Home Bias, Exchange Rate Disconnect, and Optimal Exchange Rate Policy*

Jian Wang†
Research Department
Federal Reserve Bank of Dallas

August 2006

Abstract

This paper examines how much the central bank should adjust the interest rate in response to real exchange rate fluctuations in a two-country Dynamic Stochastic General Equilibrium (DSGE) model. The model can duplicate the major real business cycle statistics observed in the data and is the first DSGE model to study the role of home bias in consumption in explaining the exchange rate disconnect puzzle. A second-order accurate solution method is employed to solve the model and compare the conditional welfare under different policy regimes. The results suggest that the monetary authority should not seek to vigorously stabilize exchange rate fluctuations. In particular, when the central bank does not take a strong stance against the inflation rate, exchange rate stabilization may induce substantial welfare loss. The model also offers a new solution to the exchange rate disconnect puzzle, namely home bias in consumption. When home bias is high, the shock to Uncovered Interest-rate Parity (UIP) can substantially drive up exchange rate volatility while leaving the volatility of real macroeconomic variables, such as GDP, almost untouched. The model predicts the volatility of the real exchange rate relative to that of GDP increases with the extent of home bias. This relation is strongly supported by the data.

JEL Classifications: E52, F31, F41
Keywords: Home bias, Exchange rate volatility, Exchange rate disconnect, Optimal monetary policy

*I am grateful to Charles Engel, Menzie Chinn, Bruce Hansen for their priceless advice and encouragement. I would like to thank Kevin Grier, Ben Keen, Nelson Mark, Yongseok Shin and Ken West for valuable discussions and comments that have substantially improved this paper. I also want to thank seminar participants at the Bank of Canada, the Federal Reserve Bank of Dallas, Louisiana State University, North Carolina State University, University of Notre Dame, University of Oklahoma, Wayne State University, and University of Wisconsin-Madison. All views are those of the author and do not necessarily reflect the views of the Federal Reserve Bank of Dallas or the Federal Reserve System.

†Email: jian.wang@dal.frb.org Office address: Research Department, Federal Reserve Bank of Dallas, 2200 N. Pearl Street, Dallas, TX 75201 Phone: (214) 922-6471.

Contents

1 Introduction **1**

2 A Two-country DSGE Model **5**
2.1 Final Goods Market . . . 5
2.2 Intermediate Goods Market . . . 9
2.2.1 Optimal Prices for Intermediate Goods . . . 10
2.2.2 Aggregate Demand for Labor and Capital . . . 11
2.3 Household's Problem . . . 12
2.4 Monetary Policy Rules and Market Clearing Conditions . . . 15

3 Calibration and Real Business Cycle Statistics **16**

4 Solution Method **17**
4.1 Second-order Accurate Solution . . . 17
4.2 Conditional Welfare and the Welfare Measure . . . 18
4.2.1 Conditional Welfare . . . 19
4.2.2 Welfare Measure . . . 20

5 Policy Evaluation **20**
5.1 CPI Inflation Targeting . . . 21
5.2 Exchange Rate Stabilization . . . 22
5.3 Home Bias and Exchange Rate Disconnect . . . 25
5.3.1 Model Prediction . . . 25
5.3.2 Empirical Support . . . 26
5.4 Robustness Analysis . . . 27
5.4.1 Habit Persistence . . . 27
5.4.2 Exchange Rate Disconnect and Monetary Policy . . . 28

6 Conclusion **29**

References **31**

Appendix **36**

7 Transformation of Equations **36**

8 Stationary Economy **36**
8.1 Final Goods Sector . . . 36
8.2 Intermediate Goods Sector . . . 38
8.3 Household and Monetary Rules . . . 38

9 Steady State **39**

10 Comparison of Inflation Targeting Regimes **41**

1 Introduction

After the collapse of the fixed exchange rate system in the early seventies and the more recent failures of pegging the exchange rate within narrow ranges, many countries switched to a monetary policy regime defined by John Taylor as a trinity: (1) a flexible exchange rate, (2) an inflation target, and (3) a monetary policy rule. The role of the exchange rate in monetary policy is an important issue for this new policy regime. If a country chooses not to "permanently" fix the exchange rate, should the instrument of its monetary policy react to the exchange rate? Economists are far from a state of consensus on this issue. After reviewing the literature, John Taylor (2001) concluded:"An important and still unsettled issue for monetary policy in open economies is how much of an interest rate reaction there should be to the exchange rate in a monetary regime of a flexible exchange rate, an inflation target, and a monetary policy rule."

There are two different strands of literature focusing upon exchange rate stabilization. The first one is based on the tradeoff between exchange rate stabilization and the stability of the whole economy. Ball (1999) and Svensson (2000) find that the inclusion of the exchange rate into monetary policy rule can stabilize output or inflation or both. In contrast, Obstfeld and Rogoff (1995) warn policy makers that the required interest rate changes for exchange rate stabilization can aggravate instability elsewhere in the economy. In an empirical study on New Zealand, West (2004) finds "...that decreasing real exchange rate volatility by about 25% would require increasing output volatility by about 10-15%, inflation volatility by about 0-15% and interest rate volatility by about 15-40%." Another strand of literature uses welfare-based New Open Economy Macroeconomic (NOEM) models to study the tradeoff between real exchange rate stabilization and the expenditure switching effect.[1] Though elegant in allowing for analytical solutions, these NOEM models are highly stylized. For example, they are usually static with no price persistence, and are therefore unable to address the tradeoff considered in the first strand of literature. Due to the prominent role of inflation stabilization for the central banks, this simplification may not be innocent in discussions of exchange rate policy. In this paper, we tend to give a more comprehensive evaluation of exchange rate policy by incorporating tradeoffs that are discussed in both strands of literature into a two-country DSGE model.

We follow a vertical production structure similar to Devereux and Engel's (2004) and Obstfeld's (2001) to model the empirical finding that while the exchange rate has important effects on import prices,[2] consumption prices do not react to exchange rate changes.[3] In each country, there are two sectors: the final consumption goods sector and the intermediate goods sector. Final consumption goods cannot be traded across countries,

[1] For example, see Devereux and Engel (2003, 2004), Obstfeld (2001, 2002) and the references cited therein. An exception is Obstfeld (2004). He defends the flexible exchange rate regime in light of its function of allowing the central bank to pursue an independent interest rate policy in a world of international capital mobility.

[2] See Campa and Goldberg (2002) and Mumtaz, Oomen and Wang (2005), for example.

[3] The fluctuations of CPI based real exchange rates almost perfectly mirror the movements of nominal exchange rates in the data. See Engel (1993, 1999), Engel and Rogers (1996), Mussa (1986) and Parsley and Wei (2001).

and are produced from internationally traded intermediate goods. Unlike Kollmann's (2003), both sectors in this paper are monopolistic competitive with staggered pricing. The multiple price stickiness helps us match the fact that the cyclical behavior of CPI inflation generally differs from that of PPI inflation. The latter is typically more volatile and less persistent than the former.[4] We assume prices are set in the producer's currency and the law of one price holds for the intermediate goods sector. In this way, the import prices vary with the movements of the exchange rate. However, the final goods prices are set in the consumer's currency and therefore do not respond to the exchange rate in the short run. By this setup, our model endogenizes an important tradeoff in exchange rate policy discussed by Devereux and Engel (2004): the flexible exchange rate facilitates the expenditure switching effect for intermediate goods, but distorts the prices of final consumption goods across countries. When the expenditure switching effect is weak, they argue, the exchange rate should be stabilized to eliminate the price distortions for the final consumption goods.

The production of final consumption goods is biased toward home intermediate goods in this paper. The home bias is critical for our model to duplicate the well-documented disconnect between exchange rates and real Macroeconomic variables.[5] Unlike the models in which the exchange rate pass-through is low for imports, the movements of the exchange rate have important impacts on the prices of internationally traded goods in this paper. Only when the foreign market is just a small portion of total output, as in the case with high home bias, could the exchange rate shock in the financial market substantially increase the volatility of the exchange rate while keeping the volatility of real variables, such as GDP, almost unchanged. Our model is the first DSGE model to point out that when the exchange rate pass-through is high for imports, the home bias in consumption is necessary to generate exchange rate volatility, and exchange rate disconnect observed in the data.[6]

We introduce price persistence in our model *a lá* Calvo (1983) and Yun (1997). Under the staggered price setting, there is an additional tradeoff for exchange rate policy: the interest rate changes required for real exchange rate stabilization may induce inflation instability and therefore lower welfare level. Our model also adapts other features, such as capital accumulation and an incomplete financial market from the literature of international real business cycles, which allow us to duplicate the major real business cycle statistics

[4]For example, see Clark(1999). This difference might be caused by the CPI smoothing policy of the central banks rather than the generic difference in price stickiness in these two sectors. However, the same pattern has also been found in the case of mid-1930s (see Means (1935)), when the role of monetary policy was not as significant as it is today. Dong (2006) also finds this difference in estimating a small-open-economy DSGE model.

[5]For empirical studies on the exchange rate disconnect, see Flood and Rose (1995) and Duarte and Stockman (2005). In their famous "Six Puzzles" paper, Obstfeld and Rogoff (2000a) suggest that the home bias generated from high trade costs can be an important factor in explaining the exchange rate disconnect puzzle.

[6]Kollmann (2004) compares an economy with a 5% export sector to that with a 20% export sector and find the exchange rate volatility has more effects on the economy with 20% export sector. However, his paper focuses on the welfare effect of openness and did not study the quantitative importance of the openness in explaining the exchange rate disconnect.

observed in the data. Most previous studies assumed that government provides an optimal production subsidy to eliminate monopoly distortion. This assumption is based more on technical consideration than on empirical relevance: it is used to justify that the first-order approximation method of solving DSGE models can actually achieve a second-order accuracy for welfare evaluation.[7] We dismiss this assumption and allow long-run distortions in our model. To avoid spurious solutions from the first-order approximation method, we employ a second-order accurate solution method developed by Schmitt-Grohé and Uribe (2004a). Furthermore, the monetary policies in our paper are evaluated in terms of conditional expected welfare instead of the unconditional one used in standard practice.

Our results suggest a very weak stance against exchange rate fluctuations in our benchmark model. In particular, if the central bank takes a weak stance against the inflation rate, exchange rate stabilization may induce substantial welfare loss. Intuitively, the real exchange rate volatility in our model is driven mainly by home bias in consumption and Uncovered Interest-rate Parity (UIP) shock. When consumptions are biased toward domestic goods, the similarity between home and foreign final consumption bundles is low. Therefore, the CPI based real exchange rate fluctuations do not necessarily imply significant price distortions across countries. In this case, the gain from exchange rate stabilization is small. However, the restriction on exchange rate flexibility obstructs terms of trade adjustment for intermediate goods. What's more, the movements of interest rate required for exchange rate stabilization induce prolonged deviations of the inflation rate from its steady-state level, which also lowers welfare level. This finding confirms the conjecture of Obsfeld and Rogoff (1995) that exchange rate stabilization will cause economic instability that may be worse than the exchange rate swings themselves.

With the decrease of home bias, we find it is more desirable to stabilize the real exchange rate. The decrease of home bias has two effects. First, the final consumption bundles become more similar, which means a higher welfare gain from real exchange rate stabilization. This is because in this case, real exchange rate changes reflect deviations from the law of one price, which are not efficient. In addition, our model predicts a more stable real exchange rate in the case of less home bias.[8] Therefore, the central banks do not have to adjust the interest rate as much as in the previous case to stabilize the real exchange rate. This reduces the cost of real exchange rate stabilization. However, we should be cautious in interpreting this result as offering support for exchange rate stabilization. In the case with little home bias, the volatility of the real exchange rate relative to that of the GDP is much smaller than what is observed in the data. This situation cannot be improved by increasing exchange rate shock (UIP shock) in our model: the volatility of

[7] See Chapter 6 of Woodford (2003) for a proof. When long-run inefficiency exists, the standard first-order accurate solution method is very likely to generate spurious welfare rankings. Please refer to Kim and Kim (2004) for an example, and Woodford (2003) for more discussion.

[8] Empirically, this is supported by the finding of Hau (2002) that the real exchange rate volatility is negatively correlated with the economic openness of a country.

GDP also increases quickly when we use UIP shock to pump up the exchange rate volatility. So the case with little home bias is contradictory to two important empirical findings: exchange rate volatility and exchange rate disconnect.

In contrast, our benchmark model is more likely to reflect current economic situations. Intuitively, when home bias is high, the foreign market is only a very small portion of the GDP. The exchange rate shock in the financial market can drive up exchange rate volatility, but has very limited impact on the GDP. In this sense, our model provides an interesting solution to the exchange rate disconnect puzzle: home bias in consumption. Our model also predicts that the volatility of real exchange rate relative to the volatility of GDP is positively correlated with the extent of home bias. We find strong support for this prediction in the data.

Our results are also robust under the preference with habit persistence. Bergin and Tchakarov (2004) report a much higher welfare loss from exchange rate variation when the preference exhibits habit persistence. Intuitively, when households are more risk sensitive, welfare loss is higher for given exogenous shocks. However this does not guarantee that the central banks should react to exchange rate variation due only to this higher welfare loss. The same tradeoff still exists as before, and the cost of real exchange rate stabilization is also higher under habit persistence. Our model shows that the cost still exceeds the benefit for real exchange rate stabilization in this case.

In this paper, we take home bias in consumption as exogenously given. Helpman (1999) argues that there is no clear evidence of home bias in preference after controlling for income. However, our treatment of home bias can be taken as a shortcut for a model with no home bias in consumption but with high international trade costs, such as the model in Atkeson and Burstein (2005). Although theoretically, exchange rate stabilization can decrease trade costs by reducing exchange rate risk, we doubt this effect would be significant enough to overturn our results. Burstein, Neves and Rebelo (2003) find a trade cost that is large enough to generate the same home bias level as in our model, even after controlling for nominal exchange rate fluctuations.

We have to admit that our results are critically contingent on the mechanism that generates real exchange rate volatility. There is no consensus among economists on this issue. Chari, Kehoe and McGrattan (2002) and Devereux and Engel (2002) attribute exchange rate volatility to nominal stickiness. In contrast, this volatility is mainly driven by home bias in final consumption bundles in our model, Atkeson and Burstein (2005) and Ghironi and Metliz (2005). Our results suggest that this debate may have very important implications for the choice of exchange rate policy. Further research on whether our results are robust under the mechanism of sticky-price-driven exchange rate volatility is desirable.

The remainder of the paper is organized as follows: Section 2 lays out the theoretical model; Section

3 provides details about calibration and compares the business cycle statistics of our model with those in the data; Section 4 discusses the solution method and related computational issues; Section 5 presents our findings for policy evaluation; and Section 6 concludes and discusses potential future research directions.

2 A Two-country DSGE Model

The world economy consists of two countries: Home and Foreign. There are two sectors of production in each country: the final goods sector and the intermediate goods sector. The final goods are internationally non-tradeable, and are produced from the internationally traded Home and Foreign intermediate goods composites. Figure 1 shows the production structure in the Home country. A symmetric structure holds for the Foreign country.

In the Home final goods sector, there is a continuum of differentiated final goods $Y_t(i)$ indexed by $i \in [0, 1]$. The representative household of Home country uses them to form a final goods composite Y_t according to equation (1) for consumption, investment, saving, and associated costs[9]:

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}. \tag{1}$$

In equation (1), θ is the elasticity of substitution between differentiated final goods $Y_t(i)$. Each variety of final goods is produced from the Home and Foreign intermediate goods composites Y_{Ht} and Y_{Ft} by a single final goods firm. The Home (Foreign) intermediate goods composite is composed of differentiated Home (Foreign) intermediate goods $Y_{Ht}(j)$ ($Y_{Ft}(j)$). In the intermediate goods sector, each variety of Home (Foreign) intermediate goods is produced by a single firm with capital and labor in the Home (Foreign) country.

2.1 Final Goods Market

The final goods market is monopolistic competitive. In the Home country, each final goods firm produces a variety of final goods from the Home and Foreign intermediate goods composites according to equation (2):

$$Y_t^s(i) = \left[\alpha^{\frac{1}{\gamma}} Y_{Ht}^d(i)^{\frac{\gamma-1}{\gamma}} + (1-\alpha)^{\frac{1}{\gamma}} Y_{Ft}^d(i)^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}}. \tag{2}$$

$Y_t^s(i)$ is the supply of final good i, and $Y_{Ht}^d(i)$ $\left(Y_{Ft}^d(i)\right)$ is the Home (Foreign) intermediate goods composite demanded by final goods firm i. α is the weight of the home intermediate goods composite required for

[9] We will give more information concerning these costs later, when we discuss the household's problem.

producing final consumption goods. We say the consumption is home biased when $\alpha > \frac{1}{2}$. γ is the elasticity of substitution between the home and foreign intermediate goods composites. Symmetrically, the production function for Foreign final goods producer i is

$$Y_t^{s*}(i) = \left[\alpha^{\frac{1}{\gamma}} Y_{Ft}^{d*}(i)^{\frac{\gamma-1}{\gamma}} + (1-\alpha)^{\frac{1}{\gamma}} Y_{Ht}^{d*}(i)^{\frac{\gamma-1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}}. \tag{3}$$

The variables with asterisks in equation (3) are foreign counterparts of the variables in Home country. Due to the symmetry between the two countries, we will focus only on the Home country to describe our model.

