Liquidity and the Market for Ideas*

Rafael Silveira
Department of Economics
University of Pennsylvania

Randall Wright
Department of Economics
University of Pennsylvania

August 2, 2005

Abstract

We study models where innovators sell ideas, or projects, to entrepreneurs who may be better at implementing them. This happens in decentralized markets with random matching and bilateral bargaining. Entrepreneurs hold liquid assets (e.g. cash) lest potentially profitable opportunities may be lost. Contributing to the literature on innovation and technology transfer, we show how bargaining and liquidity costs (e.g. interest rates) determine which ideas get traded. Contributing to monetary theory, we generalize recent models to the case where agents exchange indivisible objects (these are ideas here) with random valuations, and we allow buyers with insufficient liquidity to try to put deals on hold until they can raise the cash.

*Authors' email: rafaels@econ.upenn.edu; rwright@econ.upenn.edu. We thank Boyan Jovanovic, Tom Holmes, and members of the Penn S&M (Search and Matching) Club for their input. We thank the NSF for financial support.

1 Introduction

We take it for granted people understand that the development and implementation of new ideas is one of the major factors underlying economic performance.[1] In this vein, the concept of *technology transfer* seems especially important to many people, including innovators and entrepreneurs looking to come up with and commercialize new technologies, and governments seeking to spur economic development. The issue is this: When innovators come up with new inventions or ideas or projects, should they try to implement them themselves, say through start-up firms? Or should they try to sell them, perhaps to established firms, or more generally to entrepreneurs who are better at implementing these ideas?

If agents are heterogeneous in their abilities to come up with ideas and to extract their returns, one can imagine that some will specialize in innovation while others will specialize in implementing or commercializing the innovations. A superior allocation of resources will generally emerge when those who have the ideas are not necessarily those who implement them. People in the "knowledge transactions field" share the view that the transfer of ideas from innovators to entrepreneurs leads to a more efficient use of resources, making all parties better off and increasing the incentives for

[1] Both the inputs to and outputs of this process are important. On the input side, research and development expenditures account for 3% of US GDP, and according to a survey by the Association of University Technology Managers, the licensing of innovations just by universities, hospitals, research institutions, and patent management firms added more than $40 billion to the economy in 1999 and supported 270,000 jobs. On the output side, it is obvious that new ideas and technologies are essential to production and growth, and going back to Schumpeter (1934) it is often said that the creation of new firms is a significant mechanism through which new technologies are implemented.

investments in research.[2] Obviously, however, this requires some mechanism – say, some market – for the exchange of ideas, and the details of how this mechanism works could in principle have a big impact on outcomes. This is the subject of the current study.

Our analysis is related to the well-known model of Holmes and Schmitz (1990), although we also deviate considerably from their approach. What we share with them is, in their words, the following: "The model has two key features. The first crucial assumption is that opportunities for developing new products repeatedly arise through time... The second key feature is that we assume that individuals differ in their abilities to develop emerging opportunities." Hence, "There are two tasks in the economy, developing products and producing products previously developed" (Holmes and Schmitz 1990, p. 266-7). Where we differ is the way we envision the market where ideas get traded. While they model it as a competitive equilibrium, we take seriously the notion that there are considerable frictions in this market.[3]

First, we think it is clear that there is really no centralized market for ideas – innovators cannot simply choose a quantity of new ideas to supply to maximize profit taking as given the competitive price, and entrepreneurs

[2] A common idea in this literature is that inventor-founded startups are often second-best solutions, since innovators do not have the entrepreneurial skills to commercialize new products. Of course, one could imagine innovators trying to buy implementation expertise from entrepreneurs, but the usual view is that such expertise is largely tacit and difficult to measure, so it seems more natural for ideas to be sold to entrepreneurs. See Teece et al. (1997), Pisano and Mang (1993), and Shane (2002).

[3] In a different paper, Holmes and Schmitz (1995) consider a model where the returns to a business depend on the quality of the project and the quality of the match between the project and manager, which is quite related to (it can be considered a special case of) what we assume below. However, in this analysis, they still use competitive equilibrium, with a frictionless centralized market and Walrasian price taking.

do not simply choose how many new ideas to buy at a given price. The idea market is obviously much more *decentralized*. Hence, we model it using search theory, with random bilateral matching and bargaining between innovators and entrepreneurs. Second, we take the position that *liquidity* may be critical in this market. To capture this, we make use of some recent results in search-based monetary economics.

To expand on this, when there are imperfect markets for the exchange of ideas, it is not only critical who you meet and what they know, there is also an issue of how to pay for ideas. The fact that someone may be better than you at implementing your project clearly means little if they have nothing to offer in exchange. This is especially important in highly informal or decentralized markets. It is easy to imagine reasons why you may be reluctant to give up an idea for a promise of future payment – in particular, once you give it up it is hard to take it back. Hence, it is easy to imagine reasons why quid pro quo is the order of the day: "You want my idea? Show me the money!"

Given this, entrepreneurs may choose to keep liquid assets, cash on hand being the purest example, in case they come across a potentially profitable opportunity that may be lost if there cannot be a quick agreement. Naturally, how much liquidity they choose to keep on hand depends on its cost, e.g. the nominal interest rate, as well as other factors, including anything that affects the willingness of innovators to sell their ideas and the willingness of entrepreneurs to invest in these opportunities. Our goal is to sort out the role of some of these factors, and hence sort out what determines how many and which opportunities get traded.

The view that financial constraints impinge on entrepreneurs is by no means new. For example, Evans and Jovanovic (1989) is a well-known paper arguing for the importance of what might be called liquidity or borrowing constraints. A large related literature on the decision to become an entrepreneur generally finds a positive relation between wealth and entrepreneurial activity, and interprets this as evidence that borrowing constraints matter.[4] We believe that our approach is consistent with this position. But this literature does not incorporate liquidity the way modern monetary economics does and the way we do here; it focuses more on credit market imperfections.[5] While these models may be consistent with the idea that, e.g., entrepreneurial activity requires high wealth or savings, they do not speak to issues about liquidity costs in the sense of, say, nominal interest rates.

Since we think of ideas being traded in decentralized markets, as we said, our framework is similar to work in the search and matching literature. We think that using this approach to study technology transfer is a neat application of search theory, and a natural way to look at the substantive issues. Since we allow liquidity to potentially play a prominent part, our framework is especially close to recent monetary search theories, which are all about the role of liquidity. In particular, our environment shares features

[4] See Evans and Leighton (1989), Holtz-Eakin et al. (1994), Fairlie (1999), Quadrini (1999), Gentry and Hubbard (2000), Lel and Udell (2002), Paulson and Townsend (2000), and Guiso, Sapienza and Zingales (2001).

