Insights from an Estimated Search-Based Monetary Model with Nominal Rigidities*

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Abstract

We are developing a search-based monetary dynamic equilibrium model with nominal rigidities. We use Bayesian methods to estimate the model based on quarterly U.S. data and compare it to a money-in-the-utility (MIU) specification. We also compare steady state welfare implications of the estimated search-based and the MIU model.

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1 Introduction

Dynamic stochastic general equilibrium (DSGE) models with nominal rigidities and a monetary policy represented by interest rate feedback rules are emerging as the workhorse of applied monetary policy analysis in many central banks. Much of the empirical work with DSGE models, e.g. Smets and Wouters (2003), Del Negro, Schorfheide, Smets, and Wouters (2007), Levin, Onatski, Williams, and Williams (2005), as well as the theoretical work summarized in Woodford (2003) is based on models in which real money balances directly enter the households' utility function. Such money-in-the-utility-function (MIU) specifications are informally motivated by the insight that money balances reduce transaction costs and therefore increase utility. Once monetary policy is represented by an interest-rate-feedback rule and real money balances enter the utility function in an additively separable fashion, the model becomes block triangular and aggregate outcomes are not affected by the money stock. In fact, it has become common practice to consider cash-less models, which are obtained by letting the weight on real money balances in the utility function converge to zero. Econometric work typically excludes a measure of the money stock from the list of observables and ignores the model implied money demand equation. While the cashless approach appears reasonable if the estimated model is used to study the propagation of structural shocks other than money demand shocks, it is not innocuous for welfare analysis. To the extent that real money balances indeed affect households' utility, they are relevant for assessing the welfare consequences of changes in monetary policy.

The contribution of our paper is threefold. First, as an alternative to the commonly used MIU model, we develop an estimable DSGE model in which in every period economic activity takes place in two markets. In a decentralized market (DM), households engage in bilateral trade with a fraction of households producing and a fraction of households consuming. The centralized market (CM) resembles a standard DSGE model with nominal rigidities where production is carried out by firms. Physical capital is a factor of production in both markets. Demand for money arises because the transactions in the decentralized markets are facilitated by a medium of exchange. The basic structure of our model economy is based on recent work by Lagos and Wright (2005, henceforth LW) and Aruoba, Waller, and Wright (2007, henceforth AWW). Our specification adds nominal rigidities in the centralized market, represents monetary policy by an interest rate feedback rule, and introduces stochastic disturbances to technology, preferences, government spending, and monetary policy to make the model amenable to econometric estimation methods. While the structure of our model to a large extent resembles that of a canonical New Keynesian model with capital, the presence of the decentralized market provides a micro-founded motive for holding money and creates a non-separability between consumption and the value of real money balances. Hence, our model differs from an MIU specification both in terms of the resulting money demand equation as well as its welfare implications.

Second, using post 1984 U.S. data on output, inflation, interest rates, and the money stock we use

Bayesian techniques surveyed in An and Schorfheide (2007) to estimate our search-based DSGE model. We also fit a standard MIU model with nominal rigidities to the same set of observations. While most of the work on search-based monetary model has been theoretical, our analysis produces formal estimates of the taste and technology parameters that determine the exchange in the decentralized market. We compare the fit of the money demand equations obtained from the two estimated models. The non-separable structure of the search model creates a tight link between technology shocks and the fluctuations in the stock of money. In fact, we find that about 75% of the variation in money is due to technology shocks. Moreover, money demand shocks, represented as shocks to the taste for goods produced in the decentralized market, affect aggregate output, inflation, and interest rates. The separability between real money balances and consumption embodied in our estimated MIU model, on the other hand, creates a fairly large disconnect between fluctuations in the stock of money and the rest of the economy. About 80% of real money balances are due to money demand shocks, which in turn do not affect output, inflation, and interest rates. However, imposing the additional structure of the decentralized market in the search based model comes at a cost: its overall fit relative to the MIU specification as measured by Bayes factors is worse. While the search-based model provides tighter linkages between output, inflation, interest rates, and money, it fails to explain some of the cross correlations.

Finally, we compare the effects of changes in the central bank's target inflation rate on steady state welfare using the two estimated DSGE models. Our choice of estimation objective function requires the models to fit both the post 1984 average velocity in U.S. data as well as the fluctuations in M2. Since our estimates imply that the households place very little weight on real money balances in the MIU model, its welfare implications essentially resemble that of a cashless New Keynesian model. Since it is costly for firms to choose prices optimally and suboptimal prices lead to a distortion, a target inflation rate near zero maximizes steady state welfare. The welfare implications of the search-based model are driven by two opposing forces: on the one hand, the presence of nominal rigidities makes price changes costly and welfare reducing. On the other hand, the Friedman motive for keeping the nominal rate near zero is present in the model: inflation is a tax on money holdings. If this tax is large, then it depresses economic activity in the decentralized market, which is welfare reducing.

The remainder of the paper is organized as follows. We provide a detailed derivation and discussion of the search-based DSGE model in Section 2. A canonical MIU model with nominal rigidities and capital can be obtained by shutting down the decentralized market in the search-based model and adding a real-money-balance term to the households' utility function. This MIU model is described in Section 3. The Bayesian estimation results are presented in Section 4 and the welfare analysis is summarized in Section 5. Finally, Section 6 concludes. Detailed derivations for the two DSGE models are provided in the Appendix.

2 The Search-Based Model

The model is an extension of the two-sector model developed in LW. In every period, there is economic activity in two markets, which we label the decentralized market (DM) and the centralized market (CM). In the DM, households engage in decentralized bilateral trade with other households with one party producing and the other consuming, while the CM resembles a standard macro model where production is carried out by firms.

We extend the LW model in two dimensions. First, we include physical capital as a factor of production, following AWW. The only deviation we have from AWW in this regard is that we introduce an adjustment cost for investment to improve the empirical fit. Second, we replace the neoclassical structure on the firm side with a New Keynesian one. Intermediate goods producing firms sell their differentiated output to final good producers. The intermediate good producers face a downward sloping demand curve for their product and choose prices to maximize their profits. However, in any period only a fraction of these firms is able to re-optimize their prices. The remaining firms either adjust their prices by the lagged inflation rate or not at all. This mechanism of generating nominal rigidity is due to Calvo (1983) and widely used in the literature on New Keynesian DSGE models. Unlike in more elaborate empirical version in Smets and Wouters (2003) and Christiano, Eichenbaum, and Evans (2005), we exclude habit formation, wage stickiness, and variable capital utilization from our model specification. In turn we will describe the firms' problem in the centralized market (Section 2.1), the households' decision problems in both the centralized and the decentralized market (Section 2.2). We then characterize the behavior of fiscal and monetary policy (Section 2.3) and derive an aggregate resource constraint (Section 2.4). Our model economy is subject to aggregate disturbances, characterized in Section 2.5. A summary of all the equilibrium conditions is provided in the Appendix.

2.1 Firms in the Centralized Market

The setup of the centralized market resembles that of a New Keynesian DSGE model. Production is carried out by two types of firms in the CM: final good producers combine differentiated intermediate goods. Intermediate goods producing firms hire labor and capital services from the households to produce the inputs for the final good producers. To introduce nominal rigidity we follow Calvo (1983) by assuming that only a constant fraction of the intermediate goods producers is able to re-optimize prices.

2.1.1 Final Good Producers

The final good Y_t in the CM is a composite made of a continuum of intermediate goods $Y_t(i)$:

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{1}{1+\lambda}} di \right]^{1+\lambda}. \tag{1}$$

Note that the elasticity is $(1+\lambda)/\lambda$. $\lambda = 0$ corresponds to the linear case and $\lambda \to \infty$ corresponds to the Cobb-Douglas case. We will constrain $\lambda \in (0, \infty)$. The final good producers buy the intermediate goods on the market, package them into Y_t units of the composite good, and resell them to consumers. These firms maximize profits in a perfectly competitive environment. Their problem is:

$$\max_{Y_t, Y_t(i)} \; P_t Y_t - \int_0^1 P_t(i) Y_t(i) di \quad \text{s.t.} \quad (1) \tag{2}$$

taking $P_t(i)$ as given. The first-order condition is:

$$P_t(i) = P_t Y_t^{\frac{\lambda}{1+\lambda}} Y_t(i)^{-\frac{\lambda}{1+\lambda}}. \tag{3}$$

Therefore,

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\frac{1+\lambda}{\lambda}} Y_t. \tag{4}$$

Combining this condition with the zero profit condition one obtains an expression for the price of the composite good:

$$P_t = \left[\int_0^1 P_t(i)^{-\frac{1}{\lambda}} di\right]^{-\lambda}. \tag{5}$$

2.1.2 Intermediate Goods Producers

Intermediate goods producers, indexed by i, use the following technology:

$$Y_t(i) = \max\left\{Z_t K_t(i)^{\alpha} H_t(i)^{1-\alpha} - \mathcal{F}, 0\right\}, \tag{6}$$

Firm i's profit is given by:

$$\Pi_t(i) = P_t(i) Y_t(i) - P_t W_t H_t(i) - P_t R_t^k K_t(i). \tag{7}$$

All firms take factor prices W_t and R_t^k, as well as the prices of the other firms and the aggregate price level as given. We distinguish two types of firms: (i) firms are allowed to re-optimize their price $P_t(i)$ and (ii) firms that are not able to re-optimize their price. Firms that are not allowed to choose $P_t(i)$ optimally, satisfy the demand for their differentiated good (4) and choose capital and labor inputs to minimize costs. Firms that are able to change their price in an optimal fashion, maximize future expected profits. The profit maximization problem can be solved in two steps. First, given a desired level of output $Y_t(i)$ we determine the cost-minimizing choice of factor inputs. Second, we determine the profit maximizing price $P_t(i)$ and quantity $Y_t(i)$ that satisfies (4).

Cost minimization subject to (6) yields the conditions:

$$P_t W_t = \mu_t(i) P_t(i)(1-\alpha) Z_t K_t(i)^{\alpha} H_t(i)^{-\alpha} \tag{8}$$

$$P_t R_t^k = \mu_t(i) P_t(i) \alpha Z_t K_t(i)^{\alpha-1} H_t(i)^{1-\alpha} \tag{9}$$

where $\mu_t(i)$ is the Lagrange multiplier associated with (6). In turn, these conditions imply:

$$K_t(i) = \frac{\alpha}{1-\alpha}\frac{W_t}{R_t^k}H_t(i).$$

If we integrate both sides of the equation with respect to di and define $K_t = \int K_t(i)di$ and $H_t = \int H_t(i)di$ we obtain a relationship between aggregate labor and capital:

$$K_t = \frac{\alpha}{1-\alpha}\frac{W_t}{R_t^k}H_t. \tag{10}$$

Thus, the aggregate capital labor ratio is a linear function of the ratio of factor prices.

Total variable cost (VC_t) is given by

$$VC_t(i) = \left(W_t + R_t^k\frac{K_t(i)}{H_t(i)}\right)H_t(i) = \left(W_t + R_t^k\frac{K_t(i)}{H_t(i)}\right)Z_t^{-1}\left(\frac{K_t(i)}{H_t(i)}\right)^{-\alpha}Y_t^v(i),$$

where $Y_t^v(i) = Z_tK_t(i)^\alpha H_t(i)^{1-\alpha}$ is the "variable" part of output $Y_t(i)$. The real marginal cost MC_t is the same for all firms and equal to:

$$\begin{aligned} MC_t &= \left(W_t + R_t^k\frac{K_t(i)}{H_t(i)}\right)Z_t^{-1}\left(\frac{K_t(i)}{H_t(i)}\right)^{-\alpha} \\ &= \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}W_t^{1-\alpha}(R_t^k)^\alpha Z_t^{-1}. \end{aligned} \tag{11}$$

Conditional on the optimal choice of factor inputs, nominal profits as a function of output $Y_t(i)$ and prices $P_t(i)$ can then be expressed as

$$\Pi_t(i) = [P_t(i) - P_tMC_t]\,Y_t(i) - P_tMC_t\mathcal{F}. \tag{12}$$

Since the last part of this expression does not depend on the firm's decision, it can be safely ignored subsequently.

We assume that prices are sticky as in Calvo (1983). Specifically, each firm can re-adjust prices with probability $1-\zeta$ in each period. We depart from the Calvo-setup in assuming that for those firms that cannot adjust prices, $P_t(i)$ will increase at the geometric weighted average of the fixed rate π_{**} and of last period's inflation π_{t-1} with weights $1-\iota$ and ι, respectively. We define the price adjustment factor

$$\pi_{t+s|t}^{adj} = \prod_{l=1}^{s}\pi_{t+l-1}^{\iota}\pi_{**}^{1-\iota}$$

and adopt the convention that $\pi_{t|t}^{adj} = 1$. Firms that are unable to re-optimize their prices simply satisfy the demand for their product according to (4). For those firms that are allowed to re-optimize prices, the problem is to choose a price level $P_t^o(i)$ that maximizes the expected present discounted value of profits in all states of nature where the firm is stuck with that price in the future:

$$\max_{P_t^o(i)} \quad \mathbb{E}_t\left[\sum_{s=0}^{\infty}\zeta^s\beta^s\Xi_{t+s|t}^p\left[P_t^o(i)\pi_{t+s|t}^{adj} - P_{t+s}MC_{t+s}\right]Y_{t+s}(i)\right] \tag{13}$$

$$\text{s.t.} \quad Y_{t+s}(i) = \left[\frac{P_t^o(i)\pi_{t+s|t}^{adj}}{P_{t+s}}\right]^{-\frac{1+\lambda}{\lambda}}Y_{t+s},$$

where $\beta^s \Xi^p_{t+s|t}$ is the time t value of a dollar in period $t+s$ for the consumers. We assume that markets are complete so that $\beta^s \Xi^p_{t+s|t}$ is the same for all consumers. It is shown in the Appendix that the first-order conditions can be reduced to the following set of equations:

$$\mathcal{F}_t^{(1)} = (p_t^o)^{-\frac{1+\lambda}{\lambda}} Y_t + \zeta\beta \left(\pi_t^\iota \pi_{**}^{(1-\iota)}\right)^{-1/\lambda} \mathbb{E}_t \left[\left(\frac{p_t^o}{\pi_{t+1} p_{t+1}^o} \right)^{-\frac{1+\lambda}{\lambda}} \Xi^p_{t+1|t} \mathcal{F}_{t+1}^{(1)} \right] \tag{14}$$

$$\mathcal{F}_t^{(2)} = (p_t^o)^{-\frac{1+\lambda}{\lambda}-1} Y_t MC_t + \zeta\beta \left(\pi_t^\iota \pi_{**}^{(1-\iota)}\right)^{-\frac{1+\lambda}{\lambda}} \mathbb{E}_t \left[\left(\frac{p_t^o}{\pi_{t+1} p_{t+1}^o} \right)^{-\frac{1+\lambda}{\lambda}-1} \Xi^p_{t+1|t} \mathcal{F}_{t+1}^{(2)} \right] \tag{15}$$

$$\mathcal{F}_t^{(1)} = (1+\lambda)\mathcal{F}_t^{(2)} \tag{16}$$

Here we are considering only the symmetric equilibrium in which all firms that can readjust prices will choose the same $P_t^o(i)$. In the above formula, we dropped the i index and used the definitions $p_t^o = P_t^o/P_t$ and $\pi_t = P_t/P_{t-1}$. Equations (14) to (16) essentially determine the optimal price p_t^o as a function of marginal costs.

2.1.3 Aggregate Price Dynamics

From (5) it follows that:

$$P_t = \left[(1-\zeta)(P_t^o)^{-\frac{1}{\lambda}} + \zeta(\pi_{t-1}^\iota \pi_{**}^{1-\iota} P_{t-1})^{-\frac{1}{\lambda}} \right]^{-\lambda}. \tag{17}$$

Hence,

$$\pi_t = \left[(1-\zeta)\left(\pi_t p_t^o\right)^{-\frac{1}{\lambda}} + \zeta(\pi_{t-1}^\iota \pi_{**}^{1-\iota})^{-\frac{1}{\lambda}} \right]^{-\lambda}. \tag{18}$$

The system of equations (14) - (16) and (18) links inflation to real marginal costs and output and hence defines a so-called New Keynesian Phillips curve.

2.2 Households

There is a continuum of ex-ante identical households in the economy. These households derive utility from their activities in the two markets. A household that consumes q_t units of consumption good in the DM gets utility $\chi_t u(q_t)$ while he gets utility $U(x_t)$ by consuming x_t units in the CM. The disutility of effort in the DM for a seller and disutility of labor for a worker in the CM is linear:[1]

$$\mathcal{U}_t = U(x) - Ah_t \begin{cases} +\chi_t u(q_t) & \text{if buyer in DM} \\ -e_t & \text{if seller in DM} \end{cases} \tag{19}$$

Instead of using the disutility of effort e_t in the DM, we express the disutility as a function of output produced by the seller. To see this, we assume the following structure. For a seller, the output q_t is obtained using the

[1] This assumption, in particular the linearity of disutility of labor in the CM is a critical assumption that prevents a non-degenerate distribution of money holdings.

production function $q_t = Z_t f(e_t, k_t)$ where Z_t is a technology shock which is common across the two markets. This production function can be inverted to get $e_t = \xi(q_t, k_t, Z_t)$. Using the linear disutility in effort, we can define $c(q_t, k_t, Z_t) = \xi(q_t, k_t, Z_t)$ as the utility cost of production for the sellers. We have $c_q > 0$, $c_k < 0$, and $c_Z < 0$.

While we keep general functions in the description and the derivation of the model, we are going to use the following functional forms in our estimation

$$u(q) = \ln(q + \kappa) - \ln(\kappa), \quad U(x) = B\ln(x_t), \quad f(e, k) = e^{1-\alpha}k^{\alpha}$$

where $\kappa > 0$ is a small constant to make sure $q_t = 0$ can be handled and B determines the relative weight of the utility from consuming the CM and DM goods. We use a natural logarithm for both utility functions and use the same Cobb-Douglas production function as the function used by the intermediate good producers in the CM as these are necessary conditions for balanced-growth in this model. Also, using the derivations above, we get

$$c(q, k, Z) = \frac{1}{Z^{1/(1-\alpha)}} q^{1/(1-\alpha)} k^{-\alpha/(1-\alpha)},$$

In a given period, the households participate in the DM followed by the CM. To characterize the household's behavior in this economy, we start from the problem of the household in the CM, followed by the DM problem.