In each country, final goods prices are denominated in the consumer's currency. In contrast, all the intermediate goods are priced in the producer's currency. Here, we take the choice of price-setting currency as exogenously given.[10] However, our setup is consistent with the empirical finding that the exchange rate is highly correlated with the terms of trade,[11] while the linkage between the exchange rate and consumer prices is fairly weak.

From equation (1), it is easy for us to find the demand function for final good i

$$Y_t^d(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} Y_t^d, \tag{4}$$

where Y_t^d is the demand for the final goods composite, and its price (CPI) is defined as

$$P_t = \left[\int_0^1 P_t^{1-\theta}(i)di\right]^{\frac{1}{1-\theta}}. \tag{5}$$

From the market clearing condition for final good i, we obtain

$$Y_t^s(i) = Y_t^d(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} Y_t^d. \tag{6}$$

Final goods producers take factor prices as given when buying home and foreign intermediate goods for production. The marginal cost function of final goods firm i is obtained from the production function (2):

$$\begin{aligned} mc_t(i) &= mc_t \\ &= \left[\alpha P_{Ht}^{1-\gamma} + (1-\alpha)(S_t P_{Ft}^*)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}. \end{aligned} \tag{7}$$

P_{Ft}^* is the price of Foreign intermediate goods composite denominated in Foreign currency, and S_t represents

[10] See Devereux, Engel and Storgaard (2004) for an example of endogenous currency choice, and Engel (2004) for discussions about the optimal choice of pricing currency.

[11] See Obsfeld and Rogoff (2000b).

the nominal exchange rate, which is the Home currency price of one unit of Foreign currency. We assume the law of one price holds for the intermediate goods market. So $S_t P^*_{Ft}$ is the price of the Foreign intermediate goods composite in Home country. This marginal cost is actually identical for all final goods firms.

We introduce staggered price setting *a lá* Calvo (1983) and Yun (1997). In each period, an individual firm has a probability $1-\lambda_f$ of re-optimizing its price. Otherwise, it will just charge a price equal to last period's price multiplied by the long-run inflation rate (π). When a final goods firm re-optimizes its price, it will choose a price P_{tt} to maximize the expected life-time real profit.[12] So the profit maximization problem can be written as

$$\max_{P_{tt}} \sum_{k=0}^{\infty} E_t \left\{ \lambda_f^k \Gamma_{t,t+k} P_{t+k}^{-1} \left[\left(\pi^k P_{tt} - mc_{t+k}\right) \left(\frac{\pi^k P_{tt}}{P_{t+k}}\right)^{-\theta} Y_{t+k}^d \right] \right\},$$

where $\Gamma_{t,t+k}$ is the pricing kernel between period t and $t+k$. We assume all firms are owned by home households, and therefore $\Gamma_{t,t+k}$ is the marginal rate of substitution between time t and time $t+k$ consumption. From the first order condition, we can find that the optimal price P_{tt} satisfies

$$P_{tt} = \frac{\theta \sum_{k=0}^{\infty} (\lambda_f \pi^{-\theta})^k E_t \left[\Gamma_{t,t+k} mc_{t+k} P_{t+k}^{\theta-1} Y_{t+k}\right]}{(\theta-1) \sum_{k=0}^{\infty} \left(\lambda_f \pi^{1-\theta}\right)^k E_t \left[\Gamma_{t,t+k} P_{t+k}^{\theta-1} Y_{t+k}\right]}. \tag{8}$$

When price is flexible ($\lambda_f = 0$), the optimal price reduces to $P_{tt} = \theta mc_t/(\theta-1)$. The monopolist charges a constant markup over its marginal cost.

Under this staggered price setting environment, prices are not synchronized across firms. At any time t, only a fraction of $1-\lambda_f$ firms charge up-to-date optimal price P_{tt}. A fraction of $\lambda_f^k(1-\lambda_f)$ firms charge outdated price $P_{t-k,t}$ for $k = 1, 2, ...$ From equation (5), the price of the final goods composite P_t evolves according to equation (9)

$$P_t = \left[(1-\lambda_f) P_{tt}^{1-\theta} + \lambda_f \left(\pi P_{t-1}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}. \tag{9}$$

The price dispersion among firms is inefficient. Aggregating equation (6) across final goods firms, we find

$$\begin{aligned} Y_t^s &= \left(\int_0^1 Y_t^{s\,\frac{\theta-1}{\theta}}(i)di\right)^{\frac{\theta}{\theta-1}} \\ &= \left(\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-(\theta-1)} di\right)^{\frac{\theta}{\theta-1}} Y_t^d. \end{aligned} \tag{10}$$

[12] For the notation P_{tt}, the first time subscript denotes when the price is re-optimized, and the second time subscript gives the current time. For example, P_{tt+k} means the price is re-optimized at time t and is still effective at time $t+k$. From our setup, $P_{tt+k} = \pi^k P_{tt}$.

It is easy for us to prove that $\left(\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-(\theta-1)} di\right)^{\frac{\theta}{\theta-1}} \geq 1$, and that the equality holds only when $P_t(i)$ is identical for all firms.[13] When final goods prices vary across firms, the final goods firms have to supply more than the market demand level. Intuitively, all final goods are identical in forming the final goods composite. If their prices are different, the relative price distortion causes waste in the production of the final goods composite. This inefficiency of price dispersion provides incentive for the central banks to stabilize the inflation rate in recent studies on monetary policy using micro-structure models.

Next, we derive the demand functions for the intermediate goods composite. From the production function of final goods producers (equation (2)), we can find the demand function of final goods firm i for the Home and Foreign intermediate goods composites (factor demand functions)

$$Y_{Ht}^d(i) = \alpha \left(\frac{P_{Ht}}{mc_t}\right)^{-\gamma} Y_t^s(i) \tag{11}$$

$$Y_{Ft}^d(i) = (1-\alpha) \left(\frac{S_t P_{Ft}^*}{mc_t}\right)^{-\gamma} Y_t^s(i). \tag{12}$$

We substitute the market clearing condition in equation (6) into the above equations and aggregate them across all final goods producers (i) to find the aggregate demand for home and foreign intermediate goods composites Y_{Ht}^d and Y_{Ft}^d

$$\begin{aligned} Y_{Ht}^d &= \int_0^1 Y_{Ht}^d(i) di \\ &= \alpha \left(\frac{P_{Ht}}{mc_t}\right)^{-\gamma} \left(\frac{P_t^A}{P_t}\right)^{-\theta} Y_t^d \end{aligned} \tag{13}$$

$$\begin{aligned} Y_{Ft}^d &= \int_0^1 Y_{Ft}^d(i) di \\ &= (1-\alpha) \left(\frac{S_t P_{Ft}^*}{mc_t}\right)^{-\gamma} \left(\frac{P_t^A}{P_t}\right)^{-\theta} Y_t^d, \end{aligned} \tag{14}$$

where $P_t^A = \left(\int_0^1 P_t(i)^{-\theta} di\right)^{-1/\theta}$. From our setup of staggered pricing, we know P_t^A evolves according to

$$P_t^A = \left[(1-\lambda_f) P_{tt}^{-\theta} + \lambda_f \left(P_{t-1}^A \pi\right)^{-\theta}\right]^{-\frac{1}{\theta}}. \tag{15}$$

Due to the monopoly power, the final goods firms earn positive profits in each period. The period t profit

[13] See Schmitt-Grohé and Uribe (2004b) for a proof.

for final goods firm i is

$$\pi_t^f(i) = (P_t(i) - mc_t)\, Y_t^d(i). \tag{16}$$

The aggregate profit is obtained in equation (17) and delivered to households in each period

$$\begin{aligned} \pi_t^f &= \int_0^1 \pi_t^f(i)di \\ &= P_t Y_t^d - mc_t \left(\frac{P_t^A}{P_t}\right)^{-\theta} Y_t^d. \end{aligned} \tag{17}$$

In this section, we have 9 aggregate variables: Y_t^d, Y_{Ht}^d, Y_{Ft}^d, P_t, P_{Ht}, P_{Ft}^*, P_{tt}, P_t^A and mc_t. Correspondingly, we have 6 equations at the aggregate level: equations (7), (8), (9), (13), (14) and (15). Similarly, there are 7 aggregate variables (P_{Ht} and P_{Ft}^* are the same in these two markets.) and 6 equations in the Foreign market. In total, we have 16 aggregate variables and 12 equations.

2.2 Intermediate Goods Market

The Home intermediate goods composite used by final goods producers is made from a continuum of differentiated intermediate goods indexed by $j \in [0, 1]$ according to equation (18)

$$Y_{Ht} = \left[\int_0^1 Y_{Ht}(j)^{\frac{\phi-1}{\phi}} dj\right]^{\frac{\phi}{\phi-1}}, \tag{18}$$

where ϕ is the elasticity of substitution between differentiated intermediate goods. The Foreign intermediate goods composite is made in the same way from Foreign differentiated intermediate goods. We suppose that intermediate goods firms set prices in the producer's currency, and the law of one price holds in this market. Let $P_{Ht}(j)$ be the price of Home intermediate good j in the Home market, and let $P_{Ht}^*(j)$ be the price in the Foreign market. By LOP, we have $P_{Ht}(j) = S_t P_{Ht}^*(j)$. From equation (18), we can find the demand function for Home intermediate good j in the Home and Foreign markets

$$Y_{Ht}^d(j) = \left(\frac{P_{Ht}(j)}{P_{Ht}}\right)^{-\phi} Y_{Ht}^d \tag{19}$$

$$Y_{Ht}^{d*}(j) = \left(\frac{P_{Ht}^*(j)}{P_{Ht}^*}\right)^{-\phi} Y_{Ht}^{d*}, \tag{20}$$

where P_{Ht} and P_{Ht}^* are the prices of the Home intermediate goods composite at Home and Foreign markets respectively, which are defined as

$$P_{Ht} = \left[\int_0^1 P_{Ht}^{1-\phi}(j)dj\right]^{\frac{1}{1-\phi}} \tag{21}$$

$$P^*_{Ht} = \left[\int_0^1 \left(P^*_{Ht}(j)\right)^{1-\phi} dj \right]^{\frac{1}{1-\phi}}. \tag{22}$$

By LOP, $P_{Ht} = S_t P^*_{Ht}$. Y^d_{Ht} and Y^{d*}_{Ht} are, respectively, the aggregate demand for the Home intermediate goods composite in the Home and Foreign countries as we derived in the last section.

The intermediate goods producers rent capital and labor from households for production. The technology takes a standard Cobb-Douglas form

$$Y^s_{Ht}(j) = A_t K_t^{\psi}(j) L_t^{1-\psi}(j), \tag{23}$$

where $Y^s_{Ht}(j)$ is the supply of intermediate good j. $K_t(j)$ and $L_t(j)$ are, respectively, capital and labor used by intermediate goods company j. A_t (A^*_t) is technology shock in the Home (Foreign) country, which is identical for all firms in that country. The logarithms of the productivity shocks are assumed to follow a VAR(1) process

$$\begin{bmatrix} log(A_t) \\ log(A^*_t) \end{bmatrix} = \begin{bmatrix} \xi_{11} & \xi_{12} \\ \xi_{21} & \xi_{22} \end{bmatrix} \begin{bmatrix} log(A_{t-1}) \\ log(A^*_{t-1}) \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ \varepsilon^*_t \end{bmatrix}, \tag{24}$$

where $\xi_{12} = \xi_{21}$ are technology spillovers. The vector containing ε_t and ε^*_t is i.i.d. with zero means and variance-covariance matrix V. From this production function, we obtain the marginal cost

$$\begin{aligned} mc^{int}_t(j) &= mc^{int}_t \\ &= A_t^{-1} R_t^{\psi} W_t^{1-\psi} \psi^{-\psi} (1-\psi)^{\psi-1}, \end{aligned} \tag{25}$$

where W_t represents nominal wage, and R_t is nominal rent of capital.

2.2.1 Optimal Prices for Intermediate Goods

We follow the same way as in the final goods sector to introduce staggered prices. $1-\lambda_{int}$ is the probability for firm j to re-optimize its problem. When re-optimizing price, the intermediate goods producer j chooses a price P_{Htt} to maximize discounted lifetime real profit under the marginal cost function as in equation (25), and the aggregate demand functions as in equations (19) and (20). The profit maximization problem can be described as

$$\max_{P_{Htt}} \sum_{k=0}^{\infty} E_t \left\{ \lambda^k_{int} \Gamma_{t,t+k} P^{-1}_{Ht+k} \left[\left(\pi^k P_{Htt} - mc^{int}_{t+k}\right) \left(\frac{\pi^k P_{Htt}}{P_{Ht+k}} \right)^{-\phi} Y^w_{Ht+k} \right] \right\},$$

where $Y^w_{Ht+k} = Y^d_{Ht+k} + Y^{d*}_{Ht+k}$ is the world demand for the Home intermediate goods composite. From the first order condition, it is easy for us to find the optimal price as in equation (26)

$$P_{Htt} = \frac{\phi \sum_{k=0}^{\infty} \left(\lambda_{int}\pi^{-\phi}\right)^k E_t \left[\Gamma_{t,t+k} mc^{int}_{t+k} P^{\phi-1}_{Ht+k} Y^w_{Ht+k}\right]}{(\phi-1) \sum_{k=0}^{\infty} \left(\lambda_{int}\pi^{1-\phi}\right)^k E_t \left[\Gamma_{t,t+k} P^{\phi-1}_{Ht+k} Y^w_{Ht+k}\right]}. \tag{26}$$

From equation (21), the price of the Home intermediate goods composite in Home market evolves according to

$$P_{Ht} = \left[(1-\lambda_{int})P^{1-\phi}_{Htt} + \lambda_{int}\left(\pi P_{Ht-1}\right)^{1-\phi}\right]^{\frac{1}{1-\phi}}. \tag{27}$$

Following an argument similar to the one we used in the final goods market, we can show from equations (18), (19) and (21) that price dispersion among intermediate goods producers is also inefficient.

2.2.2 Aggregate Demand for Labor and Capital

In this section, we use the market clearing conditions of intermediate goods to derive the aggregate demand for labor and capital. From the production function of intermediate goods firms (equation(23)) and the market clearing condition

$$Y^d_{Ht}(j) + Y^{d*}_{Ht}(j) = Y^s_{Ht}(j),$$

we can get the labor demand function of intermediate goods firm j

$$L^d_t(j) = A^{-1}_t \left(\frac{1-\psi}{\psi}\right)^{\psi} \left(\frac{R_t}{W_t}\right)^{\psi} (Y^d_{Ht}(j) + Y^{d*}_{Ht}(j)). \tag{28}$$

Substituting equations (19) and (20) into the above equation, and aggregating over intermediate goods firms (j), we obtain the aggregate labor demand function

$$\begin{aligned} L^d_t &= A^{-1}_t \left(\frac{1-\psi}{\psi}\right)^{\psi} \left(\frac{R_t}{W_t}\right)^{\psi} \left(\frac{P^A_{Ht}}{P_{Ht}}\right)^{-\phi} (Y^d_{Ht} + Y^{d*}_{Ht}) \\ &= \frac{mc^{int}_t}{(1-\psi)W_t} \left(\frac{P^A_{Ht}}{P_{Ht}}\right)^{-\phi} (Y^d_{Ht} + Y^{d*}_{Ht}), \end{aligned} \tag{29}$$

where

$$\begin{aligned} P^A_{Ht} &= \left[\int_0^1 P^{-\phi}_{Ht}(j)dj\right]^{\frac{1}{-\phi}} \\ &= \left[(1-\lambda_{int})P^{-\phi}_{Htt} + \lambda_{int}\left(\pi P^A_{Ht-1}\right)^{-\phi}\right]^{\frac{1}{-\phi}}. \end{aligned} \tag{30}$$

Similarly, we can obtain the aggregate demand for capital as in equation (31)

$$\begin{aligned} K_t &= A_t^{-1}\left(\frac{\psi W_t}{(1-\psi)R_t}\right)^{1-\psi}\left(\frac{P_{Ht}^A}{P_{Ht}}\right)^{-\phi}\left(Y_{Ht}^d + Y_{Ht}^{d*}\right) \\ &= \frac{mc_t^{int}}{\psi W_t}\left(\frac{P_{Ht}^A}{P_{Ht}}\right)^{-\phi}(Y_{Ht}^d + Y_{Ht}^{d*}). \end{aligned} \tag{31}$$

Before closing this section, we derive the profit of intermediate goods firms. The profit for firm j is

$$\pi_t^{int}(j) = \left(P_{Ht}(j) - mc_t^{int}\right)\left(Y_{Ht}^d(j) + Y_{Ht}^{d*}(j)\right), \tag{32}$$

and the aggregate profit can be obtained by aggregating over intermediate goods firms as follows:

$$\begin{aligned} \pi_t^{int} &= \int_0^1 \pi_t^{int}(j)dj \\ &= P_{Ht}Y_{Ht}^w - \left(\frac{P_{Ht}^A}{P_{Ht}}\right)^{-\phi} mc_t^{int}Y_{Ht}^w. \end{aligned} \tag{33}$$

In this section, we have 7 aggregate variables: L_t, K_t, P_{Ht}^A, W_t, R_t, mc_t^{int} and P_{Htt}. We also have 6 equations at the aggregate level: equations (25), (26), (27), (29), (30) and (31). Symmetrically, there are 7 aggregate variables and 6 equations in the Foreign intermediate goods market. Therefore, we have a total of 14 aggregate variables and 12 equations in this section.

