Furthermore, the increase in the volatilities of the investment-specific and preference shocks, σ_{g^K} and σ_{U^c}, magnifies the variability of output; hence the larger reaction of the central bank to output deviations.

To understand the increase in the costly investment parameter, κ, notice that the increase in the size of the investment-specific shock, σ_{g_K}, implies more volatile investment. To make this variable compatible with the data, the model requires additional adjustment costs.

- **No Interest Rate**

Estimating the model without interest rates has a pronounced impact on all estimated parameters (except the sticky price parameter). For example, the size of the monetary shock, σ_m, increases by a factor of 3. The model seems to compensate this extra volatility with neutral and investment-specific smaller shocks so that the model's predictions are compatible with the

data. In terms of the Taylor rule, we see that the central bank becomes less responsive to lagged interest rates and more reactive to output deviations.

The last findings are hardly surprising because in the absence of interest rates, the parameters of the Taylor rule are only identified through the effects that interest rates have on marginal costs (recall that firms must pay the wage bill in advance) and the intertemporal Euler equation

$$mc_t = \left(\frac{R_t w_t}{S_t^L(1-\alpha)}\right)^{1-\alpha}\left(\frac{r_t^k}{\alpha}\right)^{\alpha}, \tag{4}$$

$$MU_t = \beta E_t \frac{R_{t+1}}{\pi_{t+1}} MU_{t+1}, \tag{5}$$

where, r_t^k is the return on capital, w_t is real wages, and MU_t is marginal utility of consumption. The lack of interest rate persistence induces more volatile marginal costs (equation 4). To counteract the additional variability, the model requires additional sluggishness. Price and wage contracts are now reviewed on average every 7 and 4.8 quarters, respectively.

The absence of interest rates implies that habit formation is 33% smaller than in the baseline scenario. To understand this result, note that habit formation helps to reconcile a declining interest rate with a smooth rising consumption following an expansionary monetary shock (Christiano et al., 2005).[10] With less persistent interest rates, this last tension decreases and therefore the need for habit formation.

- **No Output**

In the absence of output, the model has more freedom to explain consumption and investment. In an effort to match the volatile investment series, the estimation procedure pushes the costly adjustment parameter, κ, to very low values at which point the algorithm collapses. Therefore, I set this parameter to 0.1. We also see that the estimation procedure increases the size of investment-specific shock in an attempt to match the volatility of investment.

[10] In terms of equation (5), habit formation makes the change in the growth rate of consumption be related to real interest rate.

The additional volatility generated by the investment-specific shocks is accompanied by a decline in the size of the other structural shocks; presumably, in an attempt to keep the models' second moment predictions of variables such as consumption and labor compatible with the data. The smaller volatility generated by the preference shock implies less volatile consumption. Hence, less habit formation is needed to match the data.

A puzzling finding is the decline in the degree of interest rate smoothing in the Taylor rule. To understand this result, we must note that the absence of output decreases the central bank's incentives to react to deviations to this variable. Such decline can be captured either by a drop in ϕ_y or in ρ_r (equation, 2). My estimation approach seems to prefer a contraction in the later parameter. Finally, the marginal costs become less volatile due to smoother neutral technology shocks (see equation, 4). As a consequence, the need for sticky prices and wages declines. Some sluggishness, however, is still required to match the smooth profile of inflation and real wages.

- **No Inflation**

The presence of persistence parameters such as sticky prices/wages and the lagged interest rate are relevant mainly to describe the behavior of inflation. Hence, the absence of this variable removes an important source of identification of persistence in the model. Furthermore, without inflation the model must deliver less persistence, mainly to generate the inertial response of output to monetary shocks. The results in Table 3 indicate that my estimation approach reduces persistence in the model by keeping some price stickiness accompanied by a significant decline not only in sticky wages but also in the inertial component of the Taylor rule. In particular, wage contracts are reviewed on average every 7 months while persistence in the Taylor rule is 0.28.

A second important finding is the decline in the size of the monetary shock; now it is one third smaller than in the baseline scenario. Even after taking into account uncertainty, the differences remain. To understand this decline, note that the size of this shock faces a trade-off. On one hand, it must be large enough to match the reaction of inflation and output to monetary

policy. On the other hand, the Taylor rule shock must be compatible with the observed changes in interest rates. The absence of inflation alleviates the first trade-off; hence, the decline in the size of the shock we observe. Interestingly, the significant increase in this shock when interest rates are omitted suggests that matching inflation requires strong movements on monetary policy.

- **Price of Investment**

For this case, the model was re-estimated using the original seven observable variables plus the price of investment series as reported in Fisher (2006). Now there is enough information to estimate the trend of the investment-specific shock, g^K. I find its median estimate to be around 1.0045 with a 95% probability interval of $[1.0032, 1.0060]$. The presence of this additional trend should diminish the importance of the growth rate of the neutral shock as the economy now grows at the rate $g^L \left(g^K\right)^{a/(1-\alpha)}$; this is exactly what Table 3 reveals: the new point estimate of g^L drops to 1.001.

Although the inclusion of the price of investment series has some effect, especially on the Taylor rule estimates, the new estimates are similar to those found in the baseline scenario. Based solely on the median estimates, we may conclude that the price of investment is important if we are interested in identifying its trend. However, the results in the next sections suggest that including that observable might improve the model's performance along some dimensions.

- **No Wage/No Consumption**

In a recent paper, Fernandez-Villaverde and Rubio-Ramirez (2007) estimate a DSGE model excluding real wages and consumption from the set of observables. To evaluate the effect of their observable set, I re-estimate the model using the following data set: output, investment, labor, nominal interest rates, inflation, and the price of investment. For comparison purposes, the model estimated with the baseline variables and the price of investment (eight row, Table 3) may be more relevant.

Consistent with the results obtained when consumption is omitted (third row, Table 3), I find significantly larger preference shocks and larger habit formation. The new point estimates imply that price contracts are reviewed on average every 7 quarters. We also note that the Taylor rule does not change relative to that for the price-of-investment case.

Making a direct comparison with Fernandez-Villaverde and Rubio-Ramirez's results is tempting. However, we should note that our models differ in several dimensions. First, I include full indexation while their approach allows for partial indexation. Second, contrary to their specification, there is no intermediate firms in my model. More importantly, structural parameters in their study can change over time.

- **No Investment**

The major effect of the absence of investment is the substantial increase in the cost of investment adjustment. We can rationalize this result as follows. This cost of adjustment must meet two requirements. First, it must be small enough to match the volatility of investment. Second, it must be large enough to imply a sufficiently smooth investment series that when coupled with consumption is compatible with the observed series for output. In the absence of investment, the first trade-off vanishes so the model now requires a larger adjustment cost. With regards to the remaining parameters, we observe that the absence of investment has some impact on sticky wages.

- **Smaller Sample**

The last row of Table 3 presents the results from the estimation when the sample is restricted to the period 1954:III - 2001:IV. The results suggest that the point estimates are not affected by the choice of the shorter sample. We might interpret this evidence as a consequence of the relative stability the U.S. economy has enjoyed over the past decade.

In summary, I find that the following variables are quite sensitive to the set of observables: habit formation, wage stickiness, the Frisch elasticity, the scale of the preference shock, and the

investment adjustment cost. For example, the point estimate of sticky wages ranges from 0.43 to 0.79. If we take into account uncertainty, these bounds further expand to 0.32 and 0.83.

I also find that the estimation of the parameters for the steady levels of inflation, neutral technology growth, and labor is more robust to the exclusion of observable variables. Take, for example, the steady state of inflation, π. The results in Table 3 indicate that this parameter is tightly estimated around 1.010, with a difference of less that 0.4% between the largest and smallest estimates.

