Optimal Fiscal and Monetary Policy When Money is Essential [*]

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Abstract

We study optimal fiscal and monetary policy in an environment where explicit frictions give rise to valued money, making money essential in the sense that it expands the set of feasible trades. Our main results are in stark contrast to the prescriptions of earlier flexible-price Ramsey models. Two especially important findings emerge from our work: the Friedman Rule is typically not optimal, and inflation is stable over time. Inflation is not a substitute instrument for a missing tax, as is sometimes the case in standard Ramsey models. Rather, the inflation tax is exactly the right tax to use because money has a rent associated with it. Regarding the optimal dynamic policy, realized (ex-post) inflation is quite stable over time, in contrast to the very volatile ex-post inflation rates that arise in standard flexible-price Ramsey models. Taken together, these two findings turn conventional wisdom from traditional Ramsey monetary models on its head.

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[*] The views expressed here are solely those of the authors and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of any other person associated with the Federal Reserve System.

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1 Introduction

Monetary theory has made important advances of late, ones that enable researchers interested in applied policy questions to consider explicit frictions that give rise to valued money. In this paper, we build on the works of Lagos and Wright (2005) and Aruoba, Waller, and Wright (2006) and study optimal fiscal and monetary policy, following the tradition of Lucas and Stokey (1983) and Chari, Christiano, and Kehoe (1991), in such environments. Two important findings emerge from our work, both of which are opposite those of earlier flexible-price Ramsey monetary models: the Friedman Rule is typically not optimal and inflation is stable over time. Our results thus turn conventional wisdom from traditional Ramsey models on its head.

The contribution of Lagos and Wright (2005) and Aruoba, Waller, and Wright (2006) — hereafter, LW and AWW, respectively — is to integrate search-based monetary theory, in the spirit of Kiyotaki and Wright (1989, 1993), with standard dynamic general equilibrium macroeconomics. This integration makes the study of policy questions much easier and potentially more relevant than in earlier search-based models. However, these models have been criticized on two grounds. First, they superficially resemble standard cash-in-advance (CIA) or money-in-the-utility-function (MIU) models, making some question whether they really are any deeper than reduced-form models of money. This point has been raised by, among others, Howitt (2003). Second, until now, the policy questions addressed in these new models have been largely confined to the deterministic welfare costs of inflation. When parameterized to seem as close as possible to standard CIA and MIU models, the quantitative answers they have yielded to this question are similar to those obtained with CIA and MIU models, further adding to the sense that these new models simply re-invent CIA or MIU. In this paper, we ask a different policy-relevant question in these new models, and even when we parameterize the model to look very similar to standard applied models of money, we reach conclusions very different from those reached by Chari, Christiano, and Kehoe (1991) and others using typical CIA and MIU frameworks. Our results thus show that the answers to policy questions may indeed be very different once monetary frictions are treated seriously.[1]

We study the canonical Ramsey problem of optimal fiscal and monetary policy using the LW and AWW models. Our first main finding is that the optimal nominal interest rate is typically positive. This optimal deviation from the Friedman Rule is not because the inflation tax acts as a substitute instrument for a missing tax, as is sometimes the case in Ramsey models. Rather, the inflation tax is exactly the right tax for the government to use when money is essential in

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[1] In work with related motivation, albeit a somewhat different structure, Berentsen, Norman, and Wright (2005) study the optimal inflation tax in a public-finance framework. One of their main findings, which contrasts with ours, is that the Friedman Rule is optimal. A key difference between their setup and ours is that we do not allow for any lump-sum instruments.

Kocherlakota's (1998) sense that it expands the set of feasible trades. Specifically, without money, trade can only occur if there is a double coincidence of wants, whereas money allows trade to occur with only a single coincidence of wants. Money is thus a special object in this class of models and therefore has a rent associated with it. To be specific, households in this economy require the benefit of holding money to be strictly positive in order to hold it. This benefit is realized only when the household is a buyer in a bilateral match in the form of buyer's surplus and it is this surplus that we interpret as the rent associated with holding money. This money rent is not present in standard CIA or MIU models. We know from Ramsey theory that it is optimal to tax rents. The correct way to tax rents accruing to money is through inflation, hence the Friedman Rule is not optimal. Interestingly, Kocherlakota (2005) conjectured that the Friedman Rule may not be optimal in a Ramsey problem in search-based models. Our results show his conjecture is correct.

Deviations from the Friedman Rule have been obtained in other modern Ramsey models, as well. For example, Schmitt-Grohe and Uribe (2004a) show that a positive nominal interest rate can tax producers' monopoly profits, and Chugh (2006b) shows it can tax monopolistic labor suppliers' rents.[2] Both of these results, however, are examples of the Ramsey planner using a positive nominal interest rate to *indirectly* tax some rent. One may wonder whether alternative tax instruments in our model could tax the money rent. To investigate this, we introduce a sales tax on goods whose purchase can only be achieved with money. It turns out that adding this seemingly natural instrument does not admit a Ramsey equilibrium in which the Friedman Rule is optimal. Thus, if there does exist another tax instrument besides the inflation tax that can seize the money rent, it is not a straightforward one.

Our second main finding is that realized (ex-post) inflation is quite stable over time in the face of shocks, which is in contrast to the very volatile ex-post inflation rates that Chari, Christiano, and Kehoe (1991) — hereafter, CCK — find. Inflation volatility is high in CCK and the related literature because surprise movements in the price level allow the government to synthesize real state-contingent debt payments from nominally risk-free government bonds, without distorting real activity. In our model, real activity is distorted by ex-post inflation, since a change in the price level effects the composition of household's consumption. The welfare cost of this distortion dominates the insurance value of generating state-contingent debt, leaving inflation very stable. The frictions underlying monetary trade thus provide novel justification for the optimality of inflation stability, a prescription that resonates with most central bankers.

An important technical advantage of the LW and AWW frameworks is that the distribution of money-holdings across agents is simple to track: it simply collapses periodically to a point. At

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[2] In a different context, one that abstracts from public finance considerations, Rocheteau and Wright (2005) show that a positive nominal interest rate can also beneficially affect the extensive margin in the DM.

the expense of a heavier computational burden, one may want to think about optimal fiscal and monetary policy when this distribution is non-trivial. Once one goes down that route, an interesting taxation framework to apply may be the Mirrleesian one, in which idiosyncratic shocks and private information become important considerations in shaping optimal policy. However, because even the simpler step of characterizing the Ramsey-optimal policy, which assumes a representative agent, has not been studied in this class of models, we think it makes sense to begin here.

The rest of the paper is organized as follows. Section 2 lays out the baseline LW model in which we first study optimal policy. Section 3 presents the Ramsey problem for the basic model, from which it is possible to start gleaning insight as to how the results differ from those of CCK. In Section 4, we characterize and discuss the optimal policy in the basic model; we present in Section 4.1 a proof for a particularly important version of the model that the Friedman Rule is not optimal and in Section 4.2 the dynamic Ramsey allocations and policy that demonstrate optimal inflation stability. In Sections 5 and 6, we extend the Ramsey problem to the AWW model, and Section 7 shows that the main results carry over from the basic model virtually unchanged. Section 8 summarizes and offers ideas for future work.

2 Basic Model

We begin by establishing our main results in a version of the LW model. In this model, the agents in the economy participate in a centralized market (CM) where they trade general consumption goods and assets with the market and in a decentralized market (DM) where they trade specialized consumption goods bilaterally. To make the model as comparable as possible to the benchmark cash/credit environment used by CCK, we alter slightly the timing of markets in the original LW model. Specifically, in our version, the CM is the first market in a given period, followed by the DM. We make this alteration because we would like asset markets (which in the LW model meet in the CM) to convene in any period before goods markets (in particular, before goods markets in which money must be used for transactions), which is the timing assumed by CCK. We proceed by describing the activities of the government, households, and firms in our model.

2.1 Government

Government consumption is assumed to be composed entirely of goods produced in the CM. In nominal terms, the flow budget constraint of the government is

$$M_t + B_t + P_t w_t \tau_t^h H_t = P_t G_t + M_{t-1} + R_{t-1} B_{t-1}, \tag{1}$$

which states that the government has three sources of revenues to pay for its consumption: labor income tax revenues, nominal money creation, and nominal debt issuance. The notation is standard:

M_t denotes nominal money outstanding at the end of period t, B_t is nominally risk-free government debt outstanding at the end of period t, R_t is the gross nominal interest rate on bonds, τ_t^h is a proportional labor income tax on aggregate hours worked H_t in the CM, P_t is the nominal price level in the CM, and w_t is the real wage in the CM. The nominal return R_t is known at the time B_t is issued and paid in the CM of period $t+1$.

2.2 Households

Households periodically transact in markets for general goods and assets (the CM) and in markets for specialized goods (the DM). In the DM, money is essential in the sense that transactions there are infeasible without money.[3] In the CM, because markets are Walrasian trades can proceed with or without money. We describe first the timing of events in a given period and then present the household's CM and DM problems.

Events unfold for a household in a given period t as follows:

- The household begins the CM with portfolio m_{t-1} and b_{t-1}.
- The uncertainty for the current period is resolved, and the household observes government consumption G_t and the level of technology Z_t. We denote the aggregate state collectively S_t.
- The household receives the receipts from his bond holdings, $R_{t-1}b_{t-1}$.
- The household chooses its CM consumption x_t, labor supply h_t, portfolio (m_t, b_t) and pays the labor income tax.
- The household enters the DM with m_t.
- Depending on the household's trade in the DM, it exits the DM with $m_t - d_t$, $m_t + d_t$, or m_t money holdings, where d_t is the buyer's payment in bilateral trade.

2.2.1 Household CM Problem

For a household that enters the CM with money holdings m_{t-} and bond holdings b_{t-}, the CM problem is

$$W_t\left(m_{t-1}, b_{t-1}, S_t\right) = \max_{x_t, h_t, m_t, b_t} \{U(x_t) - Ah_t + V_t(m_t, b_t, S_t)\} \tag{2}$$

subject to

$$P_t x_t + m_t + b_t = P_t w_t (1-\tau_t^h) h_t + m_{t-1} + R_{t-1} b_{t-1}, \tag{3}$$

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[3] In a more general model, one can allow a double-coincidence meeting to take place where barter takes place. Doing so does not change any of the properties of the current model and we abstract from it.

where $W_t(.)$ denotes the value of entering the CM and $V_t(.)$ denotes the value of entering the DM that convenes after the CM in period t. Centralized market consumption is x_t, and the household's hours worked in the CM is h_t. Note that instantaneous utility in the CM is separable and linear in labor; it is this quasi-linearity in preferences that allows the LW model to be so tractable, as it guarantees a degenerate distribution of money holdings across households after every meeting of the CM.

Eliminating h in the objective function using the budget constraint, the first-order conditions with respect to x_t, m_t, and b_t are

$$U'(x_t) = \frac{A}{w_t(1-\tau_t^h)}, \tag{4}$$

$$\frac{A}{P_t w_t(1-\tau_t^h)} = V_{m,t}(m_t, b_t, S_t), \tag{5}$$

$$\frac{A}{P_t w_t(1-\tau_t^h)} = V_{b,t}(m_t, b_t, S_t), \tag{6}$$

familiar from LW. These optimality conditions imply the usual LW results about degeneracy of asset holdings (m_t, b_t) across households because they are independent of (m_{t-1}, b_{t-1}). All households choose the same portfolio at the end of the CM regardless of the portfolio they entered the market with. Thus, the LW result of degeneracy of money holdings readily extends to bond holdings as well. Moreover, we have standard envelope conditions

$$W_{m,t}(m_{t-1}, b_{t-1}, S_t) = \frac{A}{P_t w_t(1-\tau_t^h)}, \tag{7}$$

$$W_{b,t}(m_{t-1}, b_{t-1}, S_t) = \frac{AR_{t-1}}{P_t w_t(1-\tau_t^h)}, \tag{8}$$

which show $W_t(.)$ is linear in its arguments.

2.2.2 Household DM Problem

Now we turn to the household's DM problem. Knowing that the distribution of money holdings is degenerate in equilibrium, we will, for notational simplicity, write the household DM problem assuming that when it meets a trading partner, the trading partner has equilibrium money holdings M_t; this allows us to conserve on integrating over all possible money holdings of trading partners that a given household could meet. With probability σ, the household is a buyer in the DM; with probability σ, the household is a seller in the DM; and with probability $1-2\sigma$, the household does not participate in the DM and continues to the CM of the next period without transacting.[4] Buyers consume q in the DM, experiencing utility $u(q)$; sellers produce q in the DM, experiencing

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[4] This setup can be justified by either the search framework of the original LW model or the preference shocks setup of AWW.

disutility, which can be interpreted as the cost of production, $c(q, Z)$, where $c_Z < 0$. We assume throughout our basic model that $c(q, Z) = q/Z$.[5]

We can write the problem of a household that enters the DM with portfolio (m_t, b_t) as

$$\begin{aligned} V_t(m_t, b_t, S_t) &= \sigma\left\{u\left(q\left(m_t, M_t, S_t\right)\right) + \beta E_t W_{t+1}\left(m_t - d(m_t, M_t, S_t), b_t, S_{t+1}\right)\right\} && (9) \\ &+ \sigma\left\{-c\left(q\left(M_t, m_t, S_t\right)\right) + \beta E_t W_{t+1}\left(m_t + d(M_t, m_t, S_t), b_t, S_{t+1}\right)\right\} && (10) \\ &+ (1 - 2\sigma)\beta E_t W_{t+1}\left(m_t, b_t, S_{t+1}\right). && (11) \end{aligned}$$

The quantity $q(m_{bt}, m_{st}, S_t)$ is the quantity produced and exchanged in a bilateral meeting in the DM, where m_b denotes the money holdings of the buyer, m_s denotes the money holdings of the seller, and $d(m_b, m_{st}, S_t)$ is the amount of money that changes hands. We refer to $[q(.), d(.)]$ as the terms of trade in a single-coincidence meeting. Note that we anticipated the result that neither the buyer's nor the seller's bond holdings will matter for q and d. (Implicitly, of course, we rule out bonds as a medium of exchange.)

In the DM, we must specify the protocol by which the price and quantity in any bilateral trade are determined — that is, we must define the structure by which the terms of trade are determined. The two main alternatives in the literature are Nash bargaining and price-taking. We describe the bargaining version in detail. It turns out — see Rocheteau and Wright (2005) — that price-taking in the basic model amounts to a simple parameter restriction in the bargaining version.

