A Competitive Theory of Unemployment, Vacancies, and Labor Market Flows*

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Abstract

This paper develops a dynamic competitive model of mismatch. Workers and jobs are randomly assigned to labor markets. Each labor market clears at each instant but some labor markets have more workers than jobs, hence unemployment, and some have more jobs than workers, hence vacancies. As workers and jobs move between labor markets, some unemployed workers find vacant jobs, some employed workers lose or leave their job and become unemployed, and some employed workers move to other jobs. The model is quantitatively consistent with two central features of labor markets over the business cycle, the Beveridge curve and the matching function. It can also address a variety of labor market phenomena, including duration dependence in the job finding probability, employer-to-employer transitions, and job creation and destruction.

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1 Introduction

Why do unemployed workers and job vacancies coexist? What determines the rate at which unemployed workers find jobs or employed workers switch employers? This paper advances the proposition that at any point in time, the skills and geographical location of unemployed workers are poorly matched with the skill requirements and location of job openings. The rate at which unemployed workers find jobs depends on the rate at which they retrain or move to locations with available jobs, the rate at which jobs open in locations with available workers, and the rate at which employed workers vacate jobs in locations with suitable available unemployed workers.

My main finding is that such a model of mismatch is quantitatively consistent with two robust features of labor markets: the negative correlation between unemployment and vacancies at business cycle frequencies, the *Beveridge curve*, and the positive correlation between the rate at which unemployed workers find jobs and the vacancy-unemployment (v-u) ratio, the *matching function*. The model produces a Beveridge curve that has a slope of approximately -1, quantitatively consistent with evidence from the United States. It produces a Cobb-Douglas matching function with an elasticity of the job finding rate with respect to the v-u ratio of about 0.14. Empirically the matching function in the United States is Cobb-Douglas but the elasticity is about twice as high. I also use the model to explore important labor market phenomena like duration dependence in the exit rate from unemployment and employer-to-employer transitions.

The view of unemployment and vacancies that I advance in this paper is conceptually distinct from the one that search theory has advocated since the pioneering work of McCall (1970), Mortensen (1970), and Lucas and Prescott (1974). According to search theory, unemployed workers have left their old job and are actively searching for a new employer. In contrast, this paper emphasizes that unemployed workers are attached to an occupation and a geographic location in which jobs are currently scarce. Mismatch is a theory of former steel workers remaining near a closed plant in the hope that it reopens. Search, particularly as articulated in Lucas and Prescott (1974), is a theory of former steel workers moving to a new city to look for a positions as nurses. These two theories are complementary and it is a priori reasonable to think that mismatch may be as important as job search in understanding equilibrium unemployment.

Indeed, the mismatch view of unemployment and vacancies is not new. Tobin (1972, p. 9) advances a theory of a "stochastic macro-equilibrium" in which "excess supplies in labor markets take the form of unemployment, and excess demands the form of unfilled vacancies. At any moment, markets vary widely in excess demand or supply, and the economy as a whole shows both vacancies and unemployment."[1] Drèze and Bean (1990) discuss important subsequent developments, including conditions on the joint distribution of workers and jobs across labor markets which ensure that the aggregation of many small markets yields an aggregate CES Beveridge curve. But both of these papers link mismatch with "disequilibrium" in local labor markets. One point of this paper is to show that models of mismatch are quantitatively consistent with macro-labor facts even if each labor market is in equilibrium at every instant.

To that end, I develop an explicit model of heterogeneity. There are many local labor markets, each of which represents a particular geographic location and a particular occupation. The labor market clears within each location, but there may be unemployed workers in one location and job vacancies in another. As workers and jobs enter and exit labor markets, unemployed workers find jobs and employed workers lose jobs, sometimes moving directly to another job.

An increase in the average number of workers per location or a decrease in the number of jobs per labor market tends to raise unemployment and reduce vacancies. A proportional increase in both the average number of workers and the average number of jobs lowers the incidence of mismatch and so reduces both the unemployment and vacancy rates.

My model is deliberately parsimonious. In particular, the movement between labor markets is exogenous and random. The only economic decision is one by firms, which must decide at each instant whether to create new jobs.[2] An advantage to this approach is that it is relative spare. Nevertheless, the model is rich enough that its predictions can be confronted with U.S. data. I show that fluctuations in aggregate productivity or in the duration of jobs induce movements along a downward sloping Beveridge curve. I compare that relationship with evidence from the Job Openings and Labor Turnover Survey (JOLTS) and the Conference Board Help Wanted Index, and show that the theoretical and empirical Beveridge curves are indistinguishable.

I next show that vacancies and unemployment respond about 3.5 times as much to productivity shocks in the mismatch model as in Pissarides's (1985) matching model. Shimer (2005b) argues that the matching model only explains about ten percent of the volatility

[1] Tobin (1972) cites a number of previous authors in developing these ideas including Lipsey (1960) and Holt (1970). Hansen (1970) proposes a similar model of mismatch.

[2] This is also the only economic decision in Pissarides's (1985) matching model.

in vacancies and unemployment, so this goes a long way towards reconciling the theory and the data. Moreover, fluctuations in the expected duration of jobs induce movements along a downward sloping Beveridge curve. In the matching model, they induce a positive co-movement of unemployment and vacancies (Abraham and Katz, 1986; Shimer, 2005b).

The model also predicts a systematic relationship between the rate at which unemployed workers find jobs and the vacancy-unemployment ratio. Not only is this relationship increasing in the model, it is nearly indistinguishable from a Cobb-Douglas. An increase in productivity that raises the v-u ratio by 10 percent raises the instantaneous job finding rate by about 1.5 to 2 percent. This is roughly consistent with U.S. data, where it is impossible to reject the hypothesis of a Cobb-Douglas matching function, although the elasticity is closer to 0.3.

A careful examination of the link between the theoretical and empirical matching function requires me to account for heterogeneity in the exit rate from unemployment. The long-term unemployed are typically located in labor markets where jobs are particularly scarce, which makes their prospects for exiting unemployment particularly bleak. This dynamic sorting explains about half of the duration dependence in the job finding probability.

I then examine the model's predictions for employer-to-employer transitions. Recent U.S. data indicates that employer-to-employer transitions are more common than employment-to-unemployment transitions and are mildly procyclical. The model is quantitatively consistent with these facts.

This paper proceeds as follows. Section 2 discusses the related literature in greater detail. Section 3 develops the basic model of mismatch. Section 4 analyzes the relationship between unemployment and vacancies, including the responsiveness of the v-u ratio to shocks. Section 5 shows that the model delivers a matching function that is approximately Cobb-Douglas, although it has an elasticity somewhat lower than the U.S. matching function. Section 6 discusses duration dependence in the job finding probability. The model explains about half the measured duration dependence in U.S. data. Accounting for duration dependence significantly reduces the measured level of the job finding probability but has little effect on the elasticity of the matching function. Section 7 examines employer-to-employer flows, showing that as labor market conditions tighten, fewer workers experience a measured unemployment spell between jobs, consistent with U.S. data. I conclude in Section 8.

2 Related Literature

2.1 Mismatch Models

A number of previous authors have developed formal models of mismatch as a source of unemployment. Most early attempts used an urn-ball structure, where workers (balls) are randomly assigned to jobs (urns).[3] The random assignment ensures that some jobs are unfilled, yielding vacancies, and some jobs are assigned multiple workers, only one of whom can be hired, yielding unemployment. Hall (2000) supposes that workers are randomly assigned to locations and then matched in pairs. One worker is necessarily unemployed in any location with an odd number of workers, providing a coherent theory of the link between the importance of matching (the number of workers per location) and the unemployment rate.

Stock-flow matching models offer another sensible theory of mismatch (Taylor, 1995; Coles and Muthoo, 1998; Coles and Smith, 1998; Coles and Petrongolo, 2003). According to these models, only a small proportion of worker-job matches are feasible. When a worker loses her job, she looks among the available stock of vacancies to see if her skills are suitable for any of them. The probability that a match is suitable is independent across any two worker-job pairs, so doubling the stock of vacancies squares the probability that a worker fails to find a match. If this happens, she remains unemployed, while otherwise she is immediately paired with a suitable vacancy. Symmetrically, entering job vacancies search for a match within the stock of unemployed workers.

Perhaps the most similar models of mismatch are Lagos's (2000) model of the taxicab market and Sattinger's (2005) model of queuing. According to Lagos (2000), there are a fixed set of locations and two types of economic agents, drivers and passengers. The short side of the market is served within each location and drivers optimally relocate to the best possible location. Nevertheless, Lagos finds that empty taxis and unserved riders can coexist in equilibrium under some conditions, yielding an aggregate Beveridge curve. Sattinger (2005) assumes workers are randomly assigned to job queues and wait to be "served." A worker on a long queue experiences a longer unemployment spell. He shows that a combination of queuing and search is consistent with a downward sloping Beveridge curve.

There are many small differences between these earlier approaches to mismatch and the

[3] An incomplete list of papers using the urn-ball structure includes Butters (1977), Hall (1977), Lang (1991), Montgomery (1991), Peters (1991), Blanchard and Diamond (1994), Burdett, Shi, and Wright (2001), and Shimer (2005a).

model that I propose in this paper. For example, by making the notion of a labor market explicit, it is sensible to think about wages being determined by competition for labor within markets. The literature on urn-ball and stock-flow matching models has typically assumed that wages are either posted by firms as a recruiting device or bargained ex post by workers and firms. Prices are exogenous in Lagos (2000).

