

# Dynamic Taxation, Private Information and Money\*

Christopher Waller

University of Notre Dame

March 4, 2006 VERY PRELIMINARY

## Abstract

The objective of this paper is to study optimal fiscal and monetary policy in a dynamic Mirrlees model where the frictions giving rise to money as a medium of exchange are explicitly modeled. The framework is a three period OLG model where agents are born every other period. The young and old trade in perfectly competitive centralized markets. In ‘middle age’, agents receive preference shocks and trade amongst themselves in an anonymous search market. Money is essential in this market. Since preference shocks are private information, in a record-keeping economy without money, the planner’s allocation trades off efficient risk sharing against production efficiency in the search market and average consumption when old. For a government to replicate this outcome in a monetary economy without record-keeping, distortionary taxation of money balances is needed.

---

\*I would like to thank Aleks Berentsen, Gabriele Camera, Tom Gresik, Randy Wright and the participants of the Penn Search and Matching Conference for very helpful comments on an early draft of this paper.

# 1 Introduction

The Friedman rule states that money should not be taxed via inflation to avoid distorting its rate of return relative to other interest bearing assets. Yet we never see the Friedman rule implemented in practice. One explanation for this, dating back to Phelps (1973), is that in a second-best world governments must use distortionary taxation on goods to finance spending or transfers. So, to equate tax distortions on the margin, money should be taxed as well.

Since Friedman's and Phelps's arguments are partial equilibrium in nature, a substantial body of research developed examining the optimality of the Friedman rule in dynamic general equilibrium models where the government must resort to distortionary taxation to finance spending or transfers. Most of this literature adopts the Ramsey approach to optimal taxation – the government is assumed to be unable to use lump sum taxes and so it chooses distortionary tax rates to maximize the welfare of the representative agent. The best known use of this approach to study the inflation tax is Chari, Christiano and Kehoe (1996), denoted CCK hereafter. They show that the Friedman rule will be optimal if preferences are homothetic and weakly separable in consumption and leisure regardless of whether money is valued because of money-in-the-utility, cash-in-advance or shopping time motives.

There are two drawbacks to this approach for studying the inflation tax. First, even though non-distortionary (lump-sum) taxes achieve the first-best allocation, the Ramsey approach simply prohibits their use for some unmodeled reason. Thus, the first-best is not implementable solely because of an unspecified feature of the environment – the inability to use non-distorting taxes. Second, when it comes to studying the inflation tax, money is always 'forced' into the model via some shortcut such as money-in-the-utility function, shopping time or cash-in-advance. The frictions for why money is needed for transaction purposes are never explicitly modeled. So in addition to an unspecified reason for limiting the set of tax instruments, there is also an unmodeled friction in the environment that gives

money transaction value.

This suggests that rather than using the Ramsey model for studying optimal fiscal and monetary policy, we need to construct an environment where: 1) the government is free to use lump-sum taxes but *chooses* not to, and 2) the frictions giving rise to money as a medium of exchange are explicitly modeled.

Addressing point one above is the basis for the ‘New Dynamic Public Finance’ (NDPF) literature.<sup>1</sup> This literature studies optimal taxation in a dynamic version of Mirrlees’s (1971) model where there are heterogeneous agents and private information about agent types. In this framework, the government chooses taxes to maximize welfare subject to providing appropriate incentives for agents to reveal private information regarding their preferences and productivity. There are no restrictions whatsoever on the set of tax instruments available to the government and the standard result in this literature is that distortionary taxation is optimal.<sup>2</sup>

Modeling the frictions that provide the microfoundations for money demand is the basis of the "New Monetary Economics" literature. This literature, based on the Kiyotaki and Wright (1989,1993) monetary search framework, explicitly models the trade frictions that prevent the use of trade credit between agents and thus the need for a medium of exchange. As a result, money ‘essential’ in that it expands the set of allocations that can be achieved.<sup>3</sup>

The objective of this paper is to construct a model that combines these two literatures in order to study the dynamic taxation of money. I analyze a model where money is essential as

---

<sup>1</sup>Most notable in this area are the papers by Golosov, Kocherlakota and Tysvinski (2003) and Kocherlakota (2005).

<sup>2</sup>While most of the NDPF literature focuses on capital and labor taxation, da Costa and Werning (2005) look at the issue of distortionary taxation of money. Although they adopt the dynamic Mirrlees framework, the same short-cut motives for holding money are used as in CCK. Surprisingly, despite having heterogeneous agents and private information, da Costa and Werning essentially obtain the same result as CCK – separability of preferences makes the Friedman rule optimal for all three short-cut models. Although da Costa and Werning’s work is a good first step, it is unsatisfactory in the sense that they do not explicitly model the frictions that make money essential as a medium of exchange.

<sup>3</sup>See Kocherlakota (1998) and Wallace (2001).

a medium of exchange yet informational frictions induce the government to use distortionary taxation of money balances. The basic model is a three period OLG/search model developed by Zhu (2005) but with the addition of private information about individual preference shocks. In this framework, a new generation is born every other period. In these periods, the young and old agents can trade amongst themselves. Young agents have an endowment of labor but old agents do not. When ‘middle aged’, agents receive idiosyncratic preference shocks that make some of them producers and others consumers.

Since Kocherlakota (1998) showed that money is fundamentally a form of record-keeping, I construct allocations for three environments with differing assumptions on record-keeping. First, the allocation is derived when no durable asset or record-keeping technology exists (autarky). Second, I consider the case with a record-keeping technology where agents send reports about their preference shocks to a planner at the beginning of middle age and, based on their reports, are given a sequence of consumption/production in middle and old age. Within this environment I consider two cases: 1) when the shocks are public information and 2) when they are private information. For the latter case, the allocation must be incentive compatible with truthful revelation of the shocks. This allocation is referred to as the constrained optimum. In the constrained optimum the planner must create consumption risk for the old to induce agents to produce in middle age. Thus, the planner trades off risk sharing amongst the old against productive efficiency when middle aged. Similar to the NDPF literature, the planner wants to create a wedge between the marginal utility of consumption when young against the expected marginal utility of consumption when old to induce truthful reporting.

Finally, I consider the case where the planner (government) has no record-keeping technology and agents do not send in reports. Rather, as a substitute for record-keeping, the planner provides fiat currency to agents. Money provides no utility in and of itself and only has value by being generally accepted by agents as a medium of exchange. In this economy,

middle-aged, agents search for a suitable trading partner and bargain over the terms of trade when a match occurs. However, for comparison purposes, I also study the case where the terms of trade are determined by a Walrasian auctioneer.<sup>4</sup> Anonymity in this market makes money essential as a medium of exchange. Informational constraints prevent the government from observing an individual's preference shocks, money balances and consumption. However, the government's information set allows it to distinguish agents by age and observe aggregate consumption. Thus, it can impose distortionary labor income taxes on the young and on *aggregate* consumption. The government then chooses the growth rate of the money stock, tax rates and lump-sum transfers to achieve a desired allocation.

Using this framework I would like to address the following questions. First, can the constrained planner allocation be implemented with the use of fiat money and optimally chosen fiscal policy? Second, does the optimal fiscal policy require deviating from the Friedman rule?

Although it is difficult in general to show existence or uniqueness of the constrained optimum or monetary equilibrium, I can give a partial characterization of the allocations to see how they differ. To illustrate these differences in a more concrete fashion, analytical examples are provided where a unique steady state allocation exists for the constrained planner problem and the monetary economy. I show that in the monetary economy with bargaining, the optimal fiscal policy is to forgo the use of lump-sum taxes/transfers, equate linear tax rates on labor income and consumption (an example of the uniform taxation principle) and use distortionary taxation of money. While this does not replicate the constrained optimum in general due to holdup problems in the search market, for an interesting special case it does. With competitive pricing in the search market the holdup problems are eliminated, however the constrained optimum is not replicated under the optimal policy. Again, the

---

<sup>4</sup>Thus, one contribution of this paper is to study how bargaining affects the choice of distortionary wealth taxation in a dynamic Mirrlees model.

optimal fiscal policy requires distortionary taxation of money. While I constrain the government's instrument set to consist of lump-sum taxes/transfers and linear tax rates in these examples, I discuss how the results may be modified if non-linear taxes are allowed.

What is most striking about these examples is that distortionary taxation of money is optimal even though preferences are homothetic and separable in consumption and leisure. This suggests distortionary taxation of money is driven by the informational frictions that make money essential as a medium of exchange as opposed to assumptions on homotheticity or separability of preferences.

The structure of the paper is as follows. First, the environment is described. Then I derive the autarkic allocation and the constrained optimal stationary allocation for a planner with a record-keeping technology. Then the equilibrium steady state conditions are derived for the monetary equilibrium for the cases when agents bargain over the terms of trade in the search market and when there is Walrasian pricing.

## 2 Environment

The basic environment is the three period OLG/search model of Zhu (2005). There is a continuum of agents born with unit measure every other period. In these periods, the young and the old come together. Call this location the centralized market or CM. Young agents are endowed with labor at the start of each CM and there is a linear production technology available that converts one unit of labor into one unit of goods. Goods are perishable so there is no ability to store goods across periods. The old have no labor endowment and must receive goods from the young to consume. After meeting in the CM, the old die. In the second period of life, middle-aged agents can trade amongst themselves. Call this location the decentralized market or DM. The CM/DM structure follows from Lagos and Wright (2005).

An agent's preferences are assumed to be additively separable across consumption, labor and time. Let  $U(C)$  be utility from consuming  $C$  units of goods in the CM and  $v(h)$  is the disutility of working  $h \geq 0$  hours when young. Assume  $U', -U'', v', v'' > 0$  and  $U'(0) \rightarrow +\infty$ . Let  $\beta_1$  be the discount factor from the CM to the DM while  $\beta$  is the discount factor from the DM to the CM with  $\beta_1\beta = \hat{\beta}$ . In general  $\beta_1$  and  $\beta$  can be different. Preferences in middle age are given by  $\epsilon_b u(q) - \epsilon_s \psi(q)$  where  $\epsilon_b u(q)$  is utility from consuming  $q$  units of goods in the DM while  $-\epsilon_s \psi(q)$  is the disutility of producing  $q$  units of goods in the DM. Assume  $u', -u'', \psi', \psi'' > 0, u'(0) \rightarrow +\infty$  and  $u(0) = \psi(0) = \psi'(0) = 0$ . The parameters  $\epsilon_b$  and  $\epsilon_s$  are preference shocks such that with probability  $\sigma_b$ ,  $\epsilon_b = 1$  and  $\epsilon_s = 0$ , meaning an agent can consume but cannot produce. With probability  $\sigma_s$ ,  $\epsilon_b = 0$  and  $\epsilon_s = 1$ , meaning an agent can produce but not consume. Finally, with probability  $1 - \sigma_b - \sigma_s$ ,  $\epsilon_b = \epsilon_s = 0$  meaning they do neither. From here on I will assume  $\sigma_b = \sigma_s = \sigma$  with  $\sigma \leq 1/2$ . I will refer to consumers as buyers, producers as sellers and those doing neither as idle. Note that utility from consumption and disutility from production is allowed to differ across markets.

