Avoiding the Inflation Tax*

Huberto M. Ennis

Research Department, Federal Reserve Bank of Richmond
P.O. Box 27622, Richmond, Virginia 23261

E-mail: huberto.ennis@rich.frb.org

I study the effects of inflation on the purchasing behavior of buyers in an economy where money is essential for certain transactions (as in Lagos and Wright, 2005). I show that higher inflation induces buyers to search for transactions more intensively and buy goods of worse quality to avoid holding for a prolonged period of time a fast depreciating asset like money. The standard framework fails to capture this kind of effect and the modification proposed in this paper sheds new light on the connection between the search and the inventory models of money.

Keywords: Money, Random Matching, Search Intensity, Goods' Quality.

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1. INTRODUCTION

There are two main effects of expected inflation on the demand for money by individuals. First, when inflation increases agents tend to carry less money. Inflation acts as a tax on cash transactions and when inflation increases agents tend to shift their consumption patterns away from cash-intensive activities. This effect has been thoroughly studied in the literature and it is the basic force that determines the welfare effects of expected inflation in the canonical cash-in-advance model (see for example Cooley and Hansen, 1989).

The second effect of inflation on money demand is more subtle. When inflation increases agents try to reduce the average time they carry a given amount of money necessary for a transaction. In the words of Irving Fisher "when depreciation is anticipated, there is a tendency among owners of money to spend it speedily" (Humphrey, 1993). In this paper I use a modern search-based model of monetary exchange (Lagos and Wright, 2005) to formally study this second effect of anticipated inflation.

Recently, a powerful modeling device has become common in the monetary economics literature: A growing number of articles assume that agents have the ability (at no cost) of rebalancing their asset portfolios at the end of every monetary-trade interaction. Combined with quasilinear preferences, this assumption results in a tractable simplification of the distribution of money holdings and its dynamics. However, not everything that results from these assumptions is a gain. Counter to the intuition held by Fisher and many others, in such a model increases in inflation do not increase the willingness to trade of agents holding money. This result was highlighted, for example, by Lagos and Rocheteau (2005) who showed that in the canonical model with search intensity when inflation increases agents tend to *reduce* their effort to engage in (monetary) transactions.

I argue below that the result in Lagos and Rocheteau (2005) is an artificial consequence of the timing of rebalancing implied by their setup. I consequently change the environment to reduce the frequency of rebalancing in a way that makes it more in line with a situation where rebalancing would be costly.[1] Then I show that in such a setup increases in anticipated inflation

[1] Lagos and Rocheteau take a different route in the second part of their paper. They change the pricing mechanism to competitive price posting. They then find that for low inflation rates an increase in inflation raises

tend to make agents more willing to trade in their money holdings, hence increasing their search-for-trades intensity. I also show that a similar logic results in agents being more willing to buy low quality goods when inflation is high.[2]

This paper is related to three strands of the monetary theory literature. First, the paper attempts to contribute to our understanding of the forces at work in those search-based models of monetary exchange and inflation which follow the seminal contribution by Lagos and Wright (2005).[3] Second, the paper can be viewed as an intermediate step in establishing a connection between the Lagos-Wright literature and the modern inventory-theoretic models of the demand for money (see for example Jovanovic (1982), Alvarez and Atkeson (1997), and Chiu, 2005). Finally, the paper is closely related to the literature that focuses on production and trading inefficiencies originated in the interaction between inflation and search frictions (see for example Tommasi (1999), Peterson and Shi (2004), and Head and Kumar, 2005).

The rest of the article is organized as follows. The next section describes the model. Section 3 studies the effect of inflation on buyers search intensity. Section 4 deals with the effect of inflation on the pattern of trade between high- and low-quality goods. The last section provides some concluding remarks.

2. THE MODEL

The model is a modified version of that in Lagos and Rocheteau (2005). Time is discrete. There are two groups of agents: buyers and sellers. All agents are infinitely lived and discount the future according to the discount factor $\beta \in (0, 1)$. There is a measure one of buyers and a measure one of sellers in the economy.

buyers' search intensities. It seems still somewhat unattractive that in their model at high levels of inflation, for which the Fisher intuition would seemed to be most relevant, search intensity is decreasing in inflation.

[2]Peterson and Shi (2004) present a related search model of money with a continuum of qualities of goods and show that when inflation increases agents switch consumption towards the low quality goods. Their result hinges on the induced relative price dispersion originated in the interaction between the heterogeneous quality of goods and search frictions. They also study the determination of endogenous search intensity. However, all their results are different in nature from the ones presented here since, in their model, effective rebalancing of money holdings takes place at the end of *every* period.

[3]Precursors of this paper in the early money search literature are the papers by Li (1995, 1997).

Each period is divided into two subperiods. In the first subperiod, a subset of agents interact in a decentralized market where buyers get randomly matched with sellers and trade anonymously. When a buyer and a seller decide to trade, the buyer makes a take-it-or-leave-it offer to the seller. There are two types of goods being traded in the decentralized market: A high quality good (H) and a low quality good (L). The high quality good, when consumed, provides at least as much utility to the buyers as the low quality good but it is equally costly to produce. Let $\varepsilon_i u(q)$ be the utility for the buyer consuming q units of good i with $i = H, L$, where ε_i take the value ε_H if the good is high quality and ε_L if it is low quality. Accordingly, we have that $\varepsilon_H \geq \varepsilon_L$. The seller's cost of production is given by $c(q)$ where q is the quantity produced. A proportion σ of the sellers produce high quality goods and the rest produces low quality good.

Buyers choose a level of search intensity in the decentralized market that influences their likelihood of being matched with sellers. Let e_j be the level of search intensity chosen by buyer j and $\overline{e}$ the average search intensity in the economy. Increasing search intensity comes at a cost and we denote by $v(e)$ the utility cost of choosing search intensity e. Assume $v'(e) \geq 0$ and $v''(e) \geq 0$.

