Optimal Monetary Policy in a Channel System*

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— Preliminary —

August 3, 2005

Abstract

This paper studies monetary policy when the central bank operates a channel system of interest control. We conduct our analyses in a dynamic general equilibrium model with infinitely-lived agents where money is essential for trade. We characterize the equilibrium allocation and optimal policy under two assumptions: when the central bank has the ability to force repayment of loans and when central bank credit is subject to a default constraint. Under both assumption, it is optimal to set the deposit and lending rates equal. Under the first assumption, we find that the Friedman rule holds, i.e., it is optimal to set a nominal interest rate of zero. When credit is subject to a default constraint, however, it is optimal to choose a strictly positive interest rate, implying that the Friedman rule is not optimal.

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*We have received useful comments from the participants of several seminars and workshops. We are especially grateful to Christopher Waller for his comments. Berentsen thanks the Federal Reserve Bank of Cleveland and European Central Bank for research support. The views expressed herein are those of the authors and not those of the European Central Bank or the Eurosystem.

1 Introduction

In this paper we analyze monetary policy when the central bank operates a channel system similar to the channel system of interest-rate control used by central banks such as the Bank of Canada, the Reserve Bank of Australia, and the Reserve Bank of New Zealand. Under a channel system the central bank offers a lending facility, through which it stands ready to supply an arbitrary amount of settlement balances at a fixed interest rate and commercial banks that clear transactions through the central bank also have the right to deposit excess settlement cash overnight with the central bank at a deposit rate (see Woodford (2000) for more details).

We conduct our analyses in a dynamic general equilibrium model with infinitely-lived agents where money is essential for trade.[1] Agents face random production and consumption opportunities so that at any point of time some are liquidity constraint while other agents have idle balances.[2] Once agents have learnt their liquidity needs, they use the central bank's standing facility to either borrow or deposit money at the specified rates.

We characterize the equilibrium allocation and optimal policy under two assumptions. When the central bank has the ability to force repayment of loans and when central bank credit is subject to a default constraint. The following results emerge from the model. Under both assumption, it is optimal to set the deposit and lending rates equal. Under the first assumption, we find that

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[1] By essential we mean that the use of money expands the set of allocations (Kocherlakota (1998) and Wallace (2001)).

[2] Our paper builds on the Berentsen, Camera and Waller (2004) model of money and credit. As in Koeppl, Monnet and Temzelides (2005), we abstract from modelling commercial banks explicitly. Rather, we model them as economic agents that face random needs for liquidity and random opportunities to build reserves in order to meet these needs.

the Friedman rule holds, i.e., it is optimal to set a nominal interest rate of zero. Under the second assumption, however, it is optimal to choose a strictly positive interest rate, implying that the Friedman rule is not optimal when credit is subject to a default constraint.

An interesting aspect of our model is that money growth is endogenous unlike in most theoretical analyses of monetary policy that characterize optimal policy in terms of a path for the money supply. In practice, however, monetary policy involves rules for setting nominal interest rates and most central banks specify operating targets for overnight interest rates. The paper therefore is an attempt to break the apparent dichotomy (Goodhard, 1989) between theoretical analysis and central bank practice by investigating interest rate rule as in Woodford (2005) in a model where base money is essential.

Woodford (2003) summarizes the literature that investigates optimal interest rate rules. The channel system is investigated in Woodford (2000, 2001).

The paper is structured as follows. Section 2 outlines the environment. The equilibrium with perfect enforceability of financial claims is characterized in Section 3. Section 4 defines and characterizes the equilibria with default. Finally, welfare results are described in Section 6.

2 The environment

The environment is similar to the one introduced by Berentsen, Camera and Waller (2004). Time is discrete and in each period two perfectly competitive markets open sequentially. There is a [0,1] continuum of infinitively-lived agents. All agents produce and consume one perishable good.

At the beginning of the first market, agents get a preference shock that determines whether they can produce or consume. With probability $1-n$ an agent can consume and cannot produce. We refer to these agents as buyers. With probability n, an agent can produce and cannot consume. These are sellers.

Agent get utility $u(q)$ from q consumption in the first market, where $u'(q) > 0$, $u''(q) < 0$, $u'(0) = +\infty$ and $u'(\infty) = 0$. For certain proofs we also impose $u'''(q) \geq 0$ and that $-u''(q)q/u'(q) \geq 1$.[3] Producers incur a utility cost $c(q) = q$ from producing q units of output. All trades are anonymous and agents' trading histories are private information. Hence there is a role for money, as sellers require immediate compensation for their production effort. The discount factor is β where for technical reasons we assume that $\beta > n$.

In the second market all debt is settled. All agents can consume and produce. They get utility x from consuming x units of the general good and disutility h from producing h units of output. As in Lagos and Wright (2005), these assumptions allow us to get a degenerate distribution of money holdings at the beginning of a period.

In the first market a central bank operates a standing facility.[4] It offers

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[3] The constant elasticity of substitution utility function $q^{1-\gamma}/(1-\gamma)$ satisfies these assumptions for $\gamma \geq 1$.

[4] Under a channel system, the central bank chooses a target overnight interest rate, which is periodically adjusted in response to changing economic conditions. In addition to supplying a certain aggregate quantity of settlement cash (which can be adjusted through open-market operations), the central bank offers a lending facility, through which it stands ready to supply

nominal loans ℓ at an interest rate i and promises to pay interest $i_d d$ on nominal deposits d with $i \geq i_d$.[5] With a standing facility in the absence of open market operations the money stock evolves endogenously as follows

$$M = M_{-1} - (1-n)i\ell + n i_d d, \tag{1}$$

where M denotes the end-of-period stock of money. In the second market total loans $(1-n)\ell$ are repaid. Since interest rate payments by the private sector are $(1-n)i\ell$, the stock of money circulating in the economy shrinks by this amount. Interest payment by the central bank on total deposits are $n i_d d$. The central bank simply prints additional money to make these interest payments so the stock of money increases by this amount.

The central bank prints/burns money and operates the standing facility at zero cost. Consequently, the central bank cannot make profit or losses. It has access to a record keeping technology for financial transactions. For simplicity, we restrict financial contracts to one-period contracts only.

The existence of a standing facility implies that buyers do not face a standard cash-in-advance constraint. Before trading, they can borrow cash from the central bank to increase their money holdings at the cost of paying the interest rate i. We first assume that the central bank can enforce the repayment of loans at no cost. Then we consider the possibility of default. If an agent defaults, she will be excluded from using the standing facility in all future periods.