2.2.3 Bargaining Version

Denoting the portfolio of the buyer by (m_t, b_t), that of the seller by $(\tilde{m}_t, \tilde{b}_t)$, and the buyer's bargaining power by θ, the generalized Nash bargaining problem is

$$\max_{q_t, d_t} \left[u(q_t) + \beta E_t W_{t+1}\left(m_t - d_t, b_t, S_{t+1}\right) - \beta E_t W_{t+1}\left(m_t, b_t, S_{t+1}\right)\right]^{\theta} \quad (12)$$

$$\times \left[-c(q_t) + \beta E_t W_{t+1}\left(\tilde{m}_t + d_t, \tilde{b}_t, S_{t+1}\right) - \beta E_t W_{t+1}\left(\tilde{m}_t, \tilde{b}_t, S_{t+1}\right)\right]^{1-\theta} \quad (13)$$

subject to

$$d_t \leq m_t. \quad (14)$$

where the constraint is simply a feasibility condition stating the buyer cannot spend more than he has and the threat points are simply the values of continuing on to the next CM in period $t+1$. Using the envelope conditions from above (in particular, the implied linearity of the function

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[5] This functional form can be obtained by assuming a linear production function in effort, $q = Ze$, and a linear disutility of effort, $-e$. Inverting the production function and substituting into the disutility function gives the cost function $c(q, Z) = q/Z$.

$W_t(.)$), the bargaining problem can be written more conveniently as

$$\max_{q_t,d_t} \left\{ u(q_t) - \beta d_t E_t \left[\frac{A}{P_{t+1} w_{t+1}(1-\tau^h_{t+1})} \right] \right\}^{\theta} \left\{ -c(q_t) + \beta d_t E_t \left[\frac{A}{P_{t+1} w_{t+1}(1-\tau^h_{t+1})} \right] \right\}^{1-\theta} \tag{15}$$

subject to

$$d_t \leq m_t. \tag{16}$$

Defining $\chi_t \equiv E_t\left[A/\{P_{t+1}w_{t+1}(1-\tau^h_{t+1})\}\right]$, the Kuhn-Tucker conditions, which are necessary and sufficient, for the bargaining problem are

$$\frac{\theta u'(q_t)}{u(q_t) - \beta\chi_t d_t} - \frac{(1-\theta)c'(q_t)}{-c(q_t) + \beta\chi_t d_t} = 0, \tag{17}$$

$$-\frac{\theta\beta\chi_t}{u(q_t) - \beta\chi_t d_t} + \frac{(1-\theta)\beta\chi_t}{-c(q_t) + \beta\chi_t d_t} - \lambda_t = 0, \tag{18}$$

$$\lambda_t(m_t - d_t) = 0, \tag{19}$$

where λ_t is the multiplier associated with the constraint. If $\lambda_t = 0$, the first two conditions yield $u'(q_t) = c'(q_t)$, which defines the efficient quantity $q_t = q^*$, and $d_t = m^*_t$, which can be solved using the second equation.

If $\lambda_t > 0$, the solution will have $d_t = m_t$, meaning the buyer spends all his money in a bilateral meeting. Using the first condition, the quantity produced and traded will solve

$$\beta\chi_t m_t = g(q_t, Z_t), \tag{20}$$

where

$$g(q, Z) \equiv \frac{\theta c(q,Z)u'(q) + (1-\theta)u(q)c_q(q,Z)}{\theta u'(q) + (1-\theta)c_q(q,Z)} \tag{21}$$

as in LW. In equilibrium, $\lambda_t > 0$, which can be shown using a similar argument to the one in LW. Also, note that because the expectation in χ_t is taken with respect to S_t, we denote the bargaining problem outcomes as $q(m_t, S_t)$ and $d(m_t, S_t)$, where the first argument is understood to be the money holdings of the buyer.

Substituting this solution into the DM problem (9) and using the envelope conditions, we get

$$V_t(m_t, b_t, S_t) = \sigma\left\{u\left[q_t\left(m_t, S_t\right)\right] - c\left[q\left(M_t, S_t\right)\right] - \beta\chi_t m_t + \beta\chi_t M_t\right\} + \beta E_t W_{t+1}\left(m_t, b_t, S_{t+1}\right). \tag{22}$$

The relevant envelope conditions are

$$V_{m,t}\left(m_t, b_t, S_t\right) = \sigma\left\{u'\left[q_t\left(m_t, S_t\right)\right]\frac{\partial q_t\left(m_t, S_t\right)}{\partial m_t} - \beta\chi_t\right\} + \beta\chi_t, \tag{23}$$

$$V_{b_t,t}\left(m_t, b_t, S_t\right) = \beta R_t \chi_t. \tag{24}$$

Finally, noting

$$\frac{\partial q_t}{\partial m_t} = \frac{\beta\chi_t}{g_q(q_t, Z_t)} \tag{25}$$

from (20), (23) simplifies to

$$V_{m,t}\left(m_t, b_t, S_t\right) = \beta\chi_t \left[\sigma \frac{u'(q)}{g_q(q,Z)} + 1 - \sigma\right]. \tag{26}$$

2.2.4 Price-Taking

Although bargaining has been used almost exclusively as the pricing scheme in bilateral meetings in this class of models, more "competitive" pricing schemes have been used recently, as well. In addition to being closer to mainstream macroeconomics, competitive pricing eliminates the holdup problems inherent in bargaining. In the basic model, competitive pricing in the DM amounts to sellers and buyers solving their respective supply and demand problems taking the price as given; the market-clearing price is determined in equilibrium. Based on the results of LW and Rocheteau and Wright (2005), it follows that the price-taking version of our model is the same as the bargaining version if $\theta = 1$.

2.2.5 Solution to Household Problem

For the bargaining version we obtain the conditions that solve the household's problem as follows. First, combining the household's CM optimality conditions (4)-(6) with the envelope conditions (23) and (24) gives us:

$$U'(x_t) = \frac{A}{(1-\tau_t^h)w_t}, \tag{27}$$

$$\frac{A}{P_t w_t(1-\tau_t^h)} = \beta\chi_t \left[\sigma \frac{u'(q)}{g_q(q,Z)} + 1 - \sigma\right], \tag{28}$$

$$\frac{A}{Pw(1-\tau^h)} = \beta R_t \chi_t. \tag{29}$$

Using (27) and the definition of χ_t, we get

$$\chi_t = E_t\left[\frac{U'(x_{t+1})}{P_{t+1}}\right], \tag{30}$$

which we can use to express (28) and (29) as

$$\frac{U'(x_t)}{P_t} = \beta\left[\sigma\frac{u'(q_t)}{g_q(q_t,Z_t)} + 1 - \sigma\right] E_t\left[\frac{U'(x_{t+1})}{P_{t+1}}\right], \tag{31}$$

$$\frac{U'(x_t)}{P_t} = \beta R_t E_t\left[\frac{U'(x_{t+1})}{P_{t+1}}\right]. \tag{32}$$

We will refer to these last two equations as the household's first-order conditions with respect to money and bonds, respectively, in analogy with a standard cash/credit CCK type of model. Note that they imply a Fisher-like condition,

$$R_t = \sigma\frac{u'(q_t)}{g_q(q_t,Z_t)} + 1 - \sigma, \tag{33}$$

linking the returns on money and bonds.

Rewriting condition (33) slightly, we have

$$i_t = \sigma \left[\frac{u'(q_t)}{g_q(q_t, Z_t)} - 1 \right], \tag{34}$$

where the left hand side, $i \equiv R - 1$, is the cost of holding money (the net nominal interest rate) and the right hand side is the benefit of holding money. We note that the right-hand-side of (34) will play an important role in shaping the Ramsey allocations and hence the optimal policy; we defer discussion and intuition regarding it until we present the Ramsey problem in Section 3, in which we will see it in the context of the complete Ramsey problem and will be able to easily compare it to the benchmark Ramsey problems in Lucas and Stokey (1983) and CCK.

To finally state the solution of the household problem: given sequences $\{S_t, P_t, w_t, \tau_t^h, R_t\}$, initial condition (M_0, B_0), and appropriate transversality conditions, the solution to the household's problem is processes $\{q_t, M_t, B_t, x_t, h_t\}_{t=0}^{\infty}$ satisfying conditions (3), (20), (27), (31), and (33).

2.3 Firms

In the CM, a representative firm hires labor in a competitive labor market and operates the linear production technology $Y_t = Z_t H_t$. Profit-maximization therefore implies the wage is $w_t = Z_t$ in equilibrium.

2.4 Equilibrium

Imposing equilibrium ($m_t = M_t$, $x_t = X_t$, etc.) and combining the firms' and households' optimality conditions, we can define the equilibrium as follows. Given policy variables $\{\tau_t^h, R_t\}_{t=0}^{\infty}$, the technology realization $\{Z_t\}_{t=0}^{\infty}$, the government spending realization $\{G_t\}_{t=0}^{\infty}$, and initial condition (M_0, B_0), equilibrium is a set of processes $\{q_t, B_t, M_t, X_t, H_t, P_t\}_{t=0}^{\infty}$ satisfying

$$U'(x_t) = \frac{A}{(1 - \tau_t^h) Z_t}, \tag{35}$$

$$\beta M_t E_t \left[\frac{U'(X_{t+1})}{P_{t+1}} \right] = g(q_t, Z_t), \tag{36}$$

$$\frac{U'(X_t)}{P_t} = \beta \left[\sigma \frac{u'(q_t)}{g_q(q_t, Z_t)} + 1 - \sigma \right] E_t \left[\frac{U'(X_{t+1})}{P_{t+1}} \right], \tag{37}$$

$$R_t = \sigma \frac{u'(q_t)}{g_q(q_t, Z_t)} + 1 - \sigma, \tag{38}$$

$$X_t + G_t = Z_t H_t, \tag{39}$$

$$B_t + M_t + P_t G_t = R_{t-1} B_{t-1} + M_{t-1} + P_t Z_t \tau_t^h H_t. \tag{40}$$

For the Ramsey problem, it will be useful to combine (36) and (37) and rearrange for real money balances,

$$\frac{M_t}{P_t} = \frac{g(q_t, Z_t)}{U'(X_t)} \left[\sigma \frac{u'(q_t)}{g_q(q_t, Z_t)} + 1 - \sigma \right]. \tag{41}$$

Furthermore, in any monetary equilibrium, $R_t \geq 1$ because otherwise households could earn unbounded profits by selling bonds and buying money. We represent this restriction in terms of allocations using (38) as

$$\sigma \left(\frac{u'(q_t)}{g_q(q_t, Z_t)} - 1 \right) \geq 0. \tag{42}$$

3 Ramsey Problem in Basic Model

As is common in the Ramsey literature, we adopt the primal approach and cast the Ramsey problem as that of a planner that chooses allocations subject to feasibility and the need to raise exogenous government revenue, making sure the resulting allocations are implementable as a monetary equilibrium. We prove the following in Appendix A:

Proposition 1. *The allocations in a monetary equilibrium satisfy (39), (42), and the present-value implementability constraint (PVIC),*

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[U'(X_t)X_t - AH_t + \sigma g(q_t, Z_t) \left(\frac{u'(q_t)}{g_q(q_t, Z_t)} - 1 \right) \right] = U'(X_0) \left[\frac{M_{-1} + R_{-1}B_{-1}}{P_0} \right]. \tag{43}$$

In textbook Ramsey problems, implementability constraints typically take the form $E_0 \sum_t \beta^t \sum_i U_i(x_{1t}, ..., x_{Nt})x_{it} = a_0$, where $\{x_{it}\}_{i=1}^N$ is the set of N goods the agent consumes at time t.[6] At first glance, (43) does not seem to conform to this insight because the term related to the DM, $\sigma g(q_t, Z_t)(u'(q_t)/g'(q_t) - 1)$ does not look like marginal utility of a good times the quantity of that good. However, this term does indeed have such an interpretation; we can show that the term in the PVIC is nothing but the product of money balances and its marginal utility.

To see this, note that from the bargaining problem and (20), $S_b(q) \equiv u(q) - g(q, Z)$ is the surplus of the buyer and therefore $S_b'(q) \equiv u'(q) - g_q(q, Z)$ is the marginal surplus of the buyer. Moreover, money has no use in the DM unless the household is a buyer, which occurs with probability σ. Thus, the marginal utility of money can be expressed as $\sigma S_b'(q)\partial q/\partial m$. From (20) and (25), we have $m = g(q, Z)/\beta\chi$ and $\partial q/\partial m = \beta\chi/g_q(q, Z)$. Combining these, we obtain precisely the term in question in the PVIC. One may argue this looks like a MIU model, which would have a term mU_m in the PVIC. In our context, though, the marginal utility of money is linked to the deep frictions underlying monetary exchange.

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[6] In the CCK model, for example, instantaneous utility is defined over cash goods, credit goods, and labor, $u(c_1, c_2, l)$, and the PVIC takes the form $\sum_{t=0}^{\infty} \beta^t [u_{1t}c_{1t} + u_{2t}c_{2t} + u_{lt}l_t] = A_0$, with A_0 a function of initial money and bonds. See Chari and Kehoe (1999, p. 1676-1686) for more discussion of optimal taxation problems in general.

If $\sigma = 0$, the DM shuts down and this collapses to the usual CCK PVIC *in a real model.* That is, the model collapses not to the CCK monetary (cash-credit) economy, but to a purely real model. This is a manifestation of the "dichotomy" result the LW model displays that Aruoba and Wright (2003) pointed out. The inflation rate in the LW model does not affect CM allocations at all. In terms of the implications of the dichotomy result for the optimal financing of government spending, inflation has no ability to raise any resources for the government.[7]

Because we restrict attention to only monetary equilibria, we require that the Ramsey allocations satisfy restriction (42), which we refer to as the zero-lower-bound (ZLB) constraint. CCK show that in their model, the ZLB constraint always holds with equality under the solution of the Ramsey problem obtained by *dropping* the ZLB constraint; in other words, in the CCK model the Friedman Rule ($R_t = 1$) can be shown analytically to always be the optimal policy. Thus, in the CCK model it turns out the ZLB constraint is redundant regardless of the parameterization of the model. This is not the case in general in our model and thus we need to impose it. As a technical point, note that the ZLB constraint is an inequality constraint. Thus, when solving for the dynamics of the model, we would need to rely on nonlinear global numerical approximations to handle the occasionally binding constraint. In practice, though, for a very important parameterization of interest of the model, it turns out that the ZLB constraint can be shown to be slack — in fact, that it is always satisfied with strict inequality. We discuss this further below, and it is one of our main results.