Matching in the decentralized market takes place according to the matching function $\zeta(\overline{e}\mu_b, \mu_s)$, where μ_b is the measure of buyers in the decentralized market and μ_s the measure of sellers. Assume that $\zeta(0, \mu_s) = \zeta(\overline{e}\mu_b, 0) = 0$ and $\zeta(\overline{e}\mu_b, \mu_s) \leq \min\{\mu_b, \mu_s\}$ for all $\overline{e} \geq 0$. The probability for a buyer of being matched to a seller when choosing search intensity e_j is given by $\alpha_{bj} = \min\{e_j \zeta(\overline{e}\mu_b, \mu_s)/\overline{e}\mu_b, 1\}$ and the probability for a seller of being matched to a buyer is given by $\alpha_s = \zeta(\overline{e}\mu_b, \mu_s)/\mu_s$. Assume that $\zeta(\overline{e}\mu_b, 1) = \alpha\overline{e}\mu_b$ for $\alpha\overline{e} \leq 1$ and μ_b otherwise. This matching function rules out search externalities among buyers and simplifies matters significantly.

In the second subperiod, also a subset of agents interact in a centralized market and produce and consume a "general" good. Let $U(X)$ be the utility from consuming a quantity X of the general good. These goods can be produced one-to-one with labor, from which agents experience linear disutility.

The main modification of the environment relative to that in Lagos and Rocheteau (2005) is in the ability of buyers to access the centralized market. Assume that buyers enter the centralized

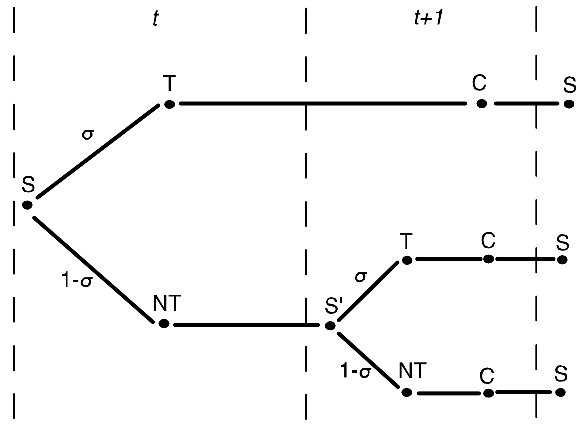

Figure 1

market every other period.[4] In particular, assume that half of the buyers visit the centralized market on odd periods and the other half on even periods. Also assume that, in between visits to the centralized market, agents can trade only once in the decentralized market, and that consumption of the goods traded in the decentralized market takes place right before the agent goes back to the centralized market.[5] Figure 1 illustrates this new timing. After a buyer exits the centralized market (node S in the figure), in the first subperiod of the following period she enters the decentralized market. If she gets matched with a seller and trades (node T), then she is done trading, waits until the next period at which point she consumes the goods bought in the decentralized market and goes back to the centralized market to rebalance her money holdings (node C). However, if trade does not take place (node NT) then the buyer does not visit the centralized market that period and goes back to the decentralized market in the following period (node S′). After not visiting the centralized market for one period, the agent has access to the centralized market regardless of whether she does or does not trade in the decentralized market.[6]

[4]Berentsen et. al. (2005) introduce a similar change in timing in the Lagos and Wright (2005) framework and study the distributional effects of inflation.

[5]These assumptions are made for simplicity and to facilitate the comparison with the analysis in Lagos and Rocheteau (2005). Later in the paper we discuss how relaxing these assumptions and allowing agents to trade and consume twice between each visit to the centralized market would change the details but not the substance of the results.

[6]A similar timing could arise endogenously from assuming a fixed cost of entering the centralized market.

This scheme describes the trading itinerary of buyers in the economy. Sellers, on the other hand, have access to both markets every period.

I maintain the standard technical assumptions on the functions u, c, and U. That is, they are twice continuously differentiable, $u(0) = c(0) = 0$, $u' > 0$, $c' > 0$, $u'' < 0$, $c'' \geq 0$, $U' > 0$, and $U'' \leq 0$. Also, there exists $q_i^* \in (0, \infty)$ such that $\varepsilon_i u'(q_i^*) = c'(q_i^*)$ for $i = H, L$. As in Lagos and Rocheteau (2005) assume that $U(X^*) = X^*$ with $U'(X^*) = 1$.

Finally, in this environment there is also an intrinsically useless, perfectly divisible and storable asset that will be called money. The stock of money in period $t = 0, 1, ...$ is given by $M_t = \gamma^t M_0 > 0$ and $\gamma \geq \beta$.

3. SEARCH INTENSITY

In this section we will concentrate our attention on the effects of inflation over the choice of search intensity. To simplify the presentation assume that $\varepsilon_H = \varepsilon_L = 1$, that is, all goods are of the same quality. As it is standard in these models the agent's holdings of real balances is the relevant monetary decision variable. Hence, define $z_t = \phi_t m_t$ as real balances, where ϕ_t is the price of money in the centralized market in period t and m_t is nominal money holdings. We denote with π_{t+1} the inflation rate between periods t and $t+1$ and we have that

$$\pi_{t+1} = \frac{\phi_t}{\phi_{t+1}} - 1.$$

The objective next is to study steady state equilibria with constant inflation and establish how the endogenous variables (and, in particular, the level of search intensity) depend on the level of inflation.

3.1. Equilibrium

Let $W_t(z)$ be the value for a buyer of entering the centralized market with z units real balances in the second subperiod of period t; and let $V_{2,t}(z)$ be the value for a buyer of entering the decentralized market in period t after holding z units of real balances, and not trading at $t-1$.[7] Also, define as e_{kt}, with $k = 1, 2$, the search intensity chosen by an agent that was last in the centralized market k periods ago. Then, the value function at time t for a buyer that went to

[7]The subscript 2 is used to indicate that the agent has not visited the centralized market in the previous period.

the centralized market in $t-1$ is given by

$$V_{1t}(z_t) = \max_{e_{1t}} V_{1t}(z_t, e_{1t}),$$

where

$$V_{1t}(z_t, e_{1t}) = \alpha e_{1t}\beta \left(u\left[q_t(z_t)\right] + W_t \left[\frac{z_t - z_t^d(z_t)}{1+\pi_{t+1}} \right] \right) + (1-\alpha e_{1t})\beta V_{2,t+1}(z_t) - v(e_{1t})$$

and the functions $q_t(z_t)$ and $z_t^d(z_t)$ are the quantity of the good traded in the decentralized market and the amount paid for that quantity, respectively. We will study the determination of these values below.