But the most important difference between this paper and the urn-ball and stock-flow literatures is one of emphasis. No previous paper has shown that a mismatch model is quantitatively consistent with the Beveridge curve. Nor has any previous paper shown that a mismatch model gives rise to a reduced-form aggregate matching function, indeed one that is quantitatively consistent with the available evidence. Instead, with the notable exception of Sattinger (2005), the literature has focused on the theoretical shortcomings of the reduced-form matching function approach by arguing that mismatch models do not deliver a structural matching function.

2.2 Search and Matching Models

The issues I examine in this paper have traditionally been the realm of search models, especially Pissarides's (1985) matching model and its variants.[4] Under appropriate restrictions on the reduced-form matching function and on the nature of shocks, the matching model is quantitatively capable of describing the Beveridge curve (Abraham and Katz, 1986; Blanchard and Diamond, 1989) and the increasing relationship between the v-u ratio and the rate at which unemployed workers find jobs (Pissarides, 1986; Blanchard and Diamond, 1989).

Despite these successes, the matching model has two significant shortcomings. The first is the matching function itself. It is intended to represent "heterogeneities, frictions, and information imperfections" and to capture "the implications of the costly trading process without the need to make the heterogeneities and other features that give rise to it explicit" (Pissarides, 2000, pp. 3–4). But if the matching function is a reduced-form relationship, one should be concerned about whether it is invariant to policy changes. We need an explicit model of heterogeneity that gives rise to an empirically successful reduced-form matching function to address this issue.

The second is wage determination. In the matching model, workers and firms are typically in a bilateral monopoly situation, and so competitive theories of wage determination are

[4] Search models based on Lucas and Prescott (1974) do not have an explicit notion of job vacancies and therefore have not been used to analyze the Beveridge curve and matching function.

inapplicable. Wages are instead set via bargaining. Some recent research has emphasized that the details of the bargaining protocol are quantitatively critical to the ability of the model to replicate business cycle fluctuations in unemployment and vacancies (Shimer, 2005b; Hall, 2005; Hall and Milgrom, 2005). The model I develop in this paper circumvents both of these issues.

3 A Model of Mismatch

3.1 Economic Agents

There are a $\mathcal{M}$ workers and a large number of firms. Both are risk-neutral, infinitely-lived, and discount future income at rate r. Time is continuous.

3.2 Stocks

I start by looking at the state of the economy at each moment of time. I then examine the flow of workers and jobs and show that this is consistent with the initial stocks described here.

At any point in time, each worker is assigned to one of $\mathcal{L}$ labor markets. These assignments are independent across workers, so the actual number of workers in a labor market is a Binomial random variable.

Each firm may have zero, one, or more jobs. Let $\mathcal{N}$ denote the total number of jobs. Each job is assigned to one labor market. Again, these assignments are independent across jobs and independent of the number of workers assigned to the labor market. Thus the actual number of jobs in a labor market is an independent Binomial random variable.

Let $M \equiv \mathcal{M}/\mathcal{L}$ and $N \equiv \mathcal{N}/\mathcal{L}$. In the remainder of this paper, I fix M and N and take the limit as $\mathcal{L} \to \infty$. In this limit, the number of workers and jobs per labor market are independent Poisson random variables. In a standard abuse of the law of large numbers, I assume that the fraction of labor markets with i workers and j jobs is deterministic, hence equal to

$$p(i,j) = \frac{e^{-(M+N)} M^i N^j}{i!j!}. \tag{1}$$

3.3 Production and Market Clearing

Workers and jobs must match in pairs in order to produce market output. One worker and one job in the same labor market can jointly produce x units of output. A single worker (an unemployed worker) produces $z < x$ at home, while a single job (a vacancy) produces nothing. Workers and jobs are indivisible. These stark assumptions give a concrete notion of unemployment and vacancies.

There is competition within each labor market. Let i denote the number of workers in some labor market and j denote the number of jobs. If $i > j$, $i - j$ workers are unemployed but all workers are indifferent about being unemployed; the wage is driven down to the value of home production, $w = z$. If $i < j$, $j - i$ jobs are vacant but all firms are indifferent about their jobs being vacant; the wage is driven up to the marginal product of labor, $w = x$. If $i = j$, there is neither unemployment nor vacancies in the market and the wage is not determined. For notational simplicity I assume that if $i = j$, the wage is equal to workers' reservation wage, $w = z$. The quantitative results are scarcely affected if I instead assume $w = x$ when $i = j$.

3.4 Flows

Each worker quits her labor market according to a Poisson process with arrival rate q. The arrival of this shock is exogenous, independent of the worker's current employment status or wage. When a worker quits her labor market, she is randomly reassigned to a new labor market, independent of conditions in the new labor market. This means that the arrival rate of workers into a labor market is qM, consistent with the number of workers per labor market distributed as a Poisson random variable.

Symmetrically, each job leaves its labor market according to a Poisson process with arrival rate l. When a job leaves its labor market, it disappears. However, a firm may create a new job by paying a fixed cost k. When it does so, the job is randomly assigned to a labor market. Again, both entry and exit of jobs is independent of conditions in the local labor market, although the decision to create a job depends on aggregate labor market conditions. To maintain a steady state stock of N jobs per labor market, the entry rate of jobs must be $n = lN$; however, out of steady state it may be higher or lower, with $\dot{N}(t) = n(t) - lN(t)$.

Whenever there are single workers and single jobs in a labor market, they are instantly paired off. For example, if a job enters a labor market with unemployed workers, one randomly selected worker is matched with the job and the pair starts producing output. I assume that a pair remains matched until either a quit or "layoff" hits the match, at rate $q + l$. This is consistent with a small unmodeled turnover cost.

3.5 Equilibrium

There is one key equilibrium condition: firms create jobs whenever doing so is profitable, analogous to the free entry condition in Pissarides (1985). If all the parameters of the model are constant over time, the number of jobs per labor market must satisfy

$$k = \frac{x - z}{r + l} \sum_{i=1}^{\infty} \sum_{j=0}^{i-1} p(i, j). \tag{2}$$

The cost of creating a job is k. It then yields profit $x - z$ whenever it is located in a labor market in which the number of other jobs, j, is strictly less than the number of workers i, a fraction $\sum_{i=1}^{\infty} \sum_{j=0}^{i-1} p(i, j)$ of the time in expectation. At other times it yields zero profits. These profits are discounted accounting both for the rate of time preference and for the rate at which jobs end, $r + l$.

If the cost of creating a job is smaller than the expected profit from a job, firms create more jobs, raising N. This reduces the fraction of labor markets in which the number of jobs is less than the number of workers, $j < i$, and hence reduces the profitability from creating a job until equilibrium is attained. In this case, equilibrium is attained instantaneously. Conversely, if the cost of creating a job is larger than the expected profit, there is no job creation. Instead, the stock of jobs falls gradually due to layoffs, $\dot{N}(t) = -lN(t)$. The fraction of labor markets in which jobs are scarce increases gradually, until in finite time the zero profit condition (2) is restored.

Comparative statics are straightforward. An increase in the cost of job creation k, a reduction in productivity x, an increase in the value of leisure z, an increase in the discount rate r, or an increase in the layoff rate l all induce a decline in the number of jobs per labor market. These results are intuitive and consistent with the predictions of a search model (Pissarides, 2000).

3.6 Normal Approximation

The exact numbers of workers and jobs per labor market are integers with Poisson distributions; however, it is computationally inconvenient to work directly with the Poisson distributions. Instead, observe that the difference between the CDF of a Poisson distribution with mean M (and hence variance M) and the CDF of a Normal distribution with mean M and variance M converges to zero in the sup-norm as M converges to infinity. For the values of M that I use in this paper, the normal approximation is extremely accurate.

This approximation suggests that fraction of labor markets with more workers than jobs, $i > j$, is approximately given by the probability that $i - j > \frac{1}{2}$ when i and j are independent Normal random variables. Since the difference between two Normal random variables is also Normal, this is simply the probability that a Normal random variable with mean $M - N$ and variance $M + N$ exceeds $\frac{1}{2}$. Then the free entry condition (2) is approximately equivalent to

$$k = \frac{x - z}{r + l}\Phi(\kappa_u), \tag{3}$$

where Φ is the CDF of the standard Normal distribution and $\kappa_u \equiv \frac{-1+2(M-N)}{2\sqrt{M+N}}$.

To see the accuracy of these calculations, let $M = 244.2$ and let N vary between 233 and 243, values that I will argue below are empirically plausible. The share of labor markets with unemployed workers falls from 69.9% to 51.3% as N increases. On the other hand, the difference between the normal approximation $\Phi(k_u)$ and the true share of labor markets with unemployed workers, $\sum_{i=1}^{\infty}\sum_{j=0}^{i-1} p(i,j)$, ranges between -0.0009% and 0.0003%, a quantitatively irrelevant difference. Given the relative ease of inverting equation (3) to compute N, I use this expression in the calculations throughout this paper.

4 The Beveridge Curve

4.1 Theoretical Concepts

The number of unemployed workers per labor market is equal to the difference between the number of workers i and the number of jobs j, summed across labor markets with more workers than jobs:

$$U = \sum_{i=1}^{\infty}\sum_{j=0}^{i-1}(i - j)p(i,j). \tag{4}$$

Likewise, the number of vacancies per labor market is

$$V = \sum_{j=1}^{\infty}\sum_{i=0}^{j-1}(j - i)p(i,j). \tag{5}$$

The unemployment and vacancy rates are $u \equiv U/M$ and $v \equiv V/N$ and the v-u ratio is $\theta \equiv V/U$. These expressions depend only on the number of workers and jobs per labor market, M and N.

