Currency Areas and Monetary Coordination*

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Abstract

In this paper we examine currency areas and monetary policy coordination in a two-country model where the value of each country's currency is determined endogenously. Exchanges in the goods markets are modelled as random bilateral matches, and households choose the amount and the frequency of purchases made in each currency area. By contrast, the currency market is centralized and Walrasian. We determine the nominal exchange rate in the equilibrium and show that the size of a currency area decreases with the growth rate of that currency and increases with the growth rate of the competing currency. We also find the following welfare results on monetary competition and coordination. First, policy coordination results in the Friedman rule being chosen for each currency. Second, policy competition increases inflation and reduces welfare in both countries relative to policy coordination. Third, under policy competition, the incentive to inflate increases with the degree of the integration of the goods markets. Fourth, currency unification delivers the same allocation as policy coordination, provided that the latter can be achieved.

Preliminary. Do not quote.

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1. Introduction

Whether countries should coordinate their monetary policy has been an important issue in international economics and in policy debate. The creation of European monetary union did not settle the debate, but rather stimulated more interests. In the center of the debate are the following questions: (i) Does monetary policy coordination enhance welfare relative to policy competition? (ii) Does the integration of markets increase the need for monetary coordination? We try to address these questions in this paper. Deviating substantially from the literature, we integrate the microfoundation of monetary theory into international economics. In particular, we derive the value of each country's currency endogenously from the frictions in the goods market.

To motivate our analysis, it is useful to briefly review the large literature on monetary policy coordination (see Obstfeld and Rogoff, 2002, for some references). The foundation of this literature is the Phillips curve, which gives monetary policy a role in stabilizing the economy in the event of shocks. The early part of this literature used linear models of rational expectations to explore the trade-off between unanticipated inflation and output fluctuations. The late part of the literature rationalizes the Phillips curve in the utility-maximization framework with nominal rigidity, which is referred to as the "new open economy macroeconomic models". Both parts of this literature have obtained mixed results on the need for monetary coordination. For example, Rogoff (1985) shows that, relative to policy competition, monetary policy coordination increases inflation and reduces welfare by reducing the credibility of monetary policy in fighting inflation. Similarly, Obstfeld and Rogoff (2002) have shown that, even when nominal prices are rigid, there is no need for monetary coordination if each country uses its monetary policy to implement the flexible-price allocation. Some other authors (e.g., Canzoneri, et al., 2004) have found that monetary policy coordination can increase welfare.

While this debate is interesting, our view is that all models in this literature share the same flaw that makes their welfare analyses questionable. That is, they do not have a strong microfoundation for money. Invariably, these models rely on ad hoc assumptions to support the value of money. The most prevalent one is that the real money balance of a country's currency generates utility to that country's households but not to other countries' households (e.g., Obstfeld and Rogoff, 1998). Another popular assumption is the cash-in-advance constraint that requires a country's goods to be sold for a particular currency (e.g., Helpman, 1981). Sometimes, this requirement is replaced by a less stringent assumption that the transaction cost in purchasing a country's goods is a decreasing function of the real money balance of that country's currency. Although these assumptions capture part of the realism of the use of money *at each point of time*, they place money at the same level as households' preferences and technology. Such assumptions are ill suited for welfare analysis on monetary policy, because they make the analysis subject to the Lucas critique.

This problem is apparent in the analysis of currency unification. For the sake of argument, consider the specification with money in the utility function. Suppose that there are two countries, 1 and 2. Initially, country 1 prints green money and country 2 prints red money. According to the specification, green money generates utility only to country 1 households and red money only to country 2 households. Now, the two countries decide to unify their currencies into a new currency, say, blue money. The new currency is printed in both countries and is marked with the name of the printing country. The specification of money in the utility function would now require blue money to generate utility to both countries' households, regardless of where the currency is printed. This requirement is not reasonable for economic modelling since the blue money printed in each country is merely a re-labeling of the old currency printed in that country.

The same problem arises in the literature on monetary policy coordination, although to a lesser extent. Competition in monetary policy can render one currency, say green money, more acceptable than red money to importers and exporters. This advantage of green money may decrease when the two countries coordinate on monetary policy. The standard models in international economics are inadequate to capture this endogenous response of the role of currencies to monetary policy coordination.

To eliminate the above problem, we model the role of money in the economy using the recent development in monetary theory (see Shi, 1995, and Trejos and Wright, 1995). There are two countries in our model, labelled country 1 and country 2. The two countries produce the same set of goods, all of which can be traded between the two countries. Preferences and production technologies are identical between the two countries. In each country, there are many types of households, each producing one particular type of goods but wishing to consume a subset of other types of goods. The exchange in the goods market is decentralized and characterized by random, bilateral matches. There is no public record-keeping of agents' trading histories. This difficulty of double coincidence of wants generates the demand for the medium of exchange. However, the medium can be either money 1 issued by country 1 government, or money 2 issued by country 2 government, or both. The households choose the amount of holdings and the frequency of the purchases conducted with each currency. Hence, there is no requirement that buyers must use a particular currency to purchase a country's goods. In addition, the currencies do not generate direct utility to the households or facilitate production.

There are two markets for goods, which we call the *currency areas*. All transactions are conducted with currency 1 in area 1, and all with currency 2 in area 2. The size of each currency area is determined endogenously, as a household in each country chooses the fraction of buyers to go to each currency area. Generically, a buyer who goes to a currency area encounters both countries' sellers with positive probability. Thus, transactions in each currency area can involve agents from the same country and agents from different countries. In both cases, prices are perfectly flexible and determined by bargaining. For tractability, we assume that each household in country i sends a fixed fraction s of its sellers to currency area i, where $i \in \{1, 2\}$. We interpret $(1 - s)$ as the *degree of market integration* between the two countries, where the case $s = 1/2$ corresponds to perfectly integrated markets. The case $s = 1$ corresponds to the assumption that country i sellers only sell goods for currency i. Even in the latter case, buyers do not have to buy from their own country's sellers – They can choose to purchase from the other country, since all goods of the same type are perfect substitutes no matter where they are produced.

In addition to the goods markets, there is a centralized currency market. Agents can exchange the two currencies in the market at the Walrasian price. However, it is costly to make instantaneous arbitrage between the currency market and the goods markets. To simplify the analysis, we assume that the two types of markets are separated in each period. In particular, households go to the currency market first before going to the goods markets.

We model a country's monetary policy as the choice of the growth rate of its money supply, denoted γ_i for country i. Money growth is brought about by lump-sum transfers (or taxes if $\gamma_i < 1$) at the beginning of each period. However, as a defining characteristic of a country, only households in country i receive the transfers or taxes of currency i. Policy competition refers to the case in which each country chooses its money growth rate to maximize the intertemporal utility of the country's own households. Policy coordination refers to the case in which the two countries jointly set growth rates of the two monies to maximize a weighted intertemporal utility of the two countries' households. In addition, we examine the welfare effect of unifying the two currencies.

In this framework, we show that the nominal exchange rate is uniquely determined for any given degree of market integration. Each country's allocation of buyers to a currency area is also determinate. When the growth rate of a currency i increases, the currency depreciates against the other currency, and both countries send fewer buyers to currency area i. This movement of buyers between the two currency areas and the trade in the currency market equalize the two countries' *relative valuation* of each currency. However, they do not equalize the levels of the valuation between the two countries, provided that the growth rates differ for the two monies. This discrepancy between the two countries' valuations makes the real exchange rate differ from unity and depend on the differential between the two monies' growth rates.

On policy coordination, we find the following results. First, policy coordination results in the Friedman rule being chosen for each currency; that is, the growth rate of the supply of each money decreases at the rate of time preference under policy coordination. Second, policy competition generates higher inflation and lower welfare in both countries than policy coordination. Third, under policy competition, the incentive to inflate by each country increases with the degree of market integration and, in particular, the incentive is the strongest when markets are completely integrated (in the sense $s = 1/2$). Fourth, currency unification delivers the same allocation as policy coordination, provided that the latter can be achieved.

As said earlier, our model builds on the recent development in monetary theory. Early applications of this theory to multiple currencies and exchange rates include Matsuyama, et al. (1993), Shi (1995), Trejos and Wright (1996), and Zhou (1997). These applications have assumed money or goods, or both, to be indivisible. As a result, they are not suitable for analyzing money growth and inflation. The current model eliminates this restriction by using the construct of large households in Shi (1997).

The paper closest to ours is the one by Head and Shi (2003). Using a similar setup, Head and Shi focus on the determination of the nominal exchange rate. Although the nominal exchange rate is also an important variable in our analysis, we focus instead on the welfare consequences of policy competition and policy coordination. Also, our analysis differs from Head and Shi in two aspects of modelling. First, we model currency areas and examine how monetary policy affects the size of each currency area. Second, we assume that the currency market is centralized and Walrasian. In contrast, there is no currency area in Head and Shi (2003), and all exchanges there, including the ones between currencies, are random bilateral matches. By introducing currency areas and centralized currency exchanges, we make not only the model more realistic but also the results more robust.

Finally, notice that our analysis focuses on welfare effects of long-run money growth and features perfectly flexible prices. These contrast to most of the "new open economy macroeconomic models", which focus on welfare effects of short-run inflation and feature sticky prices. A notable exception is Cooley and Quadrini (2003), who have found that competition in setting long-run inflation rates reduces welfare in an environment with perfectly flexible prices. Their result relies on the assumption that production in each country uses inputs from both countries which are not perfectly substitutable. By contrast, labor is the only input in production in our model and all goods of the same type are perfect substitutes in households' utility function. Above all, Cooley and Quadrini use the cash-in-advance constraint to determine the exchange rate, which makes their analysis subjected to the same criticism articulated earlier in this introduction.

2. The Model

2.1. The Environment

Consider a discrete-time economy with an infinite horizon. There are two countries, labelled 1 and 2. Each country consists of a large number of households, the measure of which is normalized to one. These households are evenly divided into K types, where $K \geq 3$. A type i household produces only good i and consumes only good $(i+1) \pmod K$, and so type $(i+1)$ goods are called type i household's consumption goods. All goods are nonstorable. The two countries have the same preferences and production technologies. Consuming q unit of consumption goods generates utility $u(q) = Aq$, where $A > 0$ is a constant. The utility of consuming goods that are not consumption goods is zero. Production incurs a utility cost $\phi(q)$ for producing q units of goods, where $\phi(q) = q^{\sigma}$, $\sigma > 1$. All goods can be traded between the two countries.

Each country issues one currency. The residents of a country receive only monetary transfers of the domestic currency. A currency does not directly generate utility or facilitate production. However, as described later, the frictions in the goods markets create difficulties of exchange that can be alleviated by the use of a medium of exchange. This medium can be either currency or both currencies. There is no restriction on which currency should be used to purchase a country's goods.

A household consists of a continuum of infinitely-lived members, who carry out different tasks but regard the household's utility as the common objective and trade according to the rules given by the household.[1] The total measure of the members in the household is normalized to 2, with a unit measure of buyers and a unit measure of sellers. Buyers carry the currencies to buy the household's consumption goods, and sellers produce and sell goods. Since both countries' goods can be purchased with either currency, an important decision of the household is to divide the buyers into the holders of each currency. For a country i household, let $n_i \in [0, 1]$ be the household's choice of the fraction of buyers who hold the domestic currency.

There are two goods markets called market 1 and market 2. All transactions are conducted with currency 1 in market 1, and all with currency 2 in market 2. Since each market uses only one currency, we interpret a market as a currency area.[2] The size and the composition of buyers in a currency area are determined endogenously by the households' choices of n_i. For example, currency area 1 has a number n_1 of country 1 buyers and a number $(1 - n_2)$ of country 2 buyers, as well as the sellers. To emphasize the choice of the division of buyers, we simplify the division on sellers by assuming that each household sends a fixed fraction s of sellers to the market marked with the domestic currency and $(1-s)$ of sellers to the market marked with the foreign currency.[3]

One way to interpret the fixed fraction $(1 - s)$ is that it is determined by physical restrictions as the upper bound on the fraction of sellers whom a country can send to the foreign currency area. Interpreting this way, we can refer to $(1 - s)$ as the degree of market integration between the two countries. In the case $s = 1$, sellers from one country only sell goods for the domestic currency and the degree of integration is low. On the other hand, the case $s = 1/2$ corresponds to an integrated economy. In this case, there are exactly the same number of sellers from both countries in each area.

In contrast to the goods markets, the currency market is centralized and frictionless. The

[1] The large household assumption makes the distribution of money holdings degenerate across housholds and so allows for a tractable analysis for issues related to money growth, see Shi (1997, 1998, 1999), Head and Shi (2003). The modelling device is also a proxy for a single agent's time allocation during a period.

[2] This interpretation is consistent with the discussion by Mundell (1961). The separation of the trades into two currency areas implies the restriction that a buyer cannot carry both currencies into each trade match. This restriction captures the difficulty of making instantaneous arbitrage between the two currencies through the goods market, and it is important for the determination of the nominal exchange rate. However, each household can arbitrage in the currency market. More importantly, the separation of the goods markets does not give rise to a cash-in-advance constraint. To the contrary, a household can decide which market from which to buy the goods and, in each market, the household can use one currency to buy goods produced by both countries' sellers.

[3] We will also assume later that the buyer in a match has all the bargaining power. Under this assumption, each household would be indifferent about the level of s if it were the household's choice.

nominal exchange rate clears that market in equilibrium. The currency areas and the currency market are illustrated in Figure 1.