The value function $W_t(z_t)$ is given by

$$W_t(z_t) = \max_{z_{t+1}} \{-h_t + \beta V_{1t+1}(z_{t+1})\}$$

subject to

$$z_{t+1} = z_t + h_t - \pi_{t+1} z_{t+1}.$$

It is easy to see that $W_t(z) = z + W_t(0)$. Then, the value function $V_{2,t+1}(z_t)$ is given by

$$V_{2,t+1}(z_t) = \max_{e_{2,t+1}} \left\{ \alpha e_{2,t+1} \left(u\left[q_{t+1}(z_{2,t+1})\right] - z_{t+1}^d(z_{2,t+1}) \right) + z_{2,t+1} + W_{t+1}(0) - v(e_{2,t+1}) \right\}$$

where $z_{2,t+1} = z_t/(1+\pi_{t+1})$.[8]

The sellers' problem is simple. Sellers have no reason to carry money to the decentralized market (see Lagos and Rocheteau, 2005) and they produce for the buyer as long as trading does not reduce their welfare. The value function of a seller at the beginning of period t is given by

$$V_t^s = \alpha_s(\bar{e}\mu_b)\left[-c(q_t) + W_t^s(z_t^d)\right] + (1-\alpha_s(\bar{e}\mu_b))W_t^s(0),$$

where (q_t, z_t^d) is an acceptable take-it-or-leave-it offer of a buyer and $W_t^s(z)$ is the value function of entering the centralized market in period t with z units of real balances. It is easy to show that $W^s(z) = z + \beta V_{t+1}^s$.

[8] To see why $z_{2,t+1} = z_t/(1+\pi_{t+1})$ note that the buyer chose m_t nominal money holdings in the centralized market in period $t-1$. Then, in period t, the buyer does not get to trade and hence has no access to the centralized market. She then starts period $t+1$ with m_t nominal money holdings, or $z_{2,t+1} = \phi_{t+1} m_t$ real balances, which can be written as in the text.

When a buyer in the decentralized market holding z_t units of real balances meets a seller at time t the terms of trade would depend on whether the buyer has visited the centralized market the previous period or not. If she has, then she makes a take-it-or-leave-it offer (q_t, z_t^d) to the seller that solves the following problem:

$$\max_{(q_t, z_t^d)} u(q_t) - \frac{z_t^d}{1+\pi_{t+1}}$$

subject to

$$-c(q) + z_t^d \geq 0, \quad z_t^d \leq z_t, \quad q_t \geq 0.$$

Let q_π^* be the quantity of goods that satisfies $u'(q_\pi^*) = c'(q_\pi^*)/(1+\pi)$. Then, the solution to this problem is given by:

$$q_t = q_{\pi_{t+1}}^* \text{ and } z_t^d = c(q_{\pi_{t+1}}^*) \text{ if } c(q_{\pi_{t+1}}^*) < z_t,$$
$$q_t = c^{-1}(z_t) \text{ and } z_t^d = z_t \text{ if } c(q_{\pi_{t+1}}^*) \geq z_t.$$

If the buyer has not visited the centralized market in the previous period then the offer satisfies the same conditions as above with value of π fixed at zero. That is:

$$q_t = q^* \text{ and } z_t^d = c(q^*) \text{ if } c(q^*) < z_{2,t},$$
$$q_t = c^{-1}(z_{2,t}) \text{ and } z_t^d = z_{2,t} \text{ if } c(q^*) \geq z_{2,t},$$

where q^*satisfies $u'(q^*) = c'(q^*)$. This is the usual take-it-or-leave-it offer that arises in the Lagos and Rocheteau (2005) framework.

Let us define $S(z) \equiv u\left[q(z)\right] - z^d(z)$. Then, we have:

$$V_{2,t+1}(z_t) = \max_{e_{2,t+1}} \left\{\alpha e_{2,t+1} S(z_{2,t+1}) + z_{2,t+1} + W_{t+1}(0) - v(e_{2,t+1})\right\}$$

and the condition for optimality of $e_{2,t+1}$ (assuming an interior solution) is:

$$\alpha S(z_{2,t+1}) - v'(e_{2,t+1}) = 0.$$

For a given level of real balances, this condition is the same condition that determines the optimal search intensity in Lagos and Rocheteau (2005). Since, in general, the surplus function $S(z)$ is increasing in z for the relevant range of values of z we have that the level of search intensity e_2 will be increasing in $z_{2,t+1}$, and therefore, since higher inflation results in lower equilibrium real balances, the equilibrium search intensity e_2 will be decreasing in inflation. This logic is just an instance of the result in Lagos and Rocheteau (2005) (see the discussion below). For now, let us call the solution of this equation for a given level of $z_{2,t+1}$ as $\widehat{e}_{2,t+1}(z_{2,t+1})$ (to simplify notation, when clear, we will just write $\widehat{e}_{2,t+1}$).

