Markets and Relationships in a Learning Economy

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Abstract

It is generally agreed that within long-term relationships agents learn the characteristics of their market partners better than through spot transactions. Yet, little is known on how relationship-based and transactional-based markets compare when agents learn about the aggregate economy from market exchanges. In this paper, we study the structure of the credit market that arises in a decentralized economy where agents learn the aggregate productivity from market exchanges. The model allows to relate the cross-country heterogeneity in the structure of credit markets to macroeconomic fundamentals such as the aggregate and cross-sectional volatility of productivity and its persistence.

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1 Introduction

In credit, goods and labor markets exchanges occur in two different modes. In some circumstances, agents develop long-term relationships with their partners during which they repeatedly exchange with them. In others, agents engage in spot transactions, that is they frequently break matches with their current partners and form matches with new ones. The structure - relationship-based or transactional-based - of markets exhibits a sharp cross-country variation. Consider the credit market for example. While credit relationships are widespread in countries of continental Europe, such as Germany, and in Japan, transactional lending is the norm in Anglo-Saxon countries (United Kingdom and United States). Furthermore, the market structure changes over time: in the last two decades, for instance, the Japanese credit market has been evolving from a relationship-based structure to a more

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transactional-based one (Kikutani, Itoh, and Hayashida, 2006). The determinants of the market structure are at the center of an intense debate in the literature. In fact, it is often argued that its sharp variation significantly contributes to the heterogeneous performance of real economies (see, e.g., Allen and Gale, 2000, and references therein).

The traditional approach to explaining the market structure focuses on its role in information acquisition. The central tenet of this approach is that within long-term relationships agents learn the characteristics of their partners better than through spot transactions.[1] Though well grounded, this approach focuses almost solely on learning about agents' idiosyncratic characteristics. Yet, agents also learn about the aggregate economy from market exchanges and it is well established that this dimension of learning is important for their behavior. For example, when choosing whether to expand its loan portfolio and gauging the investment opportunities available in the economy, a bank can exploit the information obtained in the exchanges with its current depositors and borrowers. Analogously, when choosing whether to expand its production capacity and estimating the productivity of the workers active in the economy, a firm can exploit the information obtained in the exchanges with its current workers. Indeed, such mechanisms are deemed to be so important that, just to mention a few studies, Veldkamp and Nieuwerburgh (2006), Veldkamp (2005) and Lang and Nakamura (1990) explain properties of the business cycle in models where agents learn the aggregate productivity from market exchanges.

In light of these considerations, two questions arise naturally: how do transactional-based and relationship-based markets compare when agents learn about the aggregate economy from market exchanges? Moreover, can this rationalize the sharp variation in the market structure and relate it to the characteristics of the aggregate economy? In this paper, we take a step towards answering these questions, with an emphasis on the credit market. We build an environment where agents learn the aggregate productivity from their histories in credit matches. The environment is deliberately parsimonious. In particular, we abstract from well known frictions of the credit market (moral hazard, adverse selection, externalities) and concentrate on the lack of perfect information on the aggregate productivity. The paper investigates the impact of the market structure on agents' learning and endogenously solves for the market structure as a function of the macroeconomic fundamentals, meant as the aggregate and cross-sectional volatility of productivity and its persistence.

Our starting point is a standard overlapping generation economy. Young agents operate

[1] It is however well known in the literature that not necessarily this deeper information on partners' characteristics is welfare-enhancing. For example, problems of informational monopoly and hold-up can arise (see, e.g., Rajan, 1992, for a theoretical model of the credit market).

projects and transform a final good into an intermediate good. The probability of success of a project depends jointly on the aggregate productivity and on the productivity of the agent who financed it - for example, because the lender monitors the project or helps to select the investment strategy. Old agents finance projects and, if the projects succeed, use the intermediate good obtained by their borrowers to produce final good. In each period, old agents choose how much effort to exert in producing final good on the basis of their beliefs about the aggregate productivity. These beliefs stem from their experiences when young. In fact, a young agent learns the aggregate productivity from her history of project successes and failures. Yet, her learning is noisy because lenders' productivity is unobservable and a young agent cannot perfectly disentangle the aggregate productivity from the productivity of her lender when she observes the outcome of a project.

We let young agents choose between two lending regimes. Under relationship lending, a young agent remains matched with - and obtains finance from - the same old agent over time; under transactional lending, credit matches continuously break down and new credit matches are formed. Two forces drive agents' choice and, hence, the structure of the credit market. The first is the average precision of the beliefs that agents form through market exchanges. This matters because it determines the extent to which agents' decisions approach the optimal ones. The second force is the dispersion of agents' beliefs. This matters because agents are risk averse and, hence, dislike the dispersion of production induced by the dispersion of beliefs.

The analysis delivers rich implications regarding the impact of the macroeconomic fundamentals on the market structure. In particular, we obtain that a higher persistence of the aggregate productivity favors transactional lending. Moreover, when the aggregate productivity has high (low) persistence a higher cross-sectional volatility or a higher aggregate volatility of productivity favors transactional (relationship) lending. If instead the persistence of the aggregate productivity is in an intermediate range, the volatility of productivity has a hump-shaped effect. In particular, up to some threshold a higher (cross-sectional or aggregate) volatility favors transactional lending; beyond that threshold, it favors relationship lending.

We extensively comment these results throughout the paper and, hence, we do not dwell on their intuition here. However, to grasp the forces at work it is useful to explore one of the results more in detail. Consider the effect of the persistence of the aggregate productivity. The benefit of long-term relationships in the learning process is that, since they remain matched with the same lenders over time, agents correctly attribute "mixed histories" of alternating project failures and successes to changes in the aggregate productivity rather than to changes in the productivity of their lenders. The cost of credit relationships is instead that agents misattribute "extreme histories" of repeated successes (failures) to having stayed matched with a high (low) type lender rather than to a persistent good (bad) aggregate productivity. When the aggregate productivity has high persistence "extreme histories" are more frequent than "mixed histories" so that beliefs are on average more precise under transactional lending. The dispersion of beliefs partially dampens the effect of the precision of beliefs, though it does not revert it. In fact, when the aggregate productivity has high persistence agents who chose transactional lending tend to be very confident about the aggregate productivity after experiencing reiterated successes or failures. In turn, this radical optimism or pessimism magnifies the dispersion of their beliefs and production.

The plan of the paper is as follows. In the next section, we relate the paper to the literature. Section 3 lays out the set-up of the model. Section 4 solves for agents' actions and beliefs and for the equilibrium. In Section 5, we discuss in depth the intuition behind the results. Section 6 considers applications and extensions. In this section, we extend the model to allow for learning about agents' idiosyncratic characteristics and we show that the results carry through to this extended environment. Section 8 concludes. Proofs are relegated to the Appendices.

2 Related Literature

This paper especially relates to two strands of literature. The first investigates costs and benefits of long-term relationships (see, e.g., Dewatripont and Maskin, 1995, and Williamson and Aiyagari, 2000). In this literature, a large group of studies focus on the role of long-term relationships in information acquisition about agents' characteristics. Indeed, in the context of the credit market, Boot (2000, p. 10) defines relationship lending "as the provision of financial services by a financial intermediary [...] that invests in obtaining customer-specific information". More in general, a consensus has formed that relationship-based credit markets ease lenders' acquisition of information on borrowers because, being repeatedly matched with the same borrowers, lenders have more opportunities to learn their characteristics (see, for example, Rajan, 1992, and Dell' Ariccia and Marquez, 2004, for theoretical models and Boot, 2000, for a comprehensive literature review).[2] This strand of literature essentially leaves the aggregate economy in the background. In other words, it mostly neglects the fact that agents also learn about the aggregate economy besides the characteristics of their partners and that the market structure can play a critical role in shaping this process.

The second related strand of literature analyzes learning from market exchanges in macroeconomic environments (see, for example, Amador and Weill, 2006, and references

[2] By contrary, outside (transactional) lenders will suffer from an informational deficit.

therein). In particular, as mentioned previously, several studies have recently rationalized key properties of the aggregate economy, such as its response to shocks and its cyclical behavior, as the outcome of a learning process in the credit market. Veldkamp (2005) and Veldkamp and Nieuwerburgh (2006), for instance, explain business cycle asymmetries - meant as the fact that booms are gradual while crashes are short and sharp - in a model of the credit market with learning.[3] Lang and Nakamura (1990) develop a framework where shocks to the returns of risky projects are magnified and prolonged by a learning process. This strand of literature works in a structural vacuum. Our paper contributes to it by uncovering the impact of the market structure on macroeconomic behavior via its effect on learning.[4]

3 The Model

In this section we lay out and discuss the set-up.

3.1 Agents and Goods

Consider an infinite horizon economy. Time is divided into discrete periods indexed by $t \in \mathbb{N}$ and each period has two sub-periods, "morning" and "afternoon". The economy is populated by a sequence of overlapping generations of two-period lived agents. Each generation comprises a unit continuum of agents. There is a final good, an intermediate good, and indivisible projects. Each young agent - that is each agent in the first period of her life - is endowed with two projects, one in the morning and one in the afternoon; each old agent is endowed with an amount ω of final good. Agents consume the final good when old, deriving utility $U = c$ from consumption.

3.2 Technology

3.2.1 Intermediate Good Production

Young agents can operate their projects and transform final good into intermediate good. Consider a project in the morning (an analogous reasoning holds in the afternoon). The implementation of the project entails no effort but it requires an investment $\omega/2$ of final

[3] Chalkley and Lee (1998) develop a model where business cycle asymmetries stem jointly from a learning process and agents' risk aversion.

[4] Another difference with the aforementioned studies is that, while in our environment learning is decentralized, in their environment agents learn from public signals. For models with decentralized learning developed in different contexts, see, e.g., Araujo and Camargo (2006) and Camargo (2006).

good. At the end of the morning, the project can succeed (s) and yield one unit of intermediate good or fail (f) and yield zero. The probability of project success is given by the sum of the aggregate productivity $\pi \in \{\pi_H, \pi_L\}$ and the productivity ε_j of the agent who financed the project. For example, the lender can monitor the project or help to select the investment strategy.[5]

We let π follow a Markov process with transition probability $1-\lambda$: between the morning and the afternoon of each period the aggregate productivity changes with probability $1-\lambda$. As for ε_j, we let $\varepsilon_j = \varepsilon > 0$ for half the population (high type agents) and $\varepsilon_j = -\varepsilon$ for the other half (low type agents). We also impose the following restrictions on the parameters:

$$\begin{aligned} 1/2 &< 1-\pi_L = \pi_H, & (1)\\ \varepsilon &< \pi_L, & (2)\\ 1/2 &\leq \lambda \leq 1. & (3) \end{aligned}$$

Restriction (1) is a normalization that simplifies the algebra.[6] Restriction (2) guarantees that the probability of project success is always positive, even when the aggregate productivity is low ($\pi = \pi_L$) and the agent's type is low ($\varepsilon_j = -\varepsilon$). In conjunction with restriction (1), it also guarantees that the probability of project failure is always positive, even when the aggregate productivity is high ($\pi = \pi_H$) and the agent's type is high ($\varepsilon_j = \varepsilon$). Restriction (3) is a natural assumption that implies that the aggregate productivity is weakly positively correlated over time.

3.2.2 Final Good Production

When a project succeeds, the agent who financed it can use the intermediate good and produce final good. Let e (e') denote the production effort that at the beginning of the morning the agent chooses to exert in the morning (afternoon). The amount of final good the agent produces with the unit of intermediate good equals e (e'), while the disutility she suffers from her production effort equals $e^2/2$ ($e'^2/2$).

[5] In several studies, this ability is treated as essential for project success (see, e.g., Allen and Gale, 2000, for an overview).

[6] For the sake of tractability, we restrict the attention to histories of size two, corresponding to the morning and the afternoon of a young agent. Assumption (1) then implies that at least half of the population observes a success (failure) if the aggregate productivity is π_H (π_L). This precludes situations where an agent ends up with a relatively pessimistic (optimistic) belief about the state of the economy after facing the most (least) favorable experience, i.e., a history of two successes (failures).

3.3 Market Structure

Old agents can use their endowment of final good to finance young agents of the following generation. Old agents can also use the intermediate good obtained by their borrowers and produce final good. In each sub-period, an old agent can finance one young agent and a young agent can be financed by one old agent. For tractability, we attribute full power to an old agent in her bargaining with a young agent.

Lending can occur within long-term credit relationships (R) or spot credit transactions (T). At the beginning of the afternoon, after observing the success or failure of her morning project, each young agent chooses between relationship and transactional lending.[7] Under relationship lending, the young agent is matched with - and obtains credit from - the same old agent in the morning and in the afternoon. Under transactional lending, instead, at the beginning of the afternoon the young agent breaks the match and the agents are rematched with new partners. To guarantee that there is always an available match to an agent who chooses transactional lending, we assume that with a small probability $\epsilon > 0$ a match exogenously breaks down between the morning and the afternoon.