We assume the Ramsey planner is able to commit at time zero to a policy for $t \geq 1$. We thus sidestep here the potentially interesting issue of time-inconsistency in this model. The Ramsey problem is thus to choose $\{X_t, H_t, q_t\}$ to maximize

$$E_0 \max \sum_{t=0}^{\infty} \beta^t \left[U(X_t) - AH_t + \sigma \left(u(q_t) - c(q_t) \right) \right] \tag{44}$$

subject to the resource constraint

$$X_t + G_t = Z_t H_t, \tag{45}$$

the PVIC (43), and the ZLB constraint (42), taking as given $\{G_t, Z_t\}$.

4 Optimal Policy in Basic Model

One of our central results is that for a range of values for θ, the optimal nominal interest rate is positive. We can establish this analytically for the case $\theta = 1$, which we do next. The case $\theta = 1$ is an especially important one because Rocheteau and Wright (2005) show that for this case,

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[7] When we proceed to study the AWW model in Section 5, in which the dichotomy is broken, inflation has effects on CM variables as well.

bargaining yields the same outcomes as if there were competitive forces in the DM, making DM trades look less non-standard from the point of view of modern DGE theory. For $\theta < 1$, analytical solutions are not as easy to obtain, and we resort to numerical solutions. With numerical solutions in hand, we then proceed to show that optimal inflation is very stable in the face of shocks, in contrast to the standard results of CCK and the subsequent Ramsey literature.

4.1 Optimal Positive Nominal Interest Rate when $\theta = 1$

The Friedman Rule is not optimal when $\theta = 1$, as we now show:

Proposition 2. ***(Optimal Deviation from the Friedman Rule in Basic Model)*** *If $\theta = 1$, the optimal policy features a strictly positive net nominal interest rate in every period $t \geq 1$.*

Proof. Let ξ be the multiplier on the PVIC (43) in the Ramsey problem, and consider the Ramsey problem with the ZLB constraint dropped. The first-order condition of this problem with respect to q_t for $t \geq 1$ is

$$\sigma\left[u'(q_t) - c_q(q_t, Z_t)\right] + \xi\sigma\left[g_q(q_t, Z_t)\left(\frac{u'(q_t)}{c_q(q_t, Z_t)} - 1\right) + g(q_t, Z_t)\left(\frac{u''(q_t)g_q(q_t, Z_t) - u'(q_t)g_{qq}(q_t, Z_t)}{[g_q(q_t, Z_t)]^2}\right)\right] = 0. \tag{46}$$

With $\theta = 1$, we have that $g(q, Z) = c(q, Z) = q/Z$, so this FOC simplifies considerably,

$$u'(q_t) - c_q(q_t, Z_t) = -\left(\frac{\xi}{1+\xi}\right) q_t u''(q_t). \tag{47}$$

Because u is strictly concave, the multiplier $\xi > 0$ under the Ramsey allocation, and of course $q_t > 0$ $\forall t$, the right hand side is strictly positive. This implies $u'(q_t) > c_q(q_t, Z_t)$, which in turn implies

$$\sigma\frac{u'(q_t)}{g_q(q_t, Z_t)} + 1 - \sigma > 1, \tag{48}$$

imposing $g_q(q, Z) = c_q(q, Z) = 1/Z$ because $\theta = 1$. But this implies, by the equilibrium condition (38), that $R_t > 1$, so we have established that the Friedman Rule is not optimal. $\square$

If taxation is lump-sum, the PVIC is not a constraint on the Ramsey problem, hence $\xi = 0$ and the optimal allocation involves $u'(q_t) - c_q(q_t, Z_t) = 0$, which we know from LW achieves the socially-efficient q^*, defined by $u'(q^*) = c_q(q^*, Z)$). However, when government spending must be financed via non-lump-sum taxes, $\xi > 0$, so q^* cannot be achieved since $u'(q) > c_q(q, Z)$.

Deviations from the Friedman Rule have been obtained in other Ramsey models, as well. For example, Schmitt-Grohe and Uribe (2004a) show that a positive nominal interest can tax producers' monopoly profits, and Chugh (2006b) shows that it can tax monopolistic labor suppliers' rents. We know from Ramsey theory that taxing rents is optimal because it is non-distorting. However, the deviations from the Friedman Rule in Schmitt-Grohe and Uribe (2004a) and Chugh (2006b) are instances of the Ramsey planner using a positive nominal interest rate to *indirectly* tax some rent — in neither case is money the ultimate object the Ramsey planner wants to tax.

In contrast, in our environment, inflation *directly* taxes the rent that the Ramsey planner wants to seize, which is the rent associated with money. Money has a rent in our model because without it, certain trades simply could not occur, which would decrease welfare. A household chooses to hold money with the anticipation of being a buyer in the next DM. A household would never choose to hold money unless $S_b(q) = u(q) - g(q, Z) \geq 0$, which can be interpreted as the rent that money-holders enjoy. It is precisely this rent that the Ramsey planner wants to tax, and the inflation tax is the most obvious way of doing this.

One may still wonder, though, if there is another instrument that, if the Ramsey planner had it available and were to use it, would reinstate the optimality of the Friedman Rule. Following the logic of Schmitt-Grohe and Uribe (2004a) and Chugh (2006b), such an instrument would seemingly need to be a *direct* means of taxing DM activity. A natural candidate, then, is a sales tax in the DM. However, we show in Appendix B that allowing for a DM sales tax in what seems to be a straightforward way does not admit a Ramsey equilibrium at all.[8] This result implies that the sub-optimality of the Friedman Rule we have documented is not sensitive to the inclusion of at least this tax instrument. Admittedly, this is only one candidate alternative tax to consider, although seemingly a very natural one — but it does not restore the Friedman Rule. Even if it had restored the Friedman Rule, a practical concern is that anonymity in the DM would preclude the government from being able to enforce tax collections. This latter issue, so central to the essentiality of money, is one that any alternative taxation mechanism would have to confront. The inflation tax at least has the virtue that it gets around this anonymity problem.

Left to still consider is the quantitative degree of the departure from the Friedman Rule. The rent-seizing argument would suggest that the optimal inflation rate should be one that confiscates the entire rent, but this would imply $q = 0$. Thus, the optimal inflation rate must balance the motive to seize the money rent versus pushing q too low. Our numerical results, presented next, seem to confirm this intuition.

4.2 Numerical Results

4.2.1 Parameterization

Before presenting numerical results, we briefly describe the parameterization of the model. Our benchmark parameterization is that of LW when $\theta = 1$, so that the buyer has all the bargaining

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[8] We establish this result by assuming, as is typical in the Ramsey literature, that the first-order conditions characterize the solution to the Ramsey problem. As is well-known, however, this need not be because it cannot be shown that the Ramsey problem is a concave programming problem.

power in the DM, but, as mentioned above, we will consider $\theta < 1$ as well. The DM utility function is

$$u(q) = \frac{(q+b)^{1-\eta} - b^{1-\eta}}{1-\eta}, \tag{49}$$

with $\eta = 0.266$ and $b = 0.0001$, which is a parameter that forces $u(0) = 0$, which can occur in the DM if a household does not meet another agent with whom to trade. The probability that a household is either a buyer or a seller in the DM is $\sigma = 0.311$. The cost of production in the DM is $c(q, Z) = q/Z$. Thus, technology affects production in both the CM and the DM. In the CM, instantaneous utility is $U(X) - AH$, with $U(X) = B \ln X$, and $B = 2.13$ from LW's calibration. The parameter A is normalized to $A = 1$.

The exogenous government spending and TFP processes each evolve as an AR(1) in logs,

$$\ln G_{t+1} = (1-\rho_G)\ln \bar{G} + \rho_G \ln G_t + \epsilon^G_{t+1}, \tag{50}$$

$$\ln Z_{t+1} = \rho_Z \ln Z_t + \epsilon^Z_{t+1}, \tag{51}$$

with $\epsilon^G \sim N(0, \sigma^2_{\epsilon^G})$ and $\epsilon^Z \sim N(0, \sigma^2_{\epsilon^Z})$. We calibrate $\bar{G} = 0.94$ so that government purchases constitute about 18 percent of total output in steady-state.[9] In line with Schmitt-Grohe and Uribe (2004b) and the RBC literature, we set the parameters of the stochastic processes $\sigma_{\epsilon^G} = 0.031$, $\sigma_{\epsilon^Z} = 0.007$, $\rho_G = 0.9$, and $\rho_Z = 0.82$. With these volatility parameters, our model has a standard deviation of government purchases of about 7 percent of the mean level of government spending, and the volatility of total output is about 1.8 percent, both in line with data. The persistence parameters of the exogenous processes are for an annual calibration, thus we set the annual subjective discount factor $\beta = 0.966$, which delivers an annual real interest rate of about 3.5 percent. Finally, we choose the level of steady-state government debt, an object not pinned down by the model, so that it is 45 percent of steady-state output, consistent with the parameterizations of CCK and Schmitt-Grohe and Uribe (2004b).

4.2.2 Ramsey Steady-State

In Section 4.1, we established that the Friedman Rule is not optimal when $\theta = 1$. Obtaining analytic solutions for $\theta < 1$ is not as easy, so we study the optimal steady-state policy for this case numerically. To do so, we impose steady-state on the first-order conditions for $t > 0$ of the Ramsey problem described in Section 3 and numerically solve the resulting non-linear system. The steady-state nominal interest rate and labor income tax rate that supports the Ramsey allocation are then determined from (38) and (35), respectively.

Figure 1 shows the steady-state Ramsey policy and key allocation variables, along with the multiplier ι on the ZLB constraint (lower right panel), as functions of θ. At $\theta = 1$, the optimal

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[9] Total output takes into account both CM and DM output: $\sigma M/P + ZH$.

nominal interest rate is about 2 percent at an annual rate; the associated optimal inflation rate is thus -1.6 percent, higher than the Friedman rate of deflation, which would be -3.4 percent in our model. As θ falls below unity, the optimal nominal interest rate falls. Because seignorage revenue (not shown) falls along with it, the government's revenue shortfall must be made up with the labor tax, causing the labor tax rate to rise, as the upper middle panel of Figure 1 shows. The associated responses of the allocation variables X, H, and q are easy to understand as well. As the labor income tax rate rises with the fall in θ, hours worked and hence consumption in the CM decline.

As θ falls below unity, the optimal nominal interest rate falls. This is due to the hold up problem associated with holding money when $\theta < 1$. Specifically, when $\theta < 1$, the buyer does not get the full benefit from the match and this reduces the incentives for him to hold money, causing the equilibrium q to fall. Realizing this, the Ramsey planner reduces the inflation tax, balancing his desire to tax the buyer's surplus with the desire to reduce the effects of the holdup problem.

If θ falls far enough, however, the ZLB constraint binds, making the Friedman Rule the optimal policy. For our calibration, the ZLB constraint binds if $\theta \in (0, 0.62)$, as can be seen by the fact that the multiplier ι is positive and the net nominal interest rate is zero over that interval. The kink when the ZLB constraint binds leads to kinks in the labor tax rate and allocations as well.

In Figure 2, we try to shed some light on why the ZLB constraint eventually binds as θ becomes sufficiently small. In Figure 2, we plot the allocations and implied R and τ^h that emerge from the Ramsey problem with the ZLB constraint (42) dropped. The results for $\theta \in (0.62, 1)$ are of course identical to Figure 1 because in that region the ZLB constraint did not bind anyway. With the ZLB constraint dropped and $\theta \in (0, 0.62)$, we see that the Ramsey planner would like to implement, if it were consistent with monetary equilibrium, a negative net nominal interest rate, apparently to boost q. Of course, deflation faster than the Friedman Rule is inconsistent with monetary equilibrium. Hence, the Friedman Rule is optimal but does not achieve the first-best for $\theta < 1$: this finding is consistent with LW.

4.2.3 Ramsey Dynamics: Optimal Inflation Stability

We now turn to the dynamics of the Ramsey policy, which reveals our second central result. As we alluded to when discussing the ZLB constraint in Section 3, in general solving for the dynamics of the model requires using a method that handles occasionally-binding constraints, which the ZLB constraint is. However, we have proven that with $\theta = 1$ the optimal nominal interest rate is always strictly positive, meaning the ZLB constraint is always slack. As we mentioned in Section 4.1, there is special interest in the case $\theta = 1$ anyway because it yields outcomes consistent with competitive, as opposed to pure bargaining, trades in the DM.

To study dynamics for the case $\theta = 1$, then, we approximate decision rules of the model

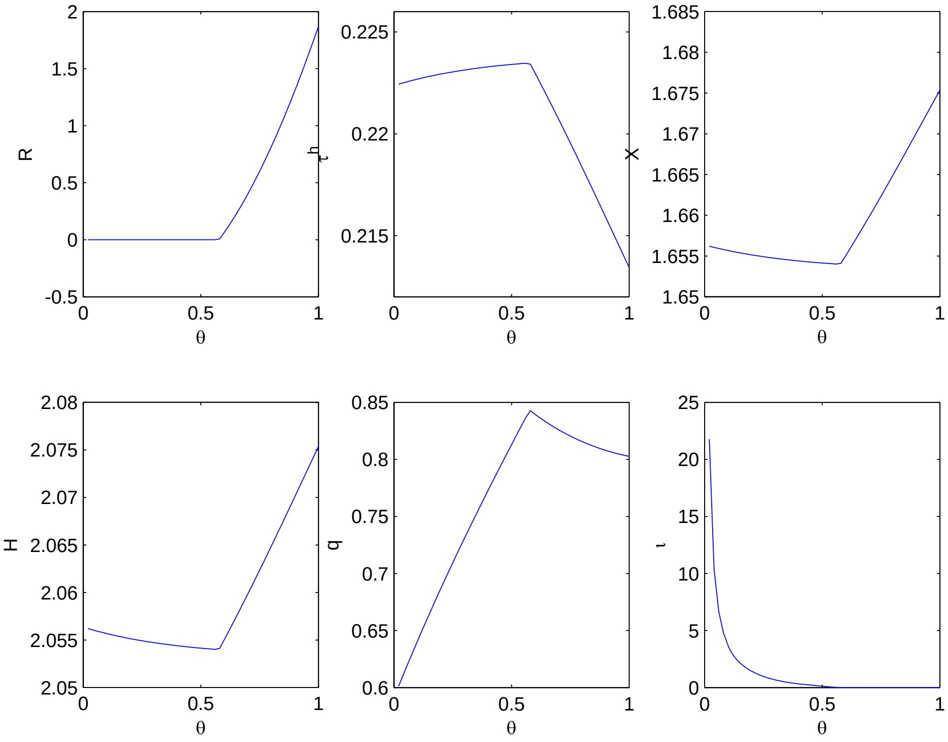

Figure 1: Ramsey steady-state policy and allocation as a function of θ with the ZLB constraint. Nominal interest rate reported in annual percentage points. Lower right panel shows the Lagrange multiplier ι on the ZLB constraint.

