# Distortionary Taxes and Public Investment when Government Promises are not Enforceable*

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**Abstract**

*We characterize Markov-perfect equilibria in a setting where the absence of government commitment affects the financing of productive public capital. We show that at any date, a government in office only considers intertemporal distortions over two consecutive periods in choosing taxes. We then use our framework to quantify the value of commitment, which we define as that obtained from binding governments to a course of actions that produce the second-best allocations. Because this calculation relies on numerical approximations, we contrast alternative approaches in the literature. We find that both linear quadratic and perturbation methods deliver accurate steady states, but that the former can yield spurious policy implications along the transition. Ultimately, our analysis suggests that very small costs of setting up a commitment technology are enough to prevent its adoption. Furthermore, while households' decision to forego government commitment may be rational at some initial date, it is nevertheless the case that consumption allocations may differ considerably in the long run.*

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# 1 Introduction

The notion that governments cannot always commit to a sequence of actions is a subject of increasing interest for economists in general and policymakers in particular. To this point, the literature on time consistent fiscal policy has confined itself to simple environments where taxes are used to finance a flow of public goods or services that are rapidly exhausted. In contrast, the benefits of government spending have been mainly documented for durable public goods that can be accumulated over time.[1] This fact is ignored in recent studies because introducing public capital (an additional state variable) significantly complicates the characterization of the optimal discretionary policy. This paper, therefore, tackles the problem of understanding how the absence of government commitment affects the provision of public infrastructure as well as the implied welfare effects over an economy's transition to its long-run equilibrium. We solve for Markov-perfect equilibria and provide a quantitative assessment of the value of commitment, which we define as that obtained from binding the government to an optimal course of actions. In doing so, we evaluate the performance of different numerical methods used in approximating time-consistent policy.

Several papers have recently analyzed optimal fiscal policy absent commitment. These include, among others, Klein, Krusell, Rios-Rull (2004), who analyze the trade off between providing a consumable public good and its financing, Hassler, Storesletten, and Zilibotti (2005), who study time-consistent redistribution under repeated voting, and Azzimonti, De Francisco and Krusell (2006), who explore the distortionary effects of income taxes on the evolution of wealth inequality. In contrast to these papers, our analysis focuses on the provision of a durable public good that expands the production frontier and which we interpret as infrastructure. Previous work on optimal public investment, including by Glomm and Ravikumar (1994, 1997), or Turnovsky (1997), characterize optimal policy only under full commitment. Thus, we contribute to the literature on public investment and discretionary policy in mainly three ways.

First, we provide an assessment of the value of government commitment for an economy whose stocks of private and public capital are initially below their long-run levels. The economy we envision is one that is still in a development phase and where the government's inability to make credible policy promises regarding infrastructure financing can affect both its transition dynamics and steady state. We find that a very small cost of setting up a binding constitution is enough to prohibit its adoption. Furthermore, although households' decision to forego government commitment may be entirely rational at some initial date, the resulting allocations can look considerably different in the long run. The disparity in steady state allocations emerges because the stocks of private and public capital take on very different paths under the two institutional alternatives. Our findings indicate that the lack of government commitment discourages long-run private capital accumulation, despite tax rates being actually lower in the Markov-perfect equilibrium. The reason is that discretionary policy also results in less public infrastructure being developed.

[1] See Fernald (1999), or Haughwout (2002), for example.

Second, at a theoretical level, we show that governments following a Markov-perfect policy choose a tax rate such that they trade off marginal inefficiencies arising in private savings with those arising in the provision of public infrastructure over two consecutive periods. The derivation of the government Euler equation (GEE) in this case is substantially more involved than those developed in previous work but remains analytically tractable. More importantly, we show that this derivation is essential for the application of numerical methods that accurately describe transition dynamics.

Finally, in computing the Markov-perfect policy problem, we compare numerical solutions obtained using GEE-based perturbation methods recently suggested in Krusell, Kuruscu, and Smith (2002), with those that emerge under the more common Linear Quadratic (LQ) approximation developed in Svensson and Woodford (2004). We know of no other papers in the literature that compare these two methods for a single problem. Because we study a model that allows for a closed-form Markov-perfect equilibrium under a particular parameterization, we can directly contrast the accuracy of these two methods. While both the LQ and perturbation methods deliver accurate steady state allocations, we find that the approximation error associated with the former can yield inaccurate estimates and misguided policy recommendations in response to changes in the state variables. An application of the perturbation method is able to essentially generate the closed-form decision and policy rules, and we use this approach to compute the evolution of discretionary policy for an example economy.

This paper is organized as follows. Section 2 describes the basic economic environment. In section 3, we define the competitive equilibrium given a stationary policy rule. Section 4 characterizes the Markov-perfect equilibrium that yields the optimal policy. Section 5 contrasts numerical solution methods, and we calculate the value of government commitment in Section 6. Section 7 offers some concluding remarks.

## 2 Economic Environment

Consider an economy populated by infinitely many households whose preferences are given by

$$\mathcal{U} = \sum_{t=0}^{\infty} \beta^t u(c_t),$$

where $0 < \beta < 1$ is a subjective discount rate, and households' period utility, $u(c_t)$, satisfies $u_c > 0$, $u_{cc} < 0$, and the usual Inada conditions. The size of the population is normalized to one.

A single consumption good is produced using the technology

$$y_t = F(k_t, l_t, k_{gt}),$$

where $k_t$ and $k_{gt}$ denote the date $t$ stocks of capital in the private and public sector respectively. We interpret $k_g$ either as public capital in the form of physical infrastructure, or as a more general

kind of public good, such as education, that enhances the productivity of other factor inputs. Labor input is denoted by $l_t$, and we assume that $F$ exhibits constant returns to scale with respect to private capital and labor. We denote the public capital elasticity of output, $F_{k_g}\frac{k_g}{y}$, by $\theta \in (0,1)$, and assume that $F_{kk_g} > 0$.

Both types of capital can be accumulated over time and evolve according to

$$k_{t+1} = i_{kt} + (1-\delta)k_t, \tag{1}$$

and

$$k_{gt+1} = i_{k_g t} + (1-\delta)k_{gt}, \tag{2}$$

where $i_k$ and $i_{k_g}$are the levels of private and public investment respectively, and $\delta \in (0,1)$ is the depreciation rate.

## 2.1 The First-Best Solution

To begin the analysis, it is useful to characterize efficient allocations in this environment. These allocations will then be used as a benchmark with which to compare the decentralized solutions that emerge under two key policy frictions: distortionary taxation and the inability to make credible promises regarding future tax policy.

Pareto-optimal allocations are found by solving the problem of a benevolent planner who chooses sequences of consumption, private capital, and public capital so as to maximize households' lifetime utility

$$\max_{\{c_t, k_{t+1}.k_{gt+1}\}} \sum_{t=0}^{\infty} \beta^t u(c_t), \tag{P*}$$

subject to

$$c_t + k_{t+1} + k_{gt+1} = F(k_t, k_{gt}, l_t) + (1-\delta)(k_t + k_{gt}).$$

The first-order necessary conditions imply that[2]

$$u_c(c_t) = \beta[F_{kt+1} + 1 - \delta]u_c(c_{t+1}) \tag{3}$$

and that

$$u_c(c_t) = \beta[F_{k_{gt+1}} + 1 - \delta]u_c(c_{t+1}). \tag{4}$$

Therefore, in the absence of frictions, the standard result obtains whereby it is optimal to invest in $k$ and $k_g$ up to the point where their marginal products are equalized at each date,

$$F_k(k_{t+1}, k_{gt+1}, l_{t+1}) = F_{k_g}(k_{t+1}, k_{gt+1}, l_{t+1}).$$

[2] We use $u_{ct}$ to indicate $u_c(c_t)$, $F_{kt+1}$ to indicate $F_k(k_{t+1}, k_{gt+1}, l_{t+1})$, etc... to simplify notation when the context is clear.

We refer to the optimal levels of private and public capital as the 'unconstrained optimum', and denote these levels by $k_t^*$ and $k_{gt}^*$ respectively.

It is straightforward to show how to obtain these allocations in a decentralized competitive equilibrium when the government has access to lump sum taxation. In the absence of such an instrument, however, a distortion emerges that creates a wedge, or gap, in conditions (3) and (4) above. In addition, to the degree that policies other than lump sum taxes are used, such policies will generally be time inconsistent[3]. In general, we define a wedge in the efficient private investment decision by

$$\Delta_{kt} = -u_{ct} + \beta u_{ct+1} \left( F_{kt+1} + 1 - \delta \right), \tag{5}$$

where $\Delta_{kt} = 0 \ \forall t$ under first-best allocations. Similarly, we define a wedge in public investment by

$$\Delta_{k_{gt}} = -u_{ct} + \beta u_{ct+1} \left( F_{k_{gt+1}} + 1 - \delta \right). \tag{6}$$

# 3 The Decentralized Economy given Policy

Because lump sum taxes are almost never observed, we focus on a decentralized economy where the government uses distortionary income taxes to finance its public capital. In that setting, a new government coming into office typically has an incentive to disregard promises made by its predecessors. Hence, setting taxes once and for all at time zero results in policy announcements that are not credible. In other words, absent a commitment technology, the Ramsey policy is time inconsistent. Our analysis centers on the determination of optimal time consistent tax rates, and contrasts the findings with those that obtain under full commitment. In particular, we study Markov-perfect equilibria whereby sequential governments choose policy optimally based on the state they inherit when taking office.[4]

Throughout the paper, we assume that the government balances its budget every period. While we recognize that the presence or absence of government debt matters importantly for the types of policies that emerge as optimal, we maintain the balanced budget assumption in this paper for mainly three reasons. First, we wish to contrast our findings regarding the lack of government commitment with previous studies of optimal public investment carried out under full commitment. We shall show that under full commitment, our framework nest findings in Glomm and Ravikumar (1994) among others. Second, Azzimonti, Sarte and Soares (2006) show that government debt can be a powerful instrument that helps mitigate the time consistency problem in questions of optimal taxation. Therefore, an environment where domestic government debt is in limited use, such as in developing economies, is also one where the time consistency problem is likely to have the greatest effects. Finally, under a specific parameterization of the model, the framework we write down is

[3] See Kydland and Prescott (1977) for an early treatment of time inconsistent policy.

[4] An alternative approach finds the set of all possible sustainable equilibria, and characterizes the problem using reputational mechanisms that rely on trigger strategies involving reversions to the worst possible equilibrium (Chari and Kehoe [1993]). Such mechanisms, however, have been criticized for not being renegotiation proof.

simple enough to admit a closed form solution that allows us to gauge the accuracy of different numerical solution methods.

Suppose that the tax rate households face at any date stems from a stationary policy rule that depends only on the states of the economy at that date, $\Psi(k, k_g)$. Since the government maintains a balanced budget by assumption, public investment satisfies $i_g = \Psi(k, k_g)y$, and we can express new outlays of public capital, $k_g'$ as

$$k_g' = \Psi(k, k_g)y + (1 - \delta)k_g. \tag{7}$$

The financing of public infrastructure, therefore, follows that of Baxter and King (1993), who explore the allocative effects of exogenous policy changes, or Glomm and Ravikumar (1994, 1997), who later derive the optimal policy with full government commitment. We contribute to these studies by exploring the effects of relaxing the commitment assumption. Throughout the analysis, we denote next period's value of a given variable $x$ by $x'$.

### 3.1 The Recursive Competitive Equilibrium

In order to address how governments might choose discretionary policy optimally in our environment, we first need to describe how households and firms make decisions given that a tax policy, $\Psi(k, k_g)$, is in place. To this end, we first describe the recursive competitive equilibrium of our economy. In this decentralized equilibrium, households and firms take $\Psi(k, k_g)$ as given when making decisions.

*Firms*

There exists a large number of homogeneous small size firms that act competitively, where $k_g$ acts as a common externality with respect to each firm's production. We denote by $r$ and $w$ the rental price of private capital and the wage. Taking these prices as given, each firm maximizes profits and solves

$$\max_{k,\, l} \ \Pi = F(k, l, k_g) - rk - wl.$$

The corresponding first-order conditions equate $r$ and $w$ to the marginal products of capital, $F_k(k, l, k_g)$, and labor, $F_l(k, l, k_g)$, respectively.