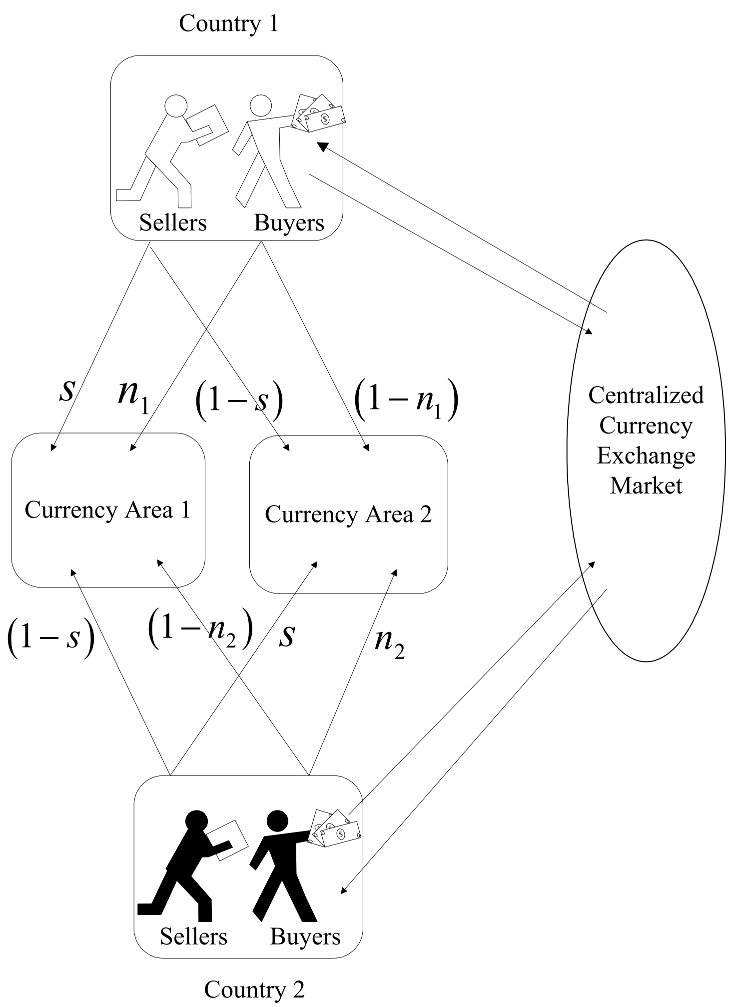

Figure 1. Illustration of the environment

2.2. Matches and Trading Quantities

Agents are matched randomly and bilaterally in each currency area. Because each household has specialized tastes and production, there is no match of a double coincidence of wants. Thus, barter is impossible.[4] The matches where a trade can occur are the ones in which the seller can

[4]This is a simplifying assumption. As is well established in the literature **(see Shi, 1997)**, barter can be easily introduced into the model without changing the main results, provided that the chance of having a barter match is not very high.

produce the consumption goods for the buyer. We call such a match a trade match. Generically, a buyer in a currency area may encounter a seller from either country. Since buyers can also come from both countries, there are four types of trade matches in each currency area. We depict them in Figure 2 for currency k $(= 1, 2)$.

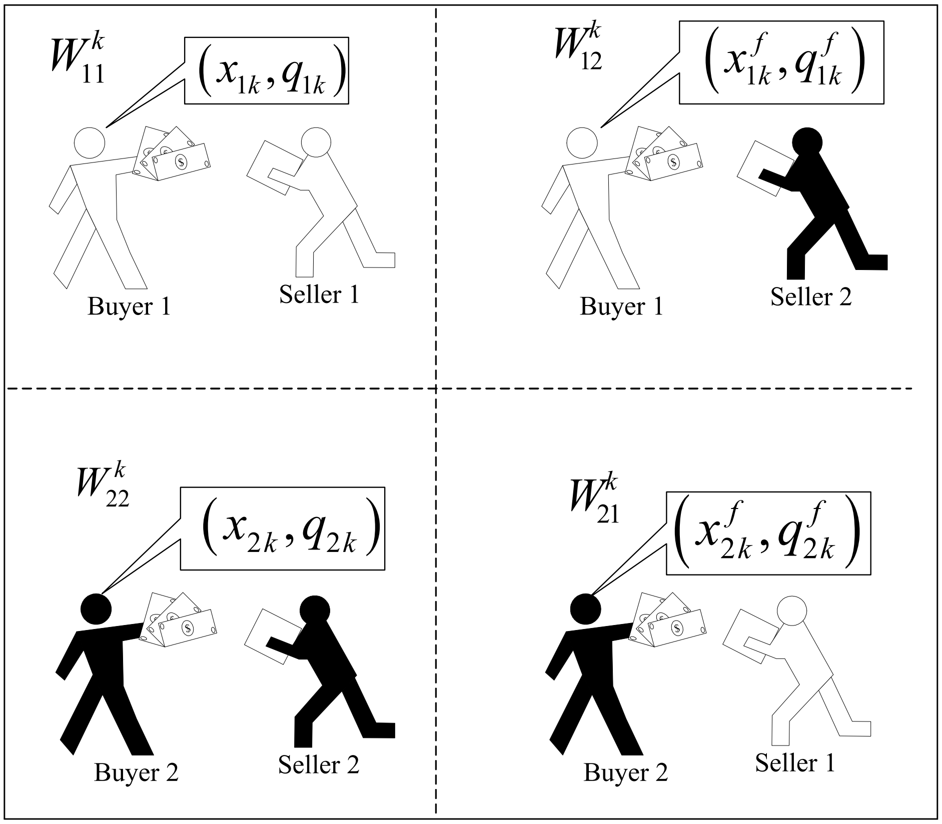

Figure 2. Trade matches in currency area k

In currency area k, the total number of country i's buyers is denoted N_{ik} and the total number of country j's sellers is S_{jk}, where i and j may or may not be the same. The total number of trade matches in area k between country i buyers and country j sellers, denoted W_{ij}^{k}, is given by the following matching function:

$$W_{ij}^{k} = L\left(N_{ik}\right) S_{jk},$$

where $L(N) = L_0(N)^{\psi}$, $\psi \in (0, 1)$, and $L_0 > 0$. Notice that, because S_{jk} is fixed, the matching function is equivalent to a Cobb-Douglas function with re-scaling the constant L_0. Thus, the matching technology has constant returns to scale, although buyers' marginal contribution to the match formation is diminishing.[5]

For a country j seller in currency area k, the probability of having a trade match with a country i buyer is W_{ij}^k/S_{jk}. Similarly, for a country i buyer in currency area k, the probability of having a trade match with a country j seller is W_{ij}^k/N_{ik}. It is convenient to define $\mu(N) = L(N)/N$. Then, $W_{ij}^k/N_{ik} = \mu(N_{ik})S_{jk}$. **F**or these matching probabilities to be indeed probabilities for all $S_{jk} \in [0,1]$, it is necessary and sufficient to restrict $L(N) \leq N$. To satisfy this restriction, we actually set $L(N) = \min\{L_0 N^{\psi}, N\}$ so that $\mu(N) \leq 1$. Notice that $L(N)$ so defined is increasing and (weakly) concave and that $\mu(N)$ is (weakly) decreasing for all $N \in [0,1]$.

In a trade match, the buyer has all the bargaining power and makes take-it-or-leave-it offers. Let x denote the amount of money paid by the buyer and q the quantity of goods sold by the seller. Thus, (x_{ik}, q_{ik}) denote the terms of trade in a match between agents from the same country, and (x_{ik}^f, q_{ik}^f) the terms between agents from different countries. The first subscript of x and q indicates the buyer's country, the second subscript indicates the currency used in the trade, and the superscript f indicates that the trade is between agents from different countries. For example, in a match with a county 2 seller in area 1, a buyer from country 1 proposes (x_{11}^f, q_{11}^f), and a buyer from country 2 proposes (x_{21}, q_{21}).

For simplicity of the notation, we normalize all prices and nominal quantities involving currency i by the total stock of currency i, M_i. For example, x_{ik} in the above description is the exchanged quantity of money normalized by M_k. Also, we normalize the nominal exchange rate by the ratio of the stocks of the two currencies and denote this normalized exchange rate as e. Thus, the actual exchange rate is eM_2/M_1, which is the number of units of currency 2 that are needed to exchange for one unit of currency 1.

2.3. Timing of Events

The events in an arbitrary period t unfold as in Figure 3, where the subscript t is suppressed. At the beginning of period t, each household in country i receives a lump-sum monetary transfer of only domestic currency i. The quantity of this transfer is $\tau_{i,t-1}M_{i,t-1}$, where

$$\tau_{i,t-1} = \gamma_{i,t-1} - 1, \quad i = 1, 2.$$

[5] Strictly speaking, if one re-scales the standard Cobb-Douglas matching function, $W_{ij}^k = W_0 (N_{ik})^{\psi} (S_{ik})^{1-\psi}$, to obtain our specification, then $L_0 = W_0/S_{ik}^{\psi}$ which varies among the four types of trade matches. By assuming that L_0 is the same for all four types of trade matches, we simplify the algebra without changing the main results.

Thus, the stock of currency k grows at the (gross) rate γ_{kt} between periods t and $t+1$. After the transfer, the household's money holdings are measured. Denote the (normalized) holdings of currency k by a country i household as m_{ik}.

Markets open sequentially. The currency market opens first.[6] A country i's household sends a (normalized) amount of currency k, f_{ik}, to the currency market. After the exchange, the currency market closes. Then, a country i's household chooses n_i. A fraction n_i of its buyers hold currency i going to currency area i, and a fraction $(1-n_i)$ of its buyers hold currency i' ($\neq i$) going to area i'. Each buyer going to a currency area is given the same amount of currency. The goods market in each area opens and matches are generated. Agents in trade matches exchange according to the instruction (x, q) given by the households, where the buyer makes a take-it-or-leave-it offer. During the exchange, matches are separated from each other and the two currency areas are also separated from each other.

After the exchange, agents bring the receipts home. Each household allocates the same amount of consumption to all members. Then time proceeds to the next period.

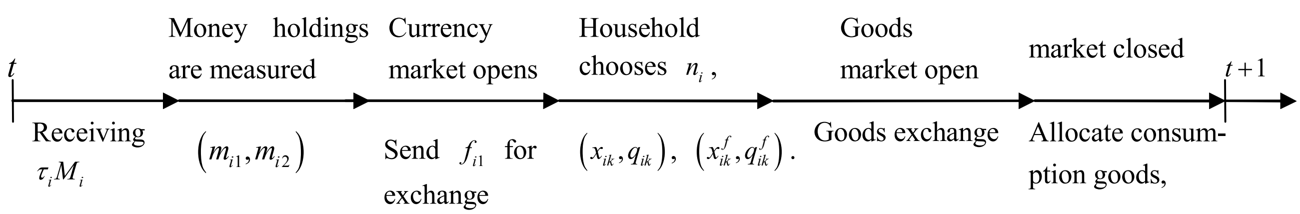

Figure 3. Timing of events in period t

2.4. The Representative Household's Decision Problem

Pick an arbitrary household in country i as the representative household of country i. Shorten the time subscript $t \pm i$ to $\pm i$, where $i > 0$. The representative household makes the following decisions: (i) the allocation of buyers to each currency area, $(n_i, 1-n_i)$; (ii) the terms of trade, (x_{ik}, q_{ik}) and (x^f_{ik}, q^f_{ik}), for a buyer to propose in a trade match in each currency area k; (iii) future

[6]One can adopt the alternative assumption that the currency market opens after the goods markets close. However, since there is no uncertainty in the economy, the model's results will essentially be the same. The algebra is slightly more complicated with the alternative assumption.

money holdings of each currency, $m_{ik,+1}$; (iv) the amount of currency k, f_{ik}, to be exchanged in the currency market. As said earlier, the nominal quantities $(x_{ik}, x^f_{ik}, f_{ik}, m_{ik})$ are all normalized by M_k. Denote these decisions as

$$h_i = \left[n_i, m_{i1,+1}, m_{i2,+1}, f_{i1}, (x_{ik}, q_{ik}, x^f_{ik}, q^f_{ik})_{k=1,2}\right].$$

Use the corresponding upper-case letters to denote other households' decisions or aggregate variables, which the representative household takes as given.

Let $V(m_{i1}, m_{i2})$ be the value function of the representative household in country i. Define:

$$\omega_{ik} = \frac{\beta}{\gamma_k} V_k(m_{i1,+1}, m_{i2,+1}), i = 1, 2; k = 1, 2.$$

where V_k denotes the derivative of the value function with respect to the kth argument. The variable ω_{ik} is the country i representative household's marginal value of next period's holdings of currency k, discounted to the current period.[7] To shorten the terminology, we call ω_{ik} the valuation of currency k of the representative household in country i. Similarly, the valuation of currency k of other households in country i is Ω_{ik}.

We formulate the maximization problem of country 1's representative household. A similar formulation applies to country 2 household's problem. To do so, notice that the household's consumption in a period is:

$$c = n_1 \left(\frac{W^1_{11}}{N_1} q_{11} + \frac{W^1_{12}}{N_1} q^f_{11} \right) + (1 - n_1) \left(\frac{W^2_{11}}{1 - N_1} q_{12} + \frac{W^2_{12}}{1 - N_1} q^f_{12} \right) \tag{2.1}$$

The terms in the first bracket are the expected amount of goods obtained by the household's buyers from currency area 1 and the terms in the second bracket are the expected amount from area 2. Since buyers can trade with domestic sellers or foreign sellers in each currency area, there are two terms inside each bracket. For example, in currency one, a buyer from the household encounters a trade match with a domestic seller with probability W^1_{11}/N_1 from whom the buyer obtains q_{11} units of goods, and with a foreign seller with probability W^1_{12}/N_1 from whom the buyer obtains q^f_{11} units of goods.

Similarly, the household's disutility of production in a period is:

$$\mathcal{P} = \left[W^1_{11} (Q_{11})^\sigma + W^1_{21} \left(Q^f_{21} \right)^\sigma \right] + \left[W^2_{11} (Q_{12})^\sigma + W^2_{21} \left(Q^f_{22} \right)^\sigma \right] \tag{2.2}$$

[7]The discounting involves the money growth rate γ_k, as well as β, because m_k is normalized by the stock of currency k. One actual unit of currency k in period $t+1$ is equivalent to only $1/\gamma_k$ units of normalized money in period t.

