Imperfect Monitoring and the Discounting of Inside Money

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Abstract

One of the fundamental questions concerning inside money is whether its issuers should be regulated and how. This paper evaluates the efficiency of one prevalent regulatory recommendation – a requirement that private issuers redeem inside money on demand at par – in a random-matching model of money where the issuers of inside money are only imperfectly monitored. I find that for sufficiently imperfect monitoring, a par redemption requirement leads to lower social welfare than if private money were redeemed at a discount. A central message of the paper is that if inside money and outside money are not perfect substitutes for one another, as is the case if there is sufficiently imperfect monitoring, a par redemption requirement may not be socially optimal because such a requirement effectively binds them to circulate as if they are. Such an outcome is a version of Gresham's law that bad money drives out good money.

Keywords: Inside and Outside Money, Imperfect monitoring, Gresham's law

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1 Introduction

The emergence of electronic money such as stored-value and prepaid cards has renewed interest in inside (or private) money schemes. One of the fundamental questions concerning inside money is whether its issuers should be regulated and how. Nineteenth century arguments, as well as more recent debates between free-banking advocates and monetarists, centered around two extremes. At one extreme was the banking school, which advocated for a relatively laizze-faire approach to private note issue. At the other exteme was the currency school, which advocated for a public monopoly on money with strict controls. More recently, the relaxation of legal restrictions on the issuance of inside money means that modern debates focus on the degree of regulation to which private issuers may be subject. This paper evaluates the efficiency of one prevalent regulatory recommendation – a requirement that private issuers redeem inside money on demand at par – in a model where inside and outside money coexist, and the issuers of inside money are imperfectly monitored. I find that for sufficiently imperfect monitoring of inside money issuers, a par redemption requirement leads to lower social welfare than if private money were redeemed at a discount.

The par redemption requirement seems common throughout recent monetary history. Such a requirement existed in most free-banking states during the Free Banking Era (1838-60) in the United States.[1] More recently, the European Central Bank (1998) has established a policy on electronic money for the Eurosystem that stipulates several minimum requirements of electronic money issuers, including par redemption.[2] Motivations for such a requirement include the safety and efficiency of the payment system. The safety argument is that par-redemption restricts the ability of inside money issuers to overissue, making an inside money scheme more stable. The efficiency argument is that currencies trading at varying exchange rates diminishes the unit-of-account function of money and so is inefficient. Implicit in this argument is the belief that inside and outside money should effectively circulate as a uniform currency, i.e. they are essentially perfect substitutes for one another. Thus, it seems natural to ask : 1) In what settings are inside money schemes stable? 2) In what settings are inside money and outside money not perfect subtitutes? and 3) Are there any welfare implications in those settings from a par redemption requirement?

Recent research in monetary theory has shed some light on the first

[1] See, for example, Rolnick-Weber (1988).

[2] Issuers are permitted to charge a small fee only to recoup costs of carrying out such a transaction. In the model of this paper, these transactions costs are assumed to be zero.

two questions. Cavalcanti-Wallace (1999b) provide a random-matching model of inside money (with no outside money) that is stable because of society's ability to perfectly monitor the trading histories of the issuers.[3] This is because monitoring enables agents to punish and reward the issuers in the future for acts that they make presently. If the long-term rewards of consistent issue and redemption exceed the short-term benefits of overissuing, inside money issuers have an incentive not to overissue. In another paper, Cavalcanti-Wallace (1999a) go on to say that in an environment in which society can perfectly monitor inside money issuers, inside money is a perfect substitute for outside money, but not vice versa. Inside money can duplicate any allocation that is achievable with outside money and can also achieve even more desirable allocations because it gives private issuers access to instant liquidity which can, in turn, lead to increased trading. In such an environment, outside money is inessential, so if both types of money were to circulate, it seems optimal that they do so as a uniform currency, and a par-redemption requirement would be innocuous.