*Households*

At each date, households decide how much to consume and save as well as how much capital to rent to firms. Each household is assumed to be endowed with one unit of time, $l = 1$, which they supply inelastically. Taking the policy rule $\Psi(k, k_g)$ as given, households maximize their lifetime utility subject to their budget constraint and the law of motion describing the accumulation of private capital. Because the policy rule $\Psi$ is stationary (i.e. it does not depend on time as a separate argument), we can write the household problem recursively as follows,

$$V(k, k_g) = \max_{c, k'} \left\{ u(c) + \beta V(k', k_g') \right\} \tag{P$^C$}$$

subject to

$$c + i = (1 - \Psi(k, k_g))\,[wl + rk]\,, \tag{8}$$

where

$$k' = i + (1 - \delta)k. \tag{9}$$

The solution to the representative household's dynamic optimization problem yields the familiar Euler equation (henceforth EE),

$$u_c(c) = \beta u_c(c')\left[(1 - \Psi(k', k'_g))r' + 1 - \delta\right], \tag{EE}$$

Having described the behavior of households and firms, we can define the recursive competitive equilibrium given taxes:

**Definition 1:** *Given the policy rule $\Psi(k, k_g)$, a recursive competitive equilibrium is a set of functions, $V(k, k_g)$, $w(k, k_g)$, $r(k, k_g)$, $l(k, k_g)$, $\mathcal{H}(k, k_g)$ and $\mathcal{K}_{\mathcal{G}}(k, k_g)$, such that*

1. *$\mathcal{H}(k, k_g)$ solves equation (EE), $l(k, k_g) = 1$ , and $V(k, k_g)$ solves ($P^C$),*

2. *prices reflect competitive factor markets, $w(k, k_g) = F_l$ and $r(k, k_g) = F_k$,*

3. *the government Budget Constraint holds,*
*$\mathcal{K}_{\mathcal{G}}(k, k_g) = (1 - \delta_{k_g})k_g + \Psi(k, k_g)[w(k, k_g)l + r(k, k_g)k]$.*

In (EE), taxes distort private incentives to consume and save, but also induce higher future returns to private investment through the development of public infrastructure. Specifically, the return to private investment, $r'$, depends on $k'_g$ through the marginal product of private capital. A question then immediately arises as to where to set the tax rate, or equivalently, the share of public investment in output.

### 3.2 A Useful Benchmark Case

It is helpful at this point to introduce a benchmark case that will be widely used throughout the analysis. Suppose that households' period utility is logarithmic, $u(c) = \log c$, that technology is Cobb-Douglas, $y = k_g^{\theta} k^{\alpha} l^{1-\alpha}$, and that capital depreciates fully within the period, $\delta = 1$. It is well known that for these specifications of preferences and technology, the household savings function in the decentralized equilibrium depends only on the current level of taxes and not the entire policy stream. Specifically, future policy changes lead to income and substitution effects on current savings that exactly offset each other. This property of our benchmark savings functions will turn out to play a key role in determining the accuracy of alternative numerical methods.

Under our benchmark assumptions, the household EE reduces to

$$c' = \alpha\beta c F'_k[1 - \Psi(k', k'_g)].$$

Given policy, we guess that the recursive competitive equilibrium is such that households save a constant proportion, $s \in (0,1)$, of after tax income, $\mathcal{H}(k,k_g) = s(1-\Psi(k,k_g))k_g^{\theta}k^{\alpha}$. It is then straightforward to show that (EE) holds if and only if $s = \alpha\beta$. It follows that

$$\mathcal{H}(k,k_g) = \alpha\beta(1-\Psi(k,k_g))y, \tag{10}$$

and that consumption is also proportional to after tax income

$$c = (1-\alpha\beta)(1-\Psi(k,k_g))y. \tag{11}$$

Using equations (10) and (11), the other equilibrium functions can be obtained easily. Furthermore, precisely because savings at any date depend only on contemporaneous taxes, and therefore cannot be directly influenced by future or past tax rates, one expects the optimal policy problem to be degenerate in the benchmark case. In fact, we shall show that optimal tax rates in the benchmark economy are constant through time irrespective of the states faced by the government.

# 4 Description of the Markov-perfect problem

We define a stationary Markov equilibrium following Klein, et al. (2004) where, in our setup, the tax rate depends on the stocks of public and private capital. Unlike Klein et al. (2004), however, and more generally models with a single state variable, the derivation of the government Euler equation – which characterizes the solution – is substantially more involved with two states. We show that such a derivation remains analytically tractable. More importantly, it is essential for the application of numerical methods that accurately describe transition dynamics.[5]

A Markov-perfect equilibrium can be described as a sequence of successive governments, each choosing a single tax rate based on the state it inherits when taking office. In making this choice, each government correctly anticipates the optimal decision rule adopted by its successors. In equilibrium, future policymakers' choices are time consistent if and only if they coincide with the rule that the current policymaker anticipated them to choose optimally. This implies that the current planner's policy choice of $\tau$ must also follow $\Psi(k,k_g)$.

In order for $\Psi(k,k_g)$ to be subgame perfect, no government must ever have an incentive to deviate from this rule. Joint deviations are not feasible since, by assumption, a new government chooses policy every period and, consequently, cannot enter binding contracts with future governments. It is sufficient, therefore, to analyze the problem of a government that is allowed to "deviate" in the current period by setting a tax rate $\tau \neq \Psi(k,k_g)$, under the assumption that $\Psi(k,k_g)$ is forever followed in the future. Thus, we now describe how an arbitrary deviation perturbs equilibrium allocations, and we allow the government to choose the best possible deviation $\tau$. In a Markov-perfect equilibrium, it must be the case that the optimal deviation $\tau$ and the policy rule $\Psi(k,k_g)$ coincide.

[5] In related work, Azzimonti, De Francisco, and Krusell (2006) study a politico-economic equilibrium also with mulitple state variables but where taxes are determined by majority voting.

From the government budget constraint, we can define the evolution of public capital as a function of current states and the tax rate, $g(k, k_g, \tau)$, such that

$$g(k, k_g, \tau) = \tau F(k, l, k_g) + (1 - \delta)k_g.$$

The representative household's budget constraint is given by

$$c = (1 - \tau)\left[w(k, k_g)l + r(k, k_g)k\right] + (1 - \delta)\,k - k',$$

where prices are written explicitly as a function of current states. Because of the representative household assumption, this constraint can also be expressed as an economy-wide resource constraint that implicitly defines a consumption function $\mathcal{C}$,

$$\mathcal{C}(k, k_g, \tau, k') \equiv (1 - \tau)F(k, l, k_g) + (1 - \delta)\,k - k'.$$

Let us define the evolution of private capital under the one-period deviation as $k' = H(k, k_g, \tau)$, where $H$ solves households' optimal consumption-savings decision when the current tax rate is given by $\tau$, and all future tax rates obey $\Psi(k, k_g)$. Since households take policy as given, the EE then becomes

$$u_c\left[\mathcal{C}(k, k_g, \tau, k')\right] = \beta u_c[\mathcal{C}(k', k_g', \Psi(k', k_g'), k'')] \cdot \left\{F_k'(1 - \Psi(k', k_g')) + 1 - \delta\right\}, \tag{12}$$

where $k_g' = g(k, k_g, \tau)$ and $k'' = \mathcal{H}(k', k_g')$.

Households' first order condition (12) is a functional equation in that it must hold for all $k$, $k_g$, and $\tau$. Although each government only chooses taxes for the period in which it is in office, this choice also affects individual and aggregate behavior in subsequent periods. As savings and investment in public capital change today following a change in the current tax rate, so do future taxes as a by-product. Put another way, since $\Psi$ is followed tomorrow onwards, variations in $\tau$ that cause $k'$ and $k_g^{'}$ to change will also cause the rate $\tau'$ to change since $\tau' = \Psi(k', k_g')$, thus affecting future savings and investment ($k''$ and $k_g''$). It follows that by affecting future states with current policy decisions, the current policymaker possesses some leverage over future governments through $\Psi$.

Formally, the Markov problem describing optimal discretionary policy at any date is

$$\max_{\tau} u(\mathcal{C}(k, k_g, \tau, H(k, k_g, \tau)) + \beta V(H(k, k_g, \tau), g(k, k_g, \tau)) \tag{P$^{\mathrm{M}}$}$$

where $V(k, k_g)$ is determined by

$$V(k, k_g) = u(\mathcal{C}) + \beta V(\mathcal{H}(k, k_g), \mathcal{K}_{\mathcal{G}}(k, k_g)), \tag{13}$$

and $\mathcal{C} \equiv (1 - \Psi(k, k_g))F(k, l, k_g) + (1 - \delta)\,k - \mathcal{H}(k, k_g)$.

A Markov-perfect equilibrium is found when, for any pair $\{k, k_g\}$, $\tau = \Psi(k, k_g)$. Formally,

$$\Psi(k, k_g) \in \underset{\tau}{\operatorname{argmax}}\; u(\mathcal{C}(k, k_g, \tau, H(k, k_g, \tau)) + \beta V(H(k, k_g, \tau), g(k, k_g, \tau)).$$

Analogously to Definition 1, we can now describe the recursive competitive equilibrium that obtains under the one-period deviation considered in this section.

**Definition 2:** *A recursive competitive equilibrium under the one-period deviation $\tau$ from $\Psi(k, k_g)$ is a set of functions, $w(k, k_g)$, $r(k, k_g)$, $l(k, k_g, \tau)$, $H(k, k_g, \tau)$, $g(k, k_g, \tau)$, $\mathcal{H}(k, k_g)$, $\mathcal{K}_{\mathcal{G}}(k, k_g)$, and $V(k, k_g)$ such that*

1. *$l(k, k_g, \tau) = 1$, and $H(k, k_g\tau)$ solves the Euler equation (12),*

2. *prices reflect competitive factor markets, $w(k, k_g) = F_l$ and $r(k, k_g) = F_k$,*

3. *the government Budget Constraint holds, $g(k, k_g, \tau) = (1 - \delta_{k_g})k_g + \tau[w(k, k_g)l + r(k, k_g)k]$,*

4. *$\mathcal{H}(k, k_g)$, $\mathcal{K}_{\mathcal{G}}(k, k_g)$ and $V(k, k_g)$ satisfy the equilibrium conditions in Definition 1.*

When the government does not deviate from the rule $\Psi$ in setting $\tau$, the resulting policy functions must be consistent with the recursive competitive equilibrium described in Definition 1, $H(k, k_g, \Psi(k, k_g)) = \mathcal{H}(k, k_g)$ and $g(k, k_g, \Psi(k, k_g)) = \mathcal{K}_{\mathcal{G}}(k, k_g)$.

## 4.1 The Government Euler Equation

As shown in Krusell and Kuruscu (2002), there potentially exist an infinite number of discontinuous equilibria (i.e. where the policy rule is discontinuous in the states) that arise only because of the infinite horizon nature of the problem. In order to select an equilibrium that is the natural limit of its finite horizon counterpart, we impose differentiability of the policy functions.

The first-order condition with respect to the current deviation $\tau$ in problem ($\mathrm{P^M}$) is

$$u_c\mathcal{C}_\tau + \beta\left[V_k' H_\tau + V_{k_g}' g_\tau\right] = 0. \tag{14}$$

Observe that expressions for $V_k$ and $V_{k_g}$ (where $V$ is given in 13) involve $\mathcal{H}_k$ and $\mathcal{H}_{k_g}$. Since households' Euler equation (EE) depends on the policy rule $\Psi(k, k_g)$, the calculation of $\mathcal{H}_k$ and $\mathcal{H}_{k_g}$— using the implicit function theorem— in turn involves derivatives of the policy function, $\Psi_k$ and $\Psi_{k_g}$. This feature of the Markov problem imposes an additional burden in solving for the Markov-perfect equilibrium since, to arrive at the unknown policy $\Psi$ (or even just the tax rate in the steady state), one already needs to take into account its derivatives with respect to the states.

We saw in section 2.1 that in the presence of lump sum taxes, the government would set all distortions to zero so that first-best allocations could be attained. In contrast, in our setting, distortionary taxes induce wedges in the intertemporal conditions describing the efficient provision of private and public capital. The following proposition states that the optimal discretionary policy is such that it sets a linear weighted sum of these distortions to zero.