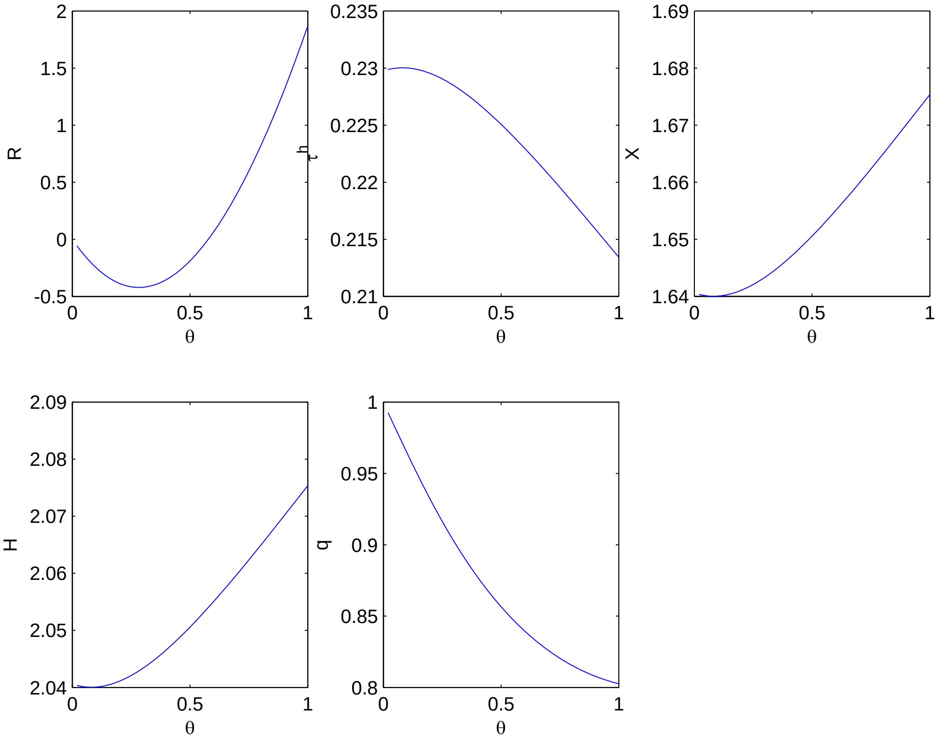

Figure 2: Ramsey steady-state policy and allocation as a function of θ without the ZLB constraint. Nominal interest rate reported in annualized percentage points.

Variable	Mean	Std. Dev.	Auto corr.	Corr(x,Y)	Corr(x,Z)	Corr(x,G)
π	-1.5925	0.2662	0.8055	0.5851	1	0.0074
τ^h	0.2135	0.0001	0.6443	-0.0145	-0.0278	-0.0027
R	1.8839	0.3933	0.8054	-0.5846	-0.9992	-0.0075
q	0.8026	0.0358	0.8055	0.5851	1	0.0074
Y	2.2755	0.0412	0.8370	1	0.7431	0.6699
X	1.6753	0.0199	0.8055	0.5851	1	0.0074
H	2.0755	0.0281	0.8748	0.7011	-0.1610	0.9854

Table 1: Simulation-based moments, using first-order approximation, in the bargaining version of the LW economy and $\theta = 1$. Driving processes are G_t and Z_t. π and R reported in annualized percentage points.

by linearizing in levels the Ramsey first-order conditions (with the ZLB constraint dropped) for time $t > 0$ around the non-stochastic steady-state of these conditions.[10] Thus, we ignore any transition from an arbitrary initial policy to the long-run policy. Our numerical method is our own implementation of the perturbation algorithm described by Schmitt-Grohe and Uribe (2004c). As in Khan, King, and Wolman (2003) and others, we assume that the initial state of the economy is the asymptotic Ramsey steady-state. Throughout, we assume that the first-order conditions of the Ramsey problem are necessary and sufficient and that all allocations are interior. We also point out that because we assume full commitment on the part of the Ramsey planner, the use of state-contingent inflation is not a manifestation of time-inconsistent policy. The "surprise" in surprise inflation is due solely to the unpredictable components of government spending and technology, and not due to a retreat on past promises.

We conduct 1000 simulations of 500 periods each and discard the first 100 periods. For each simulation, we then compute first and second moments and report the averages of these moments over the 1000 simulations.

Table 1 presents simulation-based moments for the key allocation and policy variables when the driving forces of the economy are both technology shocks and government spending shocks. We start with this case because it is the case often focused on in the existing Ramsey literature. Below, we decompose these dynamics into those induced by each of the shocks in isolation.

The first three rows of Table 1 show the dynamics of realized inflation, the labor income tax rate, and the net nominal interest rate under the Ramsey policy. We hone in first on the result that the optimal inflation rate is quite smooth over time, with a deviation of about only about 25 basis points (at an annual rate) around a mean deflation rate of 1.6 percent. The very stable

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[10] In Appendix D, we also present results from a second-order approximation to the decision rules. The results are virtually identical to the first-order-accurate results. This echoes the findings of Schmitt-Grohe and Uribe (2004b) and Chugh (2006a) that, for shocks of business cycle magnitude, first-order approximations of dynamic Ramsey allocations are almost indistinguishable from second-order approximations, in terms of both allocation variables and policy variables.

inflation rate is in sharp contrast to the extremely volatile optimal inflation rate first found by CCK in a flexible-price Ramsey model, and recently verified in, among others, the flexible-price versions of Schmitt-Grohe and Uribe (2004a, 2004b), Siu (2004), and Chugh (2006a, 2006b).[11] In all of these models, ex-post inflation imposes very small costs, but carries a welfare benefit because it allows the government to vary the real returns on its nominal government debt. In existing flexible-price Ramsey models based on fundamentally Walrasian models, then, the insurance value of state-contingent inflation is quantitatively much more valuable than the distortions caused by inflation, making the optimal inflation rate quite volatile over time.

With money essential, this result is overturned. A change in P_t changes the relative price of DM and CM goods, affecting the composition of consumption for the household. The welfare costs of distorting the composition of consumption are large enough to almost cancel out the insurance value of state-contingent inflation. The result is that realized inflation remains essentially constant at its steady-state level. Other Ramsey models make the prediction that inflation stability is optimal, for example, most notably Schmitt-Grohe and Uribe (2004b), Siu (2004) and Chugh (2006b). However, their models all rely on nominal rigidities to deliver the result; we emphasize that in our model, despite prices being fully flexible, Ramsey inflation is very stable. The real frictions underlying monetary exchange are behind our result.

Another perhaps noteworthy feature of inflation dynamics is that it displays high persistence. In the benchmark CCK model without capital, inflation persistence is virtually zero, no matter how persistent are the driving shocks. Chugh (2006a) shows that adding habit persistence or capital generates optimal inflation persistence, but clearly here we have that result with neither of these features. Further experiments reveals that in the model here, inflation persistence is inherited from the persistence of the underlying shocks, something which does not occur in a baseline CCK model without capital. It may be interesting to explore this issue further.

Finally, in line with the findings in earlier flexible-price Ramsey model, the labor income tax rate moves very little over time, exhibiting a standard deviation of only 0.01 percent around a mean of 21.35 percent. The negligible variations in τ^h are driven by the usual consumption-smoothing motive as spelled out in, say, Barro (1979).

Tables 2 and 3 present simulation-based moments when the economy is hit by shocks to either only government consumption or only technology. We perform this decomposition to demonstrate

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[11] From their simulation experiments, CCK report a mean inflation rate of -0.44 percent with a standard deviation of 19.93; Schmitt-Grohe and Uribe (2004a) report a mean inflation rate of -3.39 percent with a standard deviation of 7.47 percent; Siu (2004) reports a mean inflation rate of -2.59 percent with a standard deviation of 5.08 percent; and Chugh (2006b) reports a mean inflation rate of -4.01 percent with a standard deviation of 6.96 percent. Each of these models is calibrated in a slightly different way from the others, but the general result that comes through is clear: with flexible-prices, the Ramsey inflation rate is quite volatile.

Variable	Mean	Std. Dev.	Auto corr.	Corr(x, Y)	Corr(x, Z)	Corr(x, G)
π	-1.5919	0	—	—	—	—
τ^h	0.2134	0	—	—	—	—
R	1.8717	0	—	—	—	—
q	0.8027	0	—	—	—	—
Y	2.0755	0.0277	0.8772	1	—	1
X	1.6754	0	—	—	—	—
H	2.0755	0.0277	0.8722	1	—	1

Table 2: Simulation-based moments,, using first-order approximation, in the bargaining version of the LW economy and $\theta = 1$. Driving process is G_t. π and R reported in annualized percentage points.

Variable	Mean	Std. Dev.	Auto corr.	Corr(x, Y)	Corr(x, Z)	Corr(x, G)
π	-1.5925	0.2662	0.8055	1	1	—
τ^h	0.2135	0.0001	0.6443	-0.0217	-0.0278	—
R	1.8839	0.3933	0.8054	-0.9989	-1	—
q	0.8026	0.0358	0.8055	1	1	—
Y	2.0754	0.0199	0.8055	1	1	—
X	1.6753	0.0199	0.8055	1	1	—
H	2.0754	0.0048	0.8055	-1	-1	—

Table 3: Simulation-based moments, using first-order approximation, in the bargaining version of the LW economy and $\theta = 1$. Driving process is Z_t. π and R reported in annualized percentage points.

that the "dichotomy" result in LW has implications for which shocks affect the dynamics of optimal policy. Simply put, the dichotomy result is that CM and DM allocations have nothing to do with each other unless production in each market depends on a common capital stock (as we introduce in Section 5). Technology shocks in our model affect both CM and DM production, but government consumption shocks affect only the CM. As Table 2 shows, shocks to government spending affect only CM hours and output; they do not affect inflation at all.

5 Model with Capital

Ramsey models of optimal fiscal and monetary policy have only recently begun considering how the presence of capital accumulation affects optimal policy.[12] Here, we add capital to our baseline model following AWW: we assume that capital is accumulated in the CM and used in production in both the CM and the DM. As AWW show, with capital productive in both markets, the LW and Aruoba and Wright (2003) "dichotomy" result, in which CM and DM allocations have nothing to do with each other, disappears. We proceed by briefly describing how the model is modified to accommodate capital and then present results.

5.1 Production

The critical change from the basic model is that capital is introduced as a factor of production in both the DM and CM. In the CM, this is done in the obvious way: production takes place according to a constant returns technology subject to TFP shocks, $Z_t F(K_t, H_t)$. Profit-maximization by firms in the CM leads to standard factor-price conditions, $w_t = Z_t F_H(K_t, H_t)$ and $r_t = Z_t F_K(K_t, H_t)$.

In the period-t DM, sellers use the capital they have, which is K_{t+1} according to our timing convention.[13] As explained in AWW, this amounts to modifying the cost function to include capital as $c(q_t, K_{t+1}, Z_t)$, with $c_Z < 0$ as before.

5.2 Households

The household CM budget constraint modifies in the obvious way,

$$P_t x_t + P_t \left[k_{t+1} - (1-\tau_t^k)(r_t - \delta)k_t\right] + m_t + b_t = P_t w_t (1-\tau_t^h)h_t + m_{t-1} + R_{t-1}b_{t-1}, \tag{52}$$

where k_t is the household's capital holdings at the start of period t, r_t is the rental rate on capital, δ is its deprecation rate, and τ_t^k is the tax rate on capital income. The value functions in the CM and the DM now of course include the household's capital holdings. The household's CM problem is

$$W_t\left(m_{t-1}, b_{t-1}, k_t, S_t\right) = \max_{x_t, h_t, m_t, b_t, k_{t+1}} \left\{U(x_t) - Ah_t + V_t(m_t, b_t, k_{t+1}, S_t)\right\} \tag{53}$$

subject to (52).

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[12] Chugh (2006a) shows that the presence of capital accumulation in an otherwise-standard flexible-price Ramsey model dramatically increases the persistence of the optimal inflation rate compared to the baseline CCK model. The results of Schmitt-Grohe and Uribe (2005) show that when other frictions and rigidities are considered along with capital accumulation, this result can be mitigated.

[13] Specifically, with the CM convening before the DM, households exit the period-t CM with K_{t+1} units of capital, which is used in both period-t DM production and period-$t+1$ CM production.

The first order conditions of this problem are exactly as they were in the basic model, except of course a new condition for capital accumulation given by

$$\frac{A}{P_t w_t (1-\tau_t^h)} = V_{k,t}(m_t, b_t, k_{t+1}, S_t). \tag{54}$$

The results regarding the degenerate distribution of bonds and money readily extend to capital. $W_t(.)$ is still linear in all its arguments, with $W_{k,t}$ given by

$$W_{k,t}(m_{t-1}, b_{t-1}, k_t, S_t) = \frac{A\left[1 + (r_t - \delta)\left(1 - \tau_t^k\right)\right]}{w_t\left(1 - \tau_t^h\right)}. \tag{55}$$

In the DM, we again consider two pricing schemes: bargaining and price-taking. Unlike the basic model, price-taking cannot be obtained by a simple parameter restriction of the bargaining model and as such we briefly discuss it below. We use (q, d) to denote the terms of trade in the general discussion below and explain how it is obtained under each pricing scheme.