An agent's lifetime utility born at time  $t - 1$  is given by

$$\begin{aligned}
W_{y,t-1} &= U(C_{y,t-1}) - v(h_{t-1}) \\
&\quad + \beta_1 \sigma [u(q_{b,t}) + \beta U(C_{o,t+1}^b)] + \beta_1 \sigma [-\psi(q_{s,t}) + \beta U(C_{o,t+1}^s)] \\
&\quad + \beta_1 \beta (1 - 2\sigma) U(C_{o,t+1}^o) \\
&= U(C_{y,t-1}) - v(h_{t-1}) + \beta_1 V_t
\end{aligned}$$

where

$$V_t = \sigma [u(q_{b,t}) + \beta U(C_{o,t+1}^b)] + \sigma [-\psi(q_{s,t}) + \beta U(C_{o,t+1}^s)] + \beta (1 - 2\sigma) U(C_{o,t+1}^o) \quad (1)$$

and  $C_y$  is consumption when young,  $q_b$  is consumption of goods in the DM if a buyer,  $q_s$  is

production if a seller and  $C_o^j$  is consumption when old if the agent's trading state,  $j$ , in the DM was a buyer ( $b$ ), a seller ( $s$ ) or other ( $o$ ).

### 3 Optimal Allocations and Record-keeping

An important result in monetary theory due to Kocherlakota (1998) is that money is a form of record-keeping. So before looking at the monetary equilibrium, I consider economies with various forms of record-keeping technologies in order to compare them to the allocation that occurs in the monetary equilibrium. Consider the case where there is a complete absence of record-keeping. The age structure and the absence of durable goods or assets means the only allocation is autarky.<sup>5</sup> Consequently, young agents produce and consume for themselves and have  $q_b = q_s = C_o^j = 0$ . Lifetime welfare in the autarkic allocation is given by  $W^a = U(C_y^a) - v(C_y^a) + \hat{\beta}U(0)$  where the superscript  $a$  denotes the autarkic choice and  $C_y^a$  solves  $U'(C_y^a) = v'(C_y^a)$ .

#### 3.1 Public information and record-keeping

Now consider the case where there is a planner with a record-keeping technology. In this environment, agents send a report to the planner about the realization of their idiosyncratic preference shock in the DM and record-keeping technology allows the planner to keep track of agents' reports. Conditional on the report, the planner gives the agent a sequence of consumption in the DM and CM. If an agent reports himself as a buyer, he is given  $q_b$  units of output by the planner in the DM to consume and  $C_o^b$  when old. If he reports himself as a producer, he delivers  $q_s$  units of output to the planner in the DM and consumes  $C_o^s$  when

---

<sup>5</sup>The inability of the old to produce will eliminate the use of trade credit in the DM even if there is record-keeping and enforcement. However, if the old could produce in the CM, then trade credit in the DM is ruled out by anonymity. As usual, the age structure in OLG models prevents the young from extending trade credit to the old.

old. If he reports being idle, he neither consumes nor produces in the DM and receives  $C_o^o$  when old. As is standard, assume the planner can commit to this sequence of consumption. We would like to know what allocation can be supported by this record-keeping technology. Young agents simply report their age and receive  $C_y$  units of consumption and provide  $h$  units of labor.

A nice feature of the model is that at the start of the DM there is only one generation so the standard OLG issue of how much weight to give the initial old is avoided. So this suggests looking at the planner problem at the start of the DM at time 0 and going forward. Since the model is essentially a repeated two period static problem a stationary allocation is optimal. The planner's objective is to choose a stationary allocation to maximize the expected lifetime utility of the current middle aged agent and all future generations. With no durable goods in this economy, it is straightforward to show that the planners problem reduces to

$$\begin{aligned} \max_{q, C_o^b, C_o^s, C_o^o, C_y, h} \quad & \sigma [u(q_b) - \psi(q_s)] + \beta [\sigma U(C_o^b) + \sigma U(C_o^s) + (1 - 2\sigma)U(C_o^o) + U(C_y) - v(h)] \\ \text{s.t.} \quad & h = \sigma C_o^b + \sigma C_o^s + (1 - 2\sigma)C_o^o + C_y \\ & \sigma q_b = \sigma q_s \end{aligned}$$

Suppose that the preference shocks in the DM are public information thus observable by the planner. If the planner can force exchange to occur ex post, the optimal solution satisfies

$$\begin{aligned} u'(q^*) &= \psi'(q^*) \\ U'(C^*) &= U'(C_o^j) = U'(C_y) = v'(h) \quad j = b, s, o \\ h &= 2C^* \end{aligned}$$

where  $C^*$  solves  $U'(C^*) = v'(2C^*)$ .<sup>6</sup> Call this the *unconstrained* optimal allocation. In this allocation, the planner wants to eliminate all consumption risk, making  $U'(C_o) = EU'(C_o)$  where  $E$  is the expectations operator.

### 3.2 Private information and record-keeping

The unconstrained allocation is not feasible if the planner cannot observe agents' preference shocks. Why? Those who are producers in the DM get the same consumption in the next CM as everyone else – hence there is no reward for producing. Consequently, those agents would never reveal their true DM preference shock to the planner. So even being able to force agents to produce is useless since the planner cannot identify who the sellers are in the DM. As a result, with private information, the planner is constrained to implement an allocation that is incentive compatible, i.e., agents truthfully reveal their idiosyncratic shock in the DM and trade voluntarily. This problem is considered next.

With private information the planner has to worry about incentive constraints such that no agent has an incentive to misrepresent their true state in the DM. Truthful reporting requires:

$$\begin{aligned} u(q_b) + \beta U(C_o^b) - \beta U(C_o^o) &\geq 0 \\ -\psi(q_s) + \beta U(C_o^s) - \beta U(C_o^o) &\geq 0 \\ U(C_o^0) - U(C_o^b) &\geq 0 \end{aligned}$$

The first two constraints require that buyers and sellers in the DM have no incentive to misreport themselves as idle. The last constraint states that an idle agent must have no incentive to misreport himself as a buyer. Buyers and idle agents cannot misrepresent them-

---

<sup>6</sup>The planner can also support this allocation without resorting to force if:  $\psi(q^*) < U(C^*) - U(0)$  and  $W^a < U(C^*) - v(2C^*) + \beta_1 \sigma [u(q^*) - \psi(q^*)] + \hat{\beta} U(C^*)$ . The first condition ensures that a middle aged seller prefers to produce than starve when old, while the second ensures the young do not prefer autarky.

selves as sellers since they would be required to deliver  $q_s > 0$  units of goods, which they cannot do. Thus, incentive constraints for these cases can be dispensed with. However, an idle agent can declare himself a consumer and freely dispose of the goods (or consume them at zero utility). The third constraint also ensures that a seller would rather report himself as idle than as a buyer.

The planner's problem is:

$$\max_{q_b, q_s, C_o^b, C_o^s, C_o^o, C_y} \sigma [u(q_b) - \psi(q_s)] + \beta [\sigma U(C_o^b) + \sigma U(C_o^s) + (1 - 2\sigma)U(C_o^o) + U(C_y) - v(h)]$$

$$s.t. \quad h = \sigma C_o^b + \sigma C_o^s + (1 - 2\sigma)C_o^o + C_y$$

$$u(q_b) + \beta U(C_o^b) - \beta U(C_o^o) \geq 0 \tag{2}$$

$$-\psi(q_s) + \beta U(C_o^s) - \beta U(C_o^o) \geq 0 \tag{3}$$

$$\beta [U(C_o^o) - U(C_o^b)] \geq 0 \tag{4}$$

$$\sigma q_b = \sigma q_s \tag{5}$$

I am assuming the planner takes as given that truthtelling will be satisfied so that (5) is satisfied. Consequently let  $q = q_b = q_s$ . Let  $\sigma\lambda_b$  be the Lagrangian multiplier on (2),  $\sigma\lambda_s$  the multiplier on (3) and  $(1 - 2\sigma)\mu$  the multiplier on (4).

In general, there also must be a participation constraint on the young to induce them to produce output for both generations rather than go into autarky and produce only for themselves. Suppose a young agent decides not to participate. The worst punishment the planner can impose is to force them into autarky. Participation by the young then requires

$$W^a \leq U(\bar{C}_y) - v(\bar{h}) + \beta_1 \sigma [u(\bar{q}) - c(\bar{q})] + \hat{\beta} EU(\bar{C}_o) \tag{6}$$

The bars denote the planner's allocation. If the constraint does bind then obviously, the

planner would prefer autarky as well since his objective is to maximize the lifetime utility of the agents. I will proceed as if this constraint is not binding. Given the resulting allocation, one would then need to impose restrictions on preferences to ensure the constrained optimal allocation makes this constraint non-binding.<sup>7</sup> For example, if  $U(C) = \ln C$  then  $U(0) \rightarrow -\infty$  so this will typically be satisfied.