Here, though, we want to concentrate in the direct effect of inflation on the choice of search intensity e_1; that is, on the decision over search intensity of an agent that, depending on her trade experience, may or may not be holding the same cash balances at the beginning of the next period. Let us then rewrite the function $V_{1t}(z_t, e_{1t})$ as follows

$$V_{1t}(z_t, e_{1t}) = \alpha e_{1t}\beta \left[S(z_t) + \tfrac{\pi_{t+1}}{1+\pi_{t+1}} z_t^d\left(z_t\right)\right] +$$
$$(1-\alpha e_{1t})\beta \left[\alpha \widehat{e}_{2,t+1} S\left(\tfrac{z_t}{(1+\pi_{t+1})}\right) - v\left(\widehat{e}_{2,t+1}\right)\right] + \beta W_{t+1}\left(\tfrac{z_t}{1+\pi_{t+1}}\right) - v(e_{1t}).$$

Then, for a given value of z_t, the optimal value of e_{1t} is the one that maximizes the function $V_{1t}(z_t, e_{1t})$ where, by the Envelope Theorem, we can take the value of $\widehat{e}_{2,t+1}$ as given.

To obtain the optimal value of z_t we substitute the expression for $V_{1t}(z_t)$ in the expression for the $W_{t-1}(0)$ and solve the maximization problem involved. Similar considerations as in Lagos and Wright (2005) show that the function $V_{1t}(z_t, e_{1t})$ is concave in z_t. Also, for low levels of inflation the optimal level of real balances satisfy the inequality $z_t \le \gamma c(q^*) < c(q_\gamma^*)$.[9]

We will concentrate the attention on the set of steady state equilibria with constant inflation rate. It is easy to see that in steady state $1+\pi_t = \gamma$ for all t. Then, we consider the set of steady state equilibria indexed by γ. We need to determine three key equilibrium variables to be able to meaningfully describe a steady state. These variables are $z(\gamma)$, $e_1(\gamma)$, and $e_2(\gamma)$.

PROPOSITION 3.1. *There exists a non-empty set of monetary economies for which the steady state equilibrium search intensity $e_1(\gamma)$ is increasing in inflation.*

Proof. The steady state equilibrium variables $z(\gamma)$, $e_1(\gamma)$, and $e_2(\gamma)$ must solve the following system of equations:

$$\alpha S\left(\frac{z}{\gamma}\right) - v'(e_2) = 0$$

[9]Note that if inflation is too high the inequality $z \le \gamma c(q^*)$ may not hold because even if the surplus function $S(z)$ is constant for $z > \gamma c(q^*)$, the agent may choose a higher value of z (one between $\gamma c(q^*)$ and $c(q_\gamma^*)$) if it produces extra surplus when used for trading in the first period after visiting the centralized market.

$$\alpha\beta\left[S(z)-\alpha e_2 S\left(\frac{z}{\gamma}\right)+\left(1-\frac{1}{\gamma}\right)z-v\left(e_2\right)\right]-v'(e_1)=0$$

$$\alpha e_1\beta\left[\frac{dS(z)}{dz}+1\right]+\left(1-\alpha e_1\right)\frac{\beta}{\gamma}\left[\alpha e_2\frac{dS(z/\gamma)}{d\left(z/\gamma\right)}+1\right]-\frac{\gamma}{\beta}=0$$

This system, in general, is highly non-linear and hence difficult to characterize analytically. For this reason, we use standard functional forms to compute solutions and then show that the effort level $e_1(\gamma)$ may be increasing or decreasing in γ depending on parameter values.

Let the functional form for the utility function be $u(q)=q^{1-\eta}/\left(1-\eta\right)$ with $0<\eta<1$; also assume that $c(q)=q$, $v(e)=e^{\varphi}$ with $\varphi>1$. We fix the values of φ, β, and α and compute e_1 for different values of η and γ. Let $\varphi=2$, $\beta=0.96$ and $\alpha=0.5$. Then we have the following results:

Table 1

	$\eta=0.60$	$\eta=0.65$	$\eta=0.66$	$\eta=0.69$	$\eta=0.70$	$\eta=0.72$
$e_1(1.05)$	0.24904	0.29045	0.29783	0.31696	0.32195	0.32883
$e_1(1.07)$	0.24525	0.29009	0.29802	0.31872	0.32418	0.33195
$e_1(1.15)$	0.22390	0.28358	0.29408	0.32175	0.32937	0.34124
$e_1(1.17)$	0.21760	0.28106	0.29221	0.32171	0.32990	0.34288

Table 1 shows that for a given level of inflation search intensity tends to be increasing in inflation for higher values of η. The threshold value of η above which search intensity is increasing in inflation, in turn, depends on the level of inflation. When inflation is at around 5% the threshold value of η is between 0.65 and 0.66 while when the inflation level is around 15% the threshold value is between 0.69 and 0.70.[10] ■

This proposition stands in sharp contrast with the result in Proposition 1 of Lagos and Rocheteau (2005), where they show that in their model the equilibrium level of search intensity is *always* a decreasing function of inflation. The following discussion provides intuition for the result and assesses its generality.

[10]This is because the equilibrium value of z is increasing in η and the search intensity tends to be increasing in inflation when z is larger (because this implies a large "tax base" for the inflation tax). Similarly, when inflation is high, the equilibrium value of z is low and the search intensity is increasing for a smaller set of values of η.

3.2. Discussion

First, let us briefly review the result in Lagos and Rocheteau (2005). Since they assume that the buyer visits the centralized market every period, for interior solutions of the agent's problem, the steady state values of z and e_1 solve the following system of equations:

$$F_1(e_1, z) \equiv \alpha S(z) - v'(e_1) = 0,$$

and

$$F_2(e_1, z; \gamma) \equiv \alpha e_1 \frac{dS(z)}{dz} + 1 - \frac{\gamma}{\beta} = 0.$$

Figure 3 plots this system of equations in the (e_1, z)-plane. The system defines two implicit functions $z(\gamma)$ and $e_1(\gamma)$, and Lagos and Rocheteau show that these two functions are both decreasing in γ. In other words, they show that higher steady state inflation is associated with lower equilibrium per capita real balances and lower search intensity (from point A to point B in the figure). The logic behind this result is simple. For higher levels of inflation the buyers have lower incentives to carry money to the next period. As a result, equilibrium consumption in a match is lower. Since the net benefit of consuming $S(z)$ is increasing in z (at the equilibrium point), higher inflation implies a lower net benefit of trading. The marginal benefit of increasing search intensity is given by $\alpha S(z)$ and hence, higher inflation levels are associated with lower marginal benefits of searching, which in turn results in lower equilibrium levels of search. It is worth observing here that a very similar logic explains the behavior of e_2 in the model in this paper.