2.2.1 Centralized Market

The households take as given the aggregate price level in the CM, P_t, the nominal interest rate R_t, and the factor prices W_t and R_t^k. Using $W_t(\hat{m}_t, k_t, i_{t-1}, b_t, S_t)$ and $V_t(m_t, k_t, i_{t-1}, b_t, S_t)$ to denote the value functions in the CM and DM of period t where $\hat{m}_t$ is the money balances of the household entering the CM, the CM problem[2] is

$$\begin{aligned}
&W_t(\hat{m}_t, k_t, i_{t-1}, b_t, S_t) \\
&\quad = \max_{x_t, h_t, m_{t+1}, i_t, k_{t+1}, b_{t+1}} \{U(x_t) - Ah_t + \beta E_t[V_{t+1}(m_{t+1}, k_{t+1}, i_t, b_{t+1}, S_{t+1})]\} \\
&\quad \text{s.t.} \quad P_t x_t + P_t i_t + b_{t+1} + m_{t+1} \leq P_t W_t h_t + P_t R_t^k k_t + \Pi_t + R_{t-1} b_t + \hat{m}_t - T_t \qquad (20) \\
&\qquad\quad k_{t+1} = (1-\delta)k_t + \left[1 - S\left(\frac{i_t}{i_{t-1}}\right)\right] i_t \qquad (21)
\end{aligned}$$

given the laws of motion for the aggregate shocks, S_t. Here A is the disutility of one unit of labor, R_{t-1} is the gross nominal return on a government bond purchased in period $t-1$, T_t is a nominal lump-sum tax

[2] We could index households with j, but we will see that the assumption of complete markets implies that the index will drop out of most of these variables. In equilibrium households will make the same choice of consumption, money demand, and investment. So, we drop this index from the outset.

and Π_t denotes the total profits the household receives from intermediate good producers. (21) shows how capital is accumulated where the adjustment cost function $S(.)$ satisfies properties $S(1) = 0$, $S'(1) = 0$ and $S''(1) > 0$. Using Υ_t to denote the Lagrange multiplier for (21) and after eliminating h using (20), the FOC are

$$x_t \quad : \quad U'(x_t) = \frac{A}{W_t} \tag{22}$$

$$m_{t+1} \quad : \quad \frac{U'(x_t)}{P_t} = \beta E V_{t+1,m}(m_{t+1}, k_{t+1}, i_t, b_{t+1}, S_{t+1}) \tag{23}$$

$$i_t \quad : \quad U'(x_t) = \Upsilon_t \left[1 - S\left(\frac{i_t}{i_{t-1}}\right) + \frac{i_t}{i_{t-1}} S'\left(\frac{i_t}{i_{t-1}}\right)\right] + \beta E V_{t+1,i}(m_{t+1}, k_{t+1}, i_t, b_{t+1}, S_{t+1}) \tag{24}$$

$$k_{t+1} \quad : \quad \Upsilon_t = \beta E V_{t+1,k}(m_{t+1}, k_{t+1}, i_t, b_{t+1}, S_{t+1}) \tag{25}$$

$$b_{t+1} \quad : \quad \frac{U'(x_t)}{P_t} = \beta E V_{t+1,b}(m_{t+1}, k_{t+1}, i_t, b_{t+1}, S_{t+1}) \tag{26}$$

assuming that an interior solution exists. This leads to two key results. First, since the individual state variables, $(\hat{m}_t, k_t, i_{t-1}, b_t)$ do not appear in (23)-(26), household's decisions in the CM do not depend on his state variables. More specifically, for any distribution of assets $(\hat{m}_t, k_t, b_t)$ across agents entering the CM, the distribution of $(m_{t+1}, k_{t+1}, b_{t+1})$ is degenerate.[3] Second, we have the following envelope conditions,

$$\begin{aligned}
W_{t,m}(\hat{m}_t, k_t, i_{t-1}, b_t, S_t) &= \frac{A}{P_t W_t} \\
W_{t,k}(\hat{m}_t, k_t, i_{t-1}, b_t, S_t) &= \frac{A R_t^k}{W_t} + (1-\delta)\Upsilon_t \\
W_{t,i}(\hat{m}_t, k_t, i_{t-1}, b_t, S_t) &= \Upsilon_t \left(\frac{i_t}{i_{t-1}}\right)^2 S'\left(\frac{i_t}{i_{t-1}}\right) \\
W_{t,b}(\hat{m}_t, k_t, i_{t-1}, b_t, S_t) &= \frac{A R_{t-1}}{W_t}
\end{aligned}$$

which show that $W_t(.)$ is linear in $\hat{m}_t$ which will be important in the DM problem below. Finally, the Lagrange multiplier associated with the households' nominal budget constraint (20) is $U'(x_t)/P_t$. Under the assumption that households have access to a complete set of state-contingent claims we obtain that

$$\Xi^p_{t+1|t} = \frac{U'(x_{t+1})/P_{t+1}}{U'(x_t)/P_t}. \tag{27}$$

which the firms use to discount the future. We need to specify the details of the DM to characterize the equilibrium next. Specifically we will find V_m, V_k, V_i and V_b to obtain the equilibrium conditions.

[3] This result requires a small qualification for bond holdings. There are two parts of the argument that guarantees the degeneracy. The first part relies on the observation that $(\hat{m}_t, k_t, b_t)$ does not appear in (26). The second part relies on the strict concavity of $V(.)$ or, more specifically, the strict monotonicity of $V_b()$ which means the choice of b_{t+1} is unique. Both parts of the argument go through for money and capital in our environment, but only the first part goes through for bonds since $V_b()$ is constant as we show below. This means that in principle there could be multiple values of b_{t+1} that households choose, which can create a distribution of bond holdings. Fortunately, such a distribution of bonds holdings is not important for any of our results because bond-holdings will not affect the bargaining problem, as we show below.

2.2.2 Decentralized Market

As we said, the centralized market in this model resembles a standard New Keynesian DSGE model. It is important to recognize that transactions in the CM take place without requiring a medium of exchange. Unlike a standard monetary model where money demand is generated by constructs such as cash-in-advance, money-in-the-utility-function or transaction costs, we follow a search-based approach. The DM is critical in generating the money demand. All trades take place in bilateral meetings. The agents are anonymous in the DM which means no household would accept an IOU from another household and any trade must be *quid pro quo*. Following AWW, at the start of each DM a measure σ of households receive a taste shock that make them buyers and another σ measure of households become sellers. Alternatively, we can consider the setup in LW where each household can produce a measure σ of goods out of a measure one of all possible goods, and they like consuming another σ measure of goods.[4] When two households meet at random, with σ probability there is a single coincidence where one party likes the good the other party can produce but not vice versa. The literature started by Kiyotaki and Wright (1989) and many papers that follow show that a medium of exchange will emerge in an environment where the agents are anonymous and there is a double-coincidence problem such as the one above. In a monetary equilibrium, in such single-coincidence meetings, the party who likes what the other party has (the buyer), uses money to purchase the good from the seller.[5] The possibility to consume in the DM generate a demand for money in this model.

The value of starting the DM for a household whose taste shock has not been realized yet is given by

$$\begin{aligned} V_t(m_t, k_t, i_{t-1}, b_t, S_t) &= \sigma V_t^b(m_t, k_t, i_{t-1}, b_t, S_t) + \sigma V_t^s(m_t, k_t, i_{t-1}, b_t, S_t) \\ &\quad +(1-2\sigma) W_t(m_t, k_t, i_{t-1}, b_t, S_t), \end{aligned} \tag{28}$$

where the values of being a buyer and a seller are

$$V_t^b(m_t, k_t, i_{t-1}, b_t, S_t) = \chi_t u(q_t^b) + W_t\left(m_t - d_t^b, k_t, i_{t-1}, b_t, S_t\right) \tag{29}$$

$$V_t^s(m_t, k_t, i_{t-1}, b_t, S_t) = -c(q_t^s, k_t, Z_t) + W_t\left(m_t + d_t^s, k_t, i_{t-1}, b_t, S_t\right) \tag{30}$$

with q_t^b and d_t^b (q_t^s and d_t^s) denoting output and money exchanged when buying (selling) which are determined via bilateral bargaining as describe below. We interpret χ_t as a money demand shock as it affects the utility from consuming in the DM and money serves as a medium of exchange. Using (27) we have

$$V_t(m_t, k_t, i_{t-1}, b_t, S_t) = W_t(m_t, k_t, i_{t-1}, b_t, S_t) + \sigma\left[\chi_t u(q_t^b) - \frac{d_t^b A}{P_t W_t}\right] + \sigma\left[\frac{d_t^s A}{P_t W_t} - c(q_t^s, k_t, Z_t)\right]. \tag{31}$$

[4] As AWW argue, the setup with idiosyncratic taste shocks and the setup with search leads to the same mathematical construct which we describe below.

[5] As with any deep model of money, there is a nonmonetary equilibrium in this model which is dominated by the monetary equilibrium in terms of welfare. We focus on the monetary equilibrium.

To solve (23)-(26), we need:

$$\begin{aligned}
V_{t,m}(m_t,k_t,i_{t-1},b_t,S_t) &= \frac{A}{P_tW_t}+\sigma\left[\chi_t u'\left(q_t^b\right)\frac{\partial q_t^b}{\partial m_t}-\frac{A}{P_tW_t}\frac{\partial d_t^b}{\partial m_t}\right] \\
&\quad +\sigma\left[\frac{A}{P_tW_t}\frac{\partial d_t^s}{\partial m_t}-c_q(q_t^s,k_t,Z_t)\frac{\partial q_t^s}{\partial m_t}\right] && (32)\\
V_{t,k}(m_t,k_t,i_{t-1},b_t,S_t) &= \frac{AR_t^k}{W_t}+(1-\delta)\Upsilon_t+\sigma\left[\chi_t u'\left(q_t^b\right)\frac{\partial q_t^b}{\partial k_t}-\frac{A}{P_tW_t}\frac{\partial d_t^b}{\partial k_t}\right] \\
&\quad +\sigma\left[\frac{A}{P_tW_t}\frac{\partial d_t^s}{\partial k_t}-c_q(q_t^s,k_t,Z_t)\frac{\partial q_t^s}{\partial k_t}-c_k(q_t^s,k_t,Z_t)\right] && (33)\\
V_{t,i}(m_t,k_t,i_{t-1},b_t,S_t) &= W_{t,i}(m_t,k_t,i_{t-1},b_t,S_t) && (34)\\
V_{t,b}(m_t,k_t,i_{t-1},b_t,S_t) &= \frac{A}{P_tW_t}R_{t-1} && (35)
\end{aligned}$$

It remains to specify how the terms of trade (q,d) are determined, so that we can substitute for their derivatives in (32) and (33), for which we turn to the bargaining problem. We drop the time subscripts since everything is period t. Our bargaining problem is

$$\max_{q,d}\left[\chi u(q)-\frac{Ad}{PW}\right]^{\theta}\left[\frac{Ad}{PW}-c(q,k_s,Z)\right]^{1-\theta} \quad \text{s.t. } d\le m^b.$$

where θ is the bargaining power of the buyer, the first term is the buyer's surplus and the second term is the seller's surplus.

Omitting the t subscripts and using the insights of LW and AWW, in any monetary equilibrium $d=m^b$, and this implies $q\le q^*(k^s,Z,\chi)$ where $q^*(k_s,Z,\chi)$ is the solution to $\chi u'(q)=c_q(q,k^s,Z)$. Inserting $d=m^b$ and taking the FOC with respect to q, we get

$$\frac{m^b}{P}=\frac{g(q,k^s,\chi,Z)W}{A} \tag{36}$$

where

$$g(q,k,\chi,Z)\equiv\frac{\theta c(q,k,Z)\chi u'(q)+(1-\theta)\chi u(q)c_q(q,k,Z)}{\theta\chi u'(q)+(1-\theta)c_q(q,k,Z)}. \tag{37}$$

We write $q=q(m^b,k^s,\chi,Z)$, where $q(\cdot)$ is given by solving (36) for q as a function of (m^b,k^s,χ,Z). Turning to the partial derivatives we need, we get

$$\frac{\partial d}{\partial m^b}=1,\quad \frac{\partial q}{\partial m^b}=\frac{A}{PWg_q(q,k,\chi,Z)}>0,\quad \text{and}\quad \frac{\partial q}{\partial k^s}=-\frac{g_k(q,k,\chi,Z)}{g_q(q,k,\chi,Z)}>0$$

while the other derivatives in (32) and (33) are 0.

Now reintroducing the time subscripts and inserting these results, (32) and (33) reduce to

$$\begin{aligned}
V_{t,m}(m_t,k_t,i_{t-1},b_t,S_t) &= \frac{(1-\sigma)A}{P_tW_t}+\frac{\sigma A\chi_t u'(q_t)}{P_tW_tg_q(q_t,k_t,\chi_t,Z_t)} && (38)\\
V_{t,k}(m_t,k_t,i_{t-1},b_t,S_t) &= \frac{AR_t^k}{W_t}+(1-\delta)\Upsilon_t-\sigma\gamma(q_t,k_t,\chi_t,Z_t) && (39)
\end{aligned}$$

where

$$\gamma(q,k,\chi,Z)\equiv c_k+c_q\frac{\partial q}{\partial k}=\frac{c_k(q,k,Z)g_q(q,k,\chi,Z)-c_q\left(q,k,Z\right)g_k(q,k,\chi,Z)}{g_q(q,k,\chi,Z)}<0. \tag{40}$$

2.2.3 Household's Optimality Conditions

We obtain the optimality conditions for the household by simply substituting (34), (35), (38) and (39) in to the household's FOC to get the optimality conditions for the household. We also define $\mu_t \equiv \Upsilon_t / U' x_t$. Formally, taking as given $\left\{P_t, R_t, W_t, R_t^k, \Pi_t, T_t\right\}_{t=0}^{\infty}$ and exogenous aggregate states $\{Z_t, \chi_t\}_{t=0}^{\infty}$, the household solves for $\{x_t, m_{t+1}, k_{t+1}, i_t, b_{t+1}, \mu_t\}_{t=0}^{\infty}$ using the following equations:

$$W_t = \frac{A}{U'(x_t)} \tag{41}$$

$$1 = \beta E_t \left[\frac{U'(x_{t+1})}{U'(x_t)} \frac{R_t}{\pi_{t+1}} \right] \tag{42}$$

$$1 = \mu_t \left[1 - S\left(\frac{i_t}{i_{t-1}}\right) + \frac{i_t}{i_{t-1}} S'\left(\frac{i_t}{i_{t-1}}\right) \right] + \beta E_t \left\{ \mu_{t+1} \frac{U'(x_{t+1})}{U'(x_t)} \left(\frac{i_{t+1}}{i_t}\right)^2 S'\left(\frac{i_{t+1}}{i_t}\right) \right\} \tag{43}$$

$$k_{t+1} = (1-\delta)k_t + \left[1 - S\left(\frac{i_t}{i_{t-1}}\right) \right] \tag{44}$$

$$\mu_t = \beta E_t \left\{ \frac{U'(x_{t+1})}{U'(x_t)} \left[R_{t+1}^k + (1-\delta)\mu_{t+1} \right] - \frac{\sigma}{U'(x_t)} \gamma(q_{t+1}, k_{t+1}, \chi_{t+1}, Z_{t+1}) \right\} \tag{45}$$

$$\frac{m_{t+1}}{P_t} = \frac{\beta}{U'(x_t)} E_t \left\{ g(q_{t+1}, k_{t+1}, \chi_{t+1}, Z_{t+1}) \left[\frac{\sigma \chi_{t+1} u'(q_{t+1})}{g_q(q_{t+1}, k_{t+1}, \chi_{t+1}, Z_{t+1})} + (1-\sigma) \right] \right\} \tag{46}$$

Equations (41) to (44) resemble the optimality conditions that arise in a standard DSGE model with capital. (41) is a labor supply equation that relates the wage to the marginal utility of consumption, (42) is the Euler equation for Bond holdings. (43) describes the evolution of the shadow price of installed capital, μ_t, and (44) is the capital accumulation equation. Equations (45) and (46) reflect the presence of the decentralized market. (45) is the Euler equation for capital stock holdings. The return to capital has two components, namely the return from renting capital to intermediate good producing firms in the centralized market, R_t^k, net of capital depreciation, and the return to capital when producing in the decentralized market. Finally, (46) can be interpreted as money demand equation.

The set of equations above determines the path of money balances, given m_0 which is identical across all households assuming an interior solution. As all households start period t with the same money balances, $m_t = M_t$ where M_t is the aggregate money stock, the buyers in the DM enter the CM with $\hat{m} = 0$, the sellers with $\hat{m} = 2M$ while the remaining $1 - 2\sigma$ households carry $\hat{m} = M$. Looking at (20), this means that individual labor supply depends on the status of the agent in the previous DM as the money holdings . In particular, we have

$$h_t = \begin{cases} H_t + \frac{(M_t - 0)}{P_t W_t} & \text{buyers} \\ H_t + \frac{(M_t - 2M_t)}{P_t W_t} & \text{sellers} \\ H_t & \text{others} \end{cases} \tag{47}$$

where H_t is aggregate hours which we define below. This shows buyers in the DM work more than others since they have to make up for the money they have spent and sellers work less than others. We only care

about total hours H_t in equilibrium and will not track individual h_t.

2.3 Government

In period t, the government in this model collects a nominal lump-sum tax T_t, spends G_t on goods from the centralized market, issues one-period nominal bonds B_{t+1} that pay R_t gross interest tomorrow and supplies the money to maintain the interest rate rule. It satisfies the following budget constraint every period

$$P_t G_t + R_{t-1} B_t + M_t = T_t + B_{t+1} + M_{t+1} \tag{48}$$

Government expenditures as a fraction of real GDP (both the DM and CM output, appropriately aggregated) $\mathcal{Y}_t$, denoted by g_t are assumed to be exogenous:

$$G_t = (1 - 1/g_t)\, \mathcal{Y}_t. \tag{49}$$

Monetary policy is represented by an interest-rate feedback rule

$$\frac{R_t}{R_*} = \left(\frac{R_{t-1}}{R_*}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi_*}\right)^{\psi_1} \left(\frac{\mathcal{Y}_t}{\mathcal{Y}_*}\right)^{\psi_2}\right]^{1-\rho_R} \exp\left(\sigma_r \varepsilon_t^r\right), \tag{50}$$

where R_* is the gross steady state nominal interest rate, π^* is the steady state (or target) inflation, $\mathcal{Y}_*$ is the steady state of real GDP and ε_t^r is a policy shock.