Mills (2003) generalizes the basic model in Cavalcanti-Wallace (1999a,b) by allowing (i) both inside and outside money to circulate, and (ii) monitoring of issuers to be imperfect, and overturns their result in Cavalcanti-Wallace (1999a) by showing that both inside and outside money are necessary to implement certain allocations. Such a result suggests that inside and outside money are not perfect substitutes for one another in the case of imperfect monitoring. While inside money retains the advantage it has in Cavalcanti-Wallace (1999a), imperfecting monitoring implies that issuers have greater incentive to overissue, restricting its value. Outside money is not subject to such constraints and so can trade for higher levels of output. Due to the complexity of the general problem, Mills (2003) does not attempt to characterize the relative welfare of alternative monetary schemes.

The current paper, a simplified version of that in Mills (2003), provides some insight on the third question mentioned above. The model is a random-matching model of money in which some people (bankers) can be publicly monitored via a record-keeping technology while others (nonbankers) cannot be monitored at all because their trading histories are private information. I present a numerical example of an implementable allocation where inside money is redeemed by its issuers at a discount relative to outside money in the following sense: inside money issuers produce more to obtain outside money than they do to redeem

[3] Indeed, solid and transparent legal arrangements, which define enforceable rights and obligations of participants is another regulatory requirement of electronic money schemes in the Eurosystem.

inside money. The results are numerical because, as shown in Mills (2003), the updating lag of banker histories must be neither too short nor too long for the coexistence of both types of assets to be essential. I then compare the welfare of such an allocation to that of another allocation that differs in only one respect: outside and inside money trade at par. I find that welfare is less in such an allocation because it reduces the value of outside money. This intuition is straightfoward. The par requirement cannot raise the value of inside money in an environment with sufficiently imperfect monitoring because doing so would provide an incentive to overissue in the sense that inside money issuers do not wish to redeem notes. Thus, the only way to satisfy the par redemption requirement is to make inside money a perfect substitute for outside money by undoing the advantage that outside money has over inside money – namely that it can trade for higher levels of output. Such an outcome is a version of Gresham's law that bad money drives out good.

The connection between imperfect monitoring and the ability to issue inside money goes back at least to Klein (1974), who stresses the importance of reputations for private bankers to issue inside money, although in a model quite different from the one here[4]. King (1983) argues that imperfect information and monitoring are central to understanding inside money. Nonetheless, recent work on the coexistence of outside and inside money such as Williamson (1999), Azariadis-Bullard-Smith (2001), Bullard-Smith (2000) and Marimon-Nicolini-Teles (2003) has not focused on the challenge that imperfect history and monitoring present to the circulation of inside money. Indeed, each of these works model inside money in such a way that inside and outside money should effectively be perfect substitues for one another. This is a direct artifact of their abstraction from imperfect monitoring.

The rest of the paper is organized as follows. Section 2 presents the environment. Section 3 describes the restricted class of mechanisms I study and the conditions for implementability. Section 4 presents the main example and results, while section 5 offers concluding remarks.

2 The Economic Environment

The background environment is the random-matching model of money from Shi (1995) and Trejos-Wright (1995). Time is discrete and the horizon is infinite. There are S distinct, divisible, and perishable types of goods at each date and there is a $[0, 1]$ continuum of each of S specialization types of agents, where $S > 2$. An agent whose specialization type

[4] His model is one of competing issuers, while mine is has a great deal of cooperation in the sense that issuers redeem others' notes.

is s consumes only good s and produces only good $s+1$ (modulo S), for $s = 1, 2, \ldots, S$. Each agent maximizes expected discounted utility with a discount factor $\beta \in (0,1)$. Utility in a period is given by $u(c) - y$ where c is the amount consumed and y is the amount produced. The function u is defined on $[0, \infty)$, is increasing, twice differentiable, and satisfies $u(0) = 0$, $u' > 0, u'' < 0$, and $u'(0) = \infty$. Moreover, there exists $y^{\max} > 0$ such that $u(y^{\max}) = y^{\max}$.

Agents cannot commit to future actions. This implies that those agents who produce must expect to receive something of value for doing so.

