

# Monetary Policy in an Estimated DSGE Model with a Financial Accelerator\*

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## Preliminary and Incomplete

### Abstract

This paper estimates a sticky-price DSGE model with a financial accelerator to assess the evidence for the existence and importance of financial frictions in the amplification and propagation of the effects of transitory shocks. Structural parameters of two models, one with and one without a financial accelerator, are estimated using a maximum-likelihood procedure and post-war US data. The estimation results provide some quantitative evidence in favour of the financial accelerator model, but the simulation results do not. The sensitivity of the external finance premium to changes in the leverage ratio is found to be lower in the post-1979 period than over the two preceding decades.

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# 1 Introduction

An extensive literature has argued that credit market frictions may amplify and propagate conventional interest rate effects (for example Bernanke and Gertler 1989, 1995 and Carlstrom and Fuerst 1997). A growing number of studies have used dynamic stochastic general equilibrium (DSGE) models to analyze the role of credit market frictions in economic fluctuations. Bernanke, Gertler and Gilchrist's (1999) (BGG hereafter) financial accelerator model is the basic model underpinning much of the research on the role of financial frictions in the business cycles. Their framework links the cost of firms' external finance to the quality of their balance sheet.<sup>1</sup> Entrepreneurs, who borrow funds to undertake investment projects, face an external finance premium that rises as their personal stake in the project (net worth) falls. Declines in net worth lead to tighter financing conditions, reducing the demand for capital. This sets off an "accelerator" effect because the value of the capital held by firms (net worth) declines as the demand for capital falls resulting in a further rise in the cost of financing.

A number of studies have used this financial accelerator mechanism to account for macroeconomic developments at times of financial crisis. Cespedes, Chang and Velasco (2004), Gertler, Gilchrist and Natalucci (2003) and Tovar (2003, 2004) consider the case of open economies in emerging markets. Christiano, Motto and Rostagno (2004) use the financial accelerator in their analysis of the Great Depression in the U.S.

BGG (1999) argue that introducing a financial accelerator mechanism is not

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<sup>1</sup>An alternative approach is to introduce financial frictions by giving financial intermediaries an ability to change credit conditions without a change in borrower creditworthiness. Examples of these studies are Cook (1999), Cooper and Ejarque (2000), Atta-Mensah and Dib (2003), and Meh and Moran (2004).

only useful for understanding periods of financial crisis, but can improve the ability of otherwise standard models to explain normal cyclical fluctuations. Earlier work by Carlstrom and Fuerst (1997) demonstrated the quantitative importance of this mechanism. They include the same type of financial frictions in an otherwise standard RBC model and find that it can reproduce the hump-shaped output response to shocks as seen in the data. Using a sticky-price model calibrated to U.S. data during normal times (rather than crisis episodes), BGG show that the financial accelerator amplifies the impact of shocks and provides a quantitatively important mechanism that propagates shocks at business cycle frequencies. Subsequent work using the BGG model for other countries has found similar results. Hall (2001) finds that a model of the U.K. economy with a financial accelerator can explain the abnormally weak investment growth seen in the early 1990s. Fukunaga (2002) finds that a model with a financial accelerator calibrated to the Japanese economy is able to account for the large volatility of Japanese investment.

Much of the previous research has used calibrated models to assess the importance of the financial accelerator. In contrast, the focus of this paper is to econometrically test for the presence of credit market frictions and evaluate their importance in the amplification and propagation of transitory shocks to macroeconomic variables. To this end, we develop and estimate a sticky-price DSGE model that includes a financial accelerator à la BGG. Christiano, Motto and Rostagno (2004) and Tovar (2004) have also considered the impacts of the financial accelerator in estimated models. What distinguishes our study from these is the focus on the importance of the financial accelerator during non-crisis periods.