2.4 Aggregate Resource Constraint and National Accounting

We begin by adding the households' CM budget constraints (remember that there are three types of households as they enter the CM depending on their status in the previous DM) and the government budget constraint to obtain

$$P_t X_t + P_t I_t + P_t G_t = P_t W_t H_t + P_t R_t^k K_t + \Pi_t. \tag{51}$$

Now consider firms' profits:

$$\begin{aligned}
\Pi_t &= \int P_t(i) Y_t(i) di - P_t W_t \int H_t(i) di - P_t R_t^k \int K_t(i) di \\
&= \int P_t(i) Y_t(i) di - P_t W_t H_t - P_t R_t^k K_t \\
&= P_t Y_t - P_t W_t H_t - P_t R_t^k K_t
\end{aligned}$$

The last equality follows from the zero profit conditions for the final goods producers. Combining the expression for profits with (51) we get

$$X_t + I_t + G_t = Y_t, \tag{52}$$

which is the resource constraint in the CM. The relationship between output and the aggregate labor and capital inputs is given by

$$\bar{Y}_t = Z_t \int K_t^{\alpha}(i) H_t^{1-\alpha}(i) di - \mathcal{F} = Z_t K_t^{\alpha} H_t^{(1-\alpha)} - \mathcal{F}.$$

where $\bar{Y}$ is the output of the intermediate good producers and the second equality follows from the fact that the optimal capital labor ratio $K_t(i)/H_t(i)$ only depends on relative factor prices which are common to all firms. The relationship between $\bar{Y}_t$ and Y_t is given by

$$\bar{Y}_t = Y_t \int \left(\frac{P_t(i)}{P_t}\right)^{-\frac{1+\lambda}{\lambda}} di. \tag{53}$$

We now have to determine the distribution of prices $P_t(i)$. A fraction $1-\zeta$ of firms was allowed to re-optimize their prices in period t. For these firms $P_t(i) = P_t^o$. A fraction $\zeta(1-\zeta)$ of firms re-set their prices in period $t-1$. Hence, for these firms $P_t(i) = \pi_{t-1}^{\iota} \pi_{**}^{1-\iota} P_{t-1}^o$. Overall, we obtain

$$\int \left(\frac{P_t(i)}{P_t}\right)^{-\frac{1+\lambda}{\lambda}} di = (1-\zeta) \sum_{j=0}^{\infty} \zeta^j \left(\frac{(\pi_{t-1}\pi_{t-2}\cdots\pi_{t-j})^{\iota} \pi_{**}^{j(1-\iota)}}{\pi_t \pi_{t-1} \cdots \pi_{t-j+1}} \frac{P_{t-j}^o}{P_{t-j}} \right)^{-\frac{1+\lambda}{\lambda}}. \tag{54}$$

To capture the law of motion of the price distribution we introduce a new variable

$$D_t = (1-\zeta) \sum_{j=0}^{\infty} \zeta^j \left(\frac{(\pi_{t-1}\pi_{t-2}\cdots\pi_{t-j})^{\iota} \pi_{**}^{j(1-\iota)}}{\pi_t \pi_{t-1} \cdots \pi_{t-j+1}} \frac{P_{t-j}^o}{P_{t-j}} \right)^{-\frac{1+\lambda}{\lambda}}.$$

We verify in the Appendix that D_t follows the law of motion:

$$D_t = \zeta \left[\left(\frac{\pi_{t-1}}{\pi_t}\right)^{\iota} \left(\frac{\pi_{**}}{\pi_t}\right)^{(1-\iota)} \right]^{-\frac{1+\lambda}{\lambda}} D_{t-1} + (1-\zeta) \left[\frac{P_t^o}{P_t}\right]^{-\frac{1+\lambda}{\lambda}}. \tag{55}$$

Finally, we can write

$$Y_t D_t = Z_t K_t^{\alpha} L_t^{(1-\alpha)} - \mathcal{F}. \tag{56}$$

Turning to national accounting, nominal output is $P_t Y_t$ in the CM and σM_t in the DM since only a fraction σ of households spend all their money. Using P_t (the price of a CM good) as the unit of account, real output is Y_t for the CM and $\sigma M_t/P_t$ for the DM. We define $\mathcal{M}_t \equiv M_t/P_{t-1}$ as a measure of real money balances. The total aggregate real output for the economy then is

$$\mathcal{Y}_t \equiv Y_t + \sigma \frac{\mathcal{M}_t}{\pi_t}. \tag{57}$$

2.5 Aggregate Shocks

We consider four aggregate disturbances in our model economy: Z_t, is the random productivity term that effects production in both markets, g_t is a government spending shock, and ε_t^r is a monetary policy shock. Finally, we have a money demand shock, χ_t, which we model as a taste shock in the DM. We define

$$\tilde{Z}_t = \ln(Z_t/Z_*), \quad \tilde{\chi}_t = \ln(\chi_t/\chi_*), \quad \tilde{g}_t = \ln(g_t/g_*),$$

where Z_*, χ_* and g_* are steady state values / means of the respective random variable. We assume that the exogenous disturbances evolve according to AR(1) processes:

$$\tilde{Z}_t = \rho_z \tilde{Z}_{t-1} + \sigma_z \varepsilon_t^z, \quad \tilde{\chi}_t = \rho_\chi \tilde{\chi}_{t-1} + \sigma_\chi \varepsilon_t^\chi, \quad \tilde{g}_t = \rho_g \tilde{g}_{t-1} + \sigma_g \varepsilon_t^g$$

and we collect all innovations in $\varepsilon = [\varepsilon_t^z, \varepsilon_t^\chi, \varepsilon_t^g, \varepsilon_t^r]$ which follows a multi-variate standard normal distribution.

The law of motion for the exogenous processes completes the specification of the search-based DSGE model. The equilibrium conditions are summarized in the Appendix. We derive the deterministic steady state for this model and use a log-linear approximation to its dynamics to form a state-space representation that is used for the Bayesian estimation.

3 A Money-in-the-Utility-Function Model

The specification of the MIU model closely resembles search-theoretic model described in the previous section. The production side of the MIU economy is identical to the production sector in the centralized market. Moreover, fiscal and monetary policy are identical as well and the economy is subject to the same set of stochastic shocks. We only discuss the modifications to the household's problem.

Since there is no decentralized market, households' consumption is restricted to x_t. The instantaneous utility function is of the form

$$\mathcal{U}_t = U(x_t) - Ah_t + \frac{\chi_t}{1-\nu_m}\left(\frac{m_t}{P_t}\frac{A}{Z_*^{1/1-\alpha}}\right)^{1-\nu_m}, \tag{58}$$

where, as in the search-based model, $U(x_t) = B\ln(x_t)$. The third term on the right-hand-side of (58) captures the value of holding real money balances. The scaling by $A/Z_*^{1/(1-\alpha)}$ can be interpreted as a re-parameterization of χ_t, which has the effect that steady state velocity stays constant as we change A and Z. Here m_t are the (pre-determined) money holdings at the beginning of the period, and P_t is the price at which the final good is sold in period t. Using again $W_t(m_t, k_t, b_t, S_t)$ to denote the value function of the household in the centralized market, the household's problem is given by

$$\begin{aligned}
&W_t(m_t, k_t, i_{t-1}, b_t, S_t) \\
&= \max_{x_t, h_t, m_{t+1}, k_{t+1}, i_t, b_{t+1}} \left\{ U(x_t) - Ah_t + \frac{\chi_t}{1-\nu_m}\left(\frac{m_t}{P_t}\frac{A}{Z_t^{1/1-\alpha}}\right)^{1-\nu_m} + \beta E W_{t+1}(m_{t+1}, k_{t+1}, i_t, b_{t+1}, S_{t+1}) \right\} \\
&\text{s.t.} \quad P_t x_t + P_t i_t + b_{t+1} + m_{t+1} \leq P_t W_t h_t + P_t R_t^k k_t + \Pi_t + R_{t-1} b_t + \hat{m}_t - T_t \qquad (59) \\
&\qquad k_{t+1} = (1-\delta)k_t + \left[1 - S\left(\frac{i_t}{i_{t-1}}\right)\right] i_t \qquad (60)
\end{aligned}$$

To a large extent, the optimality conditions for the households resemble the ones derived for the centralized market in the search-based model. In fact, the labor supply function, the Euler equation for Bond

holdings, the evolution of the shadow price of installed capital, and the capital accumulation equation are identical to Equations (41) to (44). The Euler equation for capital stock holdings is given by

$$\mu_t = \beta E_t \left\{ \frac{U'(x_{t+1})}{U'(x_t)} \left[R^k_{t+1} + (1-\delta)\mu_{t+1} \right] \right\}, \tag{61}$$

which is identical to (45) except that the term related to the DM does not appear. Moreover, the money demand equation can be expressed as

$$\left(\frac{m_{t+1}}{P_t} \right)^{\nu_m} = \frac{\beta R_t}{U'(x_t)(R_t - 1)} E_t \left[\left(\frac{A}{Z_*^{1/1-\alpha}} \right)^{-\nu_m} \frac{\chi_{t+1}}{\pi_{t+1}^{1-\nu_m}} \right]. \tag{62}$$

Equations (61) and (62) replace the optimality conditions (45) and (46) in the search-based model. Notice that m_{t+1} has been chosen in period t based on the realization of time t shocks. Hence, we detrend it by P_t and define $\mathcal{M}_{t+1} = m_{t+1}/P_t$ with the understanding that $\mathcal{M}_{t+1}$ only depends on realizations of shocks dated t and earlier. Since $\mathcal{M}_{t+1}$ does neither enter in the firms' problems nor is it included in the interest-rate feedback rule of the central bank, the model has a block-diagonal structure: the determination of output, inflation, and interest rates does not depend on the money stock.

The equilibrium conditions for the MIU model are summarized in the Appendix. As we did for the search-based model, we derive the deterministic steady state for the MIU model and use a log-linear approximation to its dynamics to form a state-space representation that is used for the Bayesian estimation.

4 Empirical Analysis

We will now turn to the estimation of the search-based and the MIU model. We use a Bayesian approach discussed in detail in An and Schorfheide (2007). We begin by describing the data set (Section 4.1). We then proceed by describing the specification of the prior distributions used for the parameters of the two DSGE models (Section 4.2). Finally, we present the parameter estimates as well as implied steady states, variance decompositions, and impulse response dynamics (Section 4.3).

4.1 Data

Our empirical analysis focuses on quarterly U.S. postwar data on aggregate output, inflation, interest rates, and money. Unless otherwise noted, the data are obtained from the FRED2 database maintained by the Federal Reserve Bank of St. Louis. Our measure of per capita output is defined as real GDP (GDPC96) divided by civilian noninstitutional population (CNP16OV). The population series is provided at a monthly frequency and converted to quarterly frequency by simple averaging. Since the quarterly flow statistics reported in the National Income and Product Accounts are annualized, we divide real GDP by 4. Inflation

is defined as the log difference of the GDP deflator (GDPDEF) and multiplied by 400 to obtain annualized percentages. Our measure of nominal interest rates corresponds to the Federal Funds Rate (FEDFUNDS). The Fed Funds Rate is also provided at monthly frequency and we use simple averaging to convert it to quarterly frequency. As a measure of money we use the sweep-adjusted M2S series provided by Cynamon, Dutkowsky and Jones (2006). This series is provided at monthly frequency without seasonal adjustment. We apply the EVIEWS default version of the X12 filter to perform the seasonal adjustment and then use simple averaging to convert the series from monthly to quarterly frequency. Finally, we divide our M2 series by population and the GDP deflator to obtain a measure of per capita real money balances.

The models presented in Sections 2 and 3 are specified to capture stationary fluctuations around a deterministic steady state. Hence, we take the natural log of per capita output and real money and extract a deterministic trend by OLS regressions over the sample period 1959:I to 2006:IV. We scale the deviations from the linear trend by 100 to convert them into percentages. At this point, we have removed information about the average velocity in the postwar data, which we need to pin down some of the parameters in our DSGE model. We therefore compute the sample average of the ratio of money and nominal output, take the natural logarithm, scale it by 100 and add it to our detrended log money series. For the subsequent estimation we restrict the sample period to 1984:I to 2005:IV.

4.2 Restricted Parameters and Prior Distributions

The goal of our empirical analysis is to compare the propagation of shocks and the steady state welfare implications of the search-based DSGE model and the MIU model. Hence, it is desirable to restricted a subset of the model parameters prior to estimation. These restrictions are reported in Table 1. We fix π_* at the average inflation rate in our sample. Moreover, we let r_A be equal to the difference of the average Federal Funds Rate and the average inflation rate between 1984 and 2005 and let $\beta = 1/(1 + r_A/400)$. Using these parameter values for both DSGE models implies that the steady state inflation and nominal interest rates are equal to the post-1984 sample averages. We fix the depreciation rate δ at 0.013. This value is obtained as the average ratio of fixed asset depreciation and the stock of fixed assets between 1959 and 2005 (Source: NIPA-FAT11 (current cost net stock) and NIPA-FAT13 (current cost depreciation), fixed assets and consumer durables). We set $g_* = 1.299$, which is obtained from the average ratio of consumption plus investment and GDP. The deviations of output from a linear trend in our post-1984 sample are highly persistent. To capture this persistence we let $\rho_z = 0.98$, a value that is consistent with the stationarity assumption embodied in our theoretical models as well as the observed persistence in the data.

We also impose that the estimated models agree on the conduct of post-1984 monetary policy. To this end, we conduct a preliminary estimation of the MIU model with capital without the money series. The

mechanics of this estimation are identical to those of the subsequent estimation of the full model. The only difference is, that the monetary policy rule parameters are estimated. The preliminary analysis yields $\hat{\psi}_1 = 1.82$, $\hat{\psi}_2 = 0.18$, and $\hat{\rho}_R = 0.78$. We fix the policy rule coefficients at these estimates for the subsequent analysis with both models. Moreover, both in the search-based model as well as in the MIU model we let $\mathcal{F} = 0$ (no fixed costs) and $\pi_{**} = 1$, meaning that firms that do not re-optimize prices either keep their prices constant or adjust with lagged inflation. We estimate the degree of dynamic indexation, captured by the parameter ι.

For the MIU model, the economy-wide mark-up is given by the parameter λ, which we fix at 15%. The model dynamics do not depend on Z_*, A, and B and our estimation sample does not contain any information on these parameters. Hence, we fix them such that a posteriori the estimated MIU model and the estimated search-based model will agree on the weight B that the centralized market goods receive in the instantaneous utility function, the steady state level of GDP, and the steady state labor productivity in the centralized market.[6] Finally, we use the steady state relationships to re-parameterize the search model in terms of the steady state (inverse) velocity, rather than χ_*. The steady state velocity is then essentially estimated by the average velocity in the sample (which we added to the real money balance series). In turn, this generates an estimate of χ_*.

To estimate the search model, we fix $Z_* = 1$ and $A_* = 1$. These two parameters essentially control the level of steady state output and labor productivity in the centralized market. We also set $\chi_* = 1$ and let B adjust in the estimation such that the model captures the average velocity of money in the sample. Since the overall markup in the search model is a function of the markup in the centralized market, λ, and the markup in the decentralized market, closely related to the Nash-bargaining parameter θ, we do not fix λ in the estimation of the search model. We will describe below how we force the estimates to produce an economy-wide mark-up that is close to 15%. The utility function parameter κ, which captures the utility of households that do not consume goods produced in the decentralized market, is fixed at 0.0001.

The marginal prior distributions for the remaining parameters of the MIU model are summarized in Table 2. The prior for inverse velocity is based on pre-sample information. Similarly, the prior for α is chosen so that the implied prior for the labor share is consistent with pre-sample evidence. We use a uniform prior on the indexation parameter ι and our prior for ζ is broadly consistent with micro-evidence on the frequency of price changes. The prior distributions for ρ_g and ρ_χ reflect the belief that the government spending (demand) disturbance and the money demand shock are fairly persistent. The remaining priors were loosely chosen such that the implied distribution of the variability of the endogenous variables is broadly in line with the pre-sample variability of the observed series. We assume that all the parameters listed in

[6] In future versions of this paper, we will parameterize the empirical models in terms of steady state GDP and labor productivity so that it is more straightforward to ensure that the two models agree in these dimensions.

Table 2 are *a priori* independent. Since we fix the policy rule parameters at values that are far away from the boundary of the determinacy region, no further adjustment of the prior is needed.

Table 3 provides information on the prior distribution for the parameters of the search model. We focus our discussion on the four additional parameters[7] that appear in Table 3: B, θ, $\tilde{\sigma} = 2\sigma$ and λ. While it is difficult to fully disentangle the effect of these four parameters, it is instructive to take a look at the share of the decentralized market:

$$sh = \sigma \frac{1}{\pi_*} \frac{\mathcal{M}_*}{\mathcal{Y}_*}.$$

We can infer a plausible magnitude for $\mathcal{M}_*/\mathcal{Y}_*$ based on a pre-sample observations. Since the steady state (inverse) velocity is related to the parameter B, we pick a prior for B such that our implicit beliefs about velocity are in line with pre-sample evidence. Since σ relates velocity to the share of the decentralized market, a prior for σ is linked to prior beliefs about the DM share. The parameter θ affects the bargaining power of the seller in the decentralized market and hence the markup. The economy-wide markup in turn depends on the DM markup, the DM share, and the markup in the centralized market, λ. Since it is difficult to directly specify a prior on the structural parameters of the search model, that reflect beliefs or pre-sample information about observables, we follow the approach proposed in Del Negro and Schorfheide (2007).

We combine the marginal prior distributions reported in Table 3 and denoted as $\tilde{p}(\vartheta)$ with a function $f(\vartheta)$ that in our case reflects beliefs about the investment output ratio and the economy-wide markup. Here the vector ϑ stacks the parameters of the DSGE model. The overall prior is given by

$$p(\vartheta) \propto \tilde{p}(\vartheta) \exp\left\{ -\frac{1}{2} \left[\frac{(I_*(\vartheta)/\mathcal{Y}_*(\vartheta) - 0.16)^2}{0.01^2} - \frac{(mu(\vartheta) - 0.15)^2}{0.01^2} \right] \right\} \mathcal{I}\{\vartheta \in \Theta_D\}, \tag{63}$$

where $\propto$ signifies proportionality, I_* and $\mathcal{Y}_*$ denote the steady states of investment and output (as a function of ϑ), $mu(\vartheta)$ is the economy-wide mark-up, and $\mathcal{I}\{\vartheta \in \Theta_D\}$ is an indicator function that is one if ϑ falls in the region of the parameter space in which the linearized search model has a unique stable rational expectations solution (determinacy) The adjustment function down-weights parameter combination for which the investment output ratio deviates from 0.16 and the economy-wide mark-up deviates from 15%. We use $p(\vartheta)$ as the prior for the estimation of the search model.[8]

4.3 Parameter Estimates and Model Dynamics

We report prior and posterior means and 90% credible intervals for the parameters of the two models in Tables 4 and 2. The posterior is obtained by combining the prior distribution described in the previous subsection with the likelihood function derived from the state-space representations of the linearized DSGE

[7] Since according to our model $\sigma \in [0, 1/2]$ we introduce the transformed parameter $\tilde{\sigma}$, which lives on the unit interval.

[8] Draws from this prior can be generated using the random-walk Metropolis algorithm described in An and Schorfheide (2007). The normalization constant can be computed using Geweke's (1999) harmonic mean estimator.

models. We then use a random-walk Metropolis algorithm to generate draws from the posterior distribution of the parameters. To make inference about steady states, impulse responses, and variance decompositions, we convert the parameter draws into the statistics of interest. Further technical details are described in An and Schorfheide (2007).

The parameters ζ and ι determine the shape of the Phillips curve. According to the estimates of the MIU model, there is virtually no dynamic indexation and a moderate degree of nominal rigidity. The estimates $\hat{\zeta} = 0.737$ implies that the average duration of prices is slightly less than four quarters. Adjustment costs are fairly large, $\hat{S}''$, reducing the volatility of the return to capital and dampening its effect on the marginal costs that enter the Phillips curve relationship. Of particular interest is the estimate of the elasticity ν_m. In log-linear form, the money demand equation for the MIU model can be written as[9]

$$\begin{aligned} \tilde{M}_{t+1} &= \frac{1}{\nu_m}\tilde{X}_t - \frac{1-\nu_m}{\nu_m}E_t[\tilde{\pi}_{t+1}] - \frac{1}{\nu_m(R_*-1)}\tilde{R}_t + E_t[\tilde{\chi}_{t+1}] \\ &= 0.037\tilde{X}_t + 0.963E_t[\tilde{\pi}_{t+1}] - 2.786\tilde{R}_t + E_t[\tilde{\chi}_{t+1}]. \end{aligned} \tag{64}$$

The money demand shock is highly persistent, $\hat{\rho}_\chi = 0.974$. We will defer a discussion of the estimated shock standard deviation until we examine the impulse response dynamics and variance decompositions.