4.2 Impulse Responses

Now I turn to the implications that the different cases have on the model's economic predictions as summarized by impulse responses. As with the previous analysis, reporting a comprehensive analysis of each single impulse response requires significant amount of space and is beyond the purpose of this paper.[11] Instead, I provide a brief discussion of general patterns emphasizing the idea that the variables used to estimate the model do indeed influence the model's outcome. Figures 1 through 3 present the impulse responses of the estimated models to a one-standard deviation of the structural shocks at time 0. Values in the vertical axes correspond to percentage deviations from the variables' steady state trends. For all cases, the impulse responses are computed using the median estimates reported in Table 3. For reference, starred lines indicate the predictions using the parameter estimates from the baseline formulation; solid lines correspond to the upper and lower bounds of a 90% probability interval for the baseline case.

Consistent with the point estimates in Table 3, the impulse responses for the baseline scenario, starred lines, are broadly in line with the finding in the literature (see, for example, ACEL and Del Negro et al., 2004). From a quick look at the impulse responses we learn that the exclusion of variables during estimation can dramatically change the model's performance. A more careful analysis reveals that the model's predictions for inflation, interest rates, and real

[11] There are 4 structural shocks, 7 relevant variables, and 11 sets of parameters. As a consequence, we would need to discuss a total of 308 impulse responses!

wages are very sensitive to the set of observable variables. For all shocks, the responses associated with those variables can be either positive or negative. Interestingly, the model estimated excluding interest rates as an observable predicts a strong deflation following an expansionary monetary shock.[12] Yet the same model estimated using all variables but labor predicts a significant rise in inflation after the monetary expansion. These result is consistent with Boivin and Giannoni (2006) who find that the inclusion of additional variables during estimation can affect the model's predictions with respect to inflation.

Note that the impulses responses for the cases when we exclude consumption or we include the price of investment fall inside the 90% probability interval of the baseline case. This results is expected since the median estimates for the baseline scenario are pretty similar to those obtained including price of investment or excluding consumption (see, Table 3). Finally, we observe that omitting real wages or interest rates has the most pronounced effects on the impulse responses.

5 Ranking the Alternatives

What is the minimum set of observables a researcher should use? From the results in the previous section, we see that the answer to this question can severely influence the model's performance. Choosing few variables seems to deliver counterintuitive answers such as deflations following an expansionary monetary policy. On the other hand, there is the issue of data availability and dimensionality.[13] In this section, I propose some measures, which I hope, can be used as a guide to what variables should be used for estimation.

[12] To understand this result, note that the low interest rates persistence, $\rho_R = 0.27$, implies that households anticipate a short-lived monetary expansion. As a consequence, there is a small increase in consumption and investment. On the other hand, the strong initial decline in interest rates cuts costs by an important margin, which in turn pushes prices down. The combined effect is the persistent deflation.

[13] Each additional observable variable increases the size of the state space representation and, therefore, the time to estimate the model. One evaluation of the posterior distribution takes on average 0.234 seconds in the price of investment case and 0.218 seconds in the no wage/no consumption case. This small difference becomes relevant when the posterior must be evaluated thousands of times as in the MCMC algorithm.

In my formulation, although the model's structure is unmodified, the marginal likelihood of the data changes along the different specifications. As a consequence, direct application of posterior-odds comparisons as in Geweke (1999), and Fernandez-Villaverde and Rubio-Ramirez (2004) becomes unfeasible. Yet we would like to have a statistical way to assess the validity of the distinct alternatives. The two measures I apply to rank the alternatives are: in-sample forecast, and overall parameter uncertainty as measured by entropy.

5.1 In-Sample Fit

The absence of an observable variable during the estimation stage does not preclude the researcher from using the model to forecast the data. This is so because for any combination of observable variables, the inference strategy pursued in this paper always delivers estimates for the structural parameters in the set Ξ (see Section 3). Consequently, the model's forecast can be recovered by using the Kalman filter. For the rest of this section, I base my analysis on the following variables: the growth rates of output, consumption, investment, and real wages, and the levels of labor, inflation and interest rates. The forecasting period is 1957:1 - 2004:IV, with the last year been included as the price of investment is not part of the forecast exercise.

To study the forecasting properties of the model under the different datasets, we could compare directly the mean square errors (MSE) for the one-step ahead forecasts. Such an approach, however, ignores the changing nature of the data and tends to favor the estimation with larger datasets. Here, I opt to present a statistic based on the MSE, which also penalizes for larger datasets:

$$SBC = T \times ln(MSE) + n \times ln(T).$$

Here, T is the forecasting size and n is the number of variables used for estimation. It is not a coincidence that I labeled this statistic SBC as it resembles the Schwartz Bayesian Criterion. The first row of Table 4 presents the statistic for all seven forecasted variables. They are computed using the Kalman filter and the median estimates from the baseline scenario (Table

2). The remaining rows show the SBC for the other alternative observable sets relative to that of the baseline scenario. Each column gives the relative statistic for each of the variables under study; the column labeled Tot reports the sum of the individual SBC; all sums are expressed relative to that of the baseline case.

Casual observation of the results in Table 4 reveals important patterns. First, excluding inflation from the set of observables has the largest impact on the forecasting performance. For all variables, the SBC is at least 20% larger than those in the baseline formulation. The worsening of the forecast is quite notorious for labor, consumption, investment, and output. To understand this result, note that, in the absence of inflation, the adjustment cost parameter, κ, is estimated to be a very low number (Table 3). As a consequence, investment and output become very volatile relative to the data, which in turn justifies the worsening of the forecasts reported in Table 4.

Second, inflation, interest rates, and labor are the variables most affected by the exclusion of observable variables. In particular, the inflation forecasts significantly worsen along the alternative specifications; in 6 out of 11 cases, inflation's SBC worsened by more than any other variable. This result confirms Boivin and Giannoni's (2006) finding that the inflation forecast benefits from the inclusion of additional observable variables such as alternative measures of inflation and economic activity.

Third, the exclusion of consumption, the use of a smaller sample or the inclusion of the price of investment have relatively minor effects on the forecast performance of the model. Note that output, consumption, and investment are significantly correlated in the data. Hence, the damaging effects that the exclusion of consumption has on forecasts seems to be mitigated by the informational content of the remaining variables.

Fourth, in terms of forecasting real variables, the estimated model with the price of investment as an observable variable is the best option. In particular, investment and output benefits the most from including investment's price. Such a result is likely to be a consequence

of the additional trend introduced by the price of investment. If we turn to forecasting nominal variables (inflation and interest rates), note that the baseline formulation delivers the best results.

Finally, the forecast based on the estimated parameters from the sample 1954-2001 delivers results comparable to the forecasts based on the complete sample (first and last rows of Table 4). The baseline forecast is superior only for interest rates. Such findings are a likely sign of structural stability over the past few years.

5.2 Entropy: The Role of Uncertainty

Next, I turn to the discussion of the effects that different observable variables have on the parameters' posterior distributions. Figures (4) through (7) display the posterior densities for selected parameters under the different combinations of observable variables. The starred lines correspond to the baseline case. We clearly see that the omission of variables during the estimation stage have strong implications not only on the median estimates but also on the parameters' distributions; such results are expected in light of the findings from the RMSE and expected risk measures. Consider, for example, the discount factor, sticky wages, and the cost of investment adjustment parameter; Figures (4) and (5) show how the location and dispersion of these distributions change considerably depending on the set of measurement equations.

We observe similar effects on the distributions of Taylor rule parameters, Figure (6). Particularly notorious is the change in shape experienced by the output-reaction parameter, ϕ_y. Finally, Figure (7) also shows that the dispersion for the monetary and preference shocks is affected too.

The pervasive effects of uncertainty have been previously studied in the literature. For example, Levin et al. (2005) find that parameter uncertainty greatly affects our inference of the welfare loss associated with fluctuations. Furthermore, their results suggest that from the point of view of a policy maker, tightly estimated parameters are desirable, as they might lead to

sharper conclusions about the optimal economic policies.[14] Hence, we would prefer to estimate the model with a set of observable variables that minimizes the total variance associated with the parameters.