Assuming the young's participation constraint is satisfied, it is shown in the Appendix that  $\mu = 0$  is not incentive feasible. So it must be the case that  $\mu > 0$  and the idle agent's incentive compatibility constraint binds implying  $C_o^b = C_o^o$ . It then follows from (2) that  $u(q) > 0$  for any  $q > 0$  implying  $\lambda_b = 0$ . We then have the following first-order conditions

$$q : \frac{u'(q)}{\psi'(q)} = 1 + \lambda_s \quad (7a)$$

$$C_o^b : U'(C_o^b) = \frac{v'(h)}{1 - \mu(1 - 2\sigma)} \quad (7b)$$

$$C_o^s : U'(C_o^s) = \frac{v'(h)}{1 + \lambda_s} < 1 \quad (7c)$$

$$C_o^o : U'(C_o^o) = \frac{v'(h)}{1 - \frac{2\sigma}{1-2\sigma}\lambda_s + \mu} \quad (7d)$$

$$C_y : U'(C_y) = v'(h) \quad (7e)$$

(7b) and (7d) can be satisfied jointly for  $C_o^b = C_o^o$  if and only if

$$\lambda_s = \mu \frac{(1 - \sigma)(1 - 2\sigma)}{\sigma} > 0.$$

This implies two things: first, the seller gets no surplus in the DM and second, from (7a),  $q, q < q^*$  so the planner chooses an inefficient level of production in the DM. Using the expression for  $\lambda_s$  in (7b) and (7c), equating expressions and substituting  $U'(C_y) = v'(h)$

---

<sup>7</sup>Lagos and Wright (2005) do a similar exercise regarding non-negativity constraints on labor in the CM.

yields

$$\frac{1}{U'(C_y)} = \sigma \frac{1}{U'(C_o^s)} + (1 - \sigma) \frac{1}{U'(C_o^b)}.$$

Since  $C_o^b = C_o^o$  this is equivalent to

$$\frac{1}{U'(C_y)} = \sigma \frac{1}{U'(C_o^b)} + \sigma \frac{1}{U'(C_o^s)} + (1 - 2\sigma) \frac{1}{U'(C_o^o)} = E \left[ \frac{1}{U'(C_o)} \right] \quad (8)$$

This equation plus

$$\frac{u'(q)}{\psi'(q)} = \frac{U(C_y)}{U'(C_o^s)} \quad (9)$$

$$U'(C_y) = v' [\sigma C_o^b + \sigma C_o^s + (1 - 2\sigma) C_o^o + C_y] \quad (10)$$

$$\psi(q) = \beta [U(C_o^s) - U(C_o^o)] \quad (11)$$

give us four equations in four unknowns. Call a solution to these four equations the *constrained* optimal allocation. Any solution has  $C_o^s > C_y > C_o^b = C_o^o$  and  $q < q^*$ . It is unclear whether  $h \gtrless C^*$ . An important aspect of this allocation is that the planner needs to create consumption risk in the CM when old in order to induce truthtelling and therefore production in the DM when middle aged. Note that if  $\sigma = 0$  the information problem is effectively eliminated (the only state is idle and is publicly known) and the allocation  $C_o^s = C_o^b = C_o^o = C_y = C^*$  can be implemented subject to the young agents' participation constraint being satisfied ( $q$  is irrelevant in this case).

Finally, as a special case, suppose that  $\sigma = 1/2$  so there is no idle state in the DM. Despite eliminating a state and an incentive constraint, I show in the Appendix that setting  $\sigma = 1/2$  in (8)-(11) yields the same solution as imposing  $\sigma = 1/2$  from the start.

## 4 Monetary equilibrium: Absence of recordkeeping

Now consider the case where the government has no record-keeping technology and receives no reports. Instead it provides fiat currency to agents. Money is the only durable object in the economy and it is perfectly divisible and agents can hold unbounded amounts. It is injected in lump sum fashion to all agents at the beginning of the DM. As will be shown, since they all leave the CM with the same amount of money the lump-sum injection has equal value to all middle-aged agents.<sup>8</sup> Let  $\pi M_t$  denote the lump-sum transfer. Since monetary injections only occur every other period we have  $M_{t+2} = \gamma M_t$  where  $\gamma = 1 + \pi$  is the gross growth rate of the money supply from  $t$  to  $t + 1$ . The  $t$  subscript is suppressed for notational ease so that  $+1$  denotes  $t + 2$  and so on.

In the CM, firms hire labor and sell the output in perfectly competitive markets. Given the linear production technology the real wage paid to young labor is 1. Firms sell their output at the nominal price  $\mathcal{P} = 1/\phi$  where  $\phi$  is the goods price of money in the CM.

In the DM, the preference shocks create a double coincidence of wants problem, which combined with anonymity means that money is essential for trade – buyers give up money for goods while sellers increase their holdings of money by selling goods. As a result, there is a non-degenerate distribution of money balances among the old. However, death keeps the distribution of money holdings analytically tractable. This DM/CM structure gives money a ‘store of value’ role from young to old age and a ‘medium of exchange’ function in middle age.

How buyers and sellers are matched in the DM is left unspecified but it could be modeled as the result of random search or a random matching process that pairs each buyer to a seller who then bargain over the terms of trade.<sup>9</sup> Since this is very different from how terms

---

<sup>8</sup>This eliminates welfare gains from inflation due to a non-degenerate distribution of money balances as in Levine (1991), Molico (2005) and Berentsen, Camera and Waller (2005).

<sup>9</sup>If agents trade in a search environment in the DM, one can interpret  $\sigma$  as the *joint* probability of being a buyer (or seller) and finding a suitable trading partner. Consequently,  $1 - 2\sigma$  is the probability of not

of trade occur in the NDPF literature and in CCK, for comparison purposes I also consider the case where middle aged agents meet in an anonymous Walrasian market and prices are determined competitively. Price-taking by agents is used solely as a benchmark since it allows us to make a direct comparison to the Ramsey literature. While one may argue that looking at competitive pricing reduces this model to something akin to cash-in-advance, Rocheteau and Wright (2005) show that competitive search will replicate the Walrasian equilibrium when there are no search externalities. Thus, one can simply view the Walrasian pricing case discussed here as a ‘short cut’ to solving for the competitive search equilibrium much like one can solve a planner’s problem to get the perfectly competitive equilibrium allocation.

The government is able to observe an agent’s age, hours worked by the young, aggregate consumption and the aggregate money stock. It can impose lump sum taxes/transfers, distortionary labor taxes on the young and a sales/consumption tax in the CM. I restrict tax rates to be linear. The sales taxes and labor income taxes are collected by firms and given to the government. The government is unable to observe an *individual’s* preference shocks, consumption and money balances. Consequently, the government cannot impose taxes in the DM. Private information on money balances and consumption also rules out differential wealth taxation of the old and differential transfers to the old (i.e., transfers cannot be made ‘means tested’). The government’s budget constraint is

$$T_y + T_o = \tau_c [\sigma (C_o^b - T_o) + \sigma (C_o^s - T_o) + (1 - 2\sigma) (C_o^o - T_o) + (C_y - T_o)] + \tau_h h$$

where  $\tau_c$  is the consumption tax in the CM,  $\tau_h$  is the tax rate on real labor income and  $T_y$  is a transfer of goods from the government to the young and  $T_o$  is the lump-sum transfer to the old. The term  $C_o^j - T_o$  is consumption of the old that is subject to the sales tax.

---

trading in the DM regardless of the realization of ones’ preference shock. See Aruoba, Waller and Wright (2005) for a more detailed discussion of how the preference shocks used here can be mapped into a search model.

Rearranging yields

$$(1 + \tau_c)(T_y + T_o) = \tau_c [\sigma C_o^b + \sigma C_o^s + (1 - 2\sigma) C_o^o + C_y] + \tau_h h \quad (12)$$

In the monetary equilibrium, at the start of the DM, the lifetime expected utility of a middle-aged agent depends on how much money is brought into the DM. Thus,  $V = V(m)$  where

$$V(m) = \sigma [u(q_b) + \beta U(C_{o,+1}^b)] + \sigma [-\psi(q_s) + \beta U(C_{o,+1}^s)] + \beta(1 - 2\sigma) U(C_{o,+1}^o). \quad (13)$$

In what follows, I will solve the problem facing a representative agent of a generation born in time  $t - 1$ .

## 4.1 Young agents

An agent born at time  $t - 1$  chooses  $C_{y,-1}$ ,  $h_{-1}$  and  $m$  to

$$\begin{aligned} \max_{C_{y,-1}, h_{-1}, m} \quad & U(C_{y,-1}) - v(h_{-1}) + \beta_1 V(m) \\ \text{s.t.} \quad & (1 + \tau_c) C_{y,-1} + \phi_{-1} m = (1 - \tau_h) h_{-1} + T_y \end{aligned}$$

where  $1 - \tau_h$  is the after-tax real wage. The FOC yield

$$U'(C_{y,-1}) = \frac{1 + \tau_c}{1 - \tau_h} v'(h_{-1}) \quad (14)$$

$$\phi_{-1} U'(C_{y,-1}) = (1 + \tau_c) \beta_1 V'(m) \quad (15)$$

Since the FOC are the same for all young agents they leave with the same amount of money balances. Note that since consumption and money balances are financed by labor, the labor

tax rate does not directly appear in (15).

## 4.2 Old agents

When old, agents simply consume  $\phi_{+1}m_{+1}^j(1 + \tau_c)^{-1}$  of their real balances plus the government's lump-sum transfer of goods. Hence

$$\begin{aligned} C_{o,+1}^b &= \frac{\phi_{+1}m_{+1}^b}{1 + \tau_c} + T_o \\ C_{o,+1}^s &= \frac{\phi_{+1}m_{+1}^s}{1 + \tau_c} + T_o \\ C_{o,+1}^o &= \frac{\phi_{+1}m_{+1}^o}{1 + \tau_c} + T_o \end{aligned}$$

where  $m_{+1}^b$ ,  $m_{+1}^s$  and  $m_{+1}^o$  are respectively the money balances of an old agent who bought, sold or was idle in the DM.

## 4.3 Middle aged agents

In the second period agents receive their idiosyncratic preference shocks, match and trade.

### 4.3.1 Bargaining

Suppose that after receiving their lump-sum transfer of cash and realizing their idiosyncratic shocks, each buyer is paired with a seller via some process and they bargain over the terms of trade according to the generalized Nash solution. The terms of trade is a pair  $(q, d)$  where  $q$  is the quantity of goods exchanged while  $d$  is the quantity of money exchanged. It is assumed

agents money balances are observable to their trading partners. The bargaining problem is

$$\begin{aligned}
& \max_{q,d} S_b^\theta S_s^{1-\theta} \\
& s.t. \quad d \leq m^b + \pi M_{-1} \\
& S_b \equiv u(q) + \beta U(C_{o,+1}^b) - \beta U(C_{o,+1}^o) \geq 0 \\
& S_s \equiv -\psi(q) + \beta U(C_{o,+1}^s) - \beta U(C_{o,+1}^o) \geq 0 \\
& C_{o,+1}^b = \phi_{+1} (m^b + \pi M_{-1} - d) (1 + \tau_c)^{-1} + T_o \\
& C_{o,+1}^s = \phi_{+1} (m^s + \pi M_{-1} + d) (1 + \tau_c)^{-1} + T_o
\end{aligned}$$

where  $\beta U(C_{o,+1}^o)$  is the continuation value for both the buyer and seller. The FOC are

$$\begin{aligned}
q & : \quad \frac{\theta u'(q)}{(1-\theta)\psi'(q)} = \frac{S_b}{S_s} \\
d & : \quad \beta \phi_{+1} S_b^{\theta-1} S_s^{1-\theta} [-\theta U'(C_{o,+1}^b) + (1-\theta) U'(C_{o,+1}^s) S_b/S_s] + \nu (1 + \tau_c) = 0
\end{aligned}$$

where  $\nu$  is the Lagrangian multiplier on the buyer's cash constraint. Since Inada conditions are assumed to hold on  $U(C)$  the constraint never binds, i.e., buyers will always take some cash with them into their last period of life to ensure they consume. Consequently, we have

$$q : \quad \frac{u'(q)}{\psi'(q)} = \frac{(1-\theta) S_b}{\theta S_s} \tag{16}$$

$$d : \quad \frac{U'(C_{o,+1}^b)}{U'(C_{o,+1}^s)} = \frac{(1-\theta) S_b}{\theta S_s} \tag{17}$$

Equating these expressions and imposing the equilibrium condition  $M_{+1} = m + \pi M_{-1}$  for both buyers and sellers we obtain

$$u'(q) = \psi'(q) \frac{U'(C_{o,+1}^b)}{U'(C_{o,+1}^s)} = \psi'(q) \frac{U'[(1 + \tau_c)^{-1} (\phi_{+1} M_{+1} - \phi_{+1} d) + T_o]}{U'[(1 + \tau_c)^{-1} (\phi_{+1} M_{+1} + \phi_{+1} d) + T_o]}$$

implying  $q < q^*$  since  $U'' < 0$ . Since all middle-aged agents hold the same amount of money in equilibrium,  $d$  will be the same in all matches.