**Proposition 1.** Let $\Delta_k = -u_c + \beta u_c'(F_k' + 1 - \delta)$ and $\Delta_{k_g} = -u_c + \beta u_c'\left(F_{k_g}' + 1 - \delta\right)$. Then,

the government's first order condition (14) may be re-written as an Euler equation as follows

$$H_\tau \Delta_k + g_\tau \Delta_{k_g} + \beta\{\tilde{H}'_\tau \Delta'_k + \tilde{g}'_\tau \Delta'_{k_g}\} = 0,$$

where $\tilde{H}'_\tau = \xi B''_{k_g}$ and $\tilde{g}'_\tau = -\xi B''_k$, with $B_i = g_i - g_\tau \frac{H_i}{H_\tau}$ $i = k, k_g$ and $\xi = -H_\tau \frac{B'_k H_\tau + B'_{k_g} g_\tau}{B''_k H'_\tau + B''_{k_g} g'_\tau}$.

**Proof.** See Appendix A.

The derivation of the government Euler equation (GEE) in terms of a weighted sum of deviations from efficient intertemporal decisions is somewhat involved, but perhaps most intuitive in that form. Furthermore, it is worth noting that although the economy is dynamic, so that there are potentially an infinite number of distortions, only those in the current and subsequent period matter directly. The recursive nature of the problem together with the envelope theorem ensure that other wedges are handled optimally. Why is it that distortions in the current and subsequent period matter in proposition 1? In Klein et al. (2004), or Azzimonti et al. (2006), the government attempts to manipulate one intertemporal distortion using one policy instrument, income taxes. As a result, the GEE in Klein et al. (2004), for instance, involves trading off one intertemporal wedge, the one that distorts savings, with one intratemporal wedge, the one between the marginal utilities of private and public consumption. In our setting, however, the government attempts to manipulate two intertemporal margins, the ones that determine private and public capital, but still having access to only one instrument, income taxes. Relative to the earlier frameworks with a single state variable, therefore, the optimal policy with public investment dictates trading off intertemporal wedges over one additional period.[6]

The first term in proposition 1 depicts the increase in the inefficiency of private savings induced by a marginal increase in distortionary taxes. This inefficiency is captured by the intertemporal savings distortion that arises with distortionary taxes, $\Delta_k > 0$, scaled by the reduction in household savings that takes place when the tax rate increases, $H_\tau < 0$.[7] Similarly, the second term $g_\tau \Delta_{k_g}$ in proposition 1 summarizes how changes in current taxes affect the inefficiency of public capital provision. In particular, this effect is characterized by the wedge $\Delta_{k_g}$ present whenever $\tau > 0$, scaled by the rise in public investment implied by a marginal increase in $\tau$, $g_\tau > 0$.[8] The third and fourth terms in the weighted sum of distortions in proposition 1 may be interpreted in an analogous way. In sum, therefore, governments following a Markov-perfect policy choose a tax rate such that they trade off marginal inefficiencies arising in private savings, $H_\tau \Delta_k$, with those arising in the provision of public infrastructure, $g_\tau \Delta_{k_g}$, over two consecutive periods.

[6] In general, what affects the number of intertemporal trade offs in the GEE involves both the number of state variables and the number of independent financing instruments.

[7] To see this, simply substitute households' first-order condition (EE) into the definition of $\Delta_k$ to obtain $\Delta_k = \tau' \beta u'_c \left(F'_{k_g} + 1 - \delta\right)$. Thus, $\Delta_k > 0$ when $\tau' > 0$, which implies a level savings that is lower than optimal, $k' < k^{*\prime}$.

[8] Recall that in the first best economy, the optimal level of optimal public capital is determined by the condition $-u_c + \beta u'_c \left(F'_{k_g} + 1 - \delta\right) = 0$, evaluated at $k'^*$ and $k'^*_g$. Since $k' < k^{*\prime}$ with distortionary taxes, this equality does not necessarily hold and $\Delta_{k_g} \neq 0$.

It is worth noticing that the presence of public infrastructure alleviates the time inconsistency problem in this framework. In particular, while policy time inconsistency stems from choosing taxes that distort households' savings decisions, public investment that are financed by these taxes, by increasing the return to capital, partly offset these distortions. Consequently, being able to commit to a given sequence of tax rates may be of lesser value when these tax rates ultimately finance productive public goods.

As in proposition 1, we conjecture that Markov-perfect policies can generally be expressed in terms of offsetting distortions, and that the derivation of such an expression follows the steps laid out in Appendix A. The next section centers on a specific parametric example that makes some of the concepts we have just introduced more tangible. More importantly, the example we present allows us to derive closed-form solutions that we later use as a benchmark to assess different numerical methods in the literature on optimal discretionary policy.

## 4.2 Analytical Solution for the Benchmark economy

Consider the functional assumptions that define the benchmark economy introduced in section 3.1. In order to solve for the GEE in proposition 1 analytically, we must first find expressions for the following objects, $H_\tau$, $g_\tau$, $\widetilde{H}'_\tau$, $\widetilde{g}'_\tau$, $\Delta_k$ and $\Delta_{kg}$. Once expressions for $\Delta_k$ and $\Delta_{kg}$ are found, these can simply be updated to obtain $\Delta'_k$ and $\Delta'_{kg}$.

Since private savings depend only on current tax rates in the benchmark case, the function describing savings under the one-period Markov deviation is given by $H(k, k_g, \tau) = \alpha\beta(1-\tau)y$, so that tax increases discourage savings,

$$H_\tau = -\alpha\beta y < 0.$$

From the government's budget constraint, we have that $g(k, k_g, \tau) = \tau y$ under full depreciation, and it follows that

$$g_\tau = y > 0.$$

An increase in taxes raises public infrastructure.

Using the expressions for $H$ and $g$ to derive analytical solutions for $\tilde{H}'_\tau$ and $\tilde{g}'_\tau$, as defined in proposition 1, we obtain

$$\tilde{H}'_\tau = \frac{\alpha\beta\theta y}{\tau'} \frac{\left[-\frac{\alpha}{1-\tau} + \frac{\theta}{\tau}\right]}{\left[-\frac{\alpha}{1-\tau'} + \frac{\theta}{\tau'}\right]},$$

and

$$\tilde{g}'_\tau = -\tilde{H}'_\tau \frac{\tau'}{(1-\tau')\theta\beta}.$$

Under our benchmark functional form assumptions for preferences and technology, the wedge in private investment is given by

$$\Delta_k = -\frac{1}{c} + \beta\frac{\alpha y'}{k'}\frac{1}{c'},$$

whereas the distortion in public investment is simply

$$\Delta_{k_g} = -\frac{1}{c} + \beta \frac{\theta y'}{k'_g} \frac{1}{c'},$$

where $c = y(1-\tau)(1-\alpha\beta)$.

Given the expressions for $H_\tau$, $g_\tau$, $\widetilde{H}'_\tau$, $\widetilde{g}'_\tau$, $\Delta_k$ and $\Delta_{kg}$ we have just derived, we can substitute these expressions into the GEE in Proposition 1 to obtain, after basic manipulations, a single equation in terms of $\tau$, $\tau'$, the state variables, $k$ and $k_g$, and the structural model parameters. Suppose we conjecture a Markov-perfect tax policy that is constant over time, $\tau = \tau'$, then it is straightforward to show that $\tau = \beta\theta \ \forall t$ solves the GEE, as summarized in proposition 2.

**Proposition 2.** When household preferences are logarithmic, $u(c) = \log(c)$, technology is Cobb Douglas, $F(k, k_g) = k^\alpha k_g^\theta l^{1-\alpha}$, and capital depreciates fully within the period, $\delta = 1$, optimal discretionary income tax rates are independent of the states, $\Psi(k, k_g) = \beta\theta \ \forall \ (k, k_g)$, and thus constant through time.

Under more general parameterizations than adopted in our benchmark case, changes in current taxes, $\tau$, affect private savings, $k'$, and public investment, $k'_g$, which then impact future policy through the policy rule, $\tau' = \Psi(k', k'_g)$. The change in $\tau'$ feeds back into households' current decisions through a substitution effect, by reducing the net return to today's savings, and an income effect, by lowering disposable income tomorrow. This feedback creates a political intertemporal link between successive governments (i.e. between choices of $\tau$ and $\tau'$). The key to proposition 2, however, is that under our benchmark assumptions, private savings depend only current taxes. As a result, tomorrow's policy has no impact on today's household choices, and the intertemporal link between governments breaks down. The political decision problem, therefore, essentially collapses to a static one with a constant policy as its solution[9]

# 5 Numerical Solution Methods

The main complication in computing steady states in the Markov equilibrium results from the GEE depending not only on the levels of capital and taxes, but also on the derivatives of the policy and saving rules. Numerical work on optimal time consistent policy has generally adopted either one of two methods: (i) a Linear Quadratic Approximation Method (henceforth LQ) and (ii) a Perturbation Method. LQ approximations allow for a relatively straightforward computation of steady states and transition dynamics. They are widespread in the literature and easily handle multiple state variables. Examples of this approach are found, for instance, in Krusell and Rios-Rull (1999), and Benigno and Woodford (forthcoming), in the context of fiscal policy, but are more

[9] Under log preferences, absent the political intertemporal dimension, one can show that the policymaker's decision problem is one where the effects of policy do not interact with the state variables.

prevalent in the monetary policy literature as illustrated in Dotsey and Hornstein (2003), Svensson and Woodford (2004), King and Wolman (2004), among many others. Developments related to the Perturbation method are more recent and, to this point, have been used mostly to approximate steady state equilibria (see Azzimonti et al. 2006). In this paper, we use the Perturbation method to also approximate transitions to the steady state. To our knowledge, our work offers the first comparison of the accuracy of these different approaches.

Since our framework allows for a closed form solution in our benchmark economy, any numerical method can be assessed against the actual solution in that case. One must be cautious of this exercise because analytical solutions often abstract from important dimensions of the problem, and replicating this solution with a numerical method is no guarantee that the method works more generally. In our setting, however, considerable differences in numerical approximations emerge even in our simple benchmark case.

We find that while both the LQ and Perturbation procedures deliver very accurate estimates of steady state allocations, transition dynamics obtained using LQ approximations can be inaccurate. In particular, transition paths for taxes and public investment are noticeably different from those suggested by the actual solution. This finding stems from the inflexible parametric form assumed by the savings and income tax rules under the LQ approach. More specifically, the LQ approximation predicts that decreases in $k$ or $k_g$ away from their steady state make policymakers raise the optimal tax rate in the benchmark economy, in contrast to Proposition 2 where the tax rate was shown to be independent of the states. In this case, therefore, the poor approximation is potentially associated with misleading policy conclusions. On a more encouraging note, the Perturbation method allows us to improve upon the approximation of the policy and savings rule by adjusting the degree of the polynomial describing them. Thus, for the closed-form solution example worked out in section 4.2, the Perturbation method applied to second-order policy functions does deliver state-invariant taxes (i.e. the derivatives of the policy rule are zero, $\Psi_k = \Psi_{k_g} = 0$).

## 5.1 Properties of the Linear Quadratic and Perturbation Methods

The fundamental difference between the LQ and perturbation approaches is that, while the LQ method approximates the Markov problem with a quadratic objective function and linear constraints, the Perturbation method solves the original problem and approximates the corresponding decision rules with a polynomial. Thus, even with linear decision rules, we expect the Perturbation method to perform somewhat better than the LQ approximation.

We know from section 4.2 that the following policy functions solve the Markov-perfect problem in the benchmark case,

$$\Psi(k, k_g) = \beta\theta,$$

$$H(k, k_g) = \alpha\beta k^{\alpha} k_g^{\theta}[1 - \Psi(k, k_g)],$$

and

$$g(k, k_g) = \Psi(k, k_g) k^{\alpha} k_g^{\theta}.$$

The *Perturbation Method* consists in approximating these decision rules by an $n$-th order polynomial. Observe that conditional on $\Psi$, the government's budget constraint (7) immediately gives us the rule $g$. Consequently, we need only approximate the savings rule $H$ and the tax policy $\Psi$. Given the assumed functional forms for $H$ and $\Psi$, calculating the derivatives that appear in the GEE is now straightforward. In essence, this method recasts the problem in terms of finding the parameters of two polynomials. For comparison with the LQ approach, we use the Perturbation approach with two different approximations of the policy functions, one that assumes a linear functional form for $H$ and $\Psi$, referred to as 'method PL', and one that assumes a quadratic form for $H$, referred to as 'method PQ'.