3.4 Information

The economy features imperfect information and learning. A young agent cannot observe the productivity ε_j of her lender. Moreover, agents cannot observe the aggregate productivity π but form beliefs about it from their histories when young. In our set-up, the information a young agent obtains is the success or failure of her projects.

3.5 Timing

We summarize the timing of events and decisions by following an agent over her life-time.

Period t. In the morning of every period t, each young agent enters a match with one old agent, operates a project - transforming final good into intermediate good - and observes the project success or failure. In the afternoon, depending on the occurrence of an exogenous breakdown and on her choice of lending regime, the agent stays matched with the same old agent or enters a credit match with another old agent. The agent operates a second project and observes its success or failure.

Period $t+1$. In the morning of period $t+1$, the old agent chooses her production effort in the morning and in the afternoon. Thereafter, she lends final good to one young agent for

[7] We assume that this decision is made before the agent finds out whether her type is high or low. This occurs, for instance, if the type is only revealed to the agent when she is old. Note that the learning process of a young agent does not depend on her own type since this does not affect the probability of success.

her project. If the project succeeds, she uses the intermediate good obtained by the young agent and produces final good. In the afternoon, if no exogenous breakdown occurs and the old agent's match chooses relationship lending, the agent stays with the same young agent. If instead an exogenous breakdown occurs or if the old agent's match chooses transactional lending, the agent enters a credit match with another young agent. If the project of the young agent succeeds, the old agent uses the intermediate good obtained and produces final good.

3.6 Some Remarks

The set-up incorporates the simplifying feature that learning has only an "aggregate dimension": young agents care about learning the aggregate productivity π but not about learning the productivity ε_j of their lenders. In fact, young agents disregard the beliefs they form about their lenders because when they become old they finance young agents of the following generation and no longer deal with their lenders. The extant literature on markets versus relationships extensively investigates learning about agents' idiosyncratic characteristics and this is not the focus of our paper. Furthermore, in Section 6 we extend the model to add this dimension of learning and we show that the results carry through. Observe also that we do not allow old agents to communicate their type to young agents. Even allowing for this, several reasons could render such a communication not credible. For example, borrowing from a low type agent could entail a cost - possibly even an infinitesimal one - so that a low type agent could have no incentive to reveal her type truthfully and thereby lose some surplus to the benefit of the borrower.

4 Agents' Decisions and Equilibrium

In this section, we first solve for the production effort of an old agent taking as given her expectation about the aggregate productivity. Next, we describe the beliefs about the aggregate productivity the agent formed when young, conditional on her choice between relationship and transactional lending. Finally, we define the equilibrium concept and characterize the conditions under which there exist equilibria in which young agents choose transactional or relationship lending.

4.1 Production

Let π_j (π'_j) denote the aggregate productivity expected by old agent j in the morning (afternoon). In deciding her production effort, the old agent

$$\max_{e_j, e'_j} \left[\underbrace{(\pi_j + \varepsilon_j)e_j - \frac{\omega}{2} - \frac{(e_j)^2}{2}}_{\text{Morning}} + \underbrace{(\pi'_j + \varepsilon_j)e'_j - \frac{\omega}{2} - \frac{(e'_j)^2}{2}}_{\text{Afternoon}} + \omega \right]. \tag{4}$$

The objective function (4) is the old agent's expected consumption of final good minus the disutility from her production effort. The reader should note that we work under the assumption that the highlighted terms in (4), that is the expected returns from project financing in the morning and in the afternoon (expected output minus project financing requirement and disutility from production effort), are strictly positive. This can always be guaranteed by setting the project financing requirement $\omega/2$ sufficiently small.

The solution of the optimization problem satisfies

$$e_j = \pi_j + \varepsilon_j, \tag{5}$$

and

$$e'_j = \pi'_j + \varepsilon_j. \tag{6}$$

Therefore, the production effort of the old agent increases with the expected probability of project success $\pi_j + \varepsilon_j$ (or $\pi'_j + \varepsilon_j$).

4.2 Learning

In the morning, the aggregate productivity expected by old agent j satisfies

$$\pi_j = b_j \pi_H + (1 - b_j)\pi_L, \tag{7}$$

while in the afternoon

$$\pi'_j = b'_j \pi_H + (1 - b'_j)\pi_L, \tag{8}$$

where b_j (b'_j) denotes the agent's belief that in the morning (afternoon) the aggregate productivity is high. Thus, in order to solve for old agents' production decisions, we need to study the beliefs they form about the aggregate productivity when young.

Let $b(s, s)$ be an old agent's belief that the aggregate productivity is high in the morning if she experienced two successes when young and let $\pi(s, s)$ be the associated aggregate productivity that the old agent expects in the morning. The other beliefs and associated expected productivity are denoted in an analogous manner. In what follows, we look at equilibria where all young agents chose the same lending regime, regardless of the outcome of the morning project. Moreover, since we show that the beliefs under relationship lending coincide with the beliefs under transactional lending when $\varepsilon = 0$, we develop expressions for the former and treat the latter as a special case. Bayesian updating implies that the beliefs under relationship lending are given by (see Appendix A)

$$b(s,s) = \frac{\pi_H[\lambda\pi_H + (1-\lambda)\pi_L] + \varepsilon^2}{\pi_H[\lambda\pi_H + (1-\lambda)\pi_L] + \pi_L[\lambda\pi_L + (1-\lambda)\pi_H] + 2\varepsilon^2}, \tag{9}$$

$$b(f,s) = \frac{\pi_H[\lambda\pi_L + (1-\lambda)\pi_H] - \varepsilon^2}{\pi_H[\lambda\pi_L + (1-\lambda)\pi_H] + \pi_L[\lambda\pi_H + (1-\lambda)\pi_L] - 2\varepsilon^2}, \tag{10}$$

$$b(s,f) = \frac{\pi_L[\lambda\pi_H + (1-\lambda)\pi_L] - \varepsilon^2}{\pi_H[\lambda\pi_L + (1-\lambda)\pi_H] + \pi_L[\lambda\pi_H + (1-\lambda)\pi_L] - 2\varepsilon^2}, \tag{11}$$

$$b(f,f) = \frac{\pi_L[\lambda\pi_L + (1-\lambda)\pi_H] + \varepsilon^2}{\pi_H[\lambda\pi_H + (1-\lambda)\pi_L] + \pi_L[\lambda\pi_L + (1-\lambda)\pi_H] + 2\varepsilon^2}. \tag{12}$$

Thus, old agents' beliefs depend on all three macroeconomic fundamentals, that is the persistence λ of the aggregate productivity, the cross-sectional volatility of productivity ε, and the aggregate volatility of productivity $\pi_H - \pi_L$.

Inspection of (9)-(12) yields key insights. The belief $b(s,s)$ is stronger under transactional lending ($\varepsilon = 0$) than under relationship lending ($\varepsilon > 0$). Intuitively, under relationship lending an agent who experienced an "extreme" history of two successes when young is more cautious in attributing this history to a high aggregate productivity (H state) because she knows that her reiterated successes could stem from having stayed matched with a high type lender when young ($\varepsilon_j = \varepsilon$). A similar reasoning applies if the agent experienced an extreme history of two failures when young. By contrary, for "mixed" histories the argument is reversed: as long as $\lambda < 1$, old agents are more cautious under transactional lending than under relationship lending. For example, under transactional lending an agent who experienced first a failure and then a success when young is more cautious in attributing it to a change in the aggregate productivity from low to high because she knows that this mixed history could stem from having being matched first with a low type lender and then with a high type one. Under relationship lending, instead, an agent can control for the effect of changes in her credit match and in the type of her lender because she stays matched with the same lender in the morning and in the afternoon.

Turning to old agents' beliefs in the afternoon, these can be derived from their beliefs in the morning (9)-(12) in a straightforward manner. In fact, given her belief in the morning, an old agent expects that the aggregate productivity will change with probability $1 - \lambda$ in the afternoon. Therefore, for instance, her belief $b'(s,s)$ that the aggregate productivity is high in the afternoon if she experienced two successes when young satisfies

$$b'(s,s) = \lambda b(s,s) + (1-\lambda)[1 - b(s,s)], \tag{13}$$

and analogous expressions hold for the other histories.

Having characterized old agents' beliefs after their possible histories, we now turn to the probabilities with which histories occur. As a preliminary remark, observe that, while old agents' beliefs change from the morning to the afternoon, the distribution of histories is the same in the two sub-periods. In fact, this distribution only depends on the realizations of the aggregate productivity in the previous period and, hence, it is not affected by changes of the aggregate productivity that can take place when agents are old.

Let $p_{HH}(s,s)$ be the probability that a young agent experiences two successes when the aggregate productivity is high both in the morning and in the afternoon (that is the HH state is realized). The other probabilities are denoted in a similar way. We obtain (see Appendix A)

$$\begin{aligned} p_{HH}(s,s) &= \pi_H^2 + \varepsilon^2, && (14)\\ p_{HH}(f,s) &= p_{HH}(s,f) = \pi_L\pi_H - \varepsilon^2, && (15)\\ p_{HH}(f,f) &= \pi_L^2 + \varepsilon^2. && (16) \end{aligned}$$

Next, let $p_{LH}(s,s)$ be the probability that a young agent experiences two successes when the aggregate productivity is low in the morning and high in the afternoon (that is the LH state is realized). The other probabilities are denoted in a similar way. We obtain

$$\begin{aligned} p_{LH}(s,s) &= p_{LH}(f,f) = \pi_H\pi_L + \varepsilon^2, && (17)\\ p_{LH}(f,s) &= \pi_H^2 - \varepsilon^2, && (18)\\ p_{LH}(s,f) &= \pi_L^2 - \varepsilon^2. && (19) \end{aligned}$$

For conciseness, we do not report probabilities for the other states ($HL,\ LL$). The reader will have noted that, just like the beliefs, the probabilities of histories depend on all three macroeconomic fundamentals, λ, ε, and $\pi_H - \pi_L$. For instance, λ affects the probabilities of histories by affecting the probabilities of the states.

Expressions (14)-(19) reveal a key property of the distribution of beliefs. Consider, for instance, an old agent's belief in the morning. If the HH state occurs, the extreme history of two successes (two failures) has the highest (lowest) probability, whereas mixed histories have intermediate probabilities. Remember also that, after experiencing an extreme history of two successes when young, an agent's belief that the aggregate productivity is high is stronger under transactional lending than under relationship lending. Jointly these facts

HH State	Likely History	Induced Belief	LH State	Likely History	Induced Belief
Transactional	All Successes	Confident	*Transactional*	Failure, then success	Too cautious
Relationship	All Successes	Too cautious	*Relationship*	Failure, then success	Confident
LL State	*Likely History*	*Induced Belief*	**HL State**	*Likely History*	*Induced Belief*
Transactional	All failures	Confident	*Transactional*	Success, then failure	Too cautious
Relationship	All failures	Too cautious	*Relationship*	Success, then failure	Confident

Figure 1: Summary of Beliefs and Histories.

imply that on average under relationship lending old agents are too cautious when the state was HH in the previous period. An opposite reasoning holds in the LH state. In this state, the mixed history of a failure followed by a success is the history with the highest probability, whereas extreme histories have intermediate probabilities. Remember that, as long as $\lambda < 1$, after experiencing a history of a failure followed by a success when young, an agent's belief that the aggregate productivity is high is stronger under relationship lending than under transactional lending. Jointly these facts imply that on average under transactional lending old agents are too cautious when the state was LH in the previous period.

We can thus sum up ideas as follows (refer also to Figure 1). When the HH (LL) state is realized, under relationship lending old agents are too cautious on average: extreme histories of all successes (all failures) are very likely and agents easily misattribute these histories to having stayed matched with high (low) type lenders. When instead the LH or the HL state is realized, it is under transactional lending that old agents are too cautious on average: mixed histories of alternating failures and successes are very likely and agents easily misattribute these histories to a change in the type of their lenders rather than to a change in the aggregate productivity.