The society is able to keep a public record of the actions of a fraction B of each specialization type of agent, where $B \in [0,1]$. Agents whose histories are a part of the public record are called *bankers*. The fact that banker histories are part of a public record implies that it is possible for agents to monitor their behavior. As we shall see, this implies that bankers do not need to receive something tangible to induce them to produce in a single-coincidence meeting; bankers can be rewarded and punished in the future for actions they take currently. Society has no public record for the remaining fraction $1 - B$ of each specialization type, the *nonbankers*. The fact that nonbankers are anonymous implies that the society cannot monitor their behavior. The implication is that nonbankers must receive something tangible in order to produce. We can think of B as society's capacity for keeping track of individual trading histories.

Public information about banker histories is not perfect because the public record is not updated immediately after every action. Specifically, there is a deterministic lag of T periods, where $T \geq 0$. At the beginning of each date $t > T$, the bankers' trading histories are known up through what they did until the beginning of date $t - T$. For $t \leq T$, banker histories are unknown. Thus, if $T = 0$, then banker histories are completely publicly known and society can perfectly monitor their actions. If T is sufficiently large, then banker histories are effectively unknown and society cannot monitor their actions at all. We can think of T as society's ability to update records.

In each period, nonbankers are randomly matched in pairs with either other nonbankers or with bankers. For simplicity, it is assumed that bankers never meet each other. A single-coincidence meeting is a meeting that contains a type s agent (the producer) and a type $s+1$ agent (the consumer) for some s. There are three basic types of single coincidence meetings: meetings between nonbankers, meetings in which a banker is a producer and a nonbanker is a consumer, and meetings in which a nonbanker is a producer and a banker is a consumer. A no-coincidence meeting is a meeting in which neither agent produces what the other consumes. Because $S > 2$, there are no meetings in which there is a double-coincidence of wants.

There are two distinct assets. These assets are indivisible and agents can carry at most one unit of one asset across dates. It is not possible for an agent to simultaneously hold a unit of both assets. Each banker has a technology that permits her to create distinct, indivisible and perfectly durable objects, called notes. Because these notes are a type of credit instrument that is issued by private individuals and may circulate as a means of payment, they may serve as *inside money*. The notes issued by a single banker are distinguishable from those issued by another so that counterfeiting is not a problem. *Outside money*, on the other hand, is neither produced nor consumed. It is indivisible and perfectly durable. Without loss of generality, refer to asset 1 as the inside money and asset 2 as the outside money.

When two agents meet, the following is common knowledge: each trading partner's specialization type, asset holdings, information type (banker or nonbanker) and the past actions of the bankers in the meeting that occurred up to $t - T$ periods ago.

3 Allocations

I shall now describe the limited class of allocations. Allocations are both stationary and symmetric. Stationarity implies that there is a steady-state distribution of assets among agents. Symmetry implies two things. First, current actions are independent of an individual's specialization type and his relative position within that type. Second, notes issued by bankers are treated symmetrically by the agents in the economy. That is to say, nonbankers treat notes issued by each banker as perfect substitutes for those issued by every other banker, and bankers redeem each others' notes.

Trade takes place only in single-coincidence meetings in which there has been no discovered defection. The make-up of a single-coincidence meeting depends on four things: the information type and asset-holding of both the producer and consumer. Let the set of information types be $\{b, n\}$ where b indicates that an agent is a banker and n indicates that he is a nonbanker. I shall identify agent asset-holdings by states. Nonbankers are in one of three states in the set $A = \{0, 1, 2\}$. A nonbanker in state 0 has no asset-holdings. A nonbanker in state 1 has a unit of inside money and one in state 2 has a unit of outside money. Because of the symmetry imposed on banknotes, bankers do not gain from holding other bankers' notes and so never hold a note issued by another banker. They may, however, hold outside money. Thus, a banker is in one of two states, $\{1, 2\}$. A banker in state 1 does not have a unit of outside money and a banker in state 2 does have a unit of outside money.