### 4.3.2 Competitive pricing

Now consider the case where middle-aged agents trade in an anonymous Walrasian market. Anonymity still makes money essential. In this market, there is a market price  $p$  that agents take parametrically. With equal numbers of buyers and sellers  $q_b = q_s = q$  in equilibrium. Price-taking by agents is used here solely as a benchmark since allows us to make a direct comparison to the Ramsey literature. While one may argue that looking at competitive pricing really makes this a cash-in-advance model, Rocheteau and Wright (2005) show that competitive search will replicate the Walrasian equilibrium when there are no search externalities. Thus, one can simply view the Walrasian pricing case discussed here as a ‘short cut’ to solving for the competitive search equilibrium much like one can solve a planner’s problem to get the perfectly competitive equilibrium allocation.

A buyer’s problem is

$$\begin{aligned} \max_d \quad & u\left(\frac{d}{p}\right) + \beta U(C_{o,+1}^b) \\ \text{s.t.} \quad & d \equiv pq_b \leq m + \pi M_{-1} \\ & C_{o,+1}^b = \phi_{+1} (m + \pi M_{-1} - d) (1 + \tau_c)^{-1} + T_o \end{aligned}$$

An interior solution satisfies

$$u'(q) - \beta U'(C_{o,+1}^b) \frac{\phi_{+1}}{1 + \tau_c} p - \xi p = 0 \tag{18}$$

where  $\xi$  is the Lagrangian on the spending constraint. The Inada condition on  $U(\cdot)$  implies  $\xi = 0$  while the Inada condition on  $u(\cdot)$  ensures that the buyer spends some of his money.

A seller faces the following problem

$$\begin{aligned} \max_{\hat{d}} \quad & -\psi\left(\frac{\hat{d}}{p}\right) + \beta U(C_{o,+1}^s) \\ & \hat{d} \equiv pq_s \\ C_{o,+1}^s = & \phi_{+1} \left( m + \pi M_{-1} + \hat{d} \right) (1 + \tau_c)^{-1} + T_o \end{aligned}$$

where the FOC for an interior solution gives

$$-\psi'(q_s) + \beta U'(C_{o,+1}^s) \frac{\phi_{+1}}{1 + \tau_c} p = 0. \quad (19)$$

A sufficient condition for an interior solution is  $\psi'(0) = 0$ . Under this assumption, combining (18), (19), market clearing, i.e.,  $q_b = q_s = q$ , and  $M_{+1} = m + \pi M_{-1}$  yields

$$u'(q) = \psi'(q) \frac{U' \left[ (1 + \tau_c)^{-1} (\phi_{+1} M_{+1} - \phi_{+1} pq) + T_o \right]}{U' \left[ (1 + \tau_c)^{-1} (\phi_{+1} M_{+1} + \phi_{+1} pq) + T_o \right]} \quad (20)$$

implying once again that  $q < q^*$ .

## 5 Equilibrium

Differentiate (13) to get

$$\begin{aligned} V'(m) = & \sigma \left[ u'(q_b) \frac{\partial q_b}{\partial m} - \beta U'(C_{o,+1}^b) \frac{\phi_{+1}}{1 + \tau_c} \frac{\partial d}{\partial m} \right] \\ & + \sigma \left[ -\psi'(q_s) \frac{\partial q_s}{\partial m} + \beta U'(C_{o,+1}^s) \frac{\phi_{+1}}{1 + \tau_c} \frac{\partial \hat{d}}{\partial m} \right] \\ & + \frac{\phi_{+1}}{1 + \tau_c} \beta \left[ \sigma U'(C_{o,+1}^b) + \sigma U'(C_{o,+1}^s) + (1 - 2\sigma) U'(C_{o,+1}^o) \right] \end{aligned} \quad (21)$$

To solve this expression we need to determine the partial derivatives for each pricing scheme.

## 5.1 Bargaining

With bargaining, in general the derivatives are complicated expressions. However, since the constrained optimum allocation gives the seller zero surplus, this appears to be equivalent to buyer-take-all. Thus, in order to give the monetary equilibrium the best shot at replicating the constrained optimum, consider  $\theta = 1$ . Thus, the seller's surplus is zero and (13) becomes

$$V(m) = \sigma [u(q_b) + \beta U(C_{o,+1}^b)] + \beta(1 - \sigma)U(C_{o,+1}^o)$$

and

$$\begin{aligned} V'(m) = & \sigma \left[ u'(q_b) \frac{\partial q_b}{\partial m^b} - \beta U'(C_{o,+1}^b) \frac{\phi_{+1}}{1 + \tau_c} \frac{\partial d}{\partial m^b} \right] \\ & + \beta \frac{\phi_{+1}}{1 + \tau_c} [\sigma U'(C_{o,+1}^b) + (1 - \sigma) U'(C_{o,+1}^o)] \end{aligned} \quad (22)$$

The Appendix contains the solutions for  $\partial q_b / \partial m^b$  and  $\partial d / \partial m^b$  and shows that (22) reduces to

$$V'(m) = \frac{\phi_{+1}}{1 + \tau_c} \beta [\sigma U'(C_{o,+1}^b) + (1 - \sigma) U'(C_{o,+1}^o)] \quad (23)$$

What does this mean? Since agents do not spend all of their money in the DM, the marginal liquidity value of an additional unit of money in the DM is zero. Thus, the marginal value of money is simply its value for consumption when old. Interestingly, this means that when choosing money balances when young, agents only care about money's 'store of value' function on the margin and not its marginal liquidity value as a medium of exchange.

Combining (23) with (14) and (15) gives

$$U'(C_y) = \frac{\phi_{+1} \hat{\beta}}{\phi_{-1}} [\sigma U'(C_{o,+1}^b) + (1 - \sigma) U'(C_{o,+1}^o)]. \quad (24)$$

Since the consumption tax is assumed to be constant across time and across agents, it does

not affect the marginal value of a unit of money across time and we get something that looks like a standard consumption Euler equation. However, note that this is not a standard Euler equation because the term in brackets on the right-hand-side of this expression is not expected marginal utility from consuming when old because  $U'(C_{o,+1}^s)$  is missing. Why? Because with  $\theta = 1$  there is a holdup problem on the *sellers*. If an agent brings an additional unit of money into the DM and becomes a seller, the entire marginal surplus for the seller from this money is extracted by the buyer. So when choosing money balances, young agents ignore any value from a marginal unit of money should they be a seller in the DM.<sup>10</sup>

However, unlike holdup problems on buyers, this holdup problem induces young agents to bring in *more* money from the CM, *ceteris paribus*. Why? Young agents choose to hold money based on the expected marginal utility of consuming when old. The holdup problem eliminates a low marginal utility consumption state when old,  $U'(C_{o,+1}^s)$ , causing young agents to put more weight on the high marginal utility consumption states,  $U'(C_{o,+1}^b)$  and  $U'(C_{o,+1}^o)$ , when old. This increases the marginal value of money when old and thus the demand for money when young. Consequently, the goods price of money,  $\phi$ , falls thereby raising the real value of money in the CM. While this is ‘good’ for average old age consumption, it will also have a tendency to create greater consumption dispersion among the old, which is ‘bad’. If the latter effect dominates, then the holdup problem on the seller will tend to be welfare reducing.

Consider a steady state with  $\phi_{+1}M_{+1} = \phi_{-1}M_{-1} = \Omega$ , e.g., real balances in the CM are constant across time, and  $\phi_{+1}/\phi_{-1} = 1/\gamma = R_m$  where  $R_m$  is the gross rate of return on money. Furthermore, real spending (measured in the next CM goods price) in the DM is stationary,  $\phi_{+1}d = \delta$ . A steady state equilibrium for the bargaining model is a list

---

<sup>10</sup>In Lagos-Wright, there is a holdup problem on the buyer but not the seller since the seller’s money balances do not affect the bargaining solution due to linearity of utility in the CM. In LW then,  $\theta = 1$  eliminates the holdup problem on the buyer. Here,  $\theta = 1$  *maximizes* the holdup problem on sellers.

$\{\Omega, q, \delta, C_o^b, C_o^s, C_o^o, C_y, h\}$  solving

$$h = C_y + \sigma C_o^b + \sigma C_o^s + (1 - 2\sigma) C_o^o \quad (25)$$

$$C_o^o = \Omega (1 + \tau_c)^{-1} + T_o \quad (26)$$

$$C_o^b = (\Omega - \delta) (1 + \tau_c)^{-1} + T_o \quad (27)$$

$$C_o^s = (\Omega + \delta) (1 + \tau_c)^{-1} + T_o \quad (28)$$

$$U'(C_y) = \frac{1 + \tau_c}{1 - \tau_h} v' \left( C_y + \frac{\Omega}{1 + \tau_c} + T_o \right) \quad (29)$$

$$\psi(q) = \beta \{ U[(1 + \tau_c)^{-1} (\Omega + \delta) + T_o] - U[(1 + \tau_c)^{-1} \Omega + T_o] \} \quad (30)$$

$$\frac{u'(q)}{\psi'(q)} = \frac{U'[(1 + \tau_c)^{-1} (\Omega - \delta) + T_o]}{U'[(1 + \tau_c)^{-1} (\Omega + \delta) + T_o]} \quad (31)$$

$$U'(C_y) = \hat{\beta} R_m \{ \sigma U'[(1 + \tau_c)^{-1} (\Omega - \delta) + T_o] + (1 - \sigma) U'[(1 + \tau_c)^{-1} \Omega + T_o] \} \quad (32)$$

where (25) is the CM aggregate resource constraint and comes from substituting the government's budget constraint into the young's CM budget constraint. Using (25)-(28), the last four equations give us  $q, \Omega, \delta$  and  $C_y$  as functions of  $R_m = 1/\gamma$ .