4.3 Equilibrium

We can now define and solve for the equilibrium. Consider an agent born in period t. In period $t+1$, this agent has to decide how much effort to exert in the production of final good, contingent on her private experience when young. Let $\mathcal{H}$ be the set of such experiences. Formally, $\mathcal{H} = \{(s,s),(s,f),(f,s),(f,f)\}$.[8] The effort choice is then given by the function $\widetilde{e}_{t+1} : \{R,T\} \times \{-\varepsilon,\varepsilon\} \times \mathcal{H} \to \mathbb{R}^2$. For instance, $\left\{e_{t+1}[R,-\varepsilon,(s,f)], e'_{t+1}[R,-\varepsilon,(s,f)]\right\}$ is the morning and afternoon effort of an agent who chose relationship lending when young,

[8] In principle, we could include the experiences $(s,\emptyset)$ and $(f,\emptyset)$, corresponding to the case in which the agent breaks the match in the morning and does not form a new match in the afternoon. However, since there is no cost in forming a match and there is always the benefit of gaining additional information, an agent will never choose to be unmatched.

has found out to be of the low type and experienced a mixed history success-failure when young. Let $\widetilde{v}_t = \left\{v_t^H(s), v_t^H(f), v_t^L(s), v_t^L(f)\right\} \in [0,1]^4$, where $v_t^k(n)$ is the measure of agents who choose relationship lending in period t after observing $l \in \{s, f\}$ when the aggregate productivity is π_n, $n \in \{H, L\}$. An agent's choice between relationship and transactional lending is given by $\widetilde{c}_t : \{s, f\} \to \{R, T\}$. Finally, $\widetilde{b}_{t+1} : \mathcal{H} \to [0,1]$ are the beliefs that the current aggregate productivity is π_H after any possible history.[9]

Definition 1 *An equilibrium is a sequence $\{\widetilde{v}_t, \widetilde{c}_t, \widetilde{b}_{t+1}, \widetilde{e}_{t+1}\}_{t=1}^{\infty}$ such that, in every period, (i) given $E \in \{R, T\}$ and $\widetilde{b}_{t+1}$, $\widetilde{e}_{t+1}$ maximizes (4) for any possible type and history in $\mathcal{H}$, (ii) given $\widetilde{v}_t$ and $\widetilde{e}_{t+1}$, $\widetilde{c}_t$ maximizes the young agent's expected utility, (iii) for each realization of the aggregate productivity, the choices of the young agents induce an aggregate outcome consistent with $\widetilde{v}_t$, (iv) $\widetilde{b}_{t+1}$ is computed by Bayes rule after any history in $\mathcal{H}$.*

In Section 4.2, we computed beliefs under the assumption that in every period an agent's choice between relationship and transactional lending does not depend on the outcome of the morning project. In what follows, we prove that such equilibria exist.

The determinants of agents' choice are clearly their expected utilities from relationship lending ($\mathcal{W}^R$) and from transactional lending ($\mathcal{W}^T$) and the gap between the two expected utilities ($\Delta\mathcal{W} = \mathcal{W}^T - \mathcal{W}^R$). In particular, young agents will choose transactional (relationship) lending if and only if $\Delta\mathcal{W} > (<)0$. Lemma 1 derives the condition under which $\Delta\mathcal{W} > (<)0$ and, hence, agents choose transactional (relationship) lending. The proof is in Appendix B.

Lemma 1 *There exists an equilibrium where all agents choose transactional (relationship) lending if and only if*

$$\frac{\lambda^2}{\widetilde{\lambda}_{HL}\left(\widetilde{\lambda}_{HL} + 2\varepsilon^2\right)} > (<) \frac{(1-\lambda)^2}{\widetilde{\lambda}_{LH}\left(\widetilde{\lambda}_{LH} - 2\varepsilon^2\right)}. \tag{20}$$

Direct inspection of (20) makes it clear that the choice between transactional and relationship lending depends on the persistence of the aggregate productivity λ, on its volatility $\pi_H - \pi_L$, and on the cross-sectional volatility of productivity ε. Proposition 1, the core of the paper, relates this choice to all three macroeconomic fundamentals. The proof is in Appendix B.

Proposition 1 *The structure of the credit market depends on the macroeconomic fundamentals as follows:*

[9] Note that both $\widetilde{c}_t$ and $\widetilde{b}_{t+1}$ depend on the agent's conjecture about $\widetilde{v}_t$ but we left this dependence implicit.

(i) Persistence of aggregate productivity (λ).

For all $\varepsilon > 0$ *and* $\pi_H \in (1/2, 1)$, *there exists a unique* $\lambda(\pi_H, \varepsilon) \in (\frac{1}{2}, 1)$ *such that all agents choose transactional lending if and only if* $\lambda > \lambda(\pi_H, \varepsilon)$;

(ii) Cross – sectional volatility of productivity (ε).

(*a*) *If* $\lambda > \underline{\lambda}(\pi_H)$, *all agents choose transactional lending.*

(*b*) *If* $\underline{\lambda}(\pi_H) > \lambda > \frac{1}{2}$, *there exists a unique* $\varepsilon(\lambda, \pi_H)$ *such that for* $\varepsilon < (>) \varepsilon(\lambda, \pi_H)$ *all agents choose transactional (relationship) lending;*

(*c*) *If* $\lambda = 1/2$, *all agents choose relationship lending.*

(iii) Aggregate volatility of productivity($\pi_H - \pi_L$).

For all $\varepsilon > 0$, *there exists* $1 > \overline{\lambda}(\varepsilon) > \underline{\lambda}(\varepsilon) > \frac{1}{2}$ *such that:*

(*a*) *If* $\lambda > \overline{\lambda}(\varepsilon)$, *all agents choose transactional lending.*

(*b*) *If* $\overline{\lambda}(\varepsilon) > \lambda > \underline{\lambda}(\varepsilon)$, *there exists a unique* $\pi_H(\lambda, \varepsilon)$ *such that, for* $\pi_H < (>) \pi_H(\lambda, \varepsilon)$, *all agents choose transactional (relationship) lending.*

(*c*) *If* $\lambda < \underline{\lambda}(\varepsilon)$, *all agents choose relationship lending.*

Figures 3 and 4 display a graphical illustration of the results. Figure 2 plots the effects of λ and ε on the utility gap $\Delta\mathcal{W}$ that obtain when $\pi_H = 0.6$. The effect of λ is monotonic: a higher λ unambiguously favors transactional lending. The effect of ε on the utility gap is instead monotonic for $\lambda = 1/2$ (a higher ε favors relationship lending) and $\lambda > 1/2\pi_H$ (a higher ε favors transactional lending). When λ takes on an intermediate value the effect of ε is hump-shaped: a higher ε favors transactional lending up to some threshold $\varepsilon(\pi_H, \lambda)$, thereafter it favors relationship lending. Indeed, for sufficiently low values of λ relationship lending eventually dominates transactional lending. Figure 3 plots the effects of λ and $\pi_H - \pi_L$ that obtain when $\varepsilon = 0.1$. Although different forces are at work - see below for details - the effect of $\pi_H - \pi_L$ on the utility gap qualitatively resembles the effect of ε. A higher $\pi_H - \pi_L$ unambiguously favors relationship (transactional) lending when λ is low (high), while it has a hump-shaped effect for a small intermediate range of λ.

5 Discussion

In order to grasp the intuition behind Proposition 1, in this section we examine in turn the two forces that drive expected utility: the *production bias* and the *production volatility.* The production bias measures the extent to which agents' average production departs from the optimal one. The production volatility measures the dispersion of production across agents.

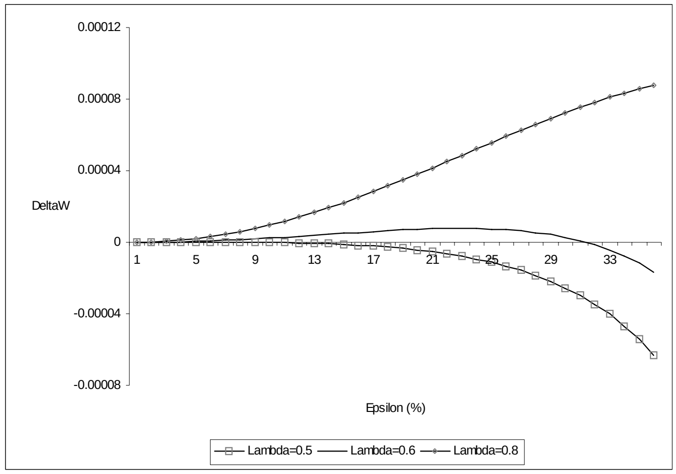

Figure 2: Welfare Gap: Effects of λ and ε.

5.1 Production Bias

We define the production bias of lending regime $E \in \{R, T\}$ as the (modulus of the) difference between an agent's expected production under the lending regime and her expected optimal production. We also define the relative production bias of transactional lending as the difference between its production bias and the production bias of relationship lending. The optimal production when the aggregate productivity is high (low) is $e_H^F = \pi_H$ ($e_L^F = \pi_L$). We can compute the production bias in the morning as

$$B^E = \frac{1}{2}\left[\lambda\left|\pi_H - \pi_{HH}^E\right| + (1-\lambda)\left|\pi_L - \pi_{HL}^E\right| + \lambda\left|\pi_L - \pi_{LL}^E\right| + (1-\lambda)\left|\pi_H - \pi_{LH}^E\right|\right], \quad (21)$$

(for $E \in \{R, T\}$), which is a weighted sum of the expected biases of the beliefs formed in the different states ($\left|\pi_H - \pi_{HH}^E\right|$, $\left|\pi_L - \pi_{HL}^E\right|$, etc.), with weights given by the probabilities of the states. Therefore, in the morning the relative production bias of transactional lending satisfies

$$B^T - B^R = \frac{1}{2}[\lambda(\pi_{HH}^R - \pi_{HH}^T + \pi_{LL}^T - \pi_{LL}^R) + (1-\lambda)(\pi_{LH}^R - \pi_{LH}^T + \pi_{HL}^T - \pi_{HL}^R). \quad (22)$$

An analogous expression holds in the afternoon so that, summing over morning and afternoon,

$$\Delta\mathcal{B} = [1 + (2\lambda - 1)^2]\left[\lambda(\pi_{HH}^R - \pi_{HH}^T) + (1-\lambda)(\pi_{LH}^R - \pi_{LH}^T)\right]. \quad (23)$$

In a scenario where $\Delta\mathcal{B}$ is positive, for example, in expectation production under transactional lending departs more than production under relationship lending from the optimal

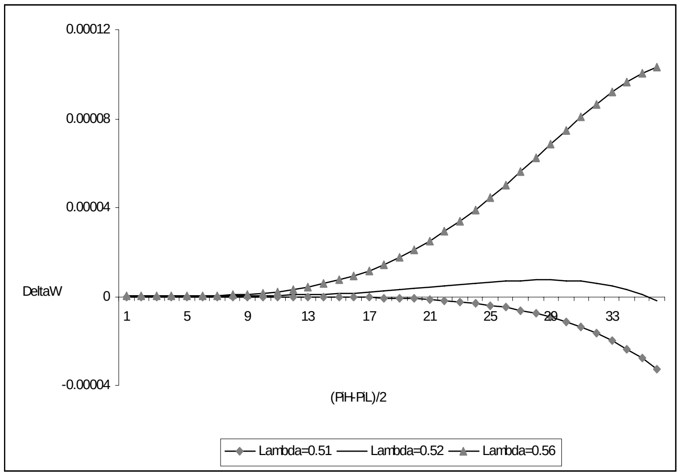

Figure 3: Welfare Gap: Effects of λ and $\pi_H - \pi_L$.

production. Expression (23) has a simple interpretation. Consider first the $\pi^R_{HH} - \pi^T_{HH}$ term. This term is negative and captures the higher precision of beliefs that occurs on average under transactional lending - or equivalently the "excess of caution" that occurs under relationship lending - when the HH or the LL state is realized. Consider next the $\pi^R_{LH} - \pi^T_{LH}$ term. This term is positive and captures the higher precision of beliefs that occurs on average under relationship lending - or equivalently the "excess of caution" that occurs under transactional lending - when the HL or the LH state is realized.

In what follows, we discuss the impact of the macroeconomic fundamentals on the relative production bias $\Delta\mathcal{B}$. The comparative statics results match those in Proposition 1 - indeed, they are proved as a part of the proposition - which reveals the critical role of the production bias in the expected utility gap between the two lending regimes. In presenting the comparative statics, we discriminate between the two channels through which each macroeconomic fundamental affects the relative production bias: the effect on the beliefs induced by histories of project successes or failures and the effect on the probabilities of histories.

Persistence of the aggregate productivity. Consider first the persistence of the aggregate productivity (λ). We obtain that, for all $\varepsilon > 0$ and $\pi_H \in (1/2, 1)$, there exists a unique $\lambda(\pi_H, \varepsilon) \in (1/2, 1)$ such that for $\lambda > (<) \lambda(\pi_H, \varepsilon)$ production is more (less) biased under relationship lending than under transactional lending. We focus first on the effect of λ on the beliefs. The higher λ, the larger is the relative precision of beliefs under transactional lending in the HH state or the LL state. Intuitively, when λ is high under transactional lending agents are very confident about the aggregate productivity if they experienced extreme histories when young. Moreover, the higher λ, the smaller is the relative precision of beliefs under relationship lending in the HL state or the LH state. Therefore, via this channel, a higher λ unambiguously renders average beliefs more precise under transactional lending. Using (23), this can be grasped by observing that a higher λ leads to a lower (more negative) $\pi_{HH}^{R} - \pi_{HH}^{T}$ term and a lower (less positive) $\pi_{LH}^{R} - \pi_{LH}^{T}$ term.