To uncover such set, I follow the information theory literature in using the concept of entropy, H, as a indicator of uncertainty. Cover (1991), for example, states that " [the] entropy of a random variable is a measure of the uncertainty of the random variable; it is a measure of the amount of information required on average to describe the random variable." Mathematically, the entropy of a random variable, X, is defined as:

$$\begin{aligned} H &= -E\left[\log_2 f(x)\right] \\ &= -\int \left[\log_2 f(x)\right] f(x) dx \geq 0. \end{aligned} \tag{6}$$

Here, $f(.)$ is the probability density function of the random variable X. For example, the entropy of random variable $x \sim \mathbb{N}(\mu, \sigma^2)$ is $H = 0.5\left(\log_2(2\pi\sigma^2) + 1\right)$. Clearly, it increases with the dispersion of the distribution. In this study, X is a multivariate object, which includes all estimated parameters but the standard deviations of the measurement errors (the ranking does not change if we include them).

Some remarks about entropy are necessary. First, a direct application of equation (6) to distributions of 6 or more variables is unfeasible (mainly due to computer memory limitations). The results reported here are based on the alternate measure: $\widetilde{H} = \sum_i \int \left[\log f_i(x_i)\right] f_i(x_i) dx_i$, where $f_i(.)$ corresponds to density function of the random variable i.[15] Second, including price of investment in the estimation adds g_{ss}^K, the steady state growth rate of the investment-specific shock, to the list of estimated parameters. To make a fair comparison across alternatives, I add the entropy of g_{ss}^K to the entropy of all cases in which this parameter is not estimated.

[14] In Guerron (2007), I find that uncertainty does affect the model's predictions for the welfare costs of inflation.

[15] Note $\widetilde{H} = H$ only when the random variables X_i are independent. For all other cases, the alternative definition is an approximation.

Finally, entropy should be taken only as an ordinal measure because its value changes with the logarithmic base used to compute it.[16]

The results from the entropy-based measure, reported in column H of Table 4, reveal some interesting features. First, the formulation that excludes consumption as an observable variable obtains the smallest total uncertainty (lowest entropy). Intuitively, the parameters' posterior distributions in the no consumption case are more tightly concentrated around the median estimates that in the other specifications. Similarly, the set of observables that includes the price of investment also achieves the lowest entropy. That these two cases have roughly the same entropy is hardly surprising. A quick look at Table 3 indicates that the size of the 95% probability intervals for the case without consumption and that including price of investment are very close.

Second, the highest entropies (largest uncertainty) are achieved when labor or investment are omitted from the estimation. Although the absence of labor has minor implications in the median estimates (relative to the baseline scenario), the resulting increase in uncertainty suggests that the baseline scenario should be preferred. A similar conclusion is applicable to the cases in which investment is excluded for estimation purposes.

Third, the use of the smaller sample 1954:III - 2001:IV has practically no effect on entropy. According to the results in Table 4, the smaller sample option dominates the baseline specification. As previously argued, this result might be a consequence of the stability of the U.S. economy over the past decade. Finally, notice that entropy does not necessarily fall when a variable is excluded. If we compare the entropy of the small sample case (7 variables) with that of the no wage/no consumption specification (6 variables), we see that the former case improves on the later.

[16] If the natural logarithm base is used, entropy is measured in *nats*. If the base is 2, then the units are *bits*.

5.3 Discussion

To conclude this section, we should note that the best results, based on the proposed measures (SBC and entropy), are obtained when the data contains the following variables: output, consumption, real wages, investment, inflation, interest rates, labor, and the price of investment. Furthermore, the use of the smaller sample $1954 - 2001$ seems to have little or no effect on the estimated parameters and the model's performance. Yet, if, for data constraints or some other specific reason, a researcher must use a smaller data set, the second best option is to exclude the price of investment. The cost of such omission is additional uncertainty, measured by the entropy of the estimated parameters, and the lack of identification of the trend associated with that price. Interestingly, the exclusion of consumption for estimation purposes seems to be a viable alternative.

Finally, my measures of performance suggest that we should preferably avoid excluding variables such as inflation, interest rates, real wages, or labor. Omitting these variables significantly worsens the SBC or the entropy or both. Furthermore, the inclusion of inflation, interest rates, and real wages is important as they bring crucial information to identify, among other things, sticky prices/wages and the parameters in the Taylor rule.

6 Conclusion

The observable variables used to estimate DSGE models have more importance than previously thought. Exclusion of variables like interest rates, real wages and inflation may severely biased the point estimates of parameters such as those describing sticky contracts on prices and wages, and habit formation. Furthermore, dropping observable variables may imply estimated parameters such that the model predicts strong deflation following an expansionary monetary shock.

Based on the posterior distribution of the structural parameters, the entropy of the estimated

parameters, and the impulse responses predicted by the model, the set of observable variables should at least consist of seven variables: output, consumption, investment, real wages, total labor, interest rates, and inflation. Ideally, the data should also include the price of investment as this variable allows the identification of two separate trends in the models, delivers good forecasts as measured by the SBC statistic, and decreases the overall uncertainty in the estimated model. Adding the price of investment requires the use of a smaller sample; however, the results in this paper indicate that omitting the post 2001:IV observations is inconsequential for both point estimates and impulse responses.

An issue not addressed in this paper is the effect of different observables on identification. In an ongoing project, I explore these identification issues following the methodology discussed in Canova (2007). Preliminary results suggest that the omission of, for example, real wages complicates the identification of the sticky price and wage parameters.

Finally, in an open-economy framework the possibilities are even larger as data from exchange rates, current account, and trade flows can be incorporated into the estimation.[17] Such analysis is left for future research.

[17] For example, Adolfson et al. (2007) use ten variables to estimate an open economy model for the European area.

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7 Appendix A: MCMC Algorithm

Let $p(\varphi)$ and $p(Y_T|\varphi)$ be the prior distribution of the parameter vector φ and the likelihood of the data conditional on the parameter vector, respectively. I use the data, a state-space representation of the model, and the Kalman Filter to evaluate the posterior distribution $p(\varphi|Y_T)$. The results reported in the paper are based on 120000 draws $\varphi_{(n)}$ from the posterior distribution generated using a random walk Metropolis-Hasting algorithm.[18] At each iteration n, a candidate parameter vector $\widetilde{\varphi}$ is drawn from the distribution $\mathbb{N}(\varphi_{(n-1)}, c_o^2 \sum)$ and the acceptance ratio, r, is computed

$$r = \frac{p(Y_T|\widetilde{\varphi})p(\widetilde{\varphi})}{p(Y_T|\widetilde{\varphi}_{(n-1)})p(\widetilde{\varphi}_{(n-1)})}.$$

The new draw $\widetilde{\varphi}$ is kept with probability $\min(r, 1)$ and rejected otherwise.

To characterize the variance of the jumping distribution, $c_o^2 \sum$, I proceed as follows. First, I apply Christopher Sims' *csminwel* code to compute the mode of the posterior distribution. To that end, 5000 draws from the prior distribution were used to evaluate the posterior. The 30 draws achieving the highest posteriors are the initial points for Sims' minimization algorithm. Second, I compute the inverse Hessian at the mode and use it as the variance of the jumping distribution. Third, the constant c_o is set to achieve an acceptance rate close to 0.35, a value typically suggested in the literature (Casella and Roberts, 2004, and Fernandez-Villaverde and Rubio-Ramirez, 2005).