2.3 Household's Problem

The representative household lives infinitely many periods and maximizes expected lifetime utility. Without losing generality, we normalize the starting time to zero. Equation (34) gives the expected lifetime utility, which is the sum of discounted period utilities

$$U = E_0\left[\sum_{t=0}^{\infty}\beta^t u_t\left(C_t, 1-L_t\right)\right], \tag{34}$$

where $E_0[\cdot]$ is the conditional expectation operator, and β is the subjective discount factor. The period utility u_t is a concave function of final goods composite C_t and leisure $1-L_t$.

The representative household sells labor and rents capital to domestic intermediate goods firms in a competitive market. The law of motion for capital takes the standard form of

$$K_{t+1} = (1-\delta)K_t + I_t, \tag{35}$$

where I_t is the investment at time t, and δ is the capital depreciation rate. Capital and investment are in the form of the final goods composite. There is a real cost for households to adjust capital stock, which is used to capture real rigidity in the economy:

$$AC_t = \frac{1}{2}\Phi\left(\frac{I_t}{K_t} - \delta\right)^2 K_t,$$

where Φ is a scale parameter of capital adjustment cost. It is easy to see that there is no adjustment cost in the steady state, where $\frac{I_t}{K_t} = \delta$.

The financial market is incomplete, in which households can trade only non-state-contingent Home and Foreign nominal bonds. There is a quadratic real cost of holding bonds

$$F_t = \frac{1}{2}\phi_d\left(\frac{B_{Ht+1}}{P_t}\right)^2 + \frac{1}{2}\phi_f\left(\frac{S_t B_{Ft+1}}{P_t}\right)^2,$$

where B_{Ht+1} (B_{Ft+1}) is the Home (Foreign) bond held by the household in Home country between period t and $t+1$. All bonds are denominated in the issuing country's currency. ϕ_d and ϕ_f are parameters of cost for, respectively, holding domestic bonds and holding foreign bonds.[14] We introduce this cost is for a technical reason: to ensure that bond holding and consumption are stationary in our model. By assigning very small values to ϕ_d and ϕ_f, this cost has a negligible effect on model dynamics.[15]

Other incomes for households include profits from intermediate and final goods firms. The representative household uses these incomes to buy differentiated final goods and aggregate them into a final goods composite (equation (1)). The final goods composite can be used for consumption, investment or paying the costs of adjusting capital stock and holding bonds. Based on the above setup, we obtain the budget constraint

$$\begin{aligned} C_t + \frac{B_{Ht+1}}{P_t} \quad &+ \quad \frac{S_t B_{Ft+1}}{P_t} + I_t + \frac{1}{2}\Phi\left(\frac{I_t}{K_t} - \delta\right)^2 K_t + \frac{1}{2}\phi_d\left(\frac{B_{Ht+1}}{P_t}\right)^2 + \frac{1}{2}\phi_f\left(\frac{S_t B_{Ft+1}}{P_t}\right)^2 \\ &\leq \quad \frac{W_t L_t}{P_t} + \frac{R_t K_t}{P_t} + \frac{B_{Ht}(1+i_{t-1})}{P_t} + \frac{B_{Ft}(1+i^*_{t-1})S_t}{P_t} + \frac{\pi_t^{int}}{P_t} + \frac{\pi_t^f}{P_t}. \end{aligned} \tag{36}$$

The left-hand side of the budget constraint includes all the expenditures: consumption, bond holdings, capital accumulation, and the costs of adjusting capital stock and holding bonds. On the right-hand side are incomes: wage income, rent from capital, interest incomes from previous bond holdings, and profits of intermediate and final goods firms. For any given initial capital stock and asset position, the representative household chooses the paths of consumption C_t, labor supply L_t, capital investment I_t, and bond holdings

[14] Note that in Foreign country, ϕ_d is the cost of holding Foreign bonds, and ϕ_f is the cost of holding Home bonds.
[15] See Schmitt-Grohé and Uribe (2003) for more details.

B_{Ht+1} and B_{Ft+1} to maximize expected life time utility subject to the above budget constraint.

In this paper, we employ the period utility function as in equation (37)

$$u_t(C_t, 1 - L_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \rho L_t. \tag{37}$$

The linear form of disutility from labor is used to capture fluctuations in the labor market and can be justified by the indivisible labor assumption as in Hansen (1985). From the first order conditions, we obtain

$$\frac{\partial u_t}{\partial L_t} + \frac{\partial u_t}{\partial C_t}\frac{W_t}{P_t} = 0 \tag{38}$$

$$1 + \Phi\left(\frac{I_t}{K_t} - \delta\right) = E_t\left\{\Gamma_{t,t+1}\left[\frac{R_{t+1}}{P_{t+1}} + 1 - \delta + \frac{\Phi}{2}\left(\left(\frac{I_{t+1}}{K_{t+1}} + 1 - \delta\right)^2 - 1\right)\right]\right\} \tag{39}$$

$$1 + \frac{\phi_d B_{Ht+1}}{P_t} = E_t\left[\Gamma_{t,t+1}\frac{(1+i_t)}{P_{t+1}/P_t}\right] \tag{40}$$

$$1 + \frac{\phi_f S_t B_{Ft+1}}{P_t} = E_t\left[\Gamma_{t,t+1}\frac{(1+i_t^*)}{(P_{t+1}/S_{t+1})/(P_t/S_t)}\right], \tag{41}$$

where $\Gamma_{t,t+1} = \beta\frac{\partial u/\partial C_{t+1}}{\partial u/\partial C_t}$ is the marginal rate of substitution between time t and $t+1$ consumption. Equation (38) gives the intra-temporal substitution condition between labor supply and consumption. Equation (39) is the first order condition for optimal capital accumulation. Equations (40) and (41) define the optimal bonds holdings for home and foreign nominal risk-free bonds. These two equations approximately imply the uncovered interest rate parity (UIP) condition:[16]

$$E_t\left[\Gamma_{t,t+1}\frac{(1+i_t)}{P_{t+1}/P_t}\right] \approx E_t\left[\Gamma_{t,t+1}\frac{(1+i_t^*)}{(P_{t+1}/S_{t+1})/(P_t/S_t)}\right]. \tag{42}$$

As it is well documented that the UIP condition is strongly rejected by data (see Engel (1996) and Lewis (1995) for surveys), we follow a similar method as Kollmann (2003) in introducing a UIP shock into equation (41):[17]

$$1 + \frac{\phi_f S_t B_{Ft+1}}{P_t} = \varphi_t E_t\left[\Gamma_{t,t+1}\frac{(1+i_t^*)}{(P_{t+1}/S_{t+1})/(P_t/S_t)}\right], \tag{43}$$

where φ_t is a UIP shock that can be interpreted as the bias of market expectation on time $t+1$ exchange

[16] To obtain this condition, we delete $\frac{\phi_d B_{Ht+1}}{P_t}$ and $\frac{\phi_f S_t B_{Ft+1}}{P_t}$ from the first order conditions since these two terms are small by assumption.

[17] There have been many other attempts to solve the UIP puzzle. For instance, McCallum (1994) argues that the puzzle can be explained by the fact that the central banks adjust the interest rate to stabilize the exchange rate.

rate.[18] It enters the bond holding condition symmetrically in Foreign country.

2.4 Monetary Policy Rules and Market Clearing Conditions

In Home country, the monetary authority follows a modified Taylor rule[19]

$$i_t = i + \Xi_\pi log\left(\frac{\pi_t}{\pi}\right) + \Xi_s log\left(\frac{\hat{S}_t}{\hat{S}}\right), \tag{44}$$

where Ξ_π and Ξ_s are policy parameters determined by the monetary authority. The variables without a time script are steady-state values. $\hat{S}_t$ is the real exchange rate defined as $S_t P_t^*/P_t$. In this modified Taylor rule, the monetary authority adjusts the interest rate to stabilize the inflation rate and the real exchange rate. Unlike the standard Taylor rule, the interest rate here does not react to the output gap. In a closed economy, Schmitt-Grohé and Uribe (2004b) find that the interest rate should not respond to the output gap. Empirical studies also show the reaction of the interest rate to the output gap is very limited in the data. Since the computation is very intensive in this paper, omitting the output gap from the policy rules can substantially reduce our computation burden. Foreign monetary authority follows a symmetric interest rate rule

$$i_t^* = i^* + \Xi_\pi^* log\left(\frac{\pi_t^*}{\pi^*}\right) - \Xi_s^* log\left(\frac{\hat{S}_t}{\hat{S}}\right). \tag{45}$$

The aggregate demand for the final goods composite can be found from resource constraint

$$C_t + I_t + \frac{1}{2}\phi_d\left(\frac{B_{Ht+1}}{P_t}\right)^2 + \frac{1}{2}\phi_f\left(\frac{S_t B_{Ft+1}}{P_t}\right)^2 + \frac{1}{2}\Phi\left(\frac{I_t}{K_t} - \delta\right)^2 K_t = Y_t. \tag{46}$$

For the bond market clearing condition, we have

$$B_{Ht} + B_{Ht}^* = 0, \tag{47}$$

and similar market clearing conditions exist for the Foreign nominal bond and the final goods composite.

In this section, we have 11 aggregate variables: C_t, B_{Ht+1}, B_{Ft+1}, i_t, I_t and their counterparts in foreign country plus exchange rate S_t. We have also found 18 equations: the law of motion of capital (35), budget constraint (36), first order conditions (equation (38)-(41)), the final goods composite constraint (46), bond market clearing conditions (47) and their foreign counterparts plus 2 interest rate rules (equation (44)

[18] Jeanne and Rose (2002) model the market bias with noise traders.

[19] We do not include monetary policy shocks in our model. Ireland (2003) finds that such shocks account for only a very small amount of variations in real and nominal variables for post-war U.S. data. McCallum (2001) also emphasizes that the policy coefficients are far more important than monetary policy shocks in shaping the dynamic behavior of key macroeconomic variables.

and (45)). Recall that we have 16 aggregate variables and 12 equations in the final goods market, and 14 variables and 12 equations in the intermediate goods market. So in total, we have 41 aggregate variables and 42 equations. By Walras' law, one market clearing condition is redundant.

3 Calibration and Real Business Cycle Statistics

To calibrate our model, we take one quarter as our time unit. The calibrated parameter values are summarized in Table (1). The annual real interest rate is given as 4%. This gives us a quarterly subjective discount factor of 0.99. The home bias (α) is set to match the fact that the ratio of import to GDP is around 15% in the U.S. θ and ϕ are set at levels such that the profit margin is 20% for intermediate and final goods firms. The value for elasticity of substitution between home and foreign goods (γ) is more controversial. Though studies based on micro-level evidence have suggested an elasticity of around 5,[20] Bergin (2004) finds that the elasticity is only slight above 1 in macro-level data. He argues that the substitution rate between home and foreign goods is lower at the aggregate level. We will follow Bergin's (2004) result to set γ at 1.1.

For the price stickiness parameters, we set λ_f at 0.75. Under this calibration, final goods firms on average re-optimize prices every four quarters. As we have mentioned, the prices of intermediate goods seem less sticky, so we set λ_{int} as equal to 0.5. Under this calibration, intermediate goods firms re-optimize every two quarters on average.[21] The production share of capital is set to 0.3, which is in line with the wage consumption ratio of the U.S. and EU countries. Following the estimate in Bergin (2004), we set consumption elasticity (σ) to unity. As in Kollmann (2003), the preference parameter ρ is equal to one. The capital adjustment cost is chosen to match the fact that the investment is about three times as volatile as output. The cost of holding domestic bonds is equal to zero, and of holding foreign bonds is equal to 0.0037 divided by steady state export. As we have mentioned, we introduce these costs to guarantee the stationarity of our model. Due to their small magnitude, they have no effect on our results. The annual capital depreciation rate is 10%, which gives us the quarterly depreciation rate of $\delta = 0.025$. Steady state quarterly inflation is set at 1.0103 in both countries, which implies an annual inflation rate of 4.2%. For the technology shocks, we follow the standard setup in the literature and set the AR(1) coefficients at $\xi_{11} = \xi_{22} = 0.9$, and the technology spillovers at ξ_{12}=ξ_{21}=0.03. The standard deviation of technology disturbance is 0.0059. For simplicity, we suppose that the disturbances are uncorrelated across countries.

We follow Kollmann (2003) to calibrate the uncovered interest rate parity (UIP) shock. He employs a two-factor structure as in equation(48), to capture the shape of autocorrelation functions of UIP shocks

[20] For instance, see Harrigan (1993).

[21] This price adjustment frequency is supported by Mumtaz, Oomen and Wang (2004). In that paper, we find the exchange rate changes are generally passed to import prices within two quarters.

between the U.S. and the EU for the peirod 1973-1994

$$log(\varphi_t) = a_t + \mu_t$$
$$a_t = \lambda_a a_{t-1} + \eta_{at}. \tag{48}$$

The estimate of parameter λ_a is 0.88 with standard deviation of white noises $\sigma_\eta = 0.0109$ and $\sigma_\mu = 0.022$.

In Table (2), we report some major business cycle statistics of our model in comparison to U.S. data. The table shows the results for our benchmark model (Benchmark), the benchmark model without UIP shock (No UIP) and the model with habit persistence preference (Habit). Under the above calibration, our model can successfully duplicate some major business cycle properties found in the data. The standard deviation of GDP is of the same order as that in the data. Consumption is less volatile than GDP, and investment is about three times as volatile as GDP. An important difference between our benchmark model and the model without UIP shock is the volatility of the real exchange rate. With UIP shock, we can duplicate the fact that real exchange rate is about four times as volatile as GDP. The duplication of this property is very important for the analysis of exchange rate policy.

4 Solution Method

A standard approach to obtaining welfare in DSGE models is using a first-order approximation of policy functions and a second-order approximation of the household's utility function. Since we allow long-run distortions in our model, the first-order approximation method is very likely to generate spurious rankings of welfare.[22] Therefore, we employ a second-order accurate solution method developed by Schmitt-Grohé and Uribe (2004a).[23]

4.1 Second-order Accurate Solution

Generally, the equilibrium conditions of a DSGE model can be cast into a system of first-order difference expectational equations as in (49)

$$E_t \left[f \left(\mathcal{Y}_{t+1}, \mathcal{Y}_t, \mathcal{X}_{t+1}, \mathcal{X}_t \right) \right] = 0, \tag{49}$$

[22] Kim and Kim (2003) show that with the standard log-linearization method, incomplete market generates even higher welfare than complete market under some plausible calibrations.

[23] Other works of second-order accurate solution method include Kim, Kim, Schaumburg and Sims (2003)

where $\mathcal{X}_t$ is an $n_x \times 1$ vector containing all predetermined variables, and $\mathcal{Y}_t$ is an $n_y \times 1$ vector containing all non-predetermined variables. The general solution to the system is in the form of:

$$\mathcal{Y}_t = G\left(\mathcal{X}_t, \sigma\right) \tag{50}$$

$$\mathcal{X}_{t+1} = H\left(\mathcal{X}_t, \sigma\right) + \eta\sigma\varepsilon_{t+1}, \tag{51}$$

where ε_t is an $n_\varepsilon \times 1$ vector of exogenous disturbances. ε_t is assumed to be i.i.d. with zero means and identity variance-covariance matrix I_{n_ε}. σ is a scale factor, which allows us to shrink the distribution of ε_t towards zero. η is a matrix used to pick up exogenous shocks. $G(\cdot)$ and $H(\cdot)$ are non-linear functions, for which we wish to find a second-order approximation around the non-stochastic steady state where $\mathcal{X}_t = \bar{\mathcal{X}}$, $\mathcal{Y}_t = \bar{\mathcal{Y}}$, and $\sigma = 0$. It is easy to see that the non-stochastic steady state is defined by

$$f\left(\bar{\mathcal{Y}}, \bar{\mathcal{Y}}, \bar{\mathcal{X}}, \bar{\mathcal{X}}\right) = 0. \tag{52}$$

Some of our equations, such as (8), are not in the format of first-order difference. In appendix, we show how to transform them into the form of (49). A second problem is nonstationarity. All real variables are stationary since there is no growth element in the technology process. However, we assume the long-run inflation rate is not zero. Therefore, all nominal variables grows at a constant rate in the steady state. In appendix, we show how to transform our model into a stationary one and list all equilibrium conditions and the solution to the steady state of our stationary system. After we write our equilibrium conditions into the form of (49), the system is solved by the second-order approximation method around the non-stochastic steady state.

4.2 Conditional Welfare and the Welfare Measure

We depart from the standard practice of policy evaluation based on unconditional expected utility. The conventional choice of unconditional expectation is usually due to its merit of computational simplicity. Following Schmitt-Grohé and Uribe (2004b), we assume that in the initial state, all state variables are in their non-stochastic steady states, and the monetary policies are evaluated by the conditional expectations of the discounted lifetime utility.[24] There are a couple of advantages in this choice of initial state. Since the non-stochastic steady state is the same across all policy regimes, our choice ensures that the economy begins from the same initial point for all policies. In addition, this choice can also significantly simplify our

[24] Generally, the ranking of monetary polices can change with the initial states of the economy. See Kim *et al.* (2003) and Schmitt-Grohé and Uribe (2004c) for more discussion. It is also of interest to investigate whether our results are robust for other initial states.

calculation: all terms containing state variables vanish in our approximation of expected lifetime utility.