Finally, note that if  $\sigma = 0$  it is straightforward to show that the unconstrained optimum  $C = C^*$  for all agents can be achieved ( $q$  is then irrelevant) by setting  $\tau_c = \tau_h = T_o = 0$  and  $\hat{\beta} R_m = 1$  in the monetary economy. However, in this case, the unconstrained optimum can also be replicated by removing money from the economy and setting  $T_o = C^* = -T_y, \tau_c = \tau_h = 0$ . In short, money is not essential if the government has access to lump-sum taxes and transfers. Thus, the OLG structure is not what makes money essential in this model. Rather it is the search market with private information on preferences that is critical for the results we obtain below regarding the optimal fiscal policy. The OLG structure itself merely keeps the distribution of money balances tractable.

## 5.2 Price taking

With price taking, differentiating the cash constraints for both buyers and sellers yields

$$\begin{aligned}\frac{\partial q_b}{\partial m} &= \frac{\partial q_b}{\partial d} \frac{\partial d}{\partial m} = \frac{1}{p} \frac{\partial d}{\partial m} \\ \frac{\partial q_s}{\partial \hat{d}} &= \frac{\partial q_s}{\partial \hat{d}} \frac{\partial \hat{d}}{\partial m} = \frac{1}{p} \frac{\partial \hat{d}}{\partial m}\end{aligned}$$

and so from (18) and (19) the square bracketed terms in (21) are zero leaving

$$V'(m) = \frac{\phi_{+1}}{1 + \tau_c} \beta [\sigma U'(C_{o,+1}^b) + \sigma U'(C_{o,+1}^s) + (1 - 2\sigma) U'(C_{o,+1}^o)]. \quad (33)$$

Combining (14), (15) and (33) yields

$$U'(C_y) = \frac{\phi_{+1} \hat{\beta}}{\phi_{-1}} [\sigma U'(C_{o,+1}^b) + \sigma U'(C_{o,+1}^s) + (1 - 2\sigma) U'(C_{o,+1}^o)] \quad (34)$$

Price taking eliminates the holdup problem on sellers associated with buyer-take-all bargaining so  $U'(C_{o,+1}^s)$  now appears in the intertemporal Euler equation since the bracketed term on the right-hand side of (34) is  $EU'(C_{o,+1})$ .

Consider a steady state with  $\phi_{+1}M_{+1} = \phi_{-1}M_{-1} = z$ , i.e., real balances in the CM are constant across time and real spending in the DM is constant,  $\phi_{+1}pq = \omega$ . A steady state

equilibrium for this economy is a list  $\{z, q, \omega, C_o^b, C_o^s, C_o^o, C_y, h\}$  solving

$$h = C_y + \sigma C_o^b + \sigma C_o^s + (1 - 2\sigma) C_o^o \quad (35)$$

$$C_o^o = z(1 + \tau_c)^{-1} + T_o \quad (36)$$

$$C_o^b = (z - \omega)(1 + \tau_c)^{-1} + T_o \quad (37)$$

$$C_o^s = (z + \omega)(1 + \tau_c)^{-1} + T_o \quad (38)$$

$$U'(C_y) = \frac{1 + \tau_c}{1 - \tau_h} v' [C_y + z(1 + \tau_c)^{-1} + T_o] \quad (39)$$

$$\psi'(q)q = (1 + \tau_c)^{-1} \beta U' [(1 + \tau_c)^{-1} (z + \omega) + T_o] \omega \quad (40)$$

$$\frac{u'(q)}{\psi'(q)} = \frac{U' [(1 + \tau_c)^{-1} (z - \omega) + T_o]}{U' [(1 + \tau_c)^{-1} (z + \omega) + T_o]} \quad (41)$$

$$U'(C_y) = \hat{\beta} R_m [\sigma U'(C_o^b) + \sigma U'(C_o^s) + (1 - 2\sigma) U'(C_o^o)] \quad (42)$$

Equation (40) is obtained by multiplying the steady state value of (19) by  $q$ . Using (35)-(38) the last four equations give us  $z, q, \omega$  and  $C_y$  as a function of  $R_m = 1/\gamma$ . Given the solutions for  $\omega$  and  $q$  we can find the steady state relative price of goods across the second and third periods as  $\omega/q = \phi_{+1}p$ . Since we have the price sequence of  $\phi$  given an initial value of the money stock  $M_0$ , we then have the price sequence  $p = \gamma\omega/q\phi$ .

## 6 Planner vs equilibrium allocations

In general, it is difficult to compare the constrained optimal allocation to the equilibrium allocations. However, we can compare the necessary conditions that must be satisfied to gain some insight as to how the allocations differ.

First, compare the intertemporal Euler equations (24) and (34) that comes out of the monetary equilibrium to the intertemporal Euler equation chosen by the planner (8). As discussed above, bargaining leads to holdup problems since agents incur sunk costs of ac-

quiring money. Consequently, when comparing the constrained optimal allocation to the equilibrium allocation, one would like to separate the effects of the informational friction from the holdup problem. Since competitive markets eliminate holdup problems, consider the competitive pricing Euler equation for now. From (34)

$$U'(\hat{C}_y) = \hat{\beta} R_m E U'(\hat{C}_o) \quad (43)$$

where  $\hat{C}$  denotes equilibrium consumption in the price taking model. At the Friedman rule,  $\hat{\beta} R_m = 1$ , so we have<sup>11</sup>

$$U'(\hat{C}_y) = E U'(\hat{C}_o). \quad (44)$$

Now compare this to (8):

$$\frac{1}{U'(\bar{C}_y)} = E \left[ \frac{1}{U'(\bar{C}_o)} \right] \quad (45)$$

where the bar denotes the planner's allocation.

Note that (45) is the same expression obtained in the models of Golosov, Kocherlakota and Tysvinski (2003) and Kocherlakota (2005).<sup>12</sup> By Jensen's inequality, evaluating (44) at the planner's allocation yields

$$U'(\bar{C}_y) < E U'(\bar{C}_o).$$

The planner wants to create a wedge between the marginal utility of consuming when young relative to the expected marginal utility of consuming when old. This wedge arises because of the incentive constraints and it is the reason why the government resorts to distortionary wealth taxation in the new dynamic public finance literature. The incentive problem in those models is to induce high productivity agents to produce a large amount of goods rather than

---

<sup>11</sup>This is the Friedman rule since for this value of  $\gamma$  the return on money is equivalent to that of a nominal bond traded only in the CM. This bond is not traded but can be priced.

<sup>12</sup>Equation (1) of GKT is  $1/U(C_t) = (\beta R_{t+1})^{-1} E[1/U'(C_{t+1})]$  which is the same as the one here for  $\beta R_{t+1} = 1$  or when the market rate of interest is equal to the time rate of discount.

lie and say they received a low productivity shock. The intuition in those models for this wedge is that if agents can save and earn interest to finance old age consumption, at some point their wealth is so large that they will lie when old and say they are ‘unproductive’. They get less consumption from the planner but make up the difference by consuming out of wealth and working less. Thus to induce truthful revelation of one’s productivity state, you need distortionary taxation of wealth (more precisely distorting the rate of return on capital) to reduce saving and future wealth.

A similar effect is happening here; the planner needs middle-aged productive agents to say they are productive and not lie. To do so the planner has to reward them with sufficiently high old age consumption from spending their acquired money balances. However, the planner does not want those real balances to earn an excessively high rate of return to ensure better risk sharing when old. So from (43) the planner allocation would require  $\hat{\beta}R_m < 1$ , i.e.,  $\gamma > \hat{\beta}$ , meaning steady state inflation above the Friedman rule. At the Friedman rule, real balances are too high meaning average consumption is too high and too diverse. By inflating, the government brings down real balances, reducing average old age consumption (and possibly  $q$ ), which is ‘bad’ but it also reduces consumption dispersion in the last period, which is good. On net, inflating above the FR raises welfare.

This trade-off between production efficiency in the DM and risk sharing in the CM does not arise in the Lagos-Wright model because of linearity of preferences in the CM. Linearity of preferences effectively makes agents ‘risk neutral’ with regards to hours/consumption fluctuations. Consequently, the optimal policy is to ensure that production in the DM is efficient and occurs under the Friedman rule.<sup>13</sup> This can easily be seen by setting  $U(C) = C$ ,  $U' = 1$  and  $\hat{\beta}R_m = 1$  in the equations above.

---

<sup>13</sup>With Nash bargaining, production is not efficient at the Friedman rule due to the non-monotonicity of the Nash solution. Under monotonic bargaining solutions, production is efficient at the Friedman rule (see Rocheteau and Waller (2005)). A similar finding applies if agents randomly received endowments in the DM, rather than produce, in the LW model (see Reed and Waller (2005)).

The Euler equation for bargaining differs not only in the fact that the planner's Euler equation involves the inverse of marginal utility when old but also because  $U'(C_o^s)$  does not appear in (24) due to the holdup problem. Nevertheless, bargaining with buyer-take-all is much closer to the constrained optimum than competitive pricing on another dimension because the planner gives a DM producer no surplus. In short, (11) and (30) are the same equation. Price taking typically does not generate this condition since (40) replaces (30) with competitive pricing.

Secondly, how does the constrained optimal value of  $q$  differ from what occurs in equilibrium? Under either bargaining or price taking we get the following equilibrium condition

$$\frac{u'(q)}{\psi'(q)} = \frac{U'(C_o^b)}{U'(C_o^s)}$$

while the constrained optimal allocation yields

$$\frac{u'(q)}{\psi'(q)} = \frac{U'(C_y)}{U'(C_o^s)}$$

Thus, for a given bundle of old age consumption (a given amount of consumption risk) in the CM, the planner wants more  $q$  to be produced than occurs in the monetary equilibrium since  $U'(C_o^b) > U'(C_y)$ . In short, buyers do not spend enough real balances, holding onto their cash in order to consume when old. Alternatively, if the value of  $q$  is the same under both the planner and equilibrium allocations, then the planner would give sellers less old age consumption and buyers more old age consumption than occurs in the equilibrium, e.g., less consumption risk. This shows that, in general, the monetary equilibrium will not achieve the optimal trade-off between productive efficiency and risk sharing.