To assess convergence of the resulting algorithm, I follow two approaches. First, as common practice in the literature, I run three chains each of size 500000 (the first 10000 iterations were discarded). Each chain started at a different random draw from the jumping distribution centered at the mode. All chains generated very similar results for the recursive means of each estimated parameter. Second, to further confirm convergence, I compute Brooks and Gelman's (1998) interval and variance potential scale reduction factors (PSRF) for each parameter and

[18] The initial 20000 iterations were discarted.

dataset. According to Brook and Gelman (1998), factors below 1.1 are indicative of convergence. For all parameters and datasets, the variance PSRF factors were less than 1.005 while the interval factors were less than 1.05.[19]

[19] For space limitations, I do not report the results, which are available upon request.

Table 1: Priors Densities for Structural Parameters

σ_m	σ^{g_L}	σ^{g_K}	σ^{U_C}	b	ξ_w	ξ_p	γ	ρ_R	ϕ_π
IG [2,1]	IG [2,1]	IG [2,1]	IG [4,2]	B [0.6,0.1]	B [0.6,0.1]	B [0.6,0.1]	N [1,0.15]	B [0.75,0.1]	N [1.60,0.3]
ϕ_y	κ	S_{Uc}	g_L	π	L	ζ	ζ_w	$\varkappa_a$	β
G [0.12,0.1]	N [3,1]	N [1.3,0.3]	N [1.01,0.003]	N [1.01,0.002]	N [52.89,3]	N [6.7,1]	N [6.7,1]	N [0.5,0.1]	B [0.99,0.002]
ρ_{g_L}	ρ_{g_K}	ρ_{Uc}	g_K	σ_{out}	σ_{cons}	σ_{invest}	σ_{labor}	σ_{wage}	σ_{inflat}
B [0.5,0.15]	B [0.5,0.15]	B [0.5,0.15]	N [1.01,0.003]	IG [0.05,0.03]	IG [0.05,0.03]	IG [0.05,0.03]	IG [0.05,0.03]	IG [0.05,0.03]	IG [0.05,0.03]

Notes: IG˜Inverse Gamma, B˜Beta, N˜Normal, G˜Gamma

Mean and Standard Deviation in square brackets

Table 2: Estimated Parameters Baseline Case

σ_m	σ^{g_L}	σ^{g_K}	σ^{U_C}	b	ξ_w	ξ_p	γ	ρ_R	ϕ_π
0.26 [0.23,0.29]	0.68 [0.59,0.78]	0.25 [0.19,0.33]	3.64 [2.48,5.46]	0.93 [0.89,0.95]	0.69 [0.60,0.78]	0.85 [0.82,0.87]	1.49 [0.88,2.33]	0.66 [0.58,0.72]	1.57 [1.41,1.73]
ϕ_y	κ	α	g_L	π	L	ζ	ζ_w	$\varkappa_a$	β
0.05 [0.03,0.07]	0.75 [0.47,1.20]	0.36 [NA]	1.003 [1.001,1.005]	1.009 [1.006,1.013]	56.83 [52.57,61.34]	7.92 [6.28,10.64]	21 [NA]	0.55 [0.38,0.74]	0.9938 [0.990,0.996]
ρ_{g_L}	ρ_{g_K}	ρ_{Uc}	g_K	σ_{out}	σ_{cons}	σ_{invest}	σ_{labor}	σ_{wage}	σ_{inflat}
069 [0.60,0.78]	0.91 [0.86,0.78]	0.98 [0.96,0.99]	1.00 [NA]	0.26 [0.13,0.35]	0.42 [0.38,0.48]	1.66 [1.43,1.90]	0.05 [0.02,0.11]	0.22 [0.19,0.25]	0.50 [0.45,0.57]

Notes: L: steady state labor; π: steady state inflation; g_L: growth rate of neutral technology

Table 3: Estimated Parameters

	σ_m	σ_{g_L}	σ_{g_K}	σ_{U_C}	b	ξ_w	ξ_p	γ
Baseline	0.26 [0.23,0.29]	0.68 [0.59,0.78]	0.25 [0.19,0.33]	3.64 [2.48,5.46]	0.93 [0.89,0.95]	0.69 [0.60,0.78]	0.85 [0.82,0.87]	1.49 [0.88,2.33]
No Labor	0.26 [0.23,0.28]	0.91 [0.63,1.26]	0.56 [0.38,0.81]	3.49 [2.29,5.39]	0.90 [0.85,0.93]	0.71 [0.63,0.77]	0.61 [0.54,0.67]	1.65 [0.98,2.65]
No Consumption	0.26 [0.23,0.30]	0.65 [0.55,0.76]	0.26 [0.20,0.33]	4.18 [2.69,6.75]	0.94 [0.89,0.97]	0.72 [0.62,0.80]	0.85 [0.81,0.87]	1.64 [0.96,2.54]
No Real Wage	0.25 [0.23,0.28]	0.60 [0.49,0.70]	0.60 [0.47,0.71]	7.32 [4.42,11.8]	0.97 [0.95,0.98]	0.44 [0.23,0.62]	0.93 [0.90,0.95]	0.99 [0.53,1.94]
No Inflation	0.18 [0.15,0.20]	0.59 [0.51,0.67]	0.22 [0.18,0.29]	2.76 [2.00,3.80]	0.91 [0.85,0.94]	0.44 [0.39,0.54]	0.86 [0.81,0.90]	1.73 [1.05,2.74]
No Interest Rate	0.72 [0.28,2.42]	0.18 [0.11,0.34]	0.17 [0.10,0.30]	4.30 [1.94,9.89]	0.70 [0.60,0.78]	0.79 [0.76,0.83]	0.86 [0.82,0.89]	1.91 [1.23,2.98]
No Output	0.20 [0.16,0.27]	0.39 [0.30,0.49]	0.91 [0.70,1.13]	2.20 [1.65,3.20]	0.85 [0.73,0.93]	0.43 [0.32,0.55]	0.82 [0.78,0.85]	1.61 [1.30,1.81]
Price Investment	0.28 [0.25,0.32]	0.57 [0.47,0.68]	0.16 [0.12,0.20]	3.26 [2.18,4.98]	0.92 [0.86,0.95]	0.64 [0.52,0.72]	0.83 [0.79,0.85]	1.72 [1.09,2.65]
No Wage/Consump	0.27 [0.24,0.30]	0.67 [0.53,0.78]	0.16 [0.13,0.20]	5.80 [3.75,9.13]	0.97 [0.95,0.99]	0.53 [0.38,0.65]	0.86 [0.84,0.89]	1.21 [0.69,2.01]
No Investment	0.25 [0.23,0.28]	0.38 [0.29,0.46]	0.31 [0.24,0.42]	4.29 [2.89,6.28]	0.93 [0.88,0.96]	0.56 [0.42,0.69]	0.83 [0.79,0.86]	1.49 [0.89,2.40]
Small Sample	0.27 [0.24,0.30]	0.70 [0.61,0.80]	0.27 [0.20,0.35]	3.21 [2.21,5.05]	0.93 [0.88,0.95]	0.65 [0.57,0.74]	0.84 [0.81,0.86]	1.66 [0.98,2.60]

Table 3: Estimated Parameters (Continued)