Notice the difference between the lower-case letters and the upper-case letters in (2.1) and (2.2).

The representative household chooses h_1 to solve the following maximization problem:

$$(PH1) \qquad V\left(m_{11,}m_{12}\right) = \max\left\{u\left(c\right) - \mathcal{P} + \beta V\left(m_{11,+1}, m_{12,+1}\right)\right\}$$

subject to following constraints:

$$\Omega_{1k}x_{1k} - \left(q_{1k}\right)^{\sigma} = 0;\ k = 1, 2; \tag{2.3}$$

$$\Omega_{2k}x_{2k}^{f} - \left(q_{2k}^{f}\right)^{\sigma} = 0;\ k = 1, 2; \tag{2.4}$$

$$\frac{m_{11} - f_{11}}{n_1} \geq x_{11};\quad \frac{m_{11} - f_{11}}{n_1} \geq x_{11}^{f}; \tag{2.5}$$

$$\frac{m_{12} + ef_{11}}{1 - n_1} \geq x_{12};\quad \frac{m_{12} + ef_{11}}{1 - n_1} \geq x_{12}^{f}. \tag{2.6}$$

In addition, the following laws of motion of money holdings must hold:

$$\begin{aligned} \gamma_1 m_{11,+1} = \quad & \left(m_{11} - f_{11}\right) - n_1\left(\tfrac{W_{11}^1}{N_1}x_{11} + \tfrac{W_{12}^1}{N_1}x_{11}^{f}\right) \\ & + \left(W_{11}^1 X_{11} + W_{21}^1 X_{21}^{f}\right) + \left(\gamma_1 - 1\right); \end{aligned} \tag{2.7}$$

$$\begin{aligned} \gamma_2 m_{12,+1} = \quad & \left(m_{12} + ef_{11}\right) - \left(1 - n_1\right)\left(\tfrac{W_{11}^2}{1-N_1}x_{12} + \tfrac{W_{12}^2}{1-N_1}x_{12}^{f}\right) \\ & + \left(W_{11}^2 X_{12} + W_{21}^2 X_{22}^{f}\right). \end{aligned} \tag{2.8}$$

The constraints (2.3) and (2.4) are the sellers' participation constraints in trade in area k. The left-hand side of each of these constraints is the seller's surplus from trade. To induce the seller to trade, the buyer's offer should give the seller a non-negative surplus. Since the buyer makes take-it-or-leave-it offer, the seller's surplus is zero, and the participation constraints are binding in equilibrium.

Buyers in trade matches also face the constraints (2.5) and (2.6). These constraints require that each buyer should not spend more money than he carries into the trade. These restrictions arise from the trading environment that the matches are separated from each other, which prevents the buyers from using the left-over money balance of other buyers in the household. Notice that, because the goods markets open after the currency market closes, the amount of money each buyers carries into the goods markets includes the amount the household received from the currency trade.

Finally, let us explain the laws of motion of the household's money holdings, given by (2.7) and (2.8). These two laws are similar, with the only key difference being that a country 1's household does not receive the monetary transfer of currency 2. Thus, we explain only (2.7). The left hand side of this constraint is the household's holdings of currency 1 in the next period.[8] The right-hand side of the constraint counts the household's receipts and expenses of currency 1. After receiving the monetary transfers at the beginning of the period, the household holds m_{11} units of currency 1. The household sells f_{11} units of currency 1 in the currency market. In the goods markets, the household's buyers spend currency 1 in area 1 and the household's sellers obtain currency 1 by selling goods. These expenses and receipts are the second and the third blocks of terms on the right-hand side of the constraint. The last term in the constraint is the amount of transfers of currency 1 that the household will receive at the beginning of the next period. After such transfers, the household's future holdings of currency 1 are measured.

3. Equilibrium

In this section, we define and characterize the equilibrium.

3.1. Definition

A monetary equilibrium consists of the representative households' decisions (h_1, h_2), other households' decisions (H_1, H_2), and the nominal exchange rate e such that the following requirements are met for all t: (i) Given (e, H_1, H_2) and other aggregate variables, h_1 solves $(PH1)$ and h_2 solves a similar problem for a country 2's household; (ii) The decisions within each country are symmetric: $h_i = H_i$ for $i = 1, 2$; (iii) The currency market clears: $f_{22} = ef_{11}$; and (iv) Money holdings add up: $m_{1k} + m_{2k} = 1$, for $k = 1, 2$.

Under symmetry within each country, we have $N_{ii} = n_i$, $N_{ii'} = 1 - n_i$, $S_{ii} = s$, and $S_{ii'} = 1 - s$ for $i = 1, 2$ and $i' \neq i$. Thus, the total number of buyers in currency area i is $(n_i + 1 - n_{i'})$ and the total number of sellers in area i is $(s + 1 - s) = 1$. We do not impose symmetry between the two countries' decisions.

[8] The money growth rate γ_1 appears on the left-hand side because $m_{11,+1}$ is normalized by $M_{1,+1}$ while the nominal terms on the right-hand side are normalized by M_1. To see this, one can derive the law of motion in terms of the actual quantities of money, rather than the normalized quantities, and then divide both sides of the equation by M_1.

3.2. Equilibrium Conditions

To characterize optimal decisions, let λ_{ik} be the shadow price of the money constraint faced by a country i buyer in a trade with a domestic seller in area k, and λ_{ik}^f be the shadow price in a trade in area k with a foreign seller. For example, λ_{11} is the shadow price of the first constraint in (2.5) and λ_{11}^f of the second constraint. It is convenient to rescale each of these multipliers by the number of corresponding trade matches that the household has. Thuat is, we multiply λ_{ii} by $n_i W_{ii}^i / N_i$, λ_{ii}^f by $n_i W_{ii'}^i / N_i$, $\lambda_{ii'}$ by $(1 - n_i) W_{ii}^{i'} / (1 - N_i)$, and $\lambda_{ii'}^f$ by $(1 - n_i) W_{ii'}^{i'} / (1 - N_i)$, where $i' \neq i$. These shadow prices are the non-pecuniary returns to the currencies generated by relaxing the trade restrictions.

The following conditions are necessary for the decisions to be optimal.

(i) Equilibrium conditions for the terms of trade (x_{ik}, q_{ik}) and (x_{ik}^f, q_{ik}^f):

$$A = (\omega_{ik} + \lambda_{ik}) \frac{\sigma (q_{ik})^{\sigma - 1}}{\Omega_{ik}};$$

$$A = \left(\omega_{ik} + \lambda_{ik}^f\right) \frac{\sigma \left(q_{ik}^f\right)^{\sigma - 1}}{\Omega_{ik}}.$$

Consider the first condition, for example. The left-hand side of the condition is the marginal utility of consumption and the right-hand side is the cost of giving up currency k to get one additional unit of consumption. By (2.3), the buyer must give up $\sigma (q_{ik})^{\sigma - 1} / \Omega_{ik}$ units of currency k in order to increase the amount of goods, q_{ik}, by one unit. The cost of giving up one unit of currency k is the sum of the future marginal value of the currency, ω_{ik}, and the shadow price of the trading restriction on the currency, λ_{ik}. The second condition above can be explained similarly.

(ii) The optimal choice of n_i, the fraction of buyers of country i sent to area i:

$$\frac{W_{ii}^i}{N_i} A q_{ii} + \frac{W_{ii'}^i}{N_i} A q_{ii}^f = \frac{W_{ii}^{i'}}{1 - N_i} A q_{ii'} + \frac{W_{ii'}^{i'}}{1 - N_i} A q_{ii'}^f; \; i' \neq i. \tag{3.1}$$

By sending one additional buyer to currency i, the household obtains more consumption goods from area i, which increases the household's utility by the amount given by the left-hand side. The cost is that the household must reduce the number of buyers sent to currency area i' by one, which reduces the utility of consumption by the amount given by the right-hand side. These marginal benefits and costs equal each other when n_i is optimal.[9]

[9] Strictly speaking, (3.1) is the result of combining these benefits and costs with those arising from the change

(iii) Optimal amounts of currency exchanges, f_{11} and f_{22}:

$$\omega_{11} + \frac{W_{11}^1}{N_1}\lambda_{11} + \frac{W_{12}^1}{N_1}\lambda_{11}^f = \left(\omega_{12} + \frac{W_{11}^2}{1-N_1}\lambda_{12} + \frac{W_{12}^2}{1-N_1}\lambda_{12}^f\right) e; \tag{3.2}$$

$$\omega_{21} + \frac{W_{22}^1}{1-N_2}\lambda_{21} + \frac{W_{21}^1}{1-N_2}\lambda_{21}^f = \left(\omega_{22} + \frac{W_{22}^2}{N_2}\lambda_{22} + \frac{W_{21}^2}{N_2}\lambda_{22}^f\right) e. \tag{3.3}$$

For example, consider a country 1 household's choice of f_{11}. By increasing f_{11}, the household reduces the amount of currency 1, and hence increases the amount of currency 2, that can be brought into the goods market. The left hand side of (3.2) is the expected cost of having one unit less of currency 1. This cost consists of the future value of the currency, ω_{11}, and the non-pecuniary returns that currency 1 can generate from relaxing the money constraints in trade matches. The expected marginal amount of non-pecuniary returns from trades with domestic sellers is $(\frac{W_{11}^1}{N_1}\lambda_{11})$ and with foreign sellers is $(\frac{W_{12}^1}{N_1}\lambda_{11}^f)$. The expected cost of selling one unit of currency 1 must be equal to the expected benefit of obtaining currency 2. The latter is given by the right-hand side of (3.2) and can be explained similarly. Note that one (normalized) unit of currency 1 exchanges for e (normalized) units of currency 2.

(iv) Envelope conditions for money holdings, m_{ii} and $m_{ii'}$ $(i' \neq i)$:

$$\frac{\gamma_i}{\beta}\omega_{ii,-1} = \omega_{ii} + \frac{W_{ii}^i}{N_i}\lambda_{ii} + \frac{W_{ii'}^i}{N_i}\lambda_{ii}^f; \tag{3.4}$$

$$\frac{\gamma_{i'}}{\beta}\omega_{ii',-1} = \omega_{ii'} + \frac{W_{ii}^{i'}}{1-N_i}\lambda_{ii'} + \frac{W_{ii'}^{i'}}{1-N_i}\lambda_{ii'}^f. \tag{3.5}$$

The left-hand side of (3.4) is the marginal value of currency i in the current period to a country i household, discounted to the previous period. The right-hand side is the discounted value of currency i in the next period plus the expected non-pecuniary return that the currency can generate in the current period. The condition (3.4) simply says that the rate of non-pecuniary return to currency i must be equal to the nominal interest rate, which is given by γ_i/β. The envelope condition for $m_{ii'}$ can be explained similarly.

in the amount of money spent. By increasing n_i, the household bears the expected cost of money spent by the additional buyer in currency area i, which is equal to $\frac{W_{ii}^i}{N_i}(\omega_{ii}+\lambda_{ii})x_{ii} + \frac{W_{ii'}^i}{N_i}(\omega_{ii}+\lambda_{ii}^f)x_{ii}^f$. By the optimal condition for (x_{ik}, q_{ik}), this amount is equal to $\frac{1}{\sigma}$ times the amount given by the left-hand side of (3.1). Similarly, the expected gain from the savings of money in currency area i' as a result of reducing $(1-n_i)$ is $\frac{1}{\sigma}$ times the amount given by the right-hand side of (3.1). After combining these costs and benefits with the ones in (3.1), we can eliminate the coefficient $(1-\frac{1}{\sigma})$.

3.3. Characterizing a Stationary Equilibrium

In this paper, we set money growth rates to be constant, i.e., $\gamma_{kt} = \gamma_k$ for all t. This enables us to focus on the stationary equilibrium. In such an equilibrium, $h_{i,+1} = h_i$ and $\omega_{ik,+1} = \omega_{ik}$ for all i and k, where h_i is the list of a country i household's choices. Also, we restrict $\gamma_1 > \beta$ and $\gamma_2 > \beta$. The reason is that, if $\gamma_1 = \gamma_2 = \beta$, the money constraints (2.5) and (2.6) do not bind in the stationary equilibrium.[10] In this case, households hold a currency at the margin for a store of value rather than a medium of exchange. By contrast, if $\gamma_1 > \beta$ and $\gamma_2 > \beta$, then the constraints (2.5) and (2.6) all bind in the stationary equilibrium (see the analysis later). That is, all λ'es are positive.

The stationary equilibrium has the following interesting properties regarding relative quantities and prices. First, the relative valuation of the two currencies must be the same in the two countries in a stationary equilibrium:

$$\frac{\gamma_1}{\gamma_2}\frac{\omega_{11}}{\omega_{12}} = \frac{\gamma_1}{\gamma_2}\frac{\omega_{21}}{\omega_{22}} = e. \tag{3.6}$$

This result comes from the conditions for optimal currency exchanges, (3.2) and (3.3), in conjunction with the envelope conditions for money holdings in the steady state. As such, the condition is necessary for preventing profitable arbitrage in the currency market.

The above condition implies that the relative valuation of a currency between the two countries is the same for the two currencies. Denote this relative valuation as

$$\theta = \frac{\omega_{11}}{\omega_{21}} = \frac{\omega_{12}}{\omega_{22}}.$$

In general, $\theta \neq 1$. That is, the absolute valuation of each currency is not equalized between the two countries. When $\theta > 1$, country 1 households value *both* currencies more highly than country 2 households; when $\theta < 1$, country 1 households value *both* currencies less than country 2 households. This cross-countries difference in the absolute valuation is caused by the inability of households to arbitrage instantaneously between the goods markets and the currency market. For convenience, we sometimes use the notation $\theta_1 = \theta$ and $\theta_2 = 1/\theta$.