The result in Lagos and Rocheteau (2005) is more general than it may appear. In fact, modifying a standard cash-in-advance model to introduce a choice of trade effort delivers the same conclusion: higher steady state inflation results in lower trading effort (see Appendix A for details). In the standard cash-in-advance model the inflation tax applies to money holdings at the beginning of the period, and it does not depend on whether the agents spend or not their money during the period. Changing the timing so that agents can use for transactions the cash obtained in the current period (but have to hold any money not spent until next period) will change the result.[11] It is interesting to note, however, that in the Lagos and Wright (2005) framework the same change in timing does not revert the Lagos and Rocheteau (2005) result. This is the case because the buyers have all the bargaining power and when the sellers find themselves holding cash in an inflationary environment, all the inflation costs associated with it are passed to the buyers during price negotiation. In the end, the buyer bears the entire inflation tax and cannot avoid it by searching more.[12]

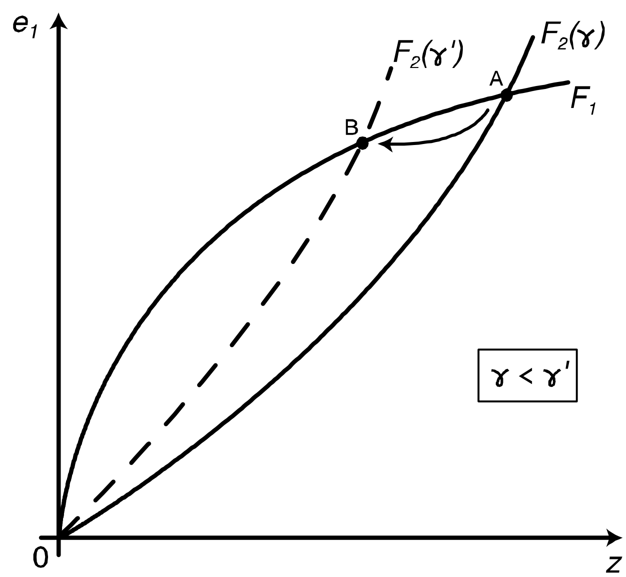

Figure 3: Lagos and Rocheteau (2005)

Let us now go back to the model in this paper and explain why $e_1(\gamma)$ can be increasing. Since the logic behind the behavior of e_2 has already been studied in Lagos and Rocheteau (2005), to simplify the presentation in what follows we assume that buyers cannot choose the value of e_2. To minimize notation assume that the probability of getting matched after not being matched the previous period is α_2 and that there is no utility cost associated with the fixed effort e_2.

[11] McCallum and Goodfriend (1987) set up a model where time devoted to transactions and money are substitutes (see also the discussion in Lucas, 2000). For a given level of consumption, by holding more money the agent can reduce the required amount of time devoted to transaction. Higher inflation will increase transaction time in such a model. The result, though, hinges on a reduce-form transactions function, which may be regarded as somewhat arbitrary.

[12] This is yet another instance where explicitly modeling transactions as in Lagos and Wright (2005) implies a different result from that in the reduce-form cash-in-advance model.

The equilibrium conditions are then:

$$\alpha\beta\left[S(z)-\alpha_2 S\left(\frac{z}{\gamma}\right)+\left(1-\frac{1}{\gamma}\right)z\right]-v'(e_1)=0,$$

and

$$\alpha e_1\beta\left(\frac{dS(z)}{dz}+1\right)+(1-\alpha e_1)\frac{\beta}{\gamma}\left(\alpha_2\frac{dS(z/\gamma)}{d(z/\gamma)}+1\right)-\frac{\gamma}{\beta}=0.$$

For the subject of this paper, the key term in the system of first order condition is $[1-(1/\gamma)]z$ in the first equation.[13] Note that when inflation increases (that is, when γ increases) this term increases, in turn increasing the marginal benefit of searching more intensively.

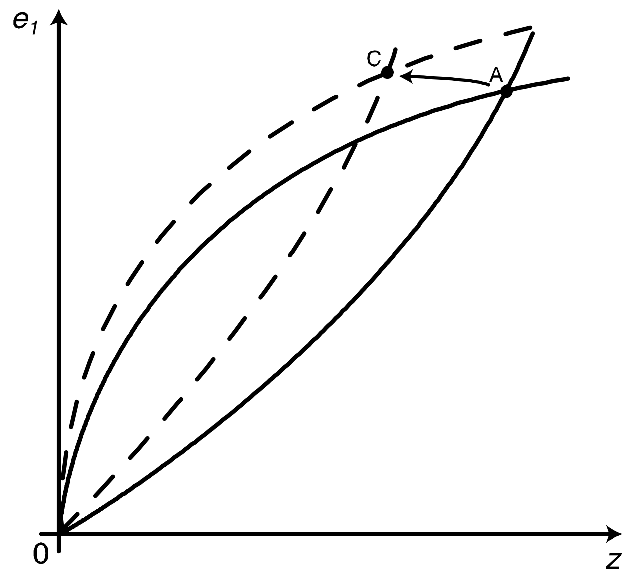

Figure 4: New Timing - Effort Increasing in Inflation

To see the way this direct incentive to search due to inflation works, consider the extreme case where $\alpha_2 \approx 0$. Clearly, by continuity, the ideas also apply to the more general case. The first order conditions then reduce to

$$\alpha\beta\left[S(z)+\left(1-\frac{1}{\gamma}\right)z\right]-v'(e_1)=0, \tag{1}$$

and

$$\alpha e_1\frac{dS(z)}{dz}+\left[\alpha e_1+(1-\alpha e_1)\frac{1}{\gamma}\right]-\frac{\gamma}{\beta^2}=0. \tag{2}$$

[13]For high values of η the implicit function $e_1(z)$ defined by the first expression may be decreasing in the relevant range. In such case the equilibrium e_1 is always an increasing function of inflation γ (see Figure 3).