The closed-economy model developed here is based on Dib (2002) and Ireland

(2001, 2003) to which we introduce a financial accelerator à la BGG.<sup>2</sup> We assume that the economy is disturbed by four transitory shocks: technology, money demand, monetary policy, and preference shocks. Our model differs from BGG in its characterization of monetary policy by a modified Taylor-type rule. We assume that the Federal Reserve manages short-term interest rates in response to inflation, output, and money growth changes and smooths the interest rates. In addition, we allow for the possibility of debt deflation and a utility function that is non-separable in consumption and real balances.

The structural parameters of the model, including those related to the financial accelerator are estimated econometrically using post-war quarterly US data and a maximum-likelihood procedure with a Kalman filter. This estimation procedure is used in Dib (2002), Ireland (2001,2003), Christiano, Motto and Rostagno (2004), and Tovar (2004).<sup>3</sup> We also estimate a constrained version of the model in which the financial accelerator is turned-off. Estimating these two versions of the model allows us to econometrically test for the presence of the financial accelerator and assess its importance in explaining business cycles.

As shown in previous research, the behaviour of the monetary authority affects the degree to which financial frictions amplify and propagate the effects of transitory shocks on macroeconomic variables. Parameterizations of the monetary policy rule that are more aggressive with respect to output and inflation deviations can reduce the impact of the financial accelerator (see BGG 1999 and Fukunaga 2002). Clarida, Gali, Gertler (2000), among others, argue that a fundamental change in the Federal Reserve policy occurred in mid-1979. Therefore, we allow the possibility that the

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<sup>2</sup>We do not work out the microeconomic contracting problem between lender and borrower, but rely instead on the aggregation results of BGG.

<sup>3</sup>Christiano, Motto and Rostagno (2004) and Tovar (2004) add measurement errors while they estimate their models.

monetary policy parameters have changed during the post-1979 period. As in Ireland (2001, 2003), we estimate both versions of the model for two disjoint subsamples. The first sample runs from 1959:1 to 1979:2, while the second runs from 1979:3 to 2003:3.

The estimation results indicate that the parameters related to the financial accelerator and most of those of the monetary policy rule are statistically significant. However, their estimated values are different in each subsample. In the post-1979 period, the external finance premium is less sensitive to firm leverage and monetary policy responds more aggressively to inflation and more modestly to money growth. For both subsamples, the likelihood ratio test easily rejects the model without the financial accelerator in favour of the one with it. The impulse response functions show that introducing the financial accelerator helps to amplify and propagate the effects of transitory shocks to investment with little impact on output.

This paper is organized as follows. Section 2 describes the model. Section 3 describes the data and the econometric method used to estimate the models. Section 4 discusses the empirical results and Section 5 concludes.

## 2 The Model

Our basic model is a closed economy DSGE model with money and price stickiness similar to Dib (2002) and Ireland(2001,2003). The key difference with between our model and this base model is the inclusion of a financial accelerator mechanism identical to that proposed by BGG.

In this model there are three types of producers: entrepreneurs; capital producers; and retailers. Entrepreneurs produce intermediate goods. They borrow from a financial intermediary that converts household deposits into business financ-

ing for the purchase of capital. The presence of asymmetric information between entrepreneurs and lenders creates a financial friction which makes entrepreneurial demand for capital depend on their financial position. Capital producers build new capital and sell it to entrepreneurs. Changes in the supply of or demand for capital will lead the price of capital to fluctuate and further propagate the shocks. Retailers are the source of nominal frictions. They differentiate intermediate goods and sell them in monopolistically competitive retail markets. They set nominal prices in a staggered fashion à la Calvo (1983). The nominal rigidity gives monetary policy a role in this model.