Second, after controlling for the currency used, the relative quantity of goods that a buyer

[10] To see this, set $\gamma_1 = \gamma_2 = \beta$ in the envelope conditions of money holdings. Then, all the λ'es are equal to zero in the stationary equilibrium.

gets from a domestic seller and from a foreign seller is related to θ as follows:

$$\frac{q_{11}}{q_{11}^f} = \frac{q_{12}}{q_{12}^f} = \frac{q_{21}^f}{q_{21}} = \frac{q_{22}^f}{q_{22}} = \theta^{1/\sigma}. \tag{3.7}$$

These results come from the binding money constraints and the sellers' participation constraints. Each quantity ratio in the above equation is the relative amount of goods that a buyer gets in the same currency area from different countries' sellers. For example, the ratio q_{ik}/q_{ik}^f is the quantity of goods a country i buyer purchases from a domestic seller relative to the quantity from a foreign seller in the same currency area k. If $\theta \neq 1$, then there is a quantity differential across sellers even after controlling for the currency used. For example, if $\theta > 1$, then a buyer from any given country gets a higher quantity of goods from a trade with a country 1 seller than from a country 2 seller in any given currency area. As we will show later, this quantity differential implies a price differential in each currency area and the failure of the law of one price.

Third, there is also a differential in the quantity of goods that a buyer gets in the two currency areas. To see this, use the above results to solve the λ'es from the conditions for optimal (q, x) as functions of (q, θ). Then, the envelope conditions imply the following equations for $i' \neq i$ and $i, i' = 1, 2$:

$$q_{ii} = \left[\left(\frac{A}{\sigma} \right) \frac{s + (1-s)\,\theta_i^{-\frac{1}{\sigma}}}{1 + \left(\frac{\gamma_i}{\beta} - 1 \right) \Big/ \mu(n_i)} \right]^{\frac{1}{\sigma-1}}; \tag{3.8}$$

$$q_{ii'} = \left[\left(\frac{A}{\sigma} \right) \frac{(1-s) + s\theta_i^{-\frac{1}{\sigma}}}{1 + \left(\frac{\gamma_{i'}}{\beta} - 1 \right) \Big/ \mu(1-n_i)} \right]^{\frac{1}{\sigma-1}}. \tag{3.9}$$

As defined before, $\mu(n) = L(n)/n$, $\theta_1 = \theta$ and $\theta_2 = 1/\theta$. The ratio $q_{ii}/q_{ii'}$ is the relative quantity of goods that a county i buyer gets from a country i seller in the two currency areas. It is clear that this ratio depends on θ, as well as on money growth rates and the allocation of buyers between the two markets.

With the above properties, equilibrium quantities are determined once (n_1, n_2, θ) are. The fractions (n_1, n_2) are determined by the condition of the optimal allocation of buyers to the two areas, (3.1), and θ by the condition for no-arbitrage in the currency market, (3.6). In Appendix A.1, we show that these conditions lead to the following more explicit conditions:

$$\frac{1 - s + s\theta^{\frac{1}{\sigma}}}{s + (1-s)\,\theta^{\frac{1}{\sigma}}} = \left[\frac{\mu(1-n_1)}{\mu(n_1)} \right]^{1-\frac{1}{\sigma}} \left[\frac{\left(\frac{\gamma_1}{\beta} - 1 \right) \Big/ \mu(n_1) + 1}{\left(\frac{\gamma_2}{\beta} - 1 \right) \Big/ \mu(1-n_1) + 1} \right]^{\frac{1}{\sigma}}; \tag{3.10}$$

$$\theta^{\frac{\sigma}{\sigma-1}} = \frac{[(\gamma_2 - 1)(1 - n_1) + s(1 - n_1)\mu(1 - n_1)][G(n_1, \theta)]^{\sigma} + (1 - s) n_1 \mu(n_1)}{(\gamma_1 - 1) n_1 + s n_1 \mu(n_1) + (1 - s)(1 - n_1)\mu(1 - n_1)[G(n_1, \theta)]^{\sigma}}, \tag{3.11}$$

where

$$G(n_1, \theta) \equiv \frac{\mu(n_1)}{\mu(1 - n_1)} \frac{1 - s + s\theta^{\frac{1}{\sigma}}}{s + (1 - s)\theta^{\frac{1}{\sigma}}}. \tag{3.12}$$

In fact, the appendix provides a proof of the following proposition:

Proposition 3.1. *Assume $\lambda_{ik}, \lambda_{ik}^f > 0$ for $i, k = 1, 2$. Then, in equilibrium, $n_2 = 1 - n_1$, and (n_1, θ) are determined by (3.10) and (3.11). If $\gamma_1 = \gamma_2 = \gamma$, then $\theta = 1$, $n_1 = n_2 = 1/2$ and $q_{ij} = q_{ij}^f = Q(\gamma)$ for all $i, j = 1, 2$, where*

$$\mathcal{Q}(\gamma) \equiv \left(\frac{A/\sigma}{1 + \left(\frac{\gamma}{\beta} - 1\right) / \mu\left(\frac{1}{2}\right)} \right)^{\frac{1}{\sigma-1}}. \tag{3.13}$$

An interesting result here is that both countries' households send the same number of buyers to each currency area in the stationary equilibrium. i.e., $n_2 = 1 - n_1$. Two aspects of this result need be explained. First, there is a unique allocation of buyers to the two currency areas. One should not take this uniqueness lightly, since it would not arise in a standard Walrasian goods market. In particular, because the two countries' goods are assumed to be perfect substitutes for each other, all feasible allocations of the buyers to the two areas would yield the same amount of consumption and the same cost of production if the goods market is Walrasian. By contrast, in goods markets with costly search, the number of matches is important to the household, in addition to the quantity of goods obtained in each trade. The unique allocation of buyers to the two areas ensures the optimal trade-off between the two goods market in the number of trades. Second, the number of buyers allocated to each currency area is the same for the countries. This result arises from the property that the two countries have the same relative valuation of the two currencies. If it were optimal for country 1 to allocate more than a half of the buyers to area 1, then it is also optimal for country 2 to do the same.[11]

Notice that the two countries have the same valuation of a currency whenever the currencies have the same growth rate, in which case each country allocates exactly a half of the buyers to each area. We will explain this result later after Lemma 4.2.

[11]This equality may not hold if the matching function is asymmetric in the sense that, after controlling for the numbers of buyers and sellers in the matching process, a country's buyer meets domestic sellers more often than meeting foreign sellers. For such matching functions, see Matsuyama et al. (1993).

The result $n_2 = 1 - n_1$ simplifies the relative quantity of goods obtained by the two countries' buyers. More precisely, substituting $n_2 = 1 - n_1$ into (3.8) and (3.9) yields:

$$\frac{q_{21}}{q_{11}} = \frac{q_{22}}{q_{12}} = \theta^{1/[\sigma(\sigma-1)]}. \tag{3.14}$$

The ratio $q_{i'i}/q_{ii}$ is the quantity of goods obtained by a foreign buyer from a foreign seller relative to the quantity obtained by a domestic buyer from a domestic seller in the same currency area i. Although this relative quantity is not equal to one when $\theta \neq 1$, it is always equalized between the two currency areas. That is, conditional on buying from their own domestic sellers, the two countries' buyers are discriminated in quantity in the same way in the two areas. Another way to express this equalization is $q_{12}/q_{11} = q_{22}/q_{21}$. That is, the relative quantity of goods that a buyer gets from a domestic seller in the two markets is the same regardless of the buyer's origin.

Once n_1 and θ are determined, other variables can be determined accordingly. First, the q's can be recovered from (3.7), (3.8) and (3.9). Second, from (3.6) and the binding constraints (2.5) and (2.6), we can obtain the nominal exchange rate as follows:

$$e = \frac{\gamma_1}{\gamma_2}\frac{n_1}{1-n_1} \Big/ \left[G(n_1,\theta)\right]^{\sigma}. \tag{3.15}$$

Third, from the laws of motion of the household's money holdings (2.7) and (2.8), we can derive the distribution of money holdings between the two countries, which is given by:

$$m_{11} = \frac{1}{\gamma_1}\left[\gamma_1 - 1 + s\mu(n_1) + \frac{1-\mu(n_1)}{1+\theta^{\frac{\sigma}{\sigma-1}}}\right],$$

$$m_{12} = \frac{1}{\gamma_2}\left[(1-s)\mu(1-n_1) + \frac{1-\mu(1-n_1)}{1+\theta^{\frac{\sigma}{\sigma-1}}}\right],$$

where $m_{2k} = 1 - m_{1k}$ for $k = 1, 2$. Finally, the amounts of currency traded satisfy $f_{22} = ef_{11}$ and

$$f_{11} = m_{11} - \left(1+\theta^{\frac{\sigma}{\sigma-1}}\right)^{-1}.$$

Note that the nominal exchange rate and the amount of direct currency exchange are both determinate in our model. These variables depend on the two countries' fundamentals, such as money stocks and money growth rate. The determinacy relies on two restrictions on the ability to arbitrage. First, the households cannot instantaneously arbitrage between the goods markets and the currency market. Second, they cannot instantaneously arbitrage between trades in the goods market that use different currencies. We refer to Head and Shi (2003) for more discussions on such determinacy.

4. Integrated Economy

In this section and the next section, we focus on the case of an integrated economy, i.e., $s = 1/2$. This case enables us to obtain sharp predictions on the effects of inflation and the welfare consequence of monetary policy coordination. In section 6, we will extend the analysis to economies with different values of s.

4.1. The Size of a Currency Area

We measure the size of a currency area by the total number of trades that take place in the area in each period.[12] Let $\mathcal{A}^k$ be the size of currency area k. Then,

$$\mathcal{A}^k = W_{11}^k + W_{12}^k + W_{21}^k + W_{22}^k = 2n_k\mu\left(n_k\right), \quad k = 1, 2.$$

The second equality comes from the result $1 - n_2 = n_1$. Thus, the size of a currency area k increases in the number of country k buyers allocated to that area. Also, as one currency area increases in size, the other currency area must shrink.

To see how n_1 changes with money growth rates, set $s = 1/2$ in (3.10). Then,

$$\left[\frac{\mu\left(n_1\right)}{\mu\left(1-n_1\right)}\right]^{\sigma-1} = \frac{1+\left(\frac{\gamma_1}{\beta}-1\right)\Big/\mu(n_1)}{1+\left(\frac{\gamma_2}{\beta}-1\right)\Big/\mu(1-n_1)}. \tag{4.1}$$

Because $\mu(n)$ is a decreasing function and $\sigma > 1$, it is easy to establish the following lemma. (See Appendix A.2)

Lemma 4.1. *There is a unique solution for n_1 to (4.1), provided $\gamma_1, \gamma_2 \in D$, where $D = \left\{(\gamma_1,\gamma_2)\,\middle|\,\gamma_i < \beta\left(L_0\right)^{1-\sigma} + \left(\gamma_{i'}-\beta\right)\left(L_0\right)^{-\sigma}\right\}$. Denote this solution as $n_1 = N(\gamma_1,\gamma_2)$. Then $N_1(\gamma_1,\gamma_2) < 0$ and $N_2(\gamma_1,\gamma_2) > 0$. Moreover, $N(\gamma,\gamma) = 1/2$ for all $\gamma > \beta$, and $N(\gamma_1,\gamma_2) > 1/2$ iff $\gamma_1 < \gamma_2$.*

The results in this lemma are intuitive. First, when the growth rate of the supply of currency k increases, all households send fewer buyers to transact with currency k and more buyers to transact with the other currency. This is because the increased growth rate of a currency increases the inflation rate associated with that currency and reduces the purchasing power of that currency.

[12] Another way to measure the size of a market is to use the aggregate transaction value in goods market. This measure is more complicated to compute because all four types of trade in an area result in different quantities of goods traded. However, the results are similar.

As a result of this change in the allocation of buyers, currency area k shrinks and the other currency area enlarges. Second, the currency whose supply grows at a lower rate has more agents participate in it, and hence it is larger than the other area. When the two currencies have the same growth rate in supply, each household sends exactly a half of the buyers to each currency area. In this case, the two currency areas have the same size.

4.2. The Nominal Exchange Rate and the Relative Valuation of the Currencies

The nominal exchange rate, e, is determined by (3.15). When $s = 1/2$, this condition becomes:

$$e = \frac{\gamma_1}{\gamma_2} \frac{n_1}{1 - n_1} \left[\frac{\mu (1 - n_1)}{\mu (n_1)} \right]^\sigma . \tag{4.2}$$

Recall that e is the nominal exchange rate normalized by the ratio of the stocks of the two currencies. Without normalization, the nominal exchange rate is eM_2/M_1. Thus, when $\gamma_1 > \gamma_2$, currency 1 depreciates over time against currency 2 and the rate of depreciation is equal to $(\gamma_1 - \gamma_2)$. After the normalization, the exchange rate, e, is stationary. Clearly, $e = 1$ if and only if the two currencies grow at the same rate. For $\gamma_1 \neq \gamma_2$, the normalized exchange rate decreases in γ_1 and increases in γ_2, with $e > 1$ if and only if $\gamma_1 < \gamma_2$.[13]

An important variable in the equilibrium is country 1's valuation of a currency relative to country 2's. This relative valuation, θ, determined by (3.11), has the properties described in the following lemma (see Appendix A.3 for a proof.).