Figure 4 plots this system of equations in the (e_1, z)-plane. First note that, as in most monetary models, a higher level of inflation reduces the incentives of agents to use money. This effect is represented by the second term of equation (2) and was (partially) present in Lagos and Rocheteau (2005). (Here, the effect is amplified by the fact that sometimes agents will hold their money balances for two periods.) But then, if the term $[1 - (1/\gamma)]z$ were not in equation (1) the forces studied in Lagos and Rocheteau (2005) would make the equilibrium level of search intensity decreasing on the level of inflation. However, in the model in this paper a higher search intensity e_1 decreases the agents' probability of holding money for two periods, and in this way, allows them to partially avoid the inflation tax. This extra benefit of searching brings about the inflation term in equation (1) and opens the possibility that the equilibrium level of e_1 be higher when inflation is higher (from point A to point C in the figure).

4. QUALITY OF GOODS

In this section we will concentrate on the effect of inflation on the quality of goods purchased in the decentralized market. For simplicity, assume that $v(e) = 0$ for all e and hence agents set their search intensity equal to $1/\alpha$ and $\alpha_b = 1$. In this environment, buyers in the decentralized market always find a match. However, they may sometimes choose not to trade. In particular, since not all sellers produce the high quality good, under certain conditions, buyers may rather wait than trade with low-quality sellers. We provide a formal characterization of these conditions next.[14]

4.1. Equilibrium

The expression for the value function of a buyer entering the centralized market at time t with z units of real balances, $W_t(z)$, is the same as in the previous section. Then, using the linearity of W_t, the value function for a buyer with z_t units of real balances at the beginning of period t

[14] When the buyer can go to the centralized market every period this model is a special case of the model with match-specific uncertainty presented in Lagos and Wright (2004). In that model, buyers always trade. See also Lagos and Rocheteau (2005).

is now given by the following expression:

$$\begin{aligned} V_{1t}(z_t) \; &= \; \sigma\beta\left[\varepsilon_H u\left[q_H(z_t)\right] + W_{t+1}\left(\frac{z_t - z_H^d(z_t)}{1+\pi_{t+1}}\right)\right] + \\ &(1-\sigma)\beta \max\left\{\varepsilon_L u\left[q_L(z_t)\right] + W_{t+1}\left(\frac{z_t - z_L^d(z_t)}{1+\pi_{t+1}}\right), V_{2,t+1}(z_t)\right\}, \end{aligned}$$

where $V_{2,t+1}(z_t)$ is the value function of a buyer entering the decentralized market in period $t+1$ after not having traded in period t. It is easy to see that if buyers anticipate going to the centralized market in the second subperiod then they will always choose to trade regardless of the quality of the good (as long as $S_i > 0$). Then, the function $V_{2,t+1}(z_t)$ is given by

$$V_{2,t+1}(z_t) = \sigma S_H(z_{2,t+1}) + (1-\sigma)S_L(z_{2,t+1}) + W_{t+1}\left(z_{2,t+1}\right).$$

where $S_i(z) = \varepsilon_i u\left[q_i(z)\right] - z_i^d(z)$, with $i = H, L$, and $z_{2,t+1} = z_t/(1+\pi_{t+1})$. Let $\left(q_i(z), z_i^d(z)\right)$ be the take-it-or-leave-it offer that a buyer carrying z units of real balances gives to a seller in a match. Then if the buyer visited the centralized market in the previous period the offer satisfies:

$$\begin{aligned} q_i(z) &= q_{i\pi}^* \text{ and } z_i^d(z) = c(q_{i\pi}^*) \text{ if } c(q_{i\pi}^*) < z, \\ q_i(z) &= c^{-1}(z) \text{ and } z_i^d(z) = z \text{ if } c(q_{i\pi}^*) \geq z. \end{aligned}$$

On the other hand, if the buyer did not visit the centralized market in the previous period then the offer satisfies:

$$\begin{aligned} q_i(z) &= q_i^* \text{ and } z_i^d(z) = c(q_i^*) \text{ if } c(q_i^*) < z, \\ q_i(z) &= c^{-1}(z) \text{ and } z_i^d(z) = z \text{ if } c(q_i^*) \geq z. \end{aligned}$$

It can be shown that $(1+\pi)c(q_i^*) < c(q_{i\pi}^*)$ and $q_{L\pi}^* < q_{H\pi}^*$. Then, in equilibrium z_t will be always lower than $c(q_{H\pi}^*)$ but it may be larger than $c(q_{L\pi}^*)$. To see this, note that the surplus of a buyer that visited the centralized market in the previous period and met a type i seller is an increasing function of z for all z lower than $c(q_{i\pi}^*)$. Then, even if carrying additional units of money would not increase the surplus obtained when the seller produces low quality goods, it may increase the surplus when the seller produces high quality goods, and this extra surplus may be enough to give the buyers incentives to carry the extra money.[15]

As before, we concentrate in steady state equilibria with a constant inflation rate. Let z_γ be the steady state equilibrium real money balances when gross inflation equals γ. Whether potential

[15] This point was made by Lagos and Wright (2004) in a generalization of this environment, but where agents go to the central market every period.

buyers always buy the low quality good will depend on two factors: (i) the level of inflation; and (ii) the relative quality of the high-quality good. For this reason, we will fix the value of ε_L and study the (steady state) equilibrium set in the space of γ and ε_H. Recall that by assumption $\varepsilon_H \geq \varepsilon_L$.