Now consider the effect of λ on the probabilities of the histories. The higher λ, the higher is the probability of the HH state or the LL state and the lower the probability of the HL state or the LH state. Therefore, a higher λ puts more weight on extreme histories. Since extreme histories induce more precise beliefs under transactional lending, this unambiguously renders average beliefs more precise under transactional lending. Using (23), this can be grasped by observing that a higher λ leads to a larger weight on the $\pi_{HH}^{R} - \pi_{HH}^{T}$ term and a smaller weight on the $\pi_{LH}^{R} - \pi_{LH}^{T}$ term. In sum, both the effect on beliefs and the effect on histories imply that a higher λ leads to a lower relative production bias $\Delta\mathcal{B}$ under transactional lending.

$\mathsf{Cross-sectional\ volatility\ of\ productivity.}$ Consider next the cross-sectional volatility of productivity (ε). We obtain that, if $\lambda > \underline{\lambda}(\pi_H)$ the relative production bias under transactional lending is negative ($\Delta\mathcal{B} < 0$) and monotonically decreasing in ε. If $1/2 < \lambda < \underline{\lambda}(\pi_H)$, there exists a unique $\varepsilon(\pi_H, \lambda)$ such that for $\varepsilon < (>) \varepsilon(\pi_H, \lambda)$ the relative production bias under transactional lending is monotonically decreasing (increasing) in ε. Finally, if $\lambda = 1/2$, the relative production bias under transactional lending is positive ($\Delta\mathcal{B} > 0$) and monotonically increasing in ε. Focus first on the effect of ε on the beliefs. The higher ε, the higher is an agent's belief that the aggregate productivity is high (low) if she experienced a mixed (f, s) history (respectively a mixed (s, f) history) under relationship lending. Intuitively, an agent who stayed matched with the same lender over time is more confident about changes in the aggregate productivity when the difference across lenders' idiosyncratic productivity is pronounced. An opposite reasoning holds for extreme histories: for example, the higher ε, the lower is an agent's belief that the aggregate productivity is high if she experienced an extreme (s, s) history under relationship lending. Clearly, the two effects contrast each other: the former renders average beliefs more precise under relationship lending while the latter renders average beliefs more precise in the transactional-based one. We find that when $\lambda > \underline{\lambda}(\pi_H)$ ($\lambda = 1/2$) the latter (former) effect always prevails and a higher ε unambiguously favors transactional (relationship) lending. When instead $1/2 < \lambda < \underline{\lambda}(\pi_H)$ the impact of ε on $\Delta\mathcal{B}$ is U-shaped: the latter effect prevails for low values of ε while the former prevails for high values of ε. Using (23), this can be further grasped by observing that in this case as ε rises the decrease in the $\pi_{HH}^{R} - \pi_{HH}^{T}$ term dominates the increase in the $\pi_{LH}^{R} - \pi_{LH}^{T}$ term for $\varepsilon < \varepsilon(\pi_H, \lambda)$ while it is dominated for $\varepsilon > \varepsilon(\pi_H, \lambda)$.

The effect of ε on histories is instead as follows. The higher ε, the more likely are extreme histories and the less likely are mixed histories under relationship lending. In fact, a higher ε increases the persistence of successes or failures. The resulting effect on the average precision of beliefs is ambiguous. For example, consider the HH state. The extreme (s, s) history induces the belief closest to the realized productivity while the extreme (f, f) history induces an overly pessimistic belief. Putting more weight on both histories produces ambiguous consequences. In sum the effect on histories is ambiguous while the effect on beliefs generates two contrasting forces. For sufficiently extreme values of λ, one of the two forces always prevails and ε has a monotonic effect on $\Delta\mathcal{B}$. For an intermediate range of λ, the interaction between these two contrasting leads to a U-shaped effect of ε.

Aggregate volatility of productivity. Finally, consider the aggregate volatility of productivity (π_H-π_L). We obtain that if production is more biased under transactional lending ($\Delta\mathcal{B} > 0$), the relative bias of transactional lending is increasing in $\pi_H - \pi_L$. If production is more biased under relationship lending ($\Delta\mathcal{B} < 0$), the relative bias of transactional lending is decreasing in $\pi_H - \pi_L$ as long as λ is not too small. In terms of the effect of π_H-π_L on the beliefs, the larger the difference $\pi_H - \pi_L$, the larger is the difference between the beliefs that agents form in the two economies after extreme histories. This increases the relative precision of beliefs under transactional lending when the HH state or the LL state is realized. An opposite reasoning holds for mixed histories: a larger difference $\pi_H - \pi_L$ increases the relative precision of beliefs under relationship lending when the HL state or the LH state is realized. Clearly, these two effects contrast each other: the former renders average beliefs more precise under transactional lending while the latter renders average beliefs more precise in the relationship-based one.

Focus next on the effect of $\pi_H - \pi_L$ on histories. The larger the difference $\pi_H - \pi_L$, the more likely is the history that induces a belief close to the realized productivity. For example, conditional on the HH state being realized, the (s, s) history, which induces the strongest belief that the aggregate productivity is high, is more likely when $\pi_H - \pi_L$ is larger. Because the belief after a (s, s) history is more precise under transactional lending, a higher $\pi_H - \pi_L$ thus increases the relative precision of beliefs under transactional lending. Analogously, conditional on the LH state being realized, the (f, s) history, which induces the strongest belief that the aggregate productivity is high, is more likely when $\pi_H - \pi_L$ is larger. Because the belief after a (f, s) history is more precise under relationship lending, a higher $\pi_H - \pi_L$ thus increases the relative precision of beliefs under relationship lending. In summary, both the effect on beliefs and the effect on histories generate contrasting forces. All in all, a higher $\pi_H - \pi_L$ generally has a monotonic effect on $\Delta\mathcal{B}$, magnifying the production bias $\Delta\mathcal{B}$, whether $\Delta\mathcal{B}$ is positive or negative. For a small intermediate range of λ, instead, the effect of $\pi_H - \pi_L$ is U-shaped: the relative production bias of transactional lending is negative when $\pi_H - \pi_L$ is low while it becomes positive when $\pi_H - \pi_L$ is sufficiently high.

5.2 Production Volatility

The second force that drives expected utility is the production volatility. Because of the convexity of the effort cost function, this exerts a role through the disutility from production effort (higher volatility, higher disutility). The expected disutility from production effort equals a weighted sum of the agent's expected disutilities from her production effort conditional on the possible state realizations in the first period of her life (HH, HL, LH, LL), with weights given by the probabilities of the states. For example, in the morning

$$D^E = \frac{1}{2}[\lambda(\frac{e_{HH}^{2E}}{2} + \frac{e_{LL}^{2E}}{2}) + (1-\lambda)(\frac{e_{LH}^{2E}}{2} + \frac{e_{HL}^{2E}}{2})] \tag{24}$$

(for $E \in \{T, R\}$) and analogously in the afternoon. Summing over morning and afternoon, the overall difference between the expected disutility from production effort under transactional lending and under relationship lending can be written as

$$\Delta\mathcal{D} = \frac{1+(2\lambda-1)^2}{4}\left[\lambda(\pi_{HH}^{2T} + \pi_{LL}^{2T} - \pi_{HH}^{2R} - \pi_{LL}^{2R}) + (1-\lambda)(\pi_{LH}^{2T} + \pi_{HL}^{2T} - \pi_{LH}^{2R} - \pi_{HL}^{2R})\right] \tag{25}$$

where, for instance,

$$\pi_{HH}^{2E} = \sum_{(m,a)\in\{f,s\}^2} p_{HH}^{E}(m,a)\left[\pi^E(m,a)\right]^2.$$

Using (23) and (25), we can now decompose the expected utility gap into a term that depends on the relative production bias $\Delta\mathcal{B}$ and a term that depends on the difference $\Delta\mathcal{D}$ between the expected disutilities from production, that is

$$\Delta\mathcal{W} = -\frac{\pi_H - \pi_L}{2}\Delta\mathcal{B} - \Delta\mathcal{D}. \tag{26}$$

Let σ_{HH}^{2E} denote the variance of production across agents conditional on the HH state being realized (for $E \in \{T, R\}$), and similarly for the other states. Using the expression for $\Delta\mathcal{B}$ in (23) and operating algebraic manipulations on the expression for $\Delta\mathcal{D}$ in (25), we can rewrite

(26) as

$$\Delta\mathcal{W} = \frac{1+(2\lambda-1)^2}{2}\left\{\begin{array}{c}\underbrace{\left(\begin{array}{c}\lambda(\pi^T_{HH}-\pi^R_{HH})[(\pi_H-\pi_L)-(\pi^R_{HH}-\pi^T_{LL})]- \\ (1-\lambda)(\pi^R_{LH}-\pi^T_{LH})[(\pi_H-\pi_L)-(\pi^T_{LH}-\pi^R_{HL})]\end{array}\right)}_{\Delta\mathcal{W}_1 \;=\; \text{Production Bias - Volatility Average Production}}- \\ \\ \underbrace{-\frac{1}{2}\left(\begin{array}{c}\lambda[(\sigma^{2T}_{HH}-\sigma^{2R}_{HH})+(\sigma^{2T}_{LL}-\sigma^{2R}_{LL})]- \\ (1-\lambda)[(\sigma^{2R}_{LH}-\sigma^{2T}_{LH})+(\sigma^{2R}_{HL}-\sigma^{2T}_{HL})]\end{array}\right)}_{\Delta\mathcal{W}_2 \;=\; \text{Volatility Idiosyncratic Production}}\end{array}\right\}. \quad (27)$$

Expression (27) has a straightforward interpretation. Observe the $\Delta\mathcal{W}_1$ block. The terms $\pi^R_{HH}-\pi^T_{LL}$ and $\pi^T_{LH}-\pi^R_{HL}$ in the square parentheses mitigate the impact of the relative production bias $\Delta\mathcal{B}$ on the expected gap. For example, $(\pi^T_{HH}-\pi^R_{HH})(\pi^R_{HH}-\pi^T_{LL})$ is a positive term that gets subtracted from $(\pi^T_{HH}-\pi^R_{HH})(\pi_H-\pi_L)$. Intuitively, the difference between the average production in the HH state and in the LL state is larger under transactional lending - where agents are more confident and average beliefs are more extreme - than under relationship lending ($e^T_{HH}-e^T_{LL}>e^R_{HH}-e^R_{LL}$). Observe next the $\Delta\mathcal{W}_2$ block. This reflects the volatility of production conditional on a state being realized. Take for example the HH state or the LL state. When one of these states is realized old agents are very confident about the aggregate productivity under transactional lending. This radical optimism (in the HH state) or pessimism (in the LL state) renders the volatility of their production especially large under transactional lending.

All in all, we thus find that the production volatility always works in a direction opposite to that of the production bias in determining the expected utility gap, although it has only a dampening effect.

6 Extensions and Applications

This section explores extensions and applications of the model.

6.1 Idiosyncratic Learning

We now extend our set-up and consider a scenario where a young agent also cares about learning her lender's productivity ε, besides learning the aggregate productivity π. This extension allows us to check the robustness of the results and to further compare our model with the extant literature on the role of long-term relationships in information acquisition. In fact, this literature typically stresses that long-term relationships favor the acquisition of information about agents' idiosyncratic characteristics.

We assume that after completing a project a young agent can implement an action a which takes on value in the interval $[l, h]$. The utility the agent derives from this action depends only on the type of the lender she is paired with.[10] In particular, the agent enjoys utility $\overline{u}$ if she chooses $a = h$ ($a = l$) when her lender is of the high (low) type. Moreover, her utility decreases with the distance of a from this optimal choice: if her lender is of the high type her utility is $\widetilde{u}(a) = \overline{u} - (h - a)^2/2$, while if her lender is of the low type her utility is $\widetilde{u}(a) = \overline{u} - (a - l)^2/2$. We can think of such a specification as reflecting some form of complementarity between the choice of the young agent and the type of the old one. The problem of the young agent is to choose the value of her action in the morning and in the afternoon. Yet, because our objective is to analyze the utility gap between transactional and relationship lending, we can restrict our attention to the young agent's choice in the afternoon. In fact, in the morning the agent's belief about the type of her lender and, hence, her preferred action are the same under the two lending regimes.

Let b_ε denote the young agent's belief that her lender is of the high type in the afternoon. The utility the agent expects from her action is

$$E[\widetilde{u}(a)] = \overline{u} - b_\varepsilon \frac{(h-a)^2}{2} - (1 - b_\varepsilon) \frac{(a-l)^2}{2}, \tag{28}$$

so that her optimal choice of a satisfies $a = b_\varepsilon h + (1 - b_\varepsilon) l$. For the sake of conciseness, we do not present the results of this extension here. Full details are available in the extended version of the paper, which is available upon request. The results can be summarized as follows. In line with the traditional view that relationships are beneficial in that they ease learning about agents' characteristics, we find that the utility gap $\Delta\mathcal{W}$ between transactional and relationship lending is smaller than in the basic set-up. This captures the benefit that young agents derive from being repeatedly matched with the same lender and thereby being able to make a better informed action choice in the afternoon. We also find that the comparative statics results with respect to the three macroeconomic fundamentals carry through to this modified environment. For example, an increase in λ monotonically increases the welfare gap $\Delta\mathcal{W}$. This occurs not only for the reasons we uncovered previously but also because an increase in λ reduces the benefit of long-term relationships in the learning process about the lender's type.