4.2.1 Conditional Welfare

Let V_t be the conditional expectation of lifetime utility at time t

$$V_t \equiv E_t \sum_{s=t}^{\infty} \beta^{s-t} u(C_s, L_s). \tag{53}$$

To find a second-order approximation of V_t, we do not have to substitute the second-order approximations of C_s and L_s into the above equation. Instead, we can define V_t as a new control variable in our model. From equation (53), we can obtain that V_t follows a law of motion as in equation (54)

$$V_t - \beta E_t \left[V_{t+1} \right] = u \left(C_t, L_t \right). \tag{54}$$

We can put equation (54) into our system (49) and find a second-order approximation of the solution to this control variable through Schmitt-Grohé and Uribe's (2004a) algorithm[25]

$$\begin{aligned} V_t &= g\left(\mathcal{X}_t, \sigma\right) \approx g\left(\bar{\mathcal{X}}, 0\right) + \left(\mathcal{X}_t - \bar{\mathcal{X}}\right)' g_x\left(\bar{\mathcal{X}}, 0\right) + g_\sigma\left(\bar{\mathcal{X}}, 0\right)\sigma \\ &+ \frac{1}{2}\left(\mathcal{X}_t - \bar{\mathcal{X}}\right)' g_{xx}\left(\bar{\mathcal{X}}, 0\right)\left(\mathcal{X}_t - \bar{\mathcal{X}}\right) + \frac{1}{2} g_{\sigma\sigma}\left(\bar{\mathcal{X}}, 0\right)\sigma^2 \\ &+ \left(\mathcal{X}_t - \bar{\mathcal{X}}\right)' g_{x\sigma}\left(\bar{\mathcal{X}}, 0\right)\sigma. \end{aligned} \tag{55}$$

Let $V \equiv g\left(\bar{\mathcal{X}}, 0\right)$ be the non-stochastic steady state lifetime utility. It is easy for us to see that

$$V = \frac{u(\bar{C}, \bar{L})}{1-\beta}. \tag{56}$$

We have supposed that at time t, all state variables are in the non-stochastic steady state, therefore, $\mathcal{X}_t = \bar{\mathcal{X}}$. This helps us eliminate all terms containing $\mathcal{X}_t - \bar{\mathcal{X}}$. Furthermore, from Theorem 1 of Schmitt-Grohé and Uribe (2004a), we know $g_\sigma\left(\bar{\mathcal{X}}, 0\right) = 0$ and $g_{x\sigma}\left(\bar{\mathcal{X}}, 0\right) = 0$. Now we can obtain a second-order approximation of V_t in a very simple form

$$V_t = V + \frac{1}{2} g_{\sigma\sigma}\left(\bar{\mathcal{X}}, 0\right)\sigma^2. \tag{57}$$

So if the initial state of the economy is in the non-stochastic steady state, the calculation of conditional welfare is greatly simplified.

[25] The Matlab programs for the second-order approximation method are available at Uribe's website http://www.econ.duke.edu/~uribe/2nd_order.htm.

4.2.2 Welfare Measure

Following the welfare measure popularized by Lucas (1987), the welfare loss (τ) of a particular monetary policy relative to the optimal one is measured as percentage consumption obtained under the optimal monetary policy that the household is willing to give up to achieve the same welfare level obtained in an alternative monetary policy. Let V_t^{opt} be the welfare obtained under optimal monetary policy, and let $\{C_s^{opt}, L_s^{opt}\}_{s=t}^{\infty}$ be the associated consumption and labor paths

$$V_t^{opt} \equiv E_t \sum_{s=t}^{\infty} \beta^{s-t} u(C_s^{opt}, L_s^{opt}). \tag{58}$$

Let V_t^a be the welfare level obtained from an alternative monetary policy, and by the definition of τ,

$$V_t^a = E_t \sum_{s=t}^{\infty} \beta^{s-t} u((1-\tau)C_s^{opt}, L_s^{opt}). \tag{59}$$

Substituting the period utility function into the above equation, we can find the formula for calculating τ

$$\tau = 100 \times \left(1 - e^{(1-\beta)\left(V_t^a - V_t^{opt}\right)}\right). \tag{60}$$

5 Policy Evaluation

In searching for the optimal monetary policy, we limit our attention to the simple operational rules we have defined in Section (2.4). We assume that the central banks are cooperative, and we consider symmetric equilibria only. In addition, we require that the operational rules induce a locally unique equilibrium and that the nominal interest rate be non-negative. For technical reasons, we are unable to impose the non-negativity constraint into our model directly.[26] We will follow Schmitt-Grohé and Uribe (2004b) to require that the target value of nominal interest rate be at least twice as large as the standard deviation of the nominal interest rate. This constraint guarantees a positive interest rate 98% of time if the equilibrium nominal interest rate is normally distributed.

A policy rule is optimal if it satisfies the above requirements and also yields the highest level of welfare. We use the method of grid search rather than some numerical optimization routine to find out this optimal rule. The welfare surface obtained from grid search shows us how much the welfare level changes in cases of policy mistakes. We would prefer to have a policy regime in which welfare is less sensitive to policy errors. We also limit our grid search to a reasonable range for each policy parameter. The optimal value for the

[26] See Rotemberg and Woodford (1999, p.75) and Schmitt-Grohé and Uribe (2004b) for more discussion.

inflation stabilization parameter Ξ_π has usually been found to be around 1.5 in other studies. We search over a slightly broader interval of $[0, 3]$ for this parameter. The reaction of interest rate to exchange rate is relatively smaller. So we set the interval as $[0, 0.1]$.

It is well known that the equilibrium may not be determinate for some values of policy parameters. Before we search for our optimal rule, it is worthwhile to compare the indeterminacy area of our model with previous studies. We first consider the simplest rule where interest rate reacts only to CPI inflation rate

$$i_t = i + \Xi_\pi log\left(\frac{\pi_t}{\pi}\right). \tag{61}$$

In this case, the equilibrium is determinate only when Ξ_π is greater than unity. This is consistent with the theoretical prediction that when the Taylor principle[27] is satisfied, an interest rate feedback rule can be compatible with a determinate equilibrium price level.[28]

Next we consider the case in which the interest rate reacts to both inflation and output

$$i_t = i + \Xi_\pi log\left(\frac{\pi_t}{\pi}\right) + \Xi_y log\left(\frac{Y_t}{Y}\right). \tag{62}$$

We did a grid search over $[0, 3]$ in steps of 0.1 for both Ξ_π and Ξ_y. In Figure 2, we use circles to denote parameter values that induce a determinate equilibrium. For parameter values lying in the blank area of the box [(0,0) (3,3)], there is no determinate equilibrium. Figure 2 shows there is a big area of parameter values under which the equilibrium is indeterminate. The indeterminacy area we find here is similar to such areas in Rotemberg and Woodford (1999) and Schmitt-Grohé and Uribe (2004b).

5.1 CPI Inflation Targeting

As we have mentioned, there are two distinct price indices in our model: CPI and PPI. In a quick comparison between CPI and other inflation targeting regimes, we do not find obvious advantages of other regimes.[29] Furthermore, only CPI inflation is formally targeted by the central banks in practice, though both indices are available. So we will follow the central banks' practice of CPI targeting in our policy analysis. We first consider the case in which both countries target only CPI inflation. Figure 3 shows conditional welfare as a function of policy parameter Ξ_π. The welfare is obtained through a grid search over $[0, 3]$ for Ξ_π in steps of 0.1. As we have mentioned, the equilibrium is indeterminate when Ξ_π is less than or equal to unity. So the

[27] Under this principle, the central bank should raise the interest rate instrument more than one-for-one with increases in inflation. See Taylor (1999) for more discussion of the desirability of this principle.

[28] See Chapter 2 of Woodford (2003) for a proof and further discussion.

[29] Please refer to the appendix for the comparison results. See Huang and Liu (2005) for further discussion of the choice of price index for inflation targeting.

plot begins from $\Xi_\pi = 1.1$.

Figure 3 suggests a strong stance on inflation for the central banks: in optimal monetary policy, Ξ_π should be set to its highest possible level of 3. We notice, however, that the curvature of welfare is very flat after the point of $\Xi_\pi = 1.5$. Therefore, the marginal gain from stabilizing the inflation rate is very small after this point. Empirical studies show that, $\Xi_\pi = 1.5$ has been a realistic policy benchmark for the U.S. economy and other industrialized countries over the past two decades.

Figures 4 and 5 show the impulse response functions with respect to one percent technology shock in Home country and UIP shock, respectively. In Figure 5, UIP shock induces one percent real depreciation for home country. It is interesting to notice that the UIP shock does not have significant effects on the real variables, such as GDP, consumption and labor. Thus, our model exhibits the property of exchange rate disconnect. We will provide more discussion of this issue later. However, due to LOP in the intermediate goods sector, the UIP shock does have important impacts on the terms of trade.

As we have mentioned, we follow Schmitt-Grohé and Uribe (2004b) in requiring that the steady-state interest rate be at least twice as large as the standard deviation of the nominal interest rate. Table 3 shows the standard deviation of the nominal interest rate and the ratio of the steady-state interest rate to this variable under different policy parameters. All parameter values satisfy our non-negativity condition with a ratio bigger than two. Note that the standard deviation of the interest rate actually decreases with the central bank's stance on inflation rate. So when the monetary authority is more aggressive against the inflation rate, it has less opportunity to hit the zero bound of the nominal interest rate. For given inflation volatility, it should be easier to hit the zero bound if the central bank tends to adjust the interest rate more to fight against the inflation rate. However, in our rational expectation model, the volatility of the inflation rate decreases with the central bank's stance against the inflation fluctuations. Our result suggests the decrease of inflation rate volatility is the dominant effect in our model. We will follow the same method in checking the non-negativity condition in the following policy analysis.

5.2 Exchange Rate Stabilization

In this section we study whether the interest rate should directly react to real exchange rate fluctuations. We consider an interest rate rule as defined in equation (44), where the interest rate reacts to the deviations of CPI inflation and the real exchange rate. We did a grid search over [0, 0.1] in steps of 0.01 for Ξ_s.[30] Table 4 shows how the welfare level changes with exchange rate stabilization. Our results suggest a very loose stance against exchange rate stabilization: the welfare level is maximized when Ξ_s is set to 0.01, which means an increase of 40 basis points for the annual interest rate in the face of 10 percent real depreciation.

[30] We also did a grid search for Ξ_π over [0, 3] in steps of 0.1 and find the welfare level increases with Ξ_π as before.

There is only negligible welfare gain: 0.001% consumption in comparison with no exchange rate stabilization. When the central bank takes a looser stance on inflation, our results suggest no exchange rate stabilization in optimal policy. Mistakenly targeting exchange rate may be in fact very destructive in this case. For example, when $\Xi_\pi = 1.1$, the welfare loss will be more than one percent if the central banks set Ξ_s at 0.1.

To interpret our results, we first consider the behavior of the real exchange rate when all prices are flexible. When both λ_f and λ_{int} are equal to zero, all firms change prices every period, and our model reduces to the one with flexible prices. From the calculation of standard deviations and the impulse response functions, we find the real exchange rate is not constant, even when the prices are fully flexible. The standard deviation of the real exchange rate in the flexible price model is 87.23% as volatile as that in the sticky price model.[31] This result questions exchange rate stabilization as a legitimate goal of monetary policy: exchange rate stabilization does not help replicate flexible price allocations if the flexible-price exchange rate itself is not constant.

From our model it is easy for us to derive the CPI when all prices are flexible

$$P_t = \frac{\theta}{\theta - 1}\left[\alpha P_{Ht}^{1-\gamma} + (1-\alpha)(P_{Ft})^{1-\gamma}\right]^{\frac{1}{1-\gamma}} \tag{63}$$

$$P_t^* = \frac{\theta}{\theta - 1}\left[\alpha (P_{Ft}^*)^{1-\gamma} + (1-\alpha)(P_{Ht}^*)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}. \tag{64}$$

Though we have assumed the Law of One Price for the intermediate goods market with $P_{Ht} = P_{Ht}^* S_t$ and $P_{Ft} = P_{Ft}^* S_t$, the Purchasing Power Parity (PPP) is not generally satisfied except when $\alpha = \frac{1}{2}$.[32] Intuitively, when there is home bias in the final goods sector, the final consumption goods are not identical across countries. So even if all prices are flexible, we cannot expect the final goods composite to have the same value when denominated in the same currency. In this case, CPI based real exchange rate fluctuations do not necessarily suggest significant price distortions for final consumption goods across borders. Therefore, there is not much welfare gain from exchange rate stabilization.

In contrast, the restrictions on exchange rate movements obstruct the expenditure switching effect for intermediate goods: in the face of country-specific technology shocks, the nominal exchange rate cannot move freely to adjust the terms of trade. It can be seen more clearly in Figure 6 that the impulse response function of the terms of trade is closer to that of the flexible price terms of trade when there is no exchange rate stabilization.

The exchange rate stabilization also increases inflation volatility in our benchmark model, which reduces

[31] The standard deviation of the (log) real exchange rate is 0.0692 and 0.0793 for the flexible and sticky price models, respectively.

[32] Another case in which the condition of PPP holds is $P_{Ht} = P_{Ft}$. That is, the terms of trade is equal to unity all the time. This condition is obviously not true when there is country-specific productivity shock and prices are flexible.

the welfare level under staggered price setting. Figure 7(a) shows the impulse response functions to one percent technology shock in home country. We denote by circles (asterisks) the policy with (without) exchange rate stabilization. Although the real exchange rate is more stable when the interest rate directly reacts to exchange rate fluctuations, the inflations are more volatile. Figure 7(b) shows the impulse response functions to UIP shock. Home currency depreciates in the face of the shock, which induces the increase of CPI inflation. So the central bank should increase the interest rate in response. If the central bank also stabilizes the real exchange rate, the depreciation of home currency also calls for an increase of the interest rate, which will reinforce inflation stabilization. So we find that in the beginning, CPI inflation is more stable in the case with exchange rate stabilization. However, the CPI inflation rate converges back to steady state more slowly in this case. Meanwhile, the increase in the interest rate suppresses market demand so much that the prices of intermediate goods even decrease and become more volatile. The increase of inflation rate volatility induces higher price dispersions among firms and hence lowers welfare level.

The welfare loss from exchange rate stabilization is sensitive to the central bank's stance on the inflation rate. Intuitively, when the central banks take a strong stance against the inflation rate, the inflation instability caused by exchange rate stabilization will be offset by the inflation stabilization term Ξ_π in the policy rule. However, with the decrease of Ξ_π, exchange rate stabilization becomes more harmful. Our results suggest no exchange rate stabilization in this case.

As we have mentioned, the gain from exchange rate stabilization is small when the similarity of final consumption bundles is low. Intuitively, exchange rate stabilization should become more desirable when home bias declines. Table 5 shows the results when the home bias parameter α is set to the lower level of 0.6. Our results suggest a much stronger stance against exchange rate fluctuations: the annual interest rate should increase 5.6 percentage points in face of 10% real depreciation. When the home bias decreases, there are two effects on real exchange rate stabilization. The final consumption bundles become more similar, so we can gain more from real exchange rate stabilization. In addition, our model predicts a more stable real exchange rate for given exogenous shocks in the case of less home bias. Therefore, the central banks do not have to adjust the interest rate as much as before to stabilize the real exchange rate, which reduces the cost of real exchange rate stabilization.

However, we should be cautious in interpreting this result as a support for exchange rate stabilization. We have noticed that in the above case with low home bias, the volatility of the real exchange rate relative to the volatility of GDP is too small to match the data. Unlike in Devereux and Engel (2002), we cannot increase the relative volatility simply by increasing UIP shock. When we increase the UIP shock, both real exchange rate and GDP become more volatile. As a result, the relative volatility of the real exchange rate to that of GDP becomes pretty stable (at around 1.4), even with big increases in UIP shock. This can be seen

more clearly in Panel A of Table 6. Intuitively, we have assumed LOP for the intermediate goods market. So any exchange rate shock will pass through to the prices of exports immediately. If the foreign market makes up a big portion of the total output, as in the case with little home bias, increasing UIP shock also drives up the volatility of GDP when we try to pump up the volatility of the real exchange rate. This result is contradictory to the well-documented exchange rate disconnect puzzle.

We also notice that the welfare gain of directly reacting to the exchange rate is very small in both cases. As we have mentioned, it is not surprising to find negligible gains in our benchmark model, since the real exchange rate fluctuations do not imply significant price distortions across borders. In the case with less home bias, the welfare gain is higher but is still at only about 0.01% of consumption. There are two possible reasons for this case. The exchange rate stabilization helps eliminate price distortions across countries, and therefore facilitates international risk-sharing. But the gain from international risk-sharing is generally small, especially in a production economy with capital accumulation like ours. For example, Kim and Kim (2003) find the gain is between 0.005% and 0.02%.[33] However, this result may also be caused by the unrealistically small real exchange rate volatility in the case with less home bias. So we should be cautious in interpreting this finding.

5.3 Home Bias and Exchange Rate Disconnect

In the last section, we noticed that the case with little home bias is contradictory to two important empirical findings: exchange rate volatility and the exchange rate disconnect. In contrast, our benchmark model is more consistent with the data.