If trade takes place in a single-coincidence meeting, it involves the transfer of a level of output and money-holdings. If a banker chooses not to trade, then that banker is a defector. A nonbanker does not receive such a label because a defection by him would never be discovered. A defection by a banker is discovered T periods from the date it occurred (say date t). For the $T-1$ periods that follow an initial defection, a banker is an undiscovered defector. An undiscovered defector can costlessly choose to either participate in trade or to defect again. From period $t+T$ on, a defecting banker is a discovered defector and is punished with autarky in all future meetings.

There are at most 21 different single-coincidence meetings. Formally, let m_{ij}^{kl} denote a single-coincidence meeting between a producer of information type k in state i and a consumer of information type l in state j, where $i, j \in A$ and $k, l \in \{b, n\}$. I restrict the class of allocations further in several ways. First, I do not allow bankers to give gifts to nonbankers, even if the monitoring of bankers makes it incentive compatible for them to do so $(m_{10}^{bn}, m_{20}^{bn}, m_{22}^{bn})$. This assumption is meant to capture that most transactions in an economy are not based on the giving of gifts. Second, I anticipate several meetings will have no trade because of various feasibility and incentive constraints $(m_{00}^{nn}, m_{10}^{nn}, m_{20}^{nn}, m_{11}^{nn}, m_{22}^{nn}, m_{11}^{nb}, m_{22}^{nb})$. Third, I impose no trade in meetings where agents could trade assets $(m_{21}^{nn}, m_{12}^{nn}, m_{12}^{nb}, m_{21}^{nb})$. Aiyagari-Wallace-Wright (1996) has shown that the existence of two distinct assets may improve welfare in environments with the assumed indivisibility of assets and upper-bound on money-holdings because it increases the frequency of trades by allowing agents to exchange a higher-valued asset for a lower-valued asset and production. I do not want such an effect here because I do not want the benefit of inside money redeemed at a discount to be derived from it. Without this effect, it is harder for the non-par redemption of inside money to improve upon the welfare attained with par redemption.

There are four non-negative levels of output in single-coincidence meetings: y_1^n, y_2^n, y_1^b and y_2^b. The output level y_1^n is relevant in trades between nonbankers that involve inside money (m_{01}^{nn}) whereas y_2^n is relevant in trades between nonbankers that involve outside money (m_{02}^{nn}). In both these types of meetings, the nonbankers switch states (the producer acquires the asset while the consumer surrenders it).

The output level y_1^b is relevant in trades between nonbankers and bankers where inside money is either issued or redeemed $(m_{01}^{nb}, m_{11}^{bn}, m_{21}^{bn})$. In meetings m_{01}^{nb} inside money is issued (the nonbanker leaves the meeting in state 1, as does the banker, who does not switch states). In meetings m_{11}^{bn} and m_{21}^{bn}, inside money is redeemed (the nonbanker switches to state 0 while the banker does not change states).[5] When bankers redeem notes, they destroy them. Bankers redeem notes issued by any banker. Finally, the output level y_2^b is relevant in trades between nonbankers and bankers that involve outside money $(m_{02}^{nb}, m_{12}^{bn})$. In each of the meetings outside money changes hands so consumers and producers switch states. This also implies that inside money is never issued nor redeemed in such meetings.

The relationship of primary interest is that between y_1^b and y_2^b. If $y_1^b = y_2^b$ then nonbanker consumers receive the same level of output from bankers whether they redeem inside money or trade outside money. Under this relationship, I say that inside money is redeemed at par. If $y_1^b < y_2^b$, then inside money is redeemed at a discount. Also of interest is the relationship between y_1^n and y_2^n. If $y_1^n = y_2^n$ then inside money and outside money circulate at par, whereas if $y_1^n < y_2^n$ inside money circulates at a discount.

3.1 Value Functions

In this subsection, I describe the expected discounted utility for nonbankers and nondefecting bankers, and for undiscovered defecting bankers. These are all expressed given that no one else defects.