From the search model we obtain a slightly lower posterior mean estimate of the Calvo parameter, $\hat{\zeta} = 0.652$, find a higher degree of dynamic indexation, $\hat{\iota} = 0.347$, and obtain a larger estimate of the adjustment cost parameter, $\hat{S}'' = 12.97$, than in the MIU model. As stated previously, the estimate of B is by and large driven by the average velocity in the sample, and the estimates of θ, σ, and λ adjust such that economy-wide markup is about 15% and the model is able to capture the cyclical fluctuations of M2.

The implied posterior distribution of the steady states is reported in Table 6. As mentioned in Section 4.1, we fix the parameters Z_* and A in the MIU model such that near the posterior mean steady state output and steady state labor productivity in the centralized market are similar across the two models. Since our estimation sample contains information about average velocity post 1984, the model estimates imply similar $\mathcal{M}_*/\mathcal{Y}_*$ ratios. The estimation of both models is based on a tight prior on the economy-wide markup: for the MIU model we fixed λ and for the search model we included an adjustment term (63) that forces the implied overall markup to be close to 15%. Hence, the overall markup remains close to 15% *a posteriori*. Since the MIU was re-parameterized in terms of $\mathcal{M}_*/\mathcal{Y}_*$ we report the implicit posterior for χ_* in Table 6, which is very close to zero. As will become clear in the steady state welfare analysis conducted in Section 5, our estimated MIU model behaves essentially like a cashless economy.

The estimated markup in the decentralized market of the search model is about 25% and the share of the decentralized market is around 38% of the economy. Compared to AWW, we find a smaller markup in

[9] We re-normalized $\tilde{\chi}_t$ such that it has a unit coefficient in the money demand equation.

the DM, which coincides with a fairly large estimate of θ, and obtain a more sizeable DM share. While the calibration in AWW also tries to match an overall markup of 15% in the economy, it focuses on the long-run elasticity of demand for M1 to choose the remaining DM parameters. We conjecture that our larger estimate of the DM share is due to the fact that the likelihood function forces the model to capture the observed business cycle fluctuations in the stock of money.

The variance decompositions associated with the two estimated models are reported in Table 7. Most of the fluctuations of output, inflation, and the Federal Funds Rate are driven by the highly persistent technology shock. It is important to note that we are reporting decompositions of the unconditional variance in the table, rather than decompositions of h-step ahead forecast errors or decompositions of the variation at business cycle frequency. Neither in the MIU model nor in the search model, monetary policy shocks play an important role for the overall variability of output, inflation, and the Federal Funds Rate. Due to the block-triangular structure of the MIU model, money demand shocks have no effect on output, inflation, and the interest rate. However, money demand shocks in the MIU model explain almost 80% of the variance of the monetary aggregate. We interpret this as a disconnect between fluctuations in money and the shocks that drive output and inflation.

The estimated search model produces a tighter link between technology shocks and monetary aggregates: more than 75% of the variance of real money balances is due to technology shocks, which affect productivity in the decentralized market that uses money as a medium of exchange. Money demand shocks, modeled as shocks to the taste for goods produced in the decentralized market, play a much smaller role for the fluctuations of real money balances. They explain only about 18% of the variation. Unfortunately, the tight link between money and output generated by the search model comes at a cost. The overall fit of the search model relative to the MIU model, as measured by the difference of log marginal data densities is not as good. The log marginal data density for the search model is -522.36, whereas it is -434.12 for the MIU model.[10] The calculation of the marginal data densities puts the search model at a slight disadvantage, because we conducted a preliminary estimation with the MIU model to determine the values at which we fixed the policy rule coefficients.

Impulse response dynamics for the two models are depicted in Figure 1. As mentioned previously, output, inflation, and interest rates do not respond to money demand shocks in the MIU model, because monetary policy is conducted through an interest rate feedback rule and real money balances enter the utility function in an additively separable manner. The estimated response of the output to technology, monetary policy, and government spending shock is very similar for the search and the MIU model. The slightly higher nominal

[10] Someone who places equal prior probability on the two models would conclude that the odds in favor of the MIU model are exp[88.36].

rigidity in the MIU model manifests itself through a weaker response of inflation to a monetary policy shock. In other words, a nominal shock moves real output more relative to prices in the MIU model.

Money demand is more responsive to a monetary policy shock in the search model. A shock that raises interest rates by 70 basis points, lowers demand for real money balances by 50 basis points in the MIU model and about 140 basis points in the search model. A positive technology shock in the MIU model lowers the marginal costs of production, despite rising wages and rental rates of capital. Since inflation can be represented as the expected sum of discounted future marginal costs, inflation falls below its steady state value. According to the interest rate feedback rule, low inflation creates a desire to reduce interest rates whereas high output would ceteris paribus lead to in increase in nominal interest rates. On balance, nominal rates fall such that along the response real rates and the return on capital are positive. According to the MIU money demand equation (64) below steady state inflation lowers the demand for real money balances whereas below average interest rates raise the demand for real money balances. Overall, the response of money is slightly positive. In the search model prices are less rigid and inflation (and interest rates) respond more strongly. Moreover, the increased activity in the decentralized market due to the positive technology shock strongly increases the demand for real money balances.

Finally, we examine to what extent the estimated DSGE models can reproduce the observed correlation between money and output. To this extent, we conduct a posterior predictive check as follows. We take draws from the posterior distribution of the parameters and simulate the models conditional on these parameters. For each simulated trajectory, we compute some sample statistics, in particular, sample correlations between money and output. Thus, we obtain draws from the posterior predictive distribution for these sample moments. Means and 90% credible intervals are summarized in Table 8. We also report the corresponding statistics for post-1984 U.S. data. If the statistics computed from the actual data fall far in the tails of the posterior predictive distribution, then this can be interpreted as an inability of the estimated model to reproduce a particular feature of the data. The data exhibit a fairly strong correlation between lagged real money balances and output, whereas the contemporaneous correlation is only about 0.13. In both DSGE models the correlation function between output and money is rather flat, due to the very persistent technology shock. The correlation is stronger in the search model, because it attributes a larger fraction of the variation in real money balances to technology, which is the major driving force for output. While the posterior predictive distribution is rather diffuse for both models, neither one seems to be able to generate the particular shape of the observed correlation function.

5 Steady State Welfare Comparisons

In this section, we consider policy experiments where we change the inflation target of the central bank, π_*, which is the inflation rate that prevails at the steady state. For now we simply compare steady states, ignoring transitions but at the end of the section we comment on what we expect regarding the transitions. Unless noted otherwise, we fix the parameter values at the posterior mean estimates reported in Section 4 (see Tables 1, 4, and 5).

A priori, we can think about four sources of welfare cost of inflation that are present in the search-based model, two of which are also shared by the MIU model. First, inflation is a tax on money holdings in both models and as inflation rises and hance the net nominal interest rate increases welfare will be reduced. This logic underlies Friedman's prescription of a zero percent net nominal interest rate which has come to be known as the Friedman rule. We will label this channel of welfare loss the Friedman channel. This channel will reveal itself as a reduction in real money balances and hence lower utility from the MIU part in the MIU model while we will see a lower q in the search-based model, which will lower utility in the DM. Second, both models display some level of price rigidity given by positive ζ and the fact that some firms cannot optimally change their prices create a price dispersion. This price dispersion becomes more severe as the steady state inflation level goes away from 0% (in both directions), reducing welfare. We will label this channel the price dispersion channel. The remaining two channels in the search-based model are unique to this model and is explained in detail in AWW. To summarize, the bilateral nature of trade and the fact that the surplus in a meeting is split by the two parties in the DM create two holdup problems: the buyers do not bring in the optimal amount of money (a money demand holdup problem) and the sellers do not bring in the optimal amount of capital (an investment holdup problem). These holdup problems are aggravated as inflation increases as this further reduces the payoffs in the DM by reducing q. We will collectively refer to these two sources of welfare loss as the holdup problem channel.

Our estimation results reveal differences between the search-based model and the MIU model in terms of some key parameters that affect welfare. For example, χ_* is fixed at unity in the search-based model while it is estimated to be 4.52×10^{-10} in the MIU model.[11] This makes the MIU model essentially a cashless model and diminishes the importance of the Friedman channel. In fact it is well-known that in the cashless version of such models welfare is maximized at 0% inflation. On the other hand, the two model display somewhat different levels of markup of the monopolistically competitive firms and degree of price stickiness. These will affect the dominance of the price dispersion channel.

Before we turn to the results, a brief discussion about how we compute the welfare loss is in order. In

[11] Fixing χ_{ast} is a normalization in the search-based model as both models have the same implied steady state velocity at the estimated parameters.

the MIU model, the steady state value up to a constant is given by

$$W(\pi_*) = U(x_*) - Ah_* + \frac{\chi_*}{1-\nu_m}\left(\frac{A\mathcal{M}_*}{\pi_* Z_*^{1/1-\alpha}}\right)^{1-\nu_m} \tag{65}$$

and we solve for the percentage change required in x_* to make the households indifferent between two economies with different steady state inflation rates. In the search-based model, the reduced-form steady state value up to a constant is given by

$$V(\pi_*) = \sigma\left[u(q_*) - c(q_*, k_*, Z_*)\right] + U(x_*) - Ah_* \tag{66}$$

and we solve for the percentage change required in x_* and consumption in the DM which will be the q_* inside the utility function to make the households indifferent between two economies with different steady state inflation rates.[12] Finally, as a technical point, we replace $\left(1 - \frac{1}{g_*}\right)\mathcal{Y}_*$ with simply a constant G_* obtained from the estimations.

Our main results are provided in Figures 2 and 3. Figure 2 plots the welfare cost of deviating from 0% in both models in terms of consumption on a grid running from the Friedman rule up to 10% inflation.[13] We see that the welfare loss for the MIU model has the familiar u-shape which shows that welfare is maximized around 0% and deviations in either direction are costly.[14] The welfare cost of 10% inflation is as high as 12% of consumption. The search-based model tells a very different story. Welfare cost is monotonically increasing, which shows that the optimal inflation is the one implied by the Friedman rule. The welfare cost of 10% is under 6%, which is roughly half of what is implied by the MIU model. We see that the Friedman channel is dominated by the price dispersion channel in the MIU model while the Friedman and holdup problem channels dominate the price dispersion channel in the search-based model.

Figure 3 show some of the steady state allocations and prices in both models as inflation changes, relative to the values at 0% inflation. First, we want to emphasize that there is a crucial difference between the MIU and the search-based model. In the MIU model, the change in welfare due to the Friedman channel is only due to the direct effect of inflation on real money balances and this has no indirect effect. However, in the search-based model, inflation effects q, which might be interpreted in the reduced form as an MIU effect but changes in q in turn affect all the other allocations and prices in the model. As a result of this, we see that

[12] Note that we will not change the q_* term inside the cost function as it is a part of production.

[13] As a side remark, note that welfare at the Friedman rule is not defined for the MIU model as unless we put an artificial bound on money holdings, there is no solution to the household's problem. Money is costless to hold and utility is increasing in money balances so the households would like to hold arbitrarily large amounts. This is not the case in the search-based model as households would never want to hold more money than what they need to purchase q^*, the first-best quantity. As a result welfare is well-defined at the Friedman rule. For the figures, we omit the Friedman rule for the MIU model.

[14] The maximum is not exactly at 0% as one would expect based on earlier work but it is slightly positive. This is an artifact of having capital in the model, which is an observation shared by other researchers in the area.

some of the key changes, decreases in X, K and W and the increase in H are more severe in the MIU model. This is why the welfare cost of higher levels of inflation is more pronounced in the MIU model.

To emphasize the finding that at the estimated parameters different channels dominate in the two models, and to establish that both models are in principle able to generate either of the two shapes of welfare loss, we vary the level of price stickiness in both models, holding all other parameters at their estimated levels. The results for a region around 0% inflation are plotted in Figure 4. In the first panel, we start from the estimated ζ and reduce it towards zero. Around $\zeta = 0.5$ the price dispersion channel loses its dominance and the welfare loss curve becomes monotonically increasing, making Friedman rule the optimal. Similarly, in the second panel we start increasing ζ from its estimated value towards unity. Around $\zeta = 0.9$ the price dispersion channel overweighs the other two channels and we get the u-shape that shows optimal inflation is around 0%.

To investigate the contribution of the New Keynesian features of the model, we repeat the exercise in Figure 2 with $\lambda = \zeta = 0$ for both models. Figure 5 plots the results along with the curves in Figure 2 for reference. First, we see that in both models welfare cost is monotonically increasing in the rate of inflation and hence the Friedman rule is optimal. Second, the welfare cost of inflation in the MIU model is about one order of magnitude smaller than that in the search-based model. In the MIU model, the only channel that is in play is the Friedman channel, while in the search model the holdup problem channels are also active. In fact, about 3.5% of consumption steady state welfare loss for 10% annual inflation is consistent with the results of AWW. Turning on the New Keynesian features boosts the welfare cost of 10% by about 1.5% for the search-based model while the increase in the MIU model is over 10%. These results confirm that the Friedman channel is much more important for the search-based model.

As we pointed out, we did not consider transitions in this section. One interpretation of the exercise we conducted is that we compare the welfare of two economies with different inflation rates. If, however, the exercise we want to conduct involves a policy change for a given economy, it is more appropriate to consider the transition path of the economy to the new steady state. Recent results in the literature tell us that these transitions may have quite large welfare implications. For example, AWW reports welfare costs associated with going from a bad steady state (high inflation) to a good one (low inflation) can be as large as half of the steady state gain. Intuitively, the magnitude of the transition depends on the change in the level of capital between the two steady states as resources are temporarily devoted to accumulating this extra capital. Looking at Figure 3, we see that going from 10% inflation to 0% inflation, the level of steady state capital changes by more in the MIU model: capital stock goes up by less than 2% in the search-based model while the change is over 4% in the MIU model. This would mean that welfare loss during the transition will be bigger for the MIU model and the curve for the MIU model in Figure 2 will shift down more so than the curve for the search-based model.

6 Conclusion

As an alternative to the commonly used MIU model, we have developed an estimable DSGE model in which the presence of a decentralized market creates an incentive for households to hold money, because money is needed as a medium of exchange. The model specification is closely tied to the theoretical literature that is developing microfounded models of monetary exchange. In particular, we base our model on recent work by Lagos and Wright (2005), and Aruoba, Waller, and Wright (2007). Using post-1984 U.S. on output, inflation, interest rates, and real money balances, we estimate our search-based DSGE model along with a standard New Keynesian model in which real money balances enter the utility function. We obtain parameter estimates for the taste and technology parameters that determine the exchange in the decentralized market of the search model. These parameter estimates are potentially useful for the theoretically-oriented literature on microfounded monetary models. We compare the dynamics of the estimated search model and the MIU model. While the decentralized market mechanism of the search model creates a stronger linkage between technology shocks and fluctuations in the stock of money, this linkage comes at a cost in terms of overall fit. Finally, we explore the steady state welfare implications of the two models. The estimated MIU model behaves very much like a New Keynesian DSGE model and a near-zero inflation rate is optimal. According to the search model, which also has embodied some New Keynesian feature, the Friedman motive for keeping the nominal interest rate near zero dominates and negative inflation rates are optimal. This paper is part of a research agenda that tries to link the literatures on microfounded monetary models and estimable New Keynesian DSGE models that are popular at central banks. Many interesting questions are left unanswered and will hopefully be addressed in future research.

References

Aruoba, S. Borağan, Christopher Waller, and Randall Wright (2007): "Money and Capital," *Manuscript*, University of Maryland, University of Notre Dame, and University of Pennsylvania.

An, Sungbae and Frank Schorfheide (2007): "Bayesian Analysis of DSGE Models," *Econometric Reviews*, **26**(2-4), 113-172.

Calvo, Guillermo (1983): " Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics*, **12**, 383-398.

Christiano, Lawrence , Martin Eichenbaum, and Charles Evans (2005): "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, **113**, 1-45.

Cynamon, Barry, Donald Dutkowsky, and Barry Jones (2006): "Redefining the Monetary Aggregates: A Clean Sweep," *Eastern Economic Journal*, **32**, 661-672.

Del Negro, Marco and Frank Schorfheide (2007): "Forming Priors for DSGE Models (and How it Affects the Assessment of Nominal Rigidities)," *Manuscript*, Federal Reserve Bank of New York and University of Pennsylvania.

Del Negro, Marco, Frank Schorfheide, Frank Smets, and Rafael Wouters (2007): "On the Fit of New Keynesian Models," *Journal of Business and Economic Statistics*, **25**, 123-162.

Geweke, John (1999): "Using Simulation Methods for Bayesian Econometric Models: Inference, Development and Communication," *Econometric Reviews*, **18**, 1-126.

Kiyotaki, Nobu and Randall Wright (1989): "On Money as a Medium of Exchange," *Journal of Political Economy*, **97**, 927-954.

Lagos, Ricardo and Randall Wright (2005): "A Unified Framework for Monetary Theory and Policy Evaluation," *Journal of Political Economy*, **113**, 463-484.

Levin, Andrew, Alexei Onatski, John Williams, and Noah Williams (2006): "Monetary Policy Under Uncertainty in Micro-Founded Macroeconometric Models" in Mark Gertler and Kenneth Rogoff (eds.) *NBER Macroeconomics Annual 2005*, MIT Press, Cambridge, MA.

Smets, Frank and Raf Wouters (2003): "An Estimated Stochastic Dynamic General Equilibrium Model for the Euro Area," *Journal of the European Economic Association*, **1**, 1123-1175.

Woodford, Michael (2003): *Interest and Prices*, Princeton University Press, Princeton.

A The Search-Based Model

We use a slightly more general specification of the utility and production functions in the subsequent exposition:

$$U(x) = B\frac{x^{1-\gamma}}{1-\gamma}, \quad u(q) = \frac{(q+\kappa)^{1-\eta} - \kappa^{1-\eta}}{1-\eta}.$$

Moreover, we let $f(e,k) = e^{\Phi}k^{1-\Phi}$.