The timing of the model is as follows. At the beginning of the first market, agents observe their preference shocks. Then the standing facility opens and

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an arbitrary amount of additional overnight settlement cash at a fixed interest rate, which is slightly higher than the target overnight interest rate. Commercial banks that clear transactions through the central bank also have the right to deposit excess settlement cash overnight with the central bank at a deposit rate, which is positive but slightly lower than the target overnight rate (see Woodford 2000).

[5] This restriction eliminates the possibility for arbitrage where agents borrow and subsequently make a deposit at interest $i_d > i$, thus increasing their money holdings at no cost.

agents can borrow or deposit money. The standing facility closes and agents trade goods. In the second market agents trade goods, then financial claims are settled.

3 Symmetric stationary equilibrium

In period t, the real price of money is ϕ in the second market. We study stationary equilibria where end-of-period real money balances are time-invariant, i.e.,

$$\phi M = \phi_{-1} M_{-1}. \tag{2}$$

We will denote the growth rate of money by $\gamma = M/M_{-1}$. In a stationary equilibrium γ is time invariant.

We let $V(m_1)$ denote the expected value from entering market 1 with m_1 money balances. $W(m_2, \ell, d)$ denotes the expected value of entering the second market with m_2 units of money, ℓ loans, and d deposits. In what follows we look at a representative period t and work backward, from the second to the first market.

3.1 The second market

In the second market, the problem of a representative agent is:

$$\begin{aligned} W(m_2, \ell, d) \quad &= \quad \max_{x,h,m_{1,+1}} x - h + \beta V(m_{1,+1}) \\ &s.t. \quad x + \phi m_{1,+1} = h + \phi m_2 + \phi(1 + i_d)d - \phi(1 + i)\ell. \end{aligned}$$

Using the budget constraint to eliminate h in the objective function, one obtains the first-order condition

$$\beta V'(m_{1,+1}) = \phi \tag{3}$$

$V'(m_{1,+1})$ is the marginal value of taking an additional unit of money into period $t + 1$. Since the marginal disutility of working is one, $-\phi$ is the utility cost of acquiring one unit of money in the second market of period t. The implication of (3) is that all agents enter the following period with the same amount of money.

The envelope conditions are

$$\begin{aligned} W_m &= \phi \quad (4) \\ W_\ell &= -\phi(1+i) \quad (5) \\ W_d &= \phi(1+i_d). \quad (6) \end{aligned}$$

As in Lagos-Wright (2005) the value function is linear in wealth.

3.2 The first market

Let q_b and q_s respectively denote the quantities consumed by a buyer and produced by a seller trading in market 1. Let p be the nominal price of goods in market 1. It is obvious that buyers will never deposit funds in the bank and sellers will never take out loans. It is also straightforward to show that sellers will deposit all their money balances if $i_d > 0$. If $i_d = 0$, they are indifferent since they earn no money and we assume that they also deposit their money in this case.

An agent who has m_1 money at the opening of the first market has expected lifetime utility

$$V(m_1) = (1-n)[u(q_b) + W(m_1 - pq_b + \ell, \ell, 0)] + n[-q_s + W(m_1 + pq_s - d, 0, d)]$$

where q_b, q_s, ℓ and d are chosen optimally as follows.

A seller's problem is $\max_{q_s}\left[-q_s + W(pq_s, 0, d)\right]$. Using (4), the first-order conditions reduce to

$$p\phi = 1. \quad (7)$$

Note that sellers cannot deposit receipts of cash obtained from selling output.[6]

If an agent is a **buyer**, he solves the following maximization problem:

$$\max_{q_b,\ell} \quad u(q_b) + W(m_1 - pq_b + \ell, \ell, 0)$$
$$\text{s.t.} \quad pq_b \leq m_1 + \ell \text{ and } \ell \leq \bar{\ell}$$

Notice that buyers can spend more cash than what they bring into the first market since they can borrow cash to supplement their money holdings at the cost of the nominal interest rate. He also faces the constraint that the loan size is constrained by $\bar{\ell}$. This constraint captures the fact that there is a threshold $\bar{\ell}$ above which an agent would choose to default. We will see that the equilibrium threshhold is a function of the borrowing and lending rates i and i_d. Buyers however take this threshhold as given.

Using (4) the buyer's first-order conditions can be written as

$$u'(q_b) = p(\phi + \lambda_q) \tag{8}$$

$$\lambda_q = \lambda_\ell + \phi i \tag{9}$$

where λ_q is the multiplier of the buyer's budget constraint and λ_ℓ the one of the borrowing constraint. Equations (7), (8) and (9) imply that

$$u'(q_b) = 1 + i + \lambda_\ell/\phi \tag{10}$$

If the borrowing constraint is not binding, $\lambda_\ell = 0$, and so trades are efficient if the central bank sets $i = 0$. If the borrowing constraint is binding, then $u'(q_b) > 1 + i$ which means trades are inefficient even when $i = 0$. The buyer spends all of his money, i.e., $pq_b = M_{-1} + \ell$.

To derive the endogenous money growth rate we have to derive the marginal value of money at the beginning of a period. In the Appendix we show that it

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[6] This is a form of limited participation in financial markets.

is given by

$$V'(m_1) = (1-n)u'(q_b)/p + n(1+i_d)\phi \tag{11}$$

According to this equation an agent with additional unit of money becomes a buyer with probability $1-n$ in which case he acquires $1/p$ units of goods yielding utility $u'(q_b)$. With probability n he becomes a seller in which case he deposits the unit of money yielding the nominal return $1+i_d$. Note that the standing facility increases the marginal value of money because agents can earn interest on idle money as opposed to the case where there is no standing facility, i.e., $i_d = 0$.

Use the first-order condition (3) lagged by one period to rewrite the marginal value of money as $V'(m_1) = \phi_{-1}/\beta$. Then replace this value in (11) to obtain

$$\frac{\phi_{-1}}{\phi\beta} = (1-n)u'(q_b) + n(1+i_d). \tag{12}$$

This gives us an equation which determines the endogenous money growth γ:

$$\frac{\gamma-\beta}{\beta} = (1-n)[u'(q_b)-1] + ni_d. \tag{13}$$

Finally, we want to determine the price of goods traded on the first market. To do so, we rewrite (1) as follows

$$\frac{\phi_{-1}}{\phi} = 1 + (1-n)i + ni_d - i(1-n)\frac{pq_b}{M_{-1}}. \tag{14}$$

To derive this equation we use $d = m_1$ and the buyer's budget constraint $\ell = pq_b - m_1$. Also we set $m_1 = M_1$ since all agents hold identical amounts of money when they enter the first market. Furthermore in a stationary equilibrium, $M/M_{-1} = \phi_{-1}/\phi$.