	ρ_R	ϕ_π	ϕ_y	κ	β	g_L	π	L
Baseline	0.66 [0.58,0.72]	1.57 [1.41,1.73]	0.05 [0.03,0.07]	0.76 [0.47,1.20]	0.9938 [0.990,0.996]	1.003 [1.001,1.005]	1.009 [1.006,1.013]	56.83 [52.57,61.34]
No Labor	0.76 [0.71,0.81]	1.62 [1.46,1.78]	0.02 [0.007,0.05]	1.18 [0.73,1.90]	0.9946 [0.992,0.996]	1.004 [1.002,1.005]	1.012 [1.008,1.015]	52.85 [NA]
No Consumption	0.65 [0.58,0.71]	1.59 [1.43,1.74]	0.046 [0.026,0.073]	0.74 [0.46,1.19]	0.9948 [0.992,0.997]	1.003 [1.001,1.005]	1.011 [1.007,1.014]	53.93 [50.85,57.83]
No Real Wage	0.87 [0.78,0.93]	1.31 [1.12,1.52]	0.21 [0.09,0.47]	2.17 [1.19,3.63]	0.9920 [0.987,0.995]	1.004 [1.002,1.006]	1.009 [1.005,1.013]	57.01 [52.83,61.41]
No Inflation	0.28 [0.18,0.39]	1.62 [1.45,1.79]	0.23 [0.15,0.33]	0.075 [0.03,0.13]	0.9814 [0.976,0.986]	1.006 [1.005,1.008]	1.009 [1.006,1.014]	50.95 [48.82,52.79]
No Interest Rate	0.27 [0.17,0.39]	1.64 [1.48,1.81]	0.77 [0.48,1.18]	0.036 [0.02,0.08]	0.9783 [0.974,0.981]	1.007 [1.005,1.008]	1.010 [1.006,1.014]	54.14 [52.59,55.78]
No Output	0.06 [0.03,0.09]	1.38 [1.23,1.56]	0.089 [0.065,0.116]	0.10 [NA]	0.9973 [0.996,0.998]	1.006 [1.005,1.008]	1.009 [1.006,1.014]	56.30 [52.28,55.38]
Price Investment	0.57 [0.49,0.63]	1.28 [1.18,1.41]	0.05 [0.03,0.07]	0.63 [0.39,1.09]	0.9960 [0.994,0.997]	1.001 [1.00,1.003]	1.011 [1.007,1.015]	57.56 [54.16,61.22]
No Wage/Consump	0.57 [0.51,0.63]	1.25 [1.18,1.34]	0.04 [0.03,0.06]	0.53 [0.35,0.83]	0.997 [0.996,0.998]	1.004 [1.002,1.005]	1.012 [1.008,1.015]	53.01 [51.24,55.44]
No Investment	0.70 [0.64,0.76]	1.54 [1.40,1.70]	0.03 [0.016,0.051]	2.91 [1.86,4.27]	0.9974 [0.996,0.998]	1.005 [1.004,1.007]	1.013 [1.008,1.015]	53.91 [50.62,58.02]
Smaller Sample	0.64 [0.56,0.70]	1.57 [1.41,1.73]	0.047 [0.03,0.07]	0.63 [0.40,0.99]	0.9953 [0.993,0.997]	1.003 [1.002,1.005]	1.010 [1.007,1.014]	52.93 [50.13,56.20]

Table 4: One-Step Ahead Forecast (SBC)

	y	c	i	L	w	π	R	Tot	$H(bits)$
Baseline	1025	756	1514	951	825	535	534	877.15	75.43
No Labor	1.03	1.04	1.06	1.19	0.98	1.80	1.13	1.13	1.30
No Consumption	1.00	1.10	1.00	1.05	0.99	1.06	1.02	1.03	**0.95**
No Real Wage	1.04	1.06	1.02	1.22	1.06	1.35	1.03	1.09	1.06
No Inflation	1.49	1.41	1.36	1.78	1.21	1.42	1.39	1.44	1.08
No Interest Rate	1.09	1.19	1.07	1.20	1.03	1.62	2.32	1.26	1.07
No Output	1.05	1.00	1.01	1.25	0.98	1.40	1.04	1.08	0.97
Price Investment	0.98	1.00	0.99	0.99	0.99	1.04	1.39	1.03	**0.95**
No Wage/Consump	1.09	1.10	1.02	1.19	1.06	2.02	1.92	1.39	1.15
No Investment	1.08	1.01	1.03	1.06	1.12	1.21	0.96	1.06	1.31
Smaller Sample	1.00	1.00	0.99	0.99	0.99	1.00	1.01	1.00	0.99