6.2 Aggregate and Cross-Sectional Volatility

The model can be applied to a number of interesting issues. For example, a pattern observed in the United States in the last three decades or so has been the drop in aggregate output

[10] For simplicity, we let the action be implemented regardless of whether the project succeeds or not.

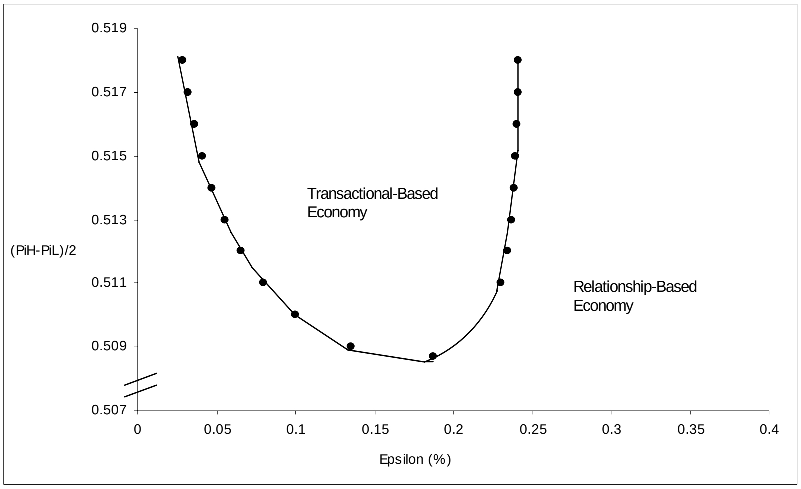

Figure 4: Fronteer (Aggregate)-(Cross-sectional) Volatility.

volatility - attributed by several scholars to a decline in the volatility of aggregate productivity - together with an increase in output idiosyncratic volatility. Can the reader use our model to analyze the impact of these macroeconomic changes on the market structure? Figure 4 plots the frontier (aggregate)-(cross-sectional) volatility. The frontier, which is constructed for $\lambda = 0.56$, connects combinations of ε and $(\pi_H - \pi_L)/2$ such that an agent's expected utility is the same under transactional and under relationship lending ($\Delta\mathcal{W} = 0$). The chosen value of λ is sufficiently high that an increase in $\pi_H - \pi_L$ unambiguously favors transactional lending ($\Delta\mathcal{W}$ increases); however, for the range of $\pi_H - \pi_L$ displayed in the figure, $\lambda < 1/2\pi_H$ so that ε has a U-shaped effect on $\Delta\mathcal{W}$.

The resulting frontier is also U-shaped. In particular, (i) transactional lending dominates for intermediate values of the cross-sectional volatility of productivity ε and (ii) the range of the cross-sectional volatilities such that transactional lending dominates becomes larger as the aggregate volatility $\pi_H - \pi_L$ rises. For reasons of space, we do not draw the frontier for other values of λ or for different ranges of $\pi_H - \pi_L$. Nevertheless, it is straightforward to infer from our results that for sufficiently larger values of λ and $\pi_H - \pi_L$ the upward sloping right branch of the frontier will shift further to the right and move outside the feasible range of ε - put differently, relationship lending will dominate only for very low values of ε. Finally, for even larger values of λ and $\pi_H - \pi_L$ transactional lending will always dominate relationship lending. As long as the persistence of productivity remains fairly stable, a macroeconomic change of the type occurred in the United States in the last thirty years can then be thought as a movement in the south-east direction of the figure (lower $(\pi_H - \pi_L)/2$ and higher ε).

7 Conclusion

In this paper, we have studied an economy where agents learn the aggregate productivity from credit market exchanges. In such an economy, the structure - relationship-based or transactional-based - of the credit market shapes the process of decentralized learning and, hence, agents' decisions. We have characterized macroeconomic conditions under which a transactional-based market structure or a relationship-based one arises. The model delivers the following distinct empirical implications: (i) Economies where the persistence of aggregate productivity is high should feature a more transactional-based market structure; (ii) As long as the persistence of aggregate productivity is sufficiently high, economies where the cross-sectional or the aggregate volatility of productivity is larger should feature a more transactional-based structure. By contrary, as long as the persistence of aggregate productivity is sufficiently low, economies where the cross-sectional or the aggregate volatility of productivity is larger should feature a more relationship-based structure. (iii) When the persistence of aggregate productivity takes on intermediate values, both the cross-sectional and the aggregate volatility of productivity should have a hump-shaped effect on the welfare gap between the transactional-based and the relationship-based structure.

In order to isolate the effects on decentralized learning, the model necessarily neglects features of the credit market which are often invoked to explain the market structure. For example, an interesting extension of the model could allow for a form of "active learning" of the type frequently introduced in the extant literature and investigate its interaction with the process of learning through exchanges. Precisely, besides learning through market exchanges, agents could also invest in a screening or monitoring technology that allows them to acquire information about the aggregate environment. We leave this and other extensions for future research.

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8 Appendix A. Beliefs and Histories

BELIEFS: Since the transition probability λ is symmetric across states, in every period a young agent enters the economy with a prior 1/2 that the economy is in the high state (H). If the project she operates in the morning is successful (s), the updated belief that the economy is in the high state equals

$$\frac{(\pi_H+\varepsilon)\frac{1}{4}+(\pi_H-\varepsilon)\frac{1}{4}}{(\pi_H+\varepsilon)\frac{1}{4}+(\pi_H-\varepsilon)\frac{1}{4}+(\pi_L+\varepsilon)\frac{1}{4}+(\pi_L-\varepsilon)\frac{1}{4}}.$$

The expression in the numerator is the probability that the economy is in the high state and a success occurs. This is given by the probability of a success given that the economy is in the high state and the lender is of the high type $(\pi_H+\varepsilon)$, times the probability that the economy is in the high state and the lender is of the high type (1/4), plus the probability of a success given that the economy is in the high state and the lender is of the low type $(\pi_H-\varepsilon)$, times the probability that the economy is in the high state and the lender is of the low type (1/4). The expression in the denominator is simply the unconditional probability of a success.

We can rewrite the expression above as

$$\frac{(\pi_H+\varepsilon)+(\pi_H-\varepsilon)}{2}=\pi_H.$$

At the beginning of the afternoon, the agent updates her belief that the economy is in the high state to $\lambda\pi_H+(1-\lambda)\pi_L\equiv\lambda_{HL}$, as she takes into account the probability $1-\lambda$ that the current state changes between the morning and the afternoon. Now consider a strategy profile where all agents choose transactional lending if and only if they observe a success. Assume that the agent observes a failure (f) of the project in the afternoon. In the morning of the following period, her posterior that the economy is in the high state is (where $\lambda_{LH}=\lambda\pi_L+(1-\lambda)\pi_H$)

$$\frac{\frac{1}{2}(\pi_L-\varepsilon)(\lambda_{HL}+\varepsilon)+\frac{1}{2}(\pi_L+\varepsilon)(\lambda_{HL}-\varepsilon)}{\frac{1}{2}(\pi_L-\varepsilon)(\lambda_{HL}+\varepsilon)+\frac{1}{2}(\pi_L+\varepsilon)(\lambda_{HL}-\varepsilon)+\frac{1}{2}(\pi_H-\varepsilon)(\lambda_{LH}+\varepsilon)+\frac{1}{2}(\pi_H+\varepsilon)(\lambda_{LH}-\varepsilon)},$$

where $\lambda_{HL}=\lambda\pi_H+(1-\lambda)\pi_L$ and $\lambda_{LH}=\lambda\pi_L+(1-\lambda)\pi_H$. Operating simple algebraic manipulations, we obtain

$$\frac{\pi_L\lambda_{HL}-\varepsilon^2}{\pi_L\lambda_{HL}+\pi_H\lambda_{LH}-2\varepsilon^2}.$$

Finally, consider a transactional-based economy. In this case, since the morning matches between old and young agents break down and new matches are formed in the afternoon, the agent's belief that the economy is in the high state in the morning of the following period after observing a failure in the afternoon is

$$\frac{\frac{1}{2}(\pi_L-\varepsilon)\lambda_{HL}+\frac{1}{2}(\pi_L+\varepsilon)\lambda_{HL}}{\frac{1}{2}(\pi_L-\varepsilon)\lambda_{HL}+\frac{1}{2}(\pi_L+\varepsilon)\lambda_{HL}+\frac{1}{2}(\pi_H-\varepsilon)\lambda_{LH}+\frac{1}{2}(\pi_H+\varepsilon)\lambda_{LH}}.$$

We can rewrite this expression as

$$\frac{\pi_L \lambda_{HL}}{\pi_L \lambda_{HL} + \pi_H \lambda_{LH}}.$$

Note that if we let $\varepsilon = 0$ in the posterior formed under relationship lending, this coincides with the posterior of the transactional-based economy. A similar reasoning can be applied to obtain the posteriors formed after the other histories of successes and failures an agent may face when young. Moreover, in all these histories, it is always the case that for $\varepsilon = 0$ the posterior formed under relationship lending equals the corresponding posterior formed in the transactional based economy. These results are available from the authors upon request.

HISTORIES: In a relationship-based economy, consider the probability that a young agent experiences a success in the morning and a failure in the afternoon when the economy is in the high state both in the morning and in the afternoon. This probability is computed as follows. First, there is a probability 1/2 that the young agent is matched with a high type lender, in which case the probability of a success followed by a failure equals

$$(\pi_H + \varepsilon)(1 - \pi_H - \varepsilon).$$

There is also a probability 1/2 that the young agent is matched with a low type lender. The probability of a success and a failure in this case equals

$$(\pi_H - \varepsilon)(1 - \pi_H + \varepsilon).$$

Summing up these two expressions we obtain

$$\pi_H \pi_L - \varepsilon^2.$$

A similar reasoning applies to the other histories of successes and failures an agent may face when young. Moreover, when $\varepsilon = 0$ the probability of a particular history under relationship lending coincides with the probability of the corresponding history under transactional lending. These results are available from the authors upon request.

9 Appendix B. Proofs

PROOF OF LEMMA 1: Notice that, when no young agent conditions her choice on the outcome of the morning project, an agent's optimal behavior only depends on the fundamentals of the economy, i.e., it does not depend on the choices made by other agents and on the agent's own experience. This feature stems from the assumption that old agents have full bargaining power in their negotiation with young agents, which implies that a young agent only cares about transactional or relationship funding to the extent that each of these mechanisms improve her learning about the aggregate productivity. In this case, if all young agents ignore their own history when making their choices, the pool of available old agents in the afternoon of every period is a random draw from the original pool of agents. As a result, beliefs are always computed as in subsection 4.2. In particular, the precision of the beliefs only depends on the fundamentals of the economy. A direct implication of this reasoning is that whenever the choice between relationship and transactional lending is made independently of the agent's own experience, Proposition 1 describes all possible equilibria in the economy. We do not formally consider the possibility of equilibria where the decision of a young agent depends on her experience in the morning. However, it is intuitive that such equilibria are unlikely. Precisely, their existence would imply that the precision of the beliefs changes depending on the occurrence of a success or a failure. Clearly, this asymmetry across distinct experiences is inconsistent with assumptions (1)-(3). Let $\mathcal{W}^R$ ($\mathcal{W}^T$) be the expected utility of a young agent when all agents choose relationship (transactional lending) and maximize (4) after any possible type and history. The expected utility of an agent in her second period of life equals a weighted sum of the agent's expected utilities conditional on the possible state realizations in her first period of life. Precisely, in the morning (where $E \in \{R, T\}$)

$$W^E = \frac{1}{2}\left[\lambda W^E_{HH} + (1-\lambda)W^E_{HL}\right] + \frac{1}{2}\left[\lambda W^E_{LL} + (1-\lambda)W^E_{LH}\right], \tag{29}$$

whereas in the afternoon

$$W'^E = \frac{1}{2}\left[\lambda W'^E_{HH} + (1-\lambda)W'^E_{HL}\right] + \frac{1}{2}\left[\lambda W'^E_{LL} + (1-\lambda)W'^E_{LH}\right]. \tag{30}$$

Moreover, $\mathcal{W}^E = W^E + W'^E$. In (29), for example, W^E_{HH} denotes the agent's expected utility in the morning conditional on the HH state being realized in the first period. In order to grasp the results, it is useful to expand expressions (29) and (30). Consider again W^E_{HH}. This depends on the agent's expected production effort e^E_{HH} conditional on the HH state being realized,

$$W^E_{HH} = \pi_H e^E_{HH} - \frac{e^{2E}_{HH}}{2}. \tag{31}$$

In turn, using (5), e^E_{HH} equals the aggregate productivity π^E_{HH} expected by the agent, as determined by the distribution of beliefs induced by the HH state. Precisely, using the beliefs and probabilities of histories from the previous subsection,

$$e^E_{HH} = \sum_{(m,a)\in\{f,s\}^2} p^E_{HH}(m,a)\pi^E(m,a). \tag{32}$$

W^E_{HH} also depends on the disutility $e^{2E}_{HH}/2$ that the agent expects from her production effort, conditional on the HH state being realized. In turn,

$$e^{2E}_{HH} = \varepsilon^2 + \sum_{(m,a)\in\{f,s\}^2} p^E_{HH}(m,a)\left[\pi^E(m,a)\right]^2. \tag{33}$$