Lemma 4.2. *Assume $s = 1/2$. Then, $\theta = 1$ if $\gamma_1 = \gamma_2$. If $\gamma_2 \leq 1$, then $\theta > 1$ for all $\beta < \gamma_1 < \gamma_2$. If $\gamma_2 > 1$, then there exists $\gamma_0 \in (1, \gamma_2]$ such that $\theta > 1$ for all $\beta < \gamma_1 < \gamma_0$.*

The two countries have the same absolute valuation of a currency only when the supplies of the two currencies grow at the same rate. If currency 2 grows at a higher rate than currency 1 and if γ_2 is not very high, then country 1 households value both currencies more than country 2 households. This difference between the two countries' valuations of a currency arises from asymmetric money transfers; that is, households only receive their own country's monetary transfers. To ease the explanation, suppose $\gamma_1 > 1$ and $\gamma_2 > 1$ so that the transfers are positive. Each country's households want to sell part of the received transfers for the other country's currency. If $\gamma_1 = \gamma_2$, then the transfers of the two currencies have the same real value. In this case, each country's

[13] See the proof of Lemma 4.2 described below.

households sell exactly a half of the received transfers for the other country's currency. The result is that each country holds a half of the total stock of each currency, and so the two countries have the same valuation of each currency. On the other hand, if $\gamma_2 > \gamma_1$ and γ_2 is low or moderate, then the real value of currency 2 transfers exceeds that of currency 1 transfers. Country 1 households sell more than a half of the received transfers for currency 2. As a result, country 1 households hold less than a half of the total stock of each currency, and they value both currencies more than country 2 households.[14]

The currency market cannot eliminate the difference between the two countries' absolute valuations of a currency; rather, it equalizes only the relative valuation of the two currencies between the two countries. The difficulty lies in the fact that, when $\gamma_2 > \gamma_1$, country 1 households value both currencies more than country 2 households. If country 1 households sell more currency 1 than they do in the currency market equilibrium, the difference between the two countries' valuations of currency 2 will narrow but the difference between the valuations of currency 1 will widen. Thus, country 1's relative valuation of currency 1 to currency 2 will exceed country 2's, which will create profitable arbitrage opportunities in the currency market.

The separation between the currency market and the goods markets is important for supporting the difference between the two countries' valuations of a currency. Such a difference would not exist if households could instantaneously arbitrage between the goods markets and the currency market. For example, if $\theta > 1$, then country 1 households would sell some goods for currency 2 directly in the currency market to drive θ down to one. It is worth mentioning that this arbitrage through the centralized (currency) market cannot be replicated by the trades of goods for currencies in the goods markets. The latter are fragmented as a result of bilateral matches and cannot ensure uniform prices in different types of matches. We now turn to these price differentials.

4.3. Relative Prices

The law of one price fails generically in the model environment. To see this, recall that there are four types of trades in each currency area k. In particular, a country i buyer can trade with a

[14] The result may be reversed when γ_2 is very high and when γ_1 is close to γ_2. With a very high growth rate, the inflation effect is so overwhelming that the real value of currency 2 transfers is lower than that of currency 1, despite the amount of currency 2 transfers is higher. In this case, country 1 households are able to buy more than a half of currency 2 transfers with less than a half of currency 1 transfers. As a result, country 1 households value both currencies less than country 2 households.

country i seller or a country i' ($\neq i$) seller. Prices in these two trades are as follows:

$$p_{ik} = \frac{\widehat{m}_{ik}}{n_{ik}q_{ik}}, \quad p^f_{ik} = \frac{\widehat{m}_{ik}}{n_{ik}q^f_{ik}},$$

where $\widehat{m}_{ik}$ is the amount of currency k that a country i household has after the trades in the currency market. For example, $\widehat{m}_{11} = m_{11} - f_{11}$ and $\widehat{m}_{12} = m_{12} + ef_{11}$. Since $\widehat{m}_{ik}$ is normalized by the total stock of currency k, prices defined above are also normalized.

We can compute the price charged by country 1's sellers relative to country 2's sellers when selling to the same buyers in the same currency area. There are four such relative prices, as given below:

$$R_1 = \frac{p_{11}}{p^f_{11}} = \frac{p_{12}}{p^f_{12}} = \frac{p^f_{21}}{p_{21}} = \frac{p^f_{22}}{p_{22}}.$$

The equalities come from (3.7). Similarly, we can compute the price paid by country 1's buyers relative to country 2's buyers when buying from the same sellers in the same currency area. These relative prices are:

$$R_2 = \frac{p_{11}}{p^f_{21}} = \frac{p_{12}}{p^f_{22}} = \frac{p^f_{11}}{p_{21}} = \frac{p^f_{12}}{p_{22}}.$$

Using (3.7) and (3.14), we can establish the following corollary.[15]

Corollary 4.3. *In a stationary equilibrium,* $R_1 = \theta^{-1/\sigma}$ *and* $R_2 = \theta^{-1}$.

The law of one price holds only when $\theta = 1$, i.e., only when the two countries have the same valuation of a currency. This case arises when the supplies of two currencies grow at the same rate. When $\theta > 1$, country 1's sellers charge lower prices than country 2's sellers when selling to the same buyers in the same currency area, and country 1's buyers pay lower prices than country 2's buyers when buying from the same sellers in the same currency area. It is easy to explain such price discrimination. When $\theta > 1$, country 1's households value both currencies more than country 2's households. For the same amount of money, country 1's sellers are willing to sell more goods than country 2's sellers. Similarly, to part with the same amount of money, country 1's buyers will demand for a higher quantity of goods.

[15] Notice that this corollary applies to all $s \in [0, 1]$, not just to the integrated economy $s = 1/2$.

5. Optimal Monetary Policy

We examine the countries' choices of long-run money growth rates (γ_1, γ_2) under three institutional regimes: the noncooperative regime, the cooperative regime, and unified currency. When analyzing each regime, we focus on whether the regime generates higher inflation and higher welfare. Only the integrated economy, $s = 1/2$, is considered in this section.

5.1. Noncooperative Monetary Policy

The first arrangement in consideration is a non-cooperative regime. Under this arrangement, each country chooses the long-run growth rate of its own currency and takes the other country's choice as given. We refer to this regime as monetary policy competition.[16]

When choosing the growth rate of its currency, a country intends to maximize the steady-state utility of the country's representative household. Denote this welfare function as $\mathcal{W}_i$ for country i. Then,

$$\begin{aligned}(1-\beta)\mathcal{W}_i = \; & A\left(W^i_{ii}q_{ii} + W^i_{ii'}q^f_{ii}\right) + A\left(W^{i'}_{ii}q_{ii'} + W^{i'}_{ii'}q^f_{ii'}\right) \\ & -\left[W^i_{ii}\left(q_{ii}\right)^\sigma + W^i_{i'i}\left(q^f_{i'i}\right)^\sigma\right] - \left[W^{i'}_{ii}\left(q_{ii'}\right)^\sigma + W^{i'}_{i'i}\left(q^f_{i'i'}\right)^\sigma\right].\end{aligned} \tag{5.1}$$

The first two terms in above expression are the total utility of consumption and the last two terms are the disutility of production. The monetary authority in country i chooses γ_i to maximize W_i, taking the other country's choice $\gamma_{i'}$ $(i' \neq i)$ as given and taking into account the dependence of individual households' choices on the policy.

To characterize the outcome under policy competition, we start with the so-called Friedman rule, $\gamma = \beta$, as the focal point because this rule is often optimal in a closed economy.[17] The following proposition describes the result (see Appendix B for a proof).

Proposition 5.1. *Given $\gamma_{i'} = \beta$ $(i' \neq i)$, country i's policy has the following effects at $\gamma_i = \beta$: $dn_i/d\gamma_i < 0$, $d\theta_i/d\gamma_i < 0$ and $d\mathcal{W}_i/d\gamma_i > 0$, where $\theta_1 = \theta$ and $\theta_2 = 1/\theta$. Thus, if there is a Nash equilibrium with policy competition, then the equilibrium generates $\gamma_1 > \beta$ and $\gamma_2 > \beta$.*

[16] As is common in the literature, the game is a one-shot game. In particular, we do not allow for trigger strategies. As is well known in game theory, allowing for trigger strategies generates a large set of equilibria, since any individually rational outcome can be supported by trigger strategies in an infinitely repeated game.

[17] Since we have imposed $\gamma_1 > \beta$ and $\gamma_2 > \beta$ to ensure that all money contraints in trade bind, we can only approach the Friedman rule by taking the limits $\gamma_i \searrow \beta$. We maintain this interpretation of the Friedman rule throughout the paper.

One country can increase its welfare by setting the money growth rate above the Friedman rule, if the other country follows the Friedman rule. The unilateral increase in the money growth rate generates such a welfare gain by redistributing money holdings between the two countries. To explain this, recall that monetary transfers are made only to domestic households. When country 1 sets its money growth rate above country 2's, the higher money injection implies that country 1 households will hold more of each currency than country 2 households (see the explanation for Lemma 4.2). This redistribution of money holdings is reflected by the fall in θ, i.e., in country 1's valuation of each currency relative to country 2's. As a result of this redistribution, country 1 households have higher real money balances than country 2 households, and hence higher purchasing power. In particular, when meeting country 2 sellers, country 1 buyers obtain higher surpluses in the trades in both areas. The gain by country 1 is at the expense of country 2. This is the classical effect of "begging thy neighbor" caused by inflation.

Let us remark on three aspects of this result. First, the effect of "begging thy neighbor" typically arises in environments in which households must use the foreign currency to purchase foreign goods. It is interesting that the effect arises in our model which does not impose such restrictions. Second, price discrimination in our model tends to constrain the redistributional gain of inflation. In the above case, for example, country 1's higher money growth induces all sellers to charge higher prices on country 1 buyers than on country 2 buyers. However, this price disadvantage does not wipe out the redistributional gain to country 1. Third, when each country is closed, there is no welfare gain from deviating above the Friedman rule. In that case, increasing money growth rate always reduces the real money balance and welfare.[18]

Also, as will be discussed in section 6, there is no welfare gain from deviating above the Friedman rule in the standard two-country model with cash-in-advance constraints, such as Helpman (1981).

[18]For search models in which the Friedman rule is not optimal, see Berentsen, Rocheteau and Shi (2002).

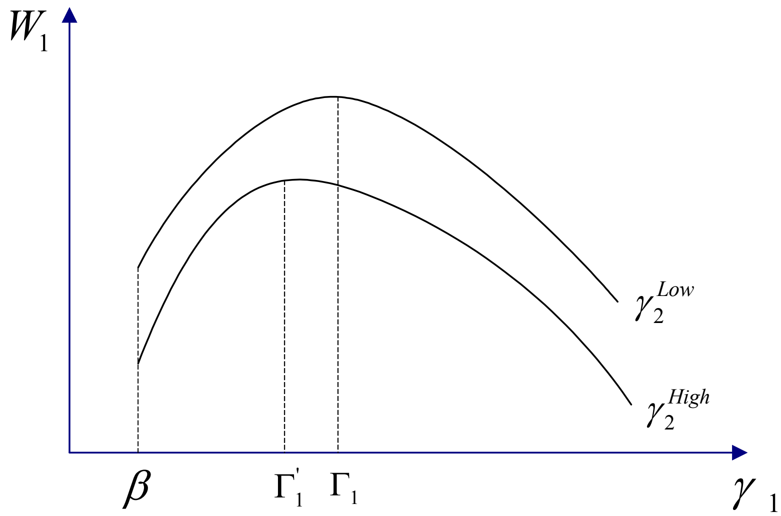

Figure 4.1. Country 1's welfare under two given values of γ_2

The welfare effect of inflation holds more generally. To see this, we compute country 1's welfare as a function of its money growth rate γ_1 under given γ_2 ($> \beta$). Figure 4.1 depicts this function for two given values of γ_2. For each given value of γ_2, country 1's welfare first increases in γ_1, reaches a peak at some level Γ_1, and then decreases in γ_1 when $\gamma_1 > \Gamma_1$. The decreasing portion of the welfare function arises from the fact that inflation reduces the overall purchasing power of money, which dominates the redistributional gain when money growth rate is high. Notice that the critical level Γ_1 is lower when γ_2 is higher. Thus, country 1's incentive to inflate is the highest when country 2 follows the Friedman rule. Moreover, the dependence of Γ_1 on γ_2 indicates that the game under policy competition has a unique equilibrium.

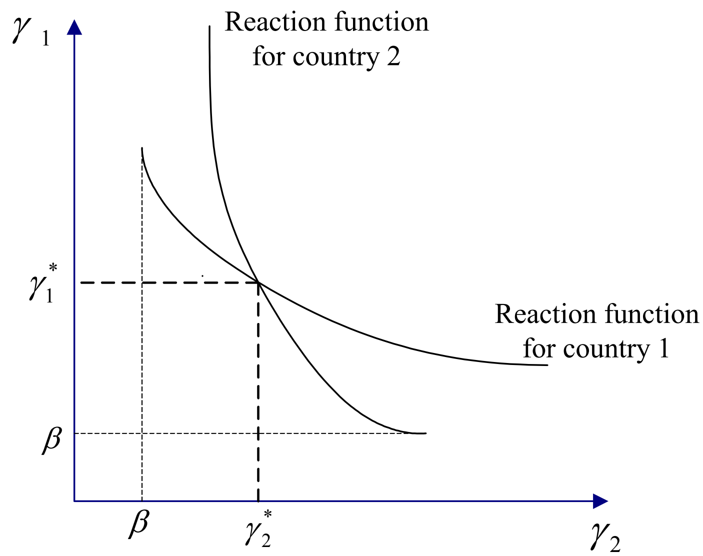

Figure 4.2. Best response functions with non-cooperation

To support the above statements, we depict the two countries' best response functions (or the reaction functions) in Figure 4.2. Indeed, both functions are decreasing and they have a unique intersection at $\gamma_1^* = \gamma_2^* > \beta$.

We setup the following Proposition. (See appendix C for a proof.)