First define $z_{L\gamma}$ as the value of real money holdings in an economy where $\varepsilon_H = \varepsilon_L$. We know that $z_{L\gamma}$ solves the following equation

$$S_L'(z_{L\gamma}) + 1 - \frac{\gamma}{\beta^2} = 0.$$

In what follows, we will only consider values of the inflation rates such that $z_{L\gamma} > 0$. Let $\overline{\gamma}$ be the highest inflation rate such that $z_{L\gamma} > 0$. The value of $\overline{\gamma}$ is always positive and depends on the properties of the utility function. For some utility functions, $z_{L\gamma}$ would be positive for all possible inflation rates and hence $\overline{\gamma} = \infty$. However, we do not need to make this assumption here.

Now note that in steady state we have that

$$V_2\left(z_{\gamma\varepsilon_H}, \gamma, \varepsilon_H\right) = \sigma S_H\left(\frac{z_{\gamma\varepsilon_H}}{\gamma}, \varepsilon_H\right) + (1-\sigma) S_L\left(\frac{z_{\gamma\varepsilon_H}}{\gamma}\right) + W_{\gamma\varepsilon_H}\left(\frac{z_{\gamma\varepsilon_H}}{\gamma}\right).$$

where $z_{\gamma\varepsilon_H}$ indicates the steady state equilibrium real balances when inflation is equal to γ and the utility from consuming the high quality good is given by $\varepsilon_H u(q)$. Let us now define the auxiliary function $G(z_{\gamma\varepsilon_H}, \gamma, \varepsilon_H) \equiv S_L\left(z_{\gamma\varepsilon_H}\right) + \left[1 - (1/\gamma)\right] z_L^d(z_{\gamma\varepsilon_H}) + W_{\gamma\varepsilon_H}\left(z_{\gamma\varepsilon_H}/\gamma\right) - V_2\left(z_{\gamma\varepsilon_H}, \gamma, \varepsilon_H\right)$. If $G(z_{\gamma\varepsilon_H}, \gamma, \varepsilon_H) \geq 0$ then buyers always trade the low quality good and if $G(z_{\gamma\varepsilon_H}, \gamma, \varepsilon_H) < 0$ then buyers only trade the low quality good after not trading for one period.

Proposition 4.1. *(1) For any $\gamma > 1$ (with $\gamma < \overline{\gamma}$) there exist $\varepsilon_H^T > \varepsilon_L$ such that for all $\varepsilon_H < \varepsilon_H^T$ (with $\varepsilon_H > \varepsilon_L$) we have that $G(z_{\gamma\varepsilon_H}, \gamma, \varepsilon_H) > 0$.*

(2) For any $\varepsilon_H > \varepsilon_L$ there exist $\gamma^T > 1$ such that for all $\gamma < \gamma^T$ (with $\gamma > 1$) we have that $G(z_{\gamma\varepsilon_H}, \gamma, \varepsilon_H) < 0$.

Proof. The function $G(z_{\gamma\varepsilon_H}, \gamma, \varepsilon_H)$ is continuous in $\gamma \geq 1$ and $\varepsilon_H \geq \varepsilon_L$. We can rewrite $G(z_{\gamma\varepsilon_H}, \gamma, \varepsilon_H)$ as follows

$$G(z_{\gamma\varepsilon_H}, \gamma, \varepsilon_H) = \left[S_L\left(z_{\gamma\varepsilon_H}\right) - S_L\left(\frac{z_{\gamma\varepsilon_H}}{\gamma}\right)\right] - \sigma\left[S_H\left(\frac{z_{\gamma\varepsilon_H}}{\gamma}, \varepsilon_H\right) - S_L\left(\frac{z_{\gamma\varepsilon_H}}{\gamma}\right)\right] + \left(1 - \frac{1}{\gamma}\right) z_L^d(z_{\gamma\varepsilon_H})$$

To prove the first statement of the proposition, note that

$$\lim_{\varepsilon_H \to \varepsilon_L} G(z_{\gamma\varepsilon_H}, \gamma, \varepsilon_H) = \left[S_L\left(z_{L\gamma}\right) - S_L\left(\frac{z_{L\gamma}}{\gamma}\right)\right] + \left(1 - \frac{1}{\gamma}\right) z_{L\gamma}.$$

Since $S_L(z)$ is increasing in z for $z < c\left(q_L^*\right)$ and constant for $z \geq c\left(q_L^*\right)$ we have that $S_L\left(z_{L\gamma}\right) - S_L\left(\frac{z_{L\gamma}}{\gamma}\right) \geq 0$ which implies that $\lim_{\varepsilon_H \to \varepsilon_L} G(z_{\gamma\varepsilon_H}, \gamma, \varepsilon_H) > 0$. The first statement of the proposition is a direct implication of this result.

To prove the second statement note that

$$\lim_{\gamma \to 1} G(z_{\gamma\varepsilon_H}, \gamma, \varepsilon_H) = -\sigma\left[S_H\left(z_{1\varepsilon_H}, \varepsilon_H\right) - S_L\left(z_{1\varepsilon_H}\right)\right].$$

It is easy to see that for any $\varepsilon_H > \varepsilon_L$ and any positive z we have that $S_H(z) - S_L(z) > 0$. Hence, it follows that $\lim_{\gamma \to 1} G(z_{\gamma\varepsilon_H}, \gamma, \varepsilon_H) < 0$ and the second statement of the proposition is a direct implication of this result. ■

The proposition shows that when the high-quality goods are not much better than the low-quality goods there is an inflation rate for which potential buyers buy the low-quality good in every opportunity they get (statement 1). Furthermore, for those relative qualities of goods there is also a low enough inflation rate such that buyers not always buy the low quality good when they have the chance (statement 2). This result captures the idea that when inflation is high, agents tend to become less choosy, hence buying goods of lower quality.