Let x_i^k denote the fraction of each specialization type with information type k in state i. Because each person must be in one of the states, the fractions of each specialization type in each state must satisfy

$$\sum_{i \in A} x_i^n = 1 - B \text{ and } \sum_{i=1}^{2} x_i^b = B. \tag{1}$$

Next, let v_i^k denote the no-defection, expected discounted utility of an agent of information type k who is in state i at the start of a period. Suppose that everyone else follows the suggested outcome. The value

[5] Implicitly, I am assuming that these notes do not bear interest by assuming that the level of output required when a note is issued is the same as the output required when a note is redeemed. This assumption is consistent with historical episodes of inside money.

functions for nonbankers can be written as:

$$
\begin{aligned}
v_0^n &= \beta v_0^n + \frac{x_1^n}{S}\{-y_1^n + \beta(v_1^n - v_0^n)\} + \frac{x_1^b}{S}\{-y_1^b + \beta(v_1^n - v_0^n)\} \\
&\quad + \frac{x_2^n}{S}\{-y_2^n + \beta(v_2^n - v_0^n)\} + \frac{x_2^b}{S}\{-y_2^b + \beta(v_2^n - v_0^n)\} \qquad (2) \\
v_1^n &= \beta v_1^n + \frac{x_0^n}{S}\{u(y_1^n) + \beta(v_0^n - v_1^n)\} + \frac{x_1^b + x_2^b}{S}\{u(y_1^b) + \beta(v_0^n - v_1^n)\} \qquad (3) \\
v_2^n &= \beta v_2^n + \frac{x_0^n}{S}\{u(y_2^n) + \beta(v_0^n - v_2^n)\} + \frac{x_1^b}{S}\{u(y_2^b) + \beta(v_0^n - v_2^n)\} \qquad (4)
\end{aligned}
$$

Consider (2). With probability $\frac{x_1^n}{S}$ the nonbanker is a producer in a single-coincidence meeting with another nonbanker and acquires a banknote in exchange for production y_1^n. With probability $\frac{x_1^b}{S}$ the nonbanker is a producer in a single-coincidence meeting with a banker and acquires banknote in exchange for production y_1^b. With probability $\frac{x_2^n}{S}$ the nonbanker is a producer in a single-coincidence meeting with another nonbanker and acquires outside money in exchange for production y_2^n. With probability $\frac{x_2^b}{S}$ the nonbanker is a producer in a single-coincidence meeting with a banker and acquires outside money in exchange for production y_1^b. Equations (3) and (4) have similar interpretations for nonbankers with a unit of inside money and outside money respectively.

The value functions for nondefecting bankers are:

$$
\begin{aligned}
v_1^b &= \beta v_1^b + \frac{x_0^n}{(1-B)S}\{u(y_1^b)\} + \frac{x_1^n}{(1-B)S}\{-y_1^b\} \\
&\quad + \frac{x_2^n}{(1-B)S}\{-y_2^b + \beta(v_2^b - v_1^b)\} \qquad (5) \\
v_2^b &= \beta v_2^b + \frac{x_0^n}{(1-B)S}\{u(y_2^b) + \beta(v_1^b - v_2^b)\} + \frac{x_1^n}{(1-B)S}\{-y_1^b\} \qquad (6)
\end{aligned}
$$

For (5), with probability $\frac{x_0^n}{(1-B)S}$ a banker without a unit of outside money consumes y_1^b and issues the nonbanker a new note. With probability $\frac{x_1^n}{(1-B)S}$ a banker without a unit of outside money produces y_1^b and redeems a unit of inside money from the nonbanker. Finally, with probability $\frac{x_2^n}{(1-B)S}$ the banker produces y_2^b and receives a unit of outside money. Equation (6) has a similar interpretation for bankers with a unit of outside money.

