Moneychangers and Commodity Money*

Vincent Bignon†
Economics Department
University of Paris 10

Richard Dutu‡
Economics Department
University of Waikato

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Abstract

We study the role played by coin experts–moneychangers–in the metallic money system and their influence on the informational version of Gresham's law. We build a model with light and heavy silver coins, bilateral bargaining, imperfect recognizability of coins and add the possibility for agents to have their coins certified by an expert before they go shopping. We show that moneychangers reduce the extent of Gresham's law (*circulation by weight*) to situations in which information on coins is good and that Gresham's law and certified heavy coins cannot coexist as equilibria. Welfare is higher with certified heavy coins than in Gresham's law. Moneychangers also restrict *circulation by tale* to regions with good information on coins, yet circulation by tale survives for high discount rates whatever the level of information on coins. Both equilibria can coexist yet agents are better off when moneychangers are inactive.

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†Department of Economics, University of Paris 10 and Sciences Po. 200 avenue de la République, 92000 Nanterre, France. Tel: +33 1 40 97 59 13. Email: vbignon@u-paris10.fr

‡Department of Economics, University of Waikato, Knighton Road, Hamilton 3240, New Zealand. Tel: +64 7 858 5164. Email: rdutu@waikato.ac.nz

1 Introduction

In today's economies, money is fiat which means that its value does not come from its intrinsic content. When money was a commodity, coins took their value partly from the value of the commodity they were made of. In the metallic money system for instance, the purchasing power of coins was in accordance with the value of its intrinsic content, through the type of metal used for the coin (gold, silver, etc.), its quantity and fineness. These attributes were therefore subject to great attention by agents. The problem was that the quality of these precious metal coins, even two supposedly identical coins, was not uniform and was difficult to assess. Sources of uncertainty on the intrinsic of coins were indeed numerous in the metallic money system. The rudimentary minting technology first, basically hammer and pile up to the 16th century (Sargent and Velde 2002). The invention of the cylinder press greatly reduced the standard error between coins but did not eliminate it. A second source of uncertainty was the well-documented temptation by kings and monarchs to manipulate (mainly downwards) the intrinsic content of coins as a way to increase their seigniorage (i.e. the difference between the price at which coins were sold and the price of the metal it contains). Even though most of these so called debasements were public information (Rolnick Velde and Weber 1996), frequent changes undoubtedly increased uncertainty. Other factors of uncertainty include wear, clipping and counterfeiting. The consequences of clipping, for instance, before the 1696 Great Recoinage in Britain have been well documented by British historian Thomas Macaulay, who reported that, because of the bad state of the coins, "*nothing could be purchased without a dispute. Over every counter there was wrangling from morning to night*" (Macaulay, 1855, p. 187).

Velde, Weber and Wright (1999), hereafter VWW, have argued that this uncertainty created an asymmetric information problem between buyers, who supposedly had more information on their own coins, and sellers who could not necessarily assess the coins offered in payment. The authors show that because of this asymmetry of information the circulation of money was impeded in two ways: either heavy coins do not circulate, or they circulate below their full information value. The authors refer to the former situation as *circulation by tale* since light and heavy coins trade at the same price when not recognized, and to the latter as *circulation by*

weight since light and heavy coins always trade at different prices and in accordance with their weight. This second case matches the informational version of Gresham's law: when coins are difficult to recognize, it is the circulation of light coins that triggers the hoarding of heavy coins ("bad money drives out good money"). Although the economics of the metallic money system displayed in VWW is very appealing, we argue in this paper that agents found a solution to this asymmetric information problem, coin experts–mainly known as moneychangers–specialized in selling information on coins to the public. The goal of this paper is to study their impact on the circulation of money and on welfare.

Coin experts, or moneychangers, were a key institution of the metallic money system. They were common in Europe by the time of the late-medieval commercial revolution. In France for instance, their commerce was mentioned as early as 1141 (Landry 1910) and they expanded all over the country from the mid-13th to the mid-15th centuries: there were 20 moneychangers in Bruges in 1308, 23 in Tours in 1344, 39 in Chartres by the end of the 13th century and 84 in Toulouse in 1337 (Chevalier 1973). There is also ample evidence that these intermediaries played an active role in trade in Ancient Greece and in the Roman Empire (Lothian, 2003), in Bizance (Kaplanis 2003) and the Islamic world (Udovitch, 1975). A first part of the moneychanger's job consisted in exchanging large denomination coins for small ones, and cried down coins for authorized ones. The other part, the one that matters here, consisted in assaying the coins by checking their weight, fineness and origin (Bompaire 2001). The verification process required in this operation involved a series of costly operations. Weight was determined using precise scales, and fineness was estimated using a set of touchstones. The touchstone test consisted in rubbing a coin on a special stone and comparing the color of the trace left with that of needles of known fineness (Gandall and Sussman, 1997). A more precise assay, called assay by fire, consisted in melting down a sample of coins to weight the quantity of pure metal. This type of assay was limited to payments involving many coins for obvious reasons.[1]

As can be well imagined, there was considerable heterogeneity in the legal statutes of money-

[1] Many moneychangers also took deposits and historians usually consider moneychangers to be the first bankers. Description of the fairs in the early thirteenth century show that many transactions were cleared by book transfers at moneychangers (Usher 1934). Also the term by which they were originally designated, *cambitor*, was often a synonym for banker, *bancherius* (Hall 1935, p. 73).

changers across Europe. In 14th century Bruges, for instance, moneychanging was the monopoly of a small group of people. Four of the offices were hereditary fiefs whereas the other moneychangers were licensed. In the neighboring city of Brussels, by contrast, the only restriction for moneychangers was a payment of a fee to the duke and to the city (De Roover 1948). In 14th century Tuscan moneychanging was performed by independent itinerant merchants (La Roncières, 1974), as for 14th century Langdoc (Bompaire 1987). But despite these differences, they all shared an expertise in coins, which is what we will focus on in this article.

The goal of this paper is to evaluate the influence of these intermediaries on the circulation of coins in the metallic money system and their impact on welfare. Precisely, we ask the following questions: 1) Do the equilibria displayed in VWW survive once moneychangers are present in the economy? 2) Under which conditions do agents have their coin certified by a moneychanger? 3) Do moneychangers increase welfare? To answer these, we embed moneychangers with an ability to expertise and certify coins into Velde Weber and Wright's (1999) model. Our method consists in refining the equilibria displayed in VWW by taking into account the possibility for agents to deviate from their equilibrium strategy and visit a moneychanger. Then we do the mirror exercise, that is characterize an economy in which heavy coins are always certified by moneychangers, and ask under which condition agents deviate and play either the by weight or by tale strategy.

First, we show that moneychangers limit Gresham's law to regions where information on coins is good. When information is good, agents are better off waiting for someone to recognize their coin rather than pay for a certificate. But when information is poor, expected gains from trade are small due to the low probability of having one's heavy coin recognized and agents are better off paying for a certificate so that Gresham's law vanishes. Things are different for circulation by tale. It survives moneychangers when information is good (similar to circulation by weight), but also for high values of the discount rate. In that case, and regardless of the level of information, gains from trade are not big enough to cover the cost of expertise so that agents prefer not visit the coin expert.[2]

[2] Circulation by weight is not an equilibrium when the discount rate is high since heavy coin holders always prefer selling it off rather than hoarding it.