A.1 Further Details: Intermediate Good Producers

The first-order condition for a intermediate good producing firm is:

$$\mathbb{E}_t\left\{\sum_{s=0}^{\infty}\zeta^s\beta^s\Xi^p_{t+s|t}\frac{1}{P^o_t(i)}\left(\frac{P^o_t(i)\pi^{adj}_{t+s|t}}{P_{t+s}}\right)^{-\frac{1+\lambda}{\lambda}}Y_{t+s}\left[P^o_t(i)\pi^{adj}_{t+s|t} - (1+\lambda)P_{t+s}MC_{t+s}\right]\right\} = 0. \tag{67}$$

Define and rewrite

$$\begin{aligned}
\mathcal{F}^{(1)}_t &= \mathbb{E}_t\left[\sum_{s=0}^{\infty}\zeta^s\beta^s\Xi^p_{t+s|t}\left(\frac{P^o_t(i)\pi^{adj}_{t+s|t}}{P_{t+s}}\right)^{-\frac{1+\lambda}{\lambda}}Y_{t+s}\pi^{adj}_{t+s|t}\right] \qquad (68)\\
&= \left(\frac{P^o_t(i)}{P_t}\right)^{-\frac{1+\lambda}{\lambda}}Y_t + \zeta\beta\mathbb{E}_t\left[\sum_{s=0}^{\infty}\zeta^s\beta^s\Xi^p_{t+1+s|t}\left(\frac{P^o_t(i)\pi^{adj}_{t+1+s|t}}{P_{t+1+s}}\right)^{-\frac{1+\lambda}{\lambda}}Y_{t+1+s}\pi^{adj}_{t+1+s|t}\right]\\
&= \left(\frac{P^o_t(i)}{P_t}\right)^{-\frac{1+\lambda}{\lambda}}Y_t + \zeta\beta\left(\pi^{\iota}_t\pi^{(1-\iota)}_{**}\right)^{-1/\lambda}\\
&\quad\times\mathbb{E}_t\left[\left(\frac{P^o_t(i)}{P^o_{t+1}(i)}\right)^{-\frac{1+\lambda}{\lambda}}\Xi^p_{t+1|t}\sum_{s=0}^{\infty}\zeta^s\beta^s\Xi^p_{t+1+s|t+1}\left(\frac{P^o_{t+1}(i)\pi^{adj}_{t+1+s|t+1}}{P_{t+1+s}}\right)^{-\frac{1+\lambda}{\lambda}}Y_{t+1+s}\pi^{adj}_{t+1+s|t+1}\right]\\
&= \left(\frac{P^o_t(i)}{P_t}\right)^{-\frac{1+\lambda}{\lambda}}Y_t + \zeta\beta\left(\pi^{\iota}_t\pi^{(1-\iota)}_{**}\right)^{-1/\lambda}\mathbb{E}_t\left[\left(\frac{P^o_t(i)}{P^o_{t+1}(i)}\right)^{-\frac{1+\lambda}{\lambda}}\Xi^p_{t+1|t}\mathcal{F}^{(1)}_{t+1}\right].
\end{aligned}$$

Similarly,

$$\begin{aligned}
\mathcal{F}^{(2)}_t &= \mathbb{E}_t\left[\sum_{s=0}^{\infty}\zeta^s\beta^s\Xi_{t+s}\left(\frac{P^o_t(i)\pi^{adj}_{t+s|t}}{P_{t+s}}\right)^{-\frac{1+\lambda}{\lambda}}Y_{t+s}\frac{P_{t+s}MC_{t+s}}{P^o_t(i)}\right] \qquad (69)\\
&= \left(\frac{P^o_t(i)}{P_t}\right)^{-\frac{1+\lambda}{\lambda}}Y_t\frac{P_tMC_t}{P^o_t(i)} + \zeta\beta\left(\pi^{\iota}_t\pi^{(1-\iota)}_{**}\right)^{-\frac{1+\lambda}{\lambda}}\mathbb{E}_t\left[\left(\frac{P^o_t(i)}{P^o_{t+1}(i)}\right)^{-\frac{1+\lambda}{\lambda}-1}\Xi^p_{t+1|t}\mathcal{F}^{(2)}_{t+1}\right].
\end{aligned}$$

and the first-order condition becomes

$$\mathcal{F}^{(1)}_t = (1+\lambda)\mathcal{F}^{(2)}_t. \tag{70}$$

A.2 Further Details: Price Dispersion

To capture the evolution of the price distribution we introduced a new variable D_t. Its law of motion can be derived as follows:

$$\begin{aligned} D_t &= (1-\zeta)\sum_{j=0}^{\infty}\zeta^j\left(\frac{(\pi_{t-1}\pi_{t-2}\cdots\pi_{t-j})^{\iota}\pi_{**}^{j(1-\iota)}}{\pi_t\pi_{t-1}\cdots\pi_{t-j+1}}\frac{P_{t-j}^o}{P_{t-j}}\right)^{-\frac{1+\lambda}{\lambda}} \\ &= (1-\zeta)\left[\frac{P_t^o}{P_t}\right]^{-\frac{1+\lambda}{\lambda}} \\ &\quad +(1-\zeta)\zeta\left[\left(\frac{\pi_{t-1}}{\pi_t}\right)^{\iota}\left(\frac{\pi_{**}}{\pi_t}\right)^{(1-\iota)}\frac{P_{t-1}^o}{P_{t-1}}\right]^{-\frac{1+\lambda}{\lambda}} \\ &\quad +(1-\zeta)\zeta^2\left[\left(\frac{\pi_{t-2}}{\pi_t}\right)^{\iota}\left(\frac{\pi_{**}^2}{\pi_t\pi_{t-1}}\right)^{(1-\iota)}\frac{P_{t-2}^o}{P_{t-2}}\right]^{-\frac{1+\lambda}{\lambda}}\cdots. \end{aligned}$$

Lagging D_t by one period yields

$$\begin{aligned} D_{t-1} &= (1-\zeta)\left[\frac{P_{t-1}^o}{P_{t-1}}\right]^{-\frac{1+\lambda}{\lambda}} \\ &\quad +(1-\zeta)\zeta\left[\left(\frac{\pi_{t-2}}{\pi_{t-1}}\right)^{\iota}\left(\frac{\pi_{**}}{\pi_{t-1}}\right)^{(1-\iota)}\frac{P_{t-2}^o}{P_{t-2}}\right]^{-\frac{1+\lambda}{\lambda}} \\ &\quad +(1-\zeta)\zeta^2\left[\left(\frac{\pi_{t-3}}{\pi_{t-1}}\right)^{\iota}\left(\frac{\pi_{**}^2}{\pi_{t-1}\pi_{t-2}}\right)^{(1-\iota)}\frac{P_{t-3}^o}{P_{t-3}}\right]^{-\frac{1+\lambda}{\lambda}}\cdots. \end{aligned}$$

Therefore, we obtain the following law of motion for the price dispersion:

$$D_t = \zeta\left[\left(\frac{\pi_{t-1}}{\pi_t}\right)^{\iota}\left(\frac{\pi_{**}}{\pi_t}\right)^{(1-\iota)}\right]^{-\frac{1+\lambda}{\lambda}}D_{t-1} + (1-\zeta)\left[\frac{P_t^o}{P_t}\right]^{-\frac{1+\lambda}{\lambda}}. \tag{71}$$

A.3 Equilibrium Conditions

We now summarize the equilibrium conditions for the search-based model. The timing is such that all t shocks are realized at the beginning of t and $\bar{S}_t = (Z_t, g_t, \chi_t)$ and R_t are observed. $\bar{S}_t$ summarizes the exogenous state variables. We define $S_t = \left(\bar{S}_t, R_t\right)$ which will be the aggregate state variables of the household's problem. In the following definitions, we do not track h_t (individual labor supply) and B_t (the bond supply of the government). We also do not track nominal money balances but instead track $\mathcal{M}_t$ as defined in the main text. Finally, we use $\pi_t \equiv P_t/P_{t-1}$ and do not track the level of prices.

Given exogenous states $\left\{\bar{S}_t\right\}_{t=0}^{\infty}$, a monetary equilibrium is defined as allocations $\{q_t, X_t, H_t, K_t, I_t, \mu_t, Y_t, \mathcal{M}_t, \mathcal{Y}_t\}_{t=0}^{\infty}$, policy $\{R_t\}_{t=0}^{\infty}$ and prices $\left\{W_t, R_t^k, p_t^0, \pi_t, D_t\right\}_{t=0}^{\infty}$ such that :

Household's Problem: Given exogenous states, policy and prices, $\left\{q_t, X_t, H_t, K_t, I_t, \mu_t, \mathcal{M}_t, \Xi^p_{t+1|t}\right\}_{t=0}^{\infty}$ satisfy

$$W_t = \frac{A}{U'(X_t)} \tag{72}$$

$$1 = \beta E_t\left[\frac{U'(X_{t+1})}{U'(X_t)}\frac{R_t}{\pi_{t+1}}\right] \tag{73}$$

$$1 = \mu_t\left[1 - S\left(\frac{I_t}{I_{t-1}}\right) + \frac{I_t}{I_{t-1}}S'\left(\frac{I_t}{I_{t-1}}\right)\right] + \beta E_t\left\{\mu_{t+1}\frac{U'(X_{t+1})}{U'(X_t)}\left(\frac{I_{t+1}}{I_t}\right)^2 S'\left(\frac{i_{t+1}}{i_t}\right)\right\} \tag{74}$$

$$K_{t+1} = (1-\delta)K_t + \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right]I_t \tag{75}$$

$$\mu_t = \beta E_t\left\{\frac{U'(X_{t+1})}{U'(x_t)}\left[R^k_{t+1} + (1-\delta)\mu_{t+1}\right] - \frac{\sigma}{U'(X_t)}\gamma(q_{t+1}, K_{t+1}, \chi_{t+1}, Z_{t+1})\right\} \tag{76}$$

$$\mathcal{M}_t = \frac{g\left(q_t, K_t, \chi_t, Z_t\right)W_t\pi_t}{A} \tag{77}$$

$$U'(X_t) = \beta E_t\left\{\frac{U'(X_{t+1})}{\pi_{t+1}}\left[\frac{\sigma\chi_{t+1}u'(q_{t+1})}{g_q(q_{t+1}, K_{t+1}, \chi_{t+1}, Z_{t+1})} + (1-\sigma)\right]\right\} \tag{78}$$

$$\Xi^p_{t+1|t} = \frac{U'(X_{t+1})}{U'(X_t)\pi_{t+1}} \tag{79}$$

Intermediate Goods Producing Firms' Problem: Intermediate goods firms choose their capital labor ratio as a function of the factor prices to minimize costs:

$$K_t = \frac{\alpha}{1-\alpha}\frac{W_t}{R^k_t}H_t. \tag{80}$$

Firms that are allowed to change prices are choosing a relative price $p^o_t(i)$ (relative to the aggregate price level) to maximize expected profits subject to the demand curve for their differentiated product, taking the aggregate price level P_t as well as the prices charged by other firms as given, which leads to

$$MC_t = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}W_t^{1-\alpha}(R^k_t)^{\alpha}Z_t^{-1} \tag{81}$$

$$\mathcal{F}^{(1)}_t = (p^o_t)^{-\frac{1+\lambda}{\lambda}}Y_t + \zeta\beta\left(\pi^{\iota}_t\pi^{(1-\iota)}_{**}\right)^{-1/\lambda}\mathbb{E}_t\left[\left(\frac{p^o_t}{\pi_{t+1}p^o_{t+1}}\right)^{-\frac{1+\lambda}{\lambda}}\Xi^p_{t+1|t}\mathcal{F}^{(1)}_{t+1}\right] \tag{82}$$

$$\mathcal{F}^{(2)}_t = (p^o_t)^{-\frac{1+\lambda}{\lambda}-1}Y_tMC_t + \zeta\beta\left(\pi^{\iota}_t\pi^{(1-\iota)}_{**}\right)^{-\frac{1+\lambda}{\lambda}}\mathbb{E}_t\left[\left(\frac{p^o_t}{\pi_{t+1}p^o_{t+1}}\right)^{-\frac{1+\lambda}{\lambda}-1}\Xi^p_{t+1|t}\mathcal{F}^{(2)}_{t+1}\right] \tag{83}$$

$$\mathcal{F}^{(1)}_t = (1+\lambda)\mathcal{F}^{(2)}_t \tag{84}$$

Final Good Producing Firms' Problem: Final goods producers take factor prices and output prices as given and choose inputs $Y_t(i)$ and output Y_t to maximize profits. Free entry ensures that final good producers make zero profits and leads to

$$\pi_t = \left[(1-\zeta)\left(\pi_t p^o_t\right)^{-\frac{1}{\lambda}} + \zeta(\pi^{\iota}_{t-1}\pi^{1-\iota}_{**})^{-\frac{1}{\lambda}}\right]^{-\lambda} \tag{85}$$

Monetary Policy: The central bank supplies the quantity of money necessary to attain the nominal interest rate

$$\frac{R_t}{R_*} = \left(\frac{R_{t-1}}{R_*}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi_*}\right)^{\psi_1} \left(\frac{\mathcal{Y}_t}{\mathcal{Y}_*}\right)^{\psi_2}\right]^{1-\rho_R} \exp\left(\sigma_r \varepsilon_t^r\right) \tag{86}$$

Aggregate Resource Constraint for CM is given by

$$Y_t = D_t^{-1}(Z_t K_t^{\alpha} H_t^{(1-\alpha)} - \mathcal{F}), \tag{87}$$

where

$$D_t = \zeta \left[\left(\frac{\pi_{t-1}}{\pi_t}\right)^{\iota} \left(\frac{\pi_{**}}{\pi_t}\right)^{(1-\iota)}\right]^{-\frac{1+\lambda}{\lambda}} D_{t-1} + (1-\zeta)\left(p_t^o\right)^{-\frac{1+\lambda}{\lambda}}. \tag{88}$$

The gross domestic product of this economy is given by

$$\mathcal{Y}_t = Y_t + \sigma \frac{\mathcal{M}_t}{\pi_t} \tag{89}$$

Market Clearing: The goods market in the CM clears:

$$X_t + I_t + \left(1 - \frac{1}{g_t}\right)\mathcal{Y}_t = Y_t \tag{90}$$

A.4 Steady States

Noting that $\mu_* = 1$, given a target inflation rate π_*, a monetary steady state is defined as allocations q_*, X_*, H_*, K_*, I_*, Y_*, $\mathcal{M}_*$, $\mathcal{Y}_*$, policy R_* and prices W_*, R_*^k, p_*^o that satisfy:

$$
\begin{aligned}
U'(X_*) &= \frac{A}{W_*} \\
R_* &= 1 + \sigma\left[\frac{\chi_* u'(q_*)}{g_q(q_*, K_*, \chi_*, 1)} - 1\right] \\
1 &= \beta\left(1 + R_*^k - \delta\right) - \sigma\beta\frac{\gamma(q_*, K_*, \chi_*, 1)}{U'(X_*)} \\
\mathcal{M}_* &= \frac{g\left(q_*, K_*, \chi_*, 1\right) W_* \pi_*}{A} \\
K_* &= \frac{\alpha}{1-\alpha}\frac{W_*}{R_*^k} H_* \\
p_*^o &= (1+\lambda)\left[\frac{1 - \zeta\beta\left(\frac{\pi_{**}}{\pi_*}\right)^{-(1-\iota)/\lambda}}{1 - \zeta\beta\left(\frac{\pi_{**}}{\pi_*}\right)^{-(1-\iota)(1+\lambda)/\lambda}}\right]\alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)} W_*^{1-\alpha}(R_*^k)^{\alpha} \\
p_*^o &= \left[\frac{1}{1-\zeta} - \frac{\zeta}{1-\zeta}\left(\frac{\pi_{**}}{\pi_*}\right)^{-\frac{1-\iota}{\lambda}}\right]^{-\lambda} \\
R_* &= \pi_*/\beta \\
D_* &= \frac{(1-\zeta)(p_*^o)^{-\frac{1+\lambda}{\lambda}}}{1 - \zeta\left(\frac{\pi_{**}}{\pi_*}\right)^{-\frac{(1+\lambda)(1-\iota)}{\lambda}}} \\
Y_* &= \frac{\bar{Y}_*}{D_*} \\
\bar{Y}_* &= Z_* K_*^{\alpha} H_*^{1-\alpha} - \mathcal{F} \\
I_* &= \delta K_* \\
Y_* &= X_* + I_* + (1 - \frac{1}{g_*})\mathcal{Y}_* \\
\mathcal{Y}_* &= Y_* + \sigma\frac{\mathcal{M}_*}{\pi_*}
\end{aligned}
$$

Note that from the firm's problem we have

$$
\begin{aligned}
\mathcal{F}_*^{(1)} &= \left(1 - \zeta\beta\pi_*\left(\frac{\pi_{**}}{\pi_*}\right)^{-(1-\iota)/\lambda}\right)^{-1}(p_*^o)^{-\frac{1+\lambda}{\lambda}} Y_* \\
\mathcal{F}_*^{(2)} &= \left(1 - \zeta\beta\pi_*\left(\frac{\pi_{**}}{\pi_*}\right)^{-(1-\iota)(1+\lambda)/\lambda}\right)^{-1}(p_*^o)^{-\frac{1+\lambda}{\lambda}-1} Y_* MC_* \\
\mathcal{F}_*^{(1)} &= (1+\lambda)\mathcal{F}_*^{(2)} \\
\pi_* &= \left[(1-\zeta)\left(\pi_* p_*^o\right)^{-\frac{1}{\lambda}} + \zeta\left(\pi_*^{\iota}\pi_{**}^{1-\iota}\right)^{-\frac{1}{\lambda}}\right]^{-\lambda}
\end{aligned}
$$

which lead to the conditions for p_*^o above. The term D_* measures the steady state price dispersion. The larger π_*/π_{**}, that is, the faster the price of the non-adjusters is eroding in real terms, the bigger D_*.

A.5 Log-Linearizations

In the subsequent presentation of the log-linearized equations we adopt the convention that we abbreviate time t expectations of a $t+1$ variable simply by a time $t+1$ subscript, omitting the expectation operator.

Firms's Problem: Marginal costs evolve according to

$$\tilde{MC}_t = (1-\alpha)\tilde{w}_t + \alpha\tilde{R}_t^k - \tilde{Z}_t.. \tag{91}$$

Conditional on capital and factor prices, the labor demand is determined according to

$$\tilde{H}_t = \tilde{K}_t + \tilde{R}_t^k - \tilde{W}_t. \tag{92}$$

Since $\mathcal{F}_t^{(1)}$ and $\mathcal{F}_t^{(2)}$ are proportional, $\tilde{\mathcal{F}}_t^{(1)} = \tilde{\mathcal{F}}_t^{(2)} = \tilde{\mathcal{F}}_t$. The remaining optimality conditions can be written as follows.

$$\begin{aligned}
\tilde{\mathcal{F}}_t &= (1-\mathcal{A})\left[-\frac{1+\lambda}{\lambda}\tilde{p}_t^o + \tilde{\mathcal{Y}}_t\right] \\
&\quad +\mathcal{A}\left[-\frac{\iota}{\lambda}\tilde{\pi}_t - \frac{1+\lambda}{\lambda}\tilde{p}_t^o + \frac{1+\lambda}{\lambda}\tilde{\pi}_{t+1} + \frac{1+\lambda}{\lambda}\tilde{p}_{t+1}^o + \tilde{\mathcal{F}}_{t+1} + \tilde{\Xi}_{t+1|t}^p\right] \\
\mathcal{A} &= \zeta\beta\left(\frac{\pi_{**}}{\pi_*}\right)^{-(1-\iota)/\lambda}
\end{aligned} \tag{93}$$

and

$$\begin{aligned}
\tilde{\mathcal{F}}_t &= (1-\mathcal{A})\left[-\left(\frac{1+\lambda}{\lambda}+1\right)\tilde{p}_t^o + \tilde{\mathcal{Y}}_t + \tilde{MC}_t\right] \\
&\quad +\mathcal{A}\Bigg[-\frac{\iota(1+\lambda)}{\lambda}\tilde{\pi}_t - \left(\frac{1+\lambda}{\lambda}+1\right)\tilde{p}_t^o + \left(\frac{1+\lambda}{\lambda}+1\right)\tilde{\pi}_{t+1} \\
&\quad + \left(\frac{1+\lambda}{\lambda}+1\right)\tilde{p}_{t+1}^o + \tilde{\mathcal{F}}_{t+1} + \tilde{\Xi}_{t+1|t}^p\Bigg] \\
\mathcal{A} &= \zeta\beta\left(\frac{\pi_{**}}{\pi_*}\right)^{-(1-\iota)(1+\lambda)/\lambda}.
\end{aligned} \tag{94}$$

The relationship between the optimal price charged by the adjusting firms and the inflation rate is given by

$$\begin{aligned}
\tilde{p}_t^o &= (\mathcal{A}-1)\tilde{\pi}_t - \mathcal{A}\iota\zeta\left(\frac{\pi_{**}}{\pi_*}\right)^{-(1-\iota)/\lambda}\tilde{\pi}_{t-1} \\
\mathcal{A} &= \frac{(p_*^o)^{1/\lambda}}{1-\zeta}
\end{aligned} \tag{95}$$

Equations (93) to (95) determine $\tilde{\pi}_t$, $\tilde{F}_t$, and $\tilde{\pi}_t^o$.