Proposition 5.2. *When the two countries choose their own money growth rate in a noncooperative fashion, there is a unique Nash equilibrium which features $\gamma_1^* = \gamma_2^* = \gamma^* > \beta$. The equilibrium allocation is Pareto inefficient.*

5.2. Monetary Policy under Coordination

The second institutional arrangement in consideration is a cooperative regime. Under this regime, the two countries choose growth rates of the two currencies jointly to maximize the world's aggregate welfare. Since the two countries are symmetric, it is meaningful to measure aggregate welfare by the average of the two countries' steady state utilities. This aggregate welfare measure is $\mathcal{W}^c = \frac{1}{2}(\mathcal{W}_1 + \mathcal{W}_2)$, where $\mathcal{W}_i$ is given by (5.1). The optimal monetary policy under the cooperative regime is a pair (γ_1, γ_2) that maximizes $\mathcal{W}^c$.

Because the two countries are symmetric, the optimal policy must have the same money growth rate for both currencies. Moreover, when $\gamma_1 = \gamma_2 = \gamma$, we have $\theta = 1$, $n_1 = n_2 = 1/2$, and

$q_{ij} = q_{ij}^f = \mathcal{Q}(\gamma)$ for all $i, j = 1, 2$, where $\mathcal{Q}(\gamma)$ is defined by (3.13) (see Proposition 3.1). Thus, the aggregate welfare function is $\mathcal{W}^c = \mathcal{W}(\gamma)$ where

$$\mathcal{W}(\gamma) \equiv \frac{2L\left(\frac{1}{2}\right)}{1-\beta}\left\{A\mathcal{Q}(\gamma) - [\mathcal{Q}(\gamma)]^{\sigma}\right\}. \tag{5.2}$$

Since $\mathcal{W}$ is increasing in $\mathcal{Q}$ and $\mathcal{Q}$ is decreasing in γ, the optimal policy under cooperation is $\gamma_1 = \gamma_2 = \beta$. That is, money growth rates of both currencies should follow the Friedman Rule.

Not only does the cooperative regime generates higher aggregate welfare than the non-cooperative regime, it also increase each country's welfare. To see this, recall that the non-cooperative equilibrium generates $\gamma_1 = \gamma_2 = \gamma^*$. Thus, it has $\theta = 1$, $n_1 = n_2 = 1/2$, and $q_{ij} = q_{ij}^f = \mathcal{Q}(\gamma^*)$ for all $i, j = 1, 2$. As a result, each country's welfare level in the non-cooperative regime is given by the same function $\mathcal{W}(\gamma^*)$ defined above. Since $\gamma^* > \beta$, then $\mathcal{W}(\gamma^*) < \mathcal{W}(\beta)$. That is, the non-cooperative outcome is Pareto inefficient, as it is dominated by the cooperative outcome.

We summarize the above results in the following proposition.

Proposition 5.3. *The optimal policy under cooperation is the Friedman rule in both countries, i.e., $\gamma_1 = \gamma_2 = \beta$. Moreover, the cooperative outcome generates higher welfare for each country than the non-cooperative outcome.*

5.3. The Regime of a Unified Currency

One difficulty in achieving the cooperative outcome is that it is not a Nash equilibrium, as each country has the incentive to deviate above the Friedman rule. To curtail the incentive to inflate, the two countries can choose to unify the currencies. We examine this regime now. Under this regime, it is no longer meaningful to assume that one country's households receive only their own country's monetary transfers. Instead, we assume that each country receives a half of the transfers. Denote the money growth rate of the unified currency as γ. Also, a direct currency exchange between the two countries is not meaningful, because households cannot bring goods to the currency market to settle the trades.

We still keep the two areas, because currency unification does not necessarily imply the integration of the goods markets. Call these areas market 1 and market 2, respectively. As before, each country i household sends an exogenous fraction s of the sellers to market i and a fraction $(1-s)$ to market $i' \neq i$. Each household in country i chooses to send a fraction ρ_i of its money holdings to market i and a fraction $(1-\rho_i)$ to market $i' \neq i$. The matching technology and bargaining are the same as before.

We can formulate a country 1 household's decision problem similarly to $(PH1)$. There are three modifications to the formulation. First, the second subscript of (x, q) now refers to the market, rather than the currency area. Second, the money constraints in trades are as follows:

$$\frac{\rho_1 m_1}{n_1} \geq x_{11}; \quad \frac{\rho_1 m_1}{n_1} \geq x_{11}^f;$$

$$\frac{(1-\rho_1)\, m_1}{1-n_1} \geq x_{12}; \quad \frac{(1-\rho_1)\, m_1}{1-n_1} \geq x_{12}^f.$$

Third, there is only one law of motion of the household's holdings (of the unified currency), in which the transfer received by the household is $(\gamma-1)/2$.

Because the two countries are symmetric, it is easy to see that $n_1 = n_2 = 1/2$ and $\rho_1 = \rho_2 = 1/2$. Then, $q_{ik} = q_{ik}^f = \mathcal{Q}(\gamma)$ for all $i, k = 1, 2$, where $\mathcal{Q}(\gamma)$ is defined by (3.13). The welfare level of each country is equal to $\mathcal{W}(\gamma)$, where $\mathcal{W}(.)$ is defined by (5.2). Therefore, setting $\gamma = \beta$ maximizes both each country's welfare and the two countries' joint welfare. We summarize this result as follows.

Proposition 5.4. *Under the unified currency regime, the optimal policy is the Friedman rule $\gamma = \beta$. With this policy, the real allocation is the same as under policy coordination, and it is efficient.*

It is not surprising that the Friedman rule is optimal with a unified currency. Because both countries get the same amount of monetary transfers, the redistributional effect of inflation does not exist in this case. Money growth above the Friedman rule only reduces the real money balance and reduces welfare.

A unified currency is more reliable than the cooperative regime as a way to achieve efficiency, although both yield the same allocation. The cooperative regime with two currencies is vulnerable to profitable deviations by each country, while the common currency provides a strong commitment for the two countries to the optimal policy.

6. Changing the Degree of Integration

In this section we examine the cases in which the two countries are not fully integrated, in the sense $s > 1/2$. That is, more sellers go to the area marked with their own country's currency than to the other area. If the policy regime is either cooperative or unified, then the optimal monetary policy is still the Friedman rule. Thus, we focus on the non-cooperative regime.

With $s > 1/2$, a country's money growth still generates a redistributional effect similar to the benchmark case $s = 1/2$. However, there is an additional effect with $s > 1/2$, which we call the *extensive effect*. This effect arises from the fact that money growth changes the total number of trades experienced by a household's sellers, by affecting the allocation of buyers between the two areas. To see this extensive effect, consider a country 1 household. The total number of trades experienced by the household's sellers in a period is

$$W_1^s \equiv W_{11}^1 + W_{21}^1 + W_{11}^2 + W_{21}^2 = 2\left[n_1\mu(n_1)s + (1-n_1)\mu(1-n_1)(1-s)\right].$$

Start from $\gamma_1 = \gamma_2$ so that $n_1 = 1/2$. A marginal increase in γ_1 affects W_1^s as follows:

$$\left.\frac{dW_1^s}{d\gamma_1}\right|_{\gamma_1=\gamma_2} = (2s-1)\mu'(\frac{1}{2})\left(\frac{dn_1}{d\gamma_1}\right)_{\gamma_1=\gamma_2}.$$

If an increase in γ_1 reduces n_1, then the total number of trades for country 1's sellers will decrease when $s > 1/2$. This is easy to explain. When n_1 (as well as $(1 - n_2)$) falls, the number of buyers going to currency area 1 falls and the number of buyers going to area 2 rises. The number of trades experienced by sellers decreases in area 1 and increases in area 2. When $s > 1/2$, more country 1 sellers are in area 1 than in area 2. Thus, for country 1 sellers, the decrease in the number of trades in area 1 exceeds the increase in trades in area 2. This reduction in the sellers' trades increases the household's welfare by reducing the disutility of production, thus creating the extensive effect of money growth.

Because the change in the sellers' number of trades is brought about by the change in the allocation of buyers between the two markets, it can be regarded as an externality generated by buyers. In contrast, the total number of trades experienced by the household's buyers does not change with a marginal increase of γ_1 from the Friedman rule.[19]

[19] This can be verified by computing the buyers' number of trades as $2\left[n_1\mu(n_1) + (1-n_1)\mu(1-n_1)\right]$. Starting from any $\gamma_1 = \gamma_2$ so that $n_1 = 1/2$, this number does not change with n_1 and hence not with γ_1.

Putting the extensive effect together with the redistributional effect, we can compute the welfare effect of country 1's money growth on country 1's welfare as follows:

$$\frac{(1-\beta)}{q^{\sigma}}\frac{d\mathcal{W}_1}{d\gamma_1} = \underbrace{\sigma\frac{2s-1}{2\beta\left(\sigma-1\right)} - \mu\left(\frac{1}{2}\right)\left[\frac{s^2+(1-s)^2}{\sigma-1}+1\right]\frac{d\theta}{d\gamma_1}}_{redistributional\ effect} \quad \underbrace{-\frac{dW_1^s}{d\gamma_1}}_{extensive\ effect} \tag{6.1}$$

Here, the quantity q is given by $\mathcal{Q}(\beta)$ and all the derivatives are evaluated at $\gamma_1 = \gamma_2 = \beta$. Since the derivatives $d\theta/d\gamma_1$ and $dn_1/d\gamma_1$ are not illuminative, they are not presented here. Notice that the redistributional effect now contains not only the effect through θ but also a direct effect of money growth on the quantities of trade.

By continuity, the welfare result in the benchmark case $s = 1/2$ extends to the cases where s is greater than but close to $1/2$. In fact, for such values of s, the extensive effect and the redistributional effect work in the same direction. Recall that, when $s = 1/2$, an increase in γ_1 reduces θ and hence redistributes purchasing power from country 2 to country 1. This is also the case when s is close to $1/2$. In addition, an increase in γ_1 reduces n_1 and hence reduces the number of trades experienced by country 1 sellers. This saves country 1 sellers' production cost and hence generates a positive extensive effect for country 1. Thus, given that country 2 follows the Friedman rule, country 1 has incentive to set the money growth rate above the Friedman rule.

For sufficiently low degrees of integration (i.e., for s sufficiently close to 1), the two effects of money growth on a country's welfare can be opposite to each other and hence the overall welfare effect is ambiguous. To depict how the effects of money growth depend on the degree of integration, we resort to numerical examples. Consider the following parameter values:

$$\beta = 0.995,\ A = 4,\ \sigma = 2,\ \psi = 0.5.$$

With the particular value of β, the length of one period can be interpreted as one month.

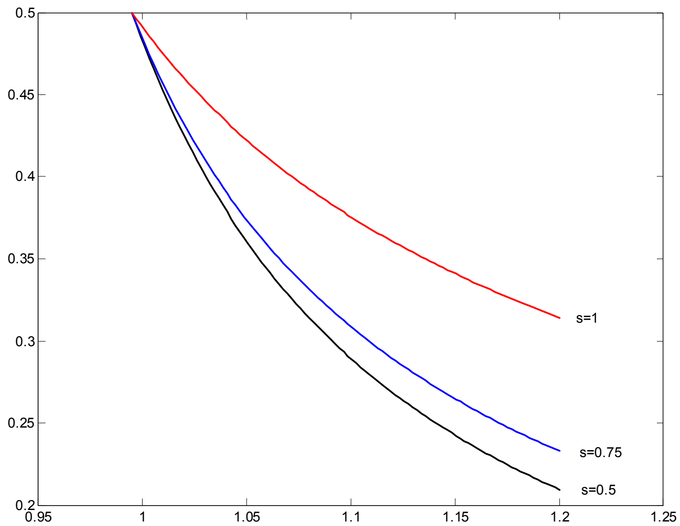

Figure 5.1. The fraction of buyers allocated to area 1, n_1

In Figure 5.1, we depict a country 1 household's allocation of buyers, n_1, as a function of γ_1, while fixing $\gamma_2 = \beta$. The horizontal axis is the money growth rate, γ_1. The three curves plot the function $n_1(\gamma_1)$ for $s = 0.5$, 0.75, and 1, respectively. At $\gamma_1 = \beta$, all three curves meet at $n_1 = 1/2$. As γ_1 increases, n_1 falls in all three cases. This indicates that the extensive effect of a country's money growth on its weflare is positive for $s > 0.5$.

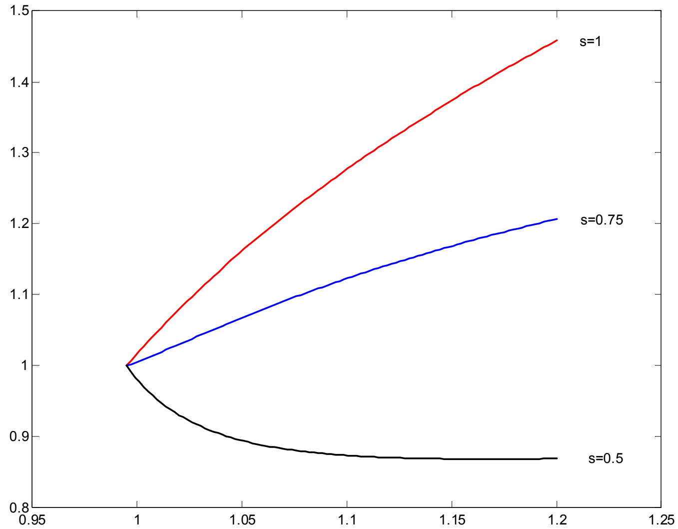

Figure 5.2. Relative valuation of a currency, $\theta(\gamma_1)$

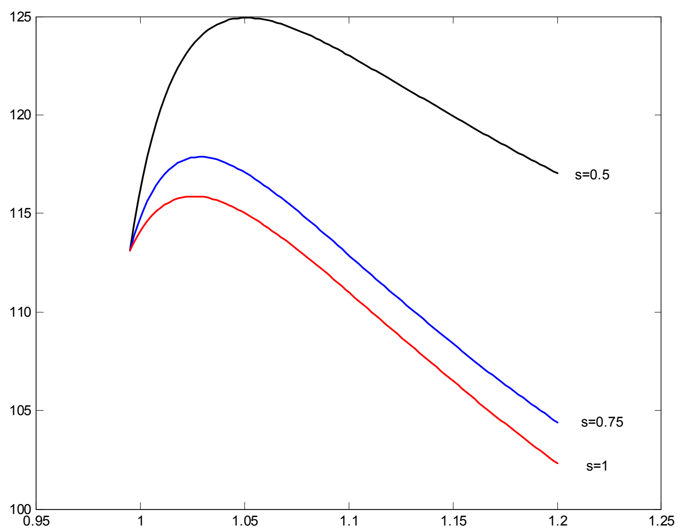

Figure 5.3. Country 1's welfare $\mathcal{W}_1(\gamma_1)$

We also dipict the relative valuation of a currency across two countries (θ) and country 1's welfare (W_1) at 3 different market integration levels. Figure 5.2 shows the relative valuation of a currency as the fuction of γ_1 for $s = 0.5$, 0.75, and 1. The three curves meet at $\theta = 1$ at $\gamma_1 = \gamma_2 = \beta$. As we have discussed in previous section, when $s = 1/2$, increasing money growth rate of currency 1 always reduces relative valuation of currencies θ and generates positive redistributional effect on country 1's welfare. This is not alway the case when the market integration level is low. In fact, when $s = 0.75$, increasing γ_1 can increase θ. And the increase in θ is even larger at $s = 1$. Therefore, when market integration level is low, the changes in the relative valuation of a currency could have negative welfare effect on country 1's household.[20] This negative welfare effect is larger when the two countries have a lower market integration level. Therefore, the incentive to increase money growth rate is lower if the market integration level is lower.