Next, let $\Delta\mathcal{W} = \mathcal{W}^T - \mathcal{W}^R$ be the utility gap between transactional lending and relationship lending. For conciseness, we do not present the computation of the utility gap in the main text. It is fully derived in the Supplement I, at the end of the paper. We obtain

$$\Delta\mathcal{W} = \frac{(\pi_H - \pi_L)^4[1 + (2\lambda - 1)^2]\varepsilon^2}{4}\left[\frac{\lambda^2}{\widetilde{\lambda}_{HL}\left(\widetilde{\lambda}_{HL} + 2\varepsilon^2\right)} - \frac{(1-\lambda)^2}{\widetilde{\lambda}_{LH}\left(\widetilde{\lambda}_{LH} - 2\varepsilon^2\right)}\right], \tag{34}$$

where

$$\begin{aligned}\widetilde{\lambda}_{HL} &= \pi_H[\lambda\pi_H + (1-\lambda)\pi_L] + \pi_L[\lambda\pi_L + (1-\lambda)\pi_H],\\ \widetilde{\lambda}_{LH} &= \pi_H[\lambda\pi_L + (1-\lambda)\pi_H] + \pi_L[\lambda\pi_H + (1-\lambda)\pi_L].\end{aligned}$$

A strategy profile in which all agents choose transactional lending corresponds to $\{\widetilde{v}_t^T, \widetilde{c}_t^T, \widetilde{b}_{t+1}^T, \widetilde{e}_{t+1}^T\}_{t=1}^{\infty}$, where $\widetilde{v}_t^T = \{0,0,0,0\}$, $\widetilde{c}_t^T(.) = T$, $\widetilde{b}_{t+1}^T$ is as in (9)-(12) with $\varepsilon = 0$, and $\widetilde{e}_{t+1}^T$ maximizes (4) after any possible type and history in $\mathcal{H}$. Consider the decision problem of a young agent. If he deviates and chooses relationship lending, his expected utility gap is given by (34). Note that, after a deviation, beliefs are updated exactly as in (9)-(12) with $\varepsilon > 0$. He does not want to deviate as long as $\Delta\mathcal{W} > 0$. This occurs if and only if

$$\frac{\lambda^2}{\widetilde{\lambda}_{HL}\left(\widetilde{\lambda}_{HL} + 2\varepsilon^2\right)} > \frac{(1-\lambda)^2}{\widetilde{\lambda}_{LH}\left(\widetilde{\lambda}_{LH} - 2\varepsilon^2\right)}.$$

Similarly, a strategy profile in which all agents choose relationship lending corresponds to $\widetilde{v}_t^R = \{(1-\epsilon)\pi_H, (1-\epsilon)(1-\pi_H), (1-\epsilon)\pi_L, (1-\epsilon)(1-\pi_L)\}$, $\widetilde{c}_t^R(.) = R$, $\widetilde{b}_{t+1}^R$ is as in (9)-(12) with $\varepsilon > 0$, and $\widetilde{e}_{t+1}^R$ maximizes (4) after any possible type and history in $\mathcal{H}$. Again, consider the decision problem of a young agent. If he deviates and chooses transactional lending, his expected utility gap is given by (34). Note that, after a deviation, beliefs are updated exactly as in (9)-(12) with $\varepsilon = 0$. He does not want to deviate as long as $\Delta\mathcal{W} < 0$. This inequality holds if and only if

$$\frac{\lambda^2}{\widetilde{\lambda}_{HL}\left(\widetilde{\lambda}_{HL} + 2\varepsilon^2\right)} < \frac{(1-\lambda)^2}{\widetilde{\lambda}_{LH}\left(\widetilde{\lambda}_{LH} - 2\varepsilon^2\right)}.$$

PROOF OF PROPOSITION 1:

(i) Since

$$\Delta\mathcal{W} = \frac{(\pi_H - \pi_L)^4[1 + (2\lambda - 1)^2]\varepsilon^2}{4}\left[\frac{\lambda^2}{\widetilde{\lambda}_{HL}\left(\widetilde{\lambda}_{HL} + 2\varepsilon^2\right)} - \frac{(1-\lambda)^2}{\widetilde{\lambda}_{LH}\left(\widetilde{\lambda}_{LH} - 2\varepsilon^2\right)}\right],$$

when $\lambda = 1/2$, we have

$$\Delta\mathcal{W} = -\frac{2(\pi_H - \pi_L)^4\varepsilon^4}{(1 + 4\varepsilon^2)(1 - 4\varepsilon^2)} < 0,$$

and when $\lambda = 1$, we obtain

$$\Delta\mathcal{W} = \frac{(\pi_H - \pi_L)^4 \varepsilon^2}{2(\pi_H^2 + \pi_L^2)(\pi_H^2 + \pi_L^2 + 2\varepsilon^2)} > 0.$$

Now, let $\lambda \in (1/2, 1)$. The partial derivative of $\Delta\mathcal{W}$ with respect to λ is

$$\frac{\partial \Delta\mathcal{W}}{\partial\lambda} = (\pi_H - \pi_L)^4 \varepsilon^2 \left\{ [1 + (2\lambda - 1)^2] \frac{\partial \Delta\Upsilon}{\partial\lambda} + 4\Upsilon(2\lambda - 1) \right\}$$

where

$$\Upsilon = \frac{\lambda^2}{(\widetilde{\lambda}_{HL} + 2\varepsilon^2)\widetilde{\lambda}_{HL}} - \frac{(1-\lambda)^2}{(\widetilde{\lambda}_{LH} - 2\varepsilon^2)\widetilde{\lambda}_{LH}}.$$

For each $\varepsilon > 0$ and $\pi_H \in (1/2, 1)$,

$$\begin{aligned} \frac{\partial\left[\frac{\lambda^2}{(\widetilde{\lambda}_{HL}+2\varepsilon^2)\widetilde{\lambda}_{HL}}\right]}{\partial\lambda} &= \frac{2\lambda\widetilde{\lambda}_{HL}(\pi_H\pi_L + \varepsilon^2)}{\left[(\widetilde{\lambda}_{HL} + 2\varepsilon^2)\widetilde{\lambda}_{HL}\right]^2} > 0, \\ \frac{\partial\left[\frac{(1-\lambda)^2}{(\widetilde{\lambda}_{LH}-2\varepsilon^2)\widetilde{\lambda}_{LH}}\right]}{\partial\lambda} &= \frac{-4(1-\lambda)(\widetilde{\lambda}_{LH} - 2\varepsilon^2)\pi_H\pi_L}{\left[(\widetilde{\lambda}_{LH} - 2\varepsilon^2)\widetilde{\lambda}_{LH}\right]^2} < 0. \end{aligned}$$

Hence $\partial\Upsilon/\partial\lambda > 0$. Therefore, whenever ΔW> 0 (so that $\Upsilon > 0$) it must be the case that $\partial\Delta\mathcal{W}/\partial\lambda > 0$. This result, combined with the fact that $\Delta\mathcal{W}$ evaluated at $\lambda = 1/2$ ($\lambda = 1$) is negative (positive), necessarily implies that, for each $\varepsilon > 0$ and $\pi_H \in (1/2, 1)$, there exists a unique $\lambda(\pi_H, \varepsilon)$ such that the utility gap is negative for $\lambda < \lambda(\pi_H, \varepsilon)$ and positive for $\lambda > \lambda(\pi_H, \varepsilon)$.

(ii) The utility gap is positive as long as

$$\frac{\lambda^2}{(\widetilde{\lambda}_{HL} + 2\varepsilon^2)\widetilde{\lambda}_{HL}} > \frac{(1-\lambda)^2}{(\widetilde{\lambda}_{LH} - 2\varepsilon^2)\widetilde{\lambda}_{LH}}.$$

We can rewrite this inequality as

$$\varepsilon^2 < \frac{1}{2}\left[\frac{\lambda^2\widetilde{\lambda}_{LH}^2 - (1-\lambda)^2\widetilde{\lambda}_{HL}^2}{\lambda^2\widetilde{\lambda}_{LH} + (1-\lambda)^2\widetilde{\lambda}_{HL}}\right] \equiv F(\lambda, \pi_H)$$

First note that $F(\frac{1}{2}, \pi_H) = 0$ and $F(1, \pi_H) = \pi_H\pi_L$. Moreover since F is continuous in λ and in π_H, the intermediate value theorem implies that there exists $\underline{\lambda}(\pi_H)$ such that $F(\underline{\lambda}(\pi_H), \pi_H) = \pi_L^2$. We can then show that, for all λ and π_H,

$$\frac{\partial^2 F(\lambda, \pi_H)}{\partial\lambda^2} < 0,$$

so that $F(.,\pi_H)$ is a strictly concave function. Precisely, if we let $\Delta \equiv \lambda^2\widetilde{\lambda}_{LH}+(1-\lambda)^2\widetilde{\lambda}_{HL}$, and $\Gamma \equiv \lambda^2\widetilde{\lambda}^2_{LH}-(1-\lambda)^2\widetilde{\lambda}^2_{HL}$,

$$\frac{\partial^2 F(\lambda,\pi_H)}{\partial\lambda^2} \equiv F_{\lambda\lambda} = \frac{1}{2}\left[\frac{\Delta^2\left(\Delta\Gamma_{\lambda\lambda}-\Gamma\Delta_{\lambda\lambda}\right)-(\Delta\Gamma_\lambda-\Gamma\Delta_\lambda)2\Delta\Delta_\lambda}{\Delta^4}\right].$$

$F_{\lambda\lambda}$ is smaller than zero if and only if

$$\Delta^2\left(\Delta\Gamma_{\lambda\lambda}-\Gamma\Delta_{\lambda\lambda}\right) < 2\Delta\Delta_\lambda(\Delta\Gamma_\lambda-\Gamma\Delta_\lambda).$$

Simple but lengthy algebra shows that

$$\begin{aligned}
\Delta &= \lambda(1-\lambda)+2(2\lambda-1)^2\pi_H\pi_L \\
\Delta_\lambda &= (2\lambda-1)(8\pi_H\pi_L-1) \\
\Delta_{\lambda\lambda} &= 2(8\pi_H\pi_L-1) \\
\Gamma &= 4(2\lambda-1)\pi_H\pi_L\left[\lambda(1-\lambda)+(2\lambda-1)^2\pi_H\pi_L\right] \\
\Gamma_\lambda &= 24\pi_H\pi_L\left[\lambda(1-\lambda)+(2\lambda-1)^2\pi_H\pi_L\right]-4\pi_H\pi_L \\
\Gamma_{\lambda\lambda} &= -24\pi_H\pi_L(2\lambda-1)(1-4\pi_H\pi_L),
\end{aligned}$$

After some tedious computation, we can rewrite the above inequality as

$$\begin{aligned}
&-\left[\lambda(1-\lambda)+2(2\lambda-1)^2\pi_H\pi_L\right]\left\{\begin{array}{c} 2\lambda(1-\lambda)+ \\ \left[5-24\lambda(1-\lambda)\right]\pi_H\pi_L-16(2\lambda-1)^2\pi_H^2\pi_L^2 \end{array}\right\} \\
< \\
&(8\pi_H\pi_L-1)\left\{\begin{array}{c} 2\lambda^2(1-\lambda)^2+\left[6\lambda(1-\lambda)-1\right](2\lambda-1)^2\pi_H\pi_L \\ +4(2\lambda-1)^4\pi_H^2\pi_L^2 \end{array}\right\}.
\end{aligned}$$

It is straightforward to show that this inequality holds as long as

$$\frac{1-\lambda(1-\lambda)}{1-4\lambda(1-\lambda)} > 2\pi_H\pi_L,$$

which is always true for any λ and π_H. Finally, since $F(1,\pi_H)=\pi_H\pi_L > F(\underline{\lambda}(\pi_H),\pi_H) = \pi_L^2$, we have that: (a) for each $\lambda < \underline{\lambda}(\pi_H)$, there exists a unique $\varepsilon(\lambda,\pi_H) \in (0,\pi_L)$ such that $\varepsilon^2 < F(\lambda,\pi_H)$ (hence $\Delta\mathcal{W} > 0$) if and only if $\varepsilon < \varepsilon(\lambda,\pi_H)$; (b) since $\varepsilon < \pi_L$, $\Delta\mathcal{W} > 0$ for all $\varepsilon > 0$ and $\lambda > \underline{\lambda}(\pi_H)$.