I calculate recursively the initial-defector expected discounted utility, given that no one else defects. The defecting banker's payoff must include the option of disagreeing to the suggested outcome in a meeting. It must also reflect the fact that she knows with certainty that her first defection will be discovered T periods after it occurs and she will be punished with autarky from that date on. Let $\widetilde{v}^b_{i\tau}$ denote the expected discounted utility of a defecting banker who enters the period in state i and who first defected τ periods ago. Then the value functions can be written as follows:

$$\widetilde{v}^b_{1\tau} = \beta\widetilde{v}^b_{1,\tau+1} + \frac{x^n_0}{(1-B)S}\{\max[u(y^b_1), 0]\} + \frac{x^n_1}{(1-B)S}\{\max[-y^b_1, 0]\}$$
$$+\frac{x^n_2}{(1-B)S}\{\max[-y^b_2 + \beta(\widetilde{v}^b_{2,\tau+1} - \widetilde{v}^b_{1,\tau+1}), 0]\} \qquad (7)$$

$$\widetilde{v}^b_{2\tau} = \beta\widetilde{v}^b_{2,\tau+1} + \frac{x^n_0}{(1-B)S}\{\max[u(y^b_2) + \beta(\widetilde{v}^b_{1,\tau+1} - \widetilde{v}^b_{2,\tau+1}), 0]\}$$
$$+\frac{x^n_1}{(1-B)S}\{\max[-y^b_1, 0]\} \qquad (8)$$

The difference between (5)-(6) and (7)-(8) is that v^b_h is replaced with $\widetilde{v}^b_{h,\tau+1}$ and there are max terms associated with the fact that an undiscovered defecting banker will defect again when it is advantageous to do so. The presence of τ reflects the fact that the continuation payoff of a defecting banker is dependent on the time left before discovery. The terminal condition is

$$\widetilde{v}^b_{iT} = 0 \qquad (9)$$

for $i = 1, 2$.

The expected discounted utility for a banker from an initial defection given that no one else defects, $\beta\widetilde{v}^b_{i1}$, is what is relevant for the incentive constraints described below. This is obtained by solving $\widetilde{v}^b_{i\tau}$ recursively from the terminal condition $\widetilde{v}^b_{iT} \equiv 0$. The terminal condition incorporates the fact that once discovered, a defecting banker is punished with autarky forever.

3.2 Constraints

Now consider the constraints that are relevant for implementation. There are three sets of constraints: participation, free-disposal and steady-state. Participation constraints require that agents are ex-post sequentially rational. This is equivalent to the requirement that they receive non-negative gains from trade. The participation constraints can be summarized as follows:

$$y_1^n \leq \beta(v_1^n - v_0^n) \tag{10}$$
$$y_1^b \leq \beta \min[v_1^n - v_0^n, v_1^b - \widetilde{v}_{1,1}^b, v_2^b - \widetilde{v}_{2,1}^b] \tag{11}$$
$$y_2^n \leq \beta(v_2^n - v_0^n) \tag{12}$$
$$y_2^b \leq \beta \min[v_2^n - v_0^n, v_2^b - \widetilde{v}_{1,1}^b] \tag{13}$$
$$u(y_1^n) \geq \beta(v_1^n - v_0^n) \tag{14}$$
$$u(y_1^b) \geq \beta \max[v_1^n - v_0^n, \widetilde{v}_{1,1}^b - v_1^b] \tag{15}$$
$$u(y_2^n) \geq \beta(v_2^n - v_0^n) \tag{16}$$
$$u(y_2^b) \geq \beta \max[v_2^n - v_0^n, \widetilde{v}_{2,1}^b - v_1^b] \tag{17}$$

Free-disposal constraints imply that nonbankers never dispose of either inside money or outside money and that bankers do not dispose of outside money. They are

$$v_i^k \geq v_0^k, \widetilde{v}_{2\tau}^b \geq \widetilde{v}_{1\tau}^b \tag{18}$$

for all $k \in \{b, n\}, i \in A, \tau \in \{1, 2, \ldots, T-1\}$.

Finally, the steady-state constraints impose restrictions on state transitions. A steady-state distribution of agents over states requires that the fraction of bankers in each state and the fraction of nonbankers in each state be constant. This can be expressed by equating the inflow and outflow of each state for nonbankers and bankers. The nonbanker inflow-equal-outflow equations are

$$x_0^n[x_1^b + x_2^b] = x_1^n[x_1^b + x_2^b] + x_2^n x_1^b \tag{19}$$
$$x_1^n[x_1^b + x_2^b] = x_0^n x_1^b \tag{20}$$
$$x_2^n x_1^b = x_0^n x_2^b \tag{21}$$

For bankers, there is one inflow-equal-outflow equation which reduces to (21).