An increase in the number of jobs per labor market N, for example due to an increase in productivity x or a decrease in the layoff rate l, reduces the unemployment rate and raises the vacancy rate since additional jobs are increasingly less likely to locate in labor markets with unemployed workers. On the other hand, a proportional increase in both M and N reduces both the unemployment and vacancy rates. For example, doubling M and N is equivalent to merging randomly selected pairs of labor markets. If both labor markets have unemployment, this merger does not affect the unemployment or vacancy rates, and similarly if both labor markets have vacancies. But merging a labor market with unemployment and a labor market with vacancies reduces the unemployment and vacancy rate in both markets. Thus higher values of M and N are associated with less mismatch.

This suggests that the mismatch construction is in other markets where the coexistence of unemployment and vacancies may be more or less common. If matching is a more severe problem, as might be the case in marriage or housing markets, M and N should be modeled as relatively small numbers. If it is a less severe problem, as in commodity markets, M and N may be thought of as very large numbers.

It is also useful to use the normal approximations to obtain simpler expressions for the unemployment and vacancy rates. The logic from the previous section suggests that the fraction of labor markets with $x > 0$ unemployed workers is approximately normally distributed with mean $M - N$ and variance $M + N$. This gives

$$U \approx \sqrt{\frac{M+N}{2\pi}} e^{-\kappa_u^2/2} + (M-N)\Phi(\kappa_u) \tag{6}$$

$$V \approx \sqrt{\frac{M+N}{2\pi}} e^{-\kappa_v^2/2} + (N-M)\Phi(\kappa_v), \tag{7}$$

where $\kappa_u = \frac{-1+2(M-N)}{2\sqrt{M+N}}$, $\kappa_v \equiv \frac{-1+2(N-M)}{2\sqrt{M+N}}$, and Φ is the CDF of the standard normal. Again let $M = 244.2$ and N vary between 233 and 243. The actual unemployment rate falls from 6.3% to 3.9% as N increases over this range, while the difference between the actual and approximate unemployment rate is smaller than 0.0002% over the entire range.

4.2 Measuring Vacancies: JOLTS

Since December 2000, the Bureau of Labor Statistics (BLS) has measured job vacancies using the JOLTS. This is the most reliable time series for vacancies in the United States. According to the BLS, "A job opening requires that 1) a specific position exists, 2) work could start within 30 days, and 3) the employer is actively recruiting from outside of the establishment to fill the position. Included are full-time, part-time, permanent, temporary, and short-term openings. Active recruiting means that the establishment is engaged in current efforts to fill the opening, such as advertising in newspapers or on the Internet, posting help-wanted signs, accepting applications, or using similar methods."[5] I measure the vacancy rate as the ratio of vacancies to vacancies plus employment.

The Bureau of Labor Statistics (BLS) uses the Current Population Survey (CPS) to measure the unemployment rate each month. The CPS measures employment and unemployment using a household questionnaire designed to determine whether an individual is working or, if she is not working, available for and actively seeking work. The ratio of unemployment to the sum of unemployment and employment is the unemployment rate. The brown dots in Figure 1 show the strong negative correlation between unemployment and vacancies over this time period, the empirical Beveridge curve.

In an average month from December 2000 to May 2005, the unemployment and vacancy rates were 5.4% and 2.3%, respectively. This is consistent with $M = 244.2$ and $N = 236.3$. I use this data to pin down the number of workers per labor market M and then consider how changes in productivity or the layoff rate affect the number of jobs per labor market N. Variation in these parameters, either deterministic or stochastic, forecastable or unexpected, induces variation in N, with U and V instantaneously adjusting so that equations (4) and (5) (or 6 and 7) hold. This traces out the solid blue line in Figure 1, the model-generated Beveridge curve.

The fact that the level of the model-generated Beveridge curve fits the data reflects a judicious choice of the number of jobs per labor market. But the fact that the slope and curvature of the model-generated Beveridge curve also fits the data comes from the structure of the model. The model cannot generate a different Beveridge curve in response to fluctuations in productivity x or in the layoff rate l.

[5] See BLS news release, July 30, 2002, available at http://www.bls.gov/jlt/jlt_nr1.pdf

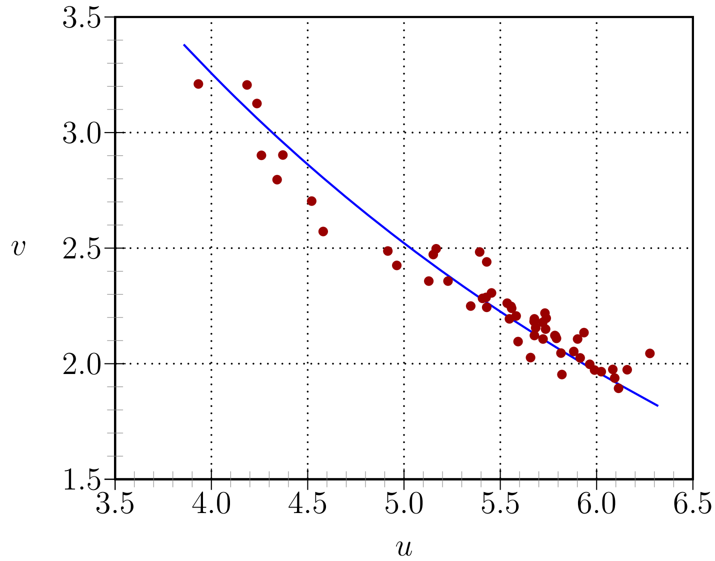

Figure 1: The brown dots show U.S. monthly data from December 2000 to May 2005. The unemployment rate is measured by the BLS from the CPS. The vacancy rate is measured by the BLS from the JOLTS. The solid blue line shows the model generated Beveridge curve with $M = 244.2$ and $N \in [233, 243]$.

4.3 Measuring Vacancies: Help-Wanted Index

A shortcoming of JOLTS is that it only covers one recession and subsequent expansion. Moreover, the recovery was unusual in that employment growth proceeded much slower than normal. While there is unfortunately no ideal measure of job vacancies over a longer time period, the Conference Board Help Wanted Index provides a crude one (Abraham, 1987). Since 1951, the Conference Board has constructed the index on a monthly basis by measuring the number of column inches of help wanted advertisements in the largest newspaper in 51 major metropolitan areas.[6] Consolidation of the newspaper industry, changes in newsprint costs, legally mandated changes in advertising like equal employment opportunity laws, and the rise of the internet likely all affected the help wanted index. To circumvent these issues, I compute the quarterly average of the help wanted advertising index, take logs, and detrend it using a low frequency HP Filter, with smoothing parameter 10^5. The red dots in Figure 2 plots this against a similarly detrended measure of the unemployment rate, generating another theoretical Beveridge curve.

I compare the detrended empirical Beveridge curve with a demeaned theoretical Beveridge curve. For a given value of $N \in [233, 243]$, I compute the unemployment and vacancy rates

[6] Since January 1951, the average unemployment rate has also been 5.4%, the same as since December 2000.

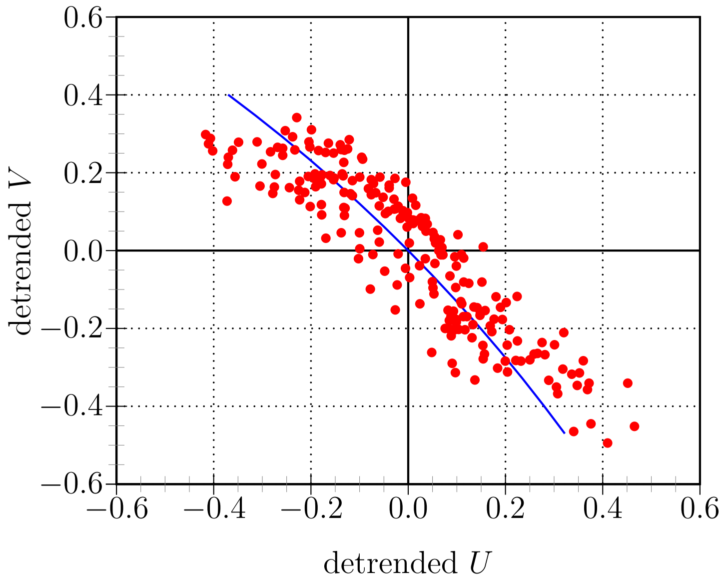

Figure 2: The red dots show U.S. quarterly data from 1951 to 2004. The unemployment rate is measured by the BLS from the CPS. The vacancy rate is measured by the BLS from the JOLTS. Both are quarterly averages of monthly data, detrended using an HP filter with smoothing parameter 10^5. The solid blue line shows the model generated Beveridge curve with $M = 244.2$ and $N \in [233, 243]$, expressed as a log deviation from $N = 238$.

using equations (6) and (7), take logs, and subtract the corresponding log unemployment and vacancy rates at $N = 238$.[7] This gives the solid blue line in Figure 2. Once again, the structure of the model determines the slope and even the slightly concave shape of the Beveridge curve, both of which are consistent with the empirical Beveridge curve.