5.3.1 Model Prediction

We have shown in Table 2 that our benchmark model can successfully duplicate exchange rate volatility. Our model also exhibits exchange rate disconnect when the home bias is high. In Panel B of Table 6, we show how the standard deviations of the real exchange rate and GDP change with UIP shock. With the increase of UIP shock, the standard deviation of the real exchange rate becomes 10 times larger than before, while the standard deviation of GDP increases by only about 25 percent. In this sense, our model provides a novel solution to the exchange rate disconnect puzzle: home bias in consumption.

Our model predicts the volatility of the real exchange rate relative to the volatility of GDP increases with the extent of home bias. Panel C of Table 6 shows how this relative volatility and the standard deviation of the real exchange rate change with home bias. A higher home bias has two effects on the relative volatility: the real exchange rate is more volatile, since the final consumption goods are more different across countries;

[33] Tesar (1995) reports similar results.

at the same time, GDP is less volatile, in that the exchange rate shock has an impact on a smaller portion of the total output. As a result, the relative volatility increases with home bias at a faster rate than does the standard deviation of the real exchange rate as shown in Panel C.

Hau (2002) studies the first effect and finds in the data that real exchange rate volatility is negatively correlated with the openness of a country (or positively correlated with home bias). In our model, we predict that a stronger relation exists between the openness and **the ratio** of real exchange rate volatility to GDP volatility. It is of interest to find out if our prediction is consistent with the data.

5.3.2 Empirical Support

We use the following data of OECD countries from International Financial Statistics (IFS): (1) real effective exchange rates, (2) imports and exports of goods and services and (3) GDP per capita.[34] Following Hau (2002), we use the data for the period between 1980 and 1998. But we choose to use quarterly data, hoping to find a strong relation between relative real exchange rate volatility and openness, even with relatively high frequency data.

We follow the same method as Hau (2002) in calculating the openness and the volatility of the real exchange rate. The openness at period t is measured by the ratio of average imports and exports to GDP. The openness of country i is the average openness during our sampling period.[35] The volatility of the real exchange rate is measured by the variation of the percentage change of the exchange rate as in equation (65)

$$vol_s^i = \left[\frac{1}{T} \sum_{t=1}^{T} \left(\frac{s_{t+1} - s_t}{s_t} \right)^2 \right]^{\frac{1}{2}}, \tag{65}$$

where s is the real effective exchange rate of country i. To obtain the volatility of GDP per capita, we first detrend the logarithm of GDP per capita with the HP filter and then calculate the standard deviation of the detrended data for each country. Our sample includes 22 countries that are members of OECD during the sample period.[36] Figure 8 shows the scatter diagrams of REER volatility and REER volatility relative to GDP volatility against openness. Both variables display some negative correlation with the openness. However, the relation between the relative volatility of REER and openness seems stronger.[37]

Table 7 reports our OLS regression results. In Panel A, we regress the volatility of the real exchange rate and relative volatility against openness. The coefficients are significant at least at a 10% level in both

[34] GDP per capita is calculated as GDP divided by the total population in each country.

[35] There is no obvious trend for openness in most countries, and our results do not change after we take out the countries in which openness seems to have a significant trend during our sampling period. These countries include Canada, Ireland, Japan and Spain.

[36] We exclude Greece and Luxembourg due to the unavailability of data.

[37] Belgium and Ireland seem to be two outliers in figure (8). Our results reported below is robust even after we remove these two countries from our sample.

regressions. In Panel B, we take logarithms for both dependent and independent variables. In this case, β is the elasticity of y_i against openness. Let β_1 be the elasticity of REER volatility over openness, and let β_2 be the elasticity of the relative volatility. We find the volatility of the real exchange rate relative to that of GDP responds more to the openness than the volatility of the real exchange rate does ($\beta_2 < \beta_1$),[38] which is consistent with our prediction.

Our results are also robust for a larger sample size. We include the same countries as Hau (2002) in our sample with 46 countries.[39] Since for most countries, GDP data are available only at annual frequency, we expand our sample period to include data between 1975 and 1998 in order to have more observations for each country. The results of the sample with 46 countries are also reported in Table 7. The findings are very similar to those from the OECD data.[40]

5.4 Robustness Analysis

In this section, we test whether our results are robust under different model setups.

5.4.1 Habit Persistence

Bergin and Tchakarov (2004) find that habit persistence has important impacts on welfare loss caused by exchange rate variations. They report a welfare loss of 4.554% consumption in the case of habit persistence in contrast to 0.144% otherwise. Will this higher welfare loss provide grounds for exchange rate stabilization?

We modify our utility function in equation (37) to accommodate habit persistence

$$u_t(C_t, 1 - L_t) = \frac{(C_t - \zeta C_{t-1})^{1-\sigma}}{1-\sigma} - \rho L_t. \tag{66}$$

In this new period utility function, the households try to smooth a weighted average of current consumption and the change of consumption. ζ is the weight put to the change of consumption. Under this preference, the households are more sensitive to exogenous risks. This form of the utility function has been widely used in the literature explaining the equity premium puzzle.

We calibrate ζ as 0.8, which is found to be a reasonable value by Deaton (1987) and Constantinides (1990) to explain aggregate consumption smoothness and the equity premium puzzle. The real business cycle statistics are reported in Table 2. Table 8 shows welfare level and loss under habit persistence preference. We find this change does not affect our results: the interest rate should not directly react to the exchange rate.

[38] The null hypothesis that $\beta_2 - \beta_1 \geq 0$ is rejected at 10% level.

[39] Ecuador is excluded due to the unavailability of data.

[40] We marginally missed the 10% significant level for the null hypothesis that $\beta_2 - \beta_1 \geq 0$ in this sample, but the point estimates are still consistent with our model prediction. We assumed a constant exchange rate shock across countries in our model prediction. This assumption is more likely to be violated when we include developing countries into our sample.

Bergin and Tchakarov (2004) measure welfare loss with the difference between the non-stochastic steady state welfare level and the second-order approximation of the expected stochastic welfare level. That is, they are comparing the welfare levels with and without exogenous shocks. Their finding is intuitive: if households are more risk averse, welfare loss is higher given the same exchange rate variation. However, this result alone cannot be used to justify the policy of exchange rate stabilization. The same tradeoff between inflation and exchange rate stabilizations still exists. Though the gains from exchange rate stabilization increase due to more risk-sensitive preference, the same is also true for the loss from inflation instability. Our results show the latter still wins out in the competition.

It is also interesting to do an exercise similar to Bergin and Tchakarov's (2004) to calculate welfare loss due to uncertainty. In our benchmark model, the loss of exchange rate variation ranges from 0.075 percent to 0.455 percent depending on the monetary policy employed by the central banks. Like Bergin and Tchakarov (2004), we find that habit persistence induces higher welfare loss. However, the difference is much smaller: it is between 0.399 percent and 0.737 percent depending on the monetary policy. An important difference between our model and Bergin and Tchakarov's (2004) is that they assume an exogenous monetary policy which does not react to endogenous variables. It would be worthwhile to study how much the discrepancy between our results can be explained by this difference.

5.4.2 Exchange Rate Disconnect and Monetary Policy

In this section, we check the extent to which our explanation of the exchange rate disconnect puzzle depends on the format of interest rate rules. We are particularly interested in whether the inclusion of exchange rate stabilization in the policy rule will change our results.

Table 9 reports how the volatility of the real exchange rate and GDP changes with UIP shock if the central banks stabilize the exchange rate. When the central banks set Ξ_s as equal to or less than 0.1, the exchange rate disconnect still exists in our model. With an increase of UIP shock, the standard deviation of GDP increases less than 25%, while the real exchange rate is over 12 times more volatile than before. Furthermore, at the volatility of GDP observed in the data (1.72%), the exchange rate is about 4 times as volatile as GDP. In this sense, our model can still duplicate both exchange rate disconnect and exchange rate volatility.

However, when the central bank puts more weight on exchange rate stabilization, the UIP shock exerts greater impact on the standard deviation of GDP. Intuitively, with a stronger stance against exchange rate fluctuations, the central bank moves the interest rate more to fight against exchange rate shock. Therefore, the financial market shock becomes more influential for real variables. With the increase of Ξ_s, we find in

Panel A that the GDP becomes more volatile, while the real exchange rate becomes less volatile.[41] As a result, at the volatility of GDP observed in the data, the volatility of the real exchange rate relative to that of GDP is much smaller than what we have seen in the data. For example, when Ξ_s is equal to 0.2, the ratio of exchange rate volatility to GDP volatility is only 1.60 if the standard deviation of GDP is equal to 1.73%.

In Panel B of Table 9, we find that the relation between home bias and the volatility of the exchange rate relative to the volatility of GDP is weaker when the central banks stabilize the exchange rate. In particular, the relative volatility decreases with home bias when α is greater than 0.8, and Ξ_s is equal to or greater than 0.2.

To sum up, our explanation of exchange rate volatility and the exchange rate disconnect puzzle is sensitive to the assumption about monetary policy. However, in empirical studies, the estimate of Ξ_s is usually well below 0.1. For example, see Clarida, Gali and Gertler (1998) and Bergin (2004).[42] Judging from empirical relevance, our explanation is still valid under exchange rate stabilization.

6 Conclusion

In this paper, we examine how much the interest rate should react to real exchange rate fluctuations in a two-country DSGE model. In our model the exchange rate volatility is mainly driven by the home bias in consumption and UIP shock to exchange rate. Our results suggest a very loose stance against exchange rate stabilization. In particular, when the central bank does not take a strong stance against the inflation rate, the inclusion of the exchange rate into monetary policy rule may induce significant welfare loss. Intuitively, when the economy exhibits high home bias in final consumption goods, real exchange rate fluctuations do not necessarily reflect significant economic inefficiency. In this case, the interest rate movements required for exchange rate stabilization will cause inflation instability that is worse than the real exchange rate swings themselves. The results are also robust for the preference with habit persistence.

Our model is the first DSGE model to study the linkage between home bias in consumption, exchange rate volatility and the exchange rate disconnect puzzle. We find strong support in the data to our explanation of the exchange rate disconnect puzzle as a result of home bias in consumption.

Our policy evaluations are admittedly contingent on the assumption that the real exchange rate volatility is mainly driven by home bias and exchange rate shock. Though the home bias in consumption can be justified by a more carefully structured model with high international trade costs, we could explore in more details

[41] We find the standard deviation of GDP decreases with Ξ_s in the first column of data. Intuitively, there is no financial market shock in this column. The real exchange rate volatility is caused by technology shock. In this case, the exchange rate and GDP stabilizations are consistent.

[42] In Clarida *et al.* (1998), the estimate of exchange rate stabilization parameter for annualized interest rate is 0.05. So the relevant one for our quarterly interest rate is $0.05 \div 4 = 0.0125$. Similarly the relevant estimate from Bergin (2004) is $0.1128 \div 4 \approx 0.03$.

the microstructure of financial market bias, and how this shock interacts with exchange rate policy. Since our results also show the extent of home bias has important implications for exchange rate policy, it is also desirable for future research to test whether optimal exchange rate policy varies with assumptions that drive exchange rate volatility in the model.

References

[1] Atkeson, A. and A. Burstein 2005. Trade Costs, Pricing to Market, and International Relative Prices, Working paper, UCLA.

[2] Ball, L. 1999. Policy Rules for Open Economies, in J. B. Taylor, ed., Monetary Policy Rules, Chicago: University of Chicago Press, 1999.

[3] Bergin, P. 2004. How Well Can the New Open Economy Macroeconomics Explain the Exchange Rate and Current Account, Working paper, 2004 University of California, Davis.

[4] Bergin, P. and I. Tchakarov 2004. Does Exchange Rate Variability Matter for Welfare? A Quantitative Investigation, Working paper, 2004 University of California, Davis.

[5] Burstein, A., J. Neves and S. Rebelo, 2003. Distribution Costs and Real Exchange Rate Dynamics during Exchange-rate-based-stabilizations, *Journal of Monetary Economics* 50, 1189-1214.

[6] Campa, M. J. and L. S. Goldberg, 2002. Exchange Rate Pass Through into Import Prices, NBER Working Paper.

[7] Calvo, G. A., 1983. Staggered Prices in a Utility-Maximizing Framework, *Journal of Monetary Economics* 12, 383-398.

[8] Chari, V., Kehoe, P. and McGrattan, E., 2002. Can Sticky Price Models Generate Volatile and Persistent Real Exchange Rates?, *Review of Economic Studies* 69(3), 533-563.

[9] Clarida, R., J. Gali, and M. Gertler, 1998. Monetary Policy Rules in Practice: Some International Evidence, *European Economic Review* 42, 1033-1067.

[10] Clark, T. E., 1999. The Responses of Prices at Different Stages of Production to Monetary Policy Shocks, *Review of Economics and Statistics* 12, 383-398.

[11] Constantinides, G. M, 1990. Habit formation: a resolution of the equity premium puzzle. *Journal of Political Economy* 98, 519-543.

[12] Deaton, A., 1987. Life-cycle models of consumption: is the evidence consistent with the theory? in Advances in Econometrics Fifth World Congress, Vol. 2, ed. by T. Bewley, Cambridge University Press, 121-148.

[13] Devereux, M. and C. Engel 2002. Exchange Rate Pass-through, Exchange Rate Volatility, and Exchange Rate Disconnect, *Journal of Monetary Economics* 49, 913-940.

[14] Devereux, M. and C. Engel 2003. Monetary Policy in the Open Economy Revisited: Price Setting and Exchange Rate Flexibility, *The Review of Economic Studies* 70, 765-783.

[15] Devereux, M. and C. Engel 2004. Expenditure Switching vs. Real Exchange Rate Stabilization: Competing Objectives for Exchange Rate Policy, Working paper.

[16] Devereux, M., C. Engel and P. Storgaard, 2004. Endogenous Exchange Rate Pass Through When Nominal Prices are Set in Advance, *Journal of International Economics* 63, 263-291.

[17] Dong, W. 2006. Expenditure-Switching Effect and the Exchange Rate Regime Debate, Working paper, the Bank of Canada.

[18] Duarte, M. and A. Stockman 2005. Rational Expectations and Exchange Rates, *Journal of Monetary Economics* 52, 3-29.

[19] Engel, C., 1993. Real Exchange Rates and Relative Prices: An Empirical Investigation, *Journal of Monetary Economics* 32, 35-50.

[20] Engel, C., 1996. The Forward Discount Anomaly and The Risk Premium: A Survey of Recent Evidence, *Journal of Empirical Finance* 3, 123-192.

[21] Engel, C., 1999. Accounting for U.S. Real Exchange Rate Changes, *Journal of Political Economics* 107, 507-538.

[22] Engel, C. and J. Rogers, 1996. How Wide is the Border?, *American Economic Review* 86, 1112-1125.

[23] Engel, C., 2004. On the Relationship between Pass-Through and Sticky Nominal Prices, Working paper, University of Wisconsin, Madison and NBER.

[24] Flood, R. and A. Rose, 1995. Fixing Exchange Rates: A Virtual Quest for Fundamentals, *Journal of Monetary Economics* 36, 3-37.

[25] Ghironi, F. and M. Melitz, 2005. International Trade and Macroeconomic Dynamics with Heterogenous Firms, *Quarterly Journal of Economics* CXX, 865-915.

[26] Hansen, G., 1985. Indivisible Labor and the Business Cycle, *Journal of Monetary Economics* 16, 309-327.

[27] Harrigan, J., 1993. OECD Imports and Trade Barriers in 1983, *Journal of International Economics* 35, 91-111.

[28] Hau, H., 2002. Real Exchange Rate Volatility and Economic Openness: Theory and Evidence, *Journal of Money, Credit and Banking* 34, 611-630.

[29] Helpman, E. 1999. The Structure of Foreign Trade, *Journal of Economics Perspectives* 13, 121-144.

[30] Huang, K. and Z. Liu, 2005. Inflation to Target: What Inflation to Target, *Journal of Monetary Economics* forthcoming.

[31] Ireland, P., 2003. Endogenous Money or Sticky Prices, *Journal of Monetary Economics* 50, 1623-1648.

[32] Jeanne, O. and A. Rose, 2002. Noise Trading and Exchange Rate Regimes, *Quarterly Journal of Economics* 117, 537-569.

[33] Kim, J. and S. Kim, 2003. Spurious Welfare Reversals in International Business Cycle Models, *Journal of International Economics* 60, 471-500.

[34] Kim, J., S. Kim, E. Schaumburg and C. A. Sims 2003. Calculating and Using Second Order Accurate Solutions of Discrete Time Dynamic Equilibrium Models, Working paper.

[35] Kollmann, R., 2003. Monetary Policy Rules in an Interdependent World, Working paper, University of Bonn and CEPR.

[36] Kollmann, R., 2004. Welfare Effects of a Monetary Union: the Role of Trade Openness, *Journal of the European Economic Association* 2, 289-301.

[37] Lewis, K., 1995. Puzzles in International Financial Markets, in: G. Grossman and K. Rogoff, eds., Handbook of International Economics, Vol. III (Elsevier, Amsterdam) 1913-1971.

[38] Lucas, R., 1987, Models of Business Cycles, Oxford: Basil Blackwell.

[39] McCallum, B., 1994. A Reconsideraton of the Uncovered Interest Rate Parity Relationship, *Journal of Monetary Economics* 33, 105-132.