How do these findings compare to the results on the optimality of the inflation tax in Ramsey models or in da Costa and Werning (2005)? CCK's (1996) key result is that if pref-

erences are homothetic and weakly separable in consumption and leisure, then the Friedman rule is the first-best policy for all three short-cut monetary models they consider (MIU, CIA, shopping time). Chari and Kehoe (1999) show that even with an OLG framework, homotheticity and separability lead to non-distortionary taxation of capital (i.e., wealth), which carries over to the issue of taxing money in an OLG framework. Indeed, as discussed above, if  $\sigma = 0$  the Friedman rule is optimal since it replicates the unconstrained optimum. This suggests it is not the OLG structure per se that drives my results.

What is surprising about the results obtained here is that the suboptimality of the Friedman rule occurs *even though* preferences are assumed to be separable in consumption and leisure and do not hinge on homotheticity. One would like to know if the different results are due to: 1) the homogeneity of agents in the Ramsey approach versus heterogeneous agents in the Mirrlees approach or 2) differences between short-cut models of money and ones in which money is essential. In general this is difficult to say. However, it cannot be the private information friction alone since, using a dynamic Mirrlees approach but the same short-cut monetary models as CCK, da Costa and Werning (2005) obtain equivalent results as CCK – the Friedman rule is optimal if preferences are weakly separable in consumption and leisure. Thus, the suboptimality of the Friedman rule in the model presented here is driven by a combination of the information friction and the record-keeping friction that makes money essential as opposed to the homotheticity and separability properties of preferences.

## 7 Examples of all three allocations

To make these findings more concrete, in this section I construct analytical examples of all three allocations to determine the optimal fiscal policy. Let preferences in the CM be homothetic and given by  $U(C) = \ln C$ ,  $v(h) = h$ ,  $u(q) = 1 - \exp^{-q}$  and  $\psi(q) = \alpha q$  with

$\alpha < 1/2$ . It follows that  $C^* = 1 = C_y$  in all examples and  $q^* = -\ln \alpha > 0$ .<sup>14</sup> For the monetary economy let  $\hat{\beta}/\gamma = \hat{\beta}R_m = x$  with  $x \leq 1$  so an increase in  $x$  means an increase in the return on money (a decrease in  $\gamma$ ) while  $x = 1$  is the Friedman rule. Finally, since  $U(0) = \ln 0 \rightarrow -\infty$ , the young's participation constraint will not be binding.

## 7.1 Constrained optimal allocation

With these preferences, from (8)-(11) we obtain

$$\bar{C}_o^s = 1 + \frac{1}{\sigma}\varepsilon; \quad \bar{C}_o^b = \bar{C}_o^o = 1 - \frac{1}{1-\sigma}\varepsilon \quad (46)$$

$$\bar{q} = q^* - \ln\left(1 + \frac{1}{\sigma}\varepsilon\right) < q^* \quad (47)$$

where the bars denote the constrained optimal allocation and  $\varepsilon$  solves

$$\frac{1}{\alpha^\alpha} = \frac{\left(1 + \frac{1}{\sigma}\varepsilon\right)^{\alpha+\beta}}{\left(1 - \frac{1}{1-\sigma}\varepsilon\right)^\beta} = f(\varepsilon) \quad (48)$$

Note that  $f(0) = 1 < 1/\alpha^\alpha$  and goes to infinity as  $\varepsilon \rightarrow 1 - \sigma$ . Furthermore, since  $f(\varepsilon)$  is continuous with  $f'(\varepsilon) > 0$  there is a unique value  $0 < \varepsilon^* < 1 - \sigma$  solving this expression. Consequently, a unique steady state solution to the planner problem exists. It then follows that  $h = 2C^*$ , which is the same as in the unconstrained optimum. So the planner does not want the young agents to bear any costs of distorting the old agents' consumption from the unconstrained optimum.

---

<sup>14</sup>Note that with these functional forms  $u'$  does not satisfy the Inada condition nor is  $\psi'(0) = 0$  satisfied, both of which were imposed in the price-taking equilibrium to ensure an interior solution. Restricting  $\alpha < 1/2$  can be shown to be a sufficient for an interior solution to exist.

## 7.2 Example of the bargaining equilibrium

From (29) we have

$$C_y = \frac{1 - \tau_h}{1 + \tau_c} \quad (49)$$

Let  $B = \Omega + (1 + \tau_c)T_o$ . From (32) we obtain  $\delta = \varphi B$  with

$$0 < \varphi \equiv \frac{B - (1 - \tau_h)x}{B - (1 - \sigma)(1 - \tau_h)x} \leq 1 \quad (50)$$

where  $\varphi$  is the fraction of real balances spend in the DM. Thus old age consumption is

$$\tilde{C}_o^{s} = \frac{1 + \varphi}{1 + \tau_c} B; \quad \tilde{C}_o^{b} = \frac{1 - \varphi}{1 + \tau_c} B; \quad \tilde{C}_o^o = \frac{1}{1 + \tau_c} B \quad (51)$$

and so

$$h = \frac{1 - \tau_h}{1 + \tau_c} + \frac{1}{1 + \tau_c} B \quad (52)$$

Using these expressions in (31) yields

$$\tilde{q} = -\ln \alpha + \ln \left( \frac{1 - \varphi}{1 + \varphi} \right) < q^*$$

where  $\tilde{q}$  denotes the equilibrium value of  $q$  in the bargaining model. Substitute this and  $C_o^o$  into (30) to obtain:

$$\frac{1}{\alpha^\alpha} = \frac{(1 + \varphi)^{\alpha + \beta}}{(1 - \varphi)^\alpha} = g(\varphi). \quad (53)$$

Note that  $g(0) = 1 < 1/\alpha^\alpha$  and goes to infinity as  $\varphi$  goes to one. Since  $g(\varphi)$  is continuous and  $g'(\varphi) > 0$  there is a unique value  $0 < \tilde{\varphi} < 1$  satisfying (53). Even though  $\varphi$  depends on  $x, \tau_c, T_o$  and  $\Omega$ , the solution  $\tilde{\varphi}$  is unchanged when  $x, \tau_h$  and  $T_o$  change. This means that the fraction of real balances spent in the DM is constant, which combined with the homotheticity of  $U(C)$  means that  $\tilde{q}$  is unaffected by changes in fiscal policy. Consequently, while inflation

reduces the value of real balances and therefore expected old age consumption, it does so in a way that leaves  $U'(C_o^b)/U'(C_o^s)$  and  $\tilde{q}$  unchanged. Contrary to standard monetary search models, inflation has no effect on the intensive margin in the DM. From (7d) we get

$$B = \left(1 + \frac{\sigma\tilde{\varphi}}{1-\tilde{\varphi}}\right) (1 - \tau_h) x. \quad (54)$$

**Optimal fiscal policy** What is the optimal fiscal policy in this economy? Since  $\tilde{\varphi}$  is independent of  $\gamma$ ,  $\tau_c, \tau_h$  or  $T_o$ ,  $\tilde{q}$  is also unaffected. Maximizing the steady-state, lifetime utility of the current middle-aged generation and future generations reduces to

$$\max_{x, \tau_c, \tau_h, T_o} \beta \left[ \sigma U(\tilde{C}_{o,+1}^s) + \sigma U(\tilde{C}_{o,+1}^b) + (1 - 2\sigma) U(\tilde{C}_{o,+1}^o) + U(C_{y,+1}) - v(h_{+1}) \right]$$

that upon substituting in (49) and (51) becomes

$$\max_{x, \tau_c, \tau_h, T_o} \beta \left[ 2 \ln \frac{1 - \tau_h}{1 + \tau_c} + \ln x - \frac{1 - \tau_h}{1 + \tau_c} \left(1 + \frac{\sigma\tilde{\varphi}}{1 - \tilde{\varphi}}\right) x - \frac{1 - \tau_h}{1 + \tau_c} + \ln \left(1 + \frac{\sigma\tilde{\varphi}}{1 - \tilde{\varphi}}\right) + \sigma \ln(1 - \varphi^2) \right]$$

Since  $T_o$  does not appear in this optimization problem it can be ignored (I will return later to why this is so). The first-order conditions yield

$$1 = \frac{1 - \tau_h}{1 + \tau_c} \left(1 + \frac{\sigma\tilde{\varphi}}{1 - \tilde{\varphi}}\right) x \quad (55)$$

$$\tau_c = -\tau_h \quad (56)$$

with the levels of  $\tau_c$  and  $-\tau_h$  being irrelevant. The condition that  $\tau_c = -\tau_h$  implies that the tax rate on consumption and ‘leisure’ should be the same, which is an application of the uniform taxation theorem since preferences are separable. Furthermore, since the tax rates are zero, it is optimal not to distort consumption and ‘leisure’ on the margin. It then follows

from (55) that  $1 = B/(1 + \tau_c)$  so

$$\tilde{C}_o^b = 1 - \tilde{\varphi}; \quad \tilde{C}_o^s = 1 + \tilde{\varphi}; \quad \tilde{C}_o^o = \tilde{C}_y = 1 = C^*; \quad \tilde{h} = 2C^* \quad (57)$$

From (54) we obtain

$$x^* = \frac{1 - \tilde{\varphi}}{1 - \tilde{\varphi} + \sigma\tilde{\varphi}} < 1$$

and since  $\hat{\beta}R_m^* = \hat{\beta}/\gamma = x^*$  the optimal gross inflation rate is

$$\gamma^* = \left(1 + \frac{\sigma\tilde{\varphi}}{1 - \tilde{\varphi}}\right) \hat{\beta} > 1$$

which is above the Friedman rule. Note that if we set  $\sigma = 0$ , which effectively shuts down the DM, then  $\gamma^* = \hat{\beta}$  and the Friedman rule is the optimal policy. So why is the Friedman rule suboptimal if  $\sigma > 0$ ? When  $\sigma > 0$  there is consumption risk when old, so the marginal value of money is higher as agents hold money for precautionary reasons. This increases the goods price of money in the CM and the amount of work the young need to provide to acquire those real balances. Running the Friedman rule makes real balances too large and so the young have to work more than is optimal to acquire those real balances. So increasing  $\gamma$  reduces old age consumption and the amount of work by the young. The quantity of goods traded in the DM is unaffected. In short, the planner is trading off average consumption when old against excessive hours worked by the young.

This intuition is very close to Aiyagari's (1995) result on positive capital taxation. Aiyagari examines capital taxation in a Bewley economy where agents have idiosyncratic trading histories and information frictions lead to incomplete markets and borrowing constraints. The idiosyncratic trading history generates a 'precautionary demand for capital' that leads to overaccumulation of capital and a rate of return that is too low. Thus, capital taxation is needed to lower the capital stock back to the optimal level. The result I obtain above on

taxation of money balances when  $\sigma > 0$  is essentially the same explanation. When there is idiosyncratic trading histories in the DM, agents have a precautionary motive for holding money to old age. This raises the real value of money above what is socially optimal. In short there is an ‘overaccumulation of real balances’ that can be eliminated by inflating above the Friedman rule.