Household's Problem: The optimality conditions for the household can be expressed as

$$\tilde{W}_t = \gamma \tilde{X}_t \tag{96}$$

$$\tilde{X}_t = \tilde{X}_{t+1} - \frac{1}{\gamma}(\tilde{R}_t - \tilde{\pi}_{t+1}) \tag{97}$$

$$\tilde{i}_t = \frac{1}{1+\beta}\tilde{i}_{t-1} + \frac{\beta}{1+\beta}\tilde{i}_{t+1} + \frac{1}{(1+\beta)S''}\tilde{\mu}_t \tag{98}$$

$$\tilde{k}_{t+1} = (1-\delta)\tilde{k}_t + \delta\tilde{i}_t \tag{99}$$

$$\begin{aligned}\tilde{\mu}_t - \gamma\tilde{X}_t &= \beta(1-\delta)\tilde{\mu}_{t+1} - \gamma\beta(1-\delta+R_*^k)\tilde{X}_{t+1} + \beta R_{*t}\tilde{R}_{t+1}^k \\ &\quad +(1-\beta(1-\delta+R_*^k))\tilde{\Gamma}_{t+1}\end{aligned} \tag{100}$$

$$\tilde{\mathcal{M}}_t = \tilde{g}_t + \tilde{W}_t + \tilde{\pi}_t \tag{101}$$

$$\tilde{R}_t = \frac{R_* - 1 + \sigma}{R_*}[\tilde{\chi}_{t+1} - \tilde{g}_{q,t+1} - \eta\frac{q_*}{(q_*+\kappa)}\tilde{q}_{t+1}] \tag{102}$$

$$\tilde{\Xi}_{t|t-1}^p = -\gamma(\tilde{X}_t - \tilde{X}_{t-1}) - \tilde{\pi}_t \tag{103}$$

Equations (96) to (103) determine wages, CM consumption, investment, capital, the shadow price of installed capital, the rental rate of capital, real money balances, the stochastic discount factor used in the firms' problem, and DM consumption.

Decentralized Market: We now determine the law of motion for $\tilde{g}_{q,t}$, $\tilde{\Gamma}_t$, and $\tilde{g}_t$. In addition, we are introducing some auxiliary variables. We begin with (omitting t subscripts),

$$\begin{aligned}
u &= \frac{(q+\kappa)^{1-\eta} - \kappa^{1-\eta}}{1-\eta} \\
u' &= (q+\kappa)^{-\eta} \\
u'' &= -\eta(q+\kappa)^{-\eta-1} \\
c &= \exp\{-\tilde{Z}\}q^{\psi}k^{1-\psi} \\
c_q &= \psi\exp\{-\tilde{Z}\}q^{\psi-1}k^{1-\psi} \\
c_k &= (1-\psi)\exp\{-\tilde{Z}\}q^{\psi}k^{-\psi} \\
c_{qq} &= \psi(\psi-1)\exp\{-\tilde{Z}\}q^{\psi-2}k^{1-\psi} \\
c_{kk} &= \psi(\psi-1)\exp\{-\tilde{Z}\}q^{\psi}k^{-\psi-1} \\
c_{qk} &= \psi(1-\psi)\exp\{-\tilde{Z}\}q^{\psi-1}k^{-\psi}
\end{aligned}$$

which can be log-linearized as follows

$$\begin{aligned}
\tilde{u}u_* &= \frac{q_*}{(q_*+\kappa)^\eta}\tilde{q} \\
\tilde{u}' &= -\eta\frac{q_*}{q_*+\kappa}\tilde{q} \\
\tilde{u}'' &= -(\eta+1)\frac{q_*}{q_*+\kappa}\tilde{q} \\
\tilde{c} &= -\tilde{Z}+\psi\tilde{q}+(1-\psi)\tilde{k} \\
\tilde{c}_q &= -\tilde{Z}+(\psi-1)\tilde{q}+(1-\psi)\tilde{k} \\
\tilde{c}_k &= -\tilde{Z}+\psi\tilde{q}-\psi\tilde{k} \\
\tilde{c}_{qq} &= -\tilde{Z}+(\psi-2)\tilde{q}+(1-\psi)\tilde{k} \\
\tilde{c}_{kk} &= -\tilde{Z}+\psi\tilde{q}-(1+\psi)\tilde{k} \\
\tilde{c}_{qk} &= -\tilde{Z}+(\psi-1)\tilde{q}+-\psi\tilde{k}
\end{aligned}$$

Recall that

$$\Gamma_t = \frac{c_{k,t}g_{q,t}-c_{q,t}g_{k,t}}{g_{q,t}}$$

which implies that $\tilde{\Gamma}_t$ evolves according to

$$\tilde{g}_{q,t}+\tilde{\Gamma}_t = \frac{c_{k*}g_{q*}}{c_{k*}g_{q*}-c_{q*}g_{k*}}[\tilde{c}_{k,t}+\tilde{g}_{q,t}] - \frac{c_{q*}g_{k*}}{c_{k*}g_{q*}-c_{q*}g_{k*}}[\tilde{c}_{q,t}+\tilde{g}_{k,t}]. \tag{104}$$

Now consider the equation

$$g_t(\theta\chi u_t'+(1-\theta)c_{q,t}) = \theta\chi c_t u_t' + (1-\theta)\chi c_{q,t}u_t,$$

which can be written in log-linear form as

$$\begin{aligned}
& [\theta\chi_* u_*' + (1-\theta)\, c_{q*}]\, g_*\tilde{g}_t \\
= \; & \theta\chi_* u_*'\,(c_*-g_*)\,\tilde{u}_t' + (1-\theta)\,\chi_* c_{q*}u_*\tilde{u}_t + (1-\theta)\,c_{q*}\,(\chi_* u_* - g_*)\,\tilde{c}_{q,t} \qquad (105) \\
& +\theta\chi_* c_* u_*'\tilde{c} + [-\theta\chi_* g_* u_*' + \theta\chi_* c_* u_*' + (1-\theta)\,\chi_* c_{q*}u_*]\,\tilde{\chi}_t \qquad (106)
\end{aligned}$$

and determines $\tilde{g}_t$.

Now consider

$$g_q = \frac{\chi u' c_q[\theta\chi u'+(1-\theta)c_q] + \theta(1-\theta)(\chi u - c)\,(\chi u' c_{qq} - c_q\chi u'')}{[\theta\chi u'+(1-\theta)c_q]^2}$$

In log-linear form, the equation can be rewritten as

$$\begin{aligned}
& g_{q*}\left[\theta\chi_* u_*' + (1-\theta)\, c_{q*}\right]^2 \tilde{g}_{q,t} \\
= \; & -\eta g_{q*}\left[\theta\chi_* u_*' + (1-\theta)\, c_{q*}\right]\left[\theta\chi_* u_*'\left(\tilde{u}_t + \tilde{\chi}_t\right) + (1-\theta)\, c_{q*}\tilde{c}_{q,t}\right] \\
& +\chi_* u_*' c_{q*}\left[\theta\chi u_*' + (1-\theta)\, c_{q*}\right]\left(\tilde{u}_t' + \tilde{\chi}_t + \tilde{c}_{q,t}\right) \\
& +\theta\left(\chi_* u_*'\right)^2 c_{q*}\left(\tilde{u}_t' + \tilde{\chi}_t\right) + \chi_*(1-\theta)\, u_*' c_{q*}^2 \tilde{c}_{q,t} \\
& +\theta(1-\theta)\,\chi_*\left(u_*' c_{qq*} - c_{q*} u_*''\right)\left[\chi_* u_*\left(\tilde{u}_t + \tilde{\chi}_t\right) - c_* \tilde{c,t}\right] \\
& +\theta(1-\theta)\,\chi_*\left(\chi_* u_* - c_*\right) u_*' c_{qq*}\left(\tilde{u}_t' + \tilde{\chi}_t + \tilde{c}_{qq,tt}\right) \\
& -\theta(1-\theta)\,\chi_*\left(\chi_* u_* - c_*\right) u_*'' c_{q*}\left(\tilde{u}_t'' + \tilde{\chi}_t + \tilde{c}_{q,t}\right).
\end{aligned} \tag{107}$$

Moreover,

$$g_k = \frac{\theta\chi u' c_k\left[\theta\chi u' + (1-\theta)c_q\right] + \theta(1-\theta)(\chi u - c)\chi u' c_{qk}}{\left[\theta\chi u' + (1-\theta)c_q\right]^2},$$

which leads to an equation for $\tilde{g}_{k,t}$:

$$\begin{aligned}
& g_{k*}[\theta\chi u_*' + (1-\theta)c_{q*}]^2 \tilde{g}_{k,t} \\
= \; & -2g_{k*}[\theta\chi_* u_*' + (1-\theta)c_{q*}]\Big(\theta\chi_* u_*'(\tilde{u}_t\tilde{\chi}_t) + (1-\theta)c_{q*}\tilde{c}_{q,t}\Big) \\
& +\theta\chi_* u_*' c_{k*}\left[\theta\chi_* u_*' + (1-\theta)\, c_{q*}\right]\left(\tilde{u}_t' + \tilde{\chi}_t + \tilde{c}_{k,t}\right) \\
& +\left(\theta\chi_* u_*'\right)^2 c_{k*}\left(\tilde{u}_t' + \tilde{\chi}_t\right) + \chi_*\theta(1-\theta)\, u_*' c_{k*} c_{q*}\tilde{c}_{q,t} \\
& +\theta(1-\theta)\,\chi_*\left(\chi_* u_* - c_*\right) u_*' c_{qk*}\left(\tilde{u}_t' + \tilde{\chi}_t + \tilde{c}_{qk,t}\right) \\
& +\theta(1-\theta)\,\chi_* u_*' c_{qk*}\left[\chi_* u_*\left(\tilde{u}_t + \tilde{\chi}_t\right) - c_*\tilde{c}_t\right].
\end{aligned} \tag{108}$$

To summarize, Equations (104) to (108) determine $\tilde{\Gamma}_t$, $\tilde{g}_t$, $\tilde{g}_{q,t}$, and $\tilde{g}_{k,t}$. The first three variables appear in the characterization of the households' problem above.

Resource Constraint, Market Clearing Conditions: Aggregate output across evolves according to

$$\tilde{\dot{Y}}_t = \tilde{Y}_t + \tilde{D}_t = (1 + \mathcal{F}/\dot{Y}_*)[\tilde{Z}_t + \alpha\tilde{K}_t + (1-\alpha)\tilde{H}_t]. \tag{109}$$

and the steady state price dispersion follows

$$\tilde{D}_t = \zeta\left(\frac{\pi_{**}}{\pi_*}\right)^{-\frac{(1+\lambda)(1-\iota)}{\lambda}}\left[\tilde{D}_{t-1} + \frac{(1+\lambda)}{\lambda}\tilde{\pi}_t - \frac{\iota(1+\lambda)}{\lambda}\tilde{\pi}_{t-1}\right] - \frac{p_*^o(1+\lambda)(1-\zeta)}{\lambda D_*}\tilde{p}_t^o \tag{110}$$

The goods market clearing condition is of the form

$$\tilde{Y}_t = \frac{X_*}{Y_*}\tilde{X}_t + \frac{I_*}{Y_*}\tilde{I}_t + \left(1 - \frac{1}{g_*}\right)\frac{\mathcal{Y}_*}{Y_*}\mathcal{Y}_t + \frac{\mathcal{Y}_*}{Y_* g_*}\tilde{g}_t \tag{111}$$

and determines investment. GDP evolves according to

$$\tilde{\mathcal{Y}}_t = \frac{Y_*}{\mathcal{Y}_*}\tilde{Y}_t + \left(1 - \frac{Y_*}{\mathcal{Y}_*}\right)(\mathcal{M}_t - \pi_t) \tag{112}$$

Government Policies: The monetary policy rule can be written as

$$\tilde{R}_t = \rho_R \tilde{R}_{t-1} + (1 - \rho_R)[\psi_1 \tilde{\pi}_t + \psi_2 \tilde{Y}_t] + \epsilon_{R,t}. \tag{113}$$

B The MIU Model

The subsequent exposition is based on a slightly more general utility function:

$$U(x) = B\frac{x^{1-\gamma}}{1-\gamma}.$$

B.1 Equilibrium Conditions

Household's Problem: Given exogenous states, policy and prices,

$$U'(x_t) = \frac{A}{W_t} \tag{114}$$

$$1 = \beta E_t\left[\frac{U'(x_{t+1})}{U'(x_t)}\frac{R_t}{\pi_{t+1}}\right] \tag{115}$$

$$1 = \mu_t\left[1 - S\left(\frac{i_t}{i_{t-1}}\right) + \frac{i_t}{i_{t-1}}S'\left(\frac{i_t}{i_{t-1}}\right)\right] + \beta E_t\left\{\mu_{t+1}\frac{U'(x_{t+1})}{U'(x_t)}\left(\frac{i_{t+1}}{i_t}\right)^2 S'\left(\frac{i_{t+1}}{i_t}\right)\right\} \tag{116}$$

$$k_{t+1} = (1-\delta)k_t + \left[1 - S\left(\frac{i_t}{i_{t-1}}\right)\right] \tag{117}$$

$$\mu_t = \beta E_t\left\{\frac{U'(x_{t+1}}{U'(x_t)}\left[R^k_{t+1} + (1-\delta)\mu_{t+1}\right]\right\} \tag{118}$$

$$\frac{U'(x_t)}{P_t} = \beta E_t\left[\frac{U'(x_{t+1})}{P_{t+1}} + \frac{\chi_{t+1}}{P_{t+1}}\left(\frac{A}{Z^{1/1-\alpha}_{t+1}}\right)^{1-\nu_m}\left(\frac{m_{t+1}}{P_{t+1}}\right)^{-\nu_m}\right] \tag{119}$$

$$\Xi^p_{t+1|t} = \frac{U'(x_{t+1})}{U'(x_t)\pi_{t+1}} \tag{120}$$

Intermediate Goods Producing Firms' Problem: Intermediate goods firms choose their capital labor ratio as a function of the factor prices to minimize costs:

$$K_t = \frac{\alpha}{1-\alpha}\frac{W_t}{R^k_t}H_t. \tag{121}$$

Firms that are allowed to change prices are choosing a relative price $p^o_t(i)$ (relative to the aggregate price level) to maximize expected profits subject to the demand curve for their differentiated product, taking the aggregate price level P_t as well as the prices charged by other firms as given, which leads to

$$MC_t = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}W_t^{1-\alpha}(R^k_t)^\alpha Z_t^{-1} \tag{122}$$

$$\mathcal{F}^{(1)}_t = (p^o_t)^{-\frac{1+\lambda}{\lambda}}Y_t + \zeta\beta\left(\pi^\iota_t\pi^{(1-\iota)}_{**}\right)^{-1/\lambda}\mathbb{E}_t\left[\left(\frac{p^o_t}{\pi_{t+1}p^o_{t+1}}\right)^{-\frac{1+\lambda}{\lambda}}\Xi^p_{t+1|t}\mathcal{F}^{(1)}_{t+1}\right] \tag{123}$$

$$\mathcal{F}^{(2)}_t = (p^o_t)^{-\frac{1+\lambda}{\lambda}-1}Y_tMC_t + \zeta\beta\left(\pi^\iota_t\pi^{(1-\iota)}_{**}\right)^{-\frac{1+\lambda}{\lambda}}\mathbb{E}_t\left[\left(\frac{p^o_t}{\pi_{t+1}p^o_{t+1}}\right)^{-\frac{1+\lambda}{\lambda}-1}\Xi^p_{t+1|t}\mathcal{F}^{(2)}_{t+1}\right] \tag{124}$$

$$\mathcal{F}^{(1)}_t = (1+\lambda)\mathcal{F}^{(2)}_t \tag{125}$$

Final Good Producing Firms' Problem: Final goods producers take factor prices and output prices as given and choose inputs $Y_t(i)$ and output Y_t to maximize profits. Free entry ensures that final good producers make zero profits and leads to

$$\pi_t = \left[(1-\zeta)\left(\pi_t p_t^o\right)^{-\frac{1}{\lambda}} + \zeta(\pi_{t-1}^{\iota}\pi_{**}^{1-\iota})^{-\frac{1}{\lambda}}\right]^{-\lambda} \tag{126}$$

Monetary Policy: The central bank supplies the quantity of money necessary to attain the nominal interest rate

$$\frac{R_t}{R_*} = \left(\frac{R_{t-1}}{R_*}\right)^{\rho_R}\left[\left(\frac{\pi_t}{\pi_*}\right)^{\psi_1}\left(\frac{\mathcal{Y}_t}{\mathcal{Y}_*}\right)^{\psi_2}\right]^{1-\rho_R}\exp\left(\sigma_r\varepsilon_t^r\right) \tag{127}$$

Aggregate Resource Constraint: is given by

$$Y_t = D_t^{-1}(Z_t K_t^{\alpha} H_t^{(1-\alpha)} - \mathcal{F}), \tag{128}$$

where

$$D_t = \zeta\left[\left(\frac{\pi_{t-1}}{\pi_t}\right)^{\iota}\left(\frac{\pi_{**}}{\pi_t}\right)^{(1-\iota)}\right]^{-\frac{1+\lambda}{\lambda}} D_{t-1} + (1-\zeta)\left(p_t^o\right)^{-\frac{1+\lambda}{\lambda}}. \tag{129}$$

The gross domestic product of this economy is given by

$$\mathcal{Y}_t = Y_t \tag{130}$$

Market Clearing: The goods market in the CM clears:

$$X_t + I_t + \left(1 - \frac{1}{g_t}\right)\mathcal{Y}_t = Y_t \tag{131}$$

B.2 Steady States

As in the search model $\mu_* = 1$ and the presence of the capital adjustment costs does not affect the derivation of the steady states. Given a target inflation rate π_*, a monetary steady state is defined as allocations X_*,