In summary, this model is quantitatively consistent with the strong negative correlation between unemployment and vacancies and with the relative magnitude of changes in the two variables over time. This is true regardless of the source of shocks to the number of jobs per labor market. In contrast, while the matching model is able to produce a negative correlation between unemployment and vacancies, doing so is not trivial. For example, Mortensen and Pissarides (1994) report a theoretical correlation between unemployment and vacancies of −0.26. Merz (1995) reports the correlation is −0.15 if search intensity is exogenous and 0.32 if it moves endogenously over the business cycle. Shimer (2005b) finds that shocks to aggregate productivity induce a strong negative correlation between unemployment and vacancies. A judicious choice of the matching function yields the correct slope of the Beveridge curve as well. But even then, fluctuations in the separation rate induce an

[7] In Section 5, I assume N follows a well-specified stochastic process and look at the behavior of the detrended unemployment and vacancy rates. This gives a very similar result.

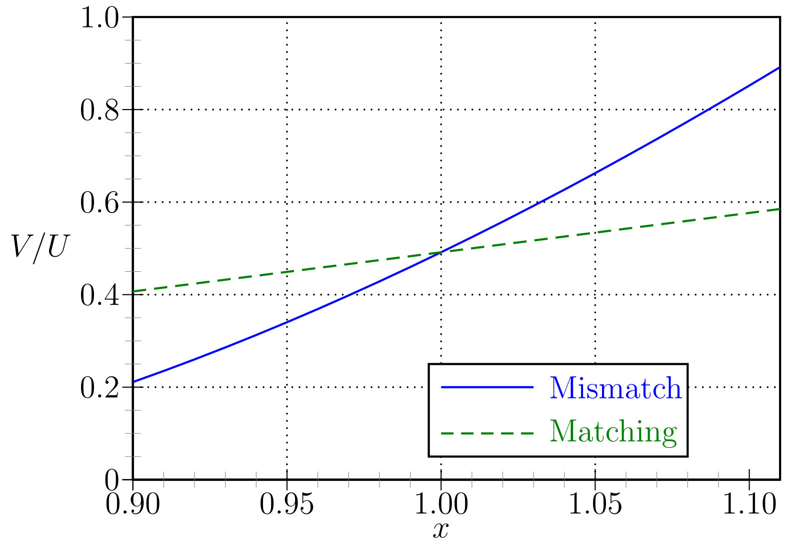

Figure 3: The solid blue line shows the v-u ratio in the mismatch model as a function of productivity x. The dashed green line shows the same relationship in the Pissarides (1985) matching model using Shimer's (2005b) calibration.

equally strong positive correlation between unemployment and vacancies.

4.4 Responsiveness to Shocks

Suppose the unemployment rate is 5 percent and there is an unanticipated permanent change in productivity x. How large are the resulting changes in unemployment and vacancies? To be concrete, suppose that initially $x = 1$ and the value of home production is $z = 0.4$, the preferred number in Shimer (2005b).[8] If $M = 244.2$, a 5 percent unemployment rate requires $N = 238.0$ (equation 6) and implies a vacancy-unemployment ratio of 0.49 (equation 7). As x varies between 0.93 and 1.11, equation (3) implies that the number of jobs per labor market varies between 233 and 243, the range depicted in Figure 1 and Figure 2. The v-u ratio varies by about 114 log points, from 0.29 to 0.89, as shown by the solid blue line in Figure 3. These comparative statics are suggestive about the responsiveness of the v-u ratio to productivity shocks.

By contrast, Shimer (2005b, p. 36) argues that in a search and matching model cali-

[8] The value of home production is a key parameter in this model and in the matching model (Shimer, 2005b; Hagedorn and Manovskii, 2005). One way to pin it down is to look at the labor share of income, equal to 1 for employed workers in markets with $i < j$ and z/x otherwise. A lower value of z reduces the labor share. In the parameterization reported here, about 60 percent of employed workers are paid $z = 0.4$ when $x = 1$, giving a labor share of $0.6 \cdot 0.4 + 0.4 \cdot 1 = 0.64$, close to the empirical value of about $\frac{2}{3}$.

brated with the same parameter values—in particular $z = 0.4$—an unanticipated permanent increase in productivity from 0.93 to 1.11 would raise the v-u ratio by about 0.32 log points. I graph this as the dashed green line in Figure 3. In words, the mismatch model yields about 3.5 times as large fluctuations in the v-u ratio in response to a given productivity shock as the search and matching model. Since Shimer (2005b) reports that productivity shocks in the search and matching model explain about ten percent of the volatility in the v-u ratio, it follows that the mismatch model closes half the gap between the model and data.

Moreover, while shocks to the separation rate induce a counterfactual upward sloping Beveridge curve in the matching model (Shimer, 2005b), shocks to the layoff rate induce movements along a downward sloping Beveridge curve in the mismatch model. Thus if the layoff rate is countercyclical, the mismatch model further helps to explain the observed movement in unemployment and vacancies. Finally, these conclusions are unaffected by the procyclicality of the quit rate, which has no effect on entry (equation 3), unemployment (equation 6), or vacancies (equation 7) in the mismatch model. In the matching model, a procyclical quit rate would again yield an upward sloping Beveridge curve.

5 The Matching Function

5.1 Theoretical Concepts

The matching function describes the relationship between the rate at which unemployed workers find jobs and the v-u ratio.[9] To describe the behavior of the matching function, I need to describe how unemployed workers find jobs.

When a worker quits her labor market and moves to a new one or a job leaves its labor market and is replaced elsewhere by a new job, this may lead to one or more transitions between employment and unemployment. If an employed worker quits her labor market, an unemployed worker may take her old job (an unemployment-to-employment or UE transition) and she may fail to find a job in her new labor market (an employment-to-unemployment or EU transition). If an unemployed worker quits his labor market, he may find a job in his new labor market (a UE transition). If a filled job leaves the labor market, its old employee may be left jobless (EU transition). But whenever a new job enters a labor market, it may

[9]This is a slight abuse of terminology. Traditionally the matching function describes the number of workers who find a job as a function of vacancies and unemployment. If this function is homogeneous of degree 1, as a substantial body of empirical evidence suggests (Petrongolo and Pissarides, 2001), the two concepts are equivalent.

hire a worker (UE transition). These events may also lead an employed worker to switch employers, a topic I defer until Section 7.

Let π_q^{UE} denote the probability that a quit leads to a UE transition. This occurs if either the quitting worker is employed in a labor market with unemployed workers or if the worker is unemployed and moves to a labor market with vacant jobs:

$$\pi_q^{UE} \equiv \frac{1}{M}\sum_{i=1}^{\infty}\sum_{j=0}^{i-1} jp(i,j) + u\sum_{i=0}^{\infty}\sum_{j=i+1}^{\infty} p(i,j). \tag{8}$$

The first term is the fraction of workers who are employed in labor markets with unemployed workers. This is equal to j workers in every labor market with $i > j$. The second term is the product of the fraction of workers who are unemployed and the fraction of labor markets with vacancies, $j > i$. Simplify the first term in equation (8) using the following identity:

$$\frac{1}{M}\sum_{i=1}^{\infty}\sum_{j=0}^{i-1} ip(i,j) = \sum_{i=1}^{\infty}\sum_{j=0}^{i-1} \frac{e^{-(M+N)}M^{i-1}N^j}{(i-1)!j!} = \sum_{i'=0}^{\infty}\sum_{j=0}^{i'} p(i',j) \tag{9}$$

The first equality uses the definition of $p(i,j)$, while the second equality is a simply a change in variables from i to $i' = i-1$ and a reapplication of the definition of $p(i,j)$. Use this and equation (4) to simplify equation (8):

$$\pi_q^{UE} = (1-u)\sum_{i=0}^{\infty}\sum_{j=0}^{i} p(i,j). \tag{10}$$

This is the product of the employment rate and the fraction of labor markets with vacancies, i.e. the probability a quit shock leads an employed worker to move to a labor market without vacancies, causing an EU transition: $\pi_q^{UE} = \pi_q^{EU}$. Since the quit rate does not affect the unemployment rate, the probability that a quit shock leads to a UE transition must be the same as the probability that it leads to an EU transition.

I similarly let π_n^{UE} denote the probability that a job entering a labor market causes a UE transition. This occurs whenever the job enters a market with unemployed workers, $i > j$:

$$\pi_n^{UE} = \sum_{i=1}^{\infty}\sum_{j=0}^{i-1} p(i,j). \tag{11}$$

Conversely, the probability that a job leaving a market causes an EU transition is equal to fraction of jobs in markets without excess jobs $i \geq j$:

$$\pi_l^{EU} = \frac{1}{N} \sum_{j=1}^{\infty} \sum_{i=j}^{\infty} jp(i,j) = \sum_{j=0}^{\infty} \sum_{i=j+1}^{\infty} p(i,j). \tag{12}$$

The second equation uses the same logic as equation (9). Reordering the sum then proves that $\pi_l^{EU} = \pi_n^{UE}$.

Putting these together gives the instantaneous transition rate from unemployment to employment in steady state, i.e. the job finding rate for unemployed workers:

$$f = \frac{qM\pi_q^{UE} + n\pi_n^{UE}}{U}.$$

There are qM quit shocks per labor market, each leading to a UE transition with probability π_q^{UE}. Similarly, jobs enter at rate n, leading to a UE transition with probability π_n^{UE}. This gives the total rate at which unemployed workers find jobs in an average labor market. Dividing by the total number of unemployed workers per labor market gives the instantaneous job finding rate for unemployed workers.