[40] McCallum, B., 2001. Analysis of the monetary transmission mechanism: methodological issues. In: Bundesbank, D. eds., The Monetary Transmission Process: Recent Developments and Lessons for Europe. (Palgrave, New York), 11C43.

[41] Means, G., 1935. Industrial Prices and Their Relative Inflexibility, US Senate Document 13, 74th Congress, 1st Session (Washington, D.C., 1935).

[42] Mumtaz, H., O. Oomen and J. Wang, 2004. Exchange Rate Pass-through into U.K. Import Prices: Evidence from Disaggregated Data. Working paper.

[43] Mussa, M., 1986. Nominal Exchange Rate Regimes and the Behavior of Real Exchange Rates: Evidence and Implications, *Carnegie-Rochester Conference Series on Public Policy* 36, 163-209.

[44] Obstfeld, M. 2001. International Macroeconomics: Beyongd the Mundell-Fleming Model, *IMF Staff Papers* 47 (Special Issue), 1-39.

[45] Obstfeld, M. 2002. Exchange Rates and Adjustment: Perspectives from the New Open-Economy Macroeconomics, *Money and Economic Studies* December 2002 (Special Edition), 23-46.

[46] Obstfeld, M. 2004. Pricing-to-Market, the Interest Rule and the Exchange Rate, working paper, University of Carlifornia, Berkeley.

[47] Obstfeld, M., and K. Rogoff, 1995. The Mirage of Fixed Exchange Rates, *Journal of Economic Perspectives* 9, 73-96.

[48] Obstfeld, M., and K. Rogoff, 2000a. The Six Major Puzzles in International Macroeconomics: Is There a Common Cause? *NBER Macroeconomics Annual* 15, 339C390.

[49] Obstfeld, M. and K. Rogoff, 2000b. New Direction for Stochastic Open Economy Models, *Journal of International Economics* 50, 117-153.

[50] Parsley, D. and S. Wei, 2001. Explaining the Border Effect: the Role of Exchange Rate Variability, Shipping Costs and Geography, *Journal of International Economics* 55, 87-105.

[51] Rotemberg, J. J. and M. Woodford, 1999. Interest Rate Rules in an Estimated Sticky Price Model, in J. B. Taylor, ed., Monetary Policy Rules, Chicago: University of Chicago Press, 1999.

[52] Schmitt-Grohé, S. and M. Uribe, 2003. Closing Small Open Economy Models. *Journal of International Economics* 61, 163-185.

[53] Schmitt-Grohé, S. and M. Uribe, 2004a. Solving Dynamic General Equilibrium Models Using a Second-Order Approximation to the Policy Function. *Journal of Economic Dynamics and Control* 28, 755-775.

[54] Schmitt-Grohé, S. and M. Uribe, 2004b. Optimal Simple and Implementable Monetary and Fiscal Rules, NBER working paper no. 10253.

[55] Schmitt-Grohé, S. and M. Uribe, 2004c. Optimal Operational Monetary Policy in the Christiano-Eichenbaum-Evans Model of the U.S. Business Cycle, Working paper.

[56] Svensson, Lars E. O., 2000. Open-Economy Inflation Targeting, *Journal of International Economics*, February, 2000, 50 (1), 155-183.

[57] Taylor, J., 1999. Robustness and Efficiencies of Monetary Policy Rules as Guidelines for Interest Rate Setting by the European Central Bank. *Journal of Monetary Economics*, 43 (3), 655-679.

[58] Taylor, J., 2001. The Role of the Exchange Rate in Monetary Policy Rules, *American Economic Review* 91, 263-267.

[59] Tesar, L., 1995. Evaluating the Gains from International Risksharing. Carnegie-Rochester Conference Series on Public Policy 42, 95-143.

[60] West, K., 2004. Monetary Policy and the Volatility of Real Exchange Rates in New Zealand, NBER Working Paper No. 10280.

[61] Woodford, M., 2003. Interest and Prices: Foundations of a Theory of Monetary Policy, Princeton University Process.

[62] Yun, T., 1996. Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles, *Journal of Monetary Economics* 37, 345-370.

APPENDIX

7 Transformation of Equations

Some of our equations, for instance equation (8), are not in the format of first order difference as in (49). This can be solved by introducing two new variables. Let P_t^n be the numerator of $P_{t,t}$ and P_t^d be the denominator. By definition,

$$P_t^n \equiv \sum_{k=0}^{\infty} (\pi^{-\theta}\lambda_f)^k E_t \left[\Gamma_{t,t+k} mc_{t+k} P_{t+k}^{\theta-1} Y_{t+k}\right]$$
$$P_t^d \equiv \sum_{k=0}^{\infty} (\pi^{1-\theta}\lambda_f)^k E_t \left[\Gamma_{t,t+k} P_{t+k}^{\theta-1} Y_{t+k}\right] \tag{67}$$

From equation (67), we can write P_t^n and P_t^d into first order difference equations by using the law of iterated expectations and the pricing kennel's property that $\Gamma_{t,t+s} = \Gamma_{t,t+1}\Gamma_{t+1,t+s}$

$$P_t^n = mc_t P_t^{\theta-1} Y_t + \lambda_f \pi^{-\theta} E_t \left[\Gamma_{t,t+1} P_{t+1}^n\right] \tag{68}$$

$$P_t^d = P_t^{\theta-1} Y_t + \lambda_f \pi^{1-\theta} E_t \left[\Gamma_{t,t+1} P_{t+1}^d\right]. \tag{69}$$

By the same token, we can also convert equation (26) in the intermediate goods market into first order difference equations.

8 Stationary Economy

We make our model stationary by dividing all equations by final or intermediate goods price index. In this section, we use the final goods market as our example to explain how we do it and also list here all other equilibrium conditions defining our stationary economy.

8.1 Final Goods Sector

For home country, there are 8 aggregate level equations in the final goods market. For reader's convenience, we reproduce them here

$$mc_t = \left[\alpha P_{Ht}^{1-\gamma} + (1-\alpha)(S_t P_{Ft}^*)^{1-\gamma}\right]^{\frac{1}{1-\gamma}} \tag{1}$$

$$P_{t,t} = \frac{P_t^n}{P_t^d} \tag{2}$$

$$P_t^n = mc_t P_t^{\theta-1} Y_t + \lambda_f \pi^{-\theta} E_t \left[\Gamma_{t,t+1} P_{t+1}^n\right] \tag{3}$$

$$P_t^d = P_t^{\theta-1} Y_t + \lambda_f \pi^{1-\theta} E_t \left[\Gamma_{t,t+1} P_{t+1}^d\right] \tag{4}$$

$$P_t^{1-\theta} = (1-\lambda_f) P_{tt}^{1-\theta} + \lambda_f \left(\pi P_{t-1}\right)^{1-\theta} \tag{5}$$

$$Y_{Ht} = \alpha \left(\frac{P_{Ht}}{mc_t}\right)^{-\gamma} \left(\frac{P_t^A}{P_t}\right)^{-\theta} Y_t \tag{6}$$

$$Y_{Ft} = (1-\alpha) \left(\frac{S_t P_{Ft}}{mc_t}\right)^{-\gamma} \left(\frac{P_t^A}{P_t}\right)^{-\theta} Y_t \tag{7}$$

$$\left(P_t^A\right)^{-\theta} = (1-\lambda_f) P_{tt}^{-\theta} + \lambda_f \left(P_{t-1}^A \pi\right)^{-\theta}. \tag{8}$$

We divide each nominal variable in the above equations by the final goods price index P_t or a function of P_t. We use a hat to denote the corresponding stationary nominal variables. Let $\hat{mc}_t = \frac{mc_t}{P_t}$, $\hat{P}_{Ht} = \frac{P_{Ht}}{P_t}$, $\hat{P}_{Ft}^* = \frac{P_{Ft}^*}{P_t^*}$, $\hat{P}_t^A = \frac{P_t^A}{P_t}$, $\hat{S}_t = S_t P_t^*/P_t$, $\hat{P}_t^n = \frac{P_t^n}{P_t^\theta}$, and $\hat{P}_t^d = \frac{P_t^d}{P_t^{\theta-1}}$. After this transformation, it is easy for us to find the following equations with the stationary nominal variables

$$\hat{mc}_t = \left[\alpha \hat{P}_{Ht}^{1-\gamma} + (1-\alpha)(\hat{S}_t \hat{P}_{Ft}^*)^{1-\gamma}\right]^{\frac{1}{1-\gamma}} \tag{9}$$

$$1 = (1-\lambda_f) \left(\frac{\hat{P}_t^n}{\hat{P}_t^d}\right)^{1-\theta} + \lambda_f \left(\frac{\pi}{\pi_t}\right)^{1-\theta} \tag{10}$$

$$\hat{P}_t^n = \theta \hat{mc}_t Y_t + \lambda_f \pi^{-\theta} E_t \left[\Gamma_{t,t+1} \hat{P}_{t+1}^n \pi_{t+1}^\theta\right] \tag{11}$$

$$\hat{P}_t^d = (\theta-1) Y_t + \lambda_f \pi^{1-\theta} E_t \left[\Gamma_{t,t+1} \hat{P}_{t+1}^d \pi_{t+1}^{\theta-1}\right] \tag{12}$$

$$Y_{Ht} = \alpha \left(\frac{\hat{P}_{Ht}}{\hat{mc}_t}\right)^{-\gamma} \left(\hat{P}_t^A\right)^{-\theta} Y_t \tag{13}$$

$$\left(\hat{P}_t^A\right)^{-\theta} = (1-\lambda_f) \left(\frac{\hat{P}_t^n}{\hat{P}_t^d}\right)^{-\theta} + \lambda_f \left(\hat{P}_{t-1}^A \frac{\pi}{\pi_t}\right)^{-\theta} \tag{14}$$

$$Y_{Ft} = (1-\alpha) \left(\frac{\hat{S}_t \hat{P}_{Ft}^*}{\hat{mc}_t}\right)^{-\gamma} \left(\hat{P}_t^A\right)^{-\theta} Y_t. \tag{15}$$

There are 7 symmetric equations in the foreign country. π_t is the inflation of final good price index (CPI). We eliminate the final goods price index from our system by introducing this new variable. In this section, we have 14 equations.

8.2 Intermediate Goods Sector

Similarly, we can transform the equations in intermediate goods market by dividing intermediate goods price index (PPI) or its function (The marginal cost function is still divided by P_t (CPI).). $\hat{mc}_t^{int} = \frac{mc_t^{int}}{P_t}$, $\hat{R}_t = \frac{R_t}{P_t}$, $\hat{W}_t = \frac{W_t}{P_t}$, $\hat{P}_{Ht}^n = \frac{P_{Ht}^n}{P_{Ht}^\theta}$, $\hat{P}_{Ht}^d = \frac{P_{Ht}^d}{P_{Ht}^{\theta-1}}$, $\hat{P}_{Ht}^A = \frac{P_{Ht}^A}{P_{Ht}}$ and $\pi_{Ht} = P_{Ht}/P_{Ht-1}$ is the inflation of PPI in home country.

The stationary economy is described by the following equilibrium conditions

$$\hat{mc}_t^{int} = A_t^{-1}\hat{R}_t^{\psi}\hat{W}_t^{1-\psi}\psi^{-\psi}(1-\psi)^{\psi-1} \tag{16}$$

$$\hat{P}_{Ht}^n = \phi\hat{mc}_t^{int}\left(Y_{Ht}+Y_{Ht}^*\right)/\hat{P}_{Ht} + \lambda_{int}\pi^{-\phi}E_t\left[\Gamma_{t,t+1}\hat{P}_{Ht+1}^n\pi_{Ht+1}^{\phi}\right] \tag{17}$$

$$\hat{P}_{Ht}^d = (\phi-1)\left(Y_{Ht}+Y_{Ht}^*\right) + \lambda_{int}\pi^{1-\phi}E_t\left[\Gamma_{t,t+1}\hat{P}_{Ht+1}^d\pi_{Ht+1}^{\phi-1}\right] \tag{18}$$

$$1 = (1-\lambda_{int})\left(\frac{\hat{P_{Ht}^n}}{\hat{P_{Ht}^d}}\right)^{1-\phi} + \lambda_{int}\left(\frac{\pi}{\pi_{Ht}}\right)^{1-\phi} \tag{19}$$

$$L_t = A_t^{-1}\left(\frac{1-\psi}{\psi}\right)^{\psi}\left(P_{Ht}^A\right)^{-\phi}\left(Y_{Ht}+Y_{Ht}^*\right) \tag{20}$$

$$\left(\hat{P}_{Ht}^A\right)^{-\phi} = (1-\lambda_{int})\left(\frac{\hat{P}_{Ht}^n}{\hat{P}_{Ht}^d}\right)^{-\phi} + \lambda_{int}\left(\hat{P}_{Ht-1}^A\frac{\pi}{\pi_{Ht}}\right)^{-\phi} \tag{21}$$

$$K_t = A_t^{-1}\left(\frac{(1-\psi)\hat{R}_t}{\psi\hat{W}_t}\right)^{\psi-1}\left(\hat{P}_{Ht}^A\right)^{-\phi}\left(Y_{Ht}+Y_{Ht}^*\right). \tag{22}$$

We introduced a new variable in this section: intermediate good price inflation (PPI). This inflation is defined equation (23)

$$\pi_{Ht} = \pi_t\hat{P}_{Ht}/\hat{P}_{Ht-1}. \tag{23}$$

There are 8 symmetric equations in the foreign country. So in this section, we have 16 equations.

8.3 Household and Monetary Rules

In this section, all nominal variables are divided by the final goods price index (CPI). The equilibrium conditions can be written in the following stationary form

$$K_{t+1} = (1-\delta)K_t + I_t \tag{24}$$

$$C_t + \hat{B}_{Ht+1} + \hat{S}_t\hat{B}_{Ft+1} + I_t + \frac{1}{2}\Phi\left(\frac{I_t}{K_t} - \delta\right)^2 K_t + \frac{1}{2}\phi_H \hat{B}^2_{Ht+1} + \frac{1}{2}\phi_F\left(\hat{S}_t\hat{B}_{Ft+1}\right)^2$$
$$= \hat{W}_t L_t + \hat{R}_t K_t + \frac{\hat{B}_{Ht}(1+i_{t-1})}{\pi_t} + \frac{\hat{S}_t\hat{B}_{Ft}(1+i^*_{t-1})}{\pi^*_t} + \hat{\pi}^{int}_t + \hat{\pi}^f_t \tag{25}$$

$$\rho = \hat{W}_t / C_t \tag{26}$$

$$\frac{1}{C_t}\left[1 + \Phi\left(\frac{I_t}{K_t} - \delta\right)\right] - E_t\left\{\beta\frac{1}{C_{t+1}}\left(\hat{R}_{t+1} + \frac{1}{2}\Phi\left[\left(\frac{I_{t+1}}{K_{t+1}}\right)^2 - \delta^2\right]\right)\right\} -$$
$$E_t\left\{\beta\frac{1}{C_{t+1}}(1-\delta)\left(1 + \Phi\left(\frac{I_{t+1}}{K_{t+1}} - \delta\right)\right)\right\} = 0 \tag{27}$$

$$1 + \phi_H \hat{B}_{Ht+1} = \beta E_t\left[\frac{C_t}{C_{t+1}}\frac{(1+i_t)}{\pi_{t+1}}\right] \tag{28}$$

$$1 + \phi_F \hat{S}_t\hat{B}_{Ft+1} = \beta E_t\left[\frac{C_t}{C_{t+1}}\frac{\hat{S}_{t+1}(1+i^*_t)}{\hat{S}_t\pi^*_{t+1}}\right] \tag{29}$$

$$\hat{B}_{Ht} + \hat{B}^*_{Ht} = 0 \tag{30}$$

$$C_t + \hat{B}_{Ht+1} + \hat{S}_t\hat{B}_{Ft+1} + I_t + \frac{1}{2}\Phi\left(\frac{I_t}{K_t} - \delta\right)^2 K_t + \frac{1}{2}\phi_H \hat{B}^2_{Ht+1} + \frac{1}{2}\phi_F\left(\hat{S}_t\hat{B}_{Ft+1}\right)^2 = Y_t \tag{31}$$

$$i_t = i + \Xi_\pi log\left(\frac{\pi_t}{\pi}\right) + \Xi_y log\left(\frac{Y_{Ht} + Y^*_{Ht}}{Y_H + YH^*}\right) + \Xi_s log\left(\frac{S_t}{S}\right). \tag{32}$$

Similarly, there are 9 symmetric equations in the foreign country. Putting these three sections together, we have 47 endogenous variables and 48 equations. After considering the exogenous technology shocks A_t and A^*_t, totally we have 49 variables and 50 equations. By Walras' Law, one market clearing condition is redundant.