Figure 5.3 dipicts country 1's welfare as a function of γ_1 for $s = 0.5$, 0.75, and 1. In all three cases, country 1's welfare first increases with the increase of γ_1, and then start to decrease after γ_1 reaches a peak. Notice that increasing money growth rate of currency 1 has the largest positive welfare effect when the two economies are fully integrated. In this case, the redistributional effect is strongest. As the result, country 1 are more likely to inflate if the market integration level is higher.

To see this result more clearly, we plot redistributional effect and extensive effect in Figure 6,

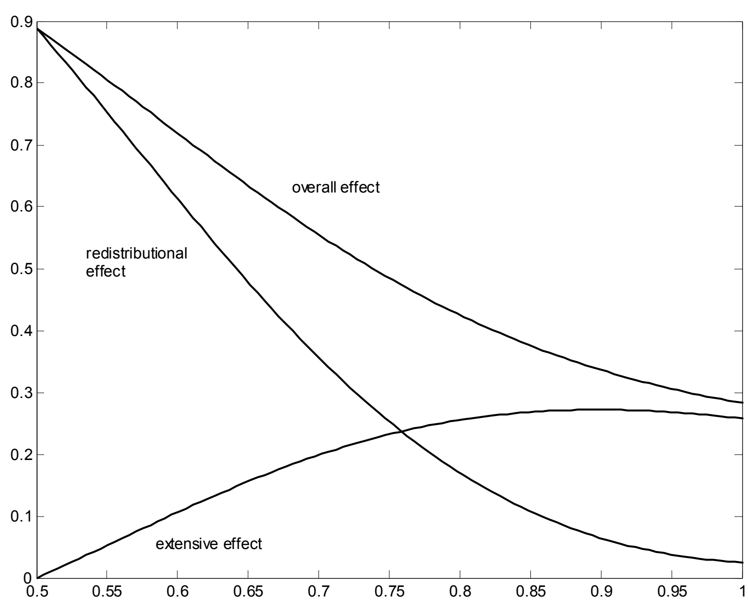

Figure 6. Welfare effect of money growth at different s

[20] **Note that, in (6.1), the welfare effect of γ_1 on country 1's welfare is negative related to the derivative $d\theta/d\gamma_1$.**

The horizontal axis is the degree of market integration level. As we analyzed before, the redistributional effect always decreases with the increase of s. In the fully integrated economy, the redistributional effect is strongest. The change in extensive effect, however, is not monotone. In fully integrated economy, there is no extensive effect since two markets are symmetric and the effect of changes in the allocation of buyers cancel out. When $s > 1/2$, changes in money growth rate change the total number of trades experienced by the sellers and create the positve extensive effect. When s is closed to 1, the extensive effect starts to decrease because the effect of money growth on the allocation of buyer become small. Since redistributional effect dominates extensive effect, the overall welfare effect is always decreasing with the increase of its money injection.

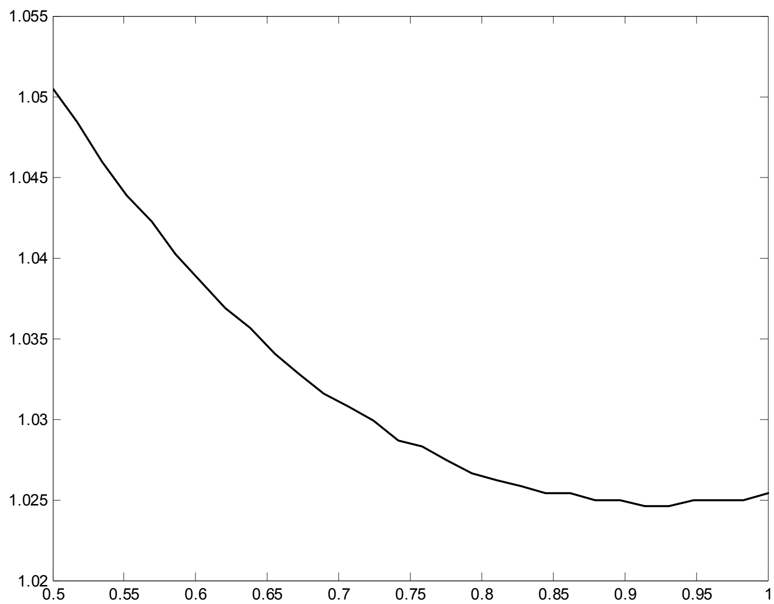

Figure 7. Optimal money growth rate (γ_1) at different s ($\gamma_2 = \beta$)

We also compute country 1's best response $\gamma_1 = \Gamma_1(\beta)$, while fixing $\gamma_2 = \beta$ $(= 0.995)$. Figure 7 depicts this best response for different values of s. The horizontal axis represents s. Three features of the diagram are worth noting. First, for all $s \geq 1/2$ (and for fixed $\gamma_2 = \beta$), it is optimal for country 1 to deviate to money growth rates above the Friedman rule. Thus, in the Nash equilibrium, both countries will have money growth rates higher than the Friedman rule, which is Pareto inefficient. Second, the optimal choice of γ_1 deviates from the Friedman rule by the largest amount at $s = 1/2$, because the redistributional gain to country 1 from inflation is the strongest when the two countries are fully integrated. Third, as the degree of integration decreases from $1 - s = 1/2$ toward 0, the optimal choice of γ_1 first falls and then rises. This non-monotonic pattern can be explain as follows. As s increases, the redistributional gain to country 1 from inflation falls and the extensive effect rises. When s is close to 1/2, the redistributional effect is the dominating welfare effect of inflation, and so its decrease in s reduces the optimal level of inflation. When s is close to 1, the extensive effect is the dominating welfare effect of inflation, and so its increase in s increases the optimal level of inflation. The least deviation of the optimal money growth γ_1 from the Friedman rule occurs when s is close to 0.88.

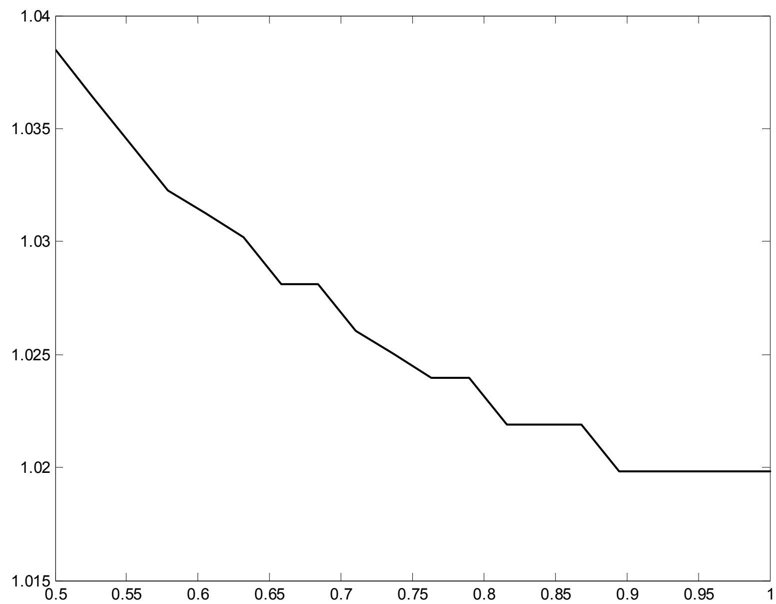

Figure 7.2. **Nash equilibrium money growth rate at different** s

We also compute the Nash equilibrium money growth rate at different market integration level. The vertical axis is the Nash equilibrium money growth rate for both countries. Figure 7.2 clearly shows that both countries will choose a higher money growth rate under policy competition when they are highly integrated with each other. As we have discussed in previous section, higher inflation reduces the purchasing power of the currencies and reduces both countries' welfare. Therefore, two highly integated economies are more likely to cooperate with each other to eliminate the inefficiency caused by higher inflation.

To conclude this section, notice that the case $s = 1$ is one in which sellers only sell their goods for their domestic currency. Even in this case, above example shows that a country's optimal policy in the non-cooperative regime is above the Friedman rule. This result contrasts sharply with that in the standard model by Helpman (1981). In a two-country setup where the two countries' goods are perfect substitutes, Helpman imposes the cash-in-advance constraint that sellers sell their goods only for their domestic currency. It can be shown in his model that the optimal policy under no cooperation is the Friedman rule.[21] The reason for this different result is that the goods market in Helpman's model is centralized and Walrasian. This centralized market implies that the law of one price always hold in his model, in contrast to our model. More importantly, the number of trades is immaterial for a household's welfare – All that matters is the total quantity of goods bought and produced. Thus, the extensive effect of inflation on welfare does not exist in Helpman's model. Because the extensive effect is important for a country's welfare gain from inflation in the case $s = 1$, it is then not surprising that Helpman's model does not generate this welfare gain.

7. Conclusion

In this paper we examine currency areas and monetary policy coordination in a two-country model where the value of each country's currency is determined endogenously. We find that, when gross money growth rate exceed the discount rate, the trading constraints bind and the nominal exchange rate is determined uniquely and depends on fundamentals. One currency has a higher price than the other currency if it has a relatively lower stock and a relatively lower growth rate in supply. There are also violations of the law of one price in equilibrium if the supply of the two currencies grow at different rate.

We also show that the size of a currency area decreases with the growth rate of that currency and increase with the growth rate of the competing currency. People tend to use the currency with lower growth rate for the money-goods exchange.

We also do the policy analysis and find that inflation has redistributional effect that can increase welfare of domestic country in the highly integrated economies. Although increasing domestic money growth rate will reduce the aggregate real balance of domestic currency, the

[21] The proof is omitted here but is available upon request.

real balance of domestic currency held by home country can increase because of the asymmetric monetary injection. This will increase domestic welfare in the cost of decreases in the foreign welfare. Therefore, if two countries choose their monetary policy unilaterally, both of them will choose a higher inflation rate that deviates from the Friedman Rule. There is inflation bias and the resulting allocation is inefficient.

When the policy coordination is feasible, both countries choose the same money growth rate that obeys the Friedman Rule. This policy maximizes the real balance of both currencies and the equilibrium allocations are Pareto efficient. Moreover, both countries are better off compared to the case under policy competition.

However, since there is welfare gain of inflation provided the other country set the money growth rate as the Friedman Rule, both countries have incentive to deviate from the Friedman Rule. Therefore, although policy coordination can eliminate the welfare loss caused by inflation and achieve the efficient result, the cooperative equilibrium is subject to collapse to the inefficient Nash equilibrium. Unified currency regime provides a strong commitment for the two countries. No country can deviate from the efficient equilibrium and choose to inflate. Therefore, unified currency regime is more reliable to ensure efficiency compared to policy coordination.

Moreover, the numerical exercise shows that the incentive to inflate increases with the degree of market integration under policy competition. Therefore, to eliminate the inefficiency, two highly integrated economies are more likely to adopt a common currency.

Appendix

A. Proof of Some Results

A.1. Proof of Proposition 3.1

First, we derive (3.10) from the condition of the optimal allocation of buyers, (3.1). Substituting (3.7) into (3.1), we have:

$$\frac{q_{12}}{q_{11}} = G(n_1, \theta), \quad \frac{q_{21}}{q_{22}} = G(n_2, \frac{1}{\theta}),$$

where the function G is defined by (3.12). Substituting (q_{11}, q_{12}) from (3.8) and (3.9), the first equation above yields (3.10). Similarly, substituting (q_{21}, q_{22}) from (3.8) and (3.9), we have:

$$\frac{1 - s + s\theta^{\frac{1}{\sigma}}}{s + (1-s)\,\theta^{\frac{1}{\sigma}}} = \left[\frac{\mu\,(n_2)}{\mu\,(1-n_2)}\right]^{1-\frac{1}{\sigma}} \left[\frac{\left(\frac{\gamma_1}{\beta} - 1\right)\Big/\,\mu(1-n_2) + 1}{\left(\frac{\gamma_2}{\beta} - 1\right)\Big/\,\mu(n_2) + 1}\right]^{\frac{1}{\sigma}}. \tag{A.1}$$

Second, we show $n_2 = 1 - n_1$. Denote the right-hand side of (3.10) temporarily as $RHS(n_1)$. Then, the right-hand side of (A.1) is equal to $RHS(1 - n_2)$. Since the left-hand sides of (3.10) and (A.1) are identical, then $RHS(1 - n_2) = RHS(n_1)$. If the function $RHS(n)$ is monotone, then $1 - n_2 = n_1$, as desired. Because $\mu(n)$ is a decreasing function, and because $\gamma_1, \gamma_2 > \beta$, the function $RHS(n)$ is indeed increasing in n.