Under method PL, we specify $H$ as follows,

$$H^{PL}(k, k_g, \tau) = \phi_0^k + \phi_1^k k + \phi_2^k k_g + \phi_3^k \tau, \tag{15}$$

Next period's public capital satisfies the government's budget constraint,

$$g^{PL}(k, k_g, \tau) = \tau k^{\alpha} k_g^{\theta}, \tag{16}$$

and taxes are linear in the state variables,

$$\Psi^{PL}(k, k_g) = \phi_0^{\tau} + \phi_1^{\tau} k + \phi_2^{\tau} k_g. \tag{17}$$

There are 7 unknown $\phi$ parameters and 2 unknown steady state values (i.e. $k_{ss}$ and $k_{gss}$). The corresponding system of 9 equations needed to solve for these parameters consists of the Euler Equation, the GEE, the derivatives of the Euler Equation with respect to $\tau$, $k$ and $k_g$, the derivatives of the GEE with respect to $k$ and $k_g$, as well as equations (15) and (16) evaluated at the steady state (see Appendix B for a detailed description of these calculations).

Under the second approximation used with the Perturbation approach, method PQ, $\Psi$ and $g$ follow equations (16) and (17), but $H$ is quadratic in its arguments. As a result, the number of unknown $\phi$ parameters rises to 15. The extra equations needed in this case are given by the second derivatives of the Euler Equation.

In contrast, the *LQ Method* re-states problem ($\mathrm{P^M}$) as one with a quadratic objective subject to a set of linear constraints. See Appendix C for a detailed description of this approach. The end result is one where all decision rules are linear in the state variables, including the decision rule for $g$ in contrast to (16) under method PL.

Table 1 summarizes the steady state values of taxes and capital allocations under optimal discretionary policy using the three methods we have just described. Table 1 also compares these approximated steady state values with those obtained from the analytical solution. The parameters

used in this example are $\theta = 0.25$, $\beta = 0.94$ $\alpha = 0.33$.[10] All methods are very accurate in approximating the steady state Markov-perfect equilibrium and, furthermore, give identical numerical solutions up to the fifth decimal place.

**Table 1.**

| *Steady States: LQ vs. Perturbation Methods* | | | |
|---|---|---|---|
| Method[b] | Policy[a], $\tau$ | $k$ | $k_g$ |
| Closed Form Solution | 23.5000 | 0.03240 | 0.3210 |
| LQ/PL/PQ | 23.5000 | 0.03237 | 0.03205 |

*a*. The tax rate is expressed in percentage terms.

*b*. Convergence of the solutions is defined up to $10^{-9}$

Also of interest is the accuracy of the approximations for equilibrium transitions to the steady state. To this end, Table 2 presents estimates of how optimal discretionary policy responds, *ceteris paribus*, when the states of the economy change, $\Psi_k$ and $\Psi_{k_g}$.

**Table 2.**

| *LQ vs. Perturbation: Policy* | | |
|---|---|---|
| Method | $\Psi_k$ | $\Psi_{k_g}$ |
| Closed Form Solution | 0.00000 | 0.00000 |
| LQ | −1.75032 | −1.33810 |
| PL | 0.36117 | 0.27629 |
| PQ | −0.00001 | −0.000001 |

Table 2 indicates that the LQ method delivers negative partial derivatives of the optimal tax rate with respect to the state variables whereas, in our closed-form solution example, the optimal discretionary policy is state invariant. In addition, the values of these derivatives are noticeably different from zero. The Perturbation method applied to a linear savings rule – method PL – overestimates the partial derivatives of the optimal tax policy with respect to the states, but the values of these derivatives are smaller. Encouragingly, an application of the Perturbation method applied to a quadratic approximation of the savings rules – method PQ – results in a discretionary tax rate that is indeed almost constant, with partial derivatives that differ from zero only at the fifth decimal place. We conclude that for this particular parameterization of the model, one needs to allow for more curvature in the savings function in order to obtain an accurate approximation to the decision rules.

[10] These parameters are discussed in the section ahead where the model is calibrated.

### 5.1.1 Implications for Transition Dynamics

While the Markov steady state appears unaffected by differences in numerical methods, these methods nevertheless yield different transition dynamics given some initial states. Figure 1 depicts the evolution of both types of capital, taxes, and consumption that arise when the economy starts off with initial levels of public and private capital that are 10 percent below their corresponding steady state values. We envision, therefore, a developing economy that is transitioning to its long-run equilibrium.

The most striking difference in these transitions concerns the evolution of taxes. While the true solution implies a constant policy $\Psi$, the LQ method predicts an increase in taxes in the first period followed by a relatively fast decline to the long-run equilibrium. The qualitative implications are thus incorrect in this case. Given the spurious behavior in taxes, the evolution of public capital also differs from its true solution. The LQ method overestimates growth in public investment in the short run, giving the time path of public capital a steeper profile than warranted initially. Observe that method PQ essentially replicates the transitions obtained under the closed form solutions. Interestingly, the predicted paths for private capital and consumption do not differ significantly across methods.

To gain better intuition for the differences in transition paths shown in Figure 1, Figure 2 shows the estimated responses of public capital and taxes to changes in the state variables. In the left-hand panels, we fix public capital at its steady state value and let private capital vary (the right-hand panels depict the opposite exercise). Note that the policy functions computed using the LQ method, shown as the dashed lines, exactly intersect the closed form solutions, shown as the dotted lines, at the steady state values of private and public capital. These figures, therefore, are consistent with all approximations giving us a very accurate estimates of the long-run equilibrium in Table 1.

In general, policy functions associated with method PQ, shown as the solid lines, are again nearly indistinguishable from the true solutions. In sharp contrast, the policy functions obtained using the LQ approach indicate obvious differences from the true government decision rules. Consequently, the LQ method is unlikely to produce accurate time paths for public capital and taxes, especially in the early stages of development when capital stocks are low. That said, both LQ and method PQ estimates of the savings rule, $H(k, k_g)$, shown in Figure 3, are relatively closer to the true solution. It is this feature of the numerical solutions that underlies the smaller error in the estimates of the time path for private capital under either method in Figure 1.

Given that the LQ approach overestimates public capital investment in the short run (the upper right-hand panel of Figure 1), but that it also overestimates tax rates initially (the lower right-hand panel of Figure 1), the net return to savings is left relatively unaffected by the LQ approximation. This implies that households' consumption-savings decisions are largely unchanged, and that potential errors in the consumption profile arising from the numerical approximation are mitigated by these offsetting effects.

Which numerical method should one select in the study of optimal discretionary policy problems? Our example suggests that this depends in part on the goal of the investigation. To the extent that one is mostly interested in long-run equilibrium properties of the Markov-perfect problem, the LQ method is easy to use, fast, and can accommodate a large number of state variables without additional complications. If the focus, however, is on transition paths, then the perturbation method seems a preferable choice. The latter method is relatively simple to implement, and allows for more flexibility in the choice of the polynomials approximating the policy functions. One disadvantage of the perturbation method is that the number of parameters characterizing the policy functions increases exponentially with the degree of the polynomial approximation and the number of state variables. This introduces a trade-off between the gains from using a larger polynomial and the error introduced by the algorithm used to solve for its parameters. For the model at hand, with only two state variables, method PQ essentially generates the true decision and policy rules in our benchmark case. For the remainder of the paper, therefore, we shall use this method in our computations.

# 6 The Value of Government Commitment

Having addressed key aspects of optimal discretionary public investment, we briefly present the more conventional Ramsey setting to assess the importance of having a commitment technology. We show that the special case of an optimal constant policy, $\tau_t = \beta\theta \ \forall t \geq 0$, described in Proposition 2 also emerges as a solution to the full commitment problem under the same functional form assumptions. In this sense, our framework nests previous results in the literature, notably Glomm and Ravikumar (1994). Moreover, this finding implies that the commitment assumption does not bind in our benchmark economy. That is not the case in a more general setting, and the objective of this section is to compare allocations under the two institutional extremes of full commitment and no commitment.

## 6.1 Qualitative Analysis

Consider a benevolent government that, at date zero, is concerned with choosing a sequence of tax rates consistent with the development of public infrastructure that maximizes household welfare. In choosing policy, this government takes as given the decentralized behavior of firms and households. We further assume that at date zero, it can credibly commit to any sequence of policy actions. The problem faced by this government would then be to maximize households' lifetime utility subject to their EE, the government budget constraint, and the economy wide resource constraint. The corresponding Lagrangian can be written as

$$\max_{\{c_t,\tau_t,k_{t+1},k_{gt+1}\}_{t=0}^{\infty}} \mathcal{L}_t = \sum_{t=0}^{\infty} \beta^t u(c_t) \qquad (\mathrm{P^R})$$

$$+\sum_{t=0}^{\infty}\beta^t\mu_{1t}\left\{\beta u_c(c_{t+1})\left[(1-\tau_{t+1})F_k(k_{t+1},k_{gt+1})+1-\delta\right]-u(c_t)\right\}$$
$$+\sum_{t=0}^{\infty}\beta^t\mu_{2t}\left\{\tau_t F(k_t,k_{gt})+(1-\delta)k_{gt}-k_{gt+1}\right\}$$
$$+\sum_{t=0}^{\infty}\beta^t\mu_{3t}\left\{(1-\tau_t)F(k_t,k_{gt})+(1-\delta)k_t-k_{t+1}-c_t\right\}.$$

The constraint associated with the multiplier $\mu_{1t}$ implies that our benevolent planner takes households' consumption-savings behavior as given. He can, however, influence the intertemporal allocations they choose by altering tax policy over time. Because the capital stocks are fixed at date 0, the first-order conditions associated with ($\mathrm{P^R}$) will generally differ at $t=0$ and $t>0$. As first noted in Kydland and Prescott (1980), this suggests an incentive to take advantage of initial conditions in the first period with the promise never to do so in the future. It is exactly in this sense that the optimal policy may not be time consistent since, once date zero has passed, a planner at date $t>0$ who re-optimizes would want to start with a tax rate, $\tau_t$, that differs from what was chosen for that date at time zero.

While Ramsey plans are typically time inconsistent, that is not necessarily so depending on the nature of household behavior. In our benchmark economy, for example, next period's private capital is determined only by current investment, $k_{t+1}=i_t$, and savings depends only on the current tax rate through disposable income as shown in section 3.2. In other words, future taxes have no impact on current investment. Consequently, in implementing a tax rate at any date, the Ramsey policymaker realizes that this rate never has an effect on past investment decisions. In this sense, period zero is no different than any other period and the government has no incentive to renege on its promises. The resulting policy, therefore, is time consistent. We summarize this finding in the following proposition.

**Proposition 3**. When preferences are logarithmic, $u(c)=\log(c)$, technology is Cobb Douglas, $F(k,k_g)=k^\alpha k_g^\theta$, and capital depreciates fully within the period, $\delta=1$, the optimal sequence of tax rates with commitment is time invariant and reproduces the Markov-perfect policy, $\tau_t=\beta\theta$ $\forall t\geq 0$.

**Proof**: See Appendix D.

An implication of proposition 3 is that in the benchmark economy, there are no welfare gains from adopting a technology that constrains governments to fulfill their promises. In other words, the commitment assumption is not binding under this parameterization. The value of commitment, however, is generally related to both the elasticity of intertemporal substitution and the depreciation rate. In particular,household savings typically depend on the entire stream of tax rates, and setting $\tau_t>0$ at some date $t$ affects the entire sequence of capital allocations. Since Markov policymakers take the states they inherit as given, they never internalize the efficiency costs of distortionary

taxes on past investment decisions. Hence, binding governments to the policy prescribed under the Ramsey plan at every date opens up the possibility of a welfare improvement.

## 6.2 Quantitative Evaluation

To gauge the importance of the friction generated by a government's inability to commit to future policy, we carry out numerical simulations of our economy with public investment. The parameters we use are standard and selected along the lines of other studies in quantitative general equilibrium theory. A time period represents a year and, following Cooley and Prescott (1995), we assume an undistorted 5.5 percent annual real interest rate, $\beta = 0.947$, a 4.8 percent capital depreciation rate, $\delta = 0.048$, and $u(c) = \log c$. The share of private capital in output in the U.S. is approximately 33 percent so that we set $\alpha$ to 1/3. The one non-standard parameter relates to the elasticity of public capital with respect to output. Estimates of $\theta$ vary significantly across studies. Glomm and Ravikumar (1997) cite values ranging from as low as 0.03 (Eberts, 1986) to as high as 0.39 (Aschauer, 1989). We set $\theta$ to 0.25 to approximate a consensus view, but alternative values ranging between 0.15 and 0.35 do not materially alter the findings below.