As before, it is possible to use the Normal approximation to the Poisson to obtain a computationally simple approximate formula for f:

$$f = \frac{q(1-u)\Phi(-\kappa_v) + \frac{n}{M}\Phi(\kappa_u)}{u}, \tag{13}$$

where again $\kappa_u = \frac{-1+2(M-N)}{2\sqrt{M+N}}$, $\kappa_v \equiv \frac{-1+2(N-M)}{2\sqrt{M+N}}$, and Φ is the CDF of the standard normal. The approximate formula is again extremely accurate.

Fluctuations in the number of jobs per labor market N induced by changes in labor productivity x induce fluctuations in the v-u ratio (equation 6 and 7) and in the job finding rate f. This gives rise to a reduced-form matching function. The next subsection briefly discusses empirical measures of the matching function before I use comparative statics and a simulation of aggregate shocks in a stochastic environment to discuss the model-generated matching function in greater detail.

5.2 Measuring the Job Finding Probability

Suppose there are U_t unemployed workers at the start of month t, indexed by $i \in \{1, \ldots, U_t\}$. Let F_t^i denote the probability that worker i finds a job by the start of month $t+1$. I assume that the randomness in the outcome of the job finding process cancels out, so $\sum_{i=1}^{U_t} F_t^i$ is the number of workers who find a job during month t. Then unemployment at the start of month $t+1$ is

$$U_{t+1} = \sum_{i=1}^{U_t}(1 - F_t^i) + U_{t+1}^s, \tag{14}$$

where U_{t+1}^s denotes the number of short-term unemployed, workers who are unemployed at the start of month $t+1$ but worked at some point during month t. Unemployment next month is equal to the number of unemployed workers who fail to obtain a job within the month, $\sum_{i=1}^{U_t}(1 - F_t^i)$, plus the number of workers who are unemployed but held a job at some time during the previous month, U_{t+1}^s. Rearrange equation (14) to get a measure of the mean job finding probability among workers who are unemployed at date t:

$$F_t \equiv \frac{\sum_{i=1}^{U_t} F_t^i}{U_t} = 1 - \frac{U_{t+1} - U_{t+1}^s}{U_t}. \tag{15}$$

The BLS measures unemployment and short-term unemployment from the CPS.[10] I use these to construct F_t from January 1951 to May 2005.

I compare this measure of the job finding probability with the v-u ratio. Figure 4 uses the measure of vacancies from JOLTS, available since December 2000. The relationship is clearly positive. Although the exact shape is difficult to discern in this short sample, the point estimate suggests a Cobb-Douglas matching function with an elasticity of about 0.37, shown as a solid line. Figure 5 uses data from the Help Wanted Index, available since 1951. The detrended data show a linear relationship between the two variables, corresponding to a Cobb-Douglas matching function with an elasticity of about 0.28. Table 1 summarizes the joint behavior of detrended unemployment, vacancies, and the job finding probability since 1951.

5.3 Comparative Statics

To compare the model with the data, fix $M = 244.2$ and $q = l = 0.027$; I discuss this choice of q and l shortly. Let N vary between 233 and 243, with the entry rate of jobs solving $n = Nl$ throughout. The solid blue line in Figure 6 and Figure 7 shows the resulting

[10]The redesign of the CPS in 1994 affected the measure of short-term unemployment. I obtain a consistent measure of U_t^s by using CPS microdata after 1994. See Shimer (2005c) for details on the properties and construction of this measure. I use a similar adjustment for the other unemployment duration series. http://home.uchicago.edu/~shimer/data/flows/ contains a time series for $f_t \equiv -\log(1 - F_t)$.

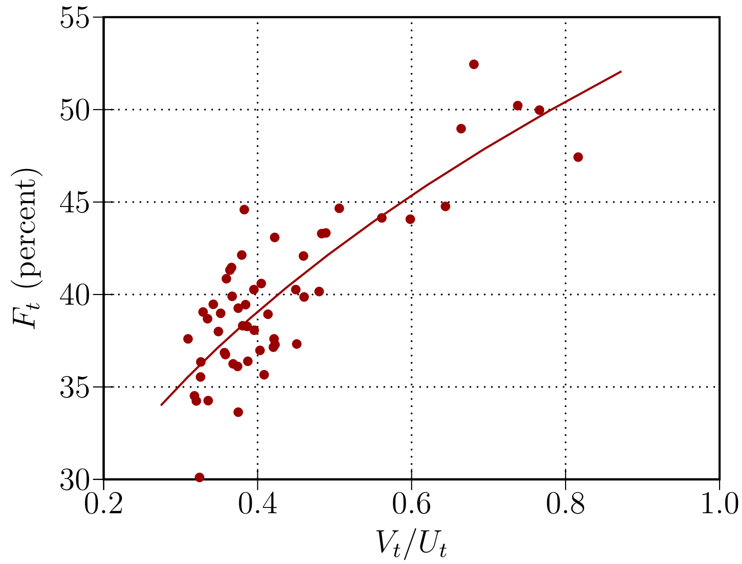

Figure 4: U.S. data, December 2000–May 2005. Vacancies are measured from JOLTS, unemployment and the job finding probability from the CPS. The line shows a Cobb-Douglas fit.

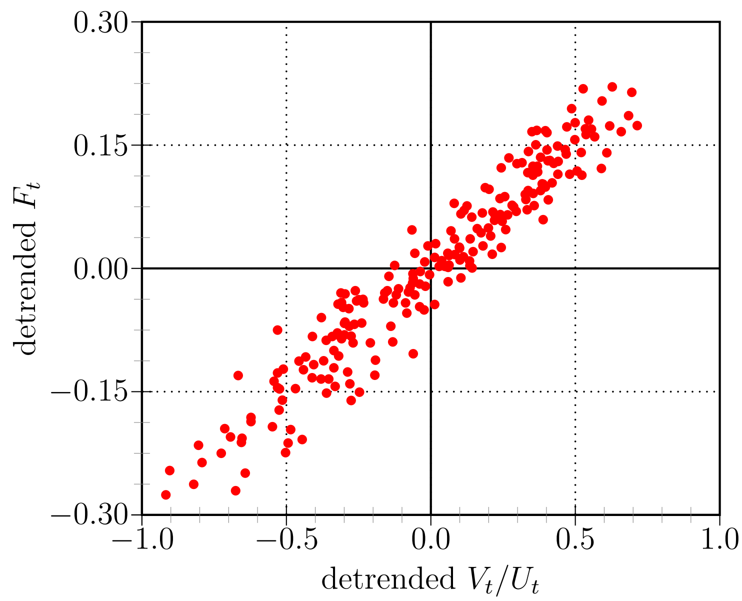

Figure 5: U.S. data, 1951Q1–2004Q4. Vacancies are measured from the help wanted advertising index, unemployment and the job finding probability from the CPS.

		u	v	v/u	F
Standard Deviation		0.189	0.201	0.380	0.118
Autocorrelation	1 Qtr	0.939	0.949	0.947	0.912
	2 Qtr	0.811	0.827	0.822	0.812
Correlation	u	1	-0.895	-0.972	-0.951
	v	—	1	0.975	0.923
	v/u	—	—	1	0.962
	F	—	—	—	1

Table 1: U.S. data, 1951–2004. Quarterly average of monthly data, detrended using an HP filter with smoothing parameter 10^5. The measure of vacancies is the help wanted index.

relationship between the instantaneous job finding rate f and the v-u ratio as N varies, the reduced-form matching function.

A proportional increase in q, l and n does not affect the number of workers or jobs per labor market but simply results in a proportional increase in f (equation 13). Hence it does not affect the curvature of the matching function. In addition, Figure 6 shows that the matching function is insensitive to the composition of the total separation rate, $s \equiv q + l$, between quits and layoffs. From equation (6) and equation (7), we know that the vacancy and unemployment rates are unaffected by q and l. This figure shows that the instantaneous job finding rate f is scarcely affected by the composition of separations.

A striking feature of the reduced-form matching function is that it is indistinguishable from a Cobb-Douglas. The dashed green line in Figure 7 shows a particular isoelastic relationship between the v-u ratio and the instantaneous job finding rate. When higher productivity raises the v-u ratio by ten percent, it also increases the instantaneous UE transition rate by just over two percent.[11] Although the theoretical relationship is not exactly Cobb-Douglas, if the model were the data generating process, it would be virtually impossible to reject the hypothesis of a Cobb-Douglas matching function empirically.

[11] Implicit differentiation of equation (6), (7), and (13) shows that if $M = N \to \infty$, the elasticity of the reduced-form matching function converges to $\frac{1}{2} - \frac{1}{\pi} \approx 0.18$; however, even in this limit it is not quite isoelastic.