9 Steady State

In this section, we solve for a symmetric steady state of the model. Under the symmetry, the asset holdings are zero for both countries, i.e. $\hat{B}_H = \hat{B}^*_H = \hat{B}_F = \hat{B}^*_F = 0$. Here we use the variables without time script to denote steady state values. It is pretty easy for us to find the steady state values for the following variables by simply dropping time script in the above equilibrium conditions

$$\hat{mc} = \hat{mc}^* = \frac{\theta - 1}{\theta}$$

$$\hat{P}^A = \hat{P}^{A*} = 1$$

$$\hat{P}_H^A = \hat{P}_F^{A*} = 1$$

$$\hat{R} = \hat{R}^* = 1/\beta - 1 + \delta$$

$$i = i^* = \pi/\beta - 1$$

$$\pi_H = \pi_F^* = \pi$$

$$A = A^* = 1.$$

The other variables are jointly determined by the remaining 29 equations. We use Matlab to solve the steady state numerically and find that solution is symmetric. So we can find the analytical solution by imposing symmetry from the beginning

$$\hat{mc} = \hat{mc}^* = \hat{P}_H = \hat{P}_F^* = \frac{\theta - 1}{\theta} \tag{1}$$

$$\hat{mc}^{int} = \hat{mc}^{int*} = \frac{(\phi - 1)(\theta - 1)}{\phi\theta} \tag{2}$$

$$\hat{W} = \hat{W}^* = \left(\left(\frac{1-\psi}{\psi} \right)^{1-\psi} \hat{R}^{-\psi} \psi \Theta \right)^{\frac{1}{1-\psi}} \tag{3}$$

where $\Theta = \frac{\theta\phi + 1 - \theta - \phi}{\theta\phi}$.

$$C = C^* = \hat{W}/\rho \tag{4}$$

$$Y = Y^* = \frac{C}{1 - \Theta\hat{W}\psi/R} \tag{5}$$

$$K = K^* = (Y - C)/\delta \tag{6}$$

$$L = L^* = \frac{1-\psi}{\psi} \frac{\hat{R}K}{\hat{W}} \tag{7}$$

$$Y_H = Y_F^* = \alpha Y \tag{8}$$

$$Y_H^* = Y_F = (1 - \alpha)Y \tag{9}$$

$$\hat{S} = 1 \tag{10}$$

$$\hat{P}^n = \hat{P}^{n*} = \frac{\theta \hat{mc} Y}{1 - \beta\lambda_f} \tag{11}$$

$$\hat{P}^d = \hat{P}^{d*} = \frac{(\theta - 1)Y}{1 - \beta\lambda_f} \tag{12}$$

$$\hat{P}_H^n = \hat{P}_F^{n*} = \frac{\phi \hat{mc}^{int} (Y_H + Y_H^*)}{(1 - \beta\lambda_{int})\hat{P}_H} \tag{13}$$

$$\hat{P}_H^d = \hat{P}_F^{d*} = \frac{(\phi - 1)(Y_H + Y_H^*)}{1 - \beta\lambda_{int}}. \tag{14}$$

This solution has been verified by the numerical solution from our Matlab programs.

10 Comparison of Inflation Targeting Regimes

In this section, we compare the CPI inflation targeting with other inflation targeting regimes. We consider 3 alternative inflation targeting regimes: (1) both countries target PPI inflation; (2) home country targets CPI while foreign country targets PPI; (3) both countries target both CPI and PPI inflations. Table (10) gives the comparison of CPI inflation targeting with PPI and asymmetric inflation targeting. When the central banks take strong stance against inflation rate, there is almost no difference in welfare level among these regimes. In all regimes, the welfare level is not sensitive to foreign country's policy mistake. Our results do not show strong support for the central banks to switch to PPI inflation targeting regime.

Now we turn to the case where the central banks target both CPI and PPI inflation rates

$$i_t = i + \Xi_{cpi} log\left(\frac{\pi_t^{cpi}}{\pi}\right) + \Xi_{ppi} log\left(\frac{\pi_t^{ppi}}{\pi}\right). \tag{1}$$

π_t^{cpi} and π_t^{ppi} are respectively CPI and PPI inflation rate at time t. Ξ_{cpi} and Ξ_{ppi} are weights that the central banks put on each inflation rate in their monetary policy. As before, we did a grid search over [0, 3] in steps of 0.1 for both Ξ_{cpi} and Ξ_{ppi}. Compared to the regime targeting single inflation rate, targeting both inflation rates has a larger parameter area that induces deterministic equilibrium. We find that a deterministic equilibrium exists whenever the sum of Ξ_{cpi} and Ξ_{ppi} is greater than unity.

However, the double-inflation-targeting only generates negligible welfare gains in our model. In comparison with CPI inflation targeting, the welfare gain is only about 0.014 percent.[43] Therefore in our paper, we only consider the CPI inflation targeting while discussing exchange rate policy.

[43]The maximum welfare level is -53.833 at the point Ξ_{cpi}=3 and Ξ_{ppi}=1.8.

Table 1: Calibration

Parameter	Value	Description
β	0.99	Subjective discount factor
α	0.85	Home bias
θ	6	Elasticity of substitution between final goods
ϕ	6	Elasticity of substitution between intermediate goods
γ	1.1	Elasticity of substitution between home and foreign goods
λ_f	0.75	Probability of not re-optimizing for final goods firm
λ_{int}	0.5	Probability of not re-optimizing for intermediate goods firm
ψ	0.3	Share of capital in production
σ	1	Preference parameter
ρ	1	Preference parameter
Φ	8	Cost parameter of capital adjustment
ϕ_d	0	Cost parameter of holding domestic bonds
ϕ_f	$0.0037/Y_H^*$	Cost parameter of holding foreign bonds
δ	0.025	Capital depreciation rate
π	1.0103	Steady state inflation rate in Home country
π^*	1.0103	Steady state inflation rate in Foreign country
A	1	Steady state technology shock in Home country
A^*	1	Steady state technology shock in Foreign country
$\xi_{11} = \xi_{22}$	0.9	Technology shock AR(1) coefficient
$\xi_{12} = \xi_{21}$	0.03	Technology spillovers
λ_a	0.88	UIP shock parameter
ζ	0.8	Habit persistence parameter

Table 2: Business Cycle Statistics for the Model

		Standard Deviations Relative to That of GDP			
	SD of GDP (%)	Consumption	Investment	Employment	RE
Data†	1.72	0.83	2.78	0.67	4.36
Benchmark	1.94	0.98	2.91	0.97	4.09
No UIP	1.89	0.70	2.88	0.35	0.49
Habit	2.08	0.75	2.84	1.10	3.84
		Autocorrelation			
	GDP	Consumption	Investment	Employment	RE
Data	0.88	0.89	0.91	0.90	0.83
Benchmark	0.96	0.94	0.91	0.95	0.92
No UIP	0.96	0.98	0.93	0.93	0.96
Habit	0.94	0.99	0.92	0.91	0.92

Note:
†–SD of GDP is from Cooley and Prescott (1995). Other data are from Chari *et al.* (2002).

Table 3: Non-negativity Check of Nominal Interest Rate

Ξ_π	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
SD (%)	0.958	0.848	0.765	0.705	0.660	0.626	0.600	0.578	0.561	0.546
Ratio†	2.119	2.394	2.653	2.879	3.074	3.241	3.386	3.511	3.620	3.715
Ξ_π	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0
SD (%)	0.534	0.524	0.516	0.508	0.502	0.496	0.491	0.487	0.484	0.480
Ratio†	3.799	3.872	3.937	3.994	4.045	4.090	4.130	4.166	4.198	4.227

Note:
†–Ratio of steady-state interest rate to standard deviation of interest rate.

Table 4: Welfare Loss under Exchange Rate Stabilization (Benchmark)

	$\Xi_\pi = 3.0$		$\Xi_\pi = 1.1$	
Ξ_s	Welfare Level	Welfare Loss (%)†	Welfare Level	Welfare Loss (%)†
0.00	-53.847	0.001	**-54.228**	**0.000**
0.01	**-53.846**	**0.000**	-54.241	0.012
0.02	-53.846	0.000‡	-54.315	0.086
0.03	-53.847	0.002	-54.421	0.192
0.04	-53.850	0.004	-54.542	0.313
0.05	-53.853	0.007	-54.668	0.439
0.06	-53.857	0.011	-54.794	0.564
0.07	-53.862	0.016	-54.917	0.687
0.08	-53.867	0.022	-55.036	0.804
0.09	-53.873	0.028	-55.149	0.917
0.10	-53.880	0.035	-55.257	1.023

Note:
†–Welfare loss is measured by the percentage consumption loss.
‡–This entry is equal to 0.0003 without rounding.

Table 5: Welfare Loss under Exchange Rate Stabilization ($\alpha = 0.6$)

$\Xi_\pi = 3.0$			$\Xi_\pi = 1.1$		
Ξ_s	Welfare Level	Welfare Loss (%)†	Ξ_s	Welfare Level	Welfare Loss (%)†
0.00	-53.9726	0.0115	**0.00**	**-54.5214**	**0.0000**
0.02	-53.9689	0.0078	0.01	-54.5502	0.0288
0.04	-53.9660	0.0050	0.02	-54.5856	0.0641
0.06	-53.9639	0.0029	0.03	-54.6294	0.1079
0.08	-53.9625	0.0014	0.04	-54.6840	0.1624
0.10	-53.9616	0.0005	0.05	-54.7526	0.2309
0.12	-53.9611	0.0001	0.06	-54.8400	0.3180
0.14	**-53.9611**	**0.0000**	0.07	-54.9526	0.4303
0.16	-53.9614	0.0003	0.08	-55.1003	0.5772
0.18	-53.9620	0.0009	0.09	-55.2979	0.7734
0.20	-53.9629	0.0018	0.10	-55.3690	0.8440

Note:
†–Welfare loss is measured by the percentage consumption loss.

Table 6: GDP and Exchange Rate Volatility

Panel A: Standard Deviation Changes with UIP Shock ($\alpha = 0.6$)								
	→ UIP shock increases →							
SD of GDP (%)	1.75	1.93	2.13	2.35	2.58	2.81	3.06	3.32
SD of RER (%)	1.54	1.97	2.40	2.83	3.27	3.70	4.14	4.57
$\frac{SD \quad of \quad RER}{SD \quad of \quad GDP}$	0.88	1.02	1.13	1.21	1.27	1.32	1.35	1.38
Panel B: Standard Deviation Changes with UIP Shock (Benchmark)								
	→ UIP shock increseses →							
SD of GDP (%)	1.49†	1.50	1.53	1.57	1.64	1.72	1.81	1.91
SD of RER (%)	0.76†	1.33	2.30	3.35	4.41	5.49	6.57	7.65
$\frac{SD \quad of \quad RER}{SD \quad of \quad GDP}$	0.51†	0.89	1.51	2.13	2.69	3.20	3.63	4.00
Panel C: Relative Volatility Changes with Home Bias								
Home Bias (α)	0.5	0.6	0.7	0.8	0.9			
SD of RER (%)	3.98	4.75	5.81	7.16	8.79			
$\frac{SD \quad of \quad RER}{SD \quad of \quad GDP}$	0.98	1.39	2.08	3.25	5.11			

Note:
†–UIP shock is zero.
Panel A: Home bias parameter (α) is set to 0.6.
Panel B: Benchmark model with home bias parameter (α) set to 0.85.
Panel C: Home bias increases while UIP shock is constant.
Interest rate rule: $i_t = i + 1.5log\left(\frac{\pi_t}{\pi}\right)$.

Table 7: OLS Regression Results

Panel A: $y_i = \alpha + \beta openness_i + \varepsilon_i$				
	$y_i = REER \quad vol$		$y_i = \frac{REER \quad vol}{GDP \quad vol}$	
	OECD	46 Countries	OECD	46 Countries
β	-0.037	-0.054	-2.550	-0.754
	(0.019)*	(0.030)*	(0.928)**	(0.327)**
R-squared	0.163	0.068	0.274	0.108
# of obs.	22	46	22	46
Panel B: $Log(y_i) = \alpha + \beta Log(openness_i) + \varepsilon_i$				
	$y_i = REER \quad vol$		$y_i = \frac{REER \quad vol}{GDP \quad vol}$	
	OECD	46 Countries	OECD	46 Countries
β	-0.481	-0.311	-0.754	-0.525
	(0.182)**	(0.159)*	(0.233)***	(0.185)***
R-squared	0.258	0.080	0.344	0.154
# of obs.	22	46	22	46

Note :
Standard deviations are reported in parentheses.
$* - 10\%$ significant $** - 5\%$ significant $*** - 1\%$ significant

Table 8: Welfare Loss under Exchange Rate Stabilization (Habit Persistence)

	$\Xi_\pi = 3.0$		$\Xi_\pi = 1.1$	
Ξ_s	Welfare Level	Welfare Loss (%)†	Welfare Level	Welfare Loss (%)†
0.00	**-287.390**	**0.000**	**-287.730**	**0.000**
0.01	-287.390	0.002	-287.760	0.026
0.02	-287.400	0.006	-287.840	0.110
0.03	-287.400	0.011	-287.960	0.223
0.04	-287.410	0.017	-288.080	0.350
0.05	-287.420	0.023	-288.220	0.480
0.06	-287.420	0.031	-288.350	0.610
0.07	-287.430	0.040	-288.470	0.736
0.08	-287.440	0.049	-288.590	0.856
0.09	-287.450	0.059	-288.710	0.971
0.10	-287.460	0.069	-288.820	1.080

Note :
†–Welfare loss is measured by the percentage consumption loss.

Table 9: GDP and Exchange Rate Volatility under Exchange Rate Targeting

Panel A: Standard Deviation Changes with UIP Shock									
	Ξ_s	→ UIP shock increseses →							
SD of GDP (%)	0.1	1.43†	1.44	1.46	1.50	1.55	1.62	1.69	1.78
	0.2	1.40†	1.42	1.49	1.59	1.73	1.88	2.06	2.25
	0.5	1.37†	1.43	1.58	1.80	2.07	2.38	2.70	3.04
SD of RER (%)	0.1	0.48†	1.05	1.92	2.83	3.75	4.68	5.61	6.53
	0.2	0.35†	0.77	1.42	2.09	2.77	3.45	4.13	4.82
	0.5	0.18†	0.42	0.79	1.17	1.55	1.93	2.31	2.69
$\frac{SD\ of\ RER}{SD\ of\ GDP}$	0.1	0.34	0.73	1.32	1.89	2.42	2.90	3.32	3.68
	0.2	0.25	0.54	0.95	1.31	1.60	1.83	2.01	2.14
	0.5	0.13	0.30	0.50	0.65	0.75	0.81	0.85	0.89
Panel B: Relative Volatility Changes with Home Bias									
Home Bias (α)		0.5	0.6	0.7	0.8	0.9			
$\frac{SD\ of\ RER}{SD\ of\ GDP}$	Ξ_s=0.1	0.729	1.023	1.637	2.714	3.028			
	Ξ_s=0.2	0.575	0.794	1.297	1.946	1.697			
	Ξ_s=0.5	0.348	0.465	0.770	0.944	0.711			

Note :
†–UIP shock is zero.
Interest rate rule: $i_t = i + 1.5log\left(\frac{\pi_t^{cpi}}{\pi}\right) + \Xi_s log\left(\frac{s_t}{s}\right)$.

Table 10: Welfare Loss under Different Targeting Regimes

	Home	Foreign	Welfare Level	Welfare Loss (%)†	Welfare Level	Welfare Loss (%)†
			Both Shocks		Tech Shock only	
CPI	3	3	-53.847	0	-53.768	0
Targeting	1.1	3	-54.208	0.361	-53.809	0.041
	3	1.1	-53.879	0.033	-53.771	0.003
	1.1	1.1	-54.228	0.381	-53.811	0.043
PPI	3	3	-53.844	0	-53.767	0
Targeting	1.1	3	-54.126	0.282	-53.912	0.144
	3	1.1	-53.864	0.020	-53.776	0.008
	1.1	1.1	-54.139	0.295	-53.915	0.148
Asymmetric	3	3	-53.847	0	-53.768	0
Targeting‡	1.1	3	-54.213	0.365	-53.809	0.041
	3	1.1	-53.848	0	-53.777	0.009
	1.1	1.1	-54.215	0.366	-53.823	0.055

†–The welfare loss is based on the optimal welfare of symmetric CPI targeting.

‡–Home country targets CPI inflation and foreign country targets PPI inflation.