H_*, K_*, Y_*, $\mathcal{M}_*$, policy R_* and prices W_*, R_*^k, p_*^o that satisfy:

$$
\begin{aligned}
U'(X_*) &= \frac{A}{W_*} \\
1 &= \beta\left(1 + R_*^k - \delta\right) \\
\mathcal{M}_* &= \left[\frac{A^{1-\nu_m}\chi_*\pi_*^{\nu_m}}{(R_*-1)U'(X_*)Z_*^{1-\nu_m/1-\alpha}}\right]^{1/\nu_m} \\
K_* &= \frac{\alpha}{1-\alpha}\frac{W_*}{R_*^k}H_* \\
p_*^o &= (1+\lambda)\left[\frac{1-\zeta\beta\left(\frac{\pi_{**}}{\pi_*}\right)^{-(1-\iota)/\lambda}}{1-\zeta\beta\left(\frac{\pi_{**}}{\pi_*}\right)^{-(1-\iota)(1+\lambda)/\lambda}}\right]\alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}W_*^{1-\alpha}(R_*^k)^{\alpha} \\
p_*^o &= \left[\frac{1}{1-\zeta} - \frac{\zeta}{1-\zeta}\left(\frac{\pi_{**}}{\pi_*}\right)^{-\frac{1-\iota}{\lambda}}\right]^{-\lambda} \\
R_* &= \pi_*/\beta \\
D_* &= \frac{(1-\zeta)(p_*^o)^{-\frac{1+\lambda}{\lambda}}}{1-\zeta\left(\frac{\pi_{**}}{\pi_*}\right)^{-\frac{(1+\lambda)(1-\iota)}{\lambda}}} \\
Y_* &= \frac{\bar{Y}_*}{D_*} \\
\bar{Y}_* &= Z_*K_*^{\alpha}H_*^{1-\alpha} - \mathcal{F} \\
\mathcal{Y}_* &= Y_* \\
Y_* &= X_* + \delta K_* + \left(1 - \frac{1}{g_*}\right)\mathcal{Y}_*
\end{aligned}
$$

The steady state equations can be solved recursively as follows:

$$
\begin{aligned}
R_* &= \pi_*/\beta \\
p_*^o &= \left[\frac{1}{1-\zeta} - \frac{\zeta}{1-\zeta}\left(\frac{\pi_{**}}{\pi_*}\right)^{-\frac{1-\iota}{\lambda}}\right]^{-\lambda} \\
R_*^k &= \frac{1}{\beta} + \delta - 1 \\
W_* &= \left[\frac{Z_* p_*^o \alpha^\alpha (1-\alpha)^{(1-\alpha)}}{(1+\lambda)\left[\frac{1-\zeta\beta\left(\frac{\pi_{**}}{\pi_*}\right)^{-(1-\iota)/\lambda}}{1-\zeta\beta\left(\frac{\pi_{**}}{\pi_*}\right)^{-(1-\iota)(1+\lambda)/\lambda}}\right](R_*^k)^\alpha}\right]^{\frac{1}{1-\alpha}} \\
X_* &= [U']^{-1}(A/W_*) \\
\mathcal{M}_* &= \left[\frac{\chi_* \pi_*^{\nu_m}}{(R_*-1)U'(X_*)}\left(\frac{A}{Z_*^{1/(1-\alpha)}}\right)^{1-\nu_m}\right]^{1/\nu_m} \\
D_* &= \frac{(1-\zeta)(p_*^o)^{-\frac{1+\lambda}{\lambda}}}{1-\zeta\left(\frac{\pi_{**}}{\pi_*}\right)^{-\frac{(1+\lambda)(1-\iota)}{\lambda}}} = (1-\zeta)^{-\lambda}\frac{\left[1-\zeta\left(\frac{\pi_*}{\pi_{**}}\right)^{\frac{1-\iota}{\lambda}}\right]^{(1+\lambda)}}{1-\zeta\left(\frac{\pi_*}{\pi_{**}}\right)^{\frac{1-\iota}{\lambda}(1+\lambda)}} \\
K_* &= \left[Z_*\left(\frac{1-\alpha}{\alpha}\frac{R_*^k}{W_*}\right)^{1-\alpha} - g_* D_* \delta\right]^{-1}(D_* g_* X_* + \mathcal{F}) \\
H_* &= K_* \frac{1-\alpha}{\alpha}\frac{R_*^k}{W_*} \\
\mathcal{Y}_* &= g_*(X_* + \delta K_*) \\
\bar{Y}_* &= Z_* K_*^\alpha H_*^{1-\alpha} - \mathcal{F}
\end{aligned}
$$

B.3 Log-Linearizations

We will frequently use equation-specific constants, such as $\mathcal{A}$ and $\mathcal{B}$. Variables dated $t+1$ refer to time t conditional expectations.

Firms's Problem: Marginal costs evolve according to

$$\tilde{MC}_t = (1-\alpha)\tilde{w}_t + \alpha\tilde{R}_t^k - \tilde{Z}_t. \tag{132}$$

Conditional on capital, the labor demand is determined according to

$$\tilde{H}_t = \tilde{K}_t + \tilde{R}_t^k - \tilde{W}_t \tag{133}$$

Since $\mathcal{F}_t^{(1)}$ and $\mathcal{F}_t^{(2)}$ are proportional, $\tilde{\mathcal{F}}_t^{(1)} = \tilde{\mathcal{F}}_t^{(2)} = \tilde{\mathcal{F}}_t$. The remaining optimality conditions can be written

as follows.

$$\begin{aligned}
\tilde{\mathcal{F}}_t &= (1-\mathcal{A})\left[-\frac{1+\lambda}{\lambda}\tilde{p}_t^o + \tilde{\mathcal{Y}}_t\right] \\
&\quad +\mathcal{A}\left[-\frac{\iota}{\lambda}\tilde{\pi}_t - \frac{1+\lambda}{\lambda}\tilde{p}_t^o + \frac{1+\lambda}{\lambda}\tilde{\pi}_{t+1} + \frac{1+\lambda}{\lambda}\tilde{p}_{t+1}^o + \tilde{\mathcal{F}}_{t+1} + \tilde{\Xi}_{t+1|t}^p\right] \\
\mathcal{A}_1 &= \zeta\beta\left(\frac{\pi_{**}}{\pi_*}\right)^{-(1-\iota)/\lambda}
\end{aligned} \tag{134}$$

and

$$\begin{aligned}
\tilde{\mathcal{F}}_t &= (1-\mathcal{A})\left[-\left(\frac{1+\lambda}{\lambda}+1\right)\tilde{p}_t^o + \tilde{\mathcal{Y}}_t + \tilde{MC}_t\right] \\
&\quad +\mathcal{A}\Bigg[-\frac{\iota(1+\lambda)}{\lambda}\tilde{\pi}_t - \left(\frac{1+\lambda}{\lambda}+1\right)\tilde{p}_t^o + \left(\frac{1+\lambda}{\lambda}+1\right)\tilde{\pi}_{t+1} \\
&\quad + \left(\frac{1+\lambda}{\lambda}+1\right)\tilde{p}_{t+1}^o + \tilde{\mathcal{F}}_{t+1} + \tilde{\Xi}_{t+1|t}^p\Bigg] \\
\mathcal{A}_2 &= \zeta\beta\left(\frac{\pi_{**}}{\pi_*}\right)^{-(1-\iota)(1+\lambda)/\lambda}.
\end{aligned} \tag{135}$$

The relationship between the optimal price charged by the adjusting firms and the inflation rate is given by

$$\begin{aligned}
\tilde{p}_t^o &= (\mathcal{A}-1)\tilde{\pi}_t - \mathcal{A}\iota\zeta\left(\frac{\pi_{**}}{\pi_*}\right)^{-(1-\iota)/\lambda}\tilde{\pi}_{t-1} \\
\mathcal{A}_p &= \frac{(p_*^o)^{1/\lambda}}{1-\zeta}
\end{aligned} \tag{136}$$

Equations (134) to (136) determine $\tilde{\pi}_t$, $\tilde{\mathcal{F}}_t$, and $\tilde{p}_t^o$.

Household's Problem The optimality conditions for the household can be expressed as

$$\tilde{W}_t = \frac{1}{\gamma}\tilde{X}_t \tag{137}$$

$$-\gamma\tilde{X}_t = -\gamma\tilde{X}_{t+1} + (\tilde{R}_t - \tilde{\pi}_{t+1}) \tag{138}$$

$$\tilde{i}_t = \frac{1}{1+\beta}\tilde{i}_{t-1} + \frac{\beta}{1+\beta}\tilde{i}_{t+1} + \frac{1}{(1+\beta)S''}\tilde{\mu}_t \tag{139}$$

$$\tilde{k}_{t+1} = (1-\delta)\tilde{k}_t + \delta\tilde{i}_t \tag{140}$$

$$\tilde{\mu}_t - \gamma\tilde{X}_t = \beta(1-\delta)\tilde{\mu}_{t+1} - \gamma\tilde{X}_{t+1} + \beta R_*^k\tilde{R}_{t+1}^k \tag{141}$$

$$\nu_m\tilde{\mathcal{M}}_{t+1} = \gamma\tilde{X}_t + \nu_m\tilde{\chi}_{t+1} - (1-\nu_m)\tilde{\pi}_{t+1} - \frac{1}{R_*-1}\tilde{R}_t \tag{142}$$

$$\tilde{\Xi}_{t|t-1}^p = -\gamma(\tilde{X}_t - \tilde{X}_{t-1}) - \tilde{\pi}_t. \tag{143}$$

Equations (137) to (143) determine wages, consumption, investment, capital, the shadow value of installed capital, the rental rate of capital, real money balances, and the stochastic discount factor.

Resource Constraint, Market Clearing Conditions Aggregate output across evolves according to

$$\tilde{\bar{Y}}_t = \tilde{Y}_t + \tilde{D}_t = (1+\mathcal{F}/\bar{Y}_*)[\tilde{Z}_t + \alpha\tilde{K}_t + (1-\alpha)\tilde{H}_t]. \tag{144}$$

and the steady state price dispersion follows

$$\tilde{D}_t = \zeta \left(\frac{\pi_{**}}{\pi_*}\right)^{-\frac{(1+\lambda)(1-\iota)}{\lambda}} \left[\tilde{D}_{t-1} + \frac{(1+\lambda)}{\lambda}\tilde{\pi}_t - \frac{\iota(1+\lambda)}{\lambda}\tilde{\pi}_{t-1}\right] - \frac{p_*^o(1+\lambda)(1-\zeta)}{\lambda D_*}\tilde{p}_t^o \tag{145}$$

The goods market clearing condition is of the form

$$\tilde{\mathcal{Y}}_t = \frac{X_*}{X_* + I_*}\tilde{X}_t + \frac{I_*}{X_* + I_*}\tilde{I}_t + \tilde{g}_t. \tag{146}$$

Government Policies The monetary policy rule can be written as

$$\tilde{R}_t = \rho_R \tilde{R}_{t-1} + (1-\rho_R)[\psi_1 \tilde{\pi}_t + \psi_2 \tilde{Y}_t] + \epsilon_{R,t}. \tag{147}$$

Table 1: Parameters Fixed During Estimation

Name	MIU Model	Search Model
Depreciation Rate δ	0.013	0.013
Steady State TFP Z_*	1.260	1.000
Persistence of TFP ρ_z	0.980	0.980
Fixed Costs $\mathcal{F}$	0.000	0.000
Markup Parameter λ	0.150	estim.
Indexation π_{**}	1.000	1.000
Preference Parameter A	0.435	1.000
Preference Parameter B	0.067	estim.
Preference Parameter χ_*	estim.	1.000
Preference Parameter κ	N/A	.0001
Steady State Real Rate r_A	2.842	2.842
Steady State Inflation Rate π_A	2.499	2.499
Policy Rule ψ_1	1.820	1.820
Policy Rule ψ_2	0.180	0.180
Policy Rule ρ_R	0.780	0.780
Share of Government Spending g_*	1.299	1.299

Notes: We use the following transformations: $\beta = 1/(1 + r_A/400)$, $\pi_* = 1 + \pi_A/400$.

Table 2: Prior Distribution for MIU Model

Name	Domain	Density	Para (1)	Para (2)
		Household		
M/PY	$I\!R^+$	Gamma	2.25	0.50
ν_m	$I\!R^+$	Gamma	20.00	5.00
		Firms		
α	$[0,1)$	Beta	0.30	0.03
ζ	$[0,1)$	Beta	0.60	0.15
ι	$[0,1)$	Beta	0.50	0.25
S''	$I\!R^+$	Gamma	4.00	1.50
		Shocks		
ρ_g	$[0,1)$	Beta	0.90	0.05
ρ_χ	$[0,1)$	Beta	0.90	0.05
σ_g	$I\!R^+$	InvGamma	1.00	4.00
σ_χ	$I\!R^+$	InvGamma	1.00	4.00
σ_R	$I\!R^+$	InvGamma	0.50	4.00
σ_Z	$I\!R^+$	InvGamma	1.00	4.00

Notes: Para (1) and Para (2) list the means and the standard deviations for Beta, Gamma, and Normal distributions; the upper and lower bound of the support for the Uniform distribution; s and ν for the Inverse Gamma distribution, where $p_{\mathcal{IG}}(\sigma|\nu, s) \propto \sigma^{-\nu-1}e^{-\nu s^2/2\sigma^2}$.

Table 3: Prior Distribution for Search Model

Name	Domain	Density	Para (1)	Para (2)
		Household		
B	$I\!R^+$	Gamma	0.20	0.10
θ	$[0,1)$	Beta	0.50	0.20
$\tilde{\sigma}$	$[0,1)$	Beta	0.20	0.10
		Firms		
α	$[0,1)$	Beta	0.30	0.03
ζ	$[0,1)$	Beta	0.60	0.15
ι	$[0,1)$	Beta	0.50	0.25
λ	$I\!R^+$	Gamma	0.15	0.05
S''	$I\!R^+$	Gamma	4.00	1.50
		Shocks		
ρ_g	$[0,1)$	Beta	0.80	0.10
ρ_χ	$[0,1)$	Beta	0.80	0.10
σ_g	$I\!R^+$	InvGamma	1.00	4.00
σ_χ	$I\!R^+$	InvGamma	1.00	4.00
σ_R	$I\!R^+$	InvGamma	0.50	4.00
σ_Z	$I\!R^+$	InvGamma	1.00	4.00

Notes: Para (1) and Para (2) list the means and the standard deviations for Beta, Gamma, and Normal distributions; the upper and lower bound of the support for the Uniform distribution; s and ν for the Inverse Gamma distribution, where $p_{\mathcal{IG}}(\sigma|\nu,s) \propto \sigma^{-\nu-1}e^{-\nu s^2/2\sigma^2}$. We multiply the product of the marginal densities reported in the table with the function $f(\cdot) = -0.5(I_*/\mathcal{Y}_* - 0.16)^2/0.01^2 - 0.5(\mu - 0.15)^2/0.01^2$ and truncate the effective prior at the boundary of the determinacy region.

Table 4: Prior and Posterior Moments for MIU Model

Name	Prior		Posterior	
	Mean	90% Intv	Mean	90% Intv
		Household		
M/PY	2.249	[1.426, 3.036]	2.273	[2.173, 2.372]
ν_m	20.002	[11.366, 27.529]	26.883	[20.754, 33.077]
		Firms		
α	0.300	[0.258, 0.340]	0.326	[0.283, 0.370]
ζ	0.595	[0.366, 0.837]	0.737	[0.681, 0.796]
ι	0.505	[0.105, 0.908]	0.058	[0.000, 0.118]
S''	3.995	[1.614, 6.308]	8.209	[5.332, 10.944]
		Shocks		
ρ_g	0.900	[0.827, 0.979]	0.895	[0.865, 0.928]
ρ_χ	0.900	[0.826, 0.977]	0.974	[0.958, 0.991]
σ_g	1.259	[0.535, 1.996]	1.229	[1.066, 1.395]
σ_χ	1.254	[0.527, 1.984]	0.814	[0.711, 0.914]
σ_R	0.627	[0.270, 0.994]	0.200	[0.175, 0.224]
σ_Z	1.255	[0.526, 1.969]	0.514	[0.439, 0.591]

Notes:

Table 5: Prior and Posterior Moments for Search Model

Name	Prior		Posterior	
	Mean	90% Intv	Mean	90% Intv
B	0.206	[0.056, 0.349]	0.067	[0.056, 0.079]
θ	0.498	[0.171, 0.825]	0.933	[0.926, 0.941]
$\tilde{\sigma}$	0.200	[0.043, 0.352]	0.338	[0.285, 0.382]
		Firms		
α	0.300	[0.259, 0.341]	0.364	[0.339, 0.387]
ζ	0.593	[0.374, 0.843]	0.652	[0.592, 0.711]
ι	0.512	[0.107, 0.909]	0.347	[0.129, 0.540]
λ	0.152	[0.072, 0.229]	0.089	[0.048, 0.125]
S''	4.987	[1.162, 8.634]	12.966	[8.411, 15.704]
		Shocks		
ρ_g	0.800	[0.651, 0.959]	0.889	[0.850, 0.926]
ρ_χ	0.799	[0.648, 0.958]	0.902	[0.879, 0.926]
σ_g	1.246	[0.532, 1.962]	0.640	[0.529, 0.747]
σ_χ	1.256	[0.541, 1.969]	1.719	[1.474, 1.976]
σ_R	0.629	[0.262, 0.991]	0.239	[0.208, 0.268]
σ_Z	1.271	[0.536, 2.030]	0.380	[0.313, 0.445]

Notes:

Table 6: Posterior Steady States

Shock	Search Model		MIU Model	
	Mean	90% Intv	Mean	90% Intv
$\mathcal{Y}_*$	0.780	[0.700, 0.854]	0.773	[0.532, 1.011]
$I_*/\mathcal{Y}_*$	0.137	[0.124, 0.151]	0.183	[0.158, 0.208]
$K_*/\mathcal{Y}_*$	10.546	[9.524, 11.591]	14.086	[12.176, 15.963]
$\mathcal{M}_*/\mathcal{Y}_*$	2.251	[2.220, 2.287]	2.275	[2.181, 2.374]
Y_*/H_*	5.118	[4.102, 6.028]	5.201	[3.474, 6.926]
W_*H_*/Y_*	0.584	[0.545, 0.618]	0.587	[0.548, 0.625]
Overall Markup	0.151	[0.135, 0.165]	0.150	fixed
χ_*	1.000	fixed	9E-6	[8E-21, 2E-7]
B	0.067	[0.056, 0.079]	0.067	fixed
DM Share	0.378	[0.326, 0.432]	N/A	N/A
DM Markup	0.252	[0.177, 0.328]	N/A	N/A

Notes: For the MIU model $\mathcal{Y}_* = Y_*$. In the estimation of the MIU model we treat the inverse velocity as "parameter" and derive the implied χ_* from the steady state relationships. In the estimation of the search model we treat B as parameter and calculate the implied inverse velocity. We can choose Z_* in the MIU model to match the level of output $\mathcal{Y}_*$ in the search model without affecting the steady state numbers reported in the table.