3.3 Implementable and Optimal Allocations

This section concludes with two definitions. The first defintion summarizes the set of implementable allocations as those that satisfy both incentive and feasibility constraints.

Definition 1 *An allocation* $(y_1^n, y_2^n, y_1^b, y_2^b)$ *is implementable if there exists* (x_i^k) *for* $k \in \{b, n\}$ *and* $i \in \{0, 1, 2\}$ *that satisfies (1)-(21).*

The second definition describes optimal allocations. These allocations are those that maximize a social welfare function subject to the constraint that they are implementable. The social welfare function is defined as the ex-ante utility of a representative agent.

Definition 2 *An allocation is optimal if maximizes*

$$W = \sum_{i,k} x_i^k v_i^k \tag{22}$$

subject to (1)-(21).

It is well-known and easy to verify that the unconstrained optimal allocation is $y_1^n = y_2^n = y_1^b = y_2^b = y^*$ where y^* is the $\arg\max u(y) - y$, or the level of output that maximizes joint-surplus in each of the single-coincidence meetings. This optimum, where inside money is redeemed at par with outside money, is the benchmark allocation against which implementable allocations with and without the par redemption requirement are compared.

4 Examples

I now provide numerical examples to demonstrate that in an environment with sufficiently imperfect monitoring, an optimal allocation where inside money is permitted to trade at a discount relative to outside money is superior to the optimal allocation where inside money trades at par with outside money. As argued in Mills (2003), the need for numerical examples stems from the complexity of an updating lag that is neither too short nor too long if both outside money and inside money are to be essential.

There are two primary examples. In the first example, the updating lag, T, is very short. In such an example, where monitoring is nearly perfect, the optimal outcome is for inside money to be redeemed at par with outside money and is implementable. In the second example, the lag is significantly longer and the optimal outcome is for inside money to be redeemed at a discount. The purpose of the two examples is to illustrate that the par redemption requirement reduces welfare only if moniotring of the issuers is sufficiently imperfect.

The examples take as given the parameters: $\{S, B, \beta, u(c)\}$, which include the number of specialization types in the economy, the fraction of each specialization type that are bankers, the discount factor, and the specification of the period utility function. The numerical values of the parameters are:

$$\{S, B, \beta, u(c)\} = \{3, .1, .99, c^{\frac{1}{2}}\}.$$

The choices of S, B, and β are somewhat arbitrary. The only importance given to the number of specialization types is that there are enough to eliminate the possibility of a double-coincidence of wants in a meeting.

The minimum number of specialization types that accomplishes this is $S = 3$. The explicit utility function for the calculations is $u(c) = c^{\frac{1}{2}}$. Such a utility function has a very simple form that satisfies all of the assumptions made in Section 2. Note $u(c) = c^{\frac{1}{2}}$ implies that the solution to $\max u(y) - y$ is $y^* = 0.25$.

In addition to the parameters above, both examples have the following distribution over states that satisfies (1) and (19)-(21):

$$x_0^n = 0.36, x_1^n = 0.18, x_2^n = 0.36, x_1^b = 0.05, x_2^b = 0.05$$

The only difference between the two examples is the choice of T, the length of the updating lag. In the first example, $T = 3$ while in the second example, $T = 100$. In both examples, the optimal allocation and welfare for allocations with and without the par-redemption requirement are compared. To find the optimal allocations under each scenario, I employ a simple grid search over the four variables $y_1^n, y_2^n, y_1^b, y_2^b$ and find the allocation that solves the problem in Definition 2.