Finally, if  $\tau_c = \tau_h = 0$  then  $T_o = -T_y$  but these transfers have no effect on the allocation since  $B^* = 1 = \Omega + T_o$ . Why? If  $T_o > 0$ , young agents know they will be getting a transfer of goods when old. This lowers the marginal value of carrying a unit of money into old age and therefore decreases the demand for money when young. It follows that the goods price of money in the CM,  $\phi$ , decreases thereby reducing the equilibrium value of real balances by exactly the value of the transfer. Consequently, consumption when old is unaffected by the transfer. More succinctly, real balances and transfers are perfect substitutes. While the current young have to work more to provide the transfer to the current old, they work less to earn the needed real balances. On net, labor hours are unchanged. So setting  $T_o = T_y = 0$  is consistent with maximizing the welfare of the representative agent.

Can the optimal fiscal policy replicate the constrained optimum? In general no but consider the special case of  $\sigma = 1/2$  so that there are only buyers and sellers in the DM.<sup>15</sup> Then from (50)

$$\varphi = 2 \frac{B - (1 - \tau_h) x}{2B - (1 - \tau_h) x}$$

and for the constrained planner allocation, (48) reduces to

$$\frac{1}{\alpha^\alpha} = \frac{(1 + 2\varepsilon)^{\alpha+\beta}}{(1 - 2\varepsilon)^\beta} \tag{58}$$

---

<sup>15</sup>In this case  $C_o^o$  is simply the old age consumption of an agent who walks away from a match.

while (53) becomes

$$\frac{1}{\alpha^\alpha} = \frac{\left(1 + 2 \frac{B - (1 - \tau_h)x}{2B - (1 - \tau_h)x}\right)^{\alpha + \beta}}{\left(1 - 2 \frac{B - (1 - \tau_h)x}{2B - (1 - \tau_h)x}\right)^\alpha}. \quad (59)$$

Notice that (58) is the same as (59) for

$$\varepsilon = \frac{B - (1 - \tau_h)x}{2B - (1 - \tau_h)x} \quad (60)$$

But since there is a unique value  $\varepsilon^*$  solving (58) this means for any change in  $x, \tau_h$  or  $T_o, \Omega$  simply adjusts proportionately such that  $\tilde{\varphi} = 2\varepsilon^*$  is unchanged. This implies  $\tilde{q} = \bar{q}$  and from (46) and (57) the consumption values in the CM are equal to the planner's choices as well. Thus, for the special case  $\sigma = 1/2$  the constrained optimum can be implemented via the use of distortionary taxation of money and uniform taxation of consumption and leisure.

How important is the restriction of linear tax rates on consumption and labor? Kocherlakota (2005) shows that the optimal tax structure is to use regressive tax rates on capital based on an agent's reported labor income. If reported labor income is low, the agent's capital tax rate is higher than if labor income is reported to be high. Why? The government has to discourage agents from strategically over-saving to finance future consumption and then shirking (reporting low labor income) when they are highly productive. It is able to do this by imposing a regressive tax on capital based on reported labor income.

In the model presented here there are some subtle differences to the standard NDPF model. First, labor is provided by the young who have no private information about their productivities. Thus non-linear taxation of labor should not be useful here as it is in Kocherlakota. Second, wealth accumulation by the middle-aged sellers does not give them an intertemporal strategy to misreport their old age productivities since they cannot work when old (i.e., their productivity when old is public information). Thus an important extension of this model would be to allow old agents to work with differing productivities that are private

information. This will allow the model to map more directly into the NDPF framework.

While I have ruled out non-linear consumption taxes, it would be nice to know if the government would prefer to run the Friedman rule and use a non-linear consumption tax scheme to replicate the constrained optimal allocation. Since money balances are private information, no progressive tax rate can do the job since agents can always break their consumption purchases from firms into ‘small’ amounts to pay the minimum consumption tax. Thus, a regressive sales tax is the only way to get agents to fully reveal their money balances and productivity through old age consumption. It seems that if the government implemented a positive sales tax on average but made it regressive it may be able to: 1) create the needed wedge between current marginal utility and expected future marginal utility, 2) create the optimal amount of consumption risk, and 3) use non-distortionary taxation of money. I leave this issue to future research.

### 7.3 Example of the perfectly competitive equilibrium

Finally, consider the monetary economy where prices are determined competitively. Let  $\mathcal{Z} = z + (1 + \tau_c) T_o$ . From (42) we get  $\omega = \rho \mathcal{Z}$  where

$$0 < \rho \equiv \left[ \frac{\mathcal{Z} - (1 - \tau_h) x}{\mathcal{Z} - (1 - 2\sigma)(1 - \tau_h) x} \right]^{1/2} < 1. \quad (61)$$

This implies  $\hat{C}_o^s = (1 + \rho) \mathcal{Z}$  and  $\hat{C}_o^b = (1 - \rho) \mathcal{Z}$ . As a result, (41) yields

$$\hat{q} = -\ln \alpha + \ln \left( \frac{1 - \rho}{1 + \rho} \right) < q^*$$

where  $\hat{q}$  is the solution for  $q$  in the monetary equilibrium with price taking. Finally (40) gives

$$\hat{q} = \frac{\beta \rho}{\alpha (1 + \rho)}.$$

Using these last two expressions for  $\hat{q}$  we get

$$-\ln \alpha + \ln \left( \frac{1 - \rho}{1 + \rho} \right) = \frac{\beta \rho}{\alpha (1 + \rho)}.$$

It is easy to show that a unique value  $0 < \hat{\rho} < 1$  solves this expression. Furthermore, while  $\rho$  depends on  $x$  from (61) for any change in  $x$ ,  $z$  simply adjusts proportionately such that  $\hat{\rho}$  continues to solve this equation. As in the bargaining case, this implies that buyers spend a constant fraction of real balances in the DM and with homothetic preferences,  $\hat{q}$  is unaffected by changes in the inflation rate. Finally, one can easily show that  $\hat{\rho} > \tilde{\varphi}$  meaning the quantity consumed in the DM is larger with competitive pricing than with bargaining.

It then follows that

$$\mathcal{Z} = (1 - \tau_h) \left( \frac{1 - \hat{\rho}^2 + 2\sigma\hat{\rho}^2}{1 - \hat{\rho}^2} \right) x.$$

**Optimal fiscal policy** Finally, what is the optimal inflation rate in this economy? Since  $\hat{q}$  is independent of  $\gamma, \tau_c, \tau_h$  and  $T_o$  maximizing the lifetime welfare of the representative young agent reduces to

$$\max_{x, \tau_c, \tau_h, T_o} = 2 \ln \frac{1 - \tau_h}{1 + \tau_c} - \frac{1 - \tau_h}{1 + \tau_c} + \ln x - \frac{1 - \tau_h}{1 + \tau_c} \left( 1 + \frac{2\sigma\hat{\rho}^2}{1 - \hat{\rho}^2} \right) x + \ln \left( 1 + \frac{2\sigma\hat{\rho}^2}{1 - \hat{\rho}^2} \right) + \sigma \ln (1 - \hat{\rho}^2)$$

This problem is equivalent to the one for bargaining. So the optimal values are

$$x^* = \frac{1 - \hat{\rho}^2}{1 - \hat{\rho}^2 + 2\sigma\hat{\rho}^2} < 1$$

$$\tau_c = -\tau_h$$

with  $T_o$  being irrelevant. We then have

$$\hat{C}_o^s = 1 + \rho; \quad \hat{C}_o^b = 1 - \rho; \quad \hat{C}_o^o = \hat{C}_y = 1 = C^*; \quad h = 2C^*.$$

Using the definition of  $x$  we obtain

$$\gamma^* = \left(1 + \frac{2\sigma\hat{\rho}^2}{1 - \hat{\rho}^2}\right)\hat{\beta} > \hat{\beta}$$

Again, the Friedman rule is not optimal and for the same reasons as discussed for the bargaining case. However, even with  $\sigma = 1/2$  the constrained optimal allocation cannot be replicated under  $\gamma^*$  since  $\hat{\rho} > \tilde{\varphi}$  implying  $\tilde{q} > \hat{q}$ . In the competitive pricing equilibrium,  $q$  is larger in the DM than the quantity desired by the planner, average old consumption is the same but the variance of old age consumption is larger. Thus, while the competitive pricing equilibrium has greater efficiency in the DM, it generates more consumption inequality and consumption risk in the CM.

## 8 Conclusion

We observe distortionary taxation of wealth by governments to redistribute wealth and to achieve a more equitable distribution of consumption across agents. Since standard Ramsey analyses suggest lump-sum taxation achieves the first best allocation, the use of distorting taxes is puzzling. Research in the NDPF literature has provided important insights as to why governments may use distortionary taxation of wealth, via capital income taxes, even though lump sum taxes are available. This paper has shown that since money is always a form of wealth, even when used for transaction purposes, it is not exempt from the same arguments.

## References

- [1] Aiyagari, S. R. (1995). “Optimal Capital Taxation with Incomplete Markets, Borrowing Constraints and Constant Discounting.” *Journal of Political Economy* 106, 1158-1175.
- [2] Berentsen, A., G. Camera and C. Waller (2005). “The Distribution of Money Balances and the Non-Neutrality of Money.” *International Economic Review* 46, 465-487.
- [3] Chari, V.V., L. Christiano and P. Kehoe (1996). “Optimality of the Friedman Rule in Economies with Distorting Taxes.” *Journal of Monetary Economics* 37, 203-223.
- [4] Chari, V.V., L. Christiano and P. Kehoe (1999). “Optimal Fiscal and Monetary Policy” in *Handbook of Macroeconomics* vol. 1c, J. Taylor and M. Woodford, eds., Amsterdam: Elsevier Press.
- [5] da Costa, C. and I. Werning (2005). “On the Optimality of the Friedman Rule with Heterogeneous Agents and Non-Linear Income Taxation.” *Mimeo*, MIT.
- [6] Golosov, M., N. Kocherlakota and A. Tsyvinski (2003). “Optimal Indirect and Capital Taxation.” *Review of Economic Studies* 70, 569-587.
- [7] Kocherlakota, N. (1998). “Money is Memory.” *Journal of Economic Theory* 81, 232-251.
- [8] Kocherlakota, N. (2004). Plenary Talk, Society for Economic Dynamics Meetings. Florence, Italy.
- [9] Kocherlakota, N. (2005). “Zero Expected Wealth Taxes: A Mirrlees Approach to Dynamic Optimal Taxation.” *Econometrica* 73, 1587-1621.
- [10] Lagos, R. and R. Wright (2005). “A Unified Framework for Monetary Theory and Policy Evaluation.” *Journal of Political Economy* 113, 463-484.