Interestingly, Gresham's law and the certification of good coins cannot coexist as equilibria, yet circulation by tale and the certification of good coins do coexist as equilibria. The reason why they coexist goes like this: when considering deviating from certification to playing by tale, buyers realize that their heavy coin is going to be treated as a light coin since all other unrecognized yet circulating coins are necessarily light coins in an equilibrium with certification. But when considering not deviating from by tale to certification, they know the unrecognized heavy coin is treated as an unknown coin in circulation by tale, and an unknown coin is priced more in a by tale equilibrium than an unknown–and therefore light–coin when deviating from certification. To summarize, two symmetric deviations that should yield the same payoff in absolute value in the end yield different payoffs. This means that *different* parameters will leave agents indifferent between deviating from by tale to certification or not deviating from certification to by tale: the frontiers between these two equilibria and their corresponding deviating threats are different so that the two equilibria overlap. Things are different for circulation by weight: whether deviating from certification to playing by weight or not deviating from by weight to certification, gains from trade on unrecognized heavy coins are one and the same and equal to zero since unrecognized heavy coins do not circulate. In the end the two symmetric deviations yield the same payoffs so that the frontiers between these two equilibria and their corresponding deviating threats merge.

In terms of welfare, it is higher with certified heavy coins than in Gresham's law. This is no surprise: whether in a by weight equilibrium or in a certified heavy coins equilibrium, light coins circulate at their full information value since unrecognized coins can only be light coins. Then light coins holders are not affected by the decision whether to certify heavy coins. So if buyers holding heavy coins decide to have them certified, they must be better off that way leaving buyers with light coin holders indifferent. So not only moneychangers are an efficient way to get rid of Gresham's law, but they also are strictly welfare improving in that case. However, welfare is *lower* with active moneychangers than with circulation by tale. The reason comes from risk aversion: in uninformed meetings a buyer gets an average quantity regardless of his coin. Since agents prefer the expected value of this average to the average of the expected values of trading with fully recognizable light and heavy coins, moving from circulation by tale to

certified coins is clearly welfare worsening. This has an interesting implication: if the economy settles on an active moneychanger equilibrium that overlaps with a by-tale equilibrium, agents would collectively be better off without moneychangers but have no incentive as individuals to deviate from that equilibrium. We realize this sounds a little bit like a late recommendation but, assuming coins were circulating by tale, it would have been a good idea to ban moneychangers from the economy.

Before we start, note that there is exist a literature on intermediaries with search frictions. Li (2002) especially studies the endogenous emergence of intermediaries selling information on the quality of goods in an environment with trading frictions, and their impact on the incentive to produce either low or high quality goods. Our paper differs is two dimensions: first Li's paper has no money, and second we focus on verifying coins' quality rather than goods quality. To our knowledge this makes our paper the first attempt to model the role played by this key player of the two-milenary long metallic money system: the moneychanger.

The paper is organized as follows. The next section presents the environment. Section 3 studies how the existence of moneychangers impacts on the equilibria displayed in VWW. Section 4 shows that an equilibrium with certified heavy coins exists for sufficiently low cost of resorting to moneychangers. Section 5 gives our welfare results and section 6 concludes.

2 The Environment

The environment is VWW to which we add coin assayers. There is a $[0, 1]$ continuum of infinitely lived agents indexed by k and there are $I \geqslant 3$ types of goods. A type $k \in I$ agent consumes good k and produces good $k+1$. Consuming q units of his consumption good yields $u(q) = q^{\eta}$ with $u(0) = 0$, $u'(q) > 0$ and $u''(q) < 0$. Producing q units of his production good costs $c(q)$ which is assumed linear for simplicity so that $c(q) = q$. Further there is a unique $\hat{q}$ such that $u(\hat{q}) = \hat{q}$. Agents discounts the future at rate $r > 0$.

Money is in the form of gold (or silver) coins coming in light (L) and heavy (H) weight. Each agent can hold at most one coin.[3] We let M_i be the measure of agents endowed with

[3] Making money divisible would make comparison with VWW less straigtforward without adding much to our story. For a model of divisible commodity money and imperfect recognizability of coins see Dutu Nosal and

coins of type $i = \{L, H\}$ and $M = M_H + M_L$ represents the fraction of buyers so that $1 - M$ represents the fraction of sellers, also called producers. Each coin yields to its owner a flow of utility γ_i per period proportional to its weight (or intrinsic content) so that $\gamma_H > \gamma_L$. This utility flow could be interpreted either as the utility one gets from possession of the metal *per se* or as a shortcoming of a more complicated story that goes as follows: with some exogenous probability a buyer holding a coin i meets some foreign trader which trades money on the basis of its intrinsic content solely. γ_i then reflects the expected return of meeting such foreign trader per period. Finally it is assumed that there is no legal exchange rate imposed by the monarch between the light and the heavy coin, or if there is one the monarch has no power to enforce it.

Agents meet bilaterally according to an anonymous random matching process with Poisson arrival rate α. Thus $\frac{\alpha}{I}(1 - M)$ is the probability per unit of time of a simple coincidence of wants, e.g. that a buyer meets a seller who produces his consumption good. In any such meeting, the buyer may or may not offer to trade his unit of money for some output. It is assumed that terms of trade are formed via bargaining in which the buyer has all the bargaining power. When he chooses to make an offer, his offer makes the seller indifferent between accepting and refusing it. If this offer is accepted by the seller, agents swap their inventories so that the buyer becomes a seller and the seller becomes a buyer with the seller's coin.

In terms of information, the buyer always knows the true quality of his coin while the seller, when presented with a coin, learns its true quality via a common knowledge signal that is informative with probability θ and uninformative with probability $1 - \theta$. That is the informational structure is the same as in Williamson and Wright (1994) but applies to coins rather than goods. In a single coincidence of wants meeting, when the signal is informative and if the two parties agree to trade, a type $i \in \{L, H\}$ coin is exchanged against q_i units of the seller's production good (which is also the buyer's consumption good). When the signal is uninformative and the parties agree to trade, the unrecognized coin is traded against a quantity noted $\bar{q}$. This quantity is a weighted average of q_L and q_H with weights coming from the probabilities to receive either a light or a heavy coin (more is said on this below). Barter is simply assumed away.

Rocheteau (2007).

To circumvent the above information problem, a buyer can visit a moneychanger who charges him δ for expertising his coin and delivering a certificate of quality. We assume that moneychangers accept all goods in payment and value them identically so that each buyer can resort to a moneychanger regardless of his production type. The certificate delivered by the moneychanger is valid only for one trading period so if a seller is to accept a certified coin, he has to pay δ to the assayer to renew the certificate and use it next period. Moneychangers act as a technology in that they always tell the truth on coins, which makes it impossible for light coin holders to bribe moneychangers.

3 Inactive Moneychangers

In this section we conduct the following exercise. Given the monetary equilibria displayed in VWW (single currency, circulation by weight and circulation by tale), we characterize for each of these three equilibria the conditions under which agents have no incentive to deviate from their equilibrium strategy by visiting a moneychanger. We start by presenting agents payoff functions and defining what an equilibrium is in a world without moneychanger.

As in VWW, we note λ_{ij} the probability (endogenously determined) that a buyer with a coin of type $i \in \{L, H\}$ wants to trade with a seller of type $j \in \{K, U\}$ where K means that the weight of the coin is known to the seller (the signal is informative), and U means that the weight of the coin is unknown.

Noting $\beta = \frac{\alpha}{I}(1-M)$, the Bellman equation for a buyer with a light coin it is

$$V_L = \frac{1}{1+r}\left\{\begin{array}{l} \gamma_L + \beta\theta \max\limits_{\lambda_{LK}} \left[\lambda_{LK}\left[u(q_L)+V_0\right] + (1-\lambda_{LK})V_L\right] \\ +\beta(1-\theta)\max\limits_{\lambda_{LU}} \left[\lambda_{LU}\left[u(\bar{q})+V_0\right] + (1-\lambda_{LU})V_L\right] \end{array}\right\}.$$

Multiplying by $(1+r)$ and rearranging yields the flow version of Bellman equation

$$rV_L = \gamma_L + \beta\theta \max_{\lambda_{LK}} \lambda_{LK}\left[u(q_L)+V_0-V_L\right] + \beta(1-\theta)\max_{\lambda_{LU}} \lambda_{LU}\left[u(\bar{q})+V_0-V_L\right]. \tag{1}$$

Eq. (4) sets the flow return to a buyer holding a light coin, rV_L, equal to three parts. The first part gives the periodic return on holding the light coin, γ_L. The second part corresponds to the probability that he meets a producer and there is a single coincidence of wants, β, multiplied

by the probability that the seller recognizes the light coin, θ, times the net gain from trading the light coin against q_L, which is equal to consuming q_L and switching from buyer with a light coin to producer, that is $u(q_L) + V_0 - V_L$, times the probability that he decides to trade with him, λ_{LK}. The last part has similar interpretation with the difference that because the coin is not recognized it is not traded for q_L but for an average quantity $\bar{q}$ defined below.

Similarly, the flow Bellman equation for a buyer holding a heavy coin is given by

$$rV_H = \gamma_H + \beta\theta \max_{\lambda_{HK}} \lambda_{HK} \left[u(q_H) + V_0 - V_H\right] + \beta\left(1-\theta\right) \max_{\lambda_{HU}} \lambda_{HU} \left[u(\bar{q}) + V_0 - V_H\right] \tag{2}$$

From the take-it-or-leave-it bargaining protocol, the informed seller is indifferent between not producing or producing q_i for the buyer and becoming a buyer with a coin of type i. Therefore the offer made by buyers satisfy

$$V_0 = -q_i + V_i \text{ for } i \in \{L, H\}. \tag{3}$$

Similarly the uninformed seller is indifferent between not producing and producing and trading $\bar{q}$ against the unknown coin so that

$$V_0 = -\bar{q} + \pi V_H + (1-\pi) V_L$$

where π is the proportion of heavy coins in circulation given by

$$\pi = \frac{\lambda_{HU} M_H}{\lambda_{HU} M_H + \lambda_{LU} M_L}.$$

Because sellers never get any utility from trade we have $V_0 = 0$ so that $V_H = q_H$ and $V_L = q_L$. Once we insert these values into (1) and (2) we obtain a simple 2-equation model

$$rq_L = \gamma_L + \beta\theta \max_{\lambda_{LK}} \lambda_{LK} \left[u(q_L) - q_L\right] + \beta\left(1-\theta\right) \max_{\lambda_{LU}} \lambda_{LU} \left[u(\bar{q}) - q_L\right] \tag{4}$$

$$rq_H = \gamma_H + \beta\theta \max_{\lambda_{HK}} \lambda_{HK} \left[u(q_H) - q_H\right] + \beta\left(1-\theta\right) \max_{\lambda_{HU}} \lambda_{HU} \left[u(\bar{q}) - q_H\right] \tag{5}$$

with $\bar{q} = \pi q_H + (1-\pi) q_L$.

Finally the values of the λ_{ij} satisfy the following incentive conditions: for $i \in \{L, H\}$,

$$\lambda_{iK} = \begin{cases} 1 & \text{if } u(q_i) - q_i \geq 0 \\ 0 & \text{otherwise,} \end{cases} \tag{6}$$

$$\lambda_{iU} = \begin{cases} 1 & \text{if } u(\bar{q}) - q_i \geq 0 \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

We can now define a monetary equilibrium in a world without moneychanger, which is the world examined in VWW.

Definition 1 *A symmetric monetary equilibrium in an economy without moneychanger is a vector of quantities* $q = (q_L, q_H)$ *and strategies* $\Psi = (\lambda_{LK}, \lambda_{HK}, \lambda_{LU}, \lambda_{HU})$ *such that:* (i) : $\lambda_{LK} = 1$ *and* $(\lambda_{HK}, \lambda_{LU}, \lambda_{HU}) \in [0,1]^3$*;* (ii) : q *satisfies (4) and (5).*

As shown by VWW, there exist three types of pure-strategy monetary equilibria in this environment: (i) : both coins circulate *by weight*; (ii) : both coins circulate *by tale*; (iii) : only the light coin circulate (*single currency equilibrium*). To show how moneychangers impact on circulation, we will consider each of these three equilibrium strategies and offer an agent the opportunity to deviate from his equilibrium strategy and have his coin certified by a moneychanger before he goes shopping. This will give birth to a non-deviation condition that refines the equilibrium. For instance, a by-weight equilibrium with inactive moneychangers will be an equilibrium in which light coins always circulate, heavy coins circulate when they are recognized, *and* no buyer holding a heavy coin deviates by visiting a moneychanger.

3.1 By-weight equilibrium

With circulation by weight, a heavy coin trades only when it is recognized by the seller. Therefore light coins always circulate whether recognized or not (and at the same price q_L) since unrecognized coins can only be light coins. Therefore $\lambda_{LK} = \lambda_{LU} = 1$. From (6) the light coins circulate in informed meetings if $\lambda_{LK} = 1$ which is equivalent to $u(q_L) \geq q_L$, and the heavy coin circulates in informed meetings if $\lambda_{HK} = 1$ which is equivalent to $u(q_H) \geq q_H$. These two conditions simplify into $r > \gamma_H$.[4] From (7) heavy coins do not circulate when not recognized if $\lambda_{HU} = 0$ which from (7) is equivalent to $u(\bar{q}) = u(q_L) \leq q_H$ since $\bar{q} = q_L$ when unrecognized heavy coins are hoarded. Inserting these values into (4) and (5), a by weight equilibrium is a

[4] Inserting $\lambda_{HK} = \lambda_{HU} = 0$ into (5) shows that the return to keeping home the heavy coin is $rq_H = \gamma_H$ so that $q_H = \gamma_H / r$. There is an incentive to deviate and trade the heavy coin if $u(q_H) - q_H \geq 0$, which is equivalent to $q_H \leq \hat{q}$, or $r \geq \gamma_H / \hat{q}$. Since $u(q_H) = q_H^{\eta}$ and $c(q_H) = q_H$, we have $\hat{q} = 1$. Finally because $\gamma_H > \gamma_L$ the condition $r \geq \gamma_H$ is sufficient for both light and heavy coins to circulate when recognized. Basically what this condition says is that the discount rate cannot be too small for the heavy coin to circulate.

list (q_L, q_H) given by

$$rq_L = \gamma_L + \beta\left[u(q_L) - q_L\right] \tag{8}$$

$$rq_H = \gamma_H + \beta\theta\left[u(q_H) - q_H\right] \tag{9}$$

that satisfy

$$r \geq \gamma_H \tag{10}$$

$$q_H \geq u(q_L). \tag{11}$$

Equations (8), (9) and (11) set to equality define the by-weight frontier (BWF). The by-weight equilibrium exists for all points in the parameter space (r, θ) to the right of $r = \gamma_H$ and to the left of the BWF (see Fig. 1).[5][6]

Now suppose that every buyer plays the by-weight equilibrium, and one buyer contemplates deviating and visiting a moneychanger. If he does not deviate he obtains the payoff $V_H = q_H$ given by (9). If he does deviate, he pays δ to a moneychanger and makes a take-it-or-leave-it (deviating) offer $\tilde{q}_H$ to the seller such that

$$-\tilde{q}_H + V_H = V_0 \tag{12}$$

That is, with this offer the seller is indifferent between producing this quantity $\tilde{q}_H$ and becoming a holder of a heavy coin, V_H, or staying as a producer, V_0. Since $V_0 = 0$ from the the take-it-or-leave-it protocol it is clear from (12) that

$$V_H = \tilde{q}_H = q_H. \tag{13}$$

That is, the deviating buyer asks the seller exactly the same quantity as if he was not deviating.

Noting $\tilde{\lambda}_H$ the deviating buyer's strategy whether to trade the certified heavy coin or not, with $\tilde{\lambda}_H = 1$ if $u(\tilde{q}_H) \geq \tilde{q}_H$, the Bellman equation for the deviator is given by

$$\tilde{V}_H = \frac{1}{1+r}\left\{-\delta + \gamma_H + \beta \max_{\tilde{\lambda}_H}\left[\tilde{\lambda}_H\left\{u(\tilde{q}_H) + V_0\right\} + \left(1 - \tilde{\lambda}_H\right)V_H\right] + (1-\beta)V_H\right\}.$$

[5] See Velde Weber and Wright (1999) for the shape of BWF.

[6] Note that in general equilibrium q_L and q_H are different across equilibria. In the developments below, unless specified, they will implicitly refer to the equilibrium in which they are determined.

Note that with probability $(1-\beta)$ he does not trade for sure so that his certificate expires and he turns back to holding an uncertified heavy coin. Using $\tilde{q}_H = q_H$ and $V_0 = 0$, the flow version (assuming the buyer wants to trade the certified heavy coin, $\tilde{\lambda}_H = 1$) is

$$r\tilde{V}_H = \gamma_H - \delta + \beta\left[u(q_H) - \tilde{V}_H\right] + (1-\beta)\left[V_H - \tilde{V}_H\right]. \tag{14}$$

This last equation says that the net gain from deviating and visiting a moneychanger is equal to the periodic return on the heavy coin minus the cost of expertise, plus the net gains from trading heavy coins in single coincidence of wants meetings, plus the net gain from swapping from deviator back to holding an uncertified heavy coin in all other circumstances. Note that this last net gain always occurs to the buyer whether he trades or not since the certificate expires at the end of the trading period.

In the end there is no incentive to deviate if the net payoff to holding an uncertified heavy coin is larger than the net payoff to deviating and holding a certified heavy coin, that is

$$V_H > \tilde{V}_H \tag{15}$$

which using (9) and (14) gives

$$\delta > \beta(1-\theta)\left[u(q_H) - q_H\right] + V_H - \tilde{V}_H \tag{16}$$

This inequality says that for a heavy coin holder *not* to deviate, the cost of expertise needs to be larger than the benefit, which is the gain from trade coming from the circulation of hoarded unrecognized heavy coins, $\beta(1-\theta)\left[u(q_H) - q_H\right]$, plus the net gain from swapping back to holding an uncertified heavy coin, $V_H - \tilde{V}_H$.

To obtain the new frontier (called $MCF\,1a$) refining the by weight equilibrium in an economy in which moneychangers are present yet inactive, we insert the indifference condition between deviating or not, $V_H = \tilde{V}_H$, into (16) and set it to equality, which yields

$$\delta = \beta(1-\theta)\left[u(q_H) - q_H\right]. \tag{17}$$

This equality indicates that a buyer is indifferent between certifying or not a heavy coin when the cost of expertise is just equal to the benefit which is the gain from trading previously hoarded unrecognized heavy coins.

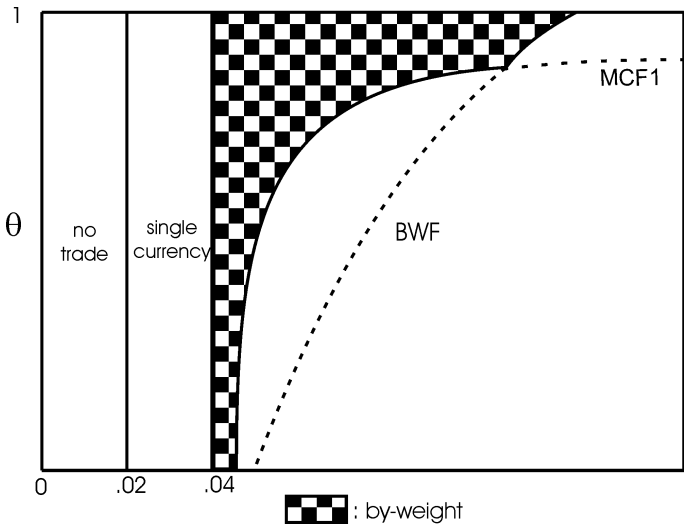

Figure 1: By-weigth equilibrium with moneychangers

Proposition 1 *Moneychangers limit Gresham's law (circulation by weight) to regions where information on coins is good.*

Proof. See Appendix. ■

The new frontier on Fig. 1, noted *MCF1a*, corresponds to the solutions to (9) that satisfy (17) for a given δ. Compared to VWW, the by-weight equilibrium zone shrinks to the North-West. By offering an alternative to the hoarding of good coins, moneychangers restrict the by weight equilibrium (and therefore the informational version of Gresham's law) to regions where information on coins is good. In that case buyers with heavy coins had better wait for another seller who is likely to recognize the coin than pay a moneychanger. However, as soon as information deteriorates, coins no longer circulate by weight and heavy coins get certified. Finally the frontier shifts up as the cost of expertise decreases.

3.2 By-tale equilibrium

With circulation by tale, unrecognized heavy coins trade at the same price $\bar{q}$ as unrecognized light coins. From (6) the two coins circulate in informed meetings if $u(q_L) \geq q_L$ and $u(q_H) \geq q_H$, which again simplify into $r > \gamma_H$. And from (7) heavy coins circulate when not recognized if

$u(\bar{q}) \geq q_H$. Inserting the corresponding $\lambda_{LK} = \lambda_{LU} = \lambda_{HK} = \lambda_{HU} = 1$ into (4) and (5), a by tale equilibrium is a list (q_L, q_H) given by

$$rq_L = \gamma_L + \beta\theta\left[u(q_L) - q_L\right] + \beta\left(1-\theta\right)\left[u\left(\bar{q}\right) - q_L\right] \tag{18}$$

$$rq_H = \gamma_H + \beta\theta\left[u(q_H) - q_H\right] + \beta\left(1-\theta\right)\left[u\left(\bar{q}\right) - q_H\right] \tag{19}$$

that satisfy the two conditions

$$r \geq \gamma_H \tag{20}$$

$$u(\bar{q}) \geq q_H. \tag{21}$$

Equations (18), (19) and (21) set to equality define the by-tale frontier (BTF). The by-tale equilibrium exists for all points in the parameter space (r, θ) to the right of $r = \gamma_H$ and to the right of the BTF (see Fig. 2).

Now suppose that every buyer plays the by-tale equilibrium, and one contemplates deviating and visiting a moneychanger. If he does not deviate he obtains the payoff $V_H = q_H$ given by (18) and (19). If he does deviate, he pays δ to a moneychanger and makes an offer $\tilde{q}_H = V_H - V_0$ to the seller that also satisfies (12) so that $V_H = \tilde{q}_H = q_H$. The continuation payoff to the deviating buyer with a heavy coin $\tilde{V}_H$ is again given by (14) so that there is no incentive to deviate from the by-tale equilibrium if $V_H > \tilde{V}_H$ which using (14) and (19) transforms into

$$\delta > \beta\left(1-\theta\right)\left[u\left(q_H\right) - u(\bar{q})\right] + V_H - \tilde{V}_H. \tag{22}$$

This inequality says that for a heavy coin holder not to deviate from playing the by-tale strategy, the cost of expertise needs to be greater than the increase in gains from trade due to the full recognizability of heavy coins plus the net gain from shifting from deviator back to playing the by tale equilibrium. Inserting the indifference condition between deviating or not, $V_H = \tilde{V}_H$, into (22) and setting it to equality yields

$$\delta = \beta\left(1-\theta\right)\left[u\left(q_H\right) - u(\bar{q})\right] \tag{23}$$

The solutions to (18) and (19) that satisfy (23) give birth to a new frontier for by tale circulation to be an equilibrium, noted *MCF2a*, and is represented on Fig. (2).

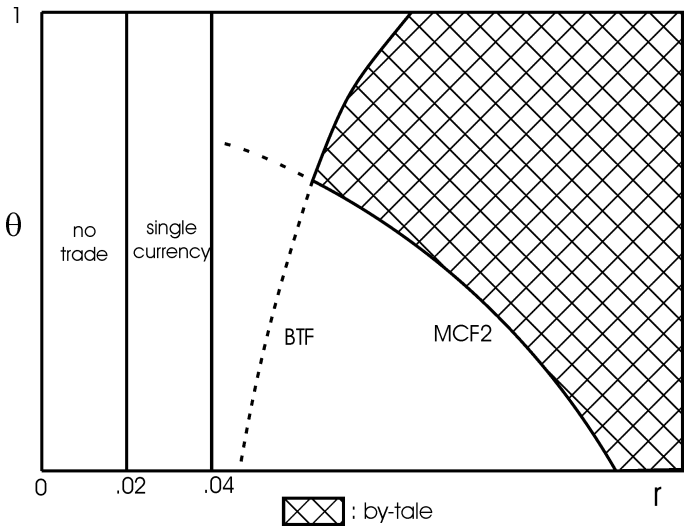

Figure 2: By-tale equilibrium with moneychangers

Proposition 2 *Moneychangers limit circulation by tale to regions where information on coins is good and/or the discount rate is high.*

Proof. See appendix. ■

From the above figure, moneychangers restrict circulation by tale to regions where information on coins is abundant for the same reason as in the by weight equilibrium. Agents had better wait for an informed seller, which are numerous, rather than pay a moneychanger. As soon as information deteriorates the by-tale equilibrium vanishes and we will see in the next section that it becomes attractive for buyers to have their heavy coin certified. In addition circulation by tale always survives in regions characterized by high discount rates. In that case, coins buy so little and gains from trade are so small, and especially gains from trade on certified heavy coins, that it does not pay to have the coin certified. Agents are better off just trading unrecognized heavy coins at the same price as light coins, saving the cost of a moneychanger. Finally the frontier shifts up as the cost of expertise decreases.

3.3 Single-currency equilibrium

In a single currency equilibrium, light coins circulate and heavy coins are hoarded whether recognized or not. From (6) the light coin circulates if $u(q_L) \geq q_L$ and the heavy one does not if $u(q_H) < q_H$ or $q_H > \hat{q}$, which simplifies into $\gamma_L \leq r \leq \gamma_H$. A single currency equilibrium is then a list (q_L, q_H) given by

$$rq_L = \gamma_L + \beta \left[u(q_L) - q_L\right] \tag{24}$$

$$rq_H = \gamma_H \tag{25}$$

such that $r \in [\gamma_L, \gamma_H]$. A key element of this equilibrium is the low discount rate. It makes it more attractive for agents to hoard the heavy coin and enjoy its high return rather than trading it. The quantity produced by the seller to compensate the buyer has to be so large that the disutility of producing this quantity overcomes the utility of consuming it so that net gains from trade are negative.

Suppose now that in this configuration a buyer hoarding his heavy coin contemplates deviating by certifying it and trading it. Again, from the buyer-takes-all protocol, he makes an offer $\tilde{q}_H$ such that a seller is indifferent between producing this quantity and becoming a non deviating buyer holding a heavy coin, and staying as a producer, that is $-\tilde{q}_H + V_H = V_0$. Since $V_0 = 0$, again $V_H = \tilde{q}_H = q_H$. He will not deviate if $V_H > \tilde{V}_H$ which using (25) and (14) yields

$$\delta > \beta \left[u(q_H) - \tilde{V}_H\right] + (1-\beta)\left[V_H - \tilde{V}_H\right]. \tag{26}$$

The frontier is given by the indifference condition $V_H = \tilde{V}_H$ yielding $\delta = \beta\left[u(q_H) - q_H\right]$ which cannot be true since $u(q_H) - q_H < 0$ in a single currency equilibrium. We conclude that there is never an incentive to deviate from the single currency equilibrium to an active moneychanger equilibrium. The intuition is simple: a deviator makes the same offer as the non-deviator, yet the non-deviating offer is not an equilibrium offer.[7]

We summarize the above discussions in the following proposition.

[7] Since Berentsen and Rocheteau (2002), it is well known that the indivisibility of money generates inefficient terms of trade. We have a clear illustration here as divisible coins or lotteries would make trade possible with low discount rates even with buyers making take-it-or-leave-it offers.

Proposition 3 *In a commodity money economy with moneychangers,*
(i) : A by weight equilibrium is characterized by a couple (q_L, q_H) *given by (8) and (9) and satisfying (10), (11) and (17);*
(ii) : A by tale equilibrium is characterized by a couple (q_L, q_H) *given by (18) and (19) and satisfying (20), (21) and (22);*
(iii) : A single currency equilibrium is characterized by a couple (q_L, q_H) *given by (24) and (25) and satisfying* $\gamma_L \leq r \leq \gamma_H$.

4 Active Moneychangers

In this section we conduct the mirror exercise: we characterize the equilibria in which heavy coins are certified by moneychangers and examine conditions under which there is no incentive for buyers to deviate and play either the by weight, by tale or single currency strategy. Let q_{HC} be the quantity traded against a certified heavy coin and V_{HC} be the Bellman equation for the buyer with a certified heavy coin. We note λ_{HC} the decision by the buyer whether to trade the certified heavy coin, with $\lambda_{HC} = 1$ if $u(q_{HC}) \geq q_{HC}$.

The payoffs to holding the light coin is

$$rq_L = \gamma_L + \beta \left[u(q_L) - q_L \right]. \tag{27}$$

Compared to the previous section, the equation for q_L is identical to the by weight case since in both cases all unrecognized coin can only be light coins so that they circulate at full information value and $\lambda_{LK} = \lambda_{LU} = 1$. For a holder of a certified heavy coin we have (assuming certificates circulate)

$$rq_{HC} = \gamma_H + \beta \left[u(q_{HC}) - q_{HC} \right] - \delta. \tag{28}$$

Note that these two equations describe a full information monetary economy with a periodic return on light coins equal to γ_L and $\gamma_H - \delta$ for heavy coins. Again, it is clear from (6) that the circulation of light coins requires $r \geq \gamma_L$. Things are slightly different now for certified heavy coins as (7) and (28) require

$$r > \gamma_H - \delta \tag{29}$$

which we assume larger than γ_L for simplicity.

We note $\tilde{\lambda}_{HK}$ the strategy whether to trade the heavy coin in informed meetings when deviating , $\tilde{\lambda}_{HU}$ the strategy whether to trade the heavy coins in uninformed meetings when deviating, and $\tilde{q}_{HC}$ the quantity purchased with a recognized yet uncertified heavy coin. The Bellman equation for the deviator is then given by

$$\tilde{V}_{HC} = \frac{1}{1+r}\left\{\begin{array}{c} \gamma_H + \beta\theta \max\limits_{\tilde{\lambda}_{HK}} \left[\tilde{\lambda}_{HK}\left\{u(\tilde{q}_{HC}) + V_0\right\} + \left(1-\tilde{\lambda}_{HK}\right) V_{HC}\right] \\ +\beta\left(1-\theta\right) \max\limits_{\tilde{\lambda}_{HU}} \left[\tilde{\lambda}_{HU}\left\{u(q_L) + V_0\right\} + \left(1-\tilde{\lambda}_{HU}\right) V_{HC}\right] \\ +(1-\beta)V_{HC} \end{array}\right\}.$$

Using $V_{HC} = q_{HC}$ and $V_0 = 0$, the flow version of the above equation is given by

$$\begin{aligned} r\tilde{V}_{HC} &= \gamma_H + \beta\theta \max_{\tilde{\lambda}_{HK}} \left\{\tilde{\lambda}_{HK}\left[u(\tilde{q}_{HC}) - \tilde{V}_{HC}\right] + \left(1-\tilde{\lambda}_{HK}\right)\left[V_{HC} - \tilde{V}_{HC}\right]\right\} \\ &\quad +\beta\left(1-\theta\right) \max_{\tilde{\lambda}_{HU}} \left\{\tilde{\lambda}_{HU}\left[u(q_L) - \tilde{V}_{HC}\right] + \left(1-\tilde{\lambda}_{HU}\right)\left[V_{HC} - \tilde{V}_{HC}\right]\right\} \\ &\quad +(1-\beta)\left[V_{HC} - \tilde{V}_{HC}\right]. \end{aligned} \tag{30}$$

First, whether in the by-weight or by-tale case, a deviator in an informed meeting makes an offer $\tilde{q}_{HC}$ that leaves the seller indifferent so that

$$-\tilde{q}_{HC} + V_{HC} = V_0$$

from which we obtain $\tilde{q}_{HC} = q_{HC}$. Second, since sellers infer that unrecognized coins can only be light coins, the payoff to the buyer with an unrecognized non certified heavy coin is $u(q_L)$ and not $u(\bar{q})$. In the end there is no incentive to deviate from the active moneychanger equilibrium and play either BW, BT or SC if

$$V_{HC} > \tilde{V}_{HC} \tag{31}$$

4.1 Deviation to by-weight

Assume first that a buyer with a heavy coin deviates and plays the by-weight strategy ($\tilde{\lambda}_{HK} = 1$ and $\tilde{\lambda}_{HU} = 0$). Inserting these values and $\tilde{q}_{HC} = q_{HC}$ into (30) his payoff is given by

$$r\tilde{V}_{HC} = \gamma_H + \beta\theta\left[u(q_{HC}) - \tilde{V}_{HC}\right] + (1-\beta\theta)\left[V_{HC} - \tilde{V}_{HC}\right].$$

Proceeding as in the previous section, the indifference condition is given by $V_{HC} = \tilde{V}_{HC}$ so that the frontier, noted *MCF1b*, is characterized by

$$\delta = \beta (1 - \theta) [u (q_{HC}) - q_{HC}]$$

which very much looks like *MCF1a* from the previous section, yet the equilibrium q_H is different. However,

Lemma 1 *MCF*1*a and MCF*1*b are the same.*

Proof. See Appendix. ■

The values of parameters that leave agents indifferent between shifting from circulation by weight to certified heavy coins also make agents indifferent between not deviating from certified heavy coins to circulation by weight. This means that the two frontiers are one and the same and that the two equilibria cannot coexist. See Fig. 1.

4.2 Deviation to by-tale

Assume now that she deviates and plays the by-tale strategy. Working as previously, the payoff to the deviator is

$$\begin{aligned} r\tilde{V}_{HC} &= \gamma_H + \beta\theta \left[u(q_{HC}) - \tilde{V}_{HC}\right] + \beta (1 - \theta) \left[u (q_L) - \tilde{V}_{HC}\right] \\ &\quad + (1 - \beta) \left[V_{HC} - \tilde{V}_{HC}\right] \end{aligned}$$

so that the *MCF2b* is given by

$$\delta = \beta (1 - \theta) [u (q_{HC}) - u(q_L)]$$

which is clearly different to *MCF2a*. Especially it stands to the right of *MCF2a* leaving some space for circulation by tale and certification to coexist. As explained in the introduction, this comes from the discrepancy, for the same values of parameters, between the payoff when agents shift from by-tale to certification and the payoff when agents do not shift from certification to by-tale.

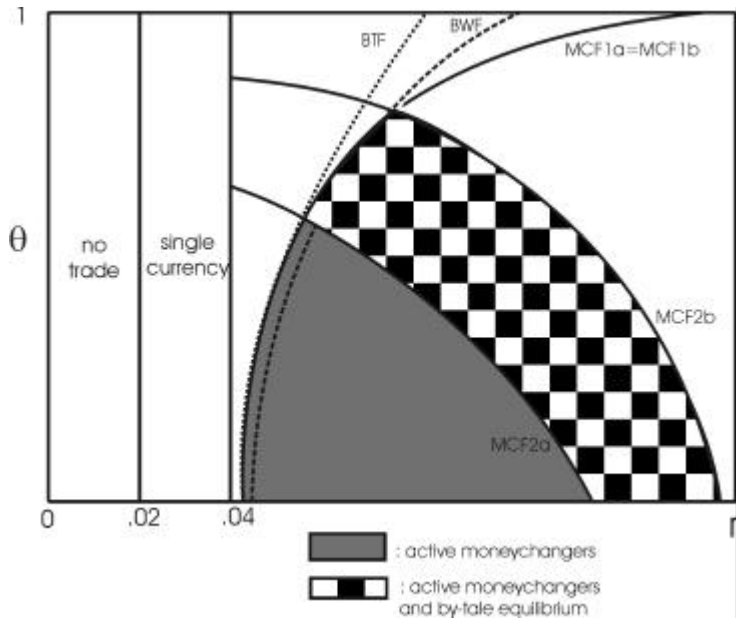

Figure 3: Coexistence of circulation by tale and active moneychangers

4.3 Deviation to single-currency

Let us note $\tilde{q}_s$ the quantity that would be traded if an uncertified coin were to circulate but does not. Because it is not an equilibrium offer for the buyer (otherwise it would circulate) this implies $u(\tilde{q}_s) + V_0 < \tilde{q}_s$ equivalent to $u(\tilde{q}_s) < \tilde{q}_s$ since $V_0 = 0$ and therefore $r < \gamma_H$. Now, substituting $\tilde{\lambda}_{HK} = \tilde{\lambda}_{HU} = 0$ into (30) and using (28) yields $u(q_{HC}) - q_{HC} > \frac{\delta}{\beta}$ which yields $r > \bar{r}$ with $\bar{r} > \gamma_H$ which contradicts $r < \gamma_H$. Therefore there is never an incentive to deviate and play the single currency equilibrium.

Proposition 4 *An "active moneychangers" equilibrium is a list (q_L, q_H) given by (27) and (28) that lie between MCF1a and MCF2b. Circulation by weight and certification never coexist. Circulation by tale and certification coexist for some values of the parameters.*

The region of "active moneychangers" equilibrium is represented on Fig. 3. When information on coins is poor and when the discount rate has intermediate values, the certification of heavy coins by moneychangers is an equilibrium. Circulation by tale and active moneychangers coexist for some values of the parameters.

5 Welfare

The welfare function in each equilibrium is defined as the ex ante weighted average of lifetime utilities across agents' types (see appendix). The following propositions resume how moneychangers affect agents' welfare:

Proposition 5 *The introduction of moneychangers that triggers a transition from a by-weight equilibrium to an active moneychangers equilibrium is always welfare improving.*

Proof. See Appendix. ■

When coins trade by weight, unrecognized coins can only be light coins so that light coins circulate at their full information value and are not impacted by the quality of the information on coins. Therefore the decision by heavy coin holders whether to visit a moneychanger will only impact buyers holding heavy coins. If they decide to opt for certification, they must be better off while buyers with light coins are indifferent. These observations imply that welfare is higher is an active moneychanger equilibrium than in a Gresham's law (circulation by weight) equilibrium.

Proposition 6 *The introduction of moneychangers that triggers a transition from a by-tale equilibrium to an active moneychangers equilibrium is always welfare worsening. When the two equilibria coexist, welfare is also lower with active moneychangers.*

Proof. See Appendix ■

In the by-tale equilibrium, the decision by heavy buyers to have their coin certified does impact on buyers holding light coins and makes the economy worse-off overall. The intuition is to be related to risk aversion: agents prefer to get $\bar{q}$ in $(1-\theta)$ meetings than q_H in $(1-\theta)\pi$ meetings and $q_L < q_H$ in the remaining $(1-\theta)(1-\pi)$ meetings. Besides, agents are better off with fully recognizable coins in circulation by tale than with active yet costly moneychangers. These observations imply that agents are always better off in a by-tale equilibrium than in an active moneychangers equilibrium.

6 Conclusion

Historical evidence show that coins experts existed throughout the metallic era and were an important part of the system. Following the literature pioneered by Avner Greif (1992), these moneychangers can be seen as one of the many institutions invented or revived during the late-medieval commercial revolution in Europe in order to solve new problems of a fast growing economy. Just like guilds of merchants solved reputation and commitment problems in a world of high monitoring costs (Greif Milgrom and Weingast, 1994), moneychangers, by offering sellers an opportunity to check the quality of the medium of payment prior to trading, constituted a possible solution the asymmetric information problem on coins.

What our research on these intermediaries has shown is that these intermediaries are helpful indeed. Both circulation by weight and circulation by tale have smaller equilibrium zones once moneychangers are added to the economy. They are also clearly welfare improving when they trigger a transition from Gresham's law (its informational version) to an active moneychangers equilibrium. However, their impact on welfare would have been negative if coins were circulating by tale. Coin experts also allow for equilibria where both coins are in full circulation and trade at different prices, equilibria that do not exist in Velde Weber Wright (1999). In a recent work Dutu Nosal and Rocheteau (2007) have shown that signaling via the offered terms of trade (rather than through certification as we do here) is possible and also welfare improving. This comes at the cost of lowering the heavy coins' velocity and the output traded against heavy coins however. Here once the cost of signaling is paid to moneychangers, this cleans the economy of any inefficiency and coins can circulate at their full information value and full velocity.

Appendix

A1. Proof of Proposition 1

To derive the shape of the *MCF1a* frontier, rewrite (17) such that

$$\delta + \beta\theta\left[u\left(q_H\right) - q_H\right] = \beta\left[u\left(q_H\right) - q_H\right]. \tag{32}$$

On the LHS of Fig 4, we plot both sides of the equality as functions of equilibrium q_H for a given θ. On the right hand side we plot the relationship between equilibrium q_H and r for the same θ. As θ decreases from 1 to 0, $\delta + \beta\theta\left[u\left(q_H\right) - q_H\right]$ gets flatter and $q(r)$ becomes steeper. Starting from $\theta = 0$ it is clear that there are two intersections between $\delta + \beta\theta\left[u\left(q_H\right) - q_H\right]$ and $\beta\left[u\left(q_H\right) - q_H\right]$ so that there are two r, one small one big, compatible with agents being indifferent. As θ increases the distance between these two r decreases and for $\theta > \bar{\theta}$ there are no more r such that $\delta = \beta\left(1-\theta\right)\left[u\left(q_H\right) - q_H\right]$. Especially at $\theta = 1$ the LHS of (32) is always larger than the RHS. Note also that for a given θ there always exists a δ that makes the indifference fall. The shape of the MCF1 follows from the above discussion. Note that we only plot the low values of r since they are the only ones that matter for the equilibrium by weight (there is no by-weight equilibrium for large value of the discount rate). Finally note that MCF1 shifts upward as δ decreases.

A2. Proof of Proposition 2

To derive the shape of the *MCF2a* frontier, rewrite (22) such that

$$\delta + \beta\theta\left[u\left(q_H\right) - u(\bar{q})\right] = \beta\left[u\left(q_H\right) - u(\bar{q})\right]. \tag{33}$$

First note that $u\left(q_H\right) - u(\bar{q})$ increases from negative to positive as q_H increases from 0 to $\hat{q}$. We use this on the LHS of Fig 5 below to plot both sides of the above equality as functions of q_H for a given θ. On the right hand side we plot the relationship between equilibrium q_H and r for the same θ. As θ increases from 0 to 1, $\delta + \beta\theta\left[u\left(q_H\right) - u(\bar{q})\right]$ rotates anti-clockwise and $q(r)$ becomes flatter. Starting from $\theta = 0$ it is clear that $\delta + \beta\theta\left[u\left(q_H\right) - u(\bar{q})\right]$ and $\beta\left[u\left(q_H\right) - u(\bar{q})\right]$ intersect once so that there is only one r that leaves agents indifferent between circulation by tale and a certified heavy coin. Note that both lines intersect iif $\delta - \beta\theta u(q_L) > -\beta u(q_L)$ which

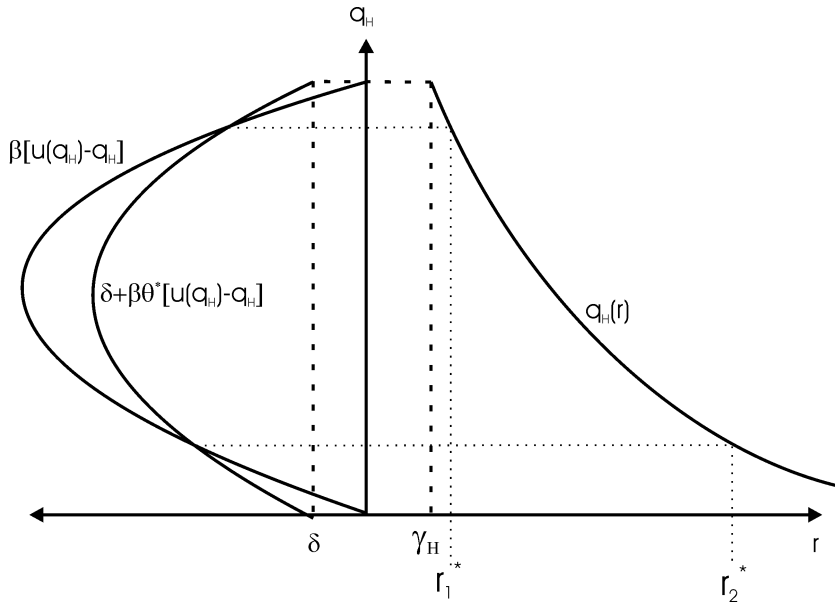

Figure 4: The MCF1 frontier.

is always satisfied. As θ increases, both lines still intersect once but for $\theta > \hat{\theta}$ it is no longer possible to have agents indifferent. Note also that at $\theta = 1$ the LHS of (33) is larger than the RHS. The shape of MCF2 follows from the above discussion. Note finally that MCF2 shifts upward as δ decreases.

A.3. Proof of Lemma 1

From (17), on *MCF 1a* we have

$$\delta = \beta (1 - \theta) \left[u \left(q_H^{bw} \right) - q_H^{bw} \right]. \tag{34}$$

Inserting this into (9) gives $rq_H^{bw} = \gamma_H + \beta \left[u(q_H^{bw}) - q_H^{bw} \right] - \delta$ identical to (28). Then $q_H^{bw} = q_H^{am}$ on MCF1a since δ always matches gains from trade on previously non circulating unrecognized heavy coins. We can therefore substitute q_H^{bw} for q_H^{am} into (34) which yields *MCF 1b*.

A.4 Proof of Proposition 3

In general, equilibrium value q_L and q_H are not the same across equilibria. We note q_i^{bw}, q_i^{bt} and q_i^{am} the equilibrium quantity traded respectively for a coin of type i in a by-weight, by-tale and active-moneychangers equilibrium. They are given by (8)-(9), (18)-(19), and (27)

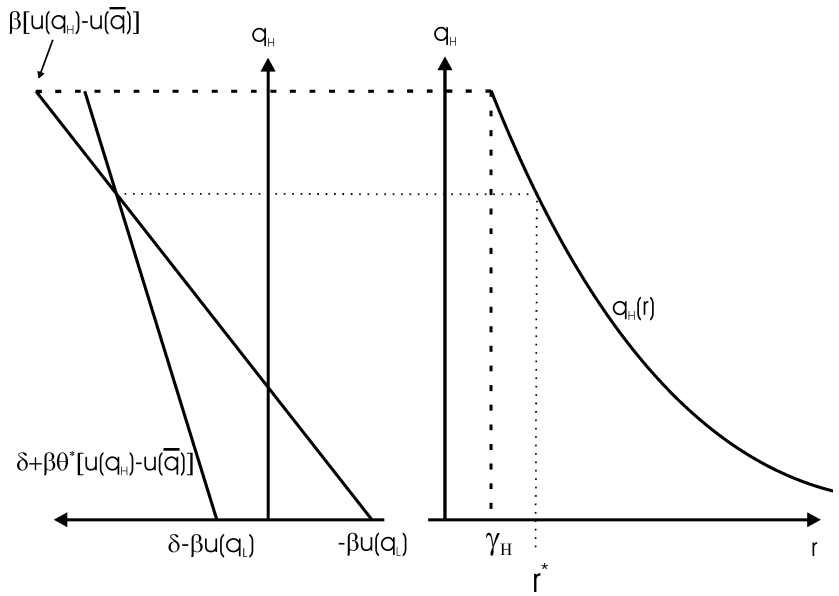

Figure 5: The MCF2 frontier.

and (28).

In each equilibrium, welfare is given by

$$rW_{bw} = M_L \left\{ \beta \left[u(q_L^{bw}) - q_L^{bw} \right] + \gamma_L \right\} + M_H \left\{ \beta\theta \left[u(q_H^{bw}) - q_H^{bw} \right] + \gamma_H \right\}, \tag{35}$$

$$\begin{aligned} rW_{bt} &= M_L \left\{ \beta\theta \left[u(q_L^{bt}) - q_L^{bt} \right] + \beta (1-\theta) \left[u(\bar{q}^{bt}) - q_L^{bt} \right] + \gamma_L \right\} \\ &\quad + M_H \left\{ \beta\theta \left[u(q_H^{bt}) - q_H^{bt} \right] + \beta (1-\theta) \left[u(\bar{q}^{bt}) - q_H^{bt} \right] + \gamma_H \right\} \end{aligned} \tag{36}$$

and

$$rW_{am} = M_L \left\{ \beta \left[u(q_L^{am}) - q_L^{am} \right] + \gamma_L \right\} + M_H \left\{ \beta \left[u(q_H^{am}) - q_H^{am} \right] + \gamma_H - \delta \right\} \tag{37}$$

First recall from Lemma 1 that $q_H^{bw} = q_H^{am}$ on MCF1a. From (8) and (27) it is clear that $q_L^{bw} = q_L^{am}$ as well whatever $(r, \theta) \in [\gamma_H, \infty] \times [0, 1]$. Therefore, using this and the proof of Lemma 1, (35) can be rewritten

$$rW_{bw} = M_L \left\{ \beta \left[u(q_L^{am}) - q_L^{am} \right] + \gamma_L \right\} + M_H \left\{ \beta \left[u(q_H^{am}) - q_H^{am} \right] + \gamma_H - \delta \right\}.$$

so that $rW_{bw} = rW_{am}$ on MCF1a.

Now note that $d\left(rW_{am}\right)/d\theta = 0$. But

$$\frac{d\left(rW_{bw}\right)}{d\theta} = \beta\left\{u\left[q\left(\theta\right) - q\left(\theta\right)\right] + \theta.q'_{\theta}.\left(u'\left[q\left(\theta\right)\right] - 1\right)\right\}.$$

Totally differentiating (9) yields $r.q'_{\theta} = \beta\left\{u\left[q\left(\theta\right) - q\left(\theta\right)\right] + \theta.q'_{\theta}.\left(u'\left[q\left(\theta\right)\right] - 1\right)\right\}$ so that $d\left(rW_{bw}\right)/d\theta = r.q'_{\theta} > 0$. Therefore, because W_{am} is constant whatever θ, $W_{bw} \geq W_{am}$ for any value of θ equal or above that of the MCF1, and $W_{bw} < W_{am}$ for any value of θ below that of the MCF1.

A.4. Proof of Proposition 4

Grouping all the terms in θ and $(1-\theta)$ in the by-tale equilibrium, we obtain

$$\begin{aligned} rW_{bt} &= \theta\left\{M_L\beta\left[u(q_L^{bt}) - q_L^{bt}\right] + M_H\beta\left[u(q_H^{bt}) - q_H^{bt}\right]\right\} \\ &\quad + (1-\theta)\beta\left\{\left(M_L + M_H\right)u(\bar{q}^{bt}) - \left(M_L q_L^{bt} + M_H q_H^{bt}\right)\right\} + M_L\gamma_L + M_H\gamma_H \end{aligned}$$

From the concavity of u, we have $u(\bar{q}^{bt}) = u\left[\pi q_H^{bt} + (1-\pi)\,q_L^{bt}\right] > \pi u(q_H^{bt}) + (1-\pi)\,u\left(q_L^{bt}\right)$. Using the definition of π we get $\left(M_L + M_H\right)u(\bar{q}^{bt}) > M_L u\left(q_L^{bt}\right) + M_H u\left(q_H^{bt}\right)$ so that $rW_{bt} > rW\left(\theta\right)$ whatever θ with

$$\begin{aligned} rW\left(\theta\right) &= \theta\left\{M_L\beta\left[u(q_L^{bt}) - q_L^{bt}\right] + M_H\beta\left[u(q_H^{bt}) - q_H^{bt}\right]\right\} \\ &\quad + (1-\theta)\beta\left\{M_L u\left(q_L^{bt}\right) + M_H u\left(q_H^{bt}\right) - \left(M_L q_L^{bt} + M_H q_H^{bt}\right)\right\} + M_L\gamma_L + M_H\gamma_H \end{aligned}$$

which simplifies into

$$rW\left(\theta\right) = M_L\left\{\beta\left[u(q_L^{bt}) - q_L^{bt}\right] + \gamma_L\right\} + M_H\left\{\beta\left[u(q_H^{bt}) - q_H^{bt}\right] + \gamma_H\right\}.$$

Note now from (18)-(19) and (27) that when $\theta = 1$, $q_L^{bt} = q_L^{am}$, and from (18)-(19) and (28) that $q_H^{bt} > q_H^{am}$ so that $rW\left(\theta = 1\right) > rW_{am}$. Because rW_{am} is invariant with θ but $rW_{bt} > rW\left(\theta = 1\right)$, we conclude $rW_{bt} > rW_{am}$.

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