Then, equating (12) and (14) we obtain expressions for real money holdings $z_m = m_1/p$ and real borrowing $z_\ell = \ell/p$. Real money holdings satisfy

$$\frac{z_m}{q_b} = \frac{(1-n)i}{n(1-\beta)(1+i_d) + (1-n)\left[1+i-\beta u'(q_b)\right]}. \tag{15}$$

Note that $S(i, i_d) \equiv \frac{z_m}{q_b}$ is the savings rate, i.e., the ratio of non-borrowed to total spending. Holding q_b constant, the saving rate is decreasing in i_d and increasing in i.

Real borrowing satisfies

$$\frac{z_\ell}{q_b} = \frac{n(1-\beta)(1+i_d) + (1-n)\left[1 - \beta u'(q_b)\right]}{n(1-\beta)(1+i_d) + (1-n)\left[1 + i - \beta u'(q_b)\right]}$$

where $B(i, i_d) = z_\ell / q_b$ is the borrowing rate, which is decreasing in i and increasing in i_d for a given q_b. In any stationary equilibrium both rates are constant since q_b is constant.

4 Perfect enforcement

In this section, we characterize the equilibrium when the central bank has the technology to enforce repayment of loans. In this case, $\bar{\ell} = +\infty$ and so $\lambda_\ell = 0$. Accordingly, from (10), we have

$$u'(q_b) = 1 + i \tag{16}$$

which implies that the endogenous rate of money growth satisfies

$$\frac{\gamma - \beta}{\beta} = (1-n)i + ni_d \tag{17}$$

Moreover, from (15) the saving rate satisfies

$$\frac{z_m}{q_b} = \frac{(1-n)i}{(1-\beta)\left[n(1+i_d) + (1-n)(1+i)\right]}, \tag{18}$$

and the borrowing rate

$$\frac{z_\ell}{q_b} = 1 - \frac{z_m}{q_b}. \tag{19}$$

We can use these four equations to define a perfect enforcement equilibrium. They determine recursively the endogenous variables $(\gamma, q_b, z_m, z_\ell)$. The first two equations yield q_b and γ. Once we have solved for q_b equation (18) yields

z_m and (19) z_ℓ. Note that all other endogenous variables can be derived from these equilibrium values.

Definition 1 *An enforcement equilibrium with credit is a list* $(\gamma, q_b, z_m, z_\ell)$ *satisfying (16)-(19) with* $z_\ell, z_m > 0$.

Proposition 1 *For any* (i, i_d) *with* $i \geq i_d \geq 0$ *there exists a critical value*

$$\bar{i}(i_d) = \frac{(1-\beta)(1+ni_d)}{\beta(1-n)}$$

such that if $0 < i < \bar{i}(i_d)$*, a unique enforcement equilibrium with credit exists.*

The critical elements to verify in the proof is whether $z_\ell, z_m > 0$. The first inequality holds if $i < \bar{i}(i_d)$. It requires that the interest rate is not too high, otherwise borrowing is too costly and buyers do not use the standing facility to borrow. The second inequality holds if $i > 0$.

An interesting aspect of the equilibrium is that, from (16), changing i_d does not affect the allocation. Nevertheless it affects borrowing. To see this, assume $i > i_d$ and consider a small increase in i_d. From (17), the rate of inflation increases and so, from (18), the ratio of non-borrowed spending to total spending decreases. Thus, by increasing the rate of inflation, increasing i_d reduces the incentives of agents to hold money across periods.[7] However, increasing i_d also increases the incentive to acquire money because the higher expected rate of interest on idle money.

We now derive the optimal policy. The central bank's objective is to maximize the welfare of the representative agent. It does so by choosing the quantities consumed and produced in each market subject to market clearing. The policy is implemented by choosing the interest rates i and i_d. The expected lifetime

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[7] This is in sharp contrast to Berentsen et. al (2004) where an increase in the deposit ratio is always welfare increasing.

utility of the representative agent at the beginning of period t is given by

$$(1-\beta)\,V=(1-n)\left[u\left(q_b\right)-q_b\right] \tag{20}$$

It is obvious that the first-best quantity satisfies $q_b = q^*$ where q^* is the value of q_b that solves $u'(q_b) = 1$.

Suppose first that the central bank is constraint to set i_d equal to some strictly positive value. What band should the channel have? Given i_d the optimal standing facility involves to choose $i = i_d$. This result follows immediately from equation (16) where q_b is decreasing in i. With a zero band we find the following results. First, from (18), the saving ratio is increasing in i since borrowing becomes more costly as i increases. Second, since the saving ratio is increasing in i it is also increasing in the gross rate of inflation γ. The intuition is that both z_m and q_b are decreasing in γ but q_b is decreasing faster than z_m.

We define the allocation that is attained under the Friedman rule as the limiting allocation that is attained when $i \rightarrow 0$.

Proposition 2 *With perfect enforcement, the Friedman rule* $i \rightarrow 0$ *implements the first-best allocation* q^**. The price level approaches infinity.*

The proof of the first part is an immediate consequence of equation (16). To understand why the price level approaches infinity under the Friedman rule note that if $i \rightarrow 0$, then borrowing becomes costless. In contrast, because of discounting holding money across periods is still costly. Consequently, the demand for money approaches zero. To encourage agents to hold the stock of money its price must approach zero, i.e. $\phi \rightarrow 0$. This immediately implies that $p \rightarrow +\infty$ and therefore $z_m = M_{-1}/p \rightarrow 0$.

5 Imperfect enforcement

We now analyze the case when the central bank cannot enforce repayment of loans. Since production is costly, those agents who borrow in market 1 have an incentive to default in market 2. To offset this short-run benefit we assume that if an agent defaults on his loan then the only punishment is permanent exclusion from the standing facility. For credit to exist, it must be the case that borrowers prefer repaying loans to being banished from using the standing facility. Given this punishment, the borrowing constraint $\bar{\ell}$ is endogenous and we need to derive conditions to ensure voluntary repayment.