4.2. Discussion

Under the standard Lagos and Wright (2005) timing agents have access to the centralized market every period, irrespective of their trading experience. When this is the case and a buyer with z_t units of real balances meets a seller producing low quality goods the value for the buyer of accepting the trade is $\varepsilon_L u\left[q_L\left(z_t\right)\right] + W_t\left[z_t - z_t^d(z_t)\right] = S_L(z_t) + W_t(z_t)$, while the value of not trading is $W_t(z_t)$. Hence, as long as $S_L(z_t)$ is positive the agent always chooses to trade and the value of holding z_t units of real balances at the beginning of period t is given by

$$V_{1t}(z_t) = \sigma S_H(z_t) + (1-\sigma) S_L(z_t) + z_t + W_t(0).$$

In other words, in the Lagos and Wright (2005) framework it is never the case that inflation changes the likelihood of trade for low-quality producers. This result is, of course, in stark contrast with the proposition in this section which shows that, under the new timing, agents become more choosy at low inflation levels and may choose to wait for high quality goods before trading.

In the model of this paper agents do not choose whether to be high- or low-quality producers. It is a straightforward extension to introduce this kind of extensive margin. Note however that to make the (potential) sellers' decision interesting we would need to change the bargaining rule being used. Basically, if buyers have all the bargaining power as it is the case in this paper, then sellers will always be indifferent between producing low- or high-quality goods: Sellers always get zero surplus from trade. However, a simple sharing rule for which the seller obtains a differential surplus from producing and trading the alternative qualities is not hard to implement (see Rocheteau and Waller, 2004).

Such a modification of the basic model would allow us to capture the kind of inefficiencies in production that Tommasi (1999) seeks to stress in his analysis of the welfare cost of inflation. If as it was shown here, under high inflation buyers are more willing to buy low quality goods in an effort to quickly spend their depreciating money holdings, then higher levels of inflation open the door to the possibility that more sellers, anticipating the favorable trading behavior of buyers, become (inefficient) low-quality producers.

One important dimension that was left unexplored in this section is the possibility that buyers respond to inflation by increasing the number of sellers that they sample every period. Note however that by not (always) trading with low-quality sellers, the buyers get to sample two sellers while holding a given batch of money balances. But the intensity of sampling is fixed in the model. Specifically, I have (exogenously) restricted each buyer to meet a single seller every period. One possibility to relax this assumption is to allow the level of search intensity to meaningfully influence the probability of a match, as in the previous section. Another alternative is to model more explicitly the within period sampling decision of buyers. In this line of research, Head and Kumar (2005) study a related economy where information frictions create a nondegenerate distribution of prices and buyers can choose the number of sellers that they wish to sample per period. With this added flexibility they are able to show that the number of sellers that a "representative" buyer samples in equilibrium increases with inflation. However, their result is a consequence of the fact that inflation increases the relative price dispersion in the economy. Combining Head and Kumar's (2005) approach with the timing in this paper may reinforce their result.

5. CONCLUDING REMARKS

Lagos and Rocheteau (2005) assume that it is costless for an agent to visit the centralized market, and hence, the agent visits this market every period. Of course, Lagos and Rocheteau take this assumption directly from the original paper by Lagos and Wright (2005). This is a simplifying assumption that has shown to be very useful to answer many interesting questions. But it comes at a cost. In particular, models with this assumption fail to capture the common Irving Fisher's intuition that when inflation is high agents want to use their money holdings more quickly. That is, the idea that when inflation is high, money becomes a "hot potato."

This paper provides a simple modification of the Lagos and Rocheteau (2005) model to illustrate a feature of the economic environment that can create a "hot potato" effect of inflation. In particular, I show that one way to generate this kind of effects is to have agents not visit the centralized market every period. Clearly, this would be the case if there were some fixed cost of visiting the centralized market. Such fixed transaction costs are not far-fetched and, although they may well be less important today than in Fisher's days, they are probably still important determinants of the reaction of the economy to relatively high levels of expected inflation. The next natural step is then to explicitly model these transaction costs.

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Appendix 1: The Cash-in-Advance Economy

Consider the following minor variation of the standard cash-in-advance economy: agents can exert some trade effort e to increase their probability of making a trade, αe. Aside from this modification, consider an economy that is similar to that presented in Cooley and Hansen (1989), but with linear production and no capital. Trade effort has a utility cost $v(e)$, with $v' > 0$ and $v'' > 0$. Also as in Cooley and Hansen (1989), the representative agent in the economy has quasilinear preferences (i.e., linear in work effort h_t).

The problem of the agent is

$$\max \sum_{t=0}^{\infty} \beta^t \left[\alpha e_t u(c_t) - h_t - v(e_t) \right]$$

subject to the budget constraint:

$$\alpha e_t c_t + (1 + \pi_{t+1}) z_{t+1} + b_{t+1} = h_t + z_t + R_t b_t + \tau_t,$$

where b_t is a real bond and τ_t is a transfer; the cash-in-advance constraint $c_t \leq z_t$; and the standard non-negativity constraints. Assume that $\pi_t = \pi$, constant for all t and concentrate in steady state equilibria. Assume also that π is high enough that the cash-in-advance constraint is binding. In particular, $\pi > \beta$. Then, the following system of equations characterizes the equilibrium levels of consumption ($c = z$) and trade effort (e):

$$\alpha e \left[u'(z) - 1\right] = \frac{\pi - \beta}{\beta},$$
$$\alpha \left[u(z) - z\right] = v'(e).$$

Denote by $[\widehat{e}(\pi), \widehat{z}(\pi)]$ the solution of this system. Then, it is easy to see that $d\widehat{e}(\pi)/d\pi < 0$. The logic is similar to the one explaining the result in Lagos and Rocheteau (2005): Since $\pi > \beta$, we have that $u'(\widehat{z}) > 1$ and hence $u(\widehat{z}) - \widehat{z}$ is increasing in $\widehat{z}$. An increase in π tends to reduce $\widehat{z}$ and hence it reduces the marginal benefit of exerting trade effort given by $\alpha \left[u(\widehat{z}) - \widehat{z}\right]$. As a consequence, agents exert less trade effort.

By increasing the trade effort in period t the agent will increase the likelihood of spending z_t. However, this effort does not reduce the inflation tax $\pi_t z_t$ paid by the agent, which in the standard cash-in-advance setup applies to cash balances at the beginning of the period (independently of whether those balances are later used during the period).