### 6.2.1 Long-Run Equilibria with and without Commitment

Table 3 summarizes properties of the Markov-perfect policy in the steady state. In contrast to the predictions in Barro (1990), or Glomm and Ravikumar (1994, 1997), the long-run tax rate, or share of public investment, is significantly lower than the output elasticity with respect public capital at only 8.92 percent. In addition, we find that, *ceteris paribus*, as private capital falls below its long-run value, so do time consistent tax rates, $\Psi_k > 0$. This suggest that Markov governments purposefully create incentives for the private sector to invest in periods when private capital is lacking. At the same time, however, optimal discretionary tax rates rise when public infrastructures fall short of their long-run equilibrium, $\Psi_{k_g} < 0$. In this case, higher taxes are necessary to fund public investments.

**Table 3.**

| *Steady State Discretionary Tax Rate* | | | |
|---|---|---|---|
| | $\Psi$ (percent) | $\Psi_k$ | $\Psi_{k_g}$ |
| Tax Rate | 8.92 | 0.125 | −0.215 |

Table 4 presents a comparison between stationary outcomes under the Markov-perfect and Ramsey equilibria. As discussed earlier, the inability of a policymaker to commit to a sequence of taxes results in too little public capital in the long run. Since such a government takes future policy as given, it does not internalize the reduction in externalities implied by not developing the constrained-efficient level of infrastructure. As a result, the accumulation of private capital is discouraged *in spite of the lower tax rates*. The combination of lower public and private capital result

in an output loss of approximately 1.4 percent associated with the lack of a commitment technology. Under this calibration, this output loss translates into a long-run consumption difference of about 1 percent between Markov-perfect and Ramsey equilibria.

**Table 5.**

| *Steady State Allocations* | | |
|---|---|---|
| Variable | **Ramsey** | **Markov-Perfect** |
| Tax Rate (percent) | 9.23 | 8.92 |
| Consumption | 2.18 | 2.16 |
| Private Investment | 0.40 | 0.39 |
| Output | 2.84 | 2.80 |
| Private Capital | 8.26 | 8.19 |
| Public Capital | 5.46 | 5.21 |
| Net Interest Rate | 5.59 | 5.59 |

### 6.3 Choices and Consequences of Alternative Tax Regimes

Figure 4 shows the transition dynamics in policy, consumption, and welfare under the two extreme institutional schemes of no commitment (Markov) and full commitment (Ramsey).[11] First, the upper left-hand panel shows that consumption paths are very close along the transition, with the largest difference occurring in the long run at one percent. However, consumption under time consistent policy is actually higher than under Ramsey policy in the early stages of development. This follows from the negative income effect associated with the higher taxes implemented under full commitment, as shown in the upper right-hand panel of Figure 4. Of course, the lower tax revenues that emerge in the Markov equilibrium constrain the development of public infrastructure and eventually result in both lower output and consumption.

Given the consumption streams shown in Figure 4, a basic question one may ask is whether the better institutional framework, in our case the one where governments are bound by commitment, would naturally arise. The lower left-hand side panel of Figure 4 depicts the instantaneous utility derived from each of the consumption sequences, $\{c_t^R\}_{t=0}^{\infty}$ and $\{c_t^M\}_{t=0}^{\infty}$, under the Ramsey and the time-consistent policy respectively. Not surprisingly, these mimic the consumption profiles. Note that while $u(c_0^R) < u(c_0^M)$ in the initial period, lifetime utility under full commitment still exceeds that under the Markovian equilibrium. The difference in welfare measures in period zero, however, is negligible as shown in the right-hand side panel of Figure 4. Hence, if households were to choose what institutional framework to implement at the beginning of time, a very small cost of setting up a commitment technology would prohibit its adoption.

[11] The initial conditions are such that the government under both institutional schemes inherits private and public capital stocks that are 5 percent below their long-run level under time-consistent policy.

Consider a household who, at date 0, is evaluating the consumptions streams $\{c_t^R\}_{t=0}^{\infty}$ and $\{c_t^M\}_{t=0}^{\infty}$ that can be achieved under our two institutional frameworks. One easy way to quantify the value of government commitment is to compute the percentage of consumption, $\zeta$, a household would be willing to give up at every date to make it indifferent between the two alternatives (Lucas 1987). Denote welfare at date 0 under the Ramsey and Markov-perfect policy by $V^R(k_0, k_{g0})$ and $V^M(k_0, k_{g0})$ respectively. Then, $\zeta$ solves

$$V^M(k_0, k_{g0}) = \sum_{t=0}^{\infty} \beta^t u(c_t^R(1-\zeta)),$$

which, under our parameterization yields

$$\zeta = 1 - (1-\beta)\left[V^R(k_0, k_{g0}) - V^M(k_0, k_{g0})\right].$$

Thus, under our calibration of the structural parameters, we find that a cost exceeding just 0.02 percent of Ramsey consumption would be enough to deter the implementation of the better institutional environment.

Although households' decision to allow for discretionary policy rather than adopt a binding institution at date 0 may be entirely rational for a small positive $\zeta$, the resulting allocations can look considerably different in the long run. The difference in continuation utility after a century, shown in Figure 4, emerges because the stocks of private and public capital take on very different paths under the two alternatives. Thus, while $V^R(k_0, k_{g0})$ and $V^M(k_0, k_{g0})$ may be too close to justify adopting the Ramsey plan initially for given $k_0$ and $k_{g0}$, $V^R(k_{ss}^R, k_{gss}^R)$ and $V^M(k_{ss}^M, k_{gss}^M)$ may be quite different in the long run because $(k_{ss}^R, k_{gss}^R)$ end up much larger than $(k_{ss}^M, k_{gss}^M)$. A country, therefore, that today has reached its steady state under no commitment would need to pay a much larger share of consumption to jump to the steady state of another country that, many years ago, adopted a commitment technology.

# 7 Concluding Remarks

We characterize Markov-perfect equilibria in a model where the absence of government commitment affects public investment in infrastructure throughout an economy's development stage. We show that in choosing the tax rate, the government trades off intertemporal distortions in the provision of public and private capital. The implied optimality condition allows us to apply numerical methods that accurately approximate the Markov-perfect equilibrium.

Our findings indicate that small costs of setting up a strong institution, which we interpret as a commitment technology, are enough to prevent its adoption. At the same time, once an institution associated with discretionary policy is adopted, the lack of government commitment can result in noticeably lower public infrastructure, private capital, output, and consumption in the long run. Hence, an economy that in the past chose to forego a binding constitution would today need to give

up a significant share of consumption to jump to the steady state it would have otherwise reached. These findings evoke the possibility of institutional traps.

Although our findings are suggestive of discretionary institutions that persist over time, we only consider the initial choice of whether to set up a commitment technology in this paper. To explore whether institutional traps actually do emerge as equilibrium phenomena, one would need to formally consider households' choice between two institutions, say between full government commitment and the absence of any commitment, at each date of an economy's transition to its long-run equilibrium. We plan to address this more complex problem in future work. Finally, our analysis focuses on two institutional extremes. In practice, different constitutional arrangements make it partially costly for governments to simply break past promises. Developing a framework that more closely captures political environments that limit the feasibility of policy change would represent a significant step towards practical policy analysis.

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# A Proof of Proposition 1

This appendix develops and interprets the Government Euler Equation (GEE), the equation that represents the optimal intertemporal allocation for a Markov policymaker. Although the derivation below applies to a generic model with two state variables, it should be clear that our generalization of the method proposed in Klein et al. (2004) can be straightforwardly extended to models with a larger state space. Throughout the derivation, we make use of the fact that in equilibrium, $H(k, k_g, \Psi(k, k_g)) = \mathcal{H}(k, k_g)$ and $g(k, k_g, \Psi(k, k_g)) = \mathcal{K}_{\mathcal{G}}(k, k_g)$.

To give an idea of the logic of underlying the derivation of the GEE (with respect to policy), consider how one derives the standard Euler equation for the representative household (with respect to savings). In the latter case, one takes the first-order condition with respect to capital next period, which involves a derivative of the value function. Using the envelope theorem, one obtains a simple expression for this derivative in terms of marginal utility and the net return to savings. The resulting expression can then updated one period and substituted in the first-order condition to yield the standard household Euler equation (EE).

In the case of the government Euler equation, the relevant first-order condition with respect to taxes involves the derivatives of the value function with respect to the two state variables, $k$ and $k_g$,

$$R_\tau + \beta \left[ V_k' H_\tau + V_{k_g}' g_\tau \right] = 0 \tag{A1}$$

where $R_\tau = u_c \left[ \mathcal{C}_{k'} H_\tau + \mathcal{C}_\tau \right]$, $V_k' = V_k(k', k_g')$, and $V_{k_g}' = V_{k_g}(k', k_g')$. However, in contrast with how the household Euler equation is derived, the envelope theorem does not hold and substituting for $V_k'$ and $V_{k_g}'$ is much more involved. The envelope theorem fails to hold because in choosing taxes today, the current government is constrained by the future policy rule, $\Psi$. The rest of this appendix, therefore, is concerned with substituting out for $V_k'$ and $V_{k_g}'$ using expressions that only involve households' decision rules and the government policy rule.

Differentiating the value function (13) with respect to each state (where we can cancel out the terms multiplying derivatives of $\Psi$ by equation A1), we obtain[12]

$$V_k = R_k + \beta \left[ V_k' H_k + V_{k_g}' g_k \right], \tag{A2}$$

$$V_{k_g} = R_{k_g} + \beta \left[ V_k' H_{k_g} + V_{k_g}' g_{k_g} \right]. \tag{A3}$$

At this stage, we have a system of three functional equations (A1), (A2), and (A3), in four unknown functions, $V_k$, $V_{k_g}$ $V_k'$, and $V_{k_g}'$. Since tomorrow's policymaker faces the same problem, however, equations (A1) through (A3) updated one period must also hold. Updating these equations then yields a system of six equations in six unknowns (with the additional unknowns being $V_k''$ and $V_{kg}''$). Solving this system gives expressions for $V_k$ and $V_{kg}$ in terms of the policy and decision rules only,

[12] In doing so, we make use of the fact that in equilibrium, $H(k, k_g, \Psi(k, k_g)) = \mathcal{H}(k, k_g)$ and $g(k, k_g, \Psi(k, k_g)) = \mathcal{K}_{\mathcal{G}}(k, k_g)$.

which we can then update one period and substitute back into the first-order condition (A1) to obtain the GEE that determines $\Psi$. Details of the actual derivations follow, in a slightly different order, in order to arrive more directly at the GEE.