5.2.1 Household DM Problem

The DM problem for the household is still given by (9) after obvious changes in arguments and using (q_t^b, d_t^b) and (q_t^s, d_t^s) to represent the terms of trade from the viewpoint of the buyers and the sellers, respectively. This simplifies to

$$V_t(m_t, b_t, k_{t+1}, S_t) = \sigma\left\{u\left(q_t^b\right) - c\left(q_t^s, k_{t+1}, Z_t\right) - \beta\chi_t d_t^b + \beta\chi_t d_t^s\right\} + \beta E_t W_{t+1}\left(m_t, b_t, k_{t+1}, S_{t+1}\right). \tag{56}$$

using the linearity of $W_{t+1}(.)$. All we have to do to characterize the solution to the household's problem is to compute the partial derivatives of $V_t(.)$, which we do for both pricing schemes below. As in AWW, capital is only a productive input in this market and cannot be used as a medium of exchange.

5.2.2 Household DM Problem - Bargaining

The bargaining problem is still given by (15) with the obvious modifications regarding capital. The solution to the problem gives $d(m_t, k_{t+1}, S_t) = m_t$; $q = d(m_t, k_{t+1}, S_t)$ solves $\beta\chi_t m_t = g(q, k_{t+1}, Z_t)$, where m refers to the buyer's money holdings and k refers to the seller's capital stock. The function $g(q_t, k_{t+1}, Z_t)$ is a straightforward modification of the one in the basic model.

The only new partial derivative we need is $V_{t,k_{t+1}}$ which is given by

$$V_{k,t}\left(m_t, b_t, k_{t+1}, S_t\right) = -\sigma\left[c_q\left(q_t, k_{t+1}, Z_t\right)\frac{\partial q}{\partial k} + c_k\left(q_t, k_{t+1}, Z_t\right)\right] + \beta A E_t\left[\frac{1 + \left(r_{t+1} - \delta\right)\left(1 - \tau_{t+1}^k\right)}{w_{t+1}\left(1 - \tau_{t+1}^h\right)}\right] \tag{57}$$

Noting $\partial q/\partial k = -g_k/g_q$, defining

$$\gamma(q_t, K_{t+1}, Z_t) \equiv \frac{c_k(q_t, K_{t+1}, Z_t)g_q(q_t, K_{t+1}, Z_t) - c_q(q_t, K_{t+1}, Z_t)g_k(q_t, K_{t+1}, Z_t)}{g_q(q_t, K_{t+1}Z_t)}, \tag{58}$$

and combining the optimality conditions (4) and (54) with the envelope condition (57), we get the household's Euler equation for capital accumulation,

$$U'(X_t) = \beta E_t \left\{U'(X_{t+1}) \left[1 + (1 - \tau^k_{t+1})(r_{t+1} - \delta)\right]\right\} - \sigma\gamma(q_t, K_{t+1}, Z_t), \tag{59}$$

which shows that investment takes into account the fact that capital affects productivity in the DM as well as in the CM. The additional term related to the DM represents the seller's payoff from carrying k_{t+1} units of capital into the DM and producing q_t units, which occurs with probability σ. It is also easy to show that we get the usual condition linking the return on bonds to the real return on capital

$$E_t \left\{\frac{1}{\pi_{t+1}} \left[1 + (1 - \tau^k_{t+1})(r_{t+1} - \delta)\right]\right\} = R_t. \tag{60}$$

5.2.3 Household DM Problem - Price-Taking

An alternative to bargaining is price taking, in which buyers and sellers each take the price of a unit of good in the DM, $\tilde{p}$, as given and solve their respective demand and supply problems. The buyer's problem is

$$V^b(m_t, b_t, k_{t+1}, S_t) = \max_q \{u(q) + \beta E_t W_{t+1}(m_t - \tilde{p}q, b_t, k_{t+1}, S_{t+1})\} \tag{61}$$

subject to $\tilde{p}q \leq m_t$. In equilibrium this constraint binds, and we have $q_t = M_t/\tilde{p}$. The seller's problem is

$$V^s(m_t, b_t, k_{t+1}, S_t) = \max_q \{-c(q, k_{t+1}, Z_t) + \beta E_t W_{t+1}(m_t + \tilde{p}q, b_t, k_{t+1}, S_{t+1})\}, \tag{62}$$

with the first order condition $c_q(q_t, k_{t+1}, Z_t) = \beta\tilde{p}\chi_t$. The two envelope conditions we need to solve the problem of the household are given by

$$V_{m,t}(m_t, b_t, k_{t+1}, S_t) = (1 - \sigma)\beta\chi_t + \sigma\beta\chi_t \frac{u'(q_t)}{c_q(q_t, k_{t+1}, Z_t)} \tag{63}$$

$$V_{k,t}(m_t, b_t, k_{t+1}, S_t) = \beta A E_t \left[\frac{1 + (r_{t+1} - \delta)\left(1 - \tau^k_{t+1}\right)}{w_{t+1}\left(1 - \tau^h_{t+1}\right)}\right] - \sigma c_k(q_t, k_{t+1}, Z_t). \tag{64}$$

It is also straightforward to show that the analog of the Fisher-like condition (33) from our baseline model is

$$R_t = \sigma \frac{u'(q_t)}{g_q(q_t, K_{t+1}, Z_t)} + 1 - \sigma, \tag{65}$$

which we need in order to write the ZLB constraint on the Ramsey problem.

5.3 Government

The government now collects revenue from capital income and labor income taxation, along with money creation and debt issuance. Its CM flow budget constraint is thus

$$M_t + B_t + P_t w_t \tau_t^h H_t + P_t \tau_t^k (r_t - \delta) K_t = P_t G_t + M_{t-1} + R_{t-1} B_{t-1}. \tag{66}$$

5.4 Equilibrium

Combining the relevant conditions derived in this section with the ones that were unchanged from the basic model, we now list the equilibrium conditions we will use in writing the Ramsey problem.

5.4.1 Bargaining

Given policy variables $\{\tau_t^h, \tau_t^k, R_t\}_{t=0}^{\infty}$, the technology realization $\{Z_t\}_{t=0}^{\infty}$, the government spending realization $\{G_t\}_{t=0}^{\infty}$, and initial condition (M_0, B_0, K_0), equilibrium is a set of processes $\{q_t, B_t, M_t, K_t, X_t, H_t, P_t\}_{t=0}^{\infty}$ satisfying

$$U'(X_t) = \frac{A}{(1-\tau_t^h) Z_t F_H(K_t, H_t)}, \tag{67}$$

$$\beta M_t E_t \left[\frac{U'(X_{t+1})}{P_{t+1}} \right] = g(q_t, K_{t+1}, Z_t), \tag{68}$$

$$\frac{U'(X_t)}{P_t} = \beta \left[\sigma \frac{u'(q_t)}{g_q(q_t, K_{t+1}, Z_t)} + 1 - \sigma \right] E_t \left[\frac{U'(X_{t+1})}{P_{t+1}} \right] - \sigma \gamma(q_t, K_{t+1}, Z_t), \tag{69}$$

$$R_t = \sigma \frac{u'(q_t)}{g_q(q_t, K_{t+1}, Z_t)} + 1 - \sigma, \tag{70}$$

$$E_t \left\{ \frac{p_t}{p_{t+1}} \left[1 + (1 - \tau_{t+1}^k)(Z_{t+1} F_K(K_{t+1}, H_{t+1}) - \delta) \right] \right\} = R_t \tag{71}$$

$$X_t + G_t + K_{t+1} = Z_t F(K_t, H_t) + (1-\delta) K_t, \tag{72}$$

$$M_t + B_t + P_t Z_t F_H(K_t, H_t) \tau_t^h H_t + P_t \tau_t^k [Z_t F_K(K_t, H_t) - \delta] K_t = P_t G_t + M_{t-1} + R_{t-1} B_{t-1} \tag{73}$$

In the bargaining version, real money balances can be expressed as

$$\frac{M_t}{P_t} = \frac{g(q_t, K_{t+1}, Z_t)}{U'(x_t)} \left[\sigma \frac{u'(q_t)}{g_q(q_t, K_{t+1}, Z_t)} + 1 - \sigma \right], \tag{74}$$

and the ZLB constraint is

$$\sigma \left(\frac{u'(q_t)}{g_q(q_t, K_{t+1}, Z_t)} - 1 \right) \geq 0. \tag{75}$$

5.4.2 Price-Taking

Given policy variables $\{\tau_t^h, \tau_t^k, R_t\}_{t=0}^{\infty}$, the technology realization $\{Z_t\}_{t=0}^{\infty}$, the government spending realization $\{G_t\}_{t=0}^{\infty}$, and initial condition (M_0, B_0, K_0), equilibrium is a set of processes $\{q_t, B_t, M_t, K_t, X_t, H_t, P_t\}_{t=0}^{\infty}$ satisfying (67), (71), (72), and (73) along with

$$\beta M_t E_t \left[\frac{U'(X_{t+1})}{P_{t+1}}\right] = q_t c_q(q_t, K_{t+1}, Z_t), \tag{76}$$

$$\frac{U'(X_t)}{P_t} = \beta \left[\sigma \frac{u'(q_t)}{c_q(q_t, K_{t+1}, Z_t)} + 1 - \sigma\right] E_t \left[\frac{U'(X_{t+1})}{P_{t+1}}\right] - \sigma c_k(q_t, K_{t+1}, Z_t), \tag{77}$$

$$R_t = \sigma \frac{u'(q_t)}{q_t c_q(q_t, K_{t+1}, Z_t)} + 1 - \sigma. \tag{78}$$

In the price-taking version, real money balances can be expressed as

$$\frac{M_t}{P_t} = \frac{q_t c_q(q_t, K_{t+1}, Z_t)}{U'(X_t)} \left[\sigma \frac{u'(q_t)}{c_q(q_t, K_{t+1}, Z_t)} + 1 - \sigma\right], \tag{79}$$

and the ZLB constraint is

$$\sigma \left(\frac{u'(q_t)}{c_q(q_t, K_{t+1}, Z_t)} - 1\right) \geq 0. \tag{80}$$

6 Ramsey Problem in Model with Capital

As we show in Appendix C, we can state the analog of Proposition 1 for the model with capital:

Proposition 3. *The allocations in a monetary equilibrium in the model with capital satisfy the resource constraint (72), the ZLB constraint (75) for the bargaining model and (80) for the price-taking model, and the present-value implementability constraint (PVIC),*

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[U'(X_t)X_t - AH_t + \sigma g(q_t, K_{t+1}, Z_t) \left(\frac{u'(q_t)}{g_q(q_t, K_{t+1}, Z_t)} - 1\right) - \sigma \gamma(q_t, K_{t+1}, Z_t) K_{t+1}\right] = U'(X_0) A_0. \tag{81}$$

for the bargaining model and

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[U'(X_t)X_t - AH_t + \sigma q_t c_q(q_t, K_{t+1}, Z_t) \left(\frac{u'(q_t)}{c_q(q_t, K_{t+1}, Z_t)} - 1\right) - \sigma c_k(q_t, K_{t+1}, Z_t) K_{t+1}\right] = U'(X_0) A_0. \tag{82}$$

for the price-taking model where the constant A_0 depends on $M_0, B_0,$ and K_0,

$$A_0 = \frac{M_{-1} + R_{-1} B_{-1}}{P_0} + \left[1 + (1 - \tau_0^k)(Z_0 F_K(K_0, H_0) - \delta)\right] K_0. \tag{83}$$

With the introduction of capital, the term in the PVIC that is related to the DM is similar to the one we derived in the basic model, augmented by a new term, $-\sigma\gamma(.)K_{t+1}$ for the bargaining model and $-\sigma c_k(.)K_{t+1}$ for the price-taking model. As we argued above, the expressions multiplying K_{t+1}

give the DM return to holding capital in each version of the model. As such, the PVICs we derive in the model with capital conform to the insight about PVICs in general Ramsey problems.

The Ramsey problem is to choose $\{X_t, H_t, K_{t+1}, q_t\}$ to maximize

$$E_0 \max \sum_{t=0}^{\infty} \beta^t \left[U(x_t) - AH_t + \sigma\left(u(q_t) - c(q_t, K_{t+1})\right)\right] \tag{84}$$

subject to the resource constraint (72), the PVIC ((81) for the bargaining model or (82) for the price-taking model), and the ZLB constraint ((75) for the bargaining model or (80) for the price-taking model), taking as given $\{G_t, Z_t\}$.

7 Optimal Policy in Model with Capital

As was the case in the basic model, we are able to prove analytically that the optimal nominal interest rate is positive for the bargaining model with $\theta = 1$. For $\theta < 1$ and for the price taking model, we must resort to numerical methods.

7.1 Optimal Positive Nominal Interest Rate

We begin by proving that under bargaining we have an optimal deviation from the Friedman Rule if $\theta = 1$.

Proposition 4. ***(Optimal Deviation from the Friedman Rule in the Model with Capital under Bargaining)*** *Under bargaining, if $\theta = 1$, the optimal policy features a strictly positive net nominal interest rate in every period $t \geq 1$.*

Proof. The proof follows very closely the one for the basic model. Let ξ be the multiplier on the PVIC (81), and consider the Ramsey problem with the ZLB constraint dropped. The first-order condition of this problem with respect to q_t for $t \geq 1$ is

$$\sigma\left[u' - c_q\right] + \sigma\xi\left[g_q\left(\frac{u'}{g_q} - 1\right) + \frac{g}{[g_q]^2}(u''g_q - u'g_{qq})\right] - \sigma\xi\gamma_q = 0, \tag{85}$$

where arguments of function have been omitted. With $\theta = 1$, we have that $g(q, K, Z) = c(q, K, Z)$ and $\gamma(q, K, Z) = 0$, so this simplifies to

$$u' - c_q = -\left(\frac{\xi}{1+\xi}\right) c \left[\frac{u''c_q - u'c_{qq}}{[c_q]^2}\right]. \tag{86}$$

Because the multiplier $\xi > 0$ under the Ramsey allocation, u is strictly concave, $c_q > 0$, and $c_{qq} > 0$, the right hand side is strictly positive. This in turn implies $u' > c_q$, which in turn implies

$$\sigma \frac{u'(q_t)}{g_q(q_t, K_{t+1}, Z_t)} + 1 - \sigma > 1, \tag{87}$$

imposing $g_q(q_t, K_{t+1}, Z_t) = c_q(q_t, K_{t+1}, Z_t)$ because $\theta = 1$. But this implies, by the equilibrium condition (75), that $R_t > 1$. $\square$

The intuition is as in the basic model: the essentiality of money creates rents, which the Ramsey planner would like to tax. The inflation tax is the most direct way of taxing money. As in the basic model, we cannot prove this result if $\theta < 1$, but can show numerically that for large enough θ, the result also holds.