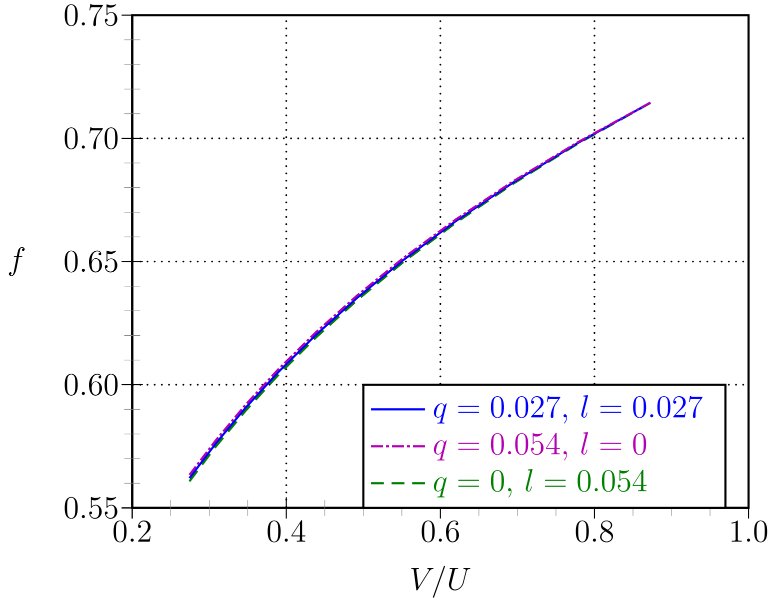

Figure 6: Comparative statics of f and V/U with respect to changes in $N \in [233, 243]$. $M = 244.2$ throughout.

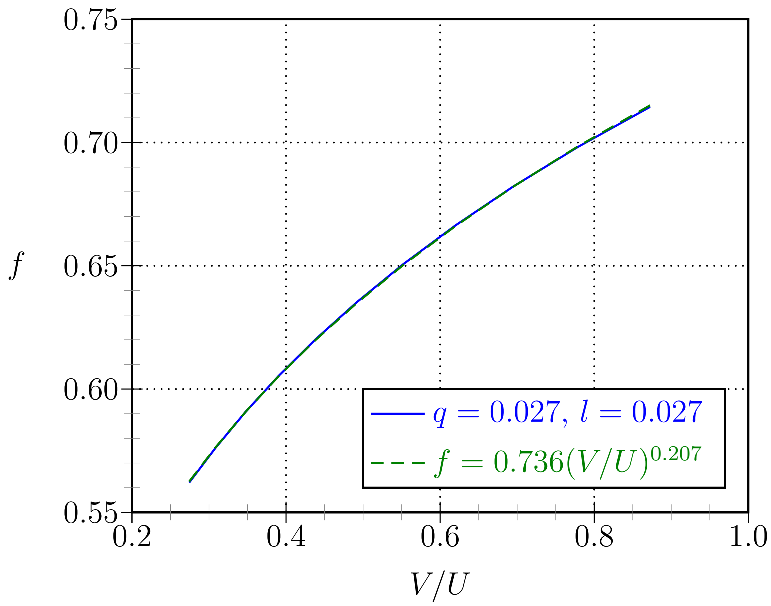

Figure 7: Comparative statics of f and V/U with respect to changes in $N \in [233, 243]$. $M = 244.2$ throughout.

5.4 Aggregate Shocks

Although the comparative statics are suggestive about the comovement between the job finding rate and the v-u ratio, they are not definitive. For example, when a positive shock hits, the number of jobs per labor market increases, resulting in a surge of workers finding jobs. Conversely, a negative shock sharply reduces hiring for a period of time until the stock of job falls to its new equilibrium value. If these dynamic responses are important, they may swamp the static relationship depicted in Figure 6 and Figure 7, reducing or even eliminating the informativeness of the v-u ratio about the job finding probability.[12]

I therefore explore the behavior of an economy subject to aggregate shocks. More precisely, I fix $M = 244.2$ and $q = l = 0.027$ throughout. There are two state variables, the actual number of jobs per labor market N and the target number of jobs per labor market N^*. I assume N^* follows an exogenous discrete state Markov process in continuous time described precisely in Appendix A. This process reflects an unmodeled shock to productivity x which, together with the free entry condition, determine the target number of jobs.

A shock hits the target N^* according to a Poisson process, i.e. at exponentially distributed time intervals, whereupon the target takes on a new value $N^{*\prime}$. If at any point in time t the actual number of jobs per labor market, $N(t)$, is smaller than the target number of jobs, $N^*(t)$, there is a surge of entry until $N(t_+) = N^*(t)$. If $N(t)$ exceeds $N^*(t)$, no new jobs enter since entering firms would lose money. During this adjustment period, the actual number of jobs per labor market declines due to layoffs, $\dot{N}(t) = -lN(t)$, until $N(t)$ reaches $N^*(t)$. Finally, if $N(t) = N^*(t)$, the entry rate of jobs exactly offsets layoffs, $n(t) = lN(t)$.

I measure $N(t)$ and hence $U(t)$ and $V(t)$ at each date $t \in \{1, 2, \ldots, \ldots 648\}$. Between measurement dates I compute the total rate at which unemployed workers find jobs. This is $qM\pi_q^{UE}(t) + n(t)\pi_n^{UE}(t)$ per unit of time at any t with $N(t) \geq N^*(t)$, since unemployed workers find jobs following either a quit or the entry of a new job. In addition, whenever $N(t) < N^*(t)$, there is a surge of entry and hence a discrete drop in unemployment. I include the positive measure of unemployed workers who find jobs at these instants. I divide the total measure of unemployed workers finding jobs during $t' \in [t, t+1)$ by unemployment at t to get a discrete time measure of the job finding rate.

The 648 time periods correspond to 54 years of monthly data. I take quarterly averages of the monthly data, detrend using an HP filter with smoothing parameter 10^5, and compute summary statistics. I repeat this process 10,000 times to obtain reliable estimates of the

[12] Coles and Petrongolo (2003) emphasize this possibility in a stock-flow matching model, although they do not demonstrate its quantitative importance in a calibrated version of the model.

		u	v	v/u	f
Standard Deviation		0.157 (0.033)	0.222 (0.040)	0.377 (0.058)	0.080 (0.012)
Autocorrelation	1 Qtr	0.944 (0.017)	0.944 (0.016)	0.944 (0.016)	0.892 (0.031)
	2 Qtr	0.844 (0.044)	0.845 (0.044)	0.845 (0.044)	0.791 (0.057)
Correlation	u	1	−0.970 (0.023)	−0.990 (0.008)	−0.931 (0.018)
	v	—	1	0.995 (0.005)	0.940 (0.017)
	v/u	—	—	1	0.943 (0.016)
	f	—	—	—	1

Table 2: Simulation of shocks to the target number of jobs N^*. The parameterization is described in the text. Bootstrapped standard errors are in parenthesis.

mean and bootstrapped standard errors. Table 2 shows the results.

The stochastic model reproduces the strong negative correlation between unemployment and vacancies depicted in Figure 2 and quantified in the first three columns of Table 1. For example, the empirical correlation between unemployment and vacancies is −0.90, while the theoretical correlation is even stronger, −0.97. The model slightly overstates the volatility of vacancies and understates the volatility of unemployment, but it is close. The theoretical autocorrelation of the two variables are about equal, consistent with the empirical evidence. This last observation is notable since the equal persistence of unemployment and vacancies is a puzzle for matching models where unemployment is a state variable and vacancies are a jump variable (Shimer, 2005b; Fujita and Ramey, 2005).

Returning to the reduced-form matching function, the correlation between the instantaneous job finding rate and the v-u ratio is strongly positive, consistent with Figure 4 and Figure 5. Since the volatility of the detrended job finding rate is only a little more than 20 percent of the volatility of the detrended v-u ratio, a regression of the detrended variables would uncover an elasticity almost exactly equal to 0.2, consistent with the comparative statics. More to the point, a regression of the detrended job finding rate on the detrended v-u ratio and its square rejects the null hypothesis that the coefficient on the squared term is zero at a 5 percent confidence level only about 1.3 percent of the time, less frequently than one would expect if the job finding rate were equal to the v-u ratio plus white noise. It is virtually impossible to distinguish the reduced form matching function from a Cobb-Douglas in time series data.

There is one major difference between the empirical and theoretical measures of the matching function. The theoretical job finding rate f is a continuous time measure while the empirical job finding probability F is a discrete time measure. Thus at this point in the paper the finding that the theoretical matching functions is Cobb-Douglas should be viewed as incomplete. The next section of the paper shows how to reconcile these numbers.

6 Duration Dependence

The job finding rate for any particular unemployed worker may differ substantially from f in equation (13), depending on the number of workers i and the number of jobs j in her labor market. This gives rise to duration dependence in the job finding rate: if an econometrician observes a worker who has been unemployed for a long time but cannot observe local labor market conditions, he should infer that the worker is probably stuck in a labor market in which jobs are scarce and workers plentiful. The worker's job finding rate is correspondingly low. Conversely, the rate at which a newly unemployed worker finds a job is higher than the average job finding rate f.

One implication is that, if at the start of the month the average unemployed worker finds a job at rate f, by the end of the month the same worker's job finding rate is less than f assuming she is still unemployed. This means that the full month job finding probability is less than $1 - e^{-f}$. This section measures the monthly job finding probability, the theoretical counterpart of F_t in equation (15). I look at the extent to which it varies with unemployment duration in the cross-section and the extent to which it varies with the v-u ratio in response to changes in productivity.