For buyers entering the second market with no money and who repay their loans, the expected discounted utility in a steady state is

$$\mathcal{U} = -h_b + \beta V\left(m_{1,+1}\right)$$

where h_b is a buyer's production in the second market if he repays his loan.

Consider the case of a buyer who reneges on his loan. The benefit of reneging is that he has more leisure in the second market because he does not work to repay the loan. The cost is that he is excluded from the standing facility. He cannot deposit because the central bank would confiscate his deposits to settle his loan arrears. Thus, a deviating buyer's expected discounted utility is

$$\widehat{\mathcal{U}} = -\widehat{h}_b + \beta \widehat{V}\left(\hat{m}_{1,+1}\right)$$

where the hat indicates the optimal choice by a deviator. The value of using the standing facility $\mathcal{U}$ as well as the expected discounted utility of defection $\widehat{\mathcal{U}}$ depend on the interest rates i and i_d. This puts constraints on the rates that the central bank can chose.

In the Appendix we show that consumption of a defector satisfies

$$(\gamma - \beta)/\beta = (1 - n)[u'(\hat{q}_b) - 1] \tag{21}$$

It is interesting to compare the defector's consumption $\hat{q}_b$ with equilibrium consumption q_b. Using (13) we find that

$$(1-n)[u'(\hat{q}_b) - u'(q_b)] = ni_d$$

Evidently, $\hat{q}_b \leq q_b$ with strict inequality if $i_d > 0$. The crucial result is that the difference in consumption is due to the fact that sellers can earn interest on idle cash. Thus increasing i_d makes it more attractive to use the standing facility. In particular, it also makes it more likely that buyers are willing to repay their loans.

The borrowing constraint $\bar{\ell}$ is derived as in Berentsen et al. (2004). It is defined to be the maximal loan size that a buyer is willing to repay in a candidate equilibrium where all buyers borrow $\ell \leq \bar{\ell}$. At the beginning of the second market, the expected discounted utility for this agent is $\mathcal{U}(\bar{\ell})$.[8] From the definition of $\bar{\ell}$, we have $\mathcal{U}(\bar{\ell}) = \hat{\mathcal{U}}$. Accordingly, $\mathcal{U} \geq \mathcal{U}(\bar{\ell}) = \widehat{\mathcal{U}}$, is a necessary condition for an unconstrained equilibrium.

In the Appendix we prove that the borrowing constraint satisfies

$$(1+i)z_{\bar{\ell}} = \gamma z_\ell + \gamma(\hat{q}_b - q_b) + \frac{\beta}{1-\beta}[(1-n)\Psi + (\gamma-1)\hat{q}_b] \tag{22}$$

where $\Psi = u(q_b) - u(\widehat{q}_b) - (q_b - \widehat{q}_b) \geq 0$ and $z_{\bar{\ell}} = \phi\bar{\ell}$.

Definition 2 *An unconstrained equilibrium with credit is a list* $(\gamma, q_b, \hat{q}_b, z_m, z_\ell, z_{\bar{\ell}})$ *that satisfies (16)-(19), (21) and (22) with* $0 < z_\ell < z_{\bar{\ell}}$ *and* $0 < z_m$.

In a constrained equilibrium we have $z_\ell = z_{\bar{\ell}}$. Accordingly, the borrowing constraint binds and so $\lambda_\ell > 0$. This implies from (8) that $u'(q_b) > 1 + i$, which

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[8] To derive the expected discounted utility of an agent who chooses $\bar{\ell}$ we consider the following one-step deviation. We consider an equilibrium where all buyers choose ℓ, and then study a deviation by one buyer who borrows $\bar{\ell}$ in one period and then returns to the equilibrium ℓ later on.

implies that the endogenous rate of money growth satisfies

$$\frac{\gamma - \beta}{\beta} = (1-n)\left[u'(q_b) - 1\right] + n i_d \tag{23}$$

Moreover, from (15), z_m satisfies

$$\frac{z_m}{q_b} = \frac{(1-n)i}{n(1-\beta)(1+i_d) + (1-n)\left[1 + i - \beta u'(q_b)\right]} \tag{24}$$

Then, in equilibrium from the budget constraint of the buyers real loans z_ℓ satisfy

$$\frac{z_\ell}{q_b} = 1 - \frac{z_m}{q_b} \tag{25}$$

Definition 3 *A constrained equilibrium is a list* $(\gamma, q_b, \hat{q}_b, z_m, z_\ell, z_{\bar{\ell}})$ *that satisfies (21)-(25) with* $z_\ell = z_{\bar{\ell}} > 0$ *and* $z_m > 0$*.*

5.1 Optimal policy

In this section we characterize the optimal policy when the central bank cannot force repayment of loans. The optimal policy involves a zero channel $i = i_d$ and a strictly positive rate implying that the Friedman rule does not hold.

The first thing to note is that if constrained and unconstrained equilibria with credit coexist, welfare is higher in the unconstrained equilibrium. To see this let q^u be the allocation in an unconstrained equilibrium and q^c the one in a constrained equilibrium. Then, from (10) for any $i > 0$ we have $q^* > q^u > q^c$ since $u'(q^u) = 1 + i < u'(q^c)$. Note that this result is also true for any i_d with $i_d \leq i$ since welfare in the unconstrained equilibrium is independent of i_d.

We now show that a necessary condition for an optimal policy is that $i = i_d$.