We can use the necessary condition (A1) to obtain

$$V_k' = -\frac{R_\tau + \beta V_{k_g}' g_\tau}{\beta H_\tau}. \tag{A4}$$

Replacing (A1) into equation (A2), and rearranging gives

$$V_k = R_k - R_\tau \frac{H_k}{H_\tau} + \beta V_{k_g}' B_k. \tag{A5}$$

where $B_k = g_k - g_\tau \frac{H_k}{H_\tau}$. Replacing (A1) once more into equation (A3), and defining $B_{k_g}$ analogously, we get

$$V_{k_g} = R_{k_g} - R_\tau \frac{H_{k_g}}{H_\tau} + \beta V_{k_g}' B_{k_g}. \tag{A6}$$

We can now rearrange equation (A5) to get

$$V_{k_g}' = \left[ V_k - R_k + R_\tau \frac{H_k}{H_\tau} \right] \frac{1}{\beta B_k},$$

Substituting this last expression into (A6) gives

$$V_{k_g} = R_{k_g} - R_\tau \frac{H_{k_g}}{H_\tau} + \frac{B_{k_g}}{B_k} \left[ V_k - R_k + R_\tau \frac{H_k}{H_\tau} \right], \tag{A7}$$

which we can update to get an expression for $V_{k_g}'$ as a function of $V_k'$,

$$V_{k_g}' = R_{k_g}' - R_\tau' \frac{H_{k_g}'}{H_\tau'} + \frac{B_{k_g}'}{B_k'} \left[ V_k' - R_k' + R_\tau' \frac{H_k'}{H_\tau'} \right]. \tag{A8}$$

As explained above, so we now update (A6) a second time in order to introduce additional equations that help us solve for the unknown derivatives of the value functions,

$$V_{k_g}'' = R_{k_g}'' - R_\tau'' \frac{H_{k_g}''}{H_\tau''} + \frac{B_{k_g}''}{B_k''} \left[ V_k'' - R_k'' + R_\tau'' \frac{H_k''}{H_\tau''} \right]. \tag{A9}$$

By updating the first-order condition (A1), we obtain another expression involving $V_k''$ as a function of $V_{k_g}''$,

$$V_k'' = -\frac{R_\tau' + \beta V_{k_g}'' g_\tau'}{\beta H_\tau'}. \tag{A10}$$

Replacing (A10) into (A9) then yields an expression for $V_{k_g}''$ that only depends on equilibrium decision rules (and not on the derivatives of the value function).

$$V_{k_g}'' = \left( R_{k_g}'' - R_k'' \frac{B_{k_g}''}{B_k''} + R_\tau'' \lambda'' - \frac{B_{k_g}''}{B_k''} \frac{R_\tau'}{\beta H_\tau'} \right) \frac{H_\tau'}{H_\tau' + g_\tau' \frac{B_{kg}''}{B_k''}}, \tag{A11}$$

where $\lambda'' = \frac{H''_k g''_{k_g} - H''_{k_g} g''_k}{H''_\tau g''_k - H''_k g''_\tau}$. By updating equation (A6) and substituting (A11) into it, one obtains an expression for $V'_{k_g}$. Following the same steps with equation (A5), one arrives at an analogous expression for $V'_k$. These last two equations can then be substituted back into the first-order condition (A1) to obtain the GEE. After cumbersome algebra, this GEE can be simplified to

$$R_\tau + \beta\left[R'_k H_\tau + R'_{k_g} g_\tau + R'_\tau A'_\tau\right] + \beta^2\left[R''_k \tilde{H}'_\tau + R''_{k_g}\tilde{g}'_\tau + R''_\tau A''_\tau\right] = 0, \qquad \text{(GEE)}$$

where

$$\begin{aligned}
R_i &= u_c\left[\mathcal{C}_{k'} H_i + \mathcal{C}_i\right] \qquad i = \tau, k, k_g, \\
A'_\tau &= -\frac{B''_k\left(H'_k H_\tau + H'_{k_g} g_\tau\right) + B''_{k_g}\left(g'_k H_\tau + g'_{k_g} g_\tau\right)}{B''_k H'_\tau + B''_{k_g} g'_\tau}, \\
\tilde{H}'_\tau &= \xi B''_{k_g},\ \tilde{g}'_\tau = -\xi B''_k, \text{ and} \\
A''_\tau &= \xi\left(H''_{k_g} B''_k - H''_k B''_{k_g}\right),
\end{aligned}$$

with $B_i = g_i - g_\tau \frac{H_i}{H_\tau}$ $i = k, k_g$ and $\xi = -H_\tau \frac{B'_k H_\tau + B'_{k_g} g_\tau}{B''_k H'_\tau + B''_{k_g} g'_\tau}$. In general, $H_\tau < 0$, and $H_k, H_{k_g} > 0$. Since $k'_g = g(k, k_g, \tau) = \tau F(k, k_g)$, then $g_i > 0$ for $i = k, k_g, \tau$.

To obtain the expression in proposition 1, substitute the definition of $R_i$, $i = k, k_g, \tau$, in equation (GEE) to obtain

$$u_c[-g_\tau - H_\tau] + \beta u'_c\left\{[f'_k + 1 - \delta_k - g'_k - H'_k]H_\tau + [f'_{k_g} + 1 - \delta_{k_g} - g'_{k_g} - H'_{k_g}]g_\tau + [-g'_\tau - H'_\tau]A'_\tau\right\}$$

$$+\beta^2 u''_c\left\{[f''_k + 1 - \delta_k - g''_k - H''_k]\tilde{H}'_\tau + [f''_{k_g} + 1 - \delta_{k_g} - g''_{k_g} - H''_{k_g}]A''_{k_g} + [-g''_\tau - H''_\tau]A''_\tau\right\} = 0.$$

Using the definition of the wedges provided in proposition 1, we can collect terms and write this equation as

$$H_\tau \Delta_k + g_\tau \Delta_{k_g} + \beta\left\{\tilde{H}'_\tau \Delta'_k + \tilde{g}'_\tau \Delta'_{k_g} - \right.$$

$$\left.\beta u''_c\left[(g''_{k_g} + H''_{k_g})\tilde{g}'_\tau + (g''_k + H''_k)\tilde{H}'_\tau + (g''_\tau + H''_\tau)A''_\tau\right]\right\} = 0.$$

It can be shown, after straightforward algebraic manipulations, that the last row of this last equation is identically zero. Therefore, the GEE is simply

$$H_\tau \Delta_k + g_\tau \Delta_{k_g} + \beta\left\{\tilde{H}'_\tau \Delta'_k + \tilde{g}'_\tau \Delta'_{k_g}\right\} = 0.$$

# B The Perturbation Method

In this appendix, we describe the Perturbation approximation algorithm used to solve for the Markov perfect equilibrium in Section 5. As in appendix A, we make use of the fact that in equilibrium, $H(k, k_g, \Psi(k, k_g)) = \mathcal{H}(k, k_g)$ and $g(k, k_g, \Psi(k, k_g)) = \mathcal{K}_\mathcal{G}(k, k_g)$.

As shown in the main text, the functional equation that determines the policy rule $\Psi$ is given by the GEE,

$$H_\tau \Delta_k + g_\tau \Delta_{k_g} + \beta \left\{ \tilde{H}'_\tau \Delta'_k + \tilde{g}'_\tau \Delta'_{k_g} \right\} = 0,$$

where $\tilde{H}'_\tau$ and $\tilde{g}'_\tau$ are defined in proposition 1. To obtain expressions for $H_i$, $i = k, k_g, \tau$, we apply the implicit function theorem to the household Euler equation (EE), since this functional equation determines $H(k, k_g, \tau)\ \forall\, k, k_g, \tau$.[13] The EE is given by

$$-u_c\left[\mathcal{C}(k, k_g, \tau, k')\right] + \beta u_c[\mathcal{C}(k', k'_g, \Psi(k', k'_g), k'')] \cdot \left\{F'_k(1 - \Psi(k', k'_g)) + 1 - \delta\right\} = 0,$$

where $k'_g = \tau f(k, k_g) + (1 - \delta_{k_g})k_g$, $k'' = H(k', k'_g, \Psi(k', k'_g))$, and which we can summarize as an implicit function $\Phi$,

$$\Phi\left(k, k_g, \tau, k', k'_g, \Psi(k', k'_g), H(k', k'_g, \Psi(k', k'_g))\right) = 0. \tag{B1}$$

Since the Euler equation depends on the policy rule $\Psi(k, k_g)$, calculations of the derivatives $H_i$ $i = k, k_g, \tau$, involve derivatives of the policy function, $\Psi_k$ and $\Psi_{k_g}$. Consequently, the GEE will also depend on these derivatives. Letting the vector $\mathbf{x}$ summarize our state space, $\mathbf{x} = (k, k_g)$, we can then write the GEE as the function $\Omega$,

$$\Omega(\mathbf{x}, \tau, \mathbf{x}', \mathbf{x}'', \Psi', \Psi'', H_\tau, H'_\tau, H''_\tau, \left\{H'_i, H''_i, \Psi'_i, \Psi''_i\right\}_{i\in\mathbf{x}}) = 0. \tag{B2}$$

In the steady state, $k = k' = k'' = \overline{k}$, $k_g = k'_g = k''_g = \overline{k}_g$, and $\Psi(\overline{k}, \overline{k}_g) = \overline{\tau}$. Hence, equations (B1) and (B2) reduce to

$$\Phi(\overline{k}, \overline{k}_g, \overline{\tau}) = 0, \tag{B3}$$

$$\Omega(\overline{k}, \overline{k}_g, \overline{\tau}, \overline{H}_\tau, \left\{\overline{H}_i, \overline{\Psi}_i\right\}_{i\in\mathbf{x}}) = 0, \tag{B4}$$

while the government budget constraint becomes

$$\delta \overline{k}_g = \overline{\tau} f(\overline{k}, \overline{k}_g). \tag{B5}$$

It is evident from these equations that, in order to find the levels $\overline{k}$, $\overline{k}_g$, and $\overline{\tau} = \Psi(\overline{k}, \overline{k}_g)$ in the steady state, one must also solve for the derivatives $\overline{H}_\tau$ and $\left\{\overline{H}_i, \overline{\Psi}_i\right\}_{i\in\mathbf{x}}$. This implies that the system of three equations (B3) through (B5) are not sufficient to uniquely determine the steady state.

To address this problem, Krusell, Kuruscu, and Smith (2002) propose to use a 'perturbation' method based on Judd's (1998). The method consists in approximating each of the rules (in this case $H$ and $\Psi$) by an $n$-th order polynomial. Given these polynomial approximations, calculating their derivatives is straightforward. This method, therefore, recasts the problem of finding the

[13] Note that the derivatives of the unknown functions in the GEE also depend on the derivatives of the function $g$ that pin down the evolution of public capital. The derivatives of $g$ in turn can be easily obtained by differentiating the government budget constraint.

decision rules into one where we simply solve for the parameters of two polynomials. The system of equations needed to solve for these parameters consists of the EE, the GEE, the government budget constraint, and a set of derivatives of the EE and GEE with respect to their arguments, all evaluated at the steady state. The higher the order of the polynomials describing the decision rules, the more derivatives of the EE and GEE need to be taken into account. An algorithm for methods PL and PQ in the text follows:

**Step 1:**

Under method PL, $H(k, k_g, \tau) = \phi_0^k + \phi_1^k k + \phi_2^k k_g + \phi_3^k \tau$, and $\Psi(k, k_g) = \phi_0^\tau + \phi_1^\tau k + \phi_2^\tau k_g$. Hence, all derivatives of order 2 and above are zero. Under this approximation of the policy rules, we have 9 unknowns $\{\overline{k}, \overline{\tau}, \phi_0^k, \phi_1^k, \phi_2^k, \phi_3^k, \phi_0^\tau, \phi_1^\tau, \phi_2^\tau\}$, so that 9 equations are necessary to complete the system. Three of these equations are given by equations (B3) through (B5). Furthermore, since the EE must hold for all $k, k_g, \tau$, we can differentiate it with respect these arguments ($\Phi_k$, $\Phi_{k_g}$ and $\Phi_\tau$), which delivers three more equations. Two more equations are given by the derivatives of the GEE with respect to $k$ and $k_g$ ($\Omega_k$, $\Omega_{k_g}$). Finally, the last equation is given by the approximation of $H$ evaluated at the steady state, $\overline{k} = \phi_0^k + \phi_1^k \overline{k} + \phi_2^k \overline{k}_g + \phi_3^k \overline{\tau}$.

**Step 2:**

Under method PQ, $H$ is quadratic in its arguments while we keep $\Psi$ as a linear functions of the states. The number of unknown parameters increases to 15. The set of equations needed to solve the system are the 9 equations obtained from step 1, as well as the equations resulting from differentiating the EE a second time with respect to its three arguments ($\Phi_{kk}, \Phi_{\tau\tau}, \Phi_{k_g k_g}, \Phi_{k k_g}, \Phi_{\tau k}, \Phi_{\tau k_g}$), which gives the remaining 6 equations.

**Step 3:**

In principal, this procedure can be continued, while increasing the degree of the polynomials for $H$ and $\Psi$, until changes in $\overline{k}, \overline{k}_g$, and $\overline{\tau}$ become negligible.