[5] Some people simply assume there is no credit (Lloyd-Ellis and Bernhardt 2000), some assume credit is exogenously limited to a fixed multiple of wealth (Evans and Jovanovic 1989), some model it as the solution to a moral hazard problem (Aghion and Bolton 1996), and some use asymmetric information (Fazzari et al. 1988, 2000).

with the model of money in Lagos and Wright (2005), where sometimes agents trade in centralized markets and sometimes in highly decentralized markets. But while we borrow from that model, we also think we make a contribution to monetary theory, per se, independent of the application.

Thus, in addition to applying the search framework to technology transfer, we generalize it along a couple of dimensions that not only fit our application but are of independent interest. First, we extend it to the case where agents are trading indivisible objects (the ideas or projects) with random valuations. Second, we extend search models generally by allowing agents with insufficient liquidity to try to put deals on hold and raise the cash in the centralized market, something that may or may not be successful. In this way we capture both theories where our notion of liquidity is crucial, and those where it plays no role, as special cases. Indeed there is a parameter that captures the extent of liquidity constraints, and we discuss the model's predictions in terms of the empirical evidence.

The rest of the paper is organized as follows. Section 2 lays out our basic assumptions. Section 3 discusses our centralized market and Section 4 our decentralized market where ideas are traded. Section 5 puts things together to characterize equilibrium. Section 6 considers various extensions. Section 7 concludes. A few technical results are relegated to the Appendix.[6]

[6] We mention some other related work. Several studies consider the transferring of ideas as a strategic action among firms, including Katz and Shapiro (1986), Gallini and Winter (1985), and Shepard (1994). Others focus on licensing contracts in terms of incentives, including Aghion and Tirole (1994) and Arora (1995). There is also a literature that focuses on university inventions, including Lowe (2003), Shane (2002), and Jensen and Thursby (2001).

2 The Basic Model

Time is discrete and continues forever. As in Lagos and Wright (2005), we assume that alternating over time there are two distinct types of markets: a centralized market, denoted CM, where agents perform the usual activities of producing, consuming and adjusting their assets; and a decentralized market, denoted DM, where agents meet bilaterally, and potentially buy and sell ideas. Although traded in the DM, ideas are implemented in the CM. All agents have discount factor β between one DM and the next CM, and discount factor δ between one CM to the next DM, where $\delta\beta < 1$.[7] There are also two types of agents: innovators, who are relatively good at coming up with ideas; and entrepreneurs, who may be better at implementing them. For now the division of agents into these two types is exogenous.

Every time the DM opens, an innovator enters with some idea that has value $R_i \geq 0$ if he implements it himself in the next CM. We assume R_i is randomly distributed with CDF $F_i(\cdot)$, but its value is known when one enters the market. To keep things simple, if an idea is not implemented in one period, its value will be an independent draw from F_i next period. Hence there is no reason not to implement an idea in any period, since you will get a new one anyway. If an innovator meets an entrepreneur in the DM, and the former has an idea which has value R_i to him, it has value $R_e \geq 0$ to the latter. We assume R_e is randomly drawn from $F_e(\cdot|R_i)$, but

[7] If either β or δ is 1, e.g., we can interpret both markets convening within the same period; or if $\beta = \delta < 1$, e.g., we can interpret them as convening in alternate periods of equal length.

it is known to the entrepreneur in the meeting; i.e. he can see the value of an idea, though he cannot implement without the innovator giving him the details. When convenient we may assume that F_j' exists and is continuous, and sometimes that the supports have a finite upper bound $\overline{R}$, but this is only for ease of presentation.[8]

When $R_e > R_i$ in a particular meeting, the entrepreneur has a better capacity or ability to implement the project. Hence there are gains from trade. Note that an innovator will never prefer to not trade an idea for speculative reasons – i.e. in hope of meeting an entrepreneur in the future with an even bigger R_e – since he will get a new idea anyway and the value of any one idea is independent across periods (this is the same as the reason he would always implement an idea in the CM). An idea is assumed to be indivisible – either I tell you or I don't. The price at which an idea is traded will be determined by bargaining. This price is in terms of money, by which mean some liquid asset the entrepreneur has on hand. While we call it money, we do not necessarily mean cash per se, but relatively liquid assets more generally; it could include e.g. deposits in one's checking account.[9]

If the price at which they would otherwise trade is greater than the amount of money m that entrepreneur has on hand, several things can hap-

[8] A special case is where $R_i = \bar{R}_i$ with probability 1, including the case $\bar{R}_i = 0$ where innovators are purely "idea men" who cannot implement anything. Another case of interest is where R_i and R_e are independent, so the value of the project to any individual is purely idiosyncratic. Another case is where R_i is interpreted as the random return to the project and R_e as this plus the quality of the match with any particular entrepreneur, much like in Holmes and Schmitz (1995).

[9] He, Huang and Wright (2005) explicitly introduces banks and checking accounts into an otherwise standard search model of monetary exchange.

pen. The innovator could walk away, keeping the idea for himself; they could settle on exchanging the idea for a lower price; or they could agree to try to meet again in the next CM, where the entrepreneur can raise the funds. However, with probability γ a meeting in the next CM fails to happen. Rather than go into details about why this might happen, we prefer to remain agnostic, and call it an exogenous breakdown. The fact that it is not certain that they can put the deal together in the next CM provides an incentive for entrepreneurs to keep liquidity on hand, lest potentially profitable opportunities fall through.[10]

3 The CM

The CM is kept very simple here. Let $W_e(m, R)$ and $W_i(m, R)$ be the the value functions for entrepreneurs and innovators entering the CM with m dollars and a project with value R in hand (perhaps his own idea if he is an innovator and did not sell it off in the previous DM, or an idea he purchased if he is an entrepreneur). We use $R = 0$ to indicate either that the agent has a project with 0 return or has no project that period (perhaps because he sold his idea if he is an innovator, or because he failed to acquire one if he is an entrepreneur). Let $V_e(m)$ and $V_i(m)$ be the value functions for agents entering the DM with m dollars, before R is observed.

[10] The innovator will not give up his idea in the DM before he is paid – say, hoping to get paid in the future – since after the entrepreneur has the idea he will not pay; but we assume there is no problem with a simultaneous (quid pro quo) trade. As in most sensible monetary theories, it is important that we cannot use reputation to enforce payment, since otherwise there credit would work fine. One simple way to rule out reputation is to assume certain types of anonymity in the DM (Kocherlakota 1998; Wallace 2001; Corbae et al. 2003).

Then for $j = i, e$ the CM problem is

$$\begin{aligned} W_j\left(m,R\right) &= \max_{X,H,m'} \left\{U(X) - AH + \delta V_j(m')\right\} \qquad (1) \\ \text{s.t. } X &= e + wH + \phi(m - m' + \pi M) + R, \end{aligned}$$

where X is consumption, H is labor supply, and m' is money taken out of the market. Here U is a standard utility function, $A > 0$ is a parameter, e is an endowment, w is the real wage, and ϕ is the value of money (the inverse of the CM nominal price level). The term τM is a lump sum cash transfer, or tax if $\tau < 0$, so that the total money supply evolves across meetings of the CM according to $M' = (1+\tau)M$. This is essentially the CM problem in Lagos and Wright (2005), where the key assumption is that utility is quasi-linear. To reduce notation, we normalize $A = 1$, and we assume there is a representative firm with a linear technology, so the real wage is pinned down by the marginal product and can be normalized to $w = 1$.[11]

Substitute for H from the budget equation to rewrite (1) as

$$\begin{aligned} W_j\left(m,R\right) &= e + \phi m + \phi\pi M + R + \max_{X}\left\{U(X) - X\right\} \qquad (2) \\ &\quad + \max_{m'}\{-\phi m' + \delta V_j(m')\}. \end{aligned}$$

$>$From (2) several results follow immediately. First, W_j is linear in (m, R), with $\partial W_j/\partial m = \phi$ and $\partial W_j/\partial R = 1$. Second, X is given by the solution to $\partial U(X)/\partial X = 1$, independent of (m, R), prices, or any other variable. Third, m' is given by the solution to $\partial V(m')/\partial m' = \phi/\delta$, independent of

[11] It is easy to incorporate firms with general technologies (Aruoba and Wright 2003). Also, it is easy to have U, e or w differ across innovators and entrepreneurs, or across agent of a given type.

(m, R), and this implies all agents of a given type j take the same amount of money out of the CM.[12] In fact we show below that in equilibrium $m' = 0$ for innovators, and hence $m' = M'$ for all entrepreneurs if we normalize the measure of entrepreneurs to 1.

4 The DM

Let α_j be the arrival rate (the probability of a meeting) for $j = i, e$; for fixed measures of innovators N_i and entrepreneurs N_e the only restriction is $\alpha_e N_e = \alpha_i N_i$, so we can take these arrival rates to be exogenous for now. If an entrepreneur does not meet an innovator, he enters the next CM with his money but no project, $(m', 0)$. Given $m' = 0$ for an innovator, if he does not meet an entrepreneur he enters the next CM with no money and his own idea, $(0, R_i)$. If an entrepreneur and an innovator do happen to meet, the values of R_e and R_i are observed.[13] Then several things can happen. If $R_e \leq R_i$ there are no gains from trade. If $R_e > R_i$ then there are, and two subcases can be considered.

On the one hand, if $m' \geq p$ where p is the price they would agree to if there were no issues of liquidity or quid pro quo, then they trade immediately. On the other hand, suppose $m' < p$, meaning that liquidity is an issue. Then they could trade the idea for m' now, or to try to meet again in the next CM where an entrepreneur can always raise more money

[12] There are two caveats to these results: we need to have an interior solution for H, and we need to have V_j strictly concave. Lagos and Wright (2005) provide assumptions on primitives to guarantee these hold.

[13] Again, there is no private information about the value of ideas in the base model, not because we do think this is uninteresting, but so we can focus on different issues.

(from his endowment, by increasing H, and so on). We also allow them to renegotiate the price to p', if they meet again in the next CM, although we will see that in equilibrium $p' = p$. In any case, meeting in the next CM is not a sure thing: with probability γ, for whatever reason, it simply does not happen. An innovator may or may not prefer the risky chance at p' to the sure thing of m'.[14]

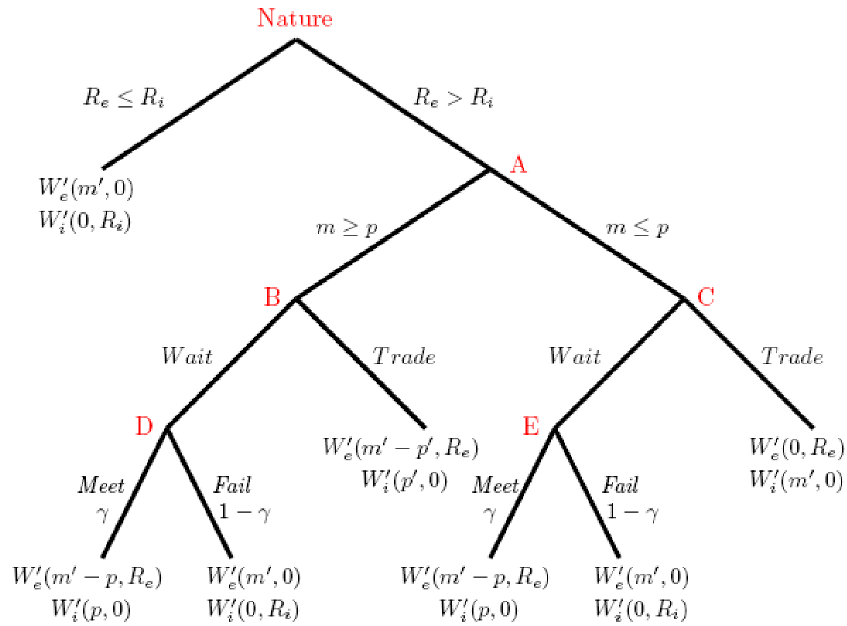
The other option is for the innovator to walk away and keep the idea for himself, but given $R_e > R_i$ this is dominated by agreeing to try to meet in the next CM, so we ignore it. The possibilities are shown in Figure 1. To describe what actually happens in the different events, we need to analyze the bargaining problems that arise. As is common in the related literature, we make a lot of use of the generalized bargaining Nash solution, where threat points are given by continuation values in case of no agreement and θ is an entrepreneur's bargaining power.

To begin, consider what happens if the pair actually do meet again in the next CM. Letting W_j' be next period's CM value function, the Nash solution is:

$$\max_{\hat{p}} [W_e'(m' - p', R_e) - W_e'(m', 0)]^{\theta} [W_i'(p', 0) - W_i'(0, R_i)]^{1-\theta} \qquad (3)$$

There is no constraint on p' – in particular, it could exceed the amount of money the entrepreneur brings to the next CM, m', since he can get more. Recall that W_j' is linear, so $W_e'(m' - p', R_e) - W_e'(m', 0) = R_e - \phi' p'$ and

[14] The innovator cannot trade the idea for m' now plus a promise of more in the next CM, since once the entrepreneur has the idea he will not honor the promise.



$W_i'(p', 0) - W_i'(0, R_i) = \phi' p' - R_i$. Hence (3) reduces to:

$$\max_{\hat{p}} \left(R_e - \phi' p'\right)^{\theta} \left(\phi' p' - R_i\right)^{1-\theta} \tag{4}$$

It is immediate that the solution is $p' = [\theta R_i + (1-\theta) R_e] / \phi'$.

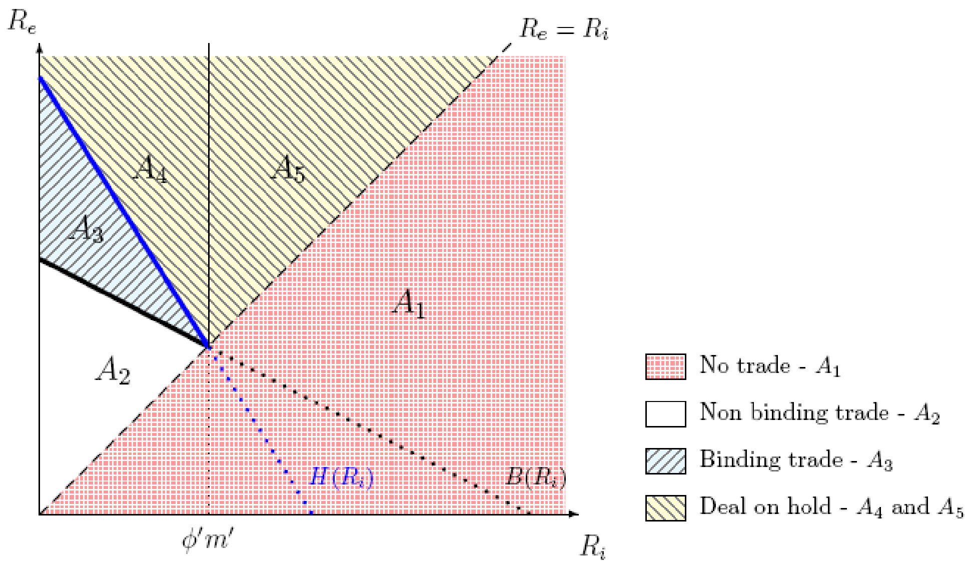
Now consider what happens in the DM. One difference from the previous bargaining problem is that the threat point is given not by the continuation value of not trading, but by the expected value of trying to reconvene in the next CM. For an entrepreneur this is $\gamma W_e'(m' - p', R_e) + (1-\gamma) W_e'(m', 0)$, and for an innovator $\gamma W_i'(p', 0) + (1-\gamma) W_i'(0, R_i)$. Using the linearity of W_j', the bargaining problem becomes:

$$\max_{p} [-\phi' p + \gamma \phi' p' + (1-\gamma) R_e]^{\theta} [\phi' p - \gamma \phi' p' - (1-\gamma) R_i]^{1-\theta} \tag{5}$$

Another difference from the previous problem is that this problem is subject to the constraint $p \leq m'$, since the entrepreneur cannot pay more money than he has in the DM.

Suppose first that the constraint $p \leq m'$ does not bind. Then it is simple to show $p = p'$, the same as the solution in the CM next period; in this case the agents settle immediately. Suppose now that the constraint binds – i.e. suppose $m' < p'$, or equivalently $R_e \geq B(R_i) = \frac{\phi' m' - \theta R_i}{1-\theta}$ (the label B stands for the fact that the constraint just *binds*). In this case the entrepreneur wants to pay m' and close the deal now, but the innovator may prefer putting things on hold and trying to meet in the next CM. Indeed, he prefers to trade now rather than wait if and only if $W_i'(m', 0) \geq \gamma W_i'(p', 0) + (1-\gamma) W_i'(0, R_i)$, or $R_e \geq H(R_i) = \frac{\phi' m' - R_i(1-\gamma+\theta\gamma)}{\gamma(1-\theta)}$ (the label H stands for the fact that the innovator is indifferent to putting the deal on *hold*).

Taking $\phi' m'$ as given for now, Figure 2 represents the possible outcomes in (R_i, R_e) space, partitioned into the following regions. Below the 45^o line, in region A_1, there is no trade. Above the 45^o line, several outcomes are possible. Below the line $R_e = B(R_i)$, in region A_2, there is immediate trade at price $p' \leq m'$. Above the B line and below the line $R_e = H(R_i)$, in region A_3, there is immediate trade at price m', since the constraint is binding but the innovator does not want to chance putting the deal on hold. Above the H line and the 45^o line, a potentially profitable deal is put on hold until the next CM, where the entrepreneur has access to more money. This occurs when R_i and R_e are both high because this means p' is high, and also because high R_i reduces the downside risk for the innovator.



When the probability of meeting in the centralized market γ is 0, the line H is vertical at $\phi' m'$. This increases the area where they trade immediately even though $m' < p'$, but the innovator still will not trade if $\phi' m' < R_i$ even though $R_e > R_i$. Therefore, due to a lack of liquidity, potentially good trades do not happen, and the project is implemented by the less efficient agent. When $\gamma = 1$, the B and H lines coincide and region A_3 disappears. However, we will see that when $\gamma = 1$ entrepreneurs carry no money, as they can always do a deal in the next CM. In this case, projects will always be implemented by the most efficient agent, with money playing no role.

We now describe the value functions for entrepreneurs in the DM. Since these are slightly messy, we break the presentation into parts by writing

$$V_e(m') = (1-\alpha_e)\beta W'_e(m',0) + \alpha_e\beta\sum_{j=1}^{5} V_e^j(m'), \tag{6}$$

where the first term is the expected value of not meeting someone and going to the next CM with $(m',0)$, and $V_e^j(m')$ is the expected value of a meeting when (R_i, R_e) falls in region A_j of Figure 2, $j = 1,...5$. We now describe each of these events.

To begin, in region A_1

$$V_e^1(m') = \int_0^\infty \int_0^{R_i} W'_e(m',0)dF_e(R_e|R_i)dF_i(R_i)$$

is the expected outcome of a meeting when $R_e < R_i$, which means no trade.[15] Now consider regions where $R_e > R_i$. In A_2

$$V_e^2(m') = \int_0^{\phi' m'} \int_{R_i}^{B(R_i)} W'_e(m'-p', R_e)dF_e(R_e|R_i)dF_i(R_i)$$

is the expected outcome when $p' \leq m'$, which means immediate trade at price p'. In A_3,

$$V_e^3(m') = \int_0^{\phi' m'} \int_{B(R_i)}^{H(R_i)} W'_e(0, R_e)dF_e(R_e|R_i)dF_i(R_i)$$

is the expected outcome when $p' > m'$, but they trade at price m' rather

[15] Since the payoff is constant over A_1, we could simplify this to $V_e^1(m') = prob(R_e \leq R_i)W'_e(m',0)$; we leave it as is, however, to facilitate comparison with the expected payoffs in other regions.

than putting the deal on hold. Finally, in A_4 and A_5,

$$\begin{aligned} V_e^4(m') &= \int_0^{\phi' m'} \int_{H(R_i)}^{\infty} \overline{W}'_e dF_e(R_e|R_i) dF_i(R_i) \\ V_e^5(m') &= \int_{\phi' m'}^{\infty} \int_{R_i}^{\infty} \overline{W}'_e dF_e(R_e|R_i) dF_i(R_i) \end{aligned}$$

are the expected outcomes, where in both cases $\overline{W}'_e = \gamma W'_e(m' - p', R_e) + (1-\gamma) W'_e(m', 0)$ is the expected payoff to putting a deal on hold.[16]

Similarly, for innovators

$$V_i(m') = (1-\alpha_i)\beta \int_0^{\infty} W'_i(m', R_i) dF_i(R_i) + \alpha_i \beta \sum_{j=1}^{5} V_i^j(m'). \tag{7}$$

Here

$$V_i^1(m') = \int_0^{\infty} \int_0^{R_i} W'_i(m', R_i) dF_e(R_e|R_i) dF_i(R_i)$$

is the expected value of not trading;

$$\begin{aligned} V_i^2(m') &= \int_0^{\phi' M'} \int_{R_i}^{B(R_i)} W'_i(m' + p, 0) dF_e(R_e|R_i) dF_i(R_i) \\ V_i^3(m') &= \int_0^{\phi' M'} \int_{B(R_i)}^{H(R_i)} W'_i(m' + M, 0) dF_e(R_e|R_i) dF_i(R_i) \end{aligned}$$

are the expected values of trading immediately at either p or m'; and

$$\begin{aligned} V_i^4(m') &= \int_0^{\phi' M'} \int_{H(R_i)}^{\infty} \overline{W}'_i dF_e(R_e|R_i) dF_i(R_i) \\ V_i^5(m') &= \int_{\phi' M'}^{\infty} \int_{R_i}^{\infty} \overline{W}'_i dF_e(R_e|R_i) dF_i(R_i), \end{aligned}$$

[16] It is to be understood that $p = [\theta R_i + (1-\theta) R_e] / \phi$ and $\overline{W}'_e$ depend on (R_i, R_e) in these integrals.

where $\overline{W}_i' = \gamma W_i'(m'+p',0) + (1-\gamma)W_i'(m',R_i)$. Note we compute $V_i(m')$ for any m', even though in equilibrium $m'=0$ for innovators, to facilitate comparison with $V_e(m')$. Also notice that innovators take as given that $m'=M'$ for all entrepreneurs.

5 Equilibrium

We now combine the DM results with the first order condition for m' from the CM. We show in the Appendix how to reduce (6) to

$$\begin{aligned} V_e(m') \;=\;& \beta W_e'(m',0) + \alpha_e\beta\theta \int_0^{\phi'm'} \int_{R_i}^{B(R_i)} (R_e - R_i) dF_e(R_e|R_i) dF_i(R_i) \\ &+\alpha_e\beta \int_0^{\phi'm'} \int_{B(R_i)}^{H(R_i)} (R_e - \phi'm') dF_e(R_e|R_i) dF_i(R_i) \\ &+\alpha_e\beta\gamma\theta \int_0^{\phi'm'} \int_{H(R_i)}^{\infty} (R_e - R_i) dF_e(R_e|R_i) dF_i(R_i) \\ &+\alpha_e\beta\gamma\theta \int_{\phi'm'}^{\infty} \int_{R_i}^{\infty} (R_e - R_i) dF_e(R_e|R_i) dF_i(R_i). \end{aligned} \tag{8}$$

We also show how to differentiate this expression (basically, by repeatedly applying Leibniz rule) to get

$$\frac{\partial V_e}{\partial m'} = \beta\phi' \left[1 + \alpha_e e(\phi'm')\right], \tag{9}$$

where for any z

$$e(z) \equiv (1-\gamma)\int_0^z \tfrac{z-R_i}{\gamma^2(1-\theta)^2} F_e'\left[\tfrac{z-R_i(1-\gamma+\theta\gamma)}{\gamma(1-\theta)}|R_i\right] dF_i(R_i) \qquad (10)$$

$$-\int_0^z \left\{F_e\left[\tfrac{z-R_i(1-\gamma+\theta\gamma)}{\gamma(1-\theta)}|R_i\right] - F_e\left[\tfrac{z-\theta R_i}{1-\theta}|R_i\right]\right\} dF_i(R_i)$$

as long as $\gamma \neq 0$, and if $\gamma = 0$

$$e(z) \equiv F_i'(z)\int_z^\infty (R_e - z)dF_e(R_e|z) - \int_0^z \left\{1 - F_e\left[\tfrac{z-\theta R_i}{1-\theta}|R_i\right]\right\} dF_i(R_i). \quad (11)$$

Note that $e(\cdot)$ is the marginal net benefit to an entrepreneur of an additional dollar of liquidity in a meeting. This is easiest to see in the case $\gamma = 0$. Then the first term in (11) is the probability of meeting an innovator with idea he values at $R_i = z$, $F_i'(z)$, times the gain from being able to buy the idea, $R_e - z$, integrated over R_e. And the second term is simply the probability of (R_i, R_e) being in the region A_3 times -1, since in this region the constraint binds and the entrepreneur pays exactly m', so the marginal dollar is taken by the innovator. A very similar intuition applies to (10), except things are complicated by the fact that sometimes the deal is put on hold.[17]

In any case, inserting (9) into the first order condition $\phi/\delta = \partial V_e/\partial m'$ we get

$$\frac{\phi}{\delta\beta\phi'} = 1 + \alpha_e e(\phi' m'). \qquad (12)$$

[17] When one changes one's real balances, most of the regions in Figure 2 change, leading to an expression for the derivative $\partial V_e/\partial m'$ with many terms as shown in the Appendix; but upon simplification most of the terms cancel for envelope-theorem-style reasons.

This is the *money demand function* for entrepreneurs, describing their choice of m' given ϕ, ϕ' and parameters. We can rewrite it in a common form by observing that $\phi/\phi' = 1+\pi$ is the inflation rate and $1/\delta\beta = 1+r$ the real interest rate, in equilibrium, across consecutive meetings of the CM. Hence the left side reduces to $(1+\pi)(1+r) = 1+i$ by the the Fisher equation for the nominal interest rate i, and (12) simplifies to $i = \alpha_e e(\phi' m')$, which equates the marginal cost of carrying money to the expected marginal benefit for entrepreneurs.

The analogous equation to (8) for innovators is

$$V_i(m') = \beta W_i'(m', 0) + v, \tag{13}$$

where v does not depend on m' (it depends instead on M', money held by entrepreneurs). Proceeding as we above now leads to $i = 0$ because, in words, the marginal benefit to an innovator of an additional dollar is 0. So for $i > 0$ there is no interior solution to the innovator's money demand problem. This proves something to which we earlier alluded: $i > 0$ implies $m' = 0$ for innovators. If $i < 0$ then $m' = \infty$; thus $i < 0$ is inconsistent with equilibrium. In the so-called Friedman Rule case, $i = 0$, as is standard, money demand is indeterminate, so we only consider what happens in the limit when $i \to 0$.

We are left with the result that $m' = 0$ for innovators and m' satisfies (12) for entrepreneurs. By market clearing, equilibrium solves (12) with $m' = M'$, or equivalently,

$$\frac{i}{\alpha_e} = e(z), \tag{14}$$

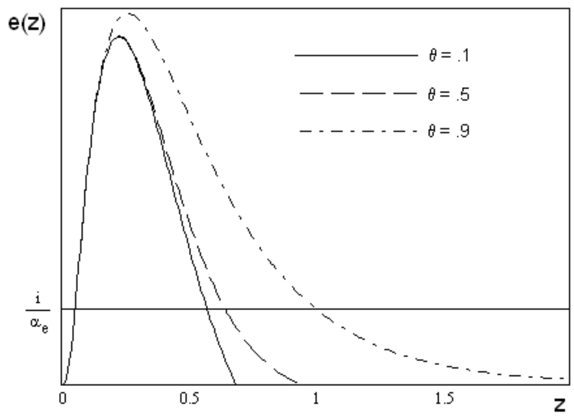
where $z = \phi' M'$ denotes real balances. In (14) we take $i = (1+\pi)(1+r) - 1$ as exogenous because we focus on steady states, where policy fixes the growth rate of the money supply τ, and real balances z are constant; this means the inflation rate is $\pi = \tau$. Hence, to find an equilibrium we simply look for a solution to $i = \alpha_e e(z)$. Given z it is easy to construct all the other endogenous variables, including the value of money in the CM, $\phi = z/M$, the unconstrained price of an idea, $p' = [\theta R_i + (1-\theta) R_e](1+\tau)M/z$, and so on.[18]

We now analyze equilibria using properties of the function $e(z)$. Consider first the case $\gamma \in (0,1)$. Then it is clear from (10) that $e(0) = 0$ and $\lim_{z\to\infty} e(z) = 0$ (the latter is especially obvious when the supports of F_i and F_e are bounded). Hence, there always exists a nonmonetary equilibrium $z = 0$. In addition, the second order condition holds only when $e'(z) < 0$. See for example Figure 5, drawn when F_i and F_e are independent lognormal distributions. Only the equilibria determined by the downward part of the curve constitute an equilibria in this model.

We now consider existence. It can be shown that $e(z) > 0$ for some z in the neighborhood of the lower bound of the support of R_i. If F_e' is continuous then it is clear that e is.[19] Hence, if $i/\alpha_e > 0$ then monetary equilibrium exists. If e is bounded, as in Figure 5, then existence requires i/α_e not too big.

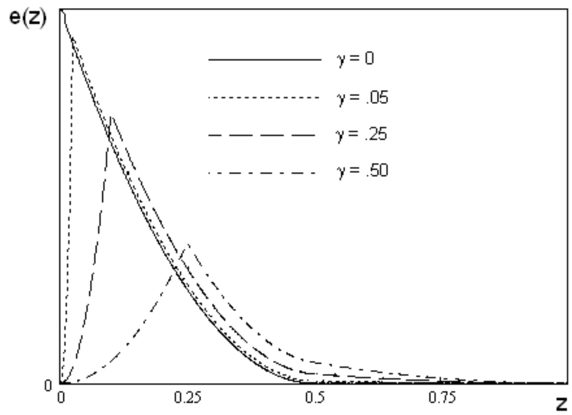
[18] We do not provide a formal definition of equilibrium here, since it should be clear that all we need to do is find z satisfying $i = \alpha_e e(z)$; see Lagos and Wright (2005) for a more rigorous presentation. Also, although we focus on steady states, in is known that there exist other equilibria in economies like this; see Lagos and Wright (2003).

[19] Continuity of F_e' is not needed to show existence but it makes the presentation easier.



The results are essentially similar when $\gamma = 0$. Although it is not clear from (11) that there is an equilibrium with $z = 0$, this is because $e(z)$ is derived as the first order condition for an interior solution for m'; in fact the nonmonetary equilibrium always exists. When $\gamma = 0$, $e(0)$ may be positive. Consider Figure 5, drawn when F_i and F_e are independent uniform distributions on $[0, 1]$ and different values of γ. When $\gamma > 0$ we have $e(0) = 0$ and, because of the second order condition, there is unique monetary equilibrium iff i/α_e is not too big. As $\gamma \to 0$, the equilibrium z does not coalesce with the nonmonetary equlibrium. See Figure **??**, which shows the equilibrium set as a function of γ: for all γ there exists an equilibrium with $z = 0$, and for $\gamma < \hat{\gamma}$ there exists one monetary equilibrium, which forms a concave function as γ varies.

In the nonmonetary equilibrium, regions other than A_1 and A_5 in Figure 2 vanish. Given $R_e > R_i$, the entrepreneur and innovator want to trade, but any deal will have to get done in the next CM, if they are able to

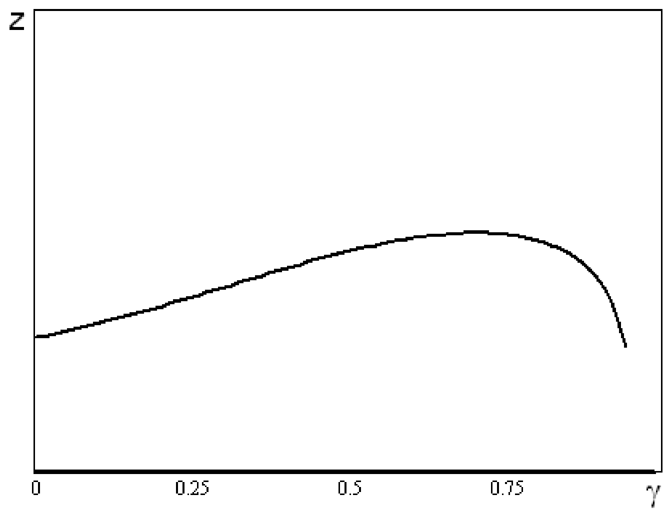


meet. Hence, with probability $1-\gamma$ the less efficient innovator implements the project himself. It is clear that there is a role for liquidity here – if the entrepreneur had the money on hand in the DM they could potneitally close the deal then and there and have the more efficient entrepreneur implements the project. Of course, if $\gamma = 1$ this is not an issue. it is easy to check thaty $\gamma = 1$ implies $e(z) = 0$ for all z, and hence the nonmonetary equilibrium is the only equilibrium. Without the friction of $\gamma < 1$ there is no role for liquidity.[20]

Consider an example where R_i and R_e are not independent: assume $R_i = \lambda R_e$ where λ distributed uniformly in $[0, 1]$ and $R_e = \overline{R}$ is constant that we can normalize to $\overline{R} = 1$. Thus, entrepreneurs are equally good at all projects, the only issue is, how good would the innovator be? In this case,

$$e(z) = (z-1)(1-2\theta)/\theta \tag{15}$$

[20] This is so even though there are still search-type frictions, in the sense that entrepreneurs and innovators only meet in the DM according to a random matching technology. Money, however, does cannot ameliorate this friction.