Table 7: Posterior Variance Decomposition

Shock	Search Model		MIU Model	
	Mean	90% Intv	Mean	90% Intv
		Output		
Gov Spending	0.079	[0.052, 0.111]	0.105	[0.064, 0.140]
Money Demand	0.004	[0.002, 0.006]	0.000	
Monetary Policy	0.008	[0.005, 0.012]	0.012	[0.006, 0.016]
Technology	0.909	[0.876, 0.938]	0.883	[0.846, 0.924]
		Inflation		
Gov Spending	0.001	[0.000, 0.003]	0.021	[0.007, 0.034]
Money Demand	0.030	[0.016, 0.048]	0.000	
Monetary Policy	0.016	[0.008, 0.024]	0.014	[0.004, 0.024]
Technology	0.952	[0.932, 0.974]	0.965	[0.948, 0.985]
		Federal Funds Rate		
Gov Spending	0.006	[0.003, 0.009]	0.001	[0.000, 0.003]
Money Demand	0.011	[0.003, 0.019]	0.000	
Monetary Policy	0.023	[0.012, 0.032]	0.041	[0.021, 0.056]
Technology	0.960	[0.945, 0.974]	0.958	[0.939, 0.976]
		Real Money Balances		
Gov Spending	0.022	[0.012, 0.032]	0.003	[0.000, 0.004]
Money Demand	0.181	[0.110, 0.237]	0.786	[0.631, 0.930]
Monetary Policy	0.029	[0.016, 0.042]	0.022	[0.007, 0.034]
Technology	0.768	[0.694, 0.840]	0.190	[0.055, 0.326]

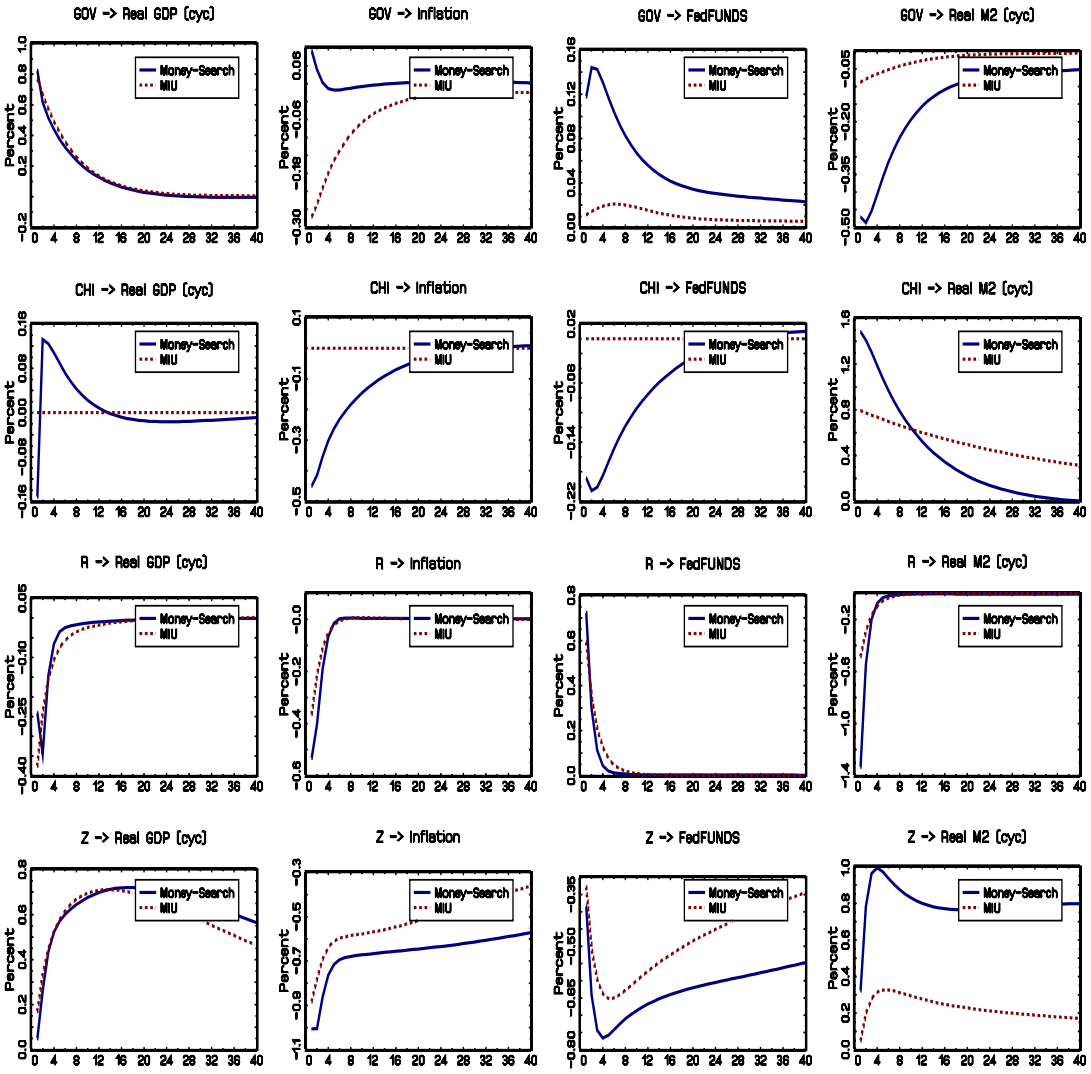
Notes: to be added.

Table 8: Posterior Predictive Check: $\text{Corr}(\hat{\mathcal{M}}_{t+h}, \hat{\mathcal{Y}}_t)$

Lag h	Data	Search Model		MIU Model	
		Mean	90% Intv	Mean	90% Intv
+4	0.198	0.476	[0.096, 0.879]	0.252	[-0.212, 0.766]
+3	0.177	0.492	[0.096, 0.861]	0.267	[-0.188, 0.785]
+2	0.152	0.510	[0.159, 0.899]	0.283	[-0.184, 0.798]
+1	0.159	0.533	[0.195, 0.904]	0.301	[-0.210, 0.775]
0	0.131	0.567	[0.207, 0.891]	0.319	[-0.169, 0.811]
−1	0.390	0.594	[0.284, 0.914]	0.310	[-0.184, 0.792]
−2	0.566	0.567	[0.248, 0.897]	0.303	[-0.135, 0.833]
−3	0.717	0.542	[0.214, 0.871]	0.295	[-0.140, 0.823]
−4	0.856	0.519	[0.176, 0.864]	0.286	[-0.158, 0.813]

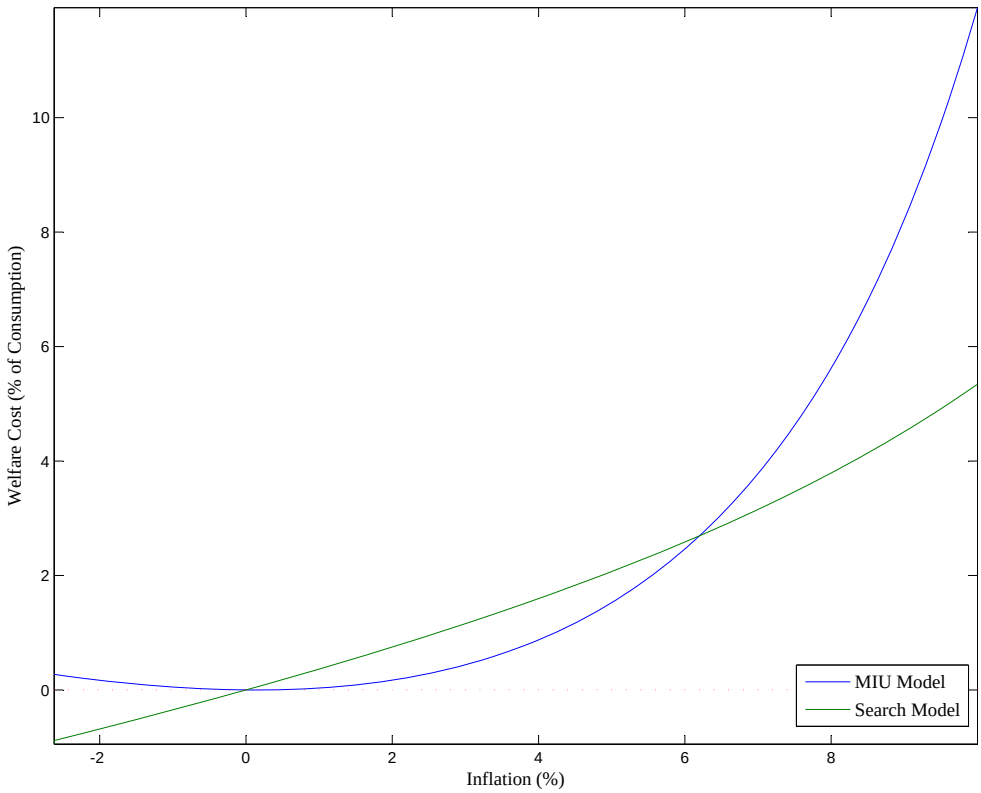
Notes: The column "Data" contains sample correlations between real money balances and output. The remaining columns summarize the posterior predictive distribution for these sample correlations based on the Search and MIU models. Draws from the posterior predictive distributions are obtained by simulating sample paths from the DSGE model conditional on the posterior parameter draws and calculating moments from the simulated sample paths. For the MIU model $\mathcal{Y}_* = Y_*$.

Figure 1: Impulse Responses, Search versus MIU Model



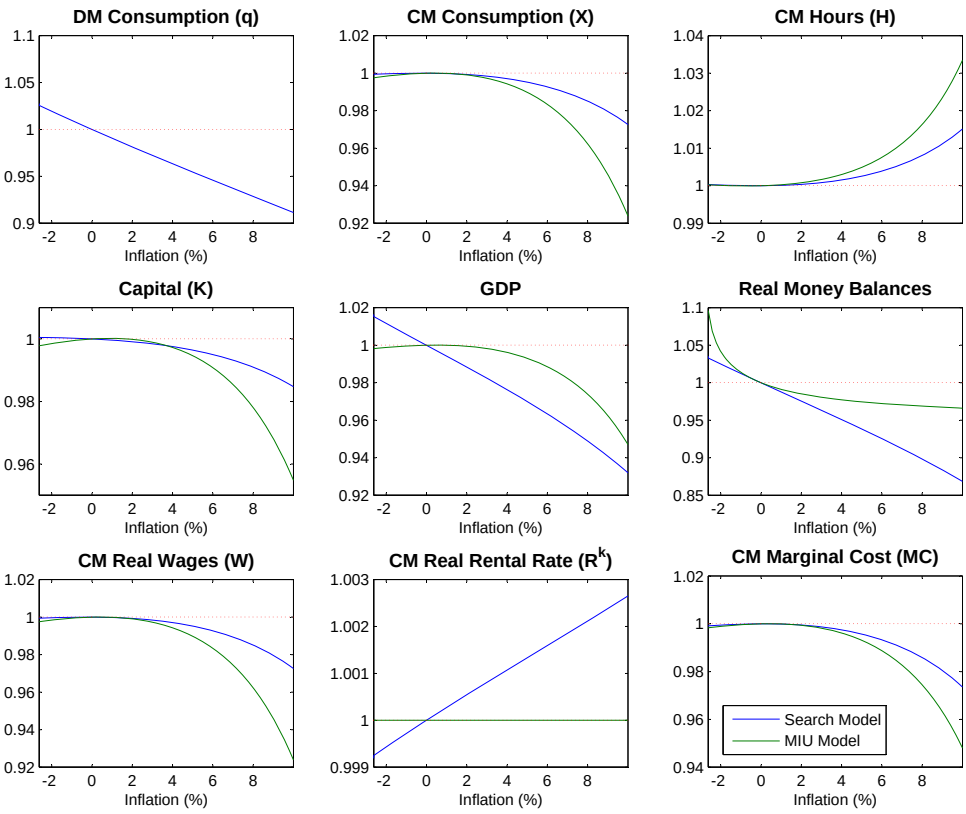
Notes: Figure depicts pointwise posterior mean of impulse response functions.

Figure 2: Welfare Comparison



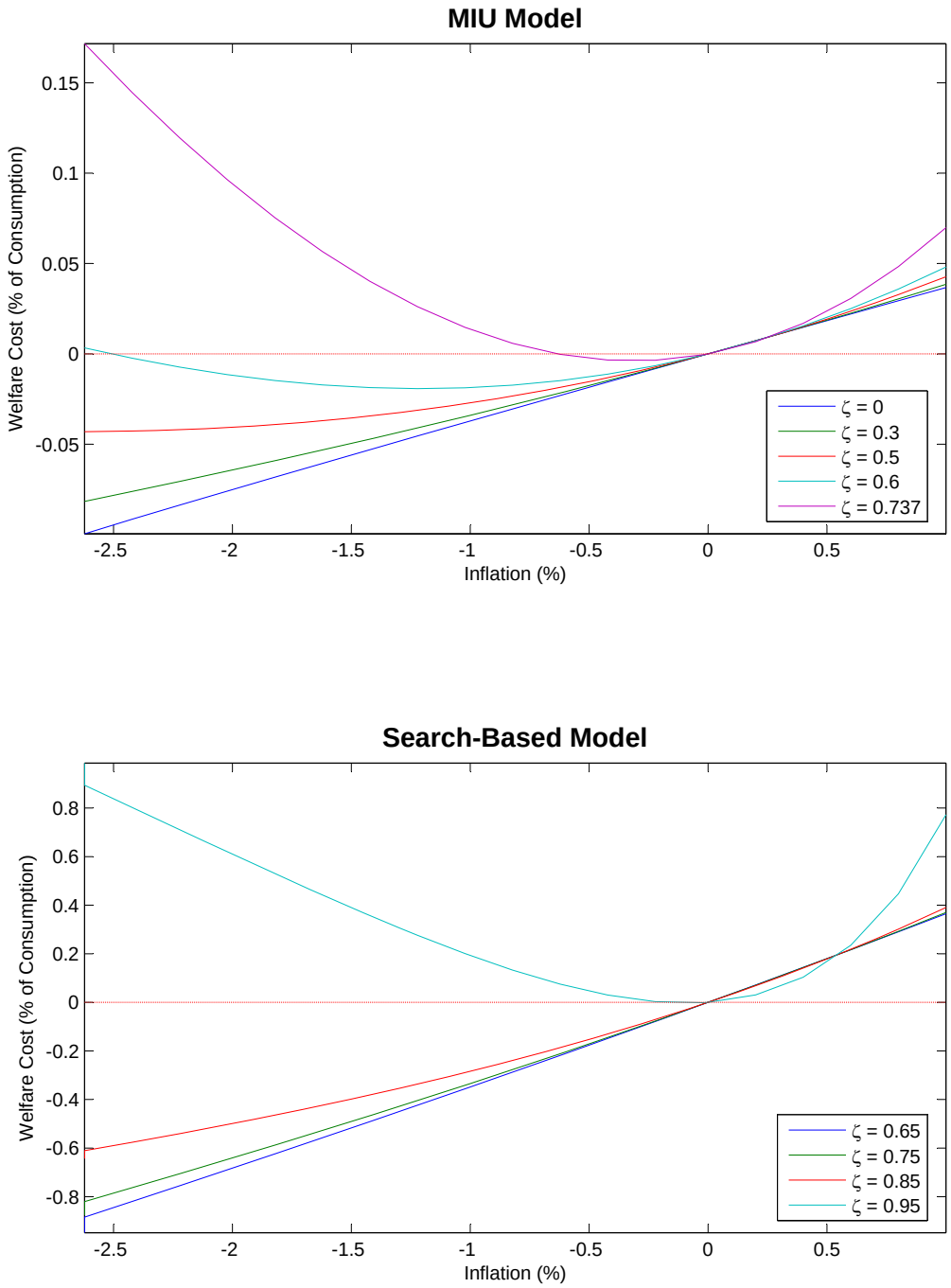
Notes:

Figure 3: Allocations in Search-Based and MIU Model



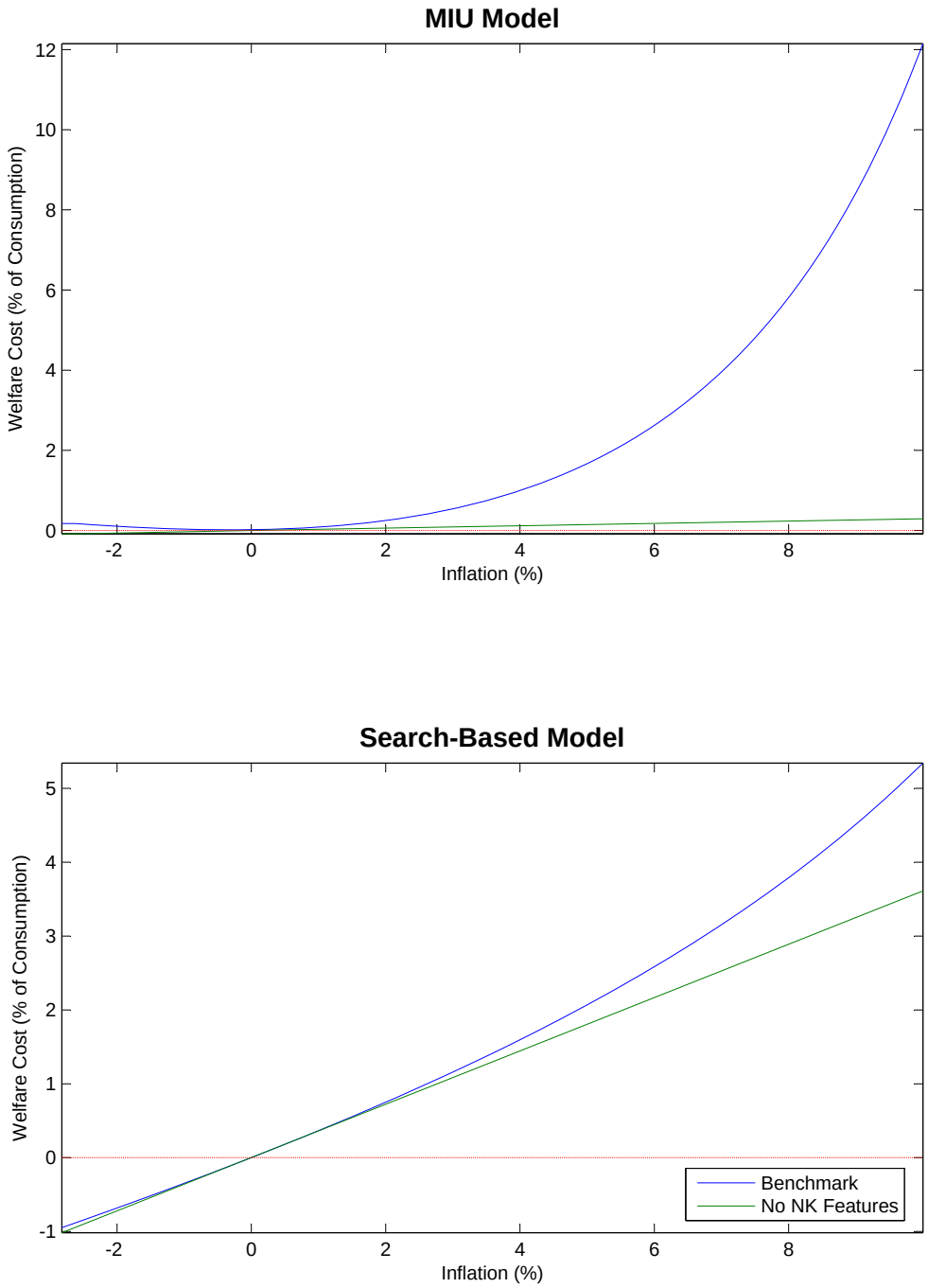
Notes:

Figure 4: Nominal Rigidities and Welfare



Notes:

Figure 5: Welfare and the New Keynesian Mechanism



Notes: