Resucitating Time-to-Build*

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Abstract

A novel specification of the time-to-build (TTB) assumption is presented whereby firms invest multiple investment projects that have complementarities, and the duration of the investment process is uncertain. The model yields to a gradual (hump-shaped) response of investment to shocks, and it is shown to be equivalent, up to first-order linearization, to investment adjustment cost models, where the cost of adjustment depends on the change in investment flow. The paper discusses how the TTB formulation is consistent with empirical features of investment decisions at the aggregate and more disaggregated levels.

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1 Introduction

Due to the time needed to build (TTB) and plan (TTP) for investment, capital accumulation is often a lengthy process: should this feature be explicitly modelled when characterizing firms' investment decisions? This paper presents a novel specification of the TTB assumption and, in contrast to recent literature, it argues that the length of the investment process should be explicitly modelled as it represents one of its most critical features.

The idea of TTB and TTP are not new in economics as they can be traced back in time at least to the work of Kalecki [1935]. Although its existence acknowledged by most economists, TTB is not usually included in either empirical or theoretical investment models (see Nickell [1978]). In their seminal contribution to quantitative macroeconomic modelling, instead, Kydland and Prescott [1982] (KP hereafter) argue that TTB is key to understanding post-World War II business cycle fluctuations in the United States. In the specification of TTB considered by KP, and almost universally by subsequent literature, firms invest in one type of good that requires a fixed amount of resources in each period up to maturity. Further, an investment project increases stock of capital only at the end of the process when it matures.

More recent studies, however, have downplayed the role of TTB. Rouwenhorst [1991] shows that KP's formulation of TTB has little effect on the response of a real business cycle (RBC) model to productivity shocks, but for generating unrealistic cyclicalities in the response of the main aggregates. Similarly, Cogley and Nason [1995] show that KP's TTB formulation has little or no impact on the persistence of output growth. Following these works, few macroeconomic studies explicitly consider the length of the investment process. The common approach, instead, is to assume a one period delay in the capital accumulation process no matter if the time unit of analysis is a quarter or a year.[1] Further, few empirical microeconomic models of investment have analyzed the TTB assumption.

The novel formulation of TTB presented in this paper departs from the one considered by KP in two ways: firms invest in many investment type of goods that have complementarities, and, the duration of each investment project is uncertain. Because, as in KP, the scale of the ongoing projects is fixed, the investment decision of the firm is in part predetermined by earlier commitments. Further, due to the complementarity of the investment goods, firms optimally choose not to fully compensate earlier commitments when adjusting the scale of the projects that are under their control.

The optimal investment decision of the firm is first analyzed in partial equilibrium. The representative firm, calibrated on US manufacturing data, is subject to productivity and interest rate shocks. The dynamic responses of the model show that investment moves only gradually after the realization of the shocks, due to the direct link between investment decisions at different dates. Two assumptions are crucial for the analytical tractability of the model. First, the duration of the investment project follows a Poisson process, so that the probability that a project matures is independent of when it started. Second, there is imperfect substitutability between the investment types but the capital stock of the firm is a homogeneous good that depreciates at a constant rate. Although these assumptions greatly simplify the analysis, neither is key for the hump-shaped response to exist. The key elements, instead, are a source of imperfect substitutability among the investment types, and the ex-post heterogeneity in the duration of the investment projects, not necessarily due to an uncertain duration. The paper makes these points by comparing the novel model with analogous deterministic specifications.

The TTB specification is then compared to capital (e.g. Hayashi [1982]) and investment adjustment cost

[1] One period delay means that current investment increases the capital stock only in the next period.

models (e.g. Christiano, Eichenbaum, and Evans [2005]). With capital adjustment costs, the firm faces costs that are function of the change in the capital stock, while with investment adjustment costs they depend on the change in investment levels. The paper shows that the TTB formulation is equivalent, up to first order linearization, to investment adjustment cost models; thus the model presented in the paper directly links investment adjustment cost models to the length of the investment process.

The response of the TTB model is, instead, different from the one of capital adjustment cost models, where investment swiftly responds to the realization of the shocks. A vast empirical literature has tested the investment model with (convex) capital adjustment costs (see e.g. Chirinko [1993]), and the model is typically rejected in the data. The level of Tobin's Q is not a sufficient statistic for the investment decision, as predicted by the model, and the estimated adjustment costs appear to be unreasonably large. Further, investment appears to be more sluggish than what is predicted by the model: past realizations of Tobin's Q enter significantly in the regression models, and the error terms are (highly) serially correlated. Finally, the current level of cash flows appear to be an important determinant of investment decisions, an observation often interpreted as indicating the existence of financial frictions (e.g. Hubbard [1998]). In this paper, I generate artificial data using the TTB model calibrated on US sectoral data, and then estimate the empirical model with convex adjustment costs. The three empirical findings discussed above are shown to be consistent with a misspecification of the empirical model such that TTB technology underlying the data-generating process is not considered in the empirical model.

Finally the TTB model is embedded in an otherwise canonical RBC model. As for the partial equilibrium model, the dynamic response of investment to a technology shock is hump shaped, and thus describes well the empirical response of aggregate investment to this shock on US data (Christiano, Eichenbaum, and Vigfusson [2004]). Further, neither investment nor the other variables display the cyclicalities highlighted by Rouwenhorst [1991] for the KP's formulation. Investment adjustment cost models have shown to help explain the response of aggregate investment and other macroeconomic variables to both fiscal (Basu and Kimball [2005], Burnside, Eichenbaum, and Fisher [2004]) and monetary policy shocks (Christiano, Eichenbaum, and Evans [2005]). Due to the equivalence result between the TTB and the investment adjustment cost model, the TTB formulation of this paper thus helps understanding the response of aggregate economies also to these type of shocks.

The remaining of the paper is organized as follows. The TTB model with uncertain duration is presented in the next Section, and then solved in Section 3. Section 4 compares the model with two alternative TTB formulations with deterministic duration. Section 5 relates the model to investment and capital adjustment cost models, while Section 6 discusses the implications of the model for empirical tests of capital adjustment cost models. Finally ,Section 7 embeds the TTB model in an otherwise canonical RBC model.

2 The Model

Consider a firm that produces the good, Y_t, using the stock of capital, K_{t-1}, and a vector of variable factor inputs $\mathbf{x}_t$ with the production function: $Y_t = f(A_t, K_{t-1}, \mathbf{x}_t)$, where A_t is the level of productivity. Let $p_{x,t}$ be the vector price of the variable factor inputs. The date t flow of firms' revenues net of variable factor costs are, after maximizing out the variable factors: $\pi_t(K_{t-1}) \equiv \max_{\mathbf{x}_t} f(A_t, K_{t-1}, \mathbf{x}_t) - p'_{x,t}\mathbf{x}_t$. The function $\pi_t(\cdot)$ is increasing and weakly concave.

To increase the stock of capital, the firm invests in a fixed but large number of perfectly symmetric investment goods indexed by their type $j \in [0, 1]$. It takes time to build and plan for the construction of each

investment good. The firm chooses the desired quantity of type-j investment *good* when it starts a type-j investment *project*. Each firm can only run one project per-investment good at a time, and the scale of the project cannot be modified once initiated. If a project matures at t a new scale may be chosen at $t+1$. Let $\iota_t(j)$ denote the date t scale of the type-j project, $M_t \subseteq [0,1]$ be the set of investment projects maturing at t, and NM_t its complement in $[0,1]$. The scale of a project that has not matured is fixed: $\iota_t(j) = \iota_{t-1}(j)$ for $j \in NM_{t-1}$. This is a key element of the model as it implies that, at each date, part of the investment decision is predetermined as some projects are still under way. It is also important to note that the notation maintains information on the scale of the investment for each type, $\iota_t(j)$, but omits an explicit indication of the date when the scale of each investment was chosen. In particular, the scale of all projects that are still under way was chosen in the past, and thus it does not include relevant information on the current realization of the shocks' innovations.

The duration of investment projects is uncertain: the maturity of each project follows a Poisson-process with arrival rate θ. A project started at date t has a probability θ of being completed at the same date t. Uncompleted projects mature with the constant probability θ at each of the subsequent dates, so that a firm expects projects to mature at date $t + (1/\theta - 1)$.

An investment project increases the capital stock only when it matures. Accordingly let the variable

$$\iota_t^m(j) = \begin{cases} \iota_t(j) & \text{if } j \in M_t, \\ 0 & \text{otherwise,} \end{cases} \tag{2.1}$$

denote the level of type j investment that will increase the capital stock at date t. The variable $\iota_t^m(j)$ is equal to the scale of the project, $\iota_t(j)$, if it matures at date t ($j \in M_t$), and it is equal to zero otherwise. Investment goods are characterized by complementarities – the return to each good is increasing with the availability of the others – and each $\iota_t^m(j)$ enters symmetrically in the date t investment *basket*

$$I_t \equiv \left(\int_0^1 \iota_t^m(j)^{1-1/\varepsilon} dj \right)^{\varepsilon/(\varepsilon-1)}, \tag{2.2}$$

where $\varepsilon > 1$, so that none of the investment goods are essential in increasing the capital stock.

The level of the investment basket I_t increases the firm's capital stock, which depreciates at the constant rate δ

$$K_t = (1-\delta)K_{t-1} + I_t. \tag{2.3}$$

The installation of capital is costly: the firm pays adjustment costs $C(I_t, K_{t-1}) \equiv c(I_t/K_{t-1})\ K_{t-1}$, where $c(\cdot)$ is increasing and convex and such that $c'(\delta) = c(\delta) = 0$ and $c''(\delta) = \psi$. Capital installation costs play no central role in the TTB technology, and only the partial equilibrium investment model includes them.

Now consider the level of investment expenditure. An investment project started at t requires $\theta\iota_t(j)$ units of Y_s at all dates $s \geq t$ up to maturity. The date t investment expenditure is equal to the sum of all projects' expenditure

$$E_t \equiv \theta \int_0^1 \iota_t(j)\ dj. \tag{2.4}$$

The firm uses on average one unit of Y for each unit of investment. Indeed, the projects last for an average of $1/\theta$ periods and the firm uses $\theta\iota_t(j)$ units of Y_t per period. A time-to-plan formulation of the model, in which investment expenditure only occurs when the project matures, would yield the same expenditure function as in (2.4). Indeed, because the maturity of the project follows a Poisson-process, it follows that

$E_t = \int_{j \in M_t} \iota_t(j) dj = \theta \int_0^1 \iota_t(j) dj$.

The firm's management acts in the interest of shareholders and maximizes at each date t the value of future dividends discounted using the gross rate R_{s+1} across subsequent dates s and $s+1$. Thus the firm solves

$$\max \ \mathbb{E}_t \left\{ \sum_{r=0}^{\infty} \left(\prod_{s=1}^{r} R_{t+r}^{-1} \right) D_{t+r} \right\}, \tag{2.5}$$

with respect to K_t and $\{\iota(j)\}_{j \in M_{t+r}}$,subject to law of motion of the capital stock (2.3) and the investment technology described above. The dividend at date $t+r$ is

$$D_{t+r} = \pi_{t+r}(K_{t-1+r}) - E_{t+r} - C(I_{t+r}, K_{t+r-1}). \tag{2.6}$$

At the beginning of each period t, the firm observes the level of productivity, A_t, the price of the variable factor inputs, the interest rate R_t and which projects have matured in the previous period $j \in M_{t-1}$. Given the stock of K_{t-1}, the firm then decides the level production, Y_t, and the corresponding vector of variable inputs $\mathbf{x}_t$. Then the firm invests, a decision that, as previously discussed, is only in part under its control. Investment in the projects that are still under way, $j \in NM_{t-1}$, cannot be modified, while the firm decides how to invest for the investment types whose projects have matured in the previous period, $j \in M_{t-1}$. The uncertainty regarding which projects matures at date t is then resolved. The level of investment for the maturing projects increases the capital stock at t, and their scale is under control of the firm in the following period, the scale of all remaining projects, instead, will remain fixed.

3 Solution of the Model

When choosing the scale of a new investment project, the firm needs to consider its effects on all levels of the investment basket and expenditure at future dates, due to the uncertain duration of the projects The optimal investment decision can be characterized using two different solution strategies.

A direct approach is to choose $\iota_t(j)$ for $j \in M_{t-1}$ by maximizing (2.5). The first order condition that characterizes the optimal decision involves the values of E_{t+s} and I_{t+s} for all $s \geq 0$. Following the literature on staggered pricing decisions (e.g. Yun [1996]) the first order condition can be linearized, and after some algebra, expressed as a second order difference equation.

A simpler solution method, instead, makes explicit use of the large number of investment projects. It is important to note that despite the uncertain duration of each investment project, there is no uncertainty in the overall investment decision of the firm. Further it is possible to show that levels of E_t and I_t depend on earlier investment decisions only through E_{t-1} and I_{t-1}. As it is shown below, this greatly simplifies the solution of the investment problem by splitting it into an intratemporal allocation problem and an intertemporal decision. The remaining of the section makes additional assumptions on the functional forms of the model. Finally, the model is calibrated on US manufacturing data and then simulated.

3.1 Optimal Investment Decision

The optimal investment decision is solved by first characterizing the date t intratemporal firm's decision of how to allocate the expenditure across the different goods that have just matured, $j \in M_{t-1}$. It is then possible to characterize the intertemporal investment decision only in terms of E_t, I_t and K_t, and not by explicitly considering the scale of the projects $\iota_t(j)$'s.

First consider the intratemporal investment decision of the firm. Because the maturity of the projects follows a Poisson process, all projects have equal probability of maturing independently of their starting date. Thereby, the date t averages of the quantities $\iota_t(j)$ and $\iota_t(j)^{1-1/\varepsilon}$ among the projects that mature at the end of the period, $j \in M_t$, and those that do not mature, $j \in NM_t$, are equal. Furthermore, because of the large number of projects, the exact fraction of projects that mature is equal to θ, and the remaining $1-\theta$ are projects that have not mature. It follows from this discussion that

$$\frac{\int_{j\in M_t} x_t(j)\ dj}{\theta} = \frac{\int_{j\in NM_t} x_t(j)\ dj}{1-\theta} = \int_0^1 x_t(j)\ dj \ \text{ for each } \ x_t(j) = \{\iota_t(j),\, \iota_t(j)^{1-1/\varepsilon}\}. \tag{3.1}$$

where the expression after the second equality is the average scale over all j's. Using (3.1), (2.2) becomes

$$I_t = \left(\theta \int_0^1 \iota_t(j)^{1-1/\varepsilon} dj\right)^{\varepsilon/(\varepsilon-1)}. \tag{3.2}$$

It now follows the crucial step of the intratemporal investment problem, which is to express E_t and I_t in terms of their respective lagged values and the date t scale of the projects that just matured. First note that I_t can be rewritten as

$$\begin{aligned} I_t &= \left(\theta\left(\int_{j\in M_{t-1}} \iota_t(j)^{1-1/\varepsilon} dj + \int_{j\in NM_{t-1}} \iota_t(j)^{1-1/\varepsilon} dj\right)\right)^{\varepsilon/(\varepsilon-1)} = \\ &= \left(\theta\left(\int_{j\in M_{t-1}} \iota_t(j)^{1-1/\varepsilon} dj + (1-\theta)\left(\int_{j\in NM_{t-1}} (\iota_{t-1}(j))^{1-1/\varepsilon}\, dj\right)/(1-\theta)\right)\right)^{\varepsilon/(\varepsilon-1)} = \\ &= \left(\theta \int_{j\in M_{t-1}} \iota_t(j)^{1-1/\varepsilon} dj + (1-\theta)\,(I_{t-1})^{1-1/\varepsilon}\right)^{\varepsilon/(\varepsilon-1)}. \end{aligned} \tag{3.3}$$

The expression after the first equality simply follows from rewriting the integral in (3.2). The second equality makes use of the fact that the scale of the investment project is fixed, or $\iota_t(j) = \iota_{t-1}(j)$, for projects that did not mature, $j \in NM_{t-1}$. The third equality follows from (3.2) lagged by one period and (3.1). Using analogous steps, (2.4) can be rewritten as

$$E_t = \theta \int_{j\in M_{t-1}} \iota_t(j) dj + (1-\theta)\ E_{t-1}. \tag{3.4}$$

The firm only controls current investment expenditure for the fraction θ of projects that had matured, while expenditure in the remaining fraction $1-\theta$ is predetermined.

So long as $\varepsilon < \infty$ the firm chooses the same scale for all maturing investment projects, because they enter symmetrically into I_t and all have the same expected cost of one unit of Y. This result follows from a simple expenditure minimization of (3.4) subject to (3.3) with respect to all projects that had matured in the previous period $\{\iota_t(j)\}_{j\in M_{t-1}}$. Let ι_t be the optimal scale that is chosen by the firm, then $\iota_t(j) = \iota_t$ for all $j \in M_{t-1}$.[2] Using this result in (3.3) and (3.4), and then substituting ι_t from (3.3) in (3.4) yields

$$E_t = \Omega(I_t, I_{t-1}) + (1-\theta)E_{t-1}, \tag{3.5}$$

[2] Because the $\iota_t(j)$'s affect future quantities only through E_t and I_t, the optimal choice of $\iota_t(j)$ in the static minization problem is equivalent to that following from the intertemporal program.

where

$$\Omega(I_t, I_{t-1}) \equiv \theta^{\frac{2}{1-\varepsilon}} \left(I_t^{1-\frac{1}{\varepsilon}} - (1-\theta)\ I_{t-1}^{1-1/\varepsilon} \right)^{\varepsilon/(\varepsilon-1)}.$$

The function $\Omega(I_t, I_{t-1})$ is increasing in its first argument and decreasing in the second. Indeed, a bigger difference between the current and the lagged level of the investment basket corresponds to a larger scale of the investment project ι_t. Condition (**??**) is the only additional condition in an otherwise standard intertemporal firm maximization problem.

The Bellman-Jacobi equation associated with the intertemporal problem is

$$V_t(K_{t-1}, E_{t-1}, I_{t-1}) = \max_{\{K_t, I_t, E_t\}} \ \pi_t(K_{t-1}) - E_t - C(I_t, K_{t-1}) + \mathbb{E}_t\ R_{t+1}^{-1}\ V_{t+1}(K_t, E_t, I_t), \tag{3.6}$$

subject to the constraint (3.5) and the law of motion of the capital stock (2.3). In the general case in which $\varepsilon < \infty$ and $\theta < 1$, the value function, as well as the optimal investment rules, depend directly on the lagged levels of E_t and I_t. As discussed below, this is the crucial difference between this model and one where investment projects mature instantaneously. The value function is also time dependent due to the temporal dependence of prices (p_{xt}, R_t) and of the productivity shock (A_t).

Let q_t and $-\mu_t$ be the date t shadow values of K_t and E_t respectively. The first order conditions associated with (3.6) are

$$\begin{array}{ll} (K_t) & q_t = \mathbb{E}_t R_{t+1}^{-1} V_{1,t+1}(K_t, I_t, E_t), \\ (E_t) & \mu_t = 1 - \mathbb{E}_t R_{t+1}^{-1} V_{2,t+1}(K_t, I_t, E_t), \\ (I_t) & \mathbb{E}_t R_{t+1}^{-1} V_{3,t+1}(K_t, I_t, E_t) + q_t = C_{1,t}(I_t, K_{t-1}) + \mu_t \Omega_1(I_t, I_{t-1}), \end{array} \tag{3.7}$$

where a numeric subscript indicates the argument with respect to which a derivative is taken. The first order condition with respect to K_t equalizes the shadow value of capital, q_t, to the partial change in the discounted value function (or marginal Q) which, using the envelope condition with respect to K_{t-1}, is

$$V_{1t}(K_{t-1}, I_{t-1}, E_{t-1}) = \pi_{1,t} + (1-\delta) q_t - C_{2,t}(I_t, K_{t-1}).$$

The first order condition with respect to E_t equates E_t's shadow *cost*, $\mu_t > 0$, to the sum of the cost of one additional unit of E_t and the increase of $(1-\theta)$ in E_{t+1} evaluated at E_{t+1}'s shadow *cost*. Indeed using the envelope condition for E_{t-1} one obtains that

$$V_{2t}(K_{t-1}, I_{t-1}, E_{t-1}) = -(1-\theta)\mu_t.$$

The first order condition with respect to I_t equates the marginal benefit and the cost of an additional unit of I_t. The marginal benefit is equal to the marginal increase in the capital stock evaluated at q_t and the discounted marginal reduction in E_{t+1} evaluated at μ_{t+1}. Indeed from the envelope condition for E_{t-1} follows that

$$V_{3t}(K_{t-1}, I_{t-1}, E_{t-1}) = -\mu_t\ \Omega_2(I_t, I_{t-1}).$$

The marginal cost of an additional unit of I_t is equal to the sum of the marginal increase in the installation cost and in the investment expenditure evaluated at μ_t.

Substituting the partial derivatives of the value function from the envelope conditions above into (3.7)

one obtains

$$\begin{aligned}
&(K_t) \quad q_t = \mathbb{E}_t R_{t+1}^{-1}(\pi_{1,t+1} + (1-\delta)q_{t+1} - C_{2,t+1}(I_{t+1}, K_t)),\\
&(E_t) \quad \mu_t = 1 + (1-\theta)\mathbb{E}_t R_{t+1}^{-1}\mu_{t+1},\\
&(I_t) \quad q_t - \mathbb{E}_t R_{t+1}^{-1}\mu_{t+1}\ \Omega_2(I_{t+1}, I_t) = C_{1,t}(I_t, K_{t-1}) + \mu_t\ \Omega_1(I_t, I_{t-1}).
\end{aligned} \tag{3.8}$$

These three conditions fully characterize the optimal investment decision problem.

3.2 Parametrization of $\pi_t(K_{t-1})$ and Steady State

The cash flow function $\pi_t(\cdot)$, left unmodelled so far, is specified in the first part of this section. Using the functional form for $\pi_t(\cdot)$ and the first order conditions (3.8), the model is then solved in its steady state.

The investment literature usually assumes that firms are perfectly competitive and operate a constant return to scale production function. These assumptions simplify the empirical test of investment models with adjustment costs, as marginal Q, a crucial determinant of the investment decision, which is unobserved to econometricians, is equal to average Q, which is, instead, easier to measure when firms are publicly traded (Hayashi [1982]).

In this section I consider, instead, a monopolistically competitive firm that operates an increasing returns to scale production function due to overhead costs. There are two advantages of this model over the competitive-constant return to scale one. The first is that this model fits well with the empirical evidence on US sectoral level data, which shows that firms tend to set prices above marginal costs and economic profits are close to zero (e.g. Hall [1988]).[3] The second, is that the resulting cash flow function $\pi_t(\cdot)$ is strictly concave, so that the scale of the firm, measured by its capital stock, is pinned down in the steady state of the model, and the model is solved in the next section by linearization around the non-stochastic steady state.[4]

The firm produces Y_t using the production function

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} - \phi, \tag{3.9}$$

with $0 < \alpha < 1$ and $\phi > 0$. The only variable factor of production is labor, L_t, and the firm takes as given the wage rate, w_t. The parameter ϕ is an overhead fixed cost, which is such that economic profits are equal to zero in steady state. The firm faces an isoelastic demand function

$$p_t = z_t Y_t^{-\eta},$$

where $0 < \eta < 1$ is the inverse of the elasticity of demand, and z_t is a demand shifter variable. Solving (3.9) for L_t, the cash flow function can be rewritten in terms of K_{t-1} as

$$\pi_t(K_{t-1}) = \max_{Y_t}\ z_t Y_t^{1-\eta} - w_t \left(\frac{Y_t + \phi}{A_t K_t^{a}}\right)^{\frac{1}{1-\alpha}}. \tag{3.10}$$

The choice of Y_t yields that the firm sets its price as a constant markup over the marginal cost

$$p_t = \frac{1}{1-\eta} MC_t, \tag{3.11}$$

[3] Aside from the empirical evidence, the assumption of zero economic profits in steady state, implies that even in the presence of imperfect competition cost and revenue based factor shares are equal. This simplifies the calculation of the production function elasticity with respect to the production factors (see Hall [1988]).

[4] With perfect competition and constant returns to scale production function, the scale of the firm is indeterminate.

where the marginal cost is

$$MC_t = w_t(1-\alpha)^{-1}((Y_t+\phi)^{\alpha}\ (A_tK_t)^{-1})^{1/(1-\alpha)}.$$

Using the envelope theorem, the derivative of the cash-flow function is then

$$\pi_t'(K_{t-1}) = \alpha\ MC_t\left(\frac{Y_t+\phi}{K_t}\right). \tag{3.12}$$

The next step is to compute the value of economic profits in steady state; the value of ϕ is found by setting profits to zero. I further normalize the steady state values of the productivity shock, the wage rate and the demand shifter to one, or $A_{ss}=1$, $w_{ss}=1$ and $z_{ss}=1$. In the remaining the subscript ss denotes the steady state level for all variables. Note that the shadow rental rate on capital is given by (3.12) and the total variable cost is equal to $w_tL_t = (1-\alpha)(Y_t+\phi)MC_t$. Thus using (3.11), economic profits in steady state are

$$\pi_{ss}(K_{ss}) - \pi_{ss}'(K_{ss})K_{ss} = p_{ss}(Y_{ss} - (1-\eta)(Y_{ss}+\phi)). \tag{3.13}$$

Equating (3.13) to zero, the overhead cost is

$$\phi = \frac{\eta}{1-\eta}Y_{ss}. \tag{3.14}$$

Evaluating (E_t) of (3.8) in steady state, the shadow cost of investment expenditure is equal to the discounted value of future expenditures

$$\mu_{ss} = \frac{1}{(1-R^{-1}(1-\theta))}. \tag{3.15}$$

From (I_t) of (3.8) and (3.15) the shadow value of capital is

$$q_{ss} = \theta^{-\frac{1}{\varepsilon-1}}. \tag{3.16}$$

The shadow value is larger than one in steady state for the following reason. Investment projects increase the capital stock only when they mature. For given E_{ss} the size of I_{ss} falls with θ, due to the lower fraction θ of projects maturing in each period *and* the complementarity amongst investment goods.[5] The higher shadow value reduces the value of K_{ss} in steady state. Using (3.10), (3.14), (3.16) into (I_t) of (3.8) it follows that

$$K_{ss} = \Lambda\left(\frac{\alpha\theta^{\frac{1}{\varepsilon-1}}}{R-(1-\delta)}\right)^{\frac{\alpha+\eta(1-\alpha)}{\eta}},$$

where $\Lambda \equiv \left((1-\eta)^{1-\eta}\,(1-\alpha)^{(1-\alpha)(1-\eta)}\right)^{1/\eta}$. From (3.11) evaluated in steady state then

$$Y_{ss} = \left((1-\eta)^{\frac{1}{1-\alpha}}\,(1-\alpha)\right)^{\frac{1-\alpha}{\alpha+\eta(1-\alpha)}} K_{ss}^{\frac{\alpha}{\alpha+\eta(1-\alpha)}},$$

which used into (3.14) yields the value of ϕ.

[5] It is possible to eliminate the steady state inefficiency, by assuming that each investment project costs $\theta^{\frac{\varepsilon}{\varepsilon-1}}$ per period rather than θ.

Name	Variable	Value
Gross Interest Rate in s.s.	R	1.08
Depreciation Rate	δ	.08
Demand Elasticity	η^{-1}	4
Capital Share in Production	α	.27
Capital Adjustment Cost Parameter	ψ	2
Persistency of $\log A_t$	ρ_A	.78
Persistency of R_t	ρ_R	.31
Wage Rate	w_t	1
Demand Shifter Variable	z_t	1

Table 1: Partial Equilibrium Model: Calibration of Parameter Values

3.3 Calibration and Numerical Solution

The properties of the investment model discussed so far are described in this section by means of the impulse responses to unexpected innovations in the shocks. For brevity attention is focused to interest rate and productivity shocks, while the wage rate, w_t, and the demand shifter, z_t, are kept constant. The productivity and interest rate shocks evolve according to first order autoregressive processes

$$\begin{aligned} \log A_t &= \rho_A \log A_{t-1} + \varepsilon_t^A \\ R_t &= (1-\rho_R)R + \rho_R R_{t-1} + \varepsilon_t^R \end{aligned} \tag{3.17}$$

The solution of the model is computed by loglinearizing the equilibrium conditions around the non stochastic steady state. The resulting system of expectational difference equations is solved using Anderson and Moore [1985] algorithms.

The parameters of the model are calibrated on post World War II US data at yearly frequencies, and are summarized in Table 1.

The gross interest rate R_t is the rate of return to the firm's share and debt holders. The value of R_t is measured as a weighted average of the ex-post real returns on the S&P 500 index and the Moody's Baa Corporate Bond Yield during the years 1950-2000.[6] The weight on the equity return is the median share of equity over total asset (3/4) of Compustat firms over 1960-2000 (Welch [2004]). The series A_t, adjusted to account for the monopolistically competitive setting described above, is from the NBER-CES Manufacturing Industry Database (Bartelsman, Becker, and Gray [2000]). The database reports data on the entire US manufacturing sector at the 4-digit SIC code over the years 1958-1996.[7] The capital share in production, α, is computed as the average in the database, while the depreciation rate is set to 8%. The elasticity of labor demand, η^{-1} implies a markup over marginal cost of 33% (Woodford and Rotemberg [1999]). The

[6] The index and dividend of the S&P500 are from Shiller Robert [2000] , while the S&P 500 index and the Moody's Baa Corporate Bond Yield is from pubblication H.15 of the Board of Governors. Inflation is computed using the Consumer Price Index.

[7] The (unadjusted) TFP series is the 5-factor TFP annual growth rate, computed as the difference between real sales' growth rate and the sum of production inputs' growth rate weighted by the respective production function elasticities. The elasticities are computed from the share of each factor's expenditure over total revenues. With imperfect competition and overhead costs, the revenue based shares are unbiased estimates of the true elasticities so long as economic profits are zero (Hall [1988]). In the TFP calculation, however, the weight on the sales growth depends on the markup. With labor being the only variable production factor, for example: $\dot{A}_t = (1-\eta)\dot{Y}_t - \alpha\dot{K}_t - (1-\alpha)\dot{L}_t$, where a dotted variable denotes the logarithmic growth rate. Starting with the TFP growth rate computed under perfect competition, $\dot{A}_t^{comp}$, the growth rate of TFP is then simply constructed as $\dot{A}_t = \dot{A}_t^{comp} - \eta\dot{Y}_t$. In the theoretical model TFP is constant in steady state. The TFP index is thereby detrended in each sector. The value of ρ^a and the volatility in the innovation (used in the simulations of the next sections) is estimated through a linear AR(1) model for $\log A_t$ by pooling all the available data (over time and across sector).

adjustment cost parameter is from Cummins, Hasset, and Oliner [2003].[8]

The impulse response functions to unexpected 1% innovations in the interest rate and the productivity shocks are reported in Figures 1 through 4. The Figures display the responses of investment, the scale of new investment projects ($j \in M_{t-1}$), the shadow value of capital , the capital stock and labor input.[9] All responses are expressed as percentage deviations from steady state values, and the term investment refers to both E_t and I_t, as the two variables are equal when expressed as deviations close to steady state.

Empirical evidence on the investment process in the US finds an approximate duration of 2 years for projects involving structures, and lower values for investment in equipment.[10] Section 5 discusses how one can parametrize the elasticity of substitution, ε, using empirical results of the investment adjustment cost literature.

Consider, first, the response of the model to an interest rate shock. Due to the positive serial autocorrelation, a higher realization of R_t raises future expected rates. These, in turn, reduce the firm's weight on future dividends so that the shadow value of capital q_t falls. The incentives to invest are reduced and so are the capital stock and future levels of production.

The response of investment depends on the values of ε and θ. Consider the comparative static with respect to θ of Figure 1, where ε is equal to 2. With a higher expected duration of the investment project (lower θ), investment responds less and only gradually to the shock. Two separate channels are in action. First, loglinearizing (**??**) yields to

$$\hat{I}_t = \theta\ \hat{\iota}_t + (1-\theta)\ \hat{I}_{t-1}. \tag{3.18}$$

In each period, the fraction of projects under control of the firm falls with a longer duration, and thus investment is increasingly inertial, i.e. it depends relatively more on earlier choices of I_t given ι_t. Moreover as θ falls, the scale of the new investment projects, ι_t, responds less to the current realization of the shocks. To see why this is so, consider the first order condition for the optimal scale of the investment project. After some algebra it follows the first order condition is

$$\mathbb{E}_t \sum_{r=0}^{\infty} \left(\prod_{s=1}^{r} R_{t+r}^{-1}(1-\theta) \right) \left\{ q_{t+r} \left(\frac{I_{t+r}}{\iota_t(j)} \right)^{1/\varepsilon} - 1 - C_1(I_{t+r}, K_{t+r-1}) \right\} = 0.$$

The firm equates the expected marginal increase in the basket, $(I_{t+r}/\iota_j(t))^{1/\varepsilon}$, weighted by shadow value of capital, q_{t+r}, to the expected marginal cost of the investment project, which also includes the installation cost of capital $(1 - C_1(I_{t+r}, K_{t+r-1}))$. As θ falls, the firm puts additional weight in the future trade off between costs and benefits. Since the current innovation is dissipated as time passes, the firm optimally chooses to respond less to the current realization of the shock as it carries fewer information relevant to evaluating the trade off.

The degree of complementarity among investment projects, ε, also determines the response of investment.

[8] The value is taken from Table 3, second panel. Cummins, Hasset, and Oliner [2003] assume a quadratic adjustment cost function. Close to steady state ψ is approximately equal to the sensitivity of the investment to capital ratio with respect to marginal Q, when there is no TTB. The calibration below, uses the same value of ψ in the TTB model so that the models can be easily compared. As shown in Section 6, however, the empirically estimates of ψ would in general differ with TTB.

[9] The shadow value of capital is equal to the discounted value of marginal Q as shown in (3.7). The values of marginal and average Q differ in the model. Although most of the literature uses average Q as a proxy of marginal Q, other papers construct direct measures of marginal Q (Abel and Blanchard [1986], Gilchrist and Himmelberg [1998] and Cummins, Hasset, and Oliner [2003])

[10] Empirical work on TTB is almost exclusive to US data. Three different levels of aggregation are considered: project level (Mayer and Sonenblum [1955]), firm level (Koeva [2001]) or aggregate level (e.g. Altug [1989], Oliner, Rudebusch, and Sichel [1995] and Christiano and Vigfusson [2003]).

Figure 3 displays the responses for different values of ε, holding θ fixed at 2/3. From (3.18) the degree of investment inertia depends on how ι_t adjusts to the shocks for a given θ. The average scale of investment chosen at earlier dates, which is fully summarized in the values of E_{t-1} and I_{t-1}, affects the marginal return to current investment due to the complementarity between the investment goods. With a high degree of complementarity, low ε, the firms does not have large incentives to reduce the scale of the new projects. Indeed, due to the large scale of the other projects, the return to investment remains relatively high. On the contrary, the current choice of the investment projects is hardly affected by earlier investment choices for high levels of ε. Indeed as shown in Figure 3, for $\varepsilon = 100$, the firm fully rebalances I_t by over adjusting the scale of the new investment projects, and the inertial response of I_t completely disappears.

Now consider the response of the model to a productivity shock. Due to the positive serial correlation, the expected marginal product of capital increases after an innovation, and thus the firm has incentives to invest more. The higher productivity also raises the demand for labor. Due to the higher level of the factor inputs and productivity, output increases. As shown in Figures 2 and 4, the response of investment once again depends on the values of θ and ε. As for the interest rate shock, investment's response is gradual and dampened with longer duration and higher complementarity. As for the interest rate shock, indeed, with lower values of θ, and given ι_t, investment is increasingly inertial from (3.18). Further, ι_t responds less to the realization of the shock as it carries fewer information about the future returns. The adjustment of ι_t also crucially depends on the value of ε : as the degree of complementarity falls, the firm over adjusts the scale of the new investment projects, so as to offset the portion of investment that is predetermined. In this case, investment responds swiftly to the shocks and it is no longer inertial.

4 TTB with Deterministic Duration

This Section compares the TTB model with uncertain project duration presented so far, with two analogous models in which the duration of the investment projects is deterministic. The comparison serves to highlight the key elements of the model from which follows the gradual response of investment. As discussed in Section 3.3, the hump-shaped response of investment follows from firm's commitment to earlier decisions for the uncompleted projects, and from the imperfect substitutability of the investment goods that induces the firm not to fully compensate the overall investment decision through the projects under control.

In the model presented so far, some investment projects last longer than others ex-post, but, because the maturity of the project follows a Poisson process, all projects are perfectly homogeneous before the uncertainty is resolved. This implies, for example, that projects started many periods in advance have the same probability to mature as the ones that have just started. Further, due to the uncertain maturity and the assumption that the investment projects increase the scale of the investment basket, I_t, only when the project matures, only a fraction θ of the investment goods increase the scale of I_t at each date. The Poisson assumption and the impossibility for the firm to store the investment goods, are not crucial for investment to respond gradually. The crucial element, as shown below, is the ex-post heterogeneity in the projects' duration and the imperfect substitutability among the investment goods.

In the first model presented in this Section, the firm invests in different projects that have complementarities in I_t as with the model of the previous sections. The duration of each project is certain, however, and it differs across the investment goods. As shown from the impulse responses, investment will respond gradually also in this model.

Also in the second model presented the duration of the projects is heterogeneous ex-ante. In this model,

however, the capital stock is a composite of different capital types that are imperfect substitutes. The stock of each capital type is increased when the corresponding investment project matures. The response of investment expenditure in this model is also hump-shaped, a result that highlights the fact that the key element for a hump-shaped response is a mechanism that induces the firm not to fully compensate the overall investment decision through the projects under control. The specific mechanism, whether a complementarity between investment goods or capital goods, for example, is not crucial.

I will now describe the first model with complementarity among investment goods and deterministic duration of the projects. The model is a generalization of KP's, and the notation borrows from theirs. The firm invests in $j = 1, ..., J$ investment goods, and each investment project lasts for $N_j \geq 1$ periods. Let $S_{t,j,n}$ denote the scale of the project of type j at date t which is n stages away from completion. The scale of the investment project is chosen once and for all at its initiation (i.e., at N_j stages from completion), and it is fixed thereafter

$$S_{t+1,j,n} = S_{t,j,n+1} \text{ for all } j = 1, ..J \text{ and } n = 1, ..., N_j. \tag{4.1}$$

At each point in time the expenditure for each type j of investment is equal to sum on all J expenditures at all stages of completion

$$E_{t,j} = \sum_{n=1}^{N_j} \omega_{j,n} S_{t,j,n}. \tag{4.2}$$

Following KP it is assumed that $\omega_{j,n} = \omega_j$ for all n. The value of the ω_j's is chosen below so that the firm's discounted investment expenditure on each project j is independent of N_j. The total investment expenditure is

$$E_t = \sum_{j=1}^{J} a_j \; E_{t,j}, \tag{4.3}$$

where the a_j's weigh each expenditure's importance in steady state, as it will be discussed below. The weights are such that $0 < a_j < 1$ and $\sum_{j=1}^{J} a_j = 1$. Only completed projects add on the capital stock, and similarly to Section 2 the investment basket is

$$I_t = \left(\sum_{j=1}^{J} a_j \iota_{t,j}^{1-1/\varepsilon} \right)^{\varepsilon/(\varepsilon-1)}, \tag{4.4}$$

where

$$\iota_{t,j} = S_{t,j,1}. \tag{4.5}$$

The capital stock K_t depreciates at the constant rate δ as in (2.3). Maximizing out the variable production factors, the intertemporal investment decision of the firm is to maximize (2.5), where D_{t+r} is given by (2.6), subject to the investment technology (4.1)-(4.5). Let q_t be the date t shadow value of capital. The first order conditions that characterize the optimal investment decisions are then

$$\mathbb{E}_t \left\{ \phi_j \left(\sum_{n=0}^{N_j-1} \left(\prod_{s=1}^{n} R_{t+s}^{-1} \right) \right) \right\} = \tag{4.6}$$

$$= \mathbb{E}_t \left\{ \left(\prod_{n=1}^{N_j} R_{t+n}^{-1} \right) \left(q_{t+N_j-1} - C_1 \left(I_{t+N_j-1}, K_{t+N_j-2} \right) \right) \left(\frac{I_{t+N_j-1}}{\iota_{t+N_j-1}} \right)^{\frac{1}{\varepsilon}} \right\},$$

for all $j = 1, .., J$ and equation (K_t) of (3.8). The optimal choice of $S_{t,j,1}$ equates the discounted marginal investment expenditure, shown in the first line of (4.6), to the marginal benefit. The benefit is equal to the discounted marginal increase in the investment basket $N_j - 1$ from when decision is taken, weighted by the shadow value of capital q_{t+N_j-1} net of the marginal increase in the installation cost $C_1(\cdot,\cdot)$. The interpretation of the first order condition with respect to K_t follows from that of Section 3.3. The values of ω_j's are such that the steady state is symmetric $\iota_{ss,j} = \iota_{ss,j}$ and the shadow cost of capital is equal to one. From (4.6) evaluated at steady state values it follows that

$$\omega_j = \frac{1 - R^{1+N_j}}{1 - R}, \tag{4.7}$$

for all $j = 1, ...J$. The steady state level of the capital stock and of the other variables are the same as in Section 3.3 when $\theta = 1$. The model is solved by log-linearization around the non-stochastic steady state. The parameters of the model are the same to as in Table 1. The average duration of the investment project is one and a half year, thus $J = 2$, $N_j = j$ and $a_j = 1/2$. The elasticity of substitution in the investment basket, ε, is equal to 2. The response of the model to a one-percent innovation in the interest rate shock is shown in Figure 6, which also displays the response of the uncertain duration model calibrated with analogous parameters (θ equal to 2/3). The response of investment expenditure after an interest rate shock is gradual as for the model with uncertain duration. Indeed, the key element to obtain a gradual investment response is the ex-post heterogeneity in the duration of the projects. Although not crucial for the gradual response, the uncertain duration affects the response of investment, which tends to be smoother in this model for the following reason. The response of the project's scale in the deterministic model falls with longer duration. The same was true for the model with uncertain duration. But due the uncertain duration, it is possible for some projects to last more than two periods. Thus the scale of the project with uncertain duration responds less than the projects in the deterministic models. This also results into a smaller response of investment expenditure. Further, total expenditure with uncertain duration displays a smooth pattern over time, while in the deterministic model, the response of expenditure is delayed only for the first two years, after which the model response is as in the model with no TTB. In the model with deterministic duration the expenditure averages investment decisions of the preceding two periods. In the uncertain duration model, instead, investment expenditure summarizes decisions taken at all previous dates, albeit with smaller weights on earlier decisions, due to the random duration of the projects.

Now consider the second specification of the deterministic model, which differs from the one just presented in the source of imperfect substitutability amongst the investment types. The capital stock of the firm is made of J capital goods

$$K_t = \left(\sum_{j=1}^{J} a_j \; k_{t,j}^{1-1/\varepsilon} \right)^{\varepsilon/(\varepsilon-1)},$$

where the weights a_j's are defined as before. Each type of capital good j depreciates at the constant rate δ and investment projects increase the capital stock only when they are completed, thus

$$k_{t,j} = (1 - \delta)k_{t-1,j} + \iota_{t,j},$$

where $\iota_{j,t} = S_{t,j,1}$. It is assumed that the firm pays installation costs for each type of capital, thus the

dividend flow at date t is

$$D_t = \pi_t(K_{t-1}) - E_t - \sum_{j=1}^{J} a_j C(S_{t,j,1}, k_{t-1,j}),$$

where E_t is given by (4.3). Let $a_j q_{t,j}$ be the shadow value of the type j capital stock. The first order conditions, obtained by maximizing (2.5) subject to the investment technology, are then

$$\begin{aligned}
(S_{t,j,1}) \qquad & \mathbb{E}_t \left\{ \phi_j \left(\sum_{n=0}^{N_j-1} \left(\prod_{s=1}^{n} R_{t+s}^{-1} \right) \right) \right\} = \\
& = \mathbb{E}_t \left\{ \left(\prod_{n=1}^{N_j} R_{t+n}^{-1} \right) \left(q_{t+N_j-1} - C_1 \left(\iota_{t+N_j-1,j}, k_{t+N_j-2,j} \right) \right) \right\}, \\
(k_{jt}) \qquad & q_{t,j} = \mathbb{E}_t R_{t+1}^{-1} (\pi_{1,t+1} \left(\tfrac{K_t}{k_{t,j}} \right)^{\frac{1}{\varepsilon}} + (1-\delta)\, q_{t+1,j} - C_{2,t+1}(\iota_{t+1,j}, k_{t,j})),
\end{aligned}$$

for all $j = 1, ...J$. The economic interpretation of these optimality conditions is analogous to the discussion of the other model presented in this Section. Further, as before, the values of the ω_j's are equal to 4.7, so that the steady state is symmetric: $k_{ss,j} = k_{ss}$ and $\iota_{ss,j} = \iota_{ss}$. All remaining parameters are chosen as before. The impulse responses to a one percent innovation in the interest rate shock are shown in Figure 5. Once again, the delayed response of investment expenditure lasts for the first two years after the shock. However, the overall shape of the response is hump-shaped, although investment now peaks after the shock.[11] Once again, investment expenditure is in part predetermined by previous decisions, and the firm does not adjust it completely after that the shock hits due to the imperfect substitutability among the investment types.

An interesting alternative to the model with uncertain duration is one where the capital stock of the firm is made of heterogeneous capital types, as in the model just presented. But with a random maturity, the level of the each capital type remains uncertain in steady state, and thus it is not possible to solve the model by linearization.[12] The results just presented for the deterministic models, however, highlight that the response of investment in this alternative formulation would also be hump-shaped, which is the crucial implication of the TTB model of Section 2. Indeed as long as investment in some projects is partially predetermined and there is imperfect substitutability among the different types of investment, the firm optimally decides to respond only gradually to the shocks.

The next Section compares the TTB model with uncertain maturity to capital and investment adjustment cost models. The main result of the Section is an equivalence result, up to first linearization, of the TTB and investment adjustment cost model.

5 Time-to-Build and Adjustment Cost Models

This section compares the TTB model of Section 2 with two investment adjustment cost models. The first is a capital adjustment cost model (Lucas [1967], Treadway [1969], Uzawa [1969]) in which the firm faces costs of investment adjustment that depend on the change in the capital stock. An example of this adjustment function is the installation cost function $C(I_t, K_{t-1}) = c(I_t/K_{t-1})K_{t-1}$, included in the model of Section 2. The second model is one where the investment adjustment costs depend on the difference between the current and lagged investment flows (e.g. $S(I_t, I_{t-1})$). Christiano, Eichenbaum, and Evans [2005] show that

[11] The parameters of the investment technology (ε and the a_j's) were kept constant to the ones of Figure 6 to facilitate the comparison. Note, however, that the parameters have a different meaning in the two formulations and the investment expenditure can peak in the second period for alternative choices of the parameter values.

[12] Further the analysis in this model is also complicated by the fact that one cannot express the intertemporal decision problem in terms of "aggregated" quantities as in Section 2.

investment adjustment cost models capture well the dynamic response of aggregate investment to shocks.

The main result of this Section is that, up to first order linearization around the steady state and for appropriate choice of parameter values, the TTB and investment adjustment cost models share the same steady state and local dynamics in its neighborhood. Instead, as discussed in the next paragraph, investment dynamics with capital adjustment costs differs as it lacks the inertial response of investment of the other two models.

Consider the investment model with capital adjustment costs, i.e. $\theta = 1$ in the model of Section 2. With capital adjustment costs, the firm responds less to shocks than without costs of adjustment. In this latter case, indeed, the firm fully responds to the shocks and q_t is always one. With capital adjustment costs, instead, firms only partially adjusts and thus q_t diverges from one (see Figures 2 to 3). Thus, similarly to the TTB model, capital adjustment costs reduce investment's response and volatility. The two models differ, however, in the shape of the dynamic response. Because the shocks considered are temporary, the firm has the highest benefits and costs to increase investment when the shock hits in both models. But while in the TTB, the short run adjustment is very costly and induces the firm to respond gradually, in the capital adjustment cost model the cost is not sufficiently large to induce a gradual response. This follows from the fact that, for standard parametrizations of the the depreciation rate, the level of investment is only a small fraction of the capital stock in a neighborhood of the steady state.

The adjustment costs around steady state, instead, are large when specified in terms of the change in the investment flow. This specification has been proposed byChristiano, Eichenbaum, and Evans [2005] to describe the hump-shaped response of aggregate investment to monetary policy shocks. In this model, the firm solves

$$\max \mathbb{E}_t \left\{ \sum_{r=0}^{\infty} \left(\prod_{s=1}^{r} R_{t+r}^{-1} \right) \left((\pi_{t+r}(K_{t+r-1}) - \tau \ (I_{t+r} - S(I_{t+r}, I_{t+r-1})) \right) \right\}, \tag{5.1}$$

with respect to K_{t+r} and I_{t+r}, subject to the law of motion of K_t of (2.3).[13] The cost function has the form

$$S(I_{t+r}, I_{t+r-1}) \equiv s\left(\frac{I_{t+r}}{I_{t+r-1}}\right) I_{t+r-1}, \text{ where } s(1) = s'(1) = 0 \text{ and } s''(1) = \chi, \tag{5.2}$$

and τ is a fixed parameter that measures the shadow value of capital in steady state. Because the adjustment cost depends on the change in the investment levels rather than in the capital stock, investment's response is in general hump-shaped as in the TTB model. In particular, an equivalence exists between the two models as shown in the following Proposition

Proposition 1 (Equivalence between Investment Adjustment Costs and TTB) *Consider a firm that solves (5.1) subject to (5.2) and (2.3), and one that solves (3.6) with $\psi = 0$ subject to (3.5) and (2.3). If $\tau = \theta^{\frac{1}{1-\varepsilon}}$ and $\chi = \frac{(1-\theta)}{\theta\varepsilon(1-R^{-1}(1-\theta))}$ then the two models share the same steady state and local dynamics in a neighborhood of it.*

Proof. In Appendix. ■

Although the literature has shown that investment adjustment cost models help explain the response of aggregate investment to shocks, it has not provided yet an explanation for the existence of these shocks.

[13] The formulation of (5.1) follows the investment literature in including the adjustment cost in the dividend, and thus is expressed in units of output. Christiano, Eichenbaum, and Evans [2005], instead, include the adjustment cost in the law of motion of (K_t) that becomes

$$K_t = (1-\delta)K_{t-1} + I_t - S(I_t, I_{t-1}).$$

In this formulation the cost is paid in units of capital. The discussion that follows does not hinge upon the units of measurement of the adjustment cost function.

The results of Proposition 1, instead, directly link these costs to TTB, which thus can be thought as a microfoundation for the investment adjustment cost model. It also follows from the equivalence result that given values of the project's duration, θ, and of the second derivative of the adjustment cost function, χ, it is possible to obtain the implicit value of ε. For example, starting with a value of χ equal to .36, one obtains that, for $\theta = 2/3$, $\varepsilon = 2$, which is the benchmark parameter value in this paper. To account for the higher long run elasticity of investment, the value of χ is smaller than what was estimated by Christiano, Eichenbaum, and Evans [2005] on quarterly data.[14] This is obviously only an approximation and the value of χ should be directly estimated on annual data.[15]

6 TTB and Empirical Models with Capital Adjustment Costs

A large literature in recent years has empirically tested the investment model with convex capital adjustment costs. The literature finds only weak support for the model (see Chirinko [1993] for a review) as discussed below. The objective of this section is to present some implications of the TTB model presented so far for the empirical investment literature. In order to do so, I will first generate artificial data by simulating the model calibrated on US sectoral data. I will then run the same regression models considered in the literature, and show that some of the empirical failures discussed in the literature are consistent with a specification error of the model which omits to include TTB as modelled in this paper.

Consider the benchmark empirical model considered in the literature. The model follows from the first order conditions presented in Section 3.1, when the capital adjustment cost function takes quadratic form

$$c\left(\frac{I_t}{K_{t-1}}\right) \equiv \frac{\psi}{2}\left(\frac{I_t}{K_{t-1}} - \delta - e_t\right)^2. \tag{6.1}$$

The variable e_t is a shock to the adjustment cost function assumed to be serially uncorrelated. Using (6.1) into (I_t) of (3.8) yields the linear regression model

$$\frac{I_t}{K_{t-1}} = \beta_0 + \beta_1 Q_t + e_t, \tag{6.2}$$

The model (6.2) implies that Q_t is a sufficient statistic for the investment decision, and that the capital adjustment cost parameter ψ can be estimated as $\psi = 1/\beta_1$. This simple characterization of the investment decision has been rejected in the data along different dimensions. First, the estimated costs of capital adjustment are unreasonably large (e.g. Summers, Bosworth, Tobin, and White [1981]). Further, the value of Q_t is hardly a sufficient statistic for the investment decision. Indeed, first lagged values of Q_t enter significantly in the regression model, and this evidence, taken together with the high serial correlation of the error term, e_t, is interpreted (e.g. Oliner, Rudebusch, and Sichel [1995]) as indicating that the investment decision is more inertial than what predicted by the model (6.2). Second, the literature on financial frictions has also included the ratio of a firm's cash flow scaled by the capital stock, CF_t/K_{t-1} as a right hand side variable of (6.1) (for a review of this literature see Hubbard [1998]). The value of a firm's cash flow (revenues less taxes and expenses, excluding investment), is used to approximate the change in firm's net worth. Firms with higher net worth have more internal funds, and thus, tend to invest more with financial frictions, due to

[14] Christiano, Eichenbaum, and Evans [2005], Table 2 on page 17, find $\chi = .9$.

[15] An alternative strategy to relate quarterly to annual estimates is to assume that the elasticity of substitution ε is time-independent and thus obtain its value using interest rates and duration at annual frequencies. However ε in general is also time-dependent as the ability for a firm to substitute one good for another depends in on the time available for the adjustment.

the lower costs of financing and non-binding financing constraints. The empirical literature finds that cash flows are highly significant when included in the regression model, and tend to be more important than Q_t in explaining investment in terms of economic magnitudes.

As I show next, this evidence is consistent with investment decisions underlying the data that include TTB and an empirical model that does not. To show this, I first generate data on investment decisions using the model with the parameter values of Table 1, calibrated on US sectoral data, and then regress the model (6.2) on these data. The adjustment cost shock e_t is assumed to be serially uncorrelated and has a standard deviation of one percent.[16] Each simulation is composed of $5,000$ observations, a number comparable to empirical studies at the firm (e.g. Cummins, Hasset, and Oliner [2003]) and sectoral level (Bartelsman, Becker, and Gray [2000]). I consider two data generating processes. The value of ε is equal to 2 in both models. The value of θ is equal to one in the first model (no TTB) and to 2/3 in the second (average duration of one and a half year). While the true error term is serially uncorrelated the empirical model allows first order serial correlation: $e_t = \rho e_{t-1} + \varepsilon_t^e$. The parameter of the model and ρ are estimated using the Cochrane-Orcutt estimation procedure. An important issue in empirical work is the measure of marginal Q_t. The vast majority of the empirical literature approximates the value of marginal Q with average Q, which is easily measured for publicly traded companies. Indeed as shown by Hayashi [1982], average and marginal Q are equal when the firm is perfectly competitive *and* operates a constant return to scale technology. Both assumptions are not likely to hold empirically, and Abel and Blanchard [1986] and Cummins, Hasset, and Oliner [2003] among others, construct empirical measures for marginal Q. The model presented in the previous sections implies that average and marginal Q differ. This is not just due to the monopolistically competitive framework but also because of the TTB technology. Because, the objective here is to show the implications of the TTB technology, the analysis abstracts from measurement errors on Q_t and assumes that the econometrician can directly observe it. The results of the simulations are presented in the Table 3. All numbers are means over 100 simulations. The Table has six columns: the value of θ in the data generating process in the first three columns is 1 and in the last three is 2/3. For each of the two data generating processes, I consider three regression models. The first model only includes the contemporaneous value of Q_t. The second model augments (6.2) with four lags of Q_t, while the third model also includes CF_t/K_{t-1}.

Consider the estimates of the regression model on the data generating process without TTB. As shown in the first three columns, the coefficient on Q_t is always statistically significant and close to .5, so that ψ is roughly two as in the data generating process. In model (2), only the second lag is statistically significant, but the magnitude is close to zero. The coefficients on all other lags are smaller and none of them is statistically significant. In model (3), cash flows scaled by the capital stock is not statistically significant. As shown in the bottom part of the Table the estimate of ρ is roughly zero as in the data generating process, and the Durbin Watson statistic indicates the lack of serial correlation in the model.

Now consider the regression models when the data generating process includes TTB of one and a half year. Although the coefficient on Q_t is always statistically significant, the magnitude of the estimated coefficient is less than half than its true value. As discussed in Section 3.3, investment expenditure responds less to the realization of the shocks as TTB increases, as a lower fraction of the investment projects in under the control of the firm. As shown in Figures 1 to 4, the value of Q is hardly affected by the longer duration of TTB, and thus the volatility of investment expenditure, E_t, with respect to Q_t falls. Because the empirical

[16] The standard deviation of the capital adjustment cost is not calibrated on empirical data. The value of the volatility, however, is not crucial in the results and discussion that follows. The main effect of a higher volatility in e_t is a lower fit of the regression in terms of R^2 and standard errors of the estimated coefficients.

model (6.2) does not include TTB, however, the lower volatility of E_t is erroneously estimated as higher installation costs of capital. As discussed in Section 3.3, investment responds gradually to shocks with TTB, while the value of Q_t immediately responds to the productivity and interest rate shocks. The investment model without TTB cannot account for the inertial response in investment. As a result the residuals of the regression model are serially correlated over time. Indeed from the bottom panel of Table 3, the estimated values of ρ fall between .7 and .8, and, aside from model (5), the Durbin Watson statistic indicates that the residuals have persistence of order greater than one. Moreover, in the regression model (5), all lagged value of Q_t are statistically significant, and the magnitude of the regression coefficients are comparable to that of Q_t. Finally in the regression model (6), the cash-flows to capital stock ratio is statistically significant, and is economically more important than Q_t as an explanatory variable for investment. Indeed a one standard deviation increase in the cash flow ratio (equal to .12) raises the investment to capital ratio by almost twice the amount following a one standard deviation in the cash-flow to capital ratio (equal to .07). In the model presented in this paper no financial frictions are present, and cash flows help explaining the investment decision only due to the specification error.

The results presented above make a direct estimate of the TTB model a promising venue. How is it possible to directly estimate the TTB model in the data? First note that, due to the unobservability of I_t, it is only possible to estimate linear approximations of the model. For example, by linearizing the first order conditions (3.8) around the steady state values of $\frac{E_t}{E_{t-1}}$ and $\frac{E_t}{K_{t-1}}$, one obtains that

$$\frac{E_t}{E_{t-1}} = \beta_0 + \beta_1 Q_t + \beta_2 \frac{E_{t+1}}{E_t} + \beta_3 \frac{E_t}{K_{t-1}} + \varepsilon_t, \tag{6.3}$$

where ε_t is an expectational error, which is orthogonal to the information set available at date t, i.e. $\mathbb{E}_t(\varepsilon_t) = 0$. The value of the structural parameter can be reconstructed from the estimated β's using the following equalities: $\beta_2 = R^{-1}$, $\beta_1 = \frac{\theta\varepsilon\left(1-R^{-1}(1-\theta)\right)}{(1-\theta)}\theta^{\frac{1}{\varepsilon-1}}$ and $\beta_3 = -\theta^{\frac{1}{\varepsilon-1}}\beta_1\psi$. Using the orthogonality condition of ε_t, the model in 6.3 can be estimated using a linear-GMM by instrumenting the right hand side variables with variables that belong to the date t information set. The goodness of the linear approximation remains an open question, and especially so at higher levels of aggregation (e.g. sectoral or aggregate investment) where the investment series tend to be non-stationary. Further the correct estimates of the structural parameters crucially depend on the quality of the empirical approximation to marginal Q_t.

Although an empirical test of the TTB model is beyond the scope of this work, the results on simulated data presented in this Section underscore how the TTB model has the potential of capturing the empirical behavior of firms' investment decisions.

7 Real Business Cycle Model

This section embeds the TTB model of Section 2 in an otherwise canonical real business cycle model. The response of the model to an unexpected innovation in the TFP shock is then compared with that of a model with no time-to-build and with KP's specification of TTB.

The representative household is infinitely lived. At each date t he decides how much to consume, C_t, work, L_t, and how many stocks, S_t, to hold so as to maximize the discounted value of utility flows

$$\max_{\{C_t, L_t, S_t\}} \mathbb{E}_t \sum_{r=0}^{\infty} \beta^r U\left(C_{t+r}, 1 - L_{t+r}\right),$$

where the utility function is specified as

$$U\left(C_t, 1-L_t\right) = \log C_t + \upsilon \ \log(1-L_t),$$

and $0 < \beta < 1$ is a discount factor, which measures household's rate of impatience. At every date, the household is subject to the flow budget constraint

$$C_t + S_t V_t \leq W_t L_t + S_{t-1}\left(V_t + D_t\right),$$

where $W_t L_t$ is his labor income, while V_t and D_t are the stock price and dividend of the representative firm. Finally S_t is the level of stock holdings.

The production sector of the economy is composed of large number (measure one) of firms that produce Y_t, using the production function (3.9). Date t Prices are expressed in terms of Y_t, and those of the investment good and of C_t are equal to one. For brevity, I consider the case in which the demand function is perfectly elastic, so that the representative firm's markup is one and the overhead cost (3.14) is zero.

The first order conditions of the household's maximization problem yield

$$\begin{array}{ll} (L_t) & U_{2,t} = w_t U_{1,t} \\ (S_t) & U_{c,t} S_t = \mathbb{E}_t \beta U_{c,t+1}\left(S_{t+1} + D_{t+1}\right). \end{array} \tag{7.1}$$

Integrating (S_t) of (7.1) forward and using the transversality condition $\lim_{t\to\infty} \beta^r U_{c,t+r} S_{t+r}$ yields that the firm's ex-post realization of the discount factor is

$$R_{t+1}^{-1} = \beta \frac{U_{c,t+1}}{U_{c,t}}.$$

From (3.12) it also follows that with perfect competition,

$$\pi_t'(K_{t-1}) = \alpha A_t K_{t-1}^{\alpha-1} L_t^{1-\alpha},$$

in equilibrium. The dividend of the representative firm is equal to the constant fraction α of Y_t in equilibrium. The first order conditions that characterize the firm's optimal decisions are given by (3.8) along with the inverse equilibrium demand for labor services $w_t = (1-\alpha)Y_t/L_t$. Finally market clearing in the financial market yields the aggregate resource constraint $C_t + E_t = Y_t$.

Given the initial levels of K, E and I,and a sequence of the TFP shock A_t, an equilibrium is defined as a state-contingent sequence of prices and quantities such that a) firms and households solve their respective maximization problem, b) goods, labor and the financial markets clear.

Now consider the TTB model of Kydland and Prescott [1982]. They consider an investment technology with TTB but where the duration of the investment project is certain. Further, they assume that the representative firm makes only one type of investment. Their model is easily obtained by considering the case of $J = 1$ in either models with deterministic maturity presented in Section 4. I follow their original analysis and assume that the time unit is a quarter, that the investment projects last for four quarters, and that the investment expenditure is equally split over the four quarters, or $\omega_n = 1/4$ for all n. I also assume that the duration of the project in the random maturity TTB model is one year, or $\theta = 1/4$. The remaining parameter values common to the two models are reported in Table 2.

The parameters are calibrated on post-World War II US data (e.g. Prescott [1986]). The impulse response

Name	Variable	Value
Gross Interest Rate in s.s.	R	$(1.08)^{.25}$
Depreciation Rate	δ	.02
Demand Elasticity	η^{-1}	∞
Capital Share in Production	α	.36
Capital Adjustment Cost Parameter	ψ	0
Labor weight in utility	υ	such that $L_{ss} = .28$
Persistency of log A_t	ρ_A	.95

Table 2: RBC Model: Calibration of Parameter Values

functions to a one percent innovation in the productivity shock are shown Figure 7. The Figure reports the response of the uncertain TTB (TTB) and the Kydland and Prescott [1982] (TTB KP) model, and of a canonical RBC model with no TTB (no TTB).First consider the response of the canonical RBC model. With the exception of aggregate consumption, all variable peak when the productivity shock hits and then decay exponentially to pre-shock levels. The response of aggregate consumption is, instead, hump-shaped due to households' intertemporal smoothing motive. Overall the response of all variables in the KP model follow those of the canonical RBC model. The main difference between the two models, is the existence of deterministic cycles in the responses of the KP model, as first highlighted by Rouwenhorst [1991]. The representative firm increases the scale of the new projects when the technology shock hits the economy. In the three quarters after the shock, however, the firm reduces the scale of the new projects, because it has to live up to the previous commitments. In the fourth quarter after the shock, the projects with a large scale mature raising the capital stock and households' incentives to consume and of firms' to invest. The pattern then repeats itself, creating the cyclicalities in the impulse response functions. As noted by Rouwenhorst [1991], however, the cyclicalities are small relative to the overall response of the economy, which is close to the canonical RBC model. Thus Rouwenhorst [1991] challenges the central role of TTB posed by KP.

Now consider the TTB model with uncertain duration. As for the partial equilibrium analysis, investment expenditure responds only gradually to the higher productivity levels. The representative firm only controls the scale of the investment projects that have just matured, and it optimally decides not to perfectly adjust the overall expenditure through the projects under control due to the complementarities. The gradual investment response is in stark contrast with the swift response in the canonical RBC model and in KP's, and describes well the empirical response investment to shocks. Indeed the response of investment to productivity shocks (Christiano, Eichenbaum, and Vigfusson [2004]) and to monetary policy shocks (Christiano, Eichenbaum, and Evans [1998]) is hump-shaped. For monetary policy shocks, Christiano, Eichenbaum, and Evans [2005] find that firms with investment adjustment costs capture well the response of investment, thus from the results of Section 5, so does the TTB technology proposed in this paper.

Due to the lower response of investment expenditure, the response of aggregate consumption is amplified in a closed economy. Further due to the initial spike in aggregate consumption, households substitute labor for leisure and thus labor can fall after a positive productivity shock.[17] Lucca[2006] exploits the higher response of consumption due TTB to explain the higher volatility of aggregate consumption in low versus high income countries. For high income economies, it is possible to eliminate the initial spike in consumption and the fall in hours worked using preferences that display habit-persistence (see Edge [2000] and Christiano, Eichenbaum, and Evans [2005]).

[17] For the empirical response of labor to technology shocks see the discussion in Gali [1999], Basu, Fernald, and Kimball [2004] and Christiano, Eichenbaum, and Vigfusson [2004].

8 Conclusions

This paper presented a TTB model with a novel set of implications for the theory of investment. The model departs from the one considered by KP in two ways: firms invest in many investment projects that have complementarities, and the duration of each investment project is uncertain. The main implication of the model is that investment responds only gradually to shocks. The gradual response follows from the impossibility of the firm to modify the scale of the projects that have not matured, so that the investment decision is in part predetermined, and the unwillingness of the firm to compensate earlier commitments by adjusting the scale of new projects due to the imperfect substitutability between investment types.

The TTB model is shown to be equivalent to investment adjustment cost models, recently emphasized in the macroeconomic literature for capturing well the empirical response of aggregate investment to shocks. Further, numerical simulations in this paper show that the TTB model may help explaining some of the empirical shortcomings of the investment model with capital adjustment costs.

Future work should attempt to evaluate the model empirically and possibly compare the estimated parameters at different levels of aggregation: for example, at the firm, sectoral and aggregate levels. Aside from gauging the empirical magnitudes of the parameters, this analysis will help understanding as to whether the sluggishness in the response of aggregate investment is due to TTB at a more disaggregated level or not. A failure to uncover TTB at a more disaggregated level would suggest that reinterpretations of the model at a more aggregated levels should be considered.

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A Proof of Proposition 1

First consider the investment problem of a firm that faces investment adjustment costs. The firm solves (5.1) subject to (5.2) and (**??**). The first order conditions of the maximization problem are

$$\begin{aligned} (K_t) \quad & q_t = \mathbb{E}_t R_{t+1}^{-1}(\pi_{1,t+1} + (1-\delta)q_{t+1}), \\ (I_t) \quad & q_t = \tau(1 + S_1(I_t, I_{t-1}) + \mathbb{E}_t R_{t+1}^{-1}\mu_{t+1}\ S_2(I_{t+1}, I_t)). \end{aligned} \tag{A.1}$$

From (A.1) the steady state level of capital solves

$$\pi'(K_{ss}) = \tau(R - (1-\delta)). \tag{A.2}$$

For each variable X_t, let the corresponding hatted variable denote a percentage deviation from steady state, $\hat{X}_t = (X_t - X_{ss})/X_{ss}$. Equation (I_t) of (A.1) can be log-linearized in

$$\tau\hat{q}_t = \chi\left(\Delta\hat{I}_t - \mathbb{E}_t R_{t+1}^{-1}\Delta\hat{I}_{t+1}\right), \tag{A.3}$$

where $\Delta\hat{I}_t = \hat{I}_t - \hat{I}_{t-1}$.

Now consider the TTB model. The first order conditions that characterize the investment decision are (3.8), from which the capital stock in steady state solves

$$\pi'(K_{ss}) = \theta^{\frac{1}{1-\varepsilon}}(R - (1-\delta)) \tag{A.4}$$

Loglinearizing (E) of (3.8) it follows that

$$\hat{\mu}_t = R^{-1}(1-\theta)\mathbb{E}_t\left(\hat{\mu}_{t+1} - \hat{R}_{t+1}\right). \tag{A.5}$$

Loglinearizing (I) of (3.8) and substituting $\hat{\mu}_t$ from (A.5) it follows that

$$\hat{q}_t = \frac{(1-\theta)}{\theta\varepsilon\ (1 - R^{-1}(1-\theta))}\left(\Delta\hat{I}_t - \mathbb{E}_t R_{t+1}^{-1}\Delta\hat{I}_{t+1}\right). \tag{A.6}$$

From (A.2) and (A.4), the capital stock in steady state is the same in the two models when

$$\tau = \theta^{\frac{1}{1-\varepsilon}}, \tag{A.7}$$

holds. It then easily follows that all other quantities are also equal in steady state. When (A.7) and $\chi = \frac{(1-\theta)}{\theta\varepsilon(1-R^{-1}(1-\theta))}$ hold, the loglinearized first order condition (A.3) and (A.6) are also equal. Further note that the first order condition with respect to (K) is the same in both models, absent capital adjustment costs in the TTB model ($\phi = 0$). In the investment adjustment cost model, investment expenditure is equal to τI_t, so that the log-deviations of the basket and expenditure are equal. This is also the case in the TTB model. By loglinearizing (**??**) and (**??**) it follows that

$$\hat{E}_t = \int_0^1 \hat{\imath}_t(j)\ dj = \hat{I}_t.$$

Thus the claim of the Proposition follows

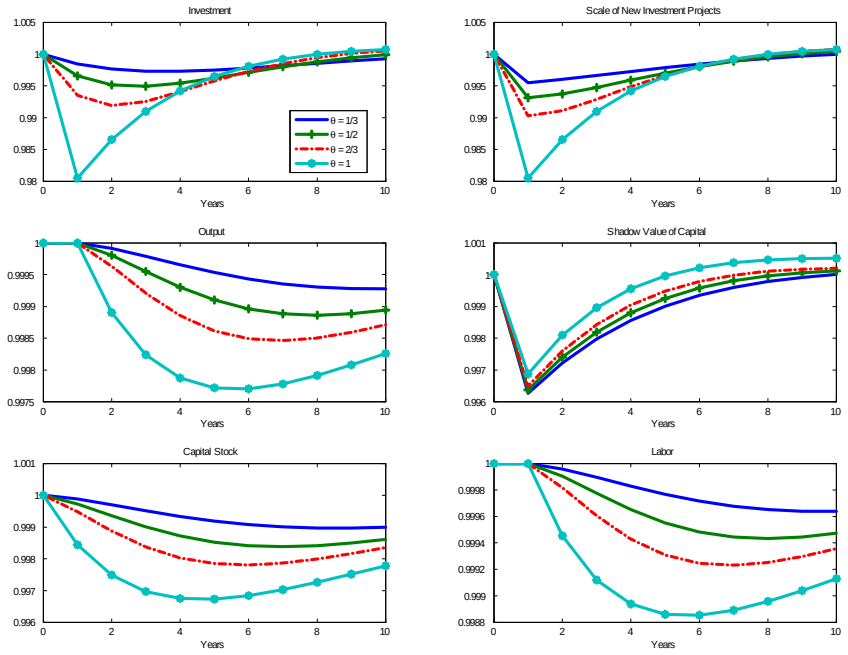

Figure 1: IRFs with $\varepsilon = 2$: Interest Rate Shock

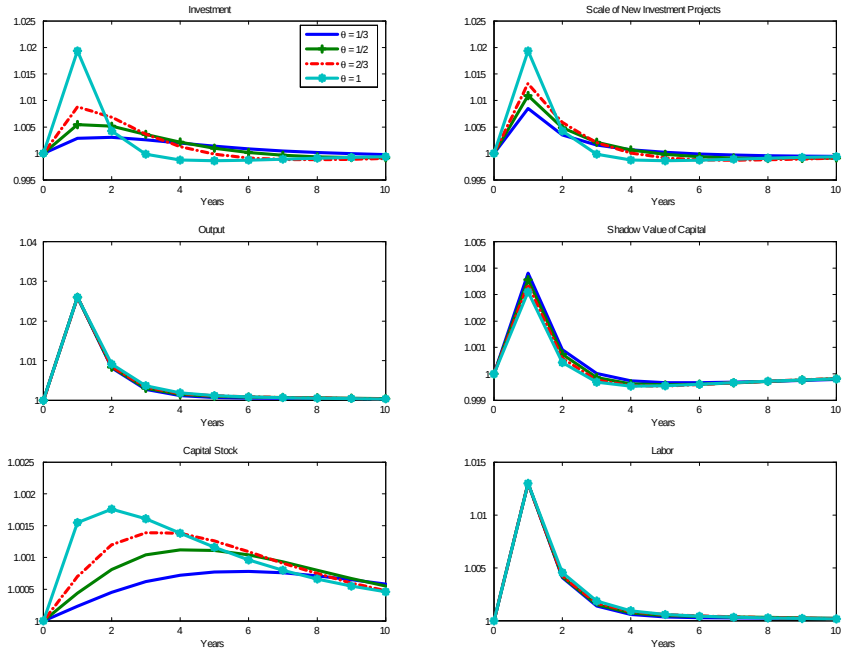

Figure 2: IRFs with $\varepsilon = 2$: Technology Shock

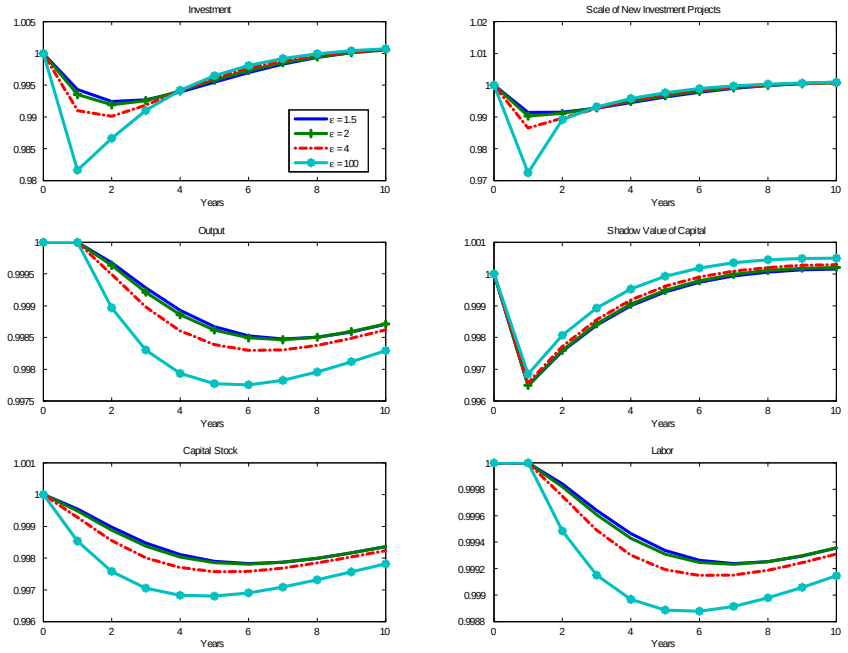

Figure 3: IRFs with $\theta = 2/3$: Interest Rate Shock

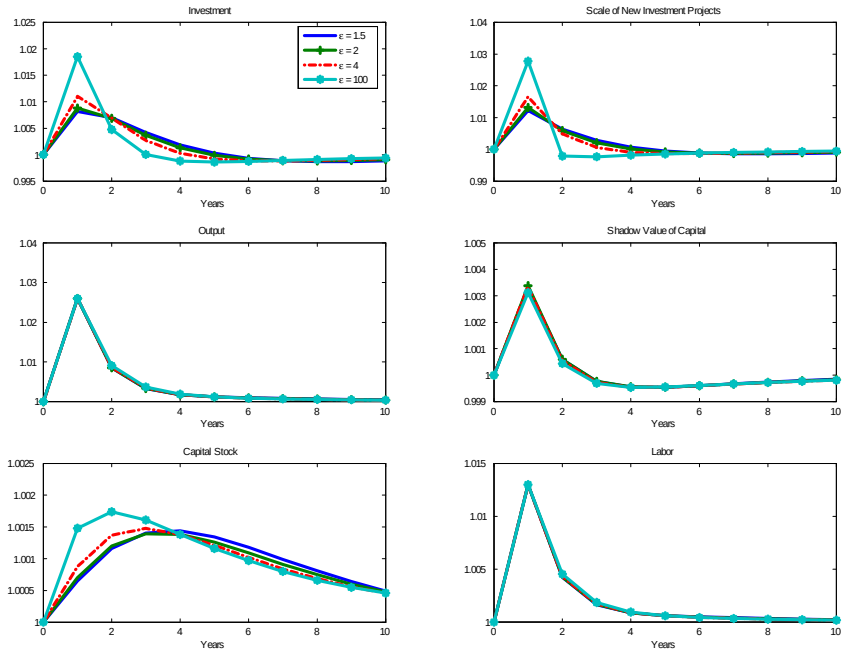

Figure 4: IRFs with $\theta = 2/3$: Technology Shock

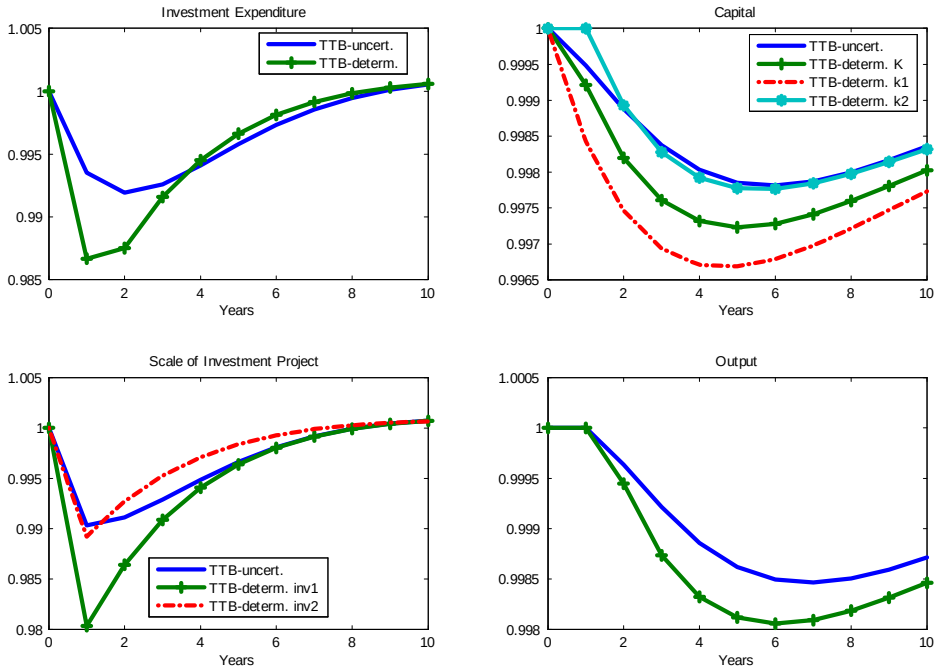

Figure 5: IRFs: Comparison with Deterministic TTB Model with Multiple Cap. Types; Interest Rate Shock

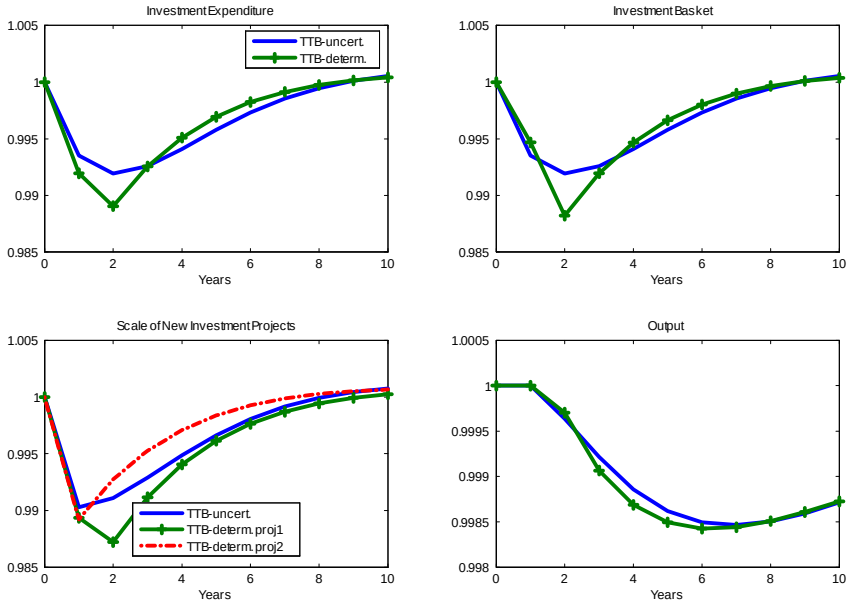

Figure 6: IRFs: Comparison with Deterministic TTB Model with Multiple Inv. Types; Interest Rate Shock

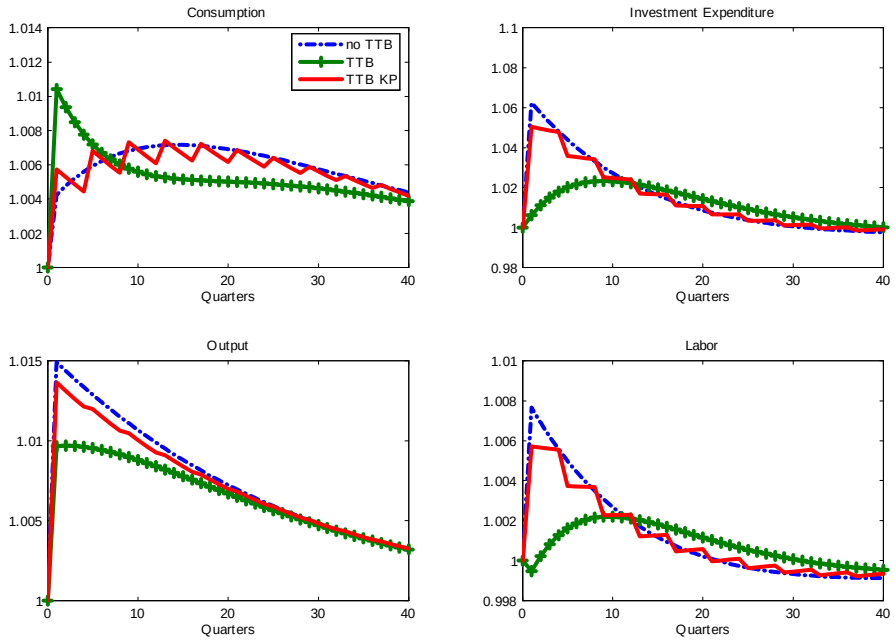

Figure 7: IRFs: RBC Model; Technology Shock

	Dependent Variable: E(t)/K(t-1)					
Theta in data generating process	1			2/3		
Model	(1)	(2)	(3)	(4)	(5)	(6)
Q(t)	.480	.478	.418	.165	.202	.066
	(.003)	(.003)	(.006)	(.001)	(.001)	(.005)
Q(t-1)		.006			.105	
		(.003)			(.001)	
Q(t-2)		-.001			.051	
		(.003)			(.001)	
Q(t-3)		-.001			.023	
		(.003)			(.001)	
Q(t-4)		-.001			.008	
		(.003)			(.001)	
CF(t)/K(t-1)			.004			.064
			(.006)			(.005)
Durbin Watson Statistic	2	2	2	1.36	2.01	1.37
Rho	.002	.001	.013	.745	.793	.773
Adj. R squared	.828	.83	.833	.732	.953	.758

Notes: Linear regression models estimated on simulated data from the models calibrated with parameters of Table 1 and sigma(e)=.01. There are 5,000 observations in each simulation, and the numbers reported are averages over 100 simulations. The value of epsilon= 2 and theta is equal to 1 or 2/3. All regression models are estimated using the Cochrane-Orcutt estimation procedure. The variable rho is the first order serial correlation in the error term. Standard errors of the estimated coefficients are reported in parenthesis below each coefficient.

Table 3: Regression Models on Artificially Generated Data