Hence $e(z)$ is linear, with $e(0) = 2 - 1/\theta$ and $e'(z) = (1 - 2\theta)/\theta$. The solution to $e(z) = i/\alpha_e$ is

$$z^* = 1 - \frac{i\theta}{\alpha_e(2\theta - 1)} \tag{16}$$

Hence a monetary equilibrium exists if and only if $i < \alpha_e(2\theta - 1)/\theta$, which can only hold if $\theta < 1/2$. In addition, when it exists, $\partial z^*/\partial\theta > 0$ and $\partial z^*/\partial i < 0$.

6 Conclusion

Our model is very much in line with Quadrini's (2000) view that "Two factors determine agents' choice of undertaking an entrepreneurial endeavor: the self-perceived ability of the agents to manage a business and their asset holdings."

"If financial constraints are important, then we expect that business start-ups will be sensitive to the wealth of potential entrepreneurs. If financial constraints were not important, then potential entrepreneurs will make the decision to start a business based solely on the expected profitability of the planned endeavor. If necessary, they will be able to get outside financing to start the project, and their own wealth not be a factor in whether or not the business is started. When financial constraints are important, however, outside financing may be unavailable or insufficient—creating a link between the wealth of the potential entrepreneur and the decision to start a business. Wealthier households will be more likely to start a business.

By allowing γ to vary, the model will be consistent with evidence such as that presented by Guiso, Sapienza, and Zingales (2001): "Ceteris paribus, an individual's odds of becoming an entrepreneur doubles if he moves from the least financially developed region to the most financially developed one. Similarly, the rate of firm's creation in the most financially developed provinces is three percentage point higher than in the least financially developed, and the number of firms divided by population 50% higher."

In fact, in example **5**, if $i = 0$ we can see from equation (16) that $z^* = R_e$, which is the efficient amount of money to carry. This means that the cost of carrying money is zero and entrepreneurs would be willing to take R_e to the centralized market and make sure that they can buy the idea of whatever kind of innovators they meet (regardless if he is good implementing it or not).

However, this is not the case in the general framework. As Claim **5** shows, there may be a combination of parameters for which, even when cost of carrying money is zero, agents do not carry the necessary amount of money to realize every efficient exchange and some projects are lost by efficient entrepreneurs.

The other source of inefficiency is the bargaining power. In the centralized market, entrepreneurs make an investment to acquire money in order to bargain with an innovator if they happen to meet. For low values of the bargaining power θ, entrepreneurs must give away significant fraction of the return from their investments. This, could hold entrepreneurs back when they decide how much to bring to the decentralized and, as before, efficient trading opportunities would be lost.

Let's refer again to the uniform distribution example. From equation (**??**), we can see that for higher values of θ (and values of $i \neq 0$), the quantity of money held by entrepreneurs is higher in equilibrium and the inefficiency region is smaller (this is the probability that a meeting and no trade happens is lessen). Also, from the derivative in (**??**), lower values of i yield higher equilibrium z^*, and this increases efficiency in the economy.

Extensions: private info (lemmons); reisk averse agents; endogenous choice of type e or i; ex ante investments.

Appendix

Here we show how to derive (8). First combine the expressions for $V_e^j(m')$, $j = 1, ...5$, to write (6) as

$$\begin{aligned} V_e(m') &= (1-\alpha_e)\beta W_e'(m',0) + \alpha_e\beta \int\limits_{A_1} W_e'(m',0) \\ &\quad + \alpha_e\beta \int\limits_{A_2} W_e'(m'-p', R_e) + \alpha_e\beta \int\limits_{A_3} W_e'(0, R_e) + \alpha_e\beta \int\limits_{A_4 \cup A_5} \overline{W}_e' \end{aligned}$$

where $\int_{A_j}(\cdot)$ denotes the integral over region A_j, and it is understood that $\int(\cdot) = \int\int(\cdot)dF_e(R_e|R_i)dF_i(R_i)$. Algebra yields

$$\begin{aligned} V_e(m') &= \beta W_e'(m',0) + \alpha_e\beta \int\limits_{A_2} \left[W_e'(m'-p', R_e) - W_e'(m',0)\right] \\ &\quad + \alpha_e\beta \int\limits_{A_3} \left[W_e'(0, R_e) - W_e'(m',0)\right] + \alpha_e\beta \int\limits_{A_4 \cup A_5} \left[\overline{W}_e' - W_e'(m',0)\right] \\ &= \beta W_e'(m',0) + \alpha_e\beta \int\limits_{A_2} (R_e - \phi' p') \\ &\quad + \alpha_e\beta \int\limits_{A_3} (R_E - \phi' m') + \alpha_e\beta \int\limits_{A_4 \cup A_5} \gamma(R_e - \phi' p'), \end{aligned}$$

using the linearity of W_e'. Inserting p', we arrive at

$$\begin{aligned} V(m') &= \beta W(m',0) + \alpha_e\beta\theta \int\limits_{A_2} (R_e - R_i) \\ &\quad + \alpha_e\beta \int\limits_{A_3} (R_e - \phi' m') + \gamma\alpha_e\beta\theta \int\limits_{A_4 \cup A_5} (R_e - R_i) \end{aligned}$$

Inserting the correct limits for the intergrals over the various regions A_1,...A_5 yields (8).

We now show how to differentiate this to get the expressin for $e(\cdot)$ in (9). We will consider two cases: $\gamma \neq 0$ and $\gamma = 0$. In the first case, using

Leibnez Rule, the derivatives of the integrals in the different regions are:

$$
\begin{aligned}
\frac{\partial}{\partial m'} \int\limits_{A_2} (\cdot) &= \phi' \int\limits_0^{\phi' m'} \frac{\phi' m' - R_i}{(1-\theta)^2} F_e'[B(R_i)|R_i] dF_i(R_i) \\
\frac{\partial}{\partial' m'} \int\limits_{A_3} (\cdot) &= \phi \int\limits_0^{\phi' m'} \frac{(\phi' m' - R_i)(1-\gamma+\theta\gamma)}{\gamma^2(1-\theta)^2} F_e'[H(R_i)|R_i] dF_i(R_i) \\
&\quad -\phi \int\limits_0^{\phi' m'} \frac{\theta(\phi' m' - R_i)}{(1-\theta)^2} F_e'[B(R_i)|R_i] dF_i(R_i) \\
&\quad -\phi \int\limits_0^{\phi' m'} \int\limits_{B(R_i)}^{H(R_i)} dF_e(R_e|R_i) dF_i(R_i) \\
\frac{\partial}{\partial m} \int\limits_{A_4} (\cdot) &= \phi \int\limits_{\phi' m'}^{\infty} (R_e - \phi' m') dF_e(R_e|\phi' m') F_i'(\phi' m') \\
&\quad -\phi \int\limits_0^{\phi' m'} \frac{\phi' m' - R_i}{\gamma^2(1-\theta)^2} F_e'[H(R_i)|R_i] dF_i(R_i) \\
\frac{\partial}{\partial m'} \int\limits_{A_5} (\cdot) &= -\phi \int\limits_{\phi' m'}^{\infty} (R_e - \phi' m') dF_e(R_e|\phi' m') F_i'(\phi' m')
\end{aligned}
$$