# C The LQ Approximation Method

This appendix describes a simple Linear Quadratic approximation algorithm used to solve for the Markov perfect equilibrium in Section 4.[14]

Substituting for consumption using the household budget constraint, we can write momentary utility as

$$u(1, k, k_g, \mid i \mid \tau) = \frac{\left[(1-\tau)k^\alpha l^{1-\alpha} k_g^\theta - i\right]^{1-\sigma} - 1}{1-\sigma},$$

[14] Versions of this algorithm applied to envionments with asymmetric information between agents and policymakers are used in Svensson and Woodford (2001), as well as Dotsey and Hornstein (2003).

where $u = \log c$ when $\sigma = 1$. We distinguish between a $3 \times 1$ vector of state variables, $s = \{1, k, k_g\}$, a $1 \times 1$ vector of flow variables, $f = \{i\}$, and a $1 \times 1$ vector of policy variables $R = \{\tau\}$. We include a constant in $s$ so as to allow the algorithm to deliver both the policy functions and the levels of the variables simultaneously. In general, period utility can be expressed as $u(Z)$, where $Z = (s, f, R)$ is a vector of size $(1 + np + nf + n\tau) \times 1$, where $np$, $nf$, and $n\tau$ denote the number of predetermined, flow, and policy variables respectively. With this notation, we can write the value function in ($\mathrm{P^M}$) as

$$V(s) = \max_{s',\, f,\, R} \{u(Z) + \beta^* V(s')\}$$

We can also express the accumulation equations in (2) and (9) as

$$s' = C_s(s, f, R), \tag{C1}$$

and the Euler equation constraint as

$$C_{f1}[s', f', R'] = C_{f0}[s, f, R]. \tag{C2}$$

Let $R = \Psi(s)$ and $f = G(s)$ denote the policy functions associated with the dynamic program ($\mathrm{P^M}$).

The solution essentially involves one main iterative procedure. We guess a solution for the policy variable in the steady state, $\widehat{R}$, which allows us to compute all steady state allocations, as well as an LQ approximation of the dynamic program around that steady state. We show below that the solution to this LQ approximation involves 3 algebraic equation in 3 unknowns, namely the linear approximations of $\Psi$, $G$, and $V$. Given the resulting $\Psi$, we then check whether the implied policy in the steady state, $\overline{R}$, coincides with the assumed guess $\widehat{R}$. If not, we update $\widehat{R}$ and repeat the process until $\widehat{R}$ and $\overline{R}$ have converged. Below is the algorithm in more detail.

**Step 1:**

Suppose that policy in the steady state is given by $\widehat{R}$. We can then use the accumulation equations, the household's resource constraint, and the Euler equation to solve for the steady state values of state and flow variables, $\widehat{s}$ and $\widehat{f}$.

**Step 2:**

Now, derive a linear approximation of (C1) and (C2),

$$s' = A_{ss}s + A_{sf}f + B_s s,$$

and

$$C_{f1s}s' + C_{f1f}f' + C_{f1R}R' = C_{y0}Z, \tag{C3}$$

respectively, as well as a quadratic approximation of period utility, $Z^T \mathbf{U} Z$, where $\mathbf{U}$ is a square symmetric matrix of size $(1 + np + nf + n\tau) \times (1 + np + nf + n\tau)$.

**Step 3:**

Guess the policy decision matrices $\Psi_j$ for $R$ and $G_j$ for $f$, as well as the value function matrix $V_j$ to get $s^T V_j s$. Substituting these guesses into the linearized constraints, equation (C3) becomes

$$\begin{aligned} C_{f1f} f' &= C_{y0} Z - C_{f1s} s' - C_{f1R} R' \\ &= C_{y0} Z - C_{f1s}[A_{ss}, A_{sf}, B_s] Z - C_{f1R} \Psi_j [A_{ss}, A_{sf}, B_s] Z, \end{aligned}$$

or

$$f' = C_{f1f}^{-1} \left\{ C_{y0} - \left[ C_{f1s} + C_{f1R} \Psi_j \right] [A_{ss}, A_{sf}, A_s] \right\} Z.$$

Write this last expression as

$$f' = [A_{fs}(\Psi_j), A_{ff}(\Psi_j), B_f(\Psi_j)] Z,$$

and use the guess $G_j$ to get

$$G_j [A_{ss} s + A_{sf} f + B_s R] = A_{fs}(\Psi_j) s + A_{ff}(\Psi_j) f + B_f(\Psi_j) R.$$

If $A_{ff} - G A_{sf}$ is invertible, then we have

$$\left[ -G_j A_{sf} + A_{ff}(\Psi_j) \right] f = [G_j A_{ss} - A_{fs}(\Psi_j)] s + [G_j B_s - B_f(\Psi_j)] R,$$

or

$$f = \underbrace{[A_{ff}(\Psi_j) - G_j A_{sf}]^{-1} [G_j A_{ss} - A_{fs}(\Psi_j)]}_{\widetilde{A}(G_j, \Psi_j)} s + \underbrace{[A_{ff}(\Psi_j) - G_j A_{sf}]^{-1} [G_j B_s - B_f(\Psi_j)]}_{\widetilde{B}(G_j, \Psi_j)} R,$$

where $\widetilde{A}$ and $\widetilde{B}$ are shown to depend explicitly on the solution guesses $G_j$ and $\Psi_j$. To save on notation, we write $\widetilde{A}(G_j, \Psi_j)$ and $\widetilde{B}(G_j, \Psi_j)$ as $\widetilde{A}_j$ and $\widetilde{B}_j$ respectively. It now follows that

$$\begin{aligned} s' &= A_{ss} s + A_{sf} [\widetilde{A}_j s + \widetilde{B}_j R] + B_s R \\ &= \underbrace{[A_{ss} + A_{sf} \widetilde{A}]}_{A^*(G_j, \Psi_j)} s + \underbrace{[B_s + A_{sf} \widetilde{B}]}_{B^*(G_j, \Psi_j)} R. \end{aligned}$$

Our dynamic program becomes

$$s^T V_{j+1} s = \max \left\{ Z^T \mathbf{U} Z + \beta^* s'^T V_j s' \right\}$$

subject to

$$s' = A_j^* s + B_j^* R.$$

Substituting for $f$ in $Z$, we write the problem as

$$s^T V_{j+1} s =$$

$$\max_{R,\, s'} \left\{ \left[ s^T,\ R^T \right] \begin{bmatrix} Q_{ss} & Q_{sR} \\ Q_{Rs} & Q_{RR} \end{bmatrix} \begin{bmatrix} s \\ R \end{bmatrix} + \beta^* \left[ A_j^* s + B_j^* R \right]^T V_j \left[ A_j^* s + B_j^* R \right] \right\}, \tag{C4}$$

where

$$Q_{ss} = \left[ I_s,\ \widetilde{A}_j^T \right] \begin{bmatrix} U_{ss} & U_{sf} \\ U_{fs} & U_{ff} \end{bmatrix} \begin{bmatrix} I_s \\ \widetilde{A}_j \end{bmatrix},$$

$$Q_{sR} = \left[ I_s,\ \widetilde{A}_j^T \right] \left\{ \begin{bmatrix} U_{sR} \\ U_{fR} \end{bmatrix} + \begin{bmatrix} U_{ss} & U_{sf} \\ U_{fs} & U_{ff} \end{bmatrix} \begin{bmatrix} 0 \\ \widetilde{B}_j \end{bmatrix} \right\},$$

and

$$Q_{RR} = U_{RR} + \left[ 0,\ \widetilde{B}_j^T \right] \begin{bmatrix} U_{ss} & U_{sf} \\ U_{fs} & U_{RR} \end{bmatrix} \begin{bmatrix} 0 \\ \widetilde{B}_j \end{bmatrix} + \left[ 0,\ \widetilde{B}_j^T \right] \begin{bmatrix} U_{sR} \\ U_{fR} \end{bmatrix} + \left[ U_{Rs},\ U_{Rf} \right] \begin{bmatrix} 0 \\ \widetilde{B}_j \end{bmatrix}.$$

Observe that (C4) is a standard discounted linear regulator problem (Sargent and Lungquist 2000, Chapter 4). The first-order conditions for the optimal choice of the policy instrument, therefore, yields

$$\Psi_{j+1} = -\left[ Q_{RR} + \beta^* B_j^{*T} V_j B_j^* \right]^{-1} \left[ Q_{Rs} + \beta^* B_j^{*T} V_j A_j^* \right]. \tag{C5}$$

Since $f = Gs$, and $f = \widetilde{A}_j s + \widetilde{B}_j R$ above, it follows that

$$G_{j+1} = \widetilde{A}_j + \widetilde{B}_j \Psi_{j+1}. \tag{C6}$$

Furthermore, by substituting for $R$ in (C4), we have that

$$V_{j+1} = \left[ I,\ \Psi_{j+1}^T \right] \left\{ Q + \beta^* \begin{bmatrix} A_j^{*T} \\ B_j^{*T} \end{bmatrix} V_j \left[ A_j^*,\ B_j^* \right] \right\} \begin{bmatrix} I \\ \Psi_{j+1} \end{bmatrix}. \tag{C7}$$

Iterating on (C5), (C6), and (C7) above then delivers solutions for $\Psi$, $G$, and $V$.

**Step 4:**

Now, recall that the solution to equations (C5), (C6), and (C7) obtains for a linear quadratic approximation of ($\mathrm{P^M}$) conditional on a guess for policy in the steady state, $\widehat{R}$, in step 1. We need to check, therefore, whether the policy implied by the solution to these equations, $\overline{R}$, coincides with $\widehat{R}$ (recall that $s$ contains a constant). If not, we update our guess for $\widehat{R}$ and repeat steps 1 through 4 until $\widehat{R} = \overline{R}$.

# D Proof of Proposition 3

Under our benchmark economy, the Ramsey Problem now reads as

$$\max_{\{\tau_t, k_{t+1}, k_{gt+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \ln((1-\tau_t) y_t - k_{t+1})$$

subject to

$$k_{t+1} = \alpha\beta y_t(1-\tau_t), \tag{D1}$$

and

$$k_{gt+1} = \tau_t y_t, \tag{D2}$$

where we have used the closed-form solution for households' savings found in section (3.2), and $y_t = k_t^{\alpha} k_{gt}^{\theta}$. Let $\lambda_t$ and $\mu_t$ denote the Lagrange multipliers associated with constraints (D1) and (D2) respectively.

The Ramsey planner's optimality conditions are

$$\lambda_t = \left(\mu_t + \frac{1}{c_t}\right)\frac{1}{\alpha\beta},$$

$$\lambda_t - \frac{1}{c_t} - \frac{\alpha\beta y_{t+1}}{k_{t+1}}\left[\lambda_{t+1}\alpha\beta(1-\tau_{t+1}) + \mu_{t+1}\tau_{t+1} - \frac{(1-\tau_{t+1})}{c_{t+1}}\right] = 0,$$

$$\mu_t + \frac{\theta\beta y_{t+1}}{k_{gt+1}}\left[\frac{(1-\tau_{t+1})}{c_{t+1}} - \lambda_{t+1}\alpha\beta(1-\tau_{t+1}) - \mu_{t+1}\tau_{t+1}\right] = 0.$$

Straightforward algebra allows us to dispose of the multipliers and obtain,

$$\frac{1}{\alpha\beta(1-\tau_t)} + \frac{\theta}{\alpha\beta(\alpha\tau_t - \theta(1-\tau_t))} - \frac{\theta}{(1-\tau_t)(\alpha\tau_{t+1} - \theta(1-\tau_{t+1}))} = 0.$$

It is then easy to see that $\tau_t = \tau_{t+1} = \beta\theta$ satisfies the above equation. Since the government faces the same first order conditions at $t = 0$ and $t > 0$, this Ramsey solution is time consistent.