## 2.1 Households

The representative household derives utility from consumption,  $c_t$ ; real money balances,  $M_t/p_t$ ; and leisure,  $1 - h_t$ . Its preferences are described by the following expected utility function:

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, M_t/p_t, h_t), \quad (1)$$

where  $\beta \in (0, 1)$  is the discount factor,  $M_t$  is holdings of nominal money balances,  $h_t$  is labour supply, and  $p_t$  is the consumer price level. The single-period utility function is specified as:

$$u(\cdot) = \frac{\gamma z_t}{\gamma - 1} \log \left[ c_t^{\frac{\gamma-1}{\gamma}} + b_t^{1/\gamma} \left( \frac{M_t}{p_t} \right)^{\frac{\gamma-1}{\gamma}} \right] + \eta \log(1 - h_t), \quad (2)$$

where  $\gamma > 0$  and  $\eta > 0$  denote the constant elasticity of substitution between consumption and real balances, and the weight on leisure in the utility function, respectively. We interpret  $z_t$  as a taste (preference) shock, while  $b_t$  is interpreted as

a money demand shock. These shocks follow first-order autoregressive processes:

$$\log(z_t) = \rho_z \log(z_{t-1}) + \varepsilon_{zt}, \quad (3)$$

and

$$\log(b_t) = (1 - \rho_b) \log(b) + \rho_b \log(b_{t-1}) + \varepsilon_{bt}, \quad (4)$$

where  $\rho_z, \rho_b \in (-1, 1)$  are autoregressive coefficients,  $b$  is constant, and the serially uncorrelated shocks  $\varepsilon_{zt}$  and  $\varepsilon_{bt}$  are normally distributed with zero means and standard deviations  $\sigma_z$  and  $\sigma_b$ , respectively.

The representative household enters period  $t$  with  $d_{t-1}$  units of real deposits in the financial intermediary; nominal money balances,  $M_{t-1}$ ; and nominal bonds,  $B_{t-1}$ . While deposits,  $d_t$ , at the financial intermediary pay interest, money balances,  $M_t$ , are money held outside of banks (cash) or low interest bearing savings instruments such as chequing accounts.<sup>4</sup> The inclusion of money balances is motivated, in part, by empirical evidence that money demand shocks matter for business cycles. During period  $t$  the household chooses to consume,  $c_t$ ; purchase new government bonds,  $B_t$ ; change money balances  $\frac{M_t}{p_t}$ ; deposit funds at the financial intermediary,  $d_t$ ; and work  $h_t$ . The budget constraint is

$$c_t + \frac{d_t}{R_t} + \frac{M_t + B_t/R_t^n}{p_t} \leq \frac{W_t}{p_t} h_t + d_{t-1} + \frac{M_{t-1} + B_{t-1} + T_t + D_t}{p_t}, \quad (5)$$

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<sup>4</sup>The real return on bonds and deposits is the same in equilibrium. We introduce nominal (bonds) and real (deposits) assets to derive explicitly the Fisher equation.

First-order conditions for the household optimization problem are:

$$\frac{z_t c_t^{-\frac{1}{\gamma}}}{c_t^{\frac{\gamma-1}{\gamma}} + b_t^{1/\gamma} m_t^{\frac{\gamma-1}{\gamma}}} = \lambda_t; \quad (6)$$

$$\frac{z_t b_t^{1/\gamma} m_t^{-\frac{1}{\gamma}}}{c_t^{\frac{\gamma-1}{\gamma}} + b_t^{1/\gamma} m_t^{\frac{\gamma-1}{\gamma}}} = \lambda_t - \beta E_t \left( \frac{\lambda_{t+1}}{\pi_{t+1}} \right); \quad (7)$$

$$\frac{\eta}{1 - h_t} = \lambda_t w_t; \quad (8)$$

$$\frac{1}{R_t} = \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \right]; \quad (9)$$

$$\frac{1}{R_t^n} = \beta E_t \left[ \frac{\lambda_{t+1}}{\pi_{t+1} \lambda_t} \right], \quad (10)$$

where  $\lambda_t$  is the Lagrangian multiplier associated with the budget constraint;  $m_t = M_t/p_t$ ,  $w_t = W_t/p_t$ ,  $\pi_{t+1} = p_{t+1}/p_t$ .