(iii) The utility gap is positive as long as

$$\frac{\lambda^2}{(\widetilde{\lambda}_{HL}+2\varepsilon^2)\widetilde{\lambda}_{HL}} > \frac{(1-\lambda)^2}{(\widetilde{\lambda}_{LH}-2\varepsilon^2)\widetilde{\lambda}_{LH}}.$$

Since

$$\begin{aligned}
\widetilde{\lambda}_{HL} &= \lambda-2(2\lambda-1)\pi_H\pi_L \\
\widetilde{\lambda}_{LH} &= 1-\lambda+2(2\lambda-1)\pi_H\pi_L,
\end{aligned}$$

after some computation, we can rewrite this inequality as

$$\gamma_1(\pi_H\pi_L)^2 + \gamma_2\pi_H\pi_L + \gamma_3 > 0,$$

where

$$\begin{aligned}
\gamma_1 &= (2\lambda-1)^2 \\
\gamma_2 &= \lambda(1-\lambda) - (2\lambda-1)\varepsilon^2 \\
\gamma_3 &= -\frac{\lambda(1-\lambda)\varepsilon^2}{2(2\lambda-1)}.
\end{aligned}$$

Moreover, the restriction $\varepsilon < \pi_L < \frac{1}{2}$ implies that the feasible range for $\pi_H\pi_L$ is given by the interval $[\varepsilon(1-\varepsilon), 1/4]$. Hence, in what follows we want to analyze the sign of the above inequality for different values of $\pi_H\pi_L$, given that $\pi_H\pi_L \in [\varepsilon(1-\varepsilon), 1/4]$. In order to do so, we compute its roots. We obtain

$$\begin{aligned}
(\pi_H\pi_L)_1 &\equiv \Pi_1(\lambda,\varepsilon) = \frac{(2\lambda-1)\varepsilon^2 - \lambda(1-\lambda) - \left\{\left[(2\lambda-1)\varepsilon^2\right]^2 + [\lambda(1-\lambda)]^2\right\}^{\frac{1}{2}}}{2(2\lambda-1)^2} \\
(\pi_H\pi_L)_2 &\equiv \Pi_2(\lambda,\varepsilon) = \frac{(2\lambda-1)\varepsilon^2 - \lambda(1-\lambda) + \left\{\left[(2\lambda-1)\varepsilon^2\right]^2 + [\lambda(1-\lambda)]^2\right\}^{\frac{1}{2}}}{2(2\lambda-1)^2}
\end{aligned}$$

Note that $\Pi_1(\lambda,\varepsilon) < 0$ since

$$\left[(2\lambda-1)\varepsilon^2 - \lambda(1-\lambda)\right]^2 < \left[(2\lambda-1)\varepsilon^2\right]^2 + [\lambda(1-\lambda)]^2.$$

Therefore, since $\gamma_1 > 0$ we only need to consider the behavior of $\Pi_2(\lambda,\varepsilon)$. Precisely,

$$\pi_H\pi_L > \Pi_2(\lambda,\varepsilon) \Leftrightarrow \gamma_1(\pi_H\pi_L)^2 + \gamma_2\pi_H\pi_L + \gamma_3 > 0.$$

First, note that

$$\Pi_2(1,\varepsilon) = \frac{\varepsilon^2+\varepsilon^2}{2} = \varepsilon^2 < \varepsilon(1-\varepsilon).$$

This implies that, when $\lambda = 1$, we have $\gamma_1(\pi_H\pi_L)^2 + \gamma_2\pi_H\pi_L + \gamma_3 > 0$ for all $\pi_H\pi_L \in [\varepsilon(1-\varepsilon), 1/4]$. Moreover,

$$\lim_{\lambda\to\frac{1}{2}} \Pi_2(\lambda,\varepsilon) = \infty,$$

which implies that, when $\lambda = 1/2$, $\gamma_1(\pi_H\pi_L)^2 + \gamma_2\pi_H\pi_L + \gamma_3 < 0$ for all $\pi_H\pi_L \in [\varepsilon(1-\varepsilon), 1/4]$. Moreover, the intermediate value theorem implies that there must exist $\underline{\lambda}(\varepsilon)$ and $\overline{\lambda}(\varepsilon)$ such that $\Pi_2(\underline{\lambda}(\varepsilon),\varepsilon) = \frac{1}{4}$ and $\Pi_2(\overline{\lambda}(\varepsilon),\varepsilon) = \varepsilon(1-\varepsilon)$. Finally, tedious but straightforward computation shows that

$$\frac{\partial^2\Pi_2(\lambda,\varepsilon)}{\partial\lambda^2} > 0.$$

As a result: (i) if $\lambda < \underline{\lambda}(\varepsilon)$, then $\Pi_2(\lambda,\varepsilon) > \frac{1}{4}$ and $\Delta\mathcal{W} < 0$ for all π_H such that $\pi_H\pi_L \in [\varepsilon(1-\varepsilon), 1/4]$; (ii) if $\lambda \in (\underline{\lambda}(\varepsilon), \underline{\lambda}(\varepsilon))$, there exists a unique $\pi_H(\lambda,\varepsilon)$ with $\pi_H(\lambda,\varepsilon)[1-\pi_H(\lambda,\varepsilon)] \in (\varepsilon(1-\varepsilon), 1/4)$ such that $\gamma_1(\pi_H\pi_L)^2 + \gamma_2\pi_H\pi_L + \gamma_3 < 0$ (hence $\Delta\mathcal{W} < 0$) if and only if $\pi_H > \pi_H(\lambda,\varepsilon)$; (iii) if $\lambda > \overline{\lambda}(\varepsilon)$, $\Pi_2(\lambda,\varepsilon) < \Pi_2(\overline{\lambda}(\varepsilon),\varepsilon) = \varepsilon(1-\varepsilon)$.

10 Supplement I. Computation of the Utility Gap

Conditional on the realization of the HH state, the utility gap in the morning is

$$W_{HH}^{T} - W_{HH}^{R} = \pi_H \left(\pi_{HH}^{T} - \pi_{HH}^{R}\right) - \frac{1}{2}\left(\pi_{HH}^{2T} - \pi_{HH}^{2R}\right),$$

while the utility gap in the afternoon is

$$W_{HH}^{\prime T} - W_{HH}^{\prime R} = \pi_H \left(\pi_{HH}^{\prime T} - \pi_{HH}^{\prime R}\right) - \frac{1}{2}\left(\pi_{HH}^{\prime 2T} - \pi_{HH}^{\prime 2R}\right),$$

where (for $E \in \{T, R\}$)

$$\begin{aligned}
\pi_{HH}^{E} &= \sum_{(m,a)\in\{f,s\}^2} p_{HH}^{E}(m,a)\pi^{E}(m,a), \\
\pi_{HH}^{2E} &= \sum_{(m,a)\in\{f,s\}^2} p_{HH}^{E}(m,a)\left[\pi^{E}(m,a)\right]^2, \\
\pi_{HH}^{\prime E} &= \sum_{(m,a)\in\{f,s\}^2} p_{HH}^{E}(m,a)\pi^{\prime E}(m,a), \\
\pi_{HH}^{\prime 2E} &= \sum_{(m,a)\in\{f,s\}^2} p_{HH}^{E}(m,a)\left[\pi^{\prime E}(m,a)\right]^2.
\end{aligned}$$

Define

$$\begin{aligned}
b_{HH}^{E} &= \sum_{(m,a)\in\{f,s\}^2} p_{HH}^{E}(m,a)b^{E}(m,a), \\
b_{HH}^{2E} &= \sum_{(m,a)\in\{f,s\}^2} p_{HH}^{E}(m,a)\left[b^{E}(m,a)\right]^2.
\end{aligned}$$

Since

$$\begin{aligned}
\pi^{\prime E}(m,a) &= b^{\prime E}(m,a)\pi_H + [1 - b^{\prime E}(m,a)]\pi_L, \\
b^{\prime E}(m,a) &= \lambda b^{E}(m,a) + (1-\lambda)[1 - b^{E}(m,a)],
\end{aligned}$$

after some algebraic manipulations, we can write

$$\begin{aligned}
\pi_{HH}^{\prime T} - \pi_{HH}^{\prime R} &= (2\lambda - 1)(\pi_{HH}^{T} - \pi_{HH}^{R}) = (2\lambda - 1)(\pi_H - \pi_L)(b_{HH}^{T} - b_{HH}^{R}), \\
\pi_{HH}^{2T} - \pi_{HH}^{2R} &= 2\pi_L(\pi_H - \pi_L)(b_{HH}^{T} - b_{HH}^{R}) + (\pi_H - \pi_L)^2(b_{HH}^{2T} - b_{HH}^{2R}), \\
\pi_{HH}^{\prime 2T} - \pi_{HH}^{\prime 2R} &= 2\lambda_{LH}(\pi_H - \pi_L)(2\lambda - 1)(b_{HH}^{T} - b_{HH}^{R}) + (\pi_H - \pi_L)^2(2\lambda - 1)^2(b_{HH}^{2T} - b_{HH}^{2R}).
\end{aligned}$$

This allows us to express the utility gap conditional on the HH state being realized as

$$(\pi_H - \pi_L)^2[1 + (2\lambda - 1)^2]\left[b_{HH}^{T} - b_{HH}^{R} - \frac{1}{2}(b_{HH}^{2T} - b_{HH}^{2R})\right].$$

A similar reasoning implies that the utility gap conditional on the LH state being realized equals

$$(\pi_H - \pi_L)^2[1 + (2\lambda - 1)^2]\left[b^T_{LH} - b^R_{LH} - \frac{1}{2}(b^{2T}_{LH} - b^{2R}_{LH})\right].$$

Now consider the LL state. The utility gap equals

$$\pi_L\left[\pi^T_{LL} - \pi^R_{LL}\right] - \frac{1}{2}[\pi^{2T}_{LL} - \pi^{2R}_{LL}] + \pi_L\left[\pi'^T_{LL} - \pi'^R_{LL}\right] - \frac{1}{2}[\pi'^{2T}_{LL} - \pi'^{2R}_{LL}].$$

Because

$$\begin{aligned}
\pi'^T_{LL} - \pi'^R_{LL} &= (2\lambda - 1)(\pi^T_{LL} - \pi^R_{LL}) = (2\lambda - 1)(\pi_H - \pi_L)(b^T_{LL} - b^R_{LL}),\\
\pi^{2T}_{LL} - \pi^{2R}_{LL} &= 2\pi_L(\pi_H - \pi_L)(b^T_{LL} - b^R_{LL}) + (\pi_H - \pi_L)^2(b^{2T}_{LL} - b^{2R}_{LL}),\\
\pi'^{2T}_{LL} - \pi'^{2R}_{LL} &= 2\lambda_{LH}(\pi_H - \pi_L)(2\lambda - 1)(b^T_{LL} - b^R_{LL}) + (\pi_H - \pi_L)^2(2\lambda - 1)^2(b^{2T}_{LL} - b^{2R}_{LL}),
\end{aligned}$$

we can express the utility gap conditional on the LL state being realized as

$$-\frac{1}{2}(\pi_H - \pi_L)^2[1 + (2\lambda - 1)^2](b^{2T}_{LL} - b^{2R}_{LL}).$$

Similarly, the utility gap conditional on the HL state being realized equals

$$-\frac{1}{2}(\pi_H - \pi_L)^2[1 + (2\lambda - 1)^2](b^{2T}_{HL} - b^{2R}_{HL}).$$

The expected utility gap is a weighted sum of the utility gaps in the different states, with weights given by the probabilities of the states. Precisely,

$$\Delta\mathcal{W} = \frac{1}{2}\left[\begin{array}{c} \lambda(W^T_{HH} - W^R_{HH} + W'^T_{HH} - W'^R_{HH} + W^T_{LL} - W^R_{LL} + W'^T_{LL} - W'^R_{LL}) + \\ (1 - \lambda)(W^T_{LH} - W^R_{LH} + W'^T_{LH} - W'^R_{LH} + W^T_{HL} - W^R_{HL} + W'^T_{HL} - W'^R_{HL}) \end{array}\right].$$

Substituting for the expressions above, we obtain

$$\begin{aligned}
\Delta\mathcal{W} \;=\; & \frac{(\pi_H - \pi_L)^2[1 + (2\lambda - 1)^2]}{2}\\
& \left[\begin{array}{c} \lambda(b^T_{HH} - b^R_{HH}) + (1 - \lambda)(b^T_{LH} - b^R_{LH}) \\ -\frac{1}{2}\lambda(b^{2T}_{HH} - b^{2R}_{HH} + b^{2T}_{LL} - b^{2R}_{LL}) - \frac{1}{2}(1 - \lambda)\left(b^{2T}_{LH} - b^{2R}_{LH} + b^{2T}_{HL} - b^{2R}_{HL}\right) \end{array}\right].
\end{aligned}$$