6.1 Cross Section

I cannot find analytical expressions for the job finding probability either unconditionally or conditional on unemployment duration. Instead I simulate 1 million unemployment spells to recover these numbers numerically. In half the spells, I start with a "job leaver," a worker who quit her labor market and moved to one in which there were more workers than jobs, $i > j$. In the other half of the spells, I start with a "job loser," a worker whose job left a labor market that previously had i workers and $j \leq i$ jobs. In both cases, I simulate the evolution of the worker's local labor market, stochastic changes in the number of workers and jobs coming from entry and exit, until the worker finds a job either because a new job enters, an employed worker leaves, or our unemployed worker quits for a labor market with available jobs. I assume that whenever a job is available, each unemployed worker is equally likely to be hired, independent of unemployment duration. For example, if at some point our unemployed worker is in a labor market with i workers and $j < i$ jobs and a new job enters, I assume that she gets the job with probability $1/(i-j)$.[13]

I use the usual values for the number of workers per labor market, $M = 244.2$, and the quit and layoff rates $q = l = 0.027$. For now I fix $N = 236.3$ giving an unemployment rate of 5.4 percent. With these values, the instantaneous job finding rate, f in equation (13), is 61.0 percent. If the job finding rate were constant, the full month job finding probability would be $1 - e^{-f} = 45.6$ percent.

Figure 8 shows the theoretical monthly probability of finding a job—the fraction of workers who find a job during the next month—as a function of the current duration of an unemployment spell and the reason for unemployment. The wiggles are due to sampling variation. The job finding probability for job losers is slightly higher than for job leavers, at least during the initial few weeks of unemployment. This reflects slight differences in initial labor market conditions for the two groups.

To compute the theoretical counterpart of the job finding probability F_t, I take a weighted average of the job finding probability in Figure 8, with weights corresponding to the fraction of spells that do not end before a particular duration. 39.8 percent of job leavers and 40.2 percent of job losers find a job in given month. The empirical counterpart of these numbers is 39.7 percent since December 2000 (Figure 4); the success of the model at matching this number is due to a judicious choice of $q = l = 0.027$. Once again, changes in q and l leaving $q + l$ constant scarcely affect this result.

The job finding probability of both job losers and job leavers declines sharply during an unemployment spell. They are about 55 percent when a worker first becomes unemployed but fall to less than 35 percent after three months of unemployment. To reduce this decline to a single dimension, I look at a weighted average of the job finding probability, where weights correspond to unemployment duration. Shimer (2005c) shows that we can measure

[13]The results in this section are sensitive to this assumption. If the most recently unemployed worker is always the first to get a job, the model generates significantly more duration dependence in the exit rate from unemployment. Conversely, if the unemployed queue for a job (Sattinger, 2005), duration dependence is inverted, with the long-term unemployed more likely to find a job than the short-term unemployed. Another approach would be to model heterogeneous workers, eliminating the current model's ambiguity on who is hired.

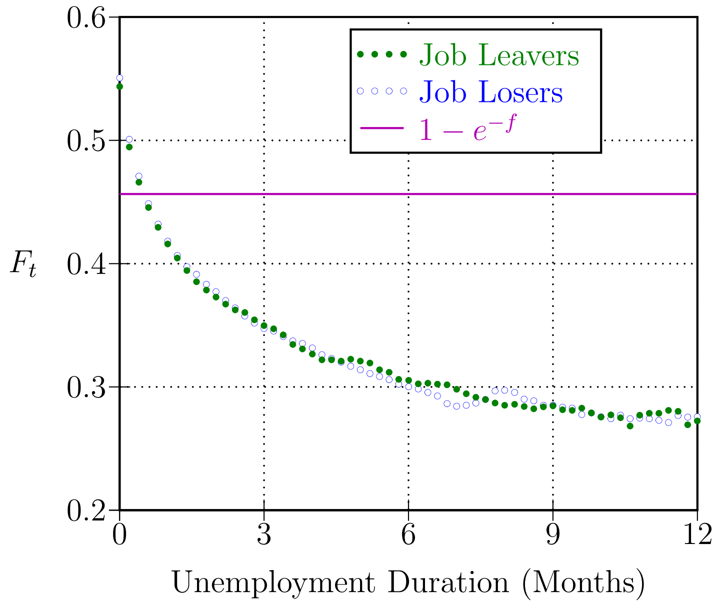

Figure 8: Monthly job finding probability as a function of unemployment duration. The number of workers per labor market is $M = 244.2$, the number of jobs per labor market is $N = 236.3$. The quit and layoff rates are $q = l = 0.027$ per month.

this using time series on unemployment U_t and mean unemployment duration $\bar{d}_t$:

$$\frac{\sum_{i=1}^{U_t} d_t^i F_t^i}{\sum_{i=1}^{U_t} d_t^i} = 1 - \frac{(\bar{d}_{t+1} - 1)U_{t+1}}{\bar{d}_t U_t},$$

where d_t^i is worker i's unemployment duration and F_t^i is her probability of finding a job. Empirically this number averaged 24.3 percent since December 2000, while in the model it is 33.0 percent for job leavers and 33.2 percent for job losers. Since the model matches $F_t = 0.40$, by this metric it explains about half of the decline in the job finding probability as a function of unemployment duration; presumably the other half is a consequence of unmodeled heterogeneity among workers within labor markets.

6.2 Comparative Statics

I now explore how time aggregation and duration dependence affect the theoretical matching function. I let N vary from 233 to 243 with M, q, and l fixed. At each value of N I compute the v-u ratio and I simulate the fraction of unemployed workers who find a job within a month. The solid blue circles in Figure 9 show the results. There is again an increasing relationship between the v-u ratio and the measured job finding probability. The solid blue

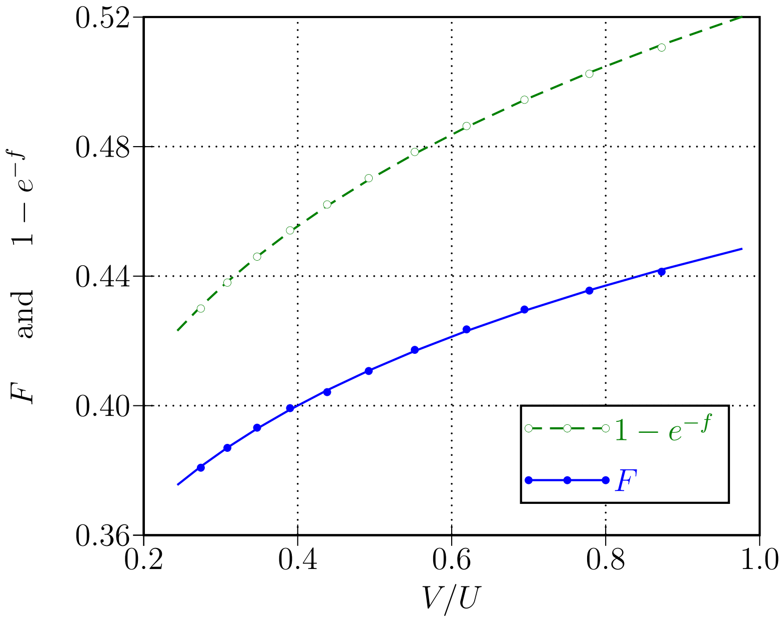

Figure 9: Theoretical job finding probability as a function of the v-u ratio. The number of workers per labor market is fixed at $M = 244.2$ and the quit and layoff rates at $q = l = 0.027$. The entry rate of jobs n varies so N takes values between 233 to 243.

line depicts a Cobb-Douglas function through these points, analogous to Figure 7. Again the fit is remarkable, although the elasticity is somewhat lower, 0.13.

The hollow green circles in Figure 9 show the relationship between $1 - e^{-f}$, a full month measure of the job finding probability ignoring duration dependence, and the v-u ratio. This is systematically about 15 percent higher than F but the quality of the Cobb-Douglas fit (dashed green line) and the elasticity (0.15) are similar. I conclude that accounting for time aggregation lowers the level of the theoretical job finding probability significantly but does not affect the main conclusion of the earlier work on the matching function: the model generates a reduced-form Cobb-Douglas matching function. The empirical elasticity of F with respect to V/U is about half the theoretical elasticity of 0.28 depicted in Figure 5.

7 Employer-to-Employer Transitions

7.1 Theoretical Concepts

This is as much a model of employer-to-employer (EE′) as it is one of UE and EU transitions. A quit shock leads to an EE' transition if it hits an employed worker and the worker moves to a labor market with vacancies, $i < j$:

$$\pi_q^{EE'} = (1-u) \sum_{i=0}^{\infty} \sum_{j=i+1}^{\infty} p(i,j). \tag{16}$$

Summing this and equation (10) implies a quit shock leads to either a EU transition or an EE′ transition with probability $1-u$, which is simply the probability that the shock hits an employed worker. Similarly, a layoff shock induces a worker to switch employers if it hits a filled job in a labor market with vacancies, $i < j$:

$$\pi_l^{EE'} = \frac{1}{N} \sum_{j=1}^{\infty} \sum_{i=0}^{j-1} ip(i,j).$$

The same logic as equation (9) implies $\frac{1}{N} \sum_{j=1}^{\infty} \sum_{i=0}^{j-1} jp(i,j) = \sum_{j=0}^{\infty} \sum_{i=0}^{j} p(i,j)$, and so using equation (5) this can be reexpressed as

$$\pi_l^{EE'} = \sum_{j=0}^{\infty} \sum_{i=0}^{j} p(i,j) - v. \tag{17}$$

Summing this and equation (12) gives $\pi_l^{EU} + \pi_l^{EE'} = 1 - v$, so the probability that a job leaving leads a worker to take a new job is equal to the probability that the shock hits a filled job.