Substituting these into $\partial V/\partial m'$ and simplifying yields (10). In the case $\gamma = 0$, the derivative of $\int\limits_{A_3} (\cdot)$ given above is not correct because $H(R_i) = \infty$

(the other expressions are fine). In this case,

$$\begin{aligned}\frac{\partial}{\partial m'}\int\limits_{A_3}(\cdot) &= \phi\int\limits_{B(R_i)}^{\infty}(R_e-\phi'm')dF_e(R_e|\phi'm')F_i'(\phi'm') \\ &\quad -\int\limits_{0}^{\phi'm'}\frac{\theta(\phi'm'-R_i)}{(1-\theta)^2}F_e'[B(R_i)|R_i]dF_i(R_i) \\ &\quad -\phi\frac{\theta(\phi'm'-R_i)}{(1-\theta)^2}\int\limits_{0}^{\phi'm'}\int\limits_{B(R_i)}^{\infty}dF_e(R_e|R_i)dF_i(R_i).\end{aligned}$$

Using this yields (11). ■

The derivative of $e(z)$ is

$$\begin{aligned}e'(z) &= \tfrac{(1-\gamma)}{\gamma^2(1-\theta)^2}\int\limits_0^z\left\{\tfrac{(z-R_i)}{\gamma(1-\theta)}\ F_e'\left[H(R_i)|R_i\right]+F_e''\left[H(R_i)|R_i\right]\right\}dF_i(R_i) \\ &\quad +\tfrac{1}{(1-\theta)}\int\limits_0^z\left\{\frac{1}{\gamma}F_e\left[H(R_i)|R_i\right]-F_e\left[B(R_i)|R_i\right]\right\}dF_i(R_i)\end{aligned}$$

Then $e_\theta'(0)=0$.

References

Aghion, P. and Bolton, P., 1996. A trickle-down theory of growth and development with debt overhang. Review of Economic Studies 64, 151– 172.

Aghion, P. and Tirole, J., 1994. The management of innovation. Quarterly Journal of Economics 109 (4), 1185-1209.

Arora, A., 1995. Licensing tacit knowledge: intellectual property rights and the market for know-how. Economics of New Technology and Innovation 4, 41-59.

Aruoba, B. and Wright, R., 2003. Search, Money, and Capital: A Neoclassical Dichotomy. Journal of Money, Credit and Banking 35, 1085-1105.

Evans, D. S. and Jovanovic, B., 1989. An Estimated Model of Entrepreneurial Choice under Liquidity Constraints. Journal of Political Economy 97 (4), 808.827.

Evans, D. S. and Leighton, S. L., 1989. Some Empirical Aspects of Entrepreneurship. American Economic Review 79 (3), 519-535.

Fairlie. R., 1999. The Absence of African-American Owned Business: An Analysis of the Dynamics of Self-Employment. Journal of Labor Economics 17, 80-108.

Fazzari, S. M., Hubbard, R. G. and Petersen, B. C., 1988. Financing Constraints and Corporate Investment. Brookings Papers on Economic

Activity 88 (1), 141.195.

Fazzari, S., Hubbard, R.G. and Petersen, B., 2000. Investment-cash flow sensitivities are useful: a comment on Kaplan and Zingales. Quarterly Journal of Economics 115, 695– 705.

Gallini, N. and Winter, R., 1985. Licensing in the theory of innovation. Rand Journal of Economics 16 (2), 237-252.

Gentry, W. and R. Hubbard, 2000. Tax Policy and Entrepreneurial Entry. American Economic Review 90, 283-287.

Guiso, L., Sapienza, P. and Zingales, L., 2001. The Real Effects of Local Financial Development. Mimeo, Chicago Business School.

He, P., Huang, L. and Wright, R., 2005. Money and Banking in Search Equilibrium. International Economic Review, 46 (2), 637-670.

Holmes, T., Schimtz, J., 1990. A theory of entrepreneurship and its application to the study of business transfers. Journal of Political Economics 98 (2), 265-294.

Holmes, T., Schimtz, J., 1995. On the turnover of business firms and business managers. Journal of Political Economy 103 (5), 1005 - 1038.

Holtz-Eakin, D., Joulfaian, D. and Rosen, H. S., 1994. Sticking It Out: Entrepreneurial Survival and Liquidity Constraints. Journal of Political Economy 102 (1), 53-75.

Jensen, R., Thursby, M., 2001. Proofs and Prototypes for sale: The licensing of university inventions. American Economic Review 91 (1), 240-259.

Katz, M., Shapiro, C., 1986. How to license intangible property. Quarterly Journal of Economics 91 (1), 240-259.

Kocherlakota, N., 1998. Money is Memory. Journal of Economic Theory 81 (2), 232-251.

Lagos, R. and Wright, R., 2003. Dynamics, Cycles and Sunspot Equilibria in 'Genuinely Dynamic, Fundamentally Disaggregative' Models of Money. Journal of Economic Theory 109, 156-171.

Lagos, R., Wright, R., 2005. A unified framework for monetary theory and policy evaluation. Journal of Political Economy 113, 463-484.

Lel, U. and Udell, G., 2002. Financial Constraints, Start-up Firms and Personal Commitments. Mimeo, Kelly School of Business, Indiana University.

Lloyd-Ellis, H. and Bernhardt, D., 2000. Enterprise, inequality and economic development. Review of Economic Studies 67, 147– 168.

Lowe, R., 2003. Entrepreneurship and information asymmetry: theory and evidence from the University of California. Working Paper, Carnegie Mellon University.

Paulson, A. and Townsend, R., 2000. Entrepreneurship and Financial Constraints in Thailand. Journal of Corporate Finance 10, 229–262.

Pisano, G., Mang, P., 1993. Collaborative product development and the market for knowhow: strategies and structures in the biotechnology industry. Res. Tech. Innovation, Management Policy 5, 109-136.

Quadrini, V., 1999. The Importance of Entrepreneurship for Wealth Concentration and Mobility. Review of Income and Wealth 45 (1), 1-19.

Quadrini, V., 2000. Entrepreneurship, Saving, and Social Mobility. Review of Economic Dynamics 3, 1-40

Schumpeter, J., 1934. The Theory of Economic Development. Harvard University Press, Cambridge.

Shane, S., 2002. Selling university technology: patterns from MIT. Management Science 48 (1), 122-138.

Shepard, A., 1994. Licensing to enhance the demand for new technologies. Rand Journal of Economics 18 (3), 360-368.

Teece, D., Pisano, G., Shuen, A., 1997. Dynamic capabilities and strategic management. Strategic Management Journal 18 (7), 509-533.

Wallace, N., 2001. Whither Monetary Economics? International Economics Review 42 (4), 847-870.