Lemma 3 *Monetary policy is welfare maximizing only if* $i = i_d$*.*

Proof of Lemma 3. A constrained equilibrium satisfies the five equations below.

$$
\begin{aligned}
[(1+i)-\gamma]z_{\bar{\ell}} - \gamma(\hat{q}-q) - \frac{\beta}{1-\beta}[(1-n)\Psi + (\gamma-1)\hat{q}] &= 0 \qquad (26)\\
u'(q)\beta(1-n) - (\gamma-\beta n) + \beta n i_d &= 0 \qquad (27)\\
u'(\hat{q})\beta(1-n) - (\gamma-\beta n) &= 0 \qquad (28)\\
q(1-n)i - [n(1+i_d) + (1-n)(1+i) - \gamma]z_m &= 0 \qquad (29)\\
q - z_{\bar{\ell}} - z_m &= 0 \qquad (30)
\end{aligned}
$$

Using (30) we can replace $z_{\bar{\ell}}$ in (26). Arranging terms, we get the following system of 4 equations, in four unknowns:

$$
\begin{aligned}
(1+i)q - (1+i-\gamma)z_m + \frac{(\beta-\gamma)\hat{q} - \beta(1-n)\Psi}{1-\beta} &= 0 \qquad (31)\\
u'(q)\beta(1-n) - (\gamma-\beta n) + \beta n i_d &= 0 \qquad (32)\\
u'(\hat{q})\beta(1-n) - (\gamma-\beta n) &= 0 \qquad (33)\\
q(1-n)i - [n(1+i_d) + (1-n)(1+i) - \gamma]z_m &= 0 \qquad (34)
\end{aligned}
$$

Using (34), we can now get rid of z_m in equation (31) to get a system of 3 equations in 3 unknowns $(q, \hat{q}, \gamma)$:

$$
\begin{aligned}
q\left[1+i - \frac{(1+i-\gamma)\,(1-n)i}{n(1+i_d)+(1-n)(1+i)-\gamma}\right] + \frac{(\beta-\gamma)\hat{q} - \beta(1-n)\Psi}{1-\beta} &= 0 \qquad (35)\\
u'(q)\beta(1-n) - (\gamma-\beta n) + \beta n i_d &= 0 \qquad (36)\\
u'(\hat{q})\beta(1-n) - (\gamma-\beta n) &= 0 \qquad (37)
\end{aligned}
$$

We can now use (37) to eliminate γ. We then have a system of 2 equations in 2 unknowns q and $\hat{q}$.

$$
\begin{aligned}
qY + \frac{\beta(1-n)}{1-\beta}\left\{\hat{q}\,[1-u'(\hat{q})] - \Psi\right\} &= 0 \qquad (38)\\
[u'(q) - u'(\hat{q})](1-n) + n i_d &= 0 \qquad (39)
\end{aligned}
$$

where

$$Y = 1 + i - \frac{[1 + i - \beta n - u'(\hat{q})\beta(1-n)]\,(1-n)i}{(1+i)(1-n) + (1+i_d)n - \beta n - u'(\hat{q})\beta(1-n)} \geq 1.$$

Using the implicit function theorem, we obtain for a given i_d, that dq/di and $d\hat{q}/di$ satisfy

$$\frac{dq}{di}\left[Y - \frac{\beta(1-n)}{(1-\beta)}[u'(q) - 1]\right] + \frac{d\hat{q}}{di}\left[q\frac{\partial Y}{\partial \hat{q}} - \hat{q}u''(\hat{q})\frac{\beta(1-n)}{(1-\beta)}\right] = -q\frac{\partial Y}{\partial i}$$

$$u''(q)\frac{dq}{di} = u''(\hat{q})\frac{d\hat{q}}{di}$$

Using the second equality to get rid off $d\hat{q}/di$ in the first equality we get

$$\frac{dq}{di}\left[Y + \frac{u''(q)}{u''(\hat{q})}q\frac{\partial Y}{\partial \hat{q}} - \frac{\beta(1-n)}{(1-\beta)}\left[u'(q) - 1 + u''(q)\hat{q}\right]\right] = -q\frac{\partial Y}{\partial i}$$

One can verify that

$$\frac{\partial Y}{\partial \hat{q}} = \frac{i(1-n)\beta(1-n)n(i-i_d)[-u''(\hat{q})]}{[(1+i)(1-n) + (1+i_d)n - \beta n - u'(\hat{q})\beta(1-n)]^2} > 0.$$

and

$$\frac{\partial Y}{\partial i} = \frac{n\,[1 + i_d - \beta n - \beta(1-n)u'(\hat{q})]\,[1 + ni_d - \beta n - \beta(1-n)u'(\hat{q})]}{[(1+i)(1-n) + (1+i_d)n - \beta n - u'(\hat{q})\beta(1-n)]^2}.$$

This can be expressed in terms of γ since $\gamma = \beta(1-n)u'(\hat{q}) + \beta n$, and we obtain

$$\frac{\partial Y}{\partial i} = \frac{n\,[1 + i_d - \gamma]\,[1 + ni_d - \gamma]}{[(1+i)(1-n) + (1+i_d)n - \gamma]^2} > 0.$$

From (14) we have $\gamma = 1 + ni_d + (1-n)i(1 - pq/M_{-1})$. Hence, $\gamma < 1 + ni_d$ (since $\bar{\ell}/M_{-1} > 0$) and so $\partial Y/\partial i > 0$.

Finally, a sufficient condition for $\frac{dq}{di} < 0$ is $u'(q)+u''(q)\hat{q} \leq 1$. Use (39) to get $u'(\hat{q})+u''(q)\hat{q} \leq 1+ni_d/\,(1-n)$ and $u''(q)\frac{dq}{di} = u''(\hat{q})\frac{d\hat{q}}{di}$ to get $u'(\hat{q})+u''(\hat{q})\hat{q}\frac{d\hat{q}}{dq} \leq 1 + ni_d/\,(1-n)$. Since the coefficient of relative risk aversion is greater than one, we have $u'(\hat{q}) + u''(\hat{q})\hat{q} < 0$ for all $\hat{q}$. Hence, a sufficient condition is $u''(\hat{q})\hat{q}\left(\frac{d\hat{q}}{dq} - 1\right) \leq 1 + ni_d/\,(1-n)$, respectively $\frac{d\hat{q}}{dq} = \frac{u''(q)}{u''(\hat{q})} > 1$. Therefore, a sufficient condition for the desired result is that $u'(.)$ is convex. ■

According to Lemma 3, if the central bank is constraint to set i_d to some particular value, it is optimal to choose the lending rate equal to the deposit rate.

We now state our main result.