7.2 Measurement

To construct a measure of EE′ transitions, I use a relatively new question from the CPS. Since the switch to dependent interviewing in 1994, the CPS has asked respondents who are employed in consecutive months, "Last month, it was reported that you worked for x. Do you still work for x (at your main job)?" Because the BLS does not tally the answer to this question, I follow Fallick and Fleischman (2004) and construct it from the CPS microdata, weighting individual answers, dividing by employment in the previous month, and seasonally adjusting using the Census X-12 algorithm.

The brown dots in Figure 10 show that in an average month from December 2000 to May 2005, 2.6 percent of employed workers switched employers. This fraction was higher at the start of the sample period when the v-u ratio was higher, generating an increasing relationship between the share of EE′ transitions in total separations and the v-u ratio.

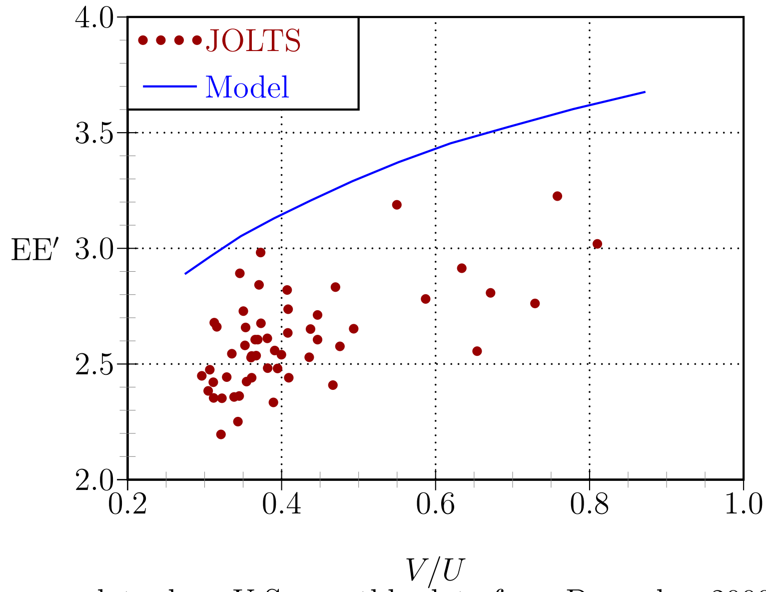

Figure 10: The brown dots show U.S. monthly data from December 2000 to May 2005. The unemployment rate is measured by the BLS from the CPS. The vacancy rate is measured by the BLS from the JOLTS. The solid blue line shows the theoretical analog with $M = 244.2$, $q = l = 0.027$, and $N \in [233, 243]$.

7.3 Comparative Statics

The data in Figure 10 is time aggregated since a worker may lose a job and become unemployed but a find a new job before the next measurement date. In order to compare the model with the data, I also have to time-aggregate the model. Indeed, the model suggests that there may be a significant number of unmeasured unemployment spells since a large fraction of workers find a job during the first few weeks of unemployment (Figure 8). To assess the importance of this, I simulate 1 million separation episodes, half for job leavers and half for job losers. In some events, a worker may move directly to another job, as described above. Otherwise, the worker has a fraction of a month, a uniform random variable on $[0, 1]$, to find a new job before experiencing a measured unemployment spell. In each case I compute the probability the worker is employed at the end of the month. Multiplying this by the quit or layoff rate gives the theoretical counterpart to the brown dots in Figure 10.

Variation in the number of jobs per labor market traces out an increasing relationship between the EE′ transition rate and the v-u ratio. Figure 10 shows that the model predicts slightly too many EE′ transitions, although the relationship between the EE′ transition rate and the v-u ratio is about right. The model is in line with the data on this measure as well.

8 Conclusions

This paper describes a competitive model of unemployment, vacancies, and labor market transitions. The basic structure is the mismatch model described by Tobin (1972) and Drèze and Bean (1990); however, while those earlier papers emphasized disequilibrium within labor markets, in this paper I have assumed that there is perfect competition within labor markets but large barriers to mobility across markets. This model provides a coherent framework for exploring important macro-labor facts in the United States economy: the downward sloping Beveridge curve, the Cobb-Douglas matching function, duration dependence in the job finding probability, and an increasing relationship between the probability of switching employers and the v-u ratio.

The matching model (Pissarides, 1985) is an obvious alternative for addressing these facts. While the matching model can deliver a Beveridge curve with the right slope, the mismatch model must deliver such a Beveridge curve. Moreover, the mismatch model delivers a downward sloping Beveridge curve even if the layoff and quit rates fluctuate cyclically and it explains why the persistence of vacancies and unemployment is similar. Both of these are problematic in the matching model. Finally, the mismatch model generates about 3.5 times as large fluctuations in the v-u ratio in response to productivity shocks as the matching model, helping to address the puzzle suggested by Shimer (2005b).

A partial success of the mismatch model is the matching function, which the matching model treats as exogenous. The mismatch model delivers a reduced-form matching function that is indistinguishable from a Cobb-Douglas, although the elasticity of the matching function is about half of the empirically relevant value. To my knowledge, this is the first explanation for the empirical regularity that the matching function appears to be a stable Cobb-Douglas function. In contrast, whether the matching model has a Cobb-Douglas matching function is a choice of the modeler.

Finally, an integral part of the mismatch model is a theory of duration dependence in the job finding probability and of employer-to-employer transitions. In principle both of these can be tacked on to the matching model (Pissarides, 2000), although there is not much work on the business cycle properties of such models. By contrast, the simplest version of the mismatch model explains half the duration dependence in the job finding probability and it explains the weak procyclicality of employer-to-employer transitions.

There are other predictions of the mismatch model that I have not explored here. Notably the mismatch model provides a coherent theory of jobs and hence a model of job flows distinct from worker flows. In principle this means that the model could simultaneously be used to address facts about labor market flows and facts about job creation and job destruction (Davis, Haltiwanger, and Schuh, 1996). Preliminary work suggests that a simple feature of the labor market, the fact that the vacancy rate is less than the unemployment rate, may explain why job flows are systematically smaller than workers flows.

I have kept this model deliberately simple in order to highlight the main forces in a model of mismatch. This also means that it is readily amenable to extensions. I mention two in closing. First, I have assumed that all workers and jobs are identical. If workers and jobs were heterogeneous, the model would make stronger predictions about who is employed and who is unemployed. This would strengthen the duration dependence in the job finding probability and may affect the model's performance along other dimensions.

Second, I have ignored the decision to exit a labor market. A worker in a labor market with $i \gg j$ has a strong incentive to move, either by relocating to another city or by retraining.[14] My analysis is applicable if mobility costs are sufficiently high that no worker would voluntarily choose to incur them. But extending the model to allow for endogenous relocation is still important, since such a model could address issues that go beyond the scope of the this paper, such as the procyclicality of quits.

A Stochastic Process

I model a second order autoregressive process for the target number of jobs per labor market N^*. The state includes the current and previous value of N^*, each of which may take on $2\nu + 1$ possible values. When an aggregate shock hits, N^* may increase or decrease by one grid point. If the previous value of N^* is higher than the current value, it is more likely that N^* will continue to fall. If N^* is lower, however, it is more likely to rise, giving a mean reverting second order autoregressive process.

More precisely, the state is a pair $\{N^*, \mu\}$ where $\mu \in \{-1, 1\}$,

$$\log\left(\frac{N^*}{N_0}\right) \in \{-\nu\Delta, -(\nu-1)\Delta, \ldots, -\Delta, 0, \Delta, \ldots, (\nu-1)\Delta, \nu\Delta\},$$

and N_0 is the geometric mean of N^*. A shock arrives at Poisson rate λ. The new value

[14] Sattinger (2005) allows for endogenous mobility in a related queueing model.

$\{N^{*\prime}, \mu'\}$ is determined as follows: If $\mu = 1$,

$$\{N^{*\prime}, \mu'\} = \begin{cases} \{N^* e^{\Delta}, 1\} & \text{with prob. } \left(\frac{1}{2} - \frac{\log N^*/N_0}{2\nu\Delta}\right)^\eta \\ \{N^* e^{-\Delta}, -1\} & \text{otherwise} \end{cases}$$

while if $\mu = -1$,

$$\{N^{*\prime}, \mu'\} = \begin{cases} \{N^* e^{-\Delta}, -1\} & \text{with prob. } \left(\frac{1}{2} + \frac{\log N^*/N_0}{2\nu\Delta}\right)^\eta \\ \{N^* e^{\Delta}, 1\} & \text{otherwise} \end{cases}$$

If $\eta = 1$, this reduces to a first order autoregressive process. In that case, the parameter $\gamma = \lambda/\nu$ determines the mean reversion of the process and $\sigma = \Delta\sqrt{\lambda}$ determines the instantaneous volatility. As ν increases for given values of γ and σ, the discrete state space process converges to an Ornstein-Uhlenbeck process (Shimer, 2005b).

If $\eta < 1$ a positive shock is more likely to be followed by another positive shock, which increases the persistence of shocks relative to the benchmark. The parameter values I use are $N_0 = 236.3$, matching the historical average unemployment and vacancy rates; $\nu = 100$; $\eta = 0.4$; $\gamma = 0.005$; and $\sigma = 0.0023$. The last three parameters are important for matching the standard deviation and first and second autocorrelation of the v-u ratio, while the results are insensitive to the choice of ν.

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