Third, we derive (3.11). To do so, first use (2.3) – (2.6) to express the ω'es as functions of (q, f). Using these expressions and the fact that $n_2 = 1 - n_1$, we obtain:

$$\begin{aligned} \theta &= \frac{\omega_{11}}{\omega_{21}} = \left(\frac{q_{11}}{q_{21}}\right)^{\sigma} \frac{m_{21} + f_{21}}{m_{11} - f_{11}}; \\ \theta &= \frac{\omega_{12}}{\omega_{22}} = \left(\frac{q_{12}}{q_{22}}\right)^{\sigma} \frac{m_{22} - ef_{21}}{m_{12} + ef_{11}}. \end{aligned}$$

From these equations we solve for (f_{11}, f_{21}). Then, the equilibrium condition in the currency market, $f_{21} = f_{11}$, can be written as:

$$e\left\{m_{11} - \left[1 + \theta\left(\frac{q_{21}}{q_{11}}\right)^{\sigma}\right]^{-1}\right\} = \left[1 + \theta\left(\frac{q_{22}}{q_{12}}\right)^{\sigma}\right]^{-1} - m_{12}. \tag{A.2}$$

We also substitute the above solution forms of (ω, f) into (3.6) to obtain an expression for e. The laws of motion of money holdings, (2.7) and (2.8), yield (m_{11}, m_{12}). The ratios between the q's can be derived from (3.8) and (3.9). Substituting these results into (A.2), we obtain (3.11).

Finally, if $\gamma_1 = \gamma_2 = \gamma$, it is easy to verify that (3.10) and (3.11) are satisfied by $(n_1, \theta) = (1/2, 1)$. Then, (3.7), (3.8) and (3.9) imply $q_{ij} = q_{ij}^f = Q(\gamma)$ for all $i, j = 1, 2$, where $Q(\gamma)$ is defined by (3.13).

A.2. Proof of Lemma 4.1

Let

$$f(n_1) = \left[\frac{\mu(n_1)}{\mu(1-n_1)}\right]^{\sigma-1} = LHS \text{ of } (4.1);$$

$$g(n_1) = \frac{1 + \left(\frac{\gamma_1}{\beta} - 1\right)\Big/\mu(n_1)}{1 + \left(\frac{\gamma_2}{\beta} - 1\right)\Big/\mu(1-n_1)} = RHS \text{ of } (4.1);$$

we know $f' < 0$, and $f(0) = (L_0)^{1-\sigma}$; $f(1) = (L_0)^{\sigma-1}$. And $g' > 0$, $g(0) = \gamma_1 / [\beta + (\gamma_2 - \beta)/L_0]$, $g(1) = [\beta + (\gamma_1 - \beta)/L_0]/\gamma_2$. Thus, n_1 is uniquely determined if and only if $f(0) > g(0)$ and $f(1) < g(1)$, which gives the range of D.

Since RHS of (4.1) is an increasing function of γ_1 and decreasing function of γ_2. It's easy to show $N_1(\gamma_1, \gamma_2) < 0$ and $N_2(\gamma_1, \gamma_2) > 0$.

Finally, if $\gamma_1 = \gamma_2 = \gamma$, it is easy to verify that $n_1 = 1/2$ solves (4.1).

A.3. Proof of Lemma 4.2

Setting $s = 1/2$ in (3.11), we have

$$\theta^{\frac{\sigma}{\sigma-1}} = 1 + \frac{\gamma_2(1-n_1)\,[G(n_1,\theta)]^\sigma}{(\gamma_1 - 1)\,n_1 + \frac{1}{2}L(n_1) + \frac{1}{2}L(1-n_1)\,[G(n_1,\theta)]^\sigma}\Delta(\gamma_1,\gamma_2),$$

where

$$\Delta(\gamma_1,\gamma_2) = 1 - \frac{1}{\gamma_2} - \left(1 - \frac{1}{\gamma_1}\right) e(\gamma_1,\gamma_2),$$

and $e(\gamma_1,\gamma_2)$ is given by (4.2). Then, $\theta > 1$ if and only if $\Delta > 0$.

To examine Δ, we examine e first. Use (4.1) to substitute for $\mu(n_1)/\mu(1-n_1)$, we rewrite (4.2) as:

$$e(\gamma_1,\gamma_2) = \frac{n}{1-n}\frac{\frac{1}{\beta} - [1-\mu(1-n)]\,\frac{1}{\gamma_2}}{\frac{1}{\beta} - [1-\mu(n)]\,\frac{1}{\gamma_1}},$$

where $n = N(\gamma_1,\gamma_2)$ is the solution to (4.1) for n_1. Since $\mu(n)$ and $\mu(1-n)$ are less than one, the right-hand side of the above expression for e is an increasing function of n and γ_2, and a decreasing function of γ_1. Since $n = N(\gamma_1,\gamma_2)$ decreases in γ_1 and increases in γ_2, then $e(\gamma_1,\gamma_2)$ decreases in γ_1 and increases in γ_2. Because $e(\gamma,\gamma) = 1$ for all $\gamma > \beta$, then $e > 1$ if and only if $\gamma_1 < \gamma_2$.

Now, return to $\Delta(\gamma_1, \gamma_2)$. Clearly, $\Delta(\gamma, \gamma) = 0$ for all $\gamma > \beta$. Suppose $\gamma_1 < \gamma_2$. Then, $e(\gamma_1, \gamma_2) > 1$. We verify the conditions in the lemma under which $\Delta > 0$, and so $\theta > 1$. If $\gamma_2 \leq 1$, then for all $\beta < \gamma_1 < \gamma_2$, we have:

$$\Delta > \left(1 - \frac{1}{\gamma_2}\right)[1 - e(\gamma_1, \gamma_2)] \geq 0.$$

The first inequality comes from $\gamma_1 < \gamma_2$ and the second inequality comes from $\gamma_2 \leq 1$ and $e(\gamma_1, \gamma_2) > 1$. If $\gamma_2 > 1$, then for all $\beta < \gamma_1 \leq 1$, we have:

$$\Delta > \left(1 - \frac{1}{\gamma_1}\right)[1 - e(\gamma_1, \gamma_2)] \geq 0.$$

For $\gamma_2 > 1$, let γ_0 be the minimum value of γ ($> \beta$) that satisfies $\Delta(\gamma, \gamma_2) = 0$. Because $\Delta(\gamma_2, \gamma_2) = 0$, then γ_0 exists and $\gamma_0 \leq \gamma_2$. Also, $\gamma_0 > 1$, because $\Delta(1, \gamma_2) > 0$ for $\gamma_2 > 1$. Thus, if $\gamma_2 > 1$, then $\Delta > 0$ for all $\gamma_1 < \gamma_0$.

B. Proof of Proposition 5.1

We show that $dW_1/d\gamma_1 > 0$ at $\gamma_1 = \beta$, given $\gamma_2 = \beta$. In the process, we also show $dn_1/d\gamma_1 < 0$ and $d\theta/d\gamma_1 < 0$. Rewrite country 1's welfare level as:

$$\begin{aligned}(1-\beta)\mathcal{W}_1 = \; & W_{11}^1 \left[Aq_{11} - (q_{11})^\sigma\right] + W_{11}^2 \left[Aq_{12} - (q_{12})^\sigma\right] \\ & + \left[W_{12}^1 A q_{11}^f - W_{21}^1 \left(q_{21}^f\right)^\sigma\right] + \left[W_{12}^2 A q_{12}^f - W_{21}^2 \left(q_{22}^f\right)^\sigma\right].\end{aligned}$$

The first two terms are net surpluses that the household's agents obtain in trades with domestic agents and the last two terms are net surpluses in trades with foreign agents. Differentiate the above welfare level with respect to γ_1 and evaluate the derivative at $\gamma_1 = \gamma_2 = \beta$. Utilizing (3.7), (3.8), (3.9), (3.14) and the result $n_2 = 1 - n_1$, we get:

$$\begin{aligned}\frac{(1-\beta)}{q^\sigma} \frac{d\mathcal{W}_1}{d\gamma_1}\bigg|_{\gamma_1=\gamma_2=\beta} = \; & \frac{(2s-1)\sigma}{2\beta(\sigma-1)} - \mu\left(\tfrac{1}{2}\right)\left[\frac{s^2+(1-s)^2}{\sigma-1} + 1\right]\frac{d\theta}{d\gamma_1} \\ & - (2s-1)\,\mu'\left(\tfrac{1}{2}\right)\frac{dn_1}{d\gamma_1}.\end{aligned} \tag{B.1}$$

Here γ_2 is taken as given in the derivatives of n_1 and θ, and both derivatives are evaluated at $\gamma_1 = \gamma_2 = \beta$. Differentiating (4.1) with respect to γ_1 and evaluating the derivative at $\gamma_1 = \gamma_2 = \beta$, we get:

$$\frac{dn_1}{d\gamma_1} = \frac{-1}{\beta\,(1-\psi)\,(\sigma-1)\,\mu\left(\frac{1}{2}\right)} < 0. \tag{B.2}$$

Then, (3.11) implies:

$$\frac{d\theta}{d\gamma_1} = -\frac{(\sigma-1)}{\sigma}\frac{(1-\beta)\left[1+2\sigma(1-\psi)\right]+2\beta(1-\psi)(\sigma-1)\mu\left(\frac{1}{2}\right)}{\beta\mu\left(\frac{1}{2}\right)(1-\psi)(\sigma-1)\left[\beta-1+\mu\left(\frac{1}{2}\right)\right]} < 0. \tag{B.3}$$

Substituting (B.1) and (B.2) into (B.1) and setting $s = 1/2$, we obtain:

$$\left.\frac{d\mathcal{W}_1}{d\gamma_1}\right|_{\gamma_1=\gamma_2=\beta} = -\frac{2\sigma-1}{2(1-\beta)(\sigma-1)}q^{\sigma}\mu\left(\frac{1}{2}\right)\frac{d\theta}{d\gamma_1} > 0. \tag{B.4}$$

Because the two countries are symmetric when $\gamma_1 = \gamma_2$, the derivative $dW_2/d\gamma_2$, evaluated at $\gamma_1 = \gamma_2 = \beta$, is given by the same expression as in (B.4). Then, $dW_2/d\gamma_2 > 0$ at $\gamma_1 = \gamma_2 = \beta$. Therefore, the Nash equilibrium generates $\gamma_1 > \beta$ and $\gamma_2 > \beta$.

C. Proof of Proposition 5.2

To show the Nash equilibrium exists, we need to show there is a pair of (γ_1, γ_2) which solves following equation:

$$\frac{d\mathcal{W}_1}{d\gamma_1} = 0; \tag{C.1}$$

$$\frac{d\mathcal{W}_2}{d\gamma_2} = 0; \tag{C.2a}$$

where (C.1) is the best response function for γ_1 and (C.2a) is the best response function for γ_2.

The strategy we use for the proof is as follows: First, we conjecture that two countries' money growth rate should be equal in the Nash equilibrium. Then, we verify there exist γ^* which can solve the two best response functions: $\left.\frac{d\mathcal{W}_i}{d\gamma_i}\right|_{\gamma_1=\gamma_2} = 0$.

Take derivative of W_1 with respect to γ_1 and evaluate the derivative at $\gamma_1 = \gamma_2$, we obtain,

$$\frac{1}{W_{11}^1}\frac{dW_1}{d\gamma_1}|_{\gamma_1=\gamma_2} = -2\left[(q)^{\sigma}+\frac{1}{\sigma-1}Aq\right]\left.\frac{d\theta}{d\gamma_1}\right|_{\gamma_1=\gamma_2} - \frac{2\left(A-\sigma(q)^{\sigma-1}\right)}{\beta\mu\left(\frac{1}{2}\right)(\sigma-1)}q^{\sigma};$$

where

$$\begin{aligned} \frac{\sigma}{\sigma-1}\left.\frac{d\theta}{d\gamma_1}\right|_{\gamma_1=\gamma_2} &= \frac{-4(\gamma-1)\left[\sigma(1-\psi)+1\right]n'-1}{(\gamma-1)+\mu\left(\frac{1}{2}\right)}; \\ n' &= -\frac{1}{4(1-\psi)\left[\beta(\sigma-1)\mu(\frac{1}{2})+(\gamma-\beta)\sigma\right]}; \end{aligned}$$

Define

$$f(\gamma) = -2\left[(q)^{\sigma} + \frac{1}{\sigma-1}Aq\right]\frac{d\theta}{d\gamma_1}\bigg|_{\gamma_1=\gamma_2} - \frac{2\left(A-\sigma(q)^{\sigma-1}\right)}{\beta\mu\left(\frac{1}{2}\right)(\sigma-1)}q^{\sigma} \equiv 0;$$

Since we have already shown that $f(\beta) > 0$ in proposition (5.1), if we can find $\gamma' > \beta$ such that $f(\gamma') < 0$, then the proof is done. Obviously, if $\frac{d\theta}{d\gamma_1}\Big|_{\gamma_1=\gamma_2} = 0$, $f(\gamma) < 0$. Thus we need to show γ_A that solves $\frac{d\theta}{d\gamma_1}\Big|_{\gamma_1=\gamma_2} = 0$ is greater than β. We can easily show $\gamma_A > 1 > \beta$. Therefore, there exists $\gamma^* = \gamma_1 = \gamma_2$ that solves the best response function for γ_1. By symmetry, γ^* also solves the best response function for γ_2. Thus, $\gamma_1 = \gamma_2 = \gamma^*$ is the Nash equilibrium.

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