7.2 Numerical Results for the Bargaining Model

We now turn to numerical results for the model with capital under bargaining.

7.2.1 Parameterization

Our benchmark parameterization here follows AWW. The DM cost function is

$$c(q_t, K_{t+1}, Z_t) = \frac{1}{Z_t} q_t^{\kappa} K_{t+1}^{1-\kappa}, \tag{88}$$

The CM production function is standard Cobb-Douglas, $F(K, H) = K^{\alpha} H^{1-\alpha}$, with $\alpha = 0.288$. We keep the annual subjective discount factor $\beta = 0.966$, and set the annual depreciation rate to $\delta = 0.07$. We maintain the CM utility function $B \ln X - AH$. We keep the parameters governing the shocks to government spending and TFP the same as in Section 4.2.1.

We set the probability of a household finding a partner in the DM to $\sigma = 0.24$, set $B = 1.53$, $A = 3.89$ and $\kappa = 1.65$. In the DM utility function, here we set $\eta = 1$, so DM utility is essentially log. We keep $b = 0.0001$.

7.2.2 Ramsey Steady-State

Figure 3 shows how the Ramsey steady-state varies with θ when the ZLB constraint is left in place. As the upper left panel shows, the range of θ over which the ZLB constraint binds is clearly larger here than in Figure 1, but this is because the parameterization here is somewhat different. Clearly, the same message as in Section 4 comes through: when θ is low enough, the ZLB constraint binds and the Friedman Rule is optimal. But for large enough θ — in particular for the interesting case $\theta = 1$ — the Friedman Rule is not optimal. Here, not only is the Friedman Rule not optimal, but the optimal inflation rate (not shown) is actually positive, around 9.5 percent, in contrast to the small deflation that was optimal in the model without capital. In the interval $\theta \in (0.90, 1)$, the rise in the optimal inflation rate and hence nominal interest rate is quite steep, although we do not have any particular intuition for this quantitative feature of the Ramsey policy.

The upper middle panel shows that the labor income tax rate is roughly constant at about 40 percent over $\theta \in (0, 0.90)$, then falls somewhat for higher values of θ. The intuition is just as before: as the inflation tax begins generating revenue, the distortionary labor income tax can be reduced.

The capital income tax rate is negative for all values of θ, as the upper right panel of Figure 3 shows. The capital income subsidy in the range 40 to 50 percent that we find is large, but not out of line with capital subsidy rates found in other Ramsey studies. For example, Schmitt-Grohe and Uribe (2005, Table 2) find an optimal capital subsidy rate of 44 percent in their benchmark model featuring a host of real and nominal rigidities and report that it can be as high as 85 percent. The Ramsey planner subsidizes capital in our model because of the capital holdup problem that AWW identify. Because capital is needed in DM production but must be accumulated in the CM by only those households that will be sellers in the DM, capital is under-accumulated. Only the seller in a DM transaction incurs the cost of acquiring capital, but both the buyer and the seller share in the surplus from production in the DM. Capital accumulation is thus too low, and the Ramsey planner encourages capital formation with a capital subsidy. If meetings in the DM occurred with low probability (low σ) or if capital use were less intensive in DM production (low κ), the capital subsidy rate would be smaller. In Figure 4, we fix $\theta = 1$ and decompose the steady-state capital income tax rate along the σ and κ dimensions. The results confirm this intuition: the larger the number of meetings in the DM or the greater capital intensity is in DM production, the larger is the capital income subsidy.

7.2.3 Ramsey Dynamics: Optimal Inflation Stability

We begin again with quantitative results for the model with $\theta = 1$ and the ZLB constraint dropped from the Ramsey problem. As in the baseline model without capital, given $\theta = 1$ and the rest of our parameterization, simulations of the model here never resulted in violation of the ZLB constraint. Also as before, our simulation-based results are obtained by linearizing the Ramsey first-order conditions for $t > 0$ around the non-stochastic steady-state (computed numerically) of those conditions, and thus ignore transitions from arbitrary initial conditions. We once again are assuming the first-order conditions of the Ramsey problem are necessary and sufficient and that all allocations are interior.

Table 4 presents simulation-based moments for the Ramsey allocation and policy variables. To enhance comparability with the results from the model without capital, we use the same realizations of shocks to government purchases and TFP to obtain results here. As above, we focus first on inflation dynamics. Inflation is once again very stable, with a standard deviation of about 23 basis points around a mean of 9.5 percent (at an annual rate). This contrasts with the results of Chugh (2006a) and Schmitt-Grohe and Uribe (2005), who find that capital accumulation does nothing to

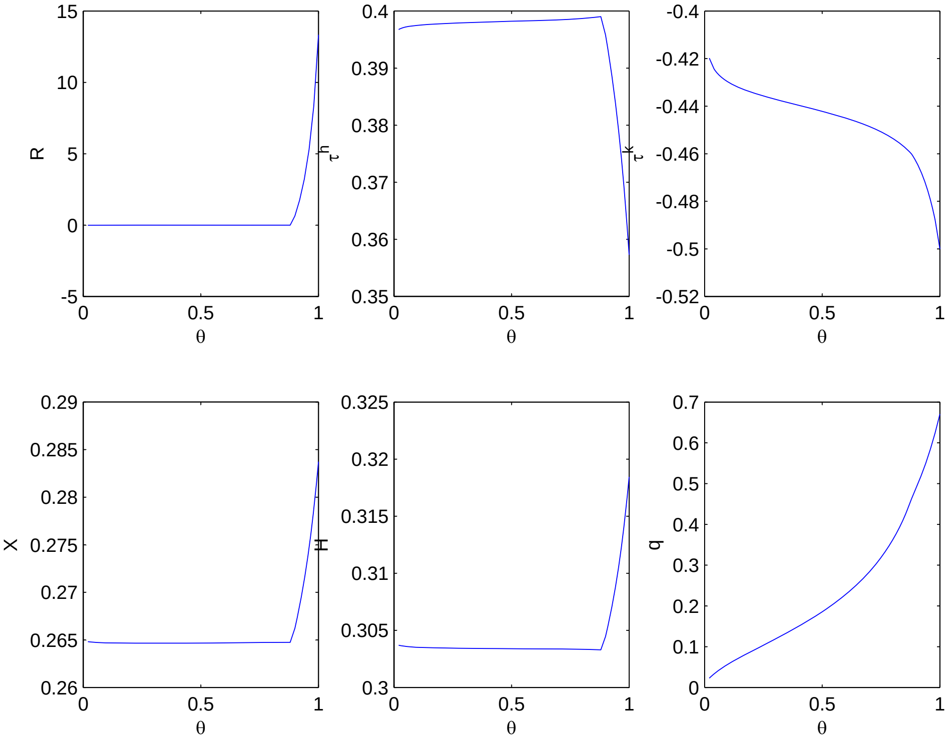

Figure 3: Ramsey steady-state policy and allocation as a function of θ with the ZLB constraint in the economy with capital. Nominal interest rate reported in annual percentage points.

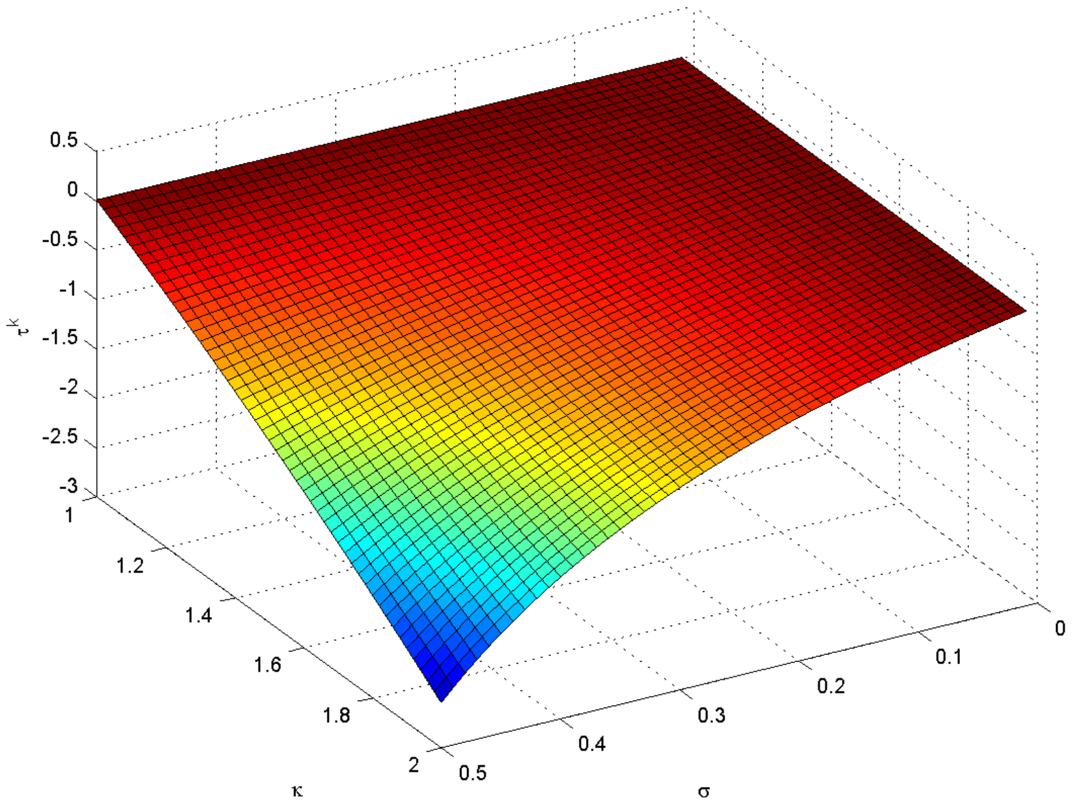

Figure 4: Optimal steady-state capital income tax rate as functions of κ and σ. Buyer bargaining power fixed at $\theta = 1$.

Variable	Mean	Std. Dev.	Auto corr.	Corr(x, Y)	Corr(x, Z)	Corr(x, G)
π	9.4861	0.2275	0.7172	-0.5755	-0.3276	-0.4351
τ^h	0.3574	0.0001	0.7472	-0.0198	-0.0258	-0.0008
τ^k	-0.5000	0.0000	—	—	—	—
R	13.3451	0.4437	0.8054	-0.8358	-0.9999	-0.0074
q	0.6706	0.0097	0.9209	0.7875	0.9315	0.0751
Y	0.5021	0.0122	0.8036	1	0.8361	0.5442
X	0.2837	0.0046	0.9389	0.5244	0.8469	-0.2635
I	0.1083	0.0070	0.6730	0.8300	0.8944	0.1041
H	0.3185	0.0067	0.7971	0.7526	0.3171	0.8384

Table 4: Simulation-based moments, using first-order approximation in the bargaining version of the AWW economy and $\theta = 1$. Driving processes are G_t and Z_t. π and R reported in annualized percentage points.

dampen the volatility of Ramsey inflation, compared to the fixed-capital CCK economy, when all markets are fundamentally Walrasian and money is not essential. The intuition for the optimality of inflation stability seems to be as in the model without capital: distorting the composition over time of DM consumption relative to CM consumption imposes welfare costs almost as large as the insurance value of state-contingent inflation, causing the Ramsey planner to largely refrain from varying inflation.

As in our baseline model, the labor income tax rate is very stable over time, with a standard deviation of just 0.01 percent around a mean of 35.74 percent. The capital income tax rate reported in Table 4 is the ex-ante tax rate, computed as in Chari and Kehoe (1999, p. 1708),

$$\tau_t^{k,e} = \frac{E_t\left[\left(\frac{\beta U'(x_{t+1})}{U'(x_t)}\right)\tau_{t+1}^k\left(F_K(K_{t+1}, H_{t+1}) - \delta\right)\right]}{E_t\left[\left(\frac{\beta U'(x_{t+1})}{U'(x_t)}\right)\left(F_K(K_{t+1}, H_{t+1}) - \delta\right)\right]}, \tag{89}$$

and our simulations show that the capital tax rate is constant at its steady-state level, a 50 percent subsidy. Chari and Kehoe (1999) also find that the ex-ante capital income tax rate is constant around its steady-state level, although in their model the steady-state level is zero for the usual reasons in the capital tax literature (for example, Chamley (1986)).

Tables 5 and 6 decompose the dynamics of the Ramsey solution into those arising from government consumption shocks and those arising from TFP shocks. Unlike in the model without capital, here we find that government consumption shocks alone do induce some variation in the inflation rate (compare the first rows of Table 2 and 5). With capital productive in both the CM and DM, fiscal shocks get transmitted across markets, leading to fluctuations in q, which implies that the policy supporting the Ramsey allocation has fluctuations in realized inflation. Technology shocks, which directly affect the DM, lead to larger volatility of inflation than do fiscal shocks (standard deviation of 0.2 vs. 0.1, comparing the first rows of Tables 5 and 6), but in any case these fluctuations are quite small.

Variable	Mean	Std. Dev.	Auto corr.	Corr(x,Y)	Corr(x,Z)	Corr(x,G)
π	9.4863	0.1066	0.8137	-0.9787	—	-0.9749
τ^h	0.3573	0.0001	0.7105	-0.0100	—	-0.0073
τ^k	-0.5000	0.0000	—	—	—	—
R	13.3402	0.0001	0.9694	0.0467	—	0.0460
q	0.6706	0.0008	0.9857	0.8232	—	0.8332
Y	0.5021	0.0068	0.8723	1	—	0.9998
X	0.2837	0.0013	0.8216	-0.9841	—	-0.9808
I	0.1083	0.0008	0.7473	0.8908	—	0.8826
H	0.3185	0.0058	0.8593	0.9989	—	0.9979

Table 5: Simulation-based moments, using first-order approximation in the bargaining version of the AWW economy and $\theta = 1$. Driving process is G_t. π and R reported in annualized percentage points.

Variable	Mean	Std. Dev.	Auto corr.	Corr(x,Y)	Corr(x,Z)	Corr(x,G)
π	9.4865	0.2032	0.6939	-0.4487	-0.3631	—
τ^h	0.3574	0.0001	0.7262	-0.0197	-0.0302	—
τ^k	-0.5000	0.0000	—	—	—	—
R	13.3452	0.4437	0.8054	-0.9952	-0.9999	—
q	0.6706	0.0097	0.9204	0.8971	0.9347	—
Y	0.5021	0.0102	0.7734	1	0.9955	—
X	0.2837	0.0044	0.9495	0.8343	0.8824	—
I	0.1083	0.0069	0.6718	0.9370	0.9001	—
H	0.3184	0.0036	0.6471	0.6494	0.5754	—

Table 6: Simulation-based moments, using first-order approximation in the bargaining version of the AWW economy and $\theta = 1$. Driving process is Z_t. π and R reported in annualized percentage points.