Consider first the linear component (which we will denote by Φ_1), i.e.,

$$\Phi_1 = \lambda(b^T_{HH} - b^R_{HH}) + (1 - \lambda)(b^T_{LH} - b^R_{LH})].$$

We can write the right hand side as

$$\begin{aligned}
&\sum_{(m,a)\in\{f,s\}^2}\left[\lambda p^T_{HH}(m,a) + (1 - \lambda)p^T_{LH}(m,a)\right]b^T(m,a) - \\
&\sum_{(m,a)\in\{f,s\}^2}\left[\lambda p^R_{HH}(m,a) + (1 - \lambda)p^R_{LH}(m,a)\right]b^R(m,a).
\end{aligned}$$

Since

$$p_{HH}^R(m,a) = \begin{cases} p_{HH}^T(m,a) + \varepsilon^2 \text{ if } m = a \\ p_{HH}^T(m,a) - \varepsilon^2 \text{ if } m \neq a \end{cases},$$

we can rewrite Φ_1 as

$$\begin{aligned} \Phi_1 &= \sum_{(m,a)\in\{f,s\}^2} \left[\lambda p_{HH}^T(m,a) + (1-\lambda) p_{LH}^T(m,a)\right] \left[b^T(m,a) - b^R(m,a)\right] - \\ &\quad \varepsilon^2 [b^R(s,s) + b^R(f,f) - b^R(f,s) - b^R(s,f)]. \end{aligned}$$

Moreover, because $b^T(s,s) - b^R(s,s) = b^R(f,f) - b^T(f,f)$ and $b^T(f,s) - b^R(f,s) = b^R(s,f) - b^T(s,f)$, after substituting for the values of p_{HH}^T and p_{LH}^T, we obtain

$$\Phi_1 = (\pi_H - \pi_L)\left\{\lambda[b^T(s,s) - b^R(s,s)] + (1-\lambda)[b^T(f,s) - b^R(f,s)]\right\}.$$

Now consider the quadratic component (which we will denote by Φ_2), i.e.,

$$\Phi_2 = -\frac{1}{2}\lambda\left(b_{HH}^{2T} - b_{HH}^{2R} + b_{LL}^{2T} - b_{LL}^{2R}\right) - \frac{1}{2}(1-\lambda)\left(b_{LH}^{2T} - b_{LH}^{2R} + b_{HL}^{2T} - b_{HL}^{2R}\right).$$

We can write

$$\begin{aligned} b_{HH}^{2T} - b_{HH}^{2R} + b_{LL}^{2T} - b_{LL}^{2R} &= \sum_{(m,a)\in\{f,s\}^2} \left[p_{HH}^T(m,a) + p_{LL}^T(m,a)\right] \left[b^T(m,a)\right]^2 - \\ &\quad \sum_{(m,a)\in\{f,s\}^2} \left[p_{HH}^R(m,a) + p_{LL}^R(m,a)\right] \left[b^R(m,a)\right]^2. \end{aligned}$$

After some manipulations, the right hand side can be rewritten as

$$\begin{aligned} &\sum_{(m,a)\in\{f,s\}^2} \left[p_{HH}^T(m,a) + p_{LL}^T(m,a)\right] \left\{\left[b^T(m,a)\right]^2 - \left[b^R(m,a)\right]^2\right\} - \\ &2\varepsilon^2 \left\{\left[b^R(s,s)\right]^2 + \left[b^R(f,f)\right]^2 - \left[b^R(f,s)\right]^2 - \left[b^R(f,f)\right]^2\right\}. \end{aligned}$$

Analogously, $\left(b_{LH}^{2T} - b_{LH}^{2R} + b_{HL}^{2T} - b_{HL}^{2R}\right)$ can be written as

$$\begin{aligned} &\sum_{(m,a)\in\{f,s\}^2} \left[p_{LH}^T(m,a) + p_{HL}^T(m,a)\right] \left\{\left[b^T(m,a)\right]^2 - \left[b^R(m,a)\right]^2\right\} - \\ &2\varepsilon^2 \left\{\left[b^R(s,s)\right]^2 + \left[b^R(f,f)\right]^2 - \left[b^R(f,s)\right]^2 - \left[b^R(f,f)\right]^2\right\}. \end{aligned}$$

After substituting for the values of p_{HH}^T, p_{LL}^T, p_{LH}^T, and p_{HL}^T, we can then express Φ_2 as

$$\begin{aligned} \Phi_2 &= \left[-\frac{\lambda}{2}(\pi_H^2 + \pi_L^2) - \frac{1-\lambda}{2} 2\pi_L\pi_H\right] \{[b^T(s,s)]^2 - [b^R(s,s)]^2 + [b^T(f,f)]^2 - [b^R(f,f)]^2\} + \\ &\quad \left[-\frac{\lambda}{2} 2(\pi_L\pi_H) - \frac{1-\lambda}{2}(\pi_H^2 + \pi_L^2)\right] \{[b^T(f,s)]^2 - [b^R(f,s)]^2 + [b^T(s,f)]^2 - [b^R(s,f)]^2\} + \\ &\quad \varepsilon^2\{[b^R(s,s)]^2 + [b^R(f,f)]^2 - [b^R(f,s)]^2 - [b^R(s,f)]^2\}. \end{aligned}$$

Let

$$\begin{aligned}
\widetilde{\lambda}_{HL} &= \pi_H[\lambda\pi_H + (1-\lambda)\pi_L] + \pi_L[\lambda\pi_L + (1-\lambda)\pi_H], \\
\widetilde{\lambda}_{LH} &= \pi_H[\lambda\pi_L + (1-\lambda)\pi_H] + \pi_L[\lambda\pi_H + (1-\lambda)\pi_L].
\end{aligned}$$

Then,

$$\begin{aligned}
\Phi_2 &= -\frac{\widetilde{\lambda}_{HL}}{2}\{[b^T(s,s)]^2 - [b^R(s,s)]^2 + [b^T(f,f)]^2 - [b^R(f,f)]^2\} \\
&\quad -\frac{\widetilde{\lambda}_{LH}}{2}\left\{[b^T(f,s)]^2 - [b^R(f,s)]^2 + [b^T(s,f)]^2 - [b^R(s,f)]^2\right\} \\
&\quad +\varepsilon^2\{[b^R(s,s)]^2 + [b^R(f,f)]^2 - [b^R(f,s)]^2 - [b^R(s,f)]^2\}.
\end{aligned}$$

Now, since

$$\left[b^T(m,a)]^2 - [b^R(m,a)\right]^2 = \left[b^T(m,a) - b^R(m,a)\right]\left[b^T(m,a) + b^R(m,a)\right],$$

we have

$$\begin{aligned}
\Phi_2 &= -\frac{\widetilde{\lambda}_{HL}}{2}[b^T(s,s) - b^R(s,s)][b^T(s,s) + b^R(s,s) - b^T(f,f) - b^R(f,f)] \\
&\quad -\frac{\widetilde{\lambda}_{LH}}{2}[b^T(f,s) - b^R(f,s)][b^T(f,s) + b^R(f,s) - b^T(s,f) - b^R(s,f)] \\
&\quad +\varepsilon^2\{[b^R(s,s)]^2 + [b^R(f,f)]^2 - [b^R(f,s)]^2 - [b^R(s,f)]^2\}.
\end{aligned}$$

Furthermore,

$$\begin{aligned}
b^T(s,s) + b^R(s,s) - b^T(f,f) - b^R(f,f) &= 2(\pi_H - \pi_L)\frac{\lambda(\widetilde{\lambda}_{HL} + \varepsilon^2)}{\widetilde{\lambda}_{HL}\left(\widetilde{\lambda}_{HL} + 2\varepsilon^2\right)} \\
b^T(f,s) + b^R(f,s) - b^T(s,f) - b^R(s,f) &= 2(\pi_H - \pi_L)\frac{(1-\lambda)(\widetilde{\lambda}_{LH} - \varepsilon^2)}{\widetilde{\lambda}_{LH}\left(\widetilde{\lambda}_{LH} - 2\varepsilon^2\right)},
\end{aligned}$$

and

$$\begin{aligned}
&[b^R(s,s)]^2 + [b^R(f,f)]^2 - [b^R(f,s)]^2 - [b^R(s,f)] \\
= \quad &\frac{\{\pi_H[\lambda\pi_H + (1-\lambda)\pi_L] + \varepsilon^2\}^2 + \{\pi_L[\lambda\pi_L + (1-\lambda)\pi_H] + \varepsilon^2\}^2}{\left(\widetilde{\lambda}_{HL} + 2\varepsilon^2\right)^2} - \\
&\frac{\{\pi_H[\lambda\pi_L + (1-\lambda)\pi_H] - \varepsilon^2\}^2 + \{\pi_L[\lambda\pi_H + (1-\lambda)\pi_L] - \varepsilon^2\}^2}{\left(\widetilde{\lambda}_{LH} - 2\varepsilon^2\right)^2}.
\end{aligned}$$

Finally, since

$$
\begin{aligned}
b^T(s,s) - b^R(s,s) &= \frac{\lambda\varepsilon^2(\pi_H - \pi_L)}{\widetilde{\lambda}_{HL}\left(\widetilde{\lambda}_{HL} + 2\varepsilon^2\right)}, \\
b^T(f,s) - b^R(f,s) &= \frac{-(1-\lambda)\varepsilon^2(\pi_H - \pi_L)}{\widetilde{\lambda}_{LH}\left(\widetilde{\lambda}_{LH} - 2\varepsilon^2\right)},
\end{aligned}
$$

we can write the utility gap as

$$
\begin{aligned}
\Delta\mathcal{W} &= \frac{(\pi_H - \pi_L)^2[1 + (2\lambda - 1)^2]\varepsilon^2}{2} \\
&\left\{ \begin{array}{c} \frac{\lambda^2(\pi_H-\pi_L)^2\varepsilon^2 + \widetilde{\lambda}_{HL}\{\pi_H[\lambda\pi_H + (1-\lambda)\pi_L] + \varepsilon^2\}^2 + \widetilde{\lambda}_{HL}\{\pi_L[\lambda\pi_L + (1-\lambda)\pi_H] + \varepsilon^2\}^2}{\widetilde{\lambda}_{HL}\left(\widetilde{\lambda}_{HL} + 2\varepsilon^2\right)^2} - \\ \frac{-(1-\lambda)^2(\pi_H-\pi_L)^2\varepsilon^2 + \widetilde{\lambda}_{LH}\{\pi_H[\lambda\pi_L + (1-\lambda)\pi_H] - \varepsilon^2\}^2 + \widetilde{\lambda}_{LH}\{\pi_L[\lambda\pi_H + (1-\lambda)\pi_L] - \varepsilon^2\}^2}{\widetilde{\lambda}_{LH}\left(\widetilde{\lambda}_{LH} - 2\varepsilon^2\right)^2} \end{array} \right\}.
\end{aligned}
$$

We now claim that

$$
\Delta\mathcal{W} = \frac{(\pi_H - \pi_L)^2[1 + (2\lambda - 1)^2]\varepsilon^2}{4} \left[\frac{\lambda^2(\pi_H - \pi_L)^2}{\widetilde{\lambda}_{HL}\left(\widetilde{\lambda}_{HL} + 2\varepsilon^2\right)} - \frac{(1-\lambda)^2(\pi_H - \pi_L)^2}{\widetilde{\lambda}_{LH}\left(\widetilde{\lambda}_{LH} - 2\varepsilon^2\right)} \right].
$$

Before proving this claim, to simplify matters, define

$$
\begin{aligned}
\pi_H[\lambda\pi_H + (1-\lambda)\pi_L] + \varepsilon^2 &\equiv \Pi_{HH}, \\
\pi_L[\lambda\pi_L + (1-\lambda)\pi_H] + \varepsilon^2 &\equiv \Pi_{LL}, \\
\pi_H[\lambda\pi_L + (1-\lambda)\pi_H] - \varepsilon^2 &\equiv \Pi_{HL}, \\
\pi_L[\lambda\pi_H + (1-\lambda)\pi_L] - \varepsilon^2 &\equiv \Pi_{LH}.
\end{aligned}
$$

Hence, we can rewrite the above equality as

$$
\frac{-(\Pi_{HH} + \Pi_{LL})\varepsilon^2 + \Pi_{HH}^2 + \Pi_{LL}^2 - \frac{(\Pi_{HH} - \Pi_{LL})^2}{2}}{(\Pi_{HH} + \Pi_{LL} - 2\varepsilon^2)(\Pi_{HH} + \Pi_{LL})} = \frac{(\Pi_{HL} + \Pi_{LH})\varepsilon^2 + \Pi_{HL}^2 + \Pi_{LH}^2 - \frac{(\Pi_{HL} - \Pi_{LH})^2}{2}}{(\Pi_{HL} + \Pi_{LH} + 2\varepsilon^2)(\Pi_{HL} + \Pi_{LH})}.
$$

After some computations we can show that this equality always holds, since it implies

$$
\frac{\Pi_{HH} + \Pi_{LL} - 2\varepsilon^2}{\Pi_{HH} + \Pi_{LL} - 2\varepsilon^2} = \frac{\Pi_{HL} + \Pi_{LH} + 2\varepsilon^2}{\Pi_{HL} + \Pi_{LH} + 2\varepsilon^2} = 1.
$$

This proves the claim.