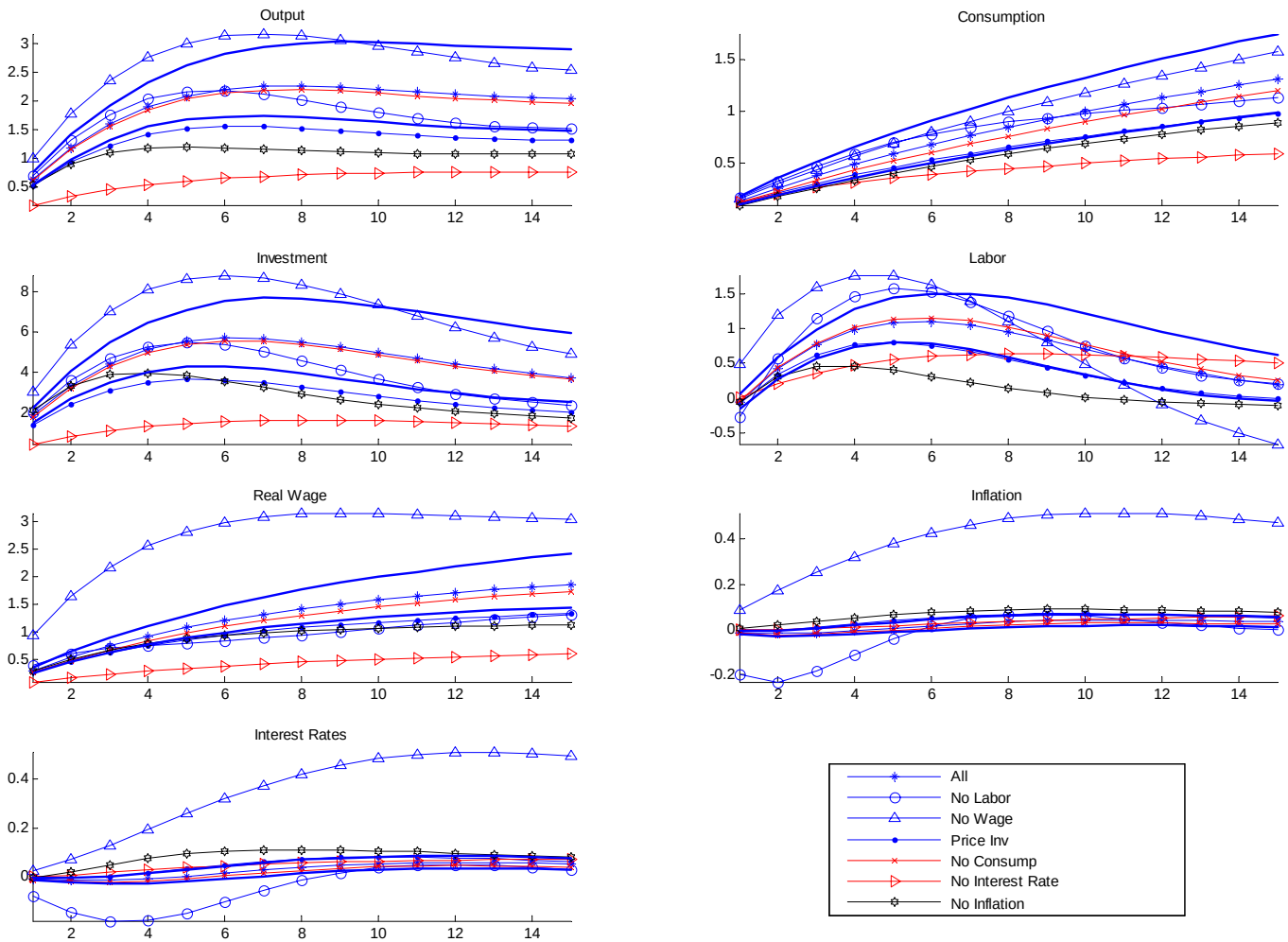

Figure 1: Efects of a Positive Investment-Specific Shock

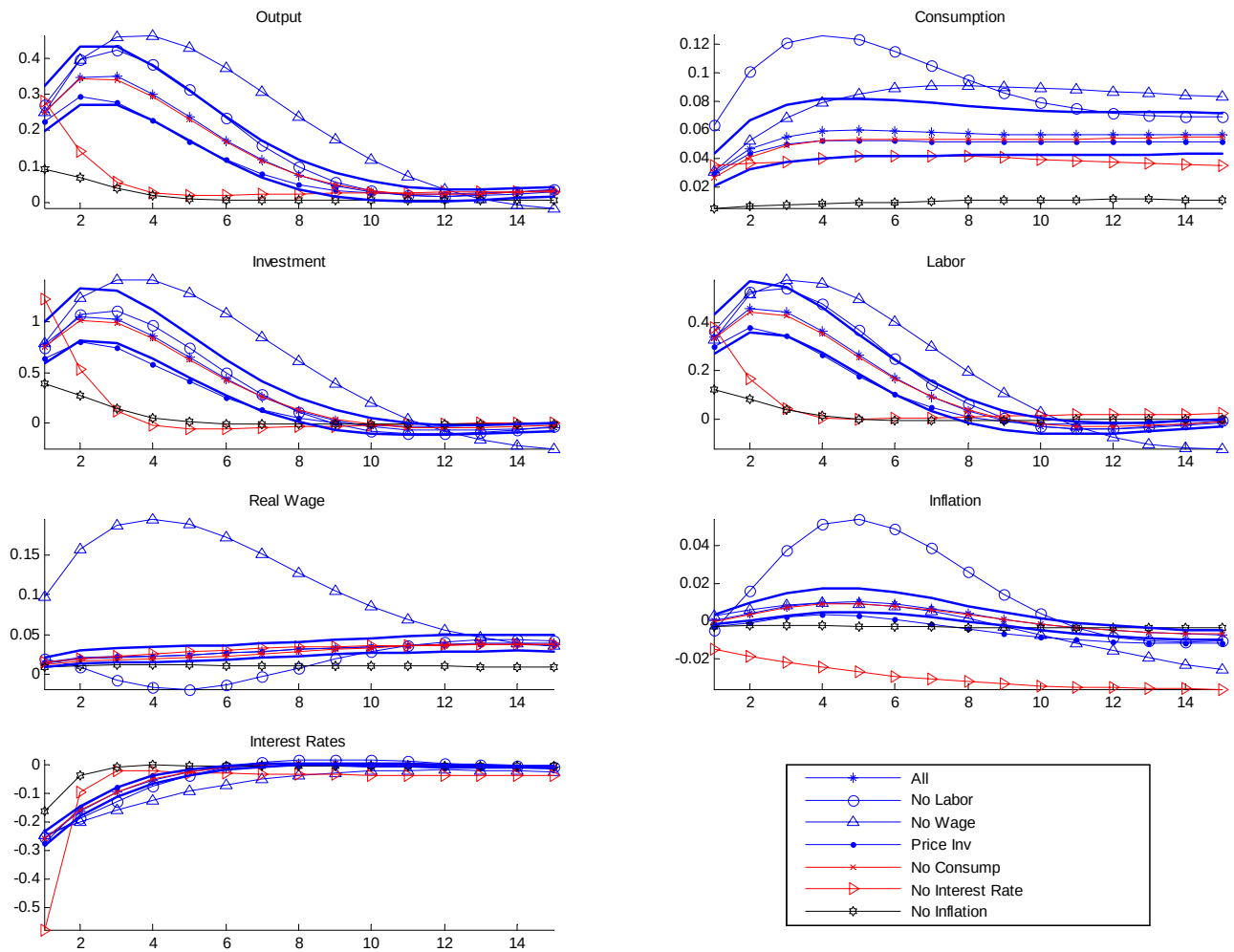

Figure 2: Effects of an Expansionary Monetary Shock