Variable	Mean	Std. Dev.	Auto corr.	Corr(x, Y)	Corr(x, Z)	Corr(x, G)
π	9.1732	0.2269	0.7169	-0.5830	-0.3303	-0.4410
τ^h	0.3254	0.0001	0.7437	-0.0292	-0.0346	-0.0042
τ^k	-0.1834	0.0000	—	—	—	—
R	13.0233	0.4751	0.8049	-0.8298	-0.9998	-0.0074
q	0.7719	0.0127	0.9521	0.7354	0.8711	0.0985
Y	0.4987	0.0122	0.8029	1	0.8301	0.5531
X	0.2800	0.0045	0.9378	0.5096	0.8439	-0.2718
I	0.1087	0.0070	0.6735	0.8305	0.8959	0.0992
H	0.3151	0.0067	0.7964	0.7585	0.3146	0.8434

Table 7: Simulation-based moments, using first-order approximation in the price-taking version of the AWW economy. Driving processes are G_t and Z_t. π and R reported in annualized percentage points.

Variable	Mean	Std. Dev.	Auto corr.	Corr(x, Y)	Corr(x, Z)	Corr(x, G)
π	9.1740	0.1048	0.8120	-0.9800	—	-0.9763
τ^h	0.3254	0.0001	0.7081	-0.0167	—	-0.0139
τ^k	-0.1834	0.0000	—	—	—	—
R	13.0169	0.0001	0.9693	0.0606	—	0.0601
q	0.7720	0.0013	0.9854	0.8199	—	0.8298
Y	0.4988	0.0067	0.8692	1	—	0.9998
X	0.2801	0.0013	0.8202	-0.9854	—	-0.9823
I	0.1087	0.0008	0.7443	0.8921	—	0.8841
H	0.3151	0.0057	0.8566	0.9990	—	0.9981

Table 8: Simulation-based moments, using first-order approximation in the price-taking version of the AWW economy. Driving process is G_t. π and R reported in annualized percentage points.

As in the model without capital, then, inflation stability characterizes the optimal policy and once again no appeal to nominal rigidities of any sort is needed.

7.3 Numerical Results for the Price-Taking Model

The story is virtually unchanged once we move to the price-taking version of the model with capital. Tables 7, 8, and 9 present moments of the dynamic Ramsey allocation and policy under price-taking. The steady-state inflation rate of about 9.2 percent is only slightly smaller than the 9.5 inflation rate in the bargaining version, and the volatility (or lack thereof) of inflation is also about the same. The level of the capital income tax subsidy is now lower, about 18 percent compared to 50 percent in the bargaining version. The reason for this result is that price-taking fixes the capital holdup problem inherent in bargaining, as AWW show. However, there still is a tendency to under-accumulate capital because the inflation tax, by taxing q, also indirectly taxes K. To mitigate this effect on K, the Ramsey planner still gives a subsidy to capital accumulation.

Variable	Mean	Std. Dev.	Auto corr.	Corr(x, Y)	Corr(x, Z)	Corr(x, G)
π	9.1736	0.2020	0.6945	-0.4537	-0.3667	—
τ^h	0.3254	0.0001	0.7280	-0.0272	-0.0377	—
τ^k	-0.1834	0.0000	—	—	—	—
R	13.0235	0.4751	0.8049	-0.9950	-0.9999	—
q	0.7719	0.0126	0.9515	0.8262	0.8764	—
Y	0.4987	0.0101	0.7725	1	0.9953	—
X	0.2800	0.0043	0.9494	0.8322	0.8815	—
I	0.1087	0.0069	0.6727	0.9384	0.9011	—
H	0.3151	0.0036	0.6481	0.6535	0.5783	—

Table 9: Simulation-based moments, using first-order approximation in the price-taking version of the AWW economy. Driving process is Z_t. π and R reported in annualized percentage points.

8 Conclusion

We view our work and results as a first step in taking more seriously the LW/AWW type of environment as a laboratory for studying policy questions. Our central findings are that the Friedman Rule is typically not the optimal long-run policy and that inflation fluctuates very little over time. These findings are opposite those of the workhorse CCK flexible-price Ramsey model. The presence of real frictions that give rise to valued money also provide completely different justification for a central bank's pursuit of inflation stability than the typically-invoked ones of nominal rigidities.

There are of course a number of ways one might want to modify our framework. Monopoly power in goods and labor markets are thought by many to be important realistic features. It would be straightforward to introduce monopoly power in the CM. The results of Schmitt-Grohe and Uribe (2004a) and Chugh (2006b) suggest that inflation in such an environment would be partly a direct tax on the money rent we identify and partly an indirect tax on producers' and labor suppliers' rents. It may be interesting to know quantitatively how these direct and indirect uses of the inflation tax interact.

Once one has monopoly power in the CM, one could go further in adding elements monetary policy makers often think are important, such as sticky prices and sticky wages. However, given our finding of optimal inflation stability (albeit around a non-zero average inflation rate), we do not see how the results could be very different with such features.

Pushing our first step in a different direction, another interesting issue to study may be the nature of and solution to the time-inconsistency problem of the Ramsey policy in this sort of environment. It is not clear how the time-consistency results of, say, Alvarez, Kehoe, and Neumeyer (2004) or Persson, Persson, and Svensson (2006), would extend to our environment.

A The Ramsey Problem in Basic Model

Using the household optimality conditions (27), (31), and (32) along with the equilibrium conditions, we now derive the present-value implementability constraint the Ramsey planner must respect. Begin as usual with the CM household flow budget constraint (we re-introduce time indices here),

$$P_tX_t + B_t + M_t = P_tw_t(1-\tau_t)H_t + M_{t-1} + R_{t-1}B_{t-1}. \tag{90}$$

To construct the present-value implementability constraint, begin by multiplying the flow budget constraint by $\beta^tU'(X_t)/P_t$ and summing from $t = 0..\infty$,

$$\begin{aligned} E_0\sum_{t=0}^{\infty}\beta^tU'(X_t)X_t + \sum_{t=0}^{\infty}\beta^tU'(X_t)\frac{B_t}{P_t} + \sum_{t=0}^{\infty}\beta^tU'(X_t)\frac{M_t}{P_t} = \\ \sum_{t=0}^{\infty}\beta^tU'(X_t)(1-\tau_t)w_tH_t + \sum_{t=0}^{\infty}\beta^tU'(X_t)\frac{M_{t-1}}{P_t} + \sum_{t=0}^{\infty}\beta^tU'(X_t)\frac{R_{t-1}B_{t-1}}{P_t}. \end{aligned} \tag{91}$$

We point out that, as usual in a dynamic Ramsey problem assuming commitment to the time-zero policy, any E_t terms that appear in intermediate expressions are eliminated by the law of iterated expectations because the entire implementability constraint is conditioned on the time-zero information set, hence the E_0. For ease of exposition, we therefore proceed dropping E_t operators that would appear in intermediate expressions as well as the E_0 operator because it is understood to be present in all subsequent expressions.

Substitute into the second term on the left-hand-side using expression (32) to get

$$\sum_{t=0}^{\infty}\beta^tU'(X_t)X_t + \sum_{t=0}^{\infty}\beta^{t+1}U'(x_{t+1})\frac{R_tB_t}{P_{t+1}} + \sum_{t=0}^{\infty}\beta^tU'(X_t)\frac{M_t}{P_t} = \tag{92}$$

$$\sum_{t=0}^{\infty}\beta^tU'(X_t)(1-\tau_t)w_tH_t + \sum_{t=0}^{\infty}\beta^tU'(X_t)\frac{M_{t-1}}{P_t} + \sum_{t=0}^{\infty}\beta^tU'(X_t)\frac{R_{t-1}B_{t-1}}{P_t}. \tag{93}$$

The second summation on the left-hand-side cancels with the the last summation on the right-hand-side to leave only the initial bond position,

$$\sum_{t=0}^{\infty}\beta^tU'(X_t)X_t + \sum_{t=0}^{\infty}\beta^tU'(X_t)\frac{M_t}{P_t} = \tag{94}$$

$$\sum_{t=0}^{\infty}\beta^tU'(X_t)(1-\tau_t)w_tH_t + \sum_{t=0}^{\infty}\beta^tU'(X_t)\frac{M_{t-1}}{P_t} + U'(x_0)\frac{R_{-1}B_{-1}}{P_0}. \tag{95}$$

Next, substitute into the second term on the left-hand-side using (31) to get

$$\sum_{t=0}^{\infty}\beta^tU'(X_t)X_t + \sum_{t=0}^{\infty}\beta^{t+1}U'(x_{t+1})\frac{M_t}{P_{t+1}}\left[\sigma\frac{u'(q_t)}{g'(q_t)} + 1 - \sigma\right] = \tag{96}$$

$$\sum_{t=0}^{\infty}\beta^tU'(X_t)(1-\tau_t)w_tH_t + \sum_{t=0}^{\infty}\beta^tU'(X_t)\frac{M_{t-1}}{P_t} + U'(x_0)\frac{R_{-1}B_{-1}}{P_0}. \tag{97}$$

Expand the second summation on the left-hand-side to get

$$\sum_{t=0}^{\infty}\beta^t U'(X_t)X_t + \sum_{t=0}^{\infty}\beta^{t+1}U'(x_{t+1})\frac{M_t}{P_{t+1}} + \sigma\sum_{t=0}^{\infty}\beta^{t+1}U'(x_{t+1})\frac{M_t}{P_{t+1}}\left[\frac{u'(q_t)}{g'(q_t)}-1\right] \tag{98}$$

$$= \sum_{t=0}^{\infty}\beta^t U'(X_t)(1-\tau_t)w_t H_t + \sum_{t=0}^{\infty}\beta^t U'(X_t)\frac{M_{t-1}}{P_t} + U'(x_0)\frac{R_{-1}B_{-1}}{P_0}. \tag{99}$$

Cancel the second summation on the left-hand-side with the second summation on the right-hand-side to leave only the initial money holdings,

$$\sum_{t=0}^{\infty}\beta^t U'(X_t)X_t + \sigma\sum_{t=0}^{\infty}\beta^{t+1}U'(x_{t+1})\frac{M_t}{P_{t+1}}\left[\frac{u'(q_t)}{g'(q_t)}-1\right] \tag{100}$$

$$= \sum_{t=0}^{\infty}\beta^t U'(X_t)(1-\tau_t)w_t H_t + U'(x_0)\left[\frac{M_{-1}+R_{-1}B_{-1}}{P_0}\right]. \tag{101}$$

Using (27), we can substitute into the first term on the right-hand-side to get

$$\sum_{t=0}^{\infty}\beta^t U'(X_t)X_t - \sum_{t=0}^{\infty}\beta^t A H_t + \sigma\sum_{t=0}^{\infty}\beta^{t+1}U'(x_{t+1})\frac{M_t}{P_{t+1}}\left[\frac{u'(q_t)}{g_q(q_t,Z_t)}-1\right] = U'(x_0)\left[\frac{M_{-1}+R_{-1}B_{-1}}{P_0}\right]. \tag{102}$$

Writing $\frac{M_t}{P_{t+1}} = \frac{M_t}{P_t}\frac{P_t}{P_{t+1}}$, express this as

$$\sum_{t=0}^{\infty}\beta^t U'(X_t)X_t - \sum_{t=0}^{\infty}\beta^t A H_t + \sigma\sum_{t=0}^{\infty}\beta^{t+1}U'(x_{t+1})\frac{M_t}{P_t}\frac{P_t}{P_{t+1}}\left[\frac{u'(q_t)}{g_q(q_t,Z_t)}-1\right] = U'(x_0)\left[\frac{M_{-1}+R_{-1}B_{-1}}{P_0}\right]. \tag{103}$$

Use (41) to substitute for M_t/P_t,

$$\sum_{t=0}^{\infty}\beta^t U'(X_t)X_t - \sum_{t=0}^{\infty}\beta^t A H_t + \sigma\sum_{t=0}^{\infty}\beta^{t+1}\frac{U'(x_{t+1})}{P_{t+1}}\frac{P_t}{U'(X_t)}g(q_t,Z_t)\left[\sigma\frac{u'(q_t)}{g_q(q_t,Z_t)}+1-\sigma\right]\left[\frac{u'(q_t)}{g_q(q_t,Z_t)}-1\right] = U'(x_0) \tag{104}$$

Finally, from (31), we can make the substitution $\beta E_t\left[\frac{U'(x_{t+1})}{P_{t+1}}\right] = \frac{U'(X_t)}{P_t}\left[\sigma\frac{u'(q_t)}{g_q(q_t,Z_t)}+1-\sigma\right]^{-1}$ in the third summation on the left-hand-side. Cancelling terms and reintroducing the E_0 operator leaves us with

$$E_0\sum_{t=0}^{\infty}\beta^t\left[U'(X_t)X_t - AH_t + \sigma g(q_t,Z_t)\left(\frac{u'(q_t)}{g_q(q_t,Z_t)}-1\right)\right] = U'(x_0)\left[\frac{M_{-1}+R_{-1}B_{-1}}{P_0}\right], \tag{105}$$

which is the present-value implementability (PVIC) constraint for the Ramsey problem in the LW model. Any allocation that satisfies this restriction, the resource constraint, and the ZLB constraint can be supported as a monetary equilibrium; furthermore, any monetary equilibrium can be described by these three conditions.

B Alternative Instruments in the DM in Basic Model

Here, we prove a claim we make in Section 4.1. To demonstrate that the deviation from the Friedman Rule in our basic model is not proxying for a sales tax in the DM, we modify our basic model. We introduce a sales tax in the DM and show that the resulting Ramsey problem does not have a solution in which the Friedman Rule holds. Our result here is only for the case $\theta = 1$ (which is equivalent to price-taking in the DM), just as is our Proposition 2. It is somewhat easier to work with the price-taking version to establish this result, which is why we use this version here.