Proposition 4 *Let β be large enough and $i = i_d$. Then, welfare is increasing in i in a constrained equilibrium and decreasing in i in an unconstrained equilibrium. The optimal value is $i = \underline{i}$, where $\underline{i}$ is defined in the proof.*

Proof of Proposition 4. We first show that if $i = i_d$, welfare is increasing in i in a constrained equilibrium and decreasing in i in an unconstrained equilibrium. In a constrained equilibrium, q and $\hat{q}$ solve

$$\begin{aligned} qY + \frac{\beta(1-n)}{1-\beta}\left\{u(\hat{q}) - \hat{q}u'(\hat{q}) - [u(q) - q]\right\} &= 0 \\ [u'(q) - u'(\hat{q})](1-n) + ni_d &= 0 \end{aligned}$$

where

$$Y = 1 + i - (1-n)i\frac{1 + i - \beta n - u'(\hat{q})\beta(1-n)}{(1+i)(1-n) + (1+i_d)n - \beta n - u'(\hat{q})\beta(1-n)}$$

If $i = i_d$, $Y = 1 + ni$ and so

$$q(1+ni) + \frac{\beta(1-n)}{1-\beta}\left\{u(\hat{q}) - \hat{q}u'(\hat{q}) - [u(q) - q]\right\} = 0 \tag{40}$$

$$u'(\hat{q}) - u'(q) = \frac{ni}{1-n} \tag{41}$$

Totally differentiate (40) and (41) to get the following expression for dq/di

$$(1-\beta)q - \beta\hat{q} = \frac{dq}{di}\frac{\beta(1-n)}{n}\left[u'(q) - 1 + u''(q)\hat{q} - \frac{(1-\beta)}{\beta(1-n)}(1+ni)\right]$$

In the proof of Lemma 3 we have shown that $u'(q) + u''(q)\hat{q} < 1$. Consequently, the term in square bracket on the right-hand side is negative. Also, for β sufficiently large, the term on the left-hand side is negative since $\hat{q}$ is bounded away from zero. Hence, $dq/di > 0$ and so welfare in a constrained equilibrium is increasing in i.

It is straightforward to show that welfare is decreasing in i in an unconstrained equilibrium. Accordingly, welfare is maximized at $i = i_d = \underline{i}$. ■

According to Proposition 4, the optimal interest rate is the one where the credit constraint starts to bind.

5.2 Existence

We now prove the existence of an equilibrium with credit. As in the case with perfect enforcement, the existence of credit requires that the interest rate on loans is not too high, i.e. $i \leq \bar{i}$. Above this interest rate, there is no credit, although a monetary equilibrium will still exist. We first prove the existence of an unconstrained equilibrium. To do this, note that given i_d, setting i as low as possible maximizes welfare. But since $i \geq i_d$, this implies $i = i_d$. Hence, we consider (for simplicity) only the existence of these unconstrained equilibria. By continuity, unconstrained equilibrium with $i_d < i$ for i sufficiently close to i_d will also exist.

Lemma 5 *Assume $i_d = i$. An unconstrained equilibrium exists if $\underline{i} \leq i \leq \bar{i}$.*

Proof of Lemma 5. In an unconstrained equilibrium $z_\ell < z_{\bar{\ell}}$, which from (22) implies that

$$z_\ell < \frac{1}{[(1+i)-\gamma]}\left[\gamma(\hat{q}-q)+\frac{\beta}{1-\beta}[(1-n)\Psi+(\gamma-1)\hat{q}]\right] \tag{42}$$

In an unconstrained equilibrium we have $\gamma = \beta\,(1+i)$, $z_\ell = \frac{q[(1+i)(1-\beta)-(1-n)i]}{(1+i)(1-\beta)}$, and $u'(\hat{q}) = 1 + \frac{i}{1-n}$. Using this information we get

$$[u(q)-qu'(q)]-[u\,(\hat{q})-\hat{q}u'(\hat{q})] \geq q\frac{(1-\beta)}{\beta(1-n)}\left[1-\frac{\beta(1-n)}{(1-\beta)}u'(q)+in+\frac{\beta(1-n)}{(1-\beta)}\right].$$

Arranging terms and using the fact that $u'(q) = 1 + i$, an unconstrained equilibrium exists if

$$\Theta(i) \equiv [u(q) - qu'(q)] - [u(\hat{q}) - \hat{q}u'(\hat{q})] - q\frac{1 - \beta + i(n - \beta)}{\beta(1 - n)} \geq 0$$

Then, $\underline{i}$ is the value of $\underline{i}$ such that the above equation holds with equality. Such a value exists, since $\Theta(i)$ is strictly negative at $i = 0$ and, since $\beta \geq n$ and since $u(q) - qu'(q)$ is increasing in q, it is strictly positive at $i = \bar{i}$ (as defined for the existence of a perfect enforcement equilibrium). The value is unique if $\Theta'(i) \geq 0$, i.e., if

$$-qu''(q)dq + \hat{q}u''(\hat{q})d\hat{q} - \frac{1 - \beta + i(n - \beta)}{\beta(1 - n)}dq - \frac{q(n - \beta)}{\beta(1 - n)}di \geq 0$$

Using the fact that $u''(q)dq = di$ and $u''(\hat{q})d\hat{q} = di/(1 - n)$ we get

$$\begin{aligned} -q + \frac{\hat{q}}{1 - n} - \frac{1 - \beta + i(n - \beta)}{\beta(1 - n)u''(q)} - \frac{q(n - \beta)}{\beta(1 - n)} &\geq 0 \\ -q\left[\frac{n(1 - \beta)}{\beta(1 - n)}\right] + \frac{\hat{q}}{1 - n} - \frac{1 - \beta + i(n - \beta)}{\beta(1 - n)u''(q)} &\geq 0 \\ \beta\hat{q} - qn(1 - \beta) - \frac{1 - \beta + i(n - \beta)}{u''(q)} &\geq 0 \end{aligned}$$

which holds if β is large enough. ■

We now prove the existence of a constrained equilibrium with credit, in the case $i = i_d$, for interest rates $i \leq \underline{i}$. We proceed in two steps. First we derive a sufficient condition for the existence of a constrained equilibrium. Then we show that this condition is satisfied when $i \leq \underline{i}$, i.e., when an unconstrained equilibrium does not exist.

Lemma 6 *Assume $i = i_d$. Let $\tilde{q}^*$ and q^* be defined by $u'(\tilde{q}^*) = 1 + ni/(1 - n)$ and $u'(q^*) = 1$. A constrained equilibrium exists if*

$$u(q^*) - u(\tilde{q}^*) \leq q^*\left[\frac{(1 - \beta)}{\beta(1 - n)}(1 + ni) + 1\right] - \tilde{q}^*\left[1 + \frac{ni}{(1 - n)}\right]. \qquad (43)$$

Proof of Lemma 6. A constrained equilibrium is characterized by equations (40) and (41). It is easy to show that there exists a $\bar{q}$ such that when $q = \hat{q} = \bar{q}$ equation (40) is satisfied. Also, we know that the locus defined by (41) is always below the 45 degree line. Hence, if the solution (say $\hat{q}^*$) for $\hat{q}$ of (40) evaluated at $q = q^*$ is lower than the solution (say $\tilde{q}^*$) of $\hat{q}$ of (41) also evaluated at $q = q^*$, then we know by continuity that there is a crossing point between the two loci defined by (40) and (41). In such case, a constrained equilibrium exists.