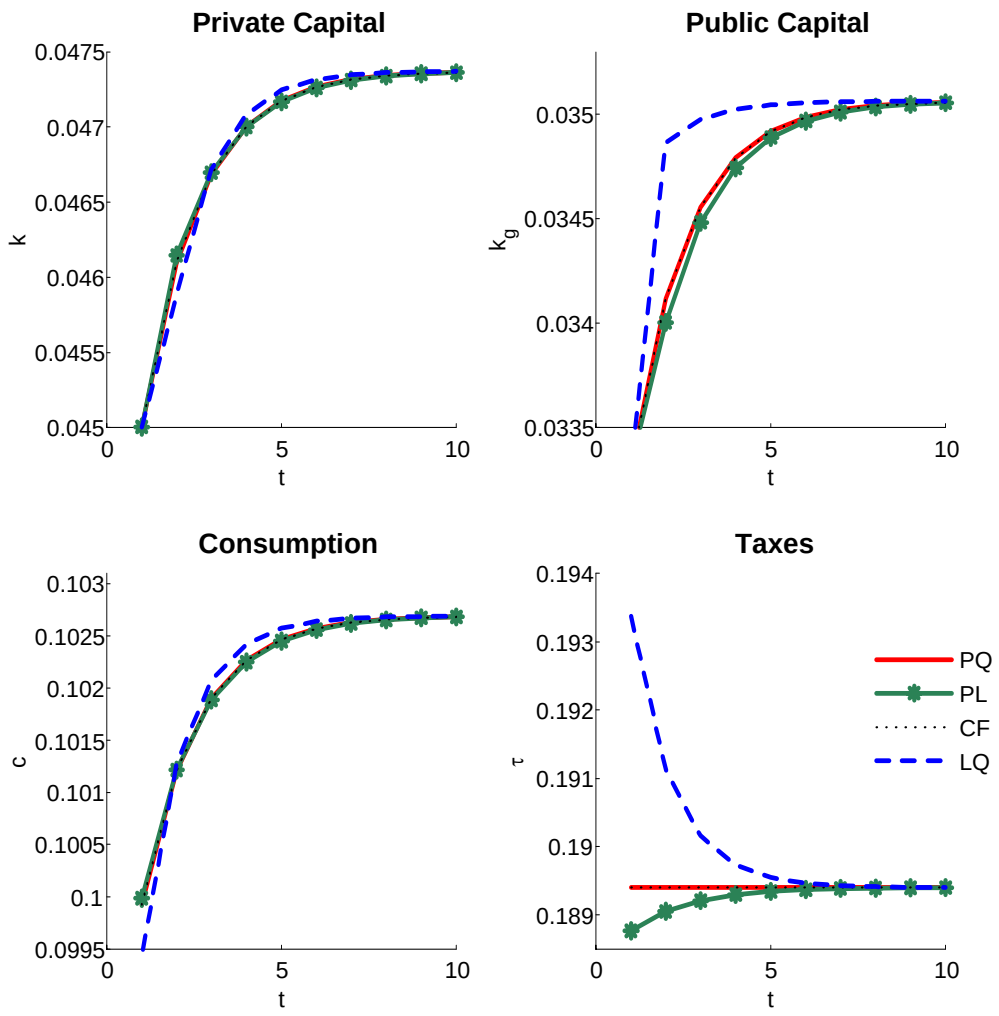


Figure 1: Evolution of allocations under the three methods ($k_0 = 0.9k_{ss}$ and $k_{g0} = 0.9k_{gss}$)

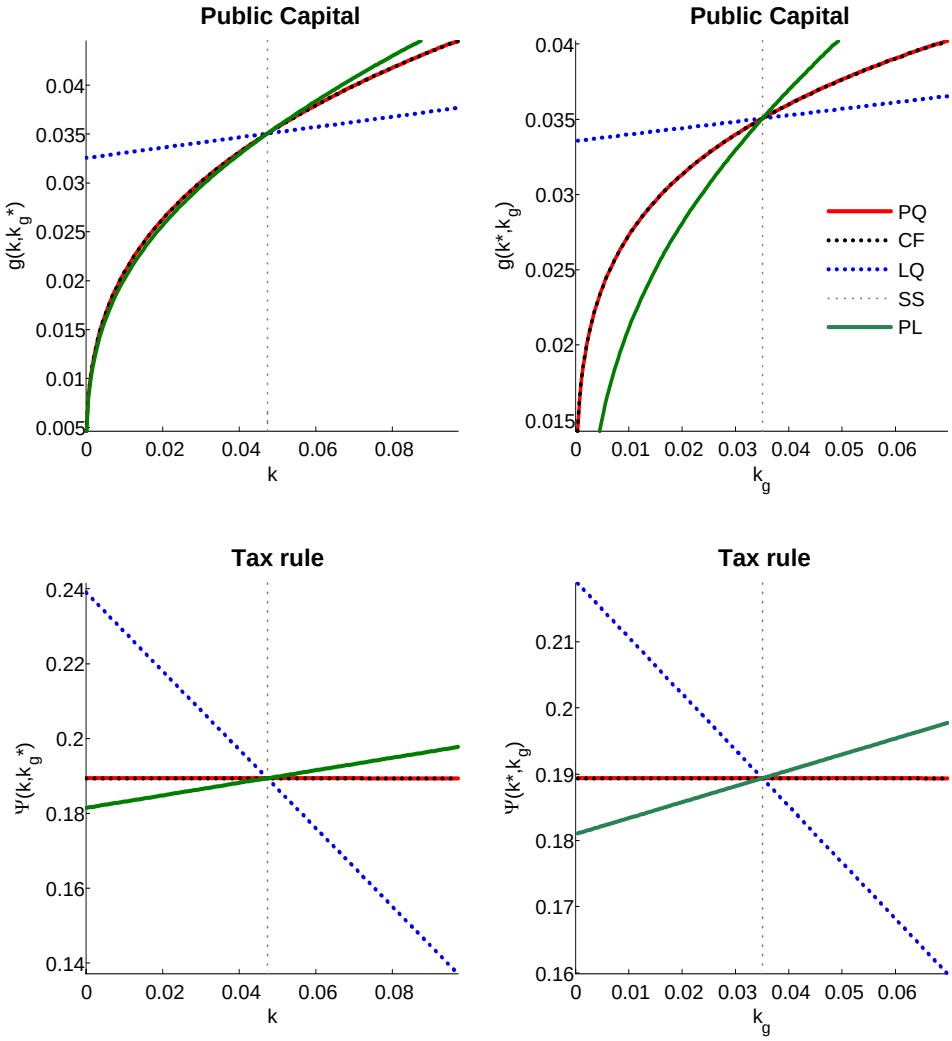


Figure 2: Public capital accumulation -$g(k, k_g)$- and tax rule -$\Psi(k, k_g)$- as functions of the states

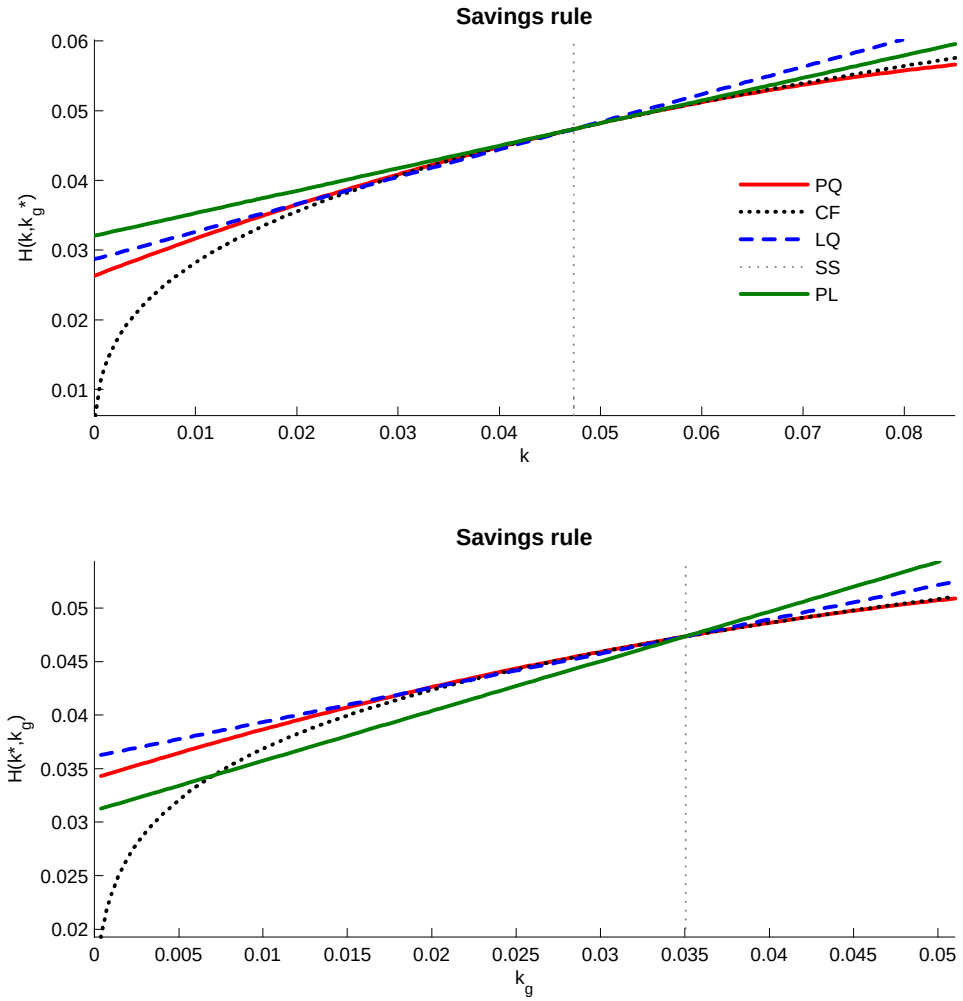


Figure 3: Private Capital Accumulation - $H(k, k_g)$ - as a function of the states

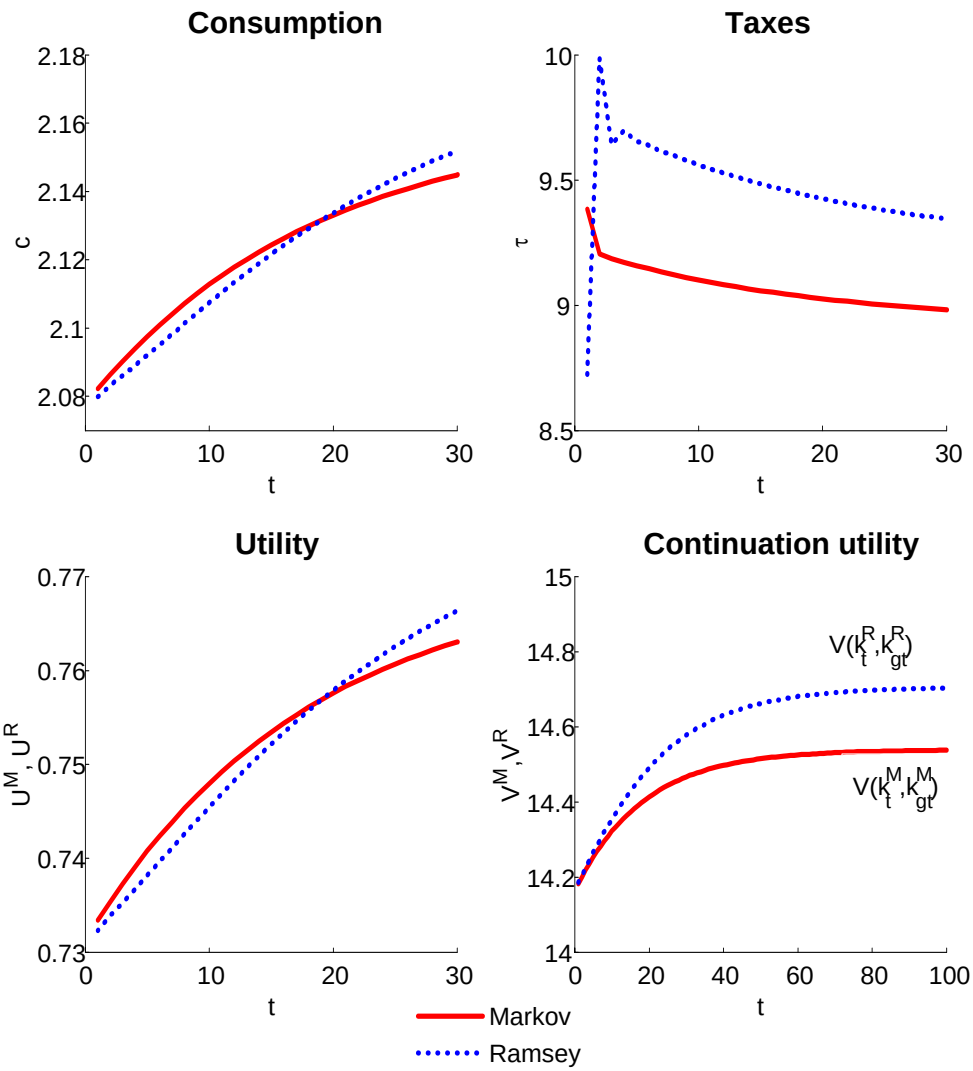


Figure 4: Dynamics of Consumption, Taxes, and Welfare, $k_0 = 0.95k_{ss}$ and $k_{g0} = 0.95k_{gss}$