We introduce a DM sales tax in the following way: with price-taking in the DM, buyers, taking $\tilde{P}_t$ as given, turn over $\tilde{P}_t q_t$ units of money in each transaction. The sellers must remit $\tau_t^d \tilde{P}_t q_t$ to the government in the next CM, which, given our timing assumptions, occurs in period $t + 1$.[14] Equivalently, we can suppose that the government receives the revenue in the DM but waits until the next CM to spend it. Because the assets markets are not open in the DM, the government cannot invest this extra revenue in an interest-bearing asset.

The government's flow budget constraint in nominal terms is thus

$$M_t + B_t + P_t w_t \tau_t^h H_t + \sigma \tau_{t-1}^d \tilde{P}_{t-1} q_{t-1} = P_t G_t + M_{t-1} + R_{t-1} B_{t-1}, \tag{106}$$

in which the σ appears because it is only DM sellers in period $t - 1$ (of which there is a measure σ) that turn over sales taxes to the government. Because in equilibrium, $M_{t-1} = \tilde{P}_{t-1} q_{t-1}$ (that is, any DM meeting in which a transaction occurs leads to the buyer turning over all his cash to the seller), we may write the period-t government budget constraint as

$$M_t + B_t + P_t w_t \tau_t^h H_t = P_t G_t + (1 - \sigma \tau_{t-1}^d) M_{t-1} + R_{t-1} B_{t-1}. \tag{107}$$

If we sum the CM budget constraints of all households (a measure σ of whom were buyers in period $t - 1$ and thus enter period t with no money; a measure σ of whom were sellers and thus enter period t with $M_{t-1} + (1 - \tau_{t-1}^d) M_{t-1}$; and a measure $1 - 2\sigma$ of whom did not trade and thus have M_{t-1}), we have in equilibrium

$$P_t X_t + M_t + B_t = P_t w_t (1 - \tau_t^h) H_t + (1 - \sigma \tau_{t-1}^d) M_{t-1} + R_{t-1} B_{t-1}. \tag{108}$$

Clearly, the government budget constraint and the summation of all the households' budget constraints yields the CM resource constraint. In the Ramsey problem, then, we can use the resource constraint and the summation of all households' budget constraints, which implies the government budget constraint is satisfied.

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[14] Thus, we assume that it is the sellers that pass along the sales tax receipts to the government; assuming that it is buyers that remit taxes would formally lead to the same analysis.

In order to solve the Ramsey problem, first let us derive the equilibrium conditions with this new tax instrument in place. To keep the discussion short, we simply point out the differences from the conditions we derive above. The CM problem summarized in (2)-(6) is unchanged. In the DM, from the buyer's problem (derived as in, say, Rocheteau and Wright (2005)), we get

$$\tilde{P}_t q_t = m_t. \tag{109}$$

The seller's problem can be written as

$$V^s\left(m_t, b_t, S_t\right) = \max_{q_t}\left\{-c\left(q_t, Z_t\right) + \beta E_t W_{t+1}\left(m_t + (1-\tau_t^d)\tilde{p}q, b_t, S_{t+1}\right)\right\}, \tag{110}$$

with the first order condition $c_q(q_t, k_{t+1}, Z_t) = \beta(1-\tau_t^d)\tilde{p}\chi_t$, which leads to the following expression for V_m:

$$V_{m,t}\left(m_t, b_t, S_t\right) = \left(1-\sigma\right)\beta\chi_t + \sigma\beta\chi_t\frac{(1-\tau_t^d)u'\left(q_t\right)}{c_q\left(q_t, Z_t\right)}. \tag{111}$$

This leads to the following equilibrium condition, which replaces (37),

$$\frac{U'(X_t)}{P_t} = \beta\left[\sigma\frac{(1-\tau_t^d)u'(q_t)}{c_q(q_t, Z_t)} + 1 - \sigma\right]E_t\left[\frac{U'(X_{t+1})}{P_{t+1}}\right]. \tag{112}$$

It is also straightforward to show that the analog of the Fisher-like condition (33) from our baseline model is

$$R_t = \sigma\frac{(1-\tau_t^d)u'(q_t)}{c_q(q_t, Z_t)} + 1 - \sigma, \tag{113}$$

which we need in order to write the ZLB constraint on the Ramsey problem. Note that in the main text we assumed $c(q, Z) = q/Z$ which is simply a normalization because $c(.)$ is in terms of utility. Using this, we have $c_q(q, Z) = 1/Z$. Also note that by construction $Z > 0$.

To construct the PVIC in this version of the model, we proceed exactly as in Appendix A, using (108). The resulting PVIC is given by

$$E_0\sum_{t=0}^{\infty}\beta^t\left[U'(X_t)X_t - AH_t + \sigma q_t\left[Z_t(1-\tau_t^d)u'(q_t) - 1\right] + \sigma\tau_t^d q_t\right] = U'(x_0)\left[\frac{M_{-1} + R_{-1}B_{-1}}{P_0}\right] - \frac{\sigma\tau_0^d q_0}{\beta}. \tag{114}$$

Associating the multiplier ξ with the PVIC and the multiplier ι_t with the ZLB constraint

$$\sigma\left[Z_t(1-\tau_t^d)u'(q_t) - 1\right] \geq 0, \tag{115}$$

the Ramsey first-order-conditions with respect to q_t and τ_t^d are, respectively,

$$\sigma\left[u'\left(q_t\right) - \frac{1}{Z_t}\right] + \sigma\xi\left\{\left[Z_t\left(1-\tau_t^d\right)u'\left(q_t\right) - 1\right] + Z_t\left(1-\tau_t^d\right)q_t u''\left(q_t\right) + \tau_t^d\right\} + \sigma Z_t\iota_t\left(1-\tau_t^d\right)u''\left(q_t\right) = 0 \tag{116}$$

and

$$\xi\sigma q_t\left[-Z_t u'\left(q_t\right) + 1\right] - \sigma Z_t\iota_t u'\left(q_t\right) = 0. \tag{117}$$

The complementary slackness condition for the ZLB constraint is

$$\iota_t \sigma \left[\left(1-\tau_t^d\right) Z_t u'\left(q_t\right) - 1 \right] = 0. \tag{118}$$

There are three cases to consider:

- $\iota_t = 0$ and ZLB binds
- $\iota_t = 0$ and ZLB does not bind
- $\iota_t > 0$ and ZLB binds

We proceed by stating and proving two lemmas. The first one rules out the first two cases above and the second lemma rules out the third. By ruling out each of the three possibilities for the solution, we establish that there is no solution to the Ramsey problem with the DM sales tax in which the Friedman Rule is satisfied.

Lemma 1. *The solution to this problem cannot have $\iota_t = 0$.*

Proof. Suppose $\iota_t = 0$. Imposing this in (116) and (117) gives

$$\left[u'\left(q_t\right) - \frac{1}{Z_t} \right] + \xi \left\{ \left[Z_t \left(1-\tau_t^d\right) u'\left(q_t\right) - 1 \right] + Z_t \left(1-\tau_t^d\right) q_t u''\left(q_t\right) + \tau_t^d \right\} = 0, \tag{119}$$

$$\xi q_t \left[-Z_t u'\left(q_t\right) + 1 \right] = 0. \tag{120}$$

Substituting $Z_t u'(q_t) = 1$ from the second expression in to the first, we get

$$\begin{aligned}
Z_t \left(1-\tau_t^d\right) u'\left(q_t\right) - 1 + Z_t \left(1-\tau_t^d\right) q_t u''\left(q_t\right) + \tau_t^d &= 0 && (121)\\
\left(1-\tau_t^d\right) \left[\underbrace{Z_t u'\left(q_t\right)}_{=1} + Z_t q_t u''\left(q_t\right) - 1 \right] &= 0 && (122)\\
\left(1-\tau_t^d\right) \left[Z_t q_t u''\left(q_t\right) \right] &= 0. && (123)
\end{aligned}$$

Given a strictly concave utility function, for any $q_t > 0$, the term in the [.] in the last line is nonzero, which means $\tau_t^d = 1$ must hold. But this would mean $Z_t u'\left(q_t\right)\left(1-\tau_t^d\right) - 1 = -1 < 0$, which violates the ZLB constraint. Therefore, $\iota_t > 0$ must hold. □

Lemma 2. *The problem does not have a solution if $\iota_t > 0$ and the ZLB constraint binds.*

Proof. Suppose the ZLB constraint binds. Simplifying (117), we get

$$\left[Z_t u'\left(q_t\right) - 1 \right] = -\frac{\iota_t Z_t u'\left(q_t\right)}{\xi q_t}. \tag{124}$$

Substituting $[Z_t u'(q_t) - 1]$ in to (116) and imposing the ZLB constraint, we get

$$-\frac{\iota_t Z_t u'(q_t)}{\xi q_t} + \xi\left[Z_t\left(1-\tau_t^d\right) q_t u''(q_t) + \tau_t^d\right] + \iota_t Z_t\left(1-\tau_t^d\right) u''(q_t) = 0. \tag{125}$$

Collecting the ι_t terms,

$$\begin{aligned}\iota_t Z_t\left[\left(1-\tau_t^d\right) u''(q_t) - \frac{u'(q_t)}{\xi q_t}\right] &= -\xi\left[Z_t\left(1-\tau_t^d\right) q_t u''(q_t) + \tau_t^d\right] &(126)\\ \iota_t &= \frac{-\xi\left[Z_t\left(1-\tau_t^d\right) q_t u''(q_t) + \tau_t^d\right]}{Z_t\left[\left(1-\tau_t^d\right) u''(q_t) - \frac{u'(q_t)}{\xi q_t}\right]}. &(127)\end{aligned}$$

Given a strictly concave and strictly increasing utility function, the denominator is strictly negative. This mean that if $\iota > 0$, we need

$$Z_t\left(1-\tau_t^d\right) q_t u''(q_t) + \tau_t^d > 0 \tag{128}$$

or

$$\tau_t^d > -Z_t\left(1-\tau_t^d\right) q_t u''(q_t) > 0, \tag{129}$$

which means the solution to this problem must have a strictly positive τ_t^d.

Finally, note that the sign of the first order condition of the Ramsey problem with respect to τ_t^d given in (117) is negative because in any monetary equilibrium we have $q_t \leq q^*$, which implies $Z_t u'(q_t) \geq 1$. This means the planner wants to choose τ_t^d as small as possible. But the condition above says that τ_t^d must be strictly positive. As such, there is no solution to the problem when the ZLB constraint is binding. (That is, a solution to the problem "find the smallest number strictly greater than zero" does not exist.) $\square$

This completes our proof that if we introduce a sales tax in the DM, the Ramsey problem has no solution.

C The Ramsey Problem in Model with Capital

Having derived the implementability constraint for the LW model in Appendix A, it is straightforward to extend it for the AWW environment. Multiplying the consumer's CM budget constraint by $\beta^t U'(X_t)/P_t$ and then summing over dates and states beginning at $t = 0$ as above, we have

$$E_0 ... + \sum_{t=0}^{\infty} \beta^t U'(X_t) K_{t+1} = \sum_{t=0}^{\infty} \beta^t U'(X_t) \left[1 + (1 - \tau_t^k)(f_K(K_t, H_t) - \delta) \right] K_t + ... \tag{130}$$

where the ellipsis indicate that the other terms are the same as those in (91). The manipulations following (91) proceed just as before (with, of course, K now included inside the function $g(.)$), so we present here only the derivation of the terms in the implementability constraint arising from the inclusion of capital.

Use the household's Euler equation for capital,

$$U'(X_t) = \beta U'(X_{t+1}) \left[1 + (1 - \tau_{t+1}^k)(f_K(K_{t+1}, H_{t+1}) - \delta) \right] - \sigma\gamma(q_t, K_{t+1}), \tag{131}$$

to substitute for $U'(X_t)$ on the left-hand-side of the previous expression to get

$$... + \sum_{t=0}^{\infty} \beta^{t+1} U'(x_{t+1}) \left[1 + (1 - \tau_{t+1}^k)(f_K(K_{t+1}, H_{t+1}) - \delta) \right] K_{t+1} \tag{132}$$

$$- \sum_{t=0}^{\infty} \beta^t \sigma\gamma(q_t, K_{t+1}) K_{t+1} = \sum_{t=0}^{\infty} \beta^t U'(X_t) \left[1 + (1 - \tau_t^k)(f_K(K_t, H_t) - \delta) \right] K_t + ... \tag{133}$$

Canceling like summations leaves

$$... - \sum_{t=0}^{\infty} \beta^t \sigma\gamma(q_t, K_{t+1}) K_{t+1} = U'(x_0) \left[1 + (1 - \tau_0^k)(f_K(K_0, H_0) - \delta) \right] K_0 + ... \tag{134}$$

Re-inserting the terms from the LW PVIC, the PVIC for the AWW model is

$$\begin{aligned} E_0 \sum_{t=0}^{\infty} \beta^t & \left[U'(X_t) X_t - AH_t + \sigma g(q_t, K_{t+1}) \left(\frac{u'(q_t)}{g_q(q_t, K_{t+1})} - 1 \right) - \sigma\gamma(q_t, K_{t+1}) K_{t+1} \right] \\ &= U'(x_0) \left[\frac{M_{-1} + R_{-1} B_{-1}}{P_0} + \left[1 + (1 - \tau_0^k)(f_K(K_0, H_0) - \delta) \right] K_0 \right], \end{aligned} \tag{135}$$

which is expression (81) in the text. Any allocation that satisfies this restriction, the resource constraint, and the ZLB constraint can be supported as a monetary equilibrium; furthermore, any monetary equilibrium can be described by these three conditions.

Variable	Mean	Std. Dev.	Auto corr.	Corr(x, Y)	Corr(x, Z)	Corr(x, G)
π	-1.5896	0.2660	0.8051	0.7428	1	0.0074
τ^h	0.2134	0.0000	0.6826	0.5870	0.7883	0.0083
R	1.8766	0.3910	0.8051	-0.7426	-0.9999	-0.0074
q	0.8032	0.0358	0.8048	0.7428	0.9995	0.0073
Y	2.2767	0.0412	0.8369	1	0.7428	0.6694
X	1.6753	0.0199	0.8051	0.7428	0.9999	0.0073
H	2.0765	0.0279	0.8715	0.5342	-0.1619	0.9841

Table 10: Simulation-based moments, using second-order approximation, in the bargaining version of the LW economy and $\theta = 1$. Driving processes are G_t and Z_t. π and R reported in annualized percentage points.

D Second-Order Approximation in Basic Model

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