- [11] Levine, D. (1991). "Asset Trading Mechanisms and Expansionary Policy." *Journal of Economic Theory* 54, 148-164.
- [12] Mirrlees, J. (1971). "An Exploration in the Theory of Optimal Income Taxation." *Review of Economic Studies* 38, 175-208.
- [13] Molico, M. (2005). "The Distribution of Money and Prices in Search Equilibrium." Forthcoming in *International Economic Review*.
- [14] Reed, R. and C. Waller (2005). "Money and Risk Sharing." Forthcoming *Journal of Money, Credit and Banking*.
- [15] Rocheteau, G. and R. Wright (2005) "Money in Search Equilibrium, in Competitive Equilibrium and in Competitive Search Equilibrium," *Econometrica* 73, 75-202.
- [16] Rocheteau, G. and C. Waller (2005) "Bargaining and the Value of Money." *Mimeo*, University of Notre Dame.
- [17] Wallace, N. (2001). "Whither Monetary Economics?" *International Economic Review*, 42, 847-869.
- [18] Zhu, T. (2005). "An Overlapping Generations Model with Search." *Mimeo*, Cornell University.

## 9 Appendix

**A1: Proof that  $\mu = 0$  is not consistent with incentive feasible allocations.** Suppose  $\mu = 0$  and  $\lambda_b, \lambda_s > 0$ . Then we have

$$\begin{aligned}
 q & : \frac{u'(q)}{\psi'(q)} = \frac{1 + \lambda_s}{1 + \lambda_b} \\
 C_o^b & : U'(C_o^b) = \frac{v'(h)}{1 + \lambda_b} \\
 C_o^s & : U'(C_o^s) = \frac{v'(h)}{1 + \lambda_s} \\
 C_o^o & : U'(C_o^o) = \frac{v'(h)}{1 - \frac{2\sigma}{(1-2\sigma)}(\lambda_b + \lambda_s)} \\
 C_y & : U(C_y) = v'(h)
 \end{aligned}$$

The second and fourth equations imply the idle agents IC constraints are violated. Hence this is not feasible.

Suppose  $\mu = 0$  and  $\lambda_b = 0, \lambda_s > 0$ . Then we have:

$$\begin{aligned}
 q & : \frac{u'(q)}{\psi'(q)} = 1 + \lambda_s = \frac{1}{U'(C_o^s)} > 1 \Rightarrow q < q^* \\
 C_o^b & : U'(C_o^b) = v'(h) \\
 C_o^s & : U'(C_o^s) = \frac{v'(h)}{1 + \lambda_s} \\
 C_o^o & : U'(C_o^o) = \frac{v'(h)}{1 - \frac{2\sigma}{(1-2\sigma)}\lambda_s} > 1 \Rightarrow C_o^o < C_o^b \\
 C_y & : U(C_y) = v'(h)
 \end{aligned}$$

Again, the second and fourth equations imply the idle agent's IC constraint is violated. Hence, this is not a feasible allocation.

Suppose  $\lambda_b > 0, \lambda_s = 0$ . We then have

$$\begin{aligned} q & : \frac{u'(q)}{\psi'(q)} = \frac{1}{1 + \lambda_b} < 1 \Rightarrow q > q^* \\ C_o^b & : U'(C_o^b) = \frac{v'(h)}{1 + \lambda_b} \\ C_o^s & : U'(C_o^s) = v'(h) \\ C_o^o & : U'(C_o^o) = \frac{v'(h)}{1 - \frac{2\sigma}{1-2\sigma}\lambda_b} \Rightarrow C_o^o < C_o^b \\ C_y & : U(C_y) = v'(h) \end{aligned}$$

Again, the second and fourth equations imply that the idle agent's IC constraint is violated, so this cannot be a feasible allocation. Thus the only feasible allocation has  $\mu > 0$ . ■

**A2: Alternate planner problem:**  $\sigma = 1/2$  In the example above  $\sigma \rightarrow 1/2$ . But at  $\sigma = 1/2$  the idle state is eliminated which also eliminates an IC constraint – there are no idle agents who can masquerade as buyers. This reduces the number of constraints on the planner. So suppose now that the non-trading state in the DM is absent, i.e.,  $\sigma = 1/2$ . Then the planner only has to worry about ensuring that sellers do not report themselves as buyers. Buyers cannot imitate sellers since they would have to produce which they cannot.

The planner's problem is now

$$\begin{aligned} \max_{q, C_o^b, C_o^s, C_o^o, C_y} & = \frac{1}{2} [u(q) - \psi(q)] \\ + \beta & \left[ \frac{1}{2} U(C_o^b) + \frac{1}{2} U(C_o^s) + U(C_y) - v \left( \frac{1}{2} C_o^b + \frac{1}{2} C_o^s + C_y \right) \right] \\ & s.t. \\ & \frac{1}{2} [u(q) + \beta U(C_o^b)] \geq 0 \\ & \frac{1}{2} [-\psi(q) + \beta U(C_o^s) - \beta U(C_o^b)] \geq 0 \end{aligned}$$

Since  $u(q) + \beta U(C_o^b) \geq 0$  for any positive  $q$  and  $C_o^b$  the constraint on the buyer cannot be binding. Furthermore, for any equilibrium with  $q > 0$  we must have  $C_o^s > C_o^b$ . The FOC are

$$\begin{aligned} q &: \frac{u'(q)}{\psi'(q)} = 1 + \lambda_s \\ C_o^b &: U'(C_o^b) = \frac{v'(h)}{1 - \lambda_s} \\ C_o^s &: U'(C_o^s) = \frac{v'(h)}{1 + \lambda_s} \\ C_y &: U(C_y) = v'(h) \end{aligned}$$

Thus,  $\lambda_s > 0$  is needed for the seller's IC to be binding. Rearranging the second and third equations gives

$$\begin{aligned} 1 - \frac{v'(h)}{U'(C_o^b)} &= \lambda_s = \frac{v'(h)}{U'(C_o^s)} - 1 \\ \frac{1}{v'(h)} &= \frac{1}{2} \left[ \frac{1}{U'(C_o^s)} + \frac{1}{U'(C_o^b)} \right] \end{aligned}$$

Thus the equilibrium solves

$$\begin{aligned} \frac{1}{v'(h)} &= \frac{1}{2} \left[ \frac{1}{U'(C_o^s)} + \frac{1}{U'(C_o^b)} \right] \\ \frac{u'(q)}{\psi'(q)} &= \frac{v'(h)}{U'(C_o^s)} \\ \psi(q) &= \beta U(C_o^s) - \beta U(C_o^b) \\ U(C_y) &= v'(h) \end{aligned}$$

Thus, the solution is exactly the same as the one above when  $\sigma \rightarrow 1/2$ . ■

**A3: Bargaining derivatives** Totally differentiating (16) and (17) yields

$$\begin{aligned}
& u''(q) U'(C_{o,+1}^s) dq + u'(q) U''(C_{o,+1}^s) \frac{\phi_{+1}}{1+\tau_c} (dm^s + dd) \\
&= \psi''(q) U'(C_{o,+1}^b) dq + \psi'(q) U''(C_{o,+1}^b) \frac{\phi_{+1}}{1+\tau_c} (dm^b - dd) \\
\psi'(q) dq &= \frac{\phi_{+1}}{1+\tau_c} \beta U'(C_{o,+1}^s) (dm^s + dd) - \frac{\phi_{+1}}{1+\tau_c} \beta U'(C_{o,+1}^o) dm^s
\end{aligned}$$

or

$$\begin{aligned}
& \begin{bmatrix} u''U'(C_{o,+1}^s) - \psi''U'(C_{o,+1}^b) & u'U''(C_{o,+1}^s) \frac{\phi_{+1}}{1+\tau_c} + \psi'U''(C_{o,+1}^b) \frac{\phi_{+1}}{1+\tau_c} \\ \psi' & -\phi_{+1}\beta U'(C_{o,+1}^s) \end{bmatrix} \begin{bmatrix} dq \\ dd \end{bmatrix} \\
&= \begin{bmatrix} \psi'U''(C_{o,+1}^b) \frac{\phi_{+1}}{1+\tau_c} dm^b - u'U''(C_{o,+1}^s) \frac{\phi_{+1}}{1+\tau_c} dm^s \\ \frac{\phi_{+1}}{1+\tau_c} \beta [U'(C_{o,+1}^s) - U'(C_{o,+1}^o)] dm^s \end{bmatrix}
\end{aligned}$$

It then follows that

$$\begin{aligned}
\frac{\partial q}{\partial m^b} &= -\frac{\phi_{+1}^2 \beta U'(C_{o,+1}^s) \psi'(q) U''(C_{o,+1}^b)}{(1+\tau_c)^2 D} \\
\frac{\partial d}{\partial m^b} &= -\frac{\phi_{+1} [\psi'(q)]^2 U''(C_{o,+1}^b)}{(1+\tau_c) D}
\end{aligned}$$

where  $D$  is the determinant of the matrix above. Substituting these expressions into (22) yields

$$\begin{aligned}
V'(m) &= -\sigma u'(q_b) \frac{\phi_{+1} \beta U'(C_{o,+1}^s) \psi'(q) U''(C_{o,+1}^b) \phi_{+1}}{(1+\tau_c)^2 D} \\
&+ \sigma \beta U'(C_{o,+1}^b) \frac{\phi_{+1}}{1+\tau_c} \frac{\psi'(q) \psi'(q) U''(C_{o,+1}^b) \phi_{+1}}{(1+\tau_c) D} \\
&+ \sigma \beta U'(C_{o,+1}^b) \phi_{+1} + (1-\sigma) \beta U'(C_{o,+1}^o) \phi_{+1} \\
&= -\frac{\beta \sigma \psi'(q) (\phi_{+1})^2 U''(C_{o,+1}^b)}{(1+\tau_c) D} [u'(q_b) U'(C_{o,+1}^s) - \psi'(q_s) U''(C_{o,+1}^b)] \\
&+ \sigma \beta U'(C_{o,+1}^b) \frac{\phi_{+1}}{1+\tau_c} + (1-\sigma) \beta U'(C_{o,+1}^o) \frac{\phi_{+1}}{1+\tau_c}
\end{aligned}$$

Setting  $q_b = q_s = q$  and using the FOC from the bargaining solution to eliminate  $U'(C_{o,+1}^b) \psi'(q)$  yields

$$V'(m) = \phi_{+1}\beta [\sigma U'(C_{o,+1}^b) + (1 - \sigma) U'(C_{o,+1}^o)]$$

Again, the marginal value of money is determined by consumption when old on the margin since the marginal liquidity value is zero. ■