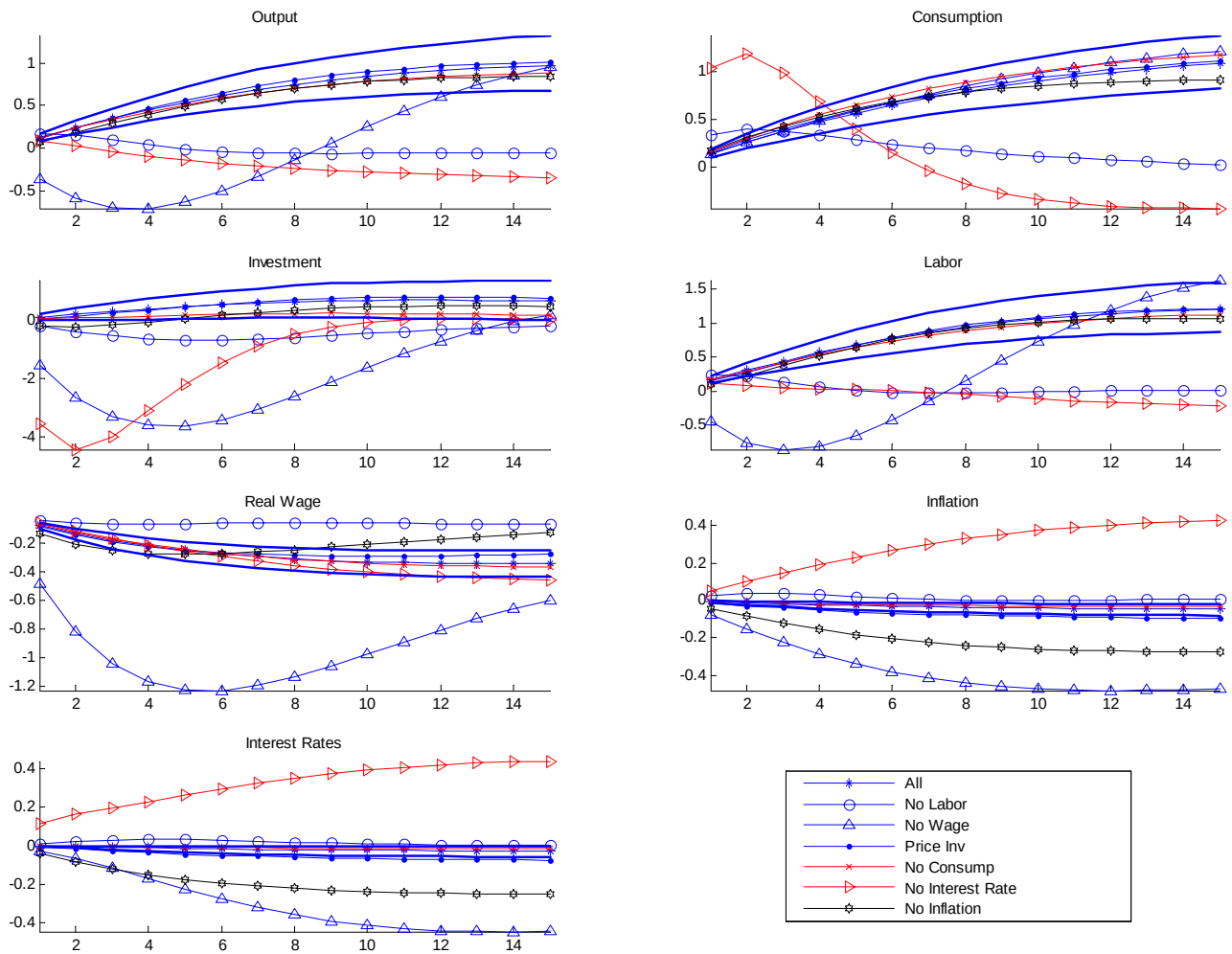

Figure 3: Effects of a Positive Preference Shock

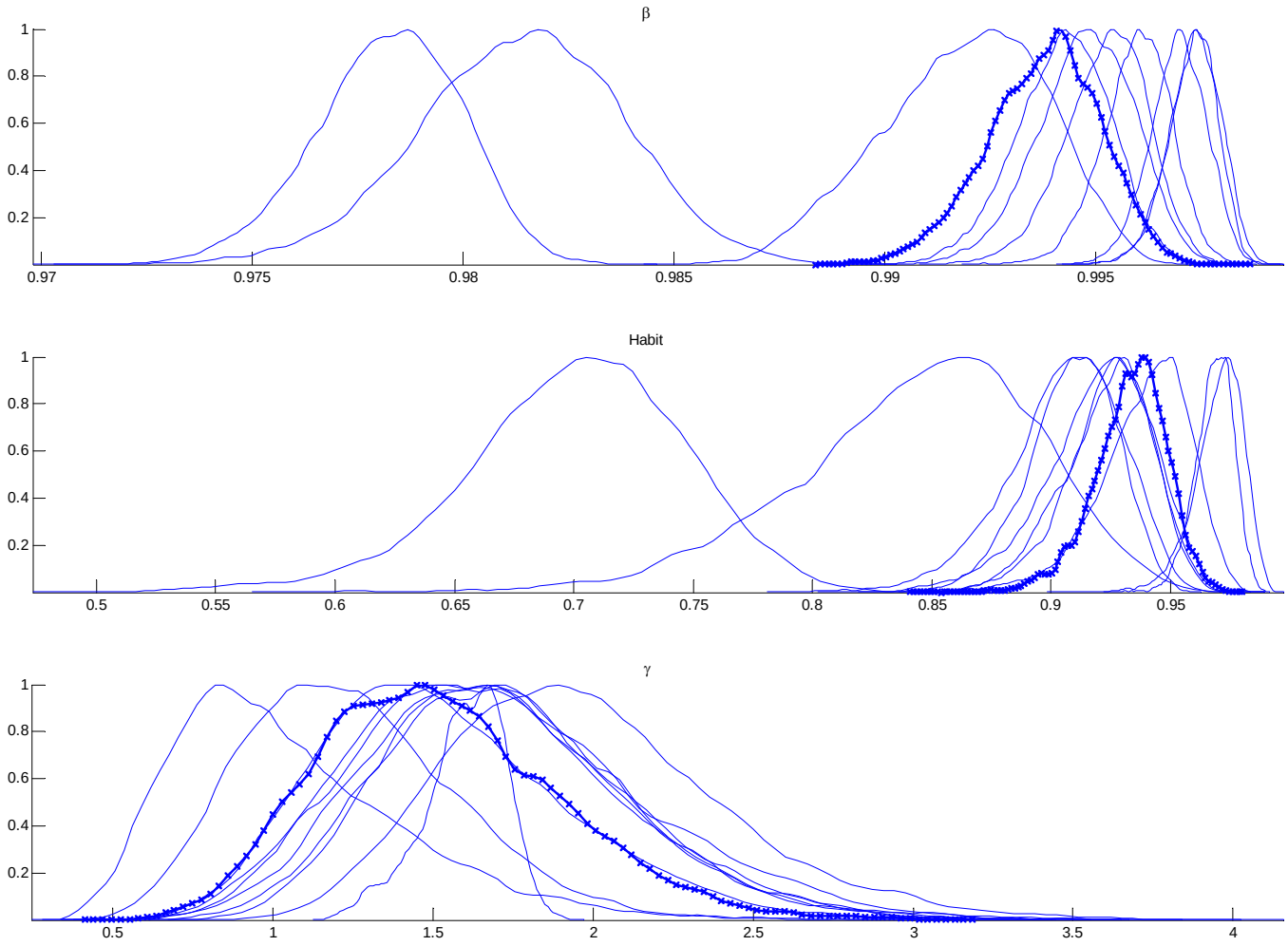

Figure 4: Parameters Posterior Densities

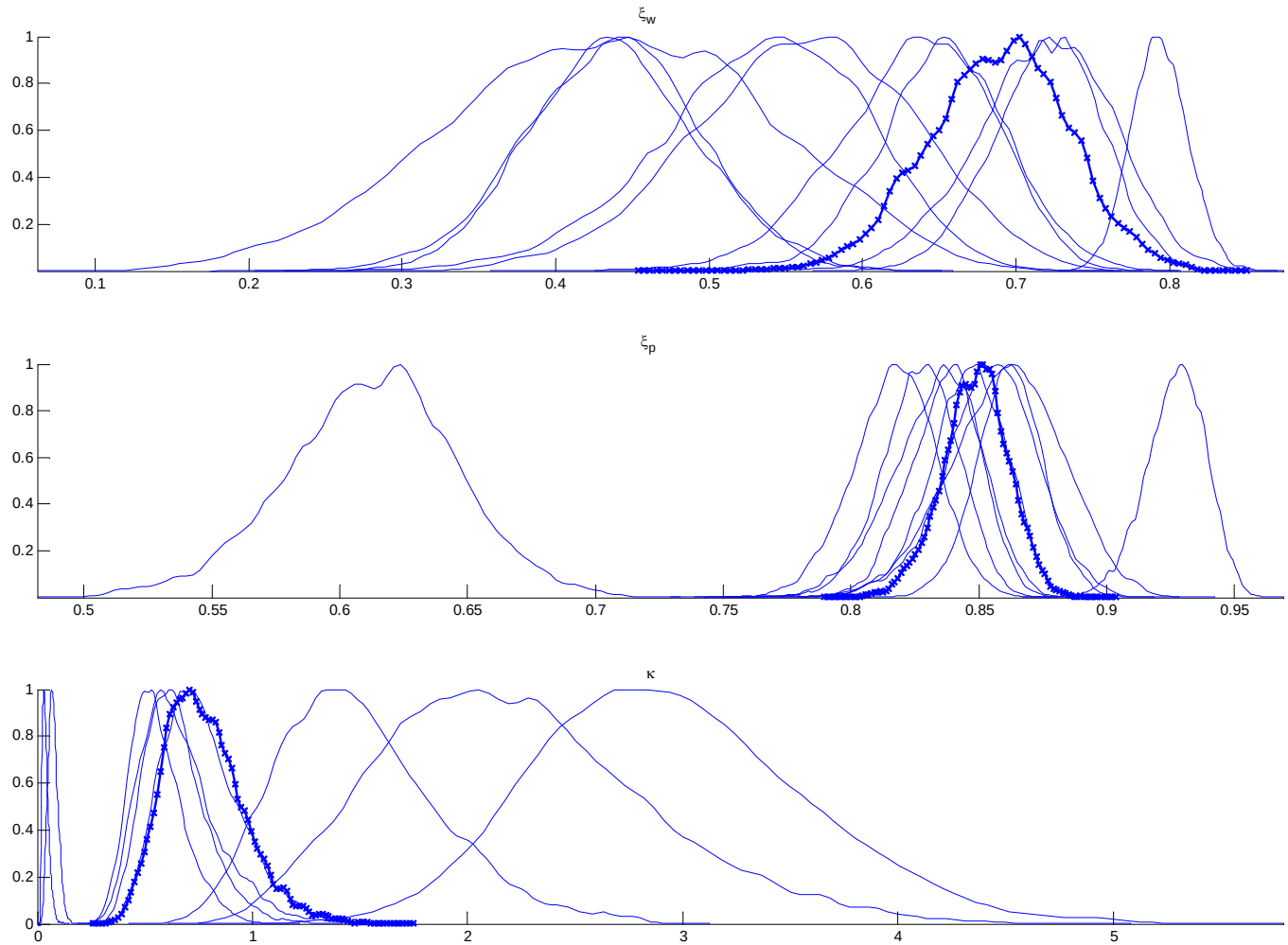