Now, from (40) evaluated at $q = q^*$, $\hat{q}^*$ solves

$$q^* \left[1 + ni\right] - \frac{\beta(1-n)}{1-\beta}[u(q^*) - q^*] = \frac{\beta(1-n)}{1-\beta}[\hat{q}^* u'(\hat{q}^*) - u(\hat{q}^*)].$$

By concavity, the right hand side is decreasing in $\hat{q}^*$. Hence if $\hat{q}^* u'(\hat{q}^*) - u(\hat{q}^*) \geq \tilde{q}^* u'(\tilde{q}^*) - u(\tilde{q}^*)$ then $\tilde{q}^* \geq \hat{q}^*$. In other words, a sufficient condition for the existence of a constrained equilibrium is

$$q^* \left[1 + ni\right] - \frac{\beta(1-n)}{1-\beta}[u(q^*) - q^*] \geq \frac{\beta(1-n)}{1-\beta}[\tilde{q}^* u'(\tilde{q}^*) - u(\tilde{q}^*)].$$

Arranging terms and using the expression for $u'(\tilde{q}^*)$, we get

$$u(q^*) - u(\tilde{q}^*) \leq q^* \frac{(1-\beta)}{\beta(1-n)} \left[\frac{(1-\beta)(1+ni) + \beta(1-n)}{(1-\beta)}\right] - \tilde{q}^* u'(\tilde{q}^*)$$

which is the required condition. ■

We are now in a position to prove that a constrained equilibrium exists in the region of parameters where an unconstrained equilibrium does not.

Lemma 7 *Assume* $i = i_d$. *Suppose* $i \leq \underline{i}$ *then a constrained equilibrium exists.*

Proof of Lemma 7. An unconstrained equilibrium exists if

$$z_\ell < \frac{1}{[(1+i) - \gamma]} \left[\gamma(\hat{q} - q) + \frac{\beta}{(1-\beta)}[(1-n)\Psi + (\gamma - 1)\hat{q}]\right]$$

In an unconstrained equilibrium as defined by (16)-(19) and (21), $\gamma = \beta(1+i)$, so that $(1+i) > \gamma$. Therefore - using (16)-(19) and (21) to simplify - an unconstrained equilibrium exists only if

$$u(q) - u(\hat{q}) \geq q\frac{(1-\beta)}{\beta(1-n)}\left[1 + ni + \frac{\beta(1-n)}{1-\beta}\right] - \hat{q}u'(\hat{q}).$$

where $u'(q) = 1+i$ and $u'(\hat{q}) = 1+i/(1-n)$. Given the coefficient of relative risk aversion is greater than one, we also know that $\tilde{q}u'(\tilde{q}) < \hat{q}u'(\hat{q})$. Now, as before define $\tilde{q}$ such that $u'(\tilde{q}) = 1 + ni/(1-n)$. By concavity of the utility function and given $u'(q^*) - u'(\tilde{q}) = u'(q) - u'(\hat{q})$ we also have the following inequality for all i (the proof is relegated to the Appendix):

$$u(q) - u(\hat{q}) \geq u(q^*) - u(\tilde{q})$$

Hence, when $i \leq \underline{i}$, an unconstrained equilibrium does *not* exist and we have the following string of inequalities, given $q^* \geq q$,

$$\begin{aligned} & q^*\frac{(1-\beta)}{\beta(1-n)}\left[1 + ni + \frac{\beta(1-n)}{1-\beta}\right] - \tilde{q}u'(\tilde{q}) \\ \geq \; & q\frac{(1-\beta)}{\beta(1-n)}\left[1 + ni + \frac{\beta(1-n)}{1-\beta}\right] - \hat{q}u'(\hat{q}) \\ \geq \; & u(q) - u(\hat{q}) \geq u(q^*) - u(\tilde{q}). \end{aligned}$$

Together with the previous lemma, this proves the claim. ■

Appendix

Lemma 8 *The marginal value of money satisfies equation (11).*

Proof of Lemma 8. To determine the price of goods being traded on the first market, we need to determine money growth. Since we only consider stationary equilibrium, money growth is also the price ratio ϕ_{-1}/ϕ. But the price of money today must equal the discounted marginal value money. We therefore need to compute this marginal value of money. In equilibrium, it is:

$$\begin{aligned} V'(m_1) \;&=\; (1-n)[u'(q_b)dq_b/dm_1 + \phi(1 - pdq_b/dm_1 + d\ell/dm_1) \\ &\quad - (1+i)\,\phi d\ell/dm_1] \\ &\quad + n[-c'(q_s)dq_s/dm_1 + \phi(1 + pdq_s/dm_1 - dd/dm_1) \\ &\quad + W_d dd/dm_1] \end{aligned}$$

When we analyze an equilibrium with perfect enforcement of financial claims the marginal value of money has a simple expression, using again the first order condition for buyers and sellers and the envelope condition gives

$$\begin{aligned} V'(m_1) \;&=\; (1-n)[u'(q_b)dq_b/dm_1 + \phi(1 - pdq_b/dm_1 + d\ell/dm_1) \\ &\quad - \phi(1+i)d\ell/dm_1] + n[\phi(1+i_d)] \end{aligned}$$

where $dd/dm_1 = 1$ and $dq_s/dm_1 = 0$ has been used. Now, when $\ell > 0$, the budget constraint of buyers is binding so that $pdq_b/dm_1 = 1 + d\ell/dm_1$. Therefore,

$$V'(m_1) = (1-n)[u'(q_b)dq_b/dm_1 - \phi(1+i)d\ell/dm_1] + n[\phi(1+i_d)].$$

Using the expression for $d\ell/dm$, together with $u'(q_b) = p\phi(1+i) + p\lambda_\ell$ and $\lambda_\ell = 0$, we have

$$V'(m_1) = (1-n)u'(q_b)/p + n(1+i_d)\phi$$

Note that

$$\begin{aligned} u'(q)dq_b/dm_1 - \phi(1+i)d\ell/dm_1 &= u'(q_b)dq_b/dm_1 - \phi(1+i)[pdq_b/dm_1 - 1] \\ &= [u'(q_b) - (1+i)]dq_b/dm_1 + \phi(1+i) \\ &= \lambda_\ell dq_b/dm_1 + \phi(1+i). \end{aligned}$$