First, consider example 1 where $T = 3$. The optimal allocation both with and without a par redemption requirement has $y_1^n = y_2^n = y_1^b = y_2^b = 0.25$, which is also the optimal allocation for the unconstrainted problem. The intution for this result is that monitoring is sufficiently close to perfect that none of the banker-producer participation constraints bind. Indeed, the monitoring of bankers works well enough that, as in Cavalcanti-Wallace (1999a), outside money is inessential; social welfare would be maximized with inside money alone.

The value functions are:

$$\begin{aligned} v_0^n &= 1.9273, v_1^n = 2.2820, v_2^n = 2.2650 \\ v_1^b &= 3.2725, v_2^b = 3.3942 \\ \widetilde{v}_1^b &= 0.1327, \widetilde{v}_2^b = 0.1327. \end{aligned}$$

Define $w^n = \sum_i x_i^n v_i^n$ as the welfare for nonbankers and $w^b = \sum_i x_i^b v_i^b$ as the welfare for bankers. Then the relative welfare levels are:

$$W = 2.2533, w^n = 2.1333, w^b = 3.3333.$$

Now, consider example 2 where $T = 100$. Here the optimal allocations with and without a par redemption requirement are

$T = 100$	y_1^n	y_2^n	y_1^b	y_2^b
Par Requirement	0.25	0.25	0.11	0.11
No Par Requirement	0.25	0.25	0.12	0.25

.

In this example, banker-producer participation constraints bind and restrict the amount of output nonbanker-consumers receive when redeeming inside money. Notice that both with and without the par requirement, inside and outside money circulate at par and at the level of output that maximizes the joint surplus in a meeting. Without the par requirement, however, inside money is redeemed at a discount relative to outside money because it is redeemed for less output than can be acquired for outside money (0.12 for inside money and 0.25 for outside money).

The value functions for the examples both with and without the par requirement are

$T = 100$	v_0^n	v_1^n	v_2^n	v_1^b	v_2^b	$\widetilde{v}_1^b$	$\widetilde{v}_2^b$
Par Requirement	1.9032	2.2248	2.2232	2.9287	2.9823	2.7872	2.7872
No Par Requirement	1.9175	2.2412	2.2559	3.0779	3.2743	2.9111	3.0553

with relative welfare levels

$T = 100$	W	w^n	w^b
Par Requirement	2.1818	2.0955	2.9555
No Par Requirement	2.2235	2.1176	3.1761

.

It is clear that the relaxation of the par requirement improves welfare in this example. The intuition for this result is straightfoward. The par requirement cannot raise the value of inside money because doing so would violate banker-producer participation constraints and bankers would have an incentive to overissue by issuing notes when convenient, but refusing to redeem notes. Thus, the only way to satisfy the par redemption requirement is to make inside money a perfect substitute for outside money. The only way to do this is by reducing the amount of output that can be traded for outside money, esentially undoing the advantage that outside money has over inside money. Such an outcome is a version of Gresham's law that bad money drives out good.

5 Concluding Remarks

This paper argues that a requirement that inside money be redeemed at par with outside money may be inefficient if there is sufficiently imperfect monitoring of the issuers of inside money. In such an environment, inside and outside money are not perfect substitutes and a par-requirement forces them to circulate as if they are. The inefficiency that results is a version of Gresham's law that bad money drives out good money; satisfaction of the par redemption requirement reduces the amount of output that can be traded for outside money rather than increasing the amount of output that can be traded for inside money.

The par-redemption requirement could be one contributing factor to the relatively low number of electronic money schemes. For the issuers of electronic money, the inability of the public to perfectly monitor them may make it difficult to estabish enough credability to make it profitable for them to redeem their money at par.

In deriving the results, I make use of the assumption that agents can only hold one unit of one asset at a time. I conjecture that this assumption is not crucial. This is because what makes both types of money essential is the trade-off between outside and inside money. This trade-off would still exist if the assumption about the unit upper bound on money-holdings were dropped. The issue and redemption of inside money would still be subject to the imperfect monitoring of bankers, implying that the value of inside money is less than that of outside money. Nonetheless, the issuance of inside money would still provide bankers with liquidity that permits them to consume more frequently.

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