Figure 5: Parameters Posterior Densities

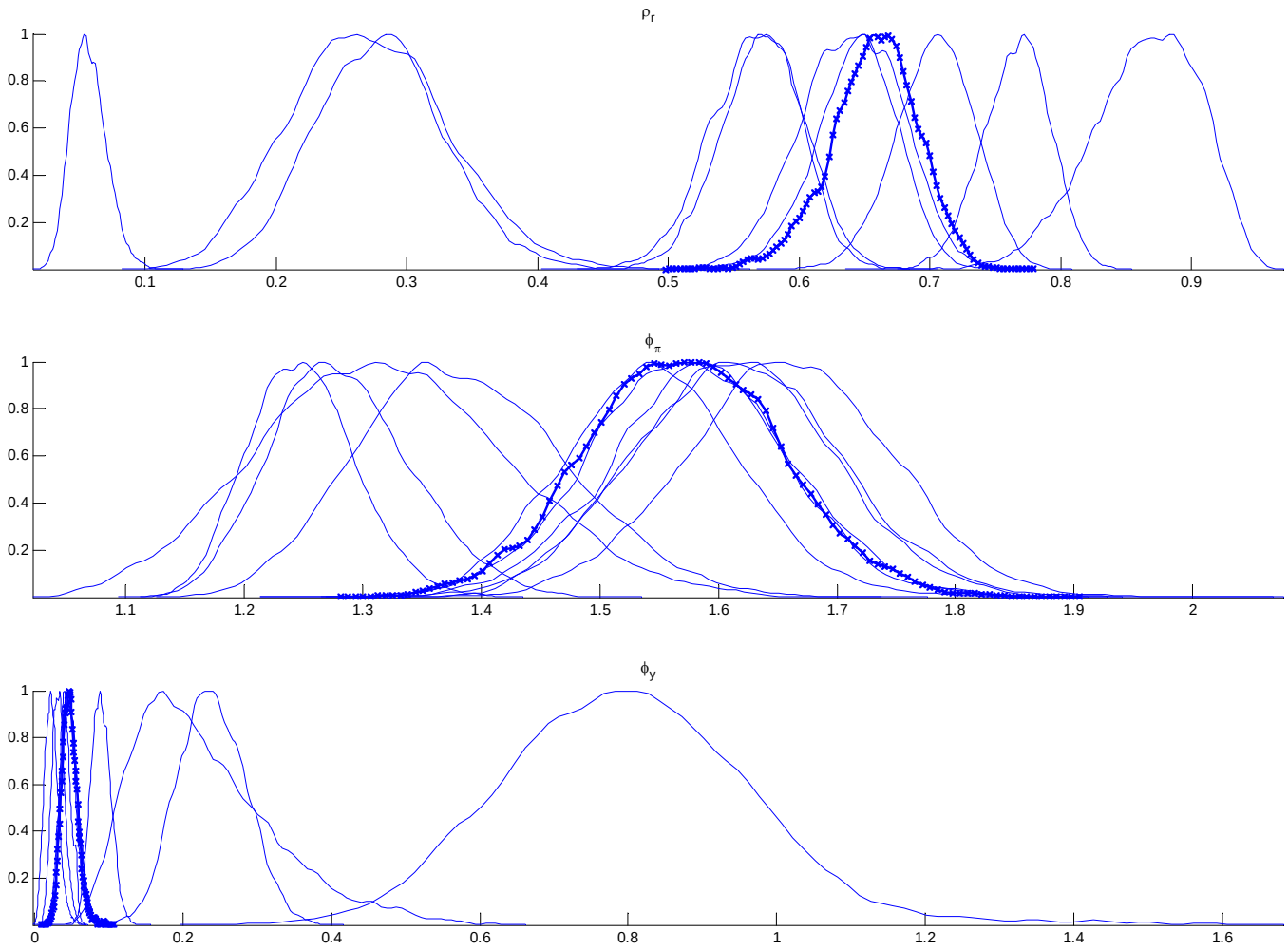

Figure 6: Parameters Posterior Densities

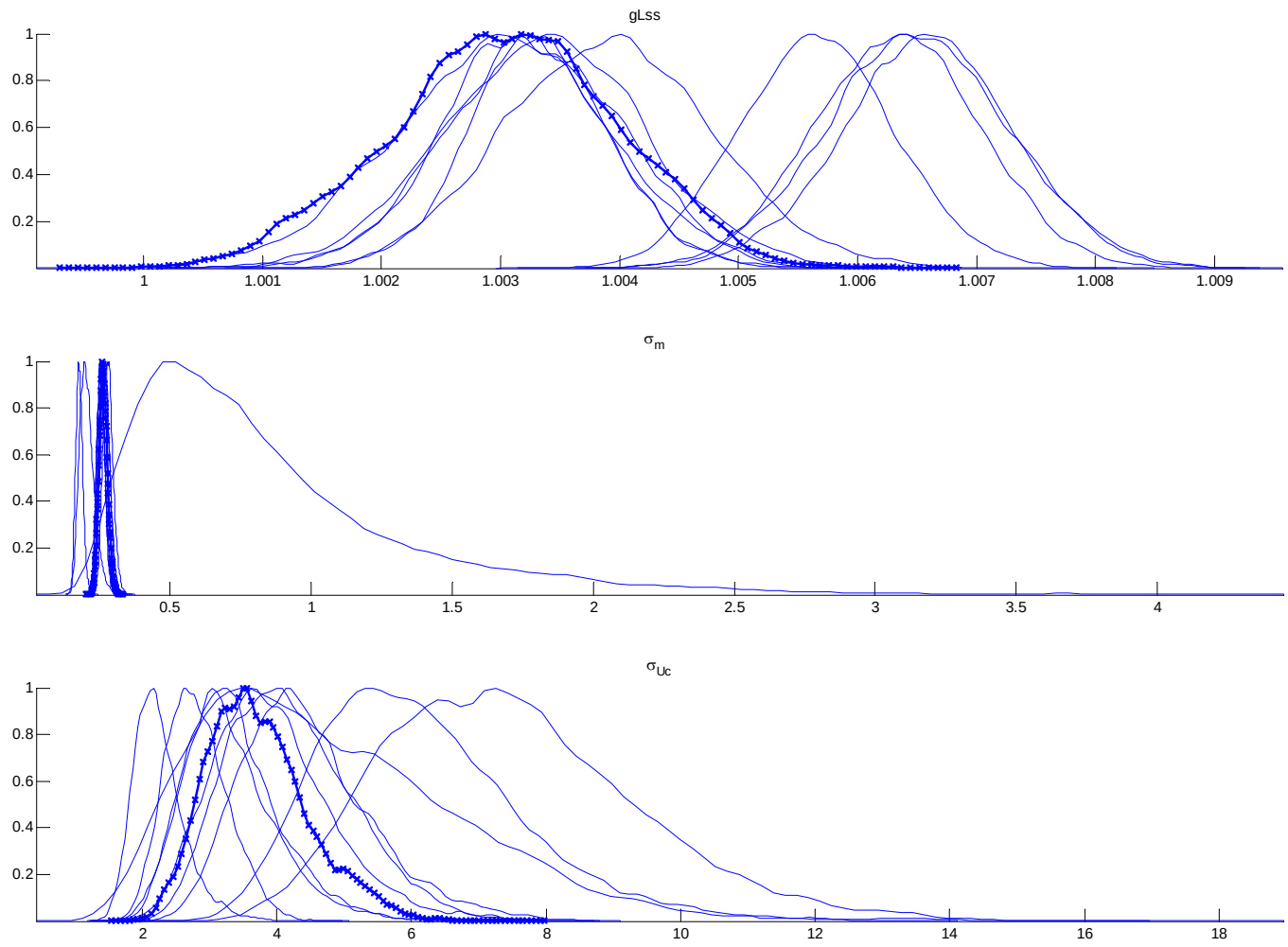

Figure 7: Parameters Posterior Densities