But the first term drops when $\lambda_\ell = 0$, while if $\lambda_\ell > 0$ we know $dq_b/dm_1 = 1$ (as we will see, $\bar{\ell}$ does not depend directly on m_1). Hence, we obtain $\lambda_\ell dq_b/dm_1 + \phi(1+i) = \lambda_\ell + \phi(1+i) = [u'(q_b) - (1+i)]/p + \phi(1+i)$. ■

Lemma 9 *The borrowing constraint satisfies*

$$(1+i)z_{\bar{\ell}} = \gamma z_\ell + \gamma(\hat{q} - q_b) + \frac{\beta}{1-\beta}[(1-n)\Psi + (\gamma - 1)\hat{q}]$$

where $\Psi = u(q_b) - u(\widehat{q_b}) - (q_b - \widehat{q_b}) \geq 0$ *and* $z_{\bar{\ell}} = \phi\bar{\ell}$.

Proof of Lemma 9. Recall that the borrowing constraint $\bar{\ell}$ satisfies $\mathcal{U}(\bar{\ell}) = \hat{\mathcal{U}}$. To derive $\mathcal{U}(\bar{\ell})$ we have to compute the hours worked of a buyer who took loan size $\bar{\ell}$ in an equilibrium where all other buyers take out loan $l \leq \bar{\ell}$. This is

$$\tilde{h}_b = x^* + \phi[m_{1,+1} + (1+i)\bar{\ell}] = x^* + \gamma q_b + \phi[(1+i)\bar{\ell} - \gamma\ell]$$

In the above derivation, we have replaced m_1 using the budget constraint of non-deviating buyer $m_1 = pq_b - \ell$.

Since we consider a one shot deviation, the deviator resumes normal behavior in future periods, so that her expected output in the second market is

$$h = nh_s + (1-n)h_b = x^*$$

Now from $\mathcal{U}(\bar{\ell}) = \hat{\mathcal{U}}$ we have

$$U(x^*) - \tilde{h}_b + \beta V(m_{1,+1}) = U(x^*) - \hat{h}_b + \beta\hat{V}(\hat{m}_{1,+1})$$

so that

$$\hat{h}_b - \tilde{h}_b = \beta[\hat{V}(\hat{m}_{1,+1}) - V(m_{1,+1})].$$

Replacing each expression for $\hat{h}_b$ and $\tilde{h}_b$, this implies

$$\gamma(\hat{q} - q_b) - \phi[(1+i)\bar{\ell} - \gamma\ell] = \beta[\hat{V}(\hat{m}_{1,+1}) - V(m_{1,+1})].$$

The difference in continuation payoff is given by

$$V(m_{1,+1}) - \hat{V}(\hat{m}_{1,+1}) = \frac{1}{1-\beta}\left\{(1-n)[u(q_b) - u(\hat{q})] + (1-n)(\hat{q} - q_b) + \hat{h} - h\right\}$$

Using the expressions for $\hat{h}$ and h, we obtain,

$$\begin{aligned}
\phi[(1+i)\bar{\ell} - \gamma\ell] &= \gamma(\hat{q} - q_b) + \beta[V(m_{1,+1}) - \hat{V}(\hat{m}_{1,+1})] \\
\phi[(1+i)\bar{\ell} - \gamma\ell] &= \gamma(\hat{q} - q_b) + \frac{\beta}{1-\beta}\{(1-n)[u(q_b) - u(\hat{q})] + [(\gamma-1)\hat{q} + (1-n)(\hat{q} - q_b)]\} \\
(1+i)z_{\bar{\ell}} &= \phi\gamma z_l + \gamma(\hat{q} - q_b) + \frac{\beta}{1-\beta}[(1-n)\Psi + (\gamma-1)\hat{q}]
\end{aligned}$$

where $\Psi \equiv u(q_b) - u(\hat{q}) - (q_b - \hat{q})$. Hence, $\bar{\ell}$ satisfies (22). ■

Lemma 10 *If u is concave, and $q^* > q > \hat{q}$, and $q^* > \tilde{q} > \hat{q}$ and $u'(q^*) - u'(\tilde{q}) = u'(q) - u'(\hat{q})$, then $u(q) - u(\hat{q}) > u(q^*) - u(\tilde{q})$.*

Proof of Lemma 10. Given the concavity of the utility function, we have the following inequality,

$$\begin{aligned}
&u(q) + u(\tilde{q}) - [u(q^*) + u(\hat{q})] \\
&> u(q^*) - u'(q)(q^* - q) + u(\hat{q}) - u'(\tilde{q})(\hat{q} - \tilde{q}) - u(q^*) - u(\hat{q}) \\
&= u'(\tilde{q})(\tilde{q} - \hat{q}) - u'(q)(q^* - q)
\end{aligned}$$

Also, we have

$$\begin{aligned}
&u(q^*) + u(\hat{q}) - [u(q) + u(\tilde{q})] \\
&> u(q) - u'(q^*)(q - q^*) + u(\tilde{q}) - u'(\hat{q})(\tilde{q} - \hat{q}) - u(q) - u(\tilde{q}) \\
&= u'(\hat{q})(\tilde{q} - \hat{q}) + u'(q^*)(q^* - q)
\end{aligned}$$

Combining both inequalities, we obtain

$$0 > [u'(\tilde{q} - u'(\hat{q})](\tilde{q} - \hat{q}) + [u'(q^*) - u'(q)](q^* - q)$$

But since $u'(q^*) - u'(\tilde{q}) = u'(q) - u'(\hat{q})$, we get that

$$q - \hat{q} \geq q^* - \tilde{q}$$

But from concavity and $q^* > q$ and $\tilde{q} > \hat{q}$, we know

$$\frac{u(q) - u(\hat{q})}{q - \hat{q}} > \frac{u(q^*) - u(\tilde{q})}{q^* - \tilde{q}}$$

Hence, it must be that $u(q) - u(\hat{q}) > u(q^*) - u(\tilde{q})$. ■

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