Figure 1: Structure of Production in Home Country

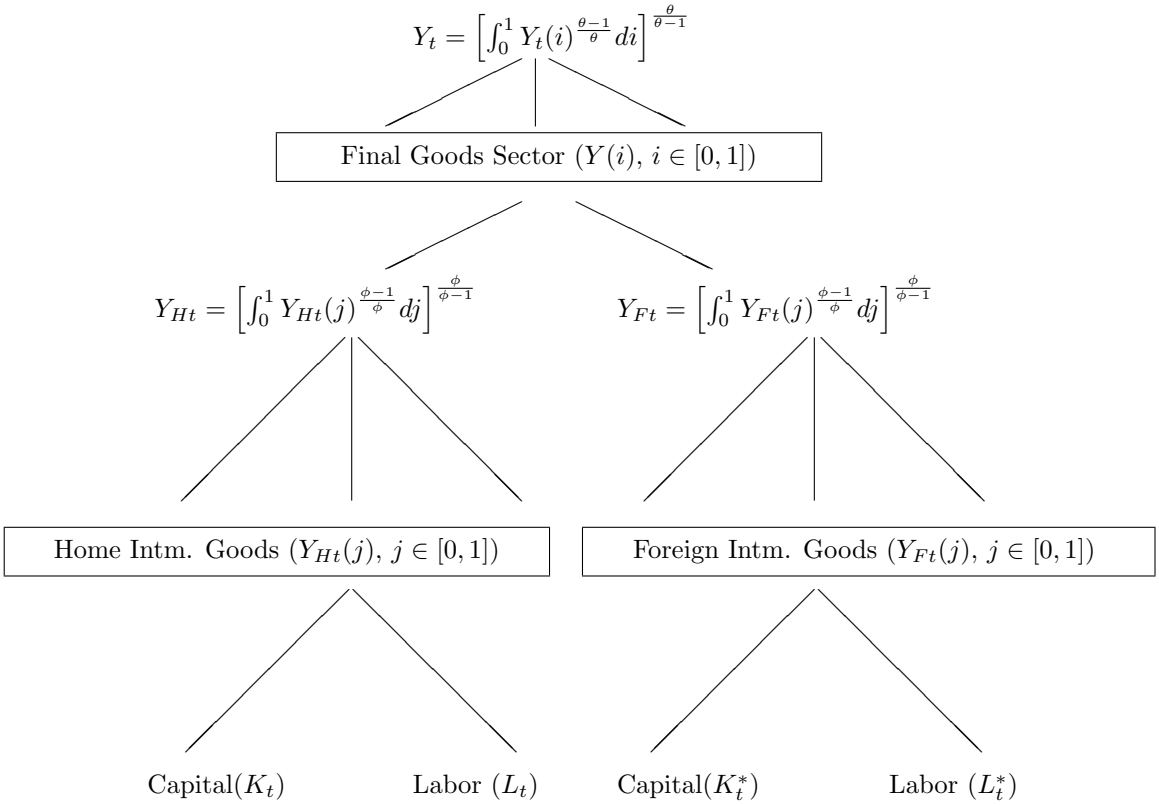


Figure 2: Determinate Equilibria

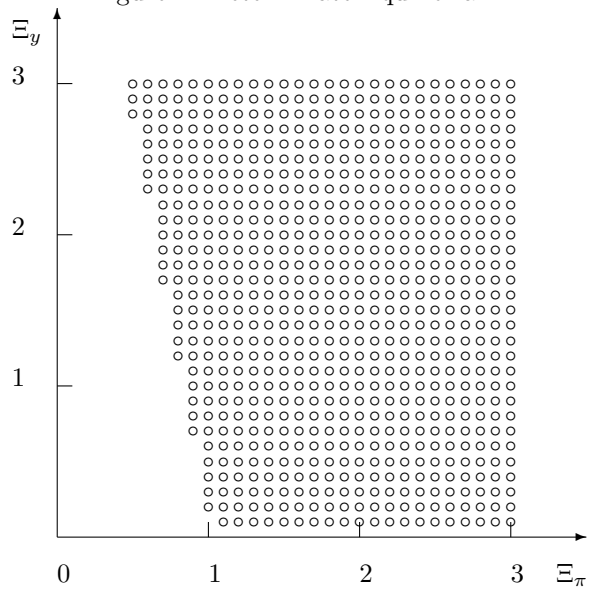


Figure 3: Welfare as a function of Ξ_π

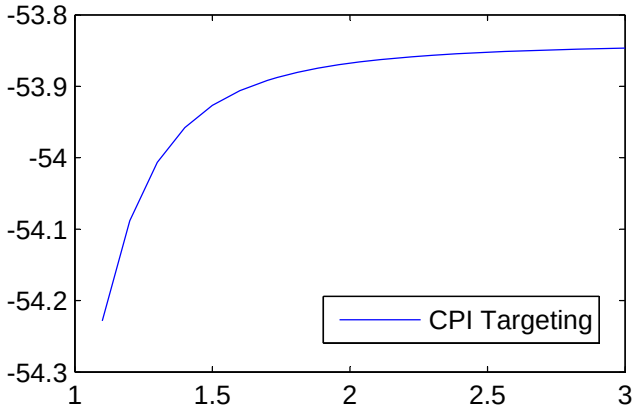


Figure 4: Impulse Response Functions to Technology Shock

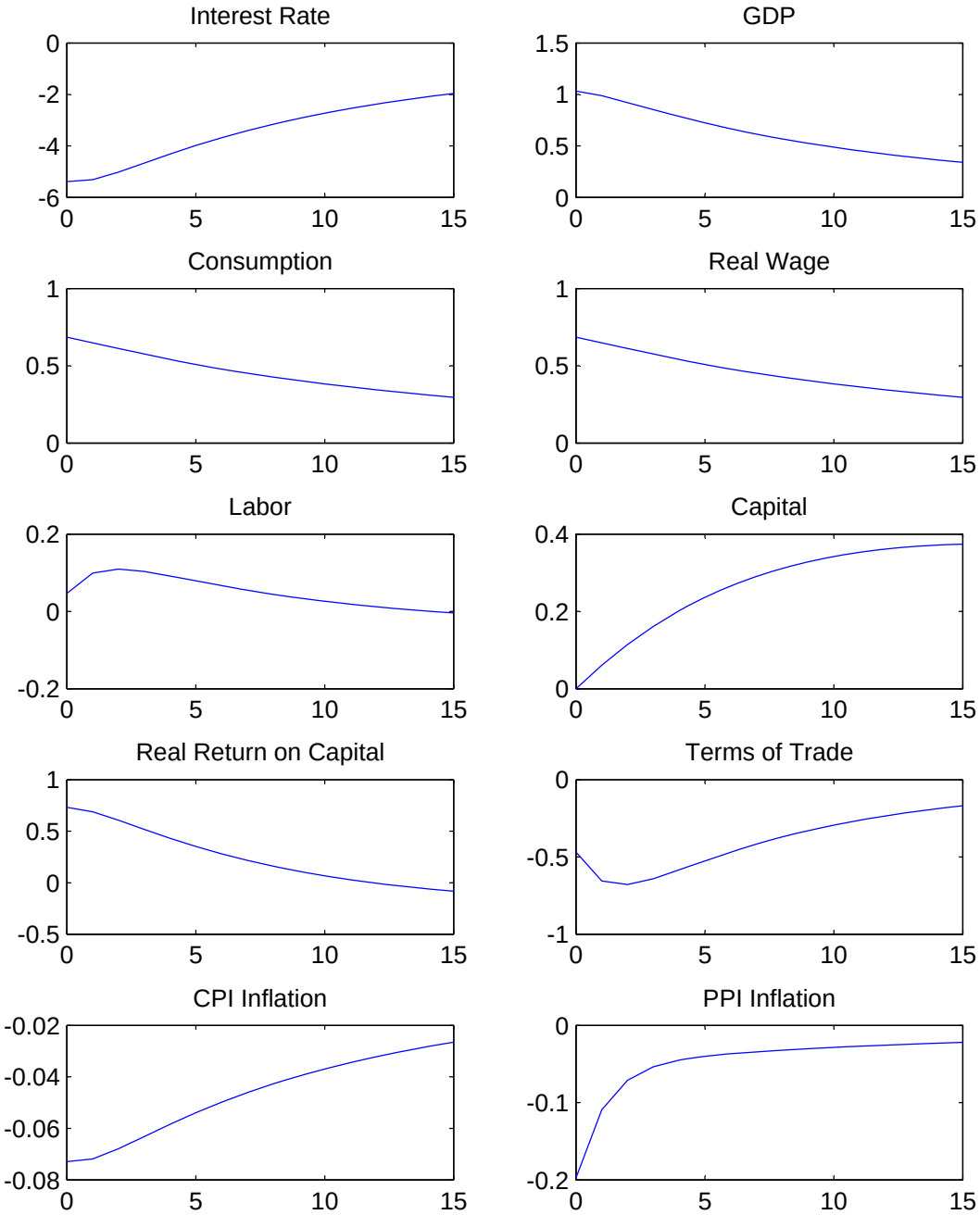


Figure 5: Impulse Response Functions to UIP Shock

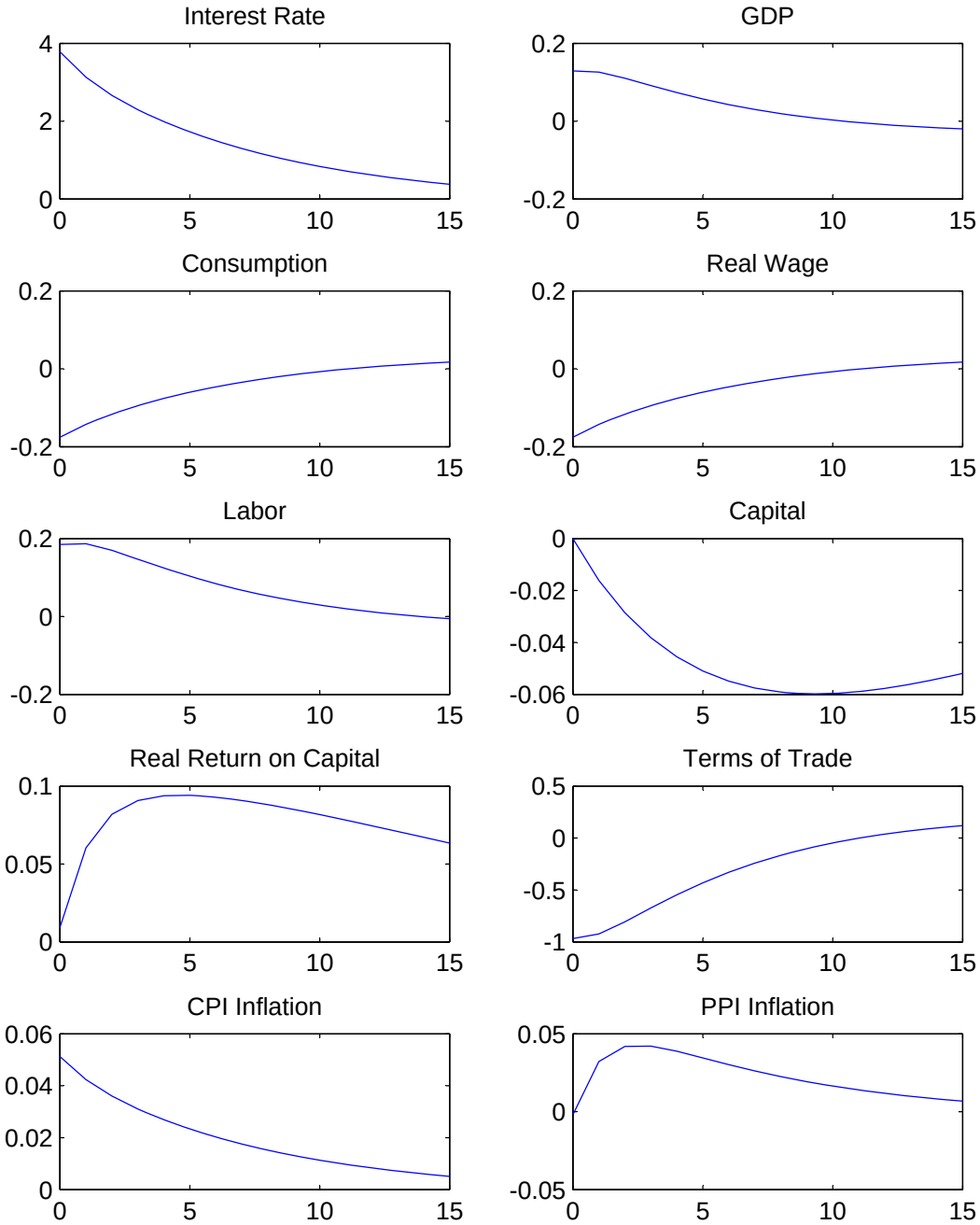


Figure 6: Impulse Response Functions of the Terms of Trade to Technology Shock

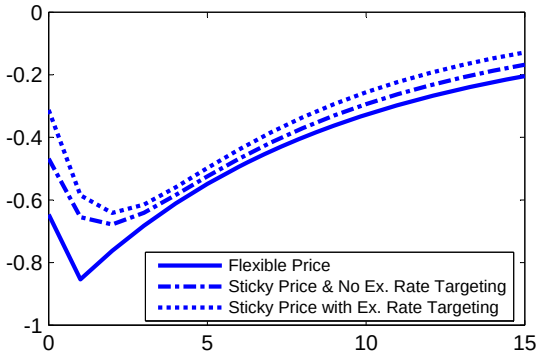
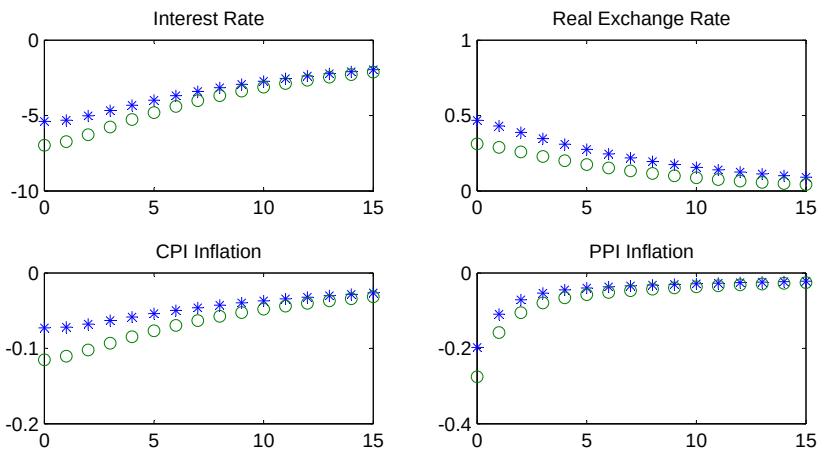
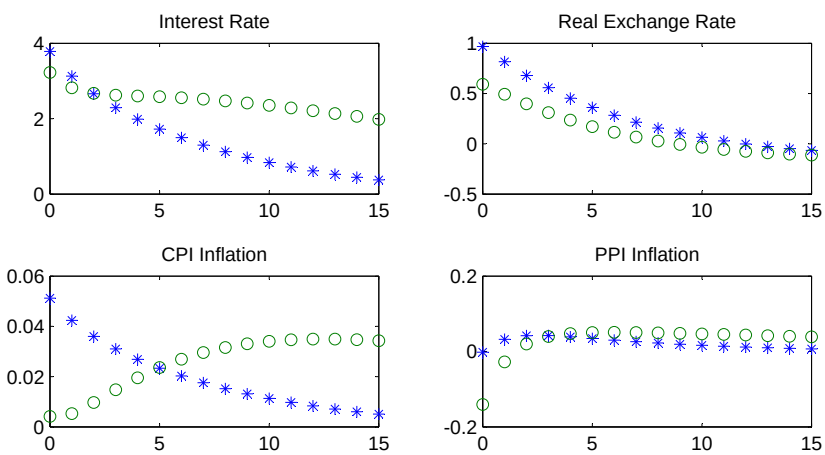


Figure 7: IRFs with and without Exchange Rate Stabilization



(a) Impulse Response Functions to Technology Shock



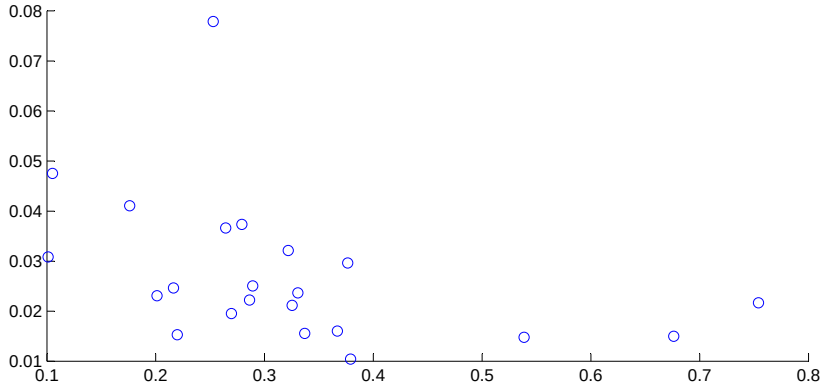
(b) Impulse Response Functions to UIP shock

Note:

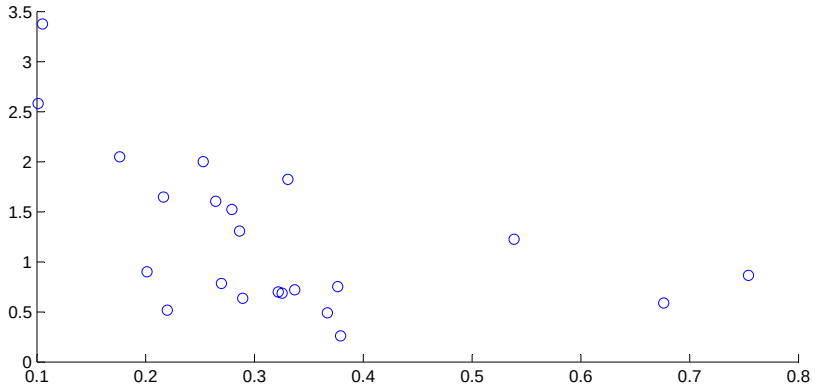
The asterisk represents the monetary policy without exchange rate stabilization, and the circle stands for the one with exchange rate stabilization.

In exchange rate stabilization policy, $\Xi_\pi = 1.5$ and $\Xi_s = 0.1$.

Figure 8: Scatter Diagram of REER and Relative REER Volatility against Openness



(a) Volatility of REER vs. Openness



(b) Volatility of relative REER vs. Openness