## 2.2 Production sector

### 2.2.1 Entrepreneurs

The entrepreneurs' behaviour is similar to that proposed by Bernanke, Gertler and Gilchrist (1999). Entrepreneurs manage firms that produce wholesale goods and borrow to finance the capital used in the production process. Entrepreneurs are risk neutral and have a finite expected horizon for planning purposes. The probability that an entrepreneur will survive until the next period is  $\nu$ , so the expected lifetime horizon is  $1/(1 - \nu)$ . This assumption ensures that entrepreneurs' net worth (the firm equity) will never be enough to fully finance the new capital acquisition. In essence, they issue debt contracts to finance their desired investment expenditures in excess of net worth.

At the end of each period, entrepreneurs purchase capital that will be used in the next period,  $q_t k_{t+1}$ . The capital acquisition is financed partly by their net worth

$n_{t+1}$  and by borrowing  $q_t k_{t+1} - n_{t+1}$  from a financial intermediary. This intermediary obtains its funds from household deposits and faces an opportunity cost of funds equal to the economy's riskless rate of return,  $R_t^n$ .

The entrepreneurs' demand for capital depends on the expected marginal return and the expected marginal external financing cost. Consequently,

$$E_t f_{t+1} = E_t \left[ \frac{r_{kt+1} + (1 - \delta)q_{t+1}}{q_t} \right], \quad (11)$$

where  $f_{t+1}$  is the external funds rate and  $r_{kt+1}$  is the marginal productivity of capital at  $t+1$ . Following BGG (1999), we assume the existence of an agency problem that makes external finance more expensive than internal funds. The entrepreneurs costlessly observe their output which is subject to a random outcome. The financial intermediaries incur an auditing cost to observe an entrepreneur's output. After observing his project outcome, an entrepreneur decides whether to repay his debt or to default. If he defaults the financial intermediary audits the loan and recovers the project outcome less monitoring costs.

Accordingly, the marginal external financing cost is equal to a gross premium for external funds plus the gross real opportunity costs equivalent to the riskless interest rate. Thus, the demand for capital should satisfy the following optimality condition:

$$E_t f_{t+1} = E_t [S(\cdot)R_t], \quad (12)$$

where  $E_t R_t = E_t (R_t^n / \pi_{t+1})$  is a riskless real interest rate and

$$S(\cdot) = E_t \left( \frac{n_{t+1}}{q_t k_{t+1}} \right)^{-\psi}, \quad (13)$$

with  $S'(\cdot) < 0$  and  $S(1) = 1$ . The parameter  $\psi$  is the elasticity of the external finance premium with respect to the leverage ratio, i.e., the borrower's share of project. Let

$\kappa = qk/n$  be the leverage ratio in the steady-state equilibrium.  $\kappa$  will be estimated among the model's structural parameters.

The gross external finance premium  $S(\cdot)$  depends on the size of the borrowers equity stake in project (or alternatively the borrowers leverage ratio). As  $n_{t+1}/q_t k_{t+1}$  falls, the borrower relies on uncollateralized borrowing (higher leverage) to a larger extent to fund his project. Since this increases the incentive to misreport the outcome of the project the loan becomes riskier and the cost of borrowing rises.<sup>5</sup>

Aggregate entrepreneurial net worth evolves according to

$$n_{t+1} = \nu v_t + (1 - \nu)g_t, \quad (14)$$

where  $v_t$  denotes the net worth of surviving entrepreneurs net of borrowing costs carried over from the previous period.  $1 - \nu$  is the share of new entrepreneurs entering the economy and  $g_t$  is the transfer or "seed money" that newly entering entrepreneurs receive from entrepreneurs that die and depart from the scene.  $v_t$  is given by

$$v_t = [f_t q_{t-1} k_t - E_{t-1} f_t (q_{t-1} k_t - n_t)] \quad (15)$$

where  $f_t$  is the ex post real return on capital held in  $t$ , and  $E_{t-1} f_t$  is the ex post cost of borrowing. Earnings from operations this period become next period's net worth.

To produce output  $y_t$ , the entrepreneurs use  $k_t$  units of capital and  $h_t$  units of labour following constant-returns-to-scale technology:

$$y_t \leq k_t^\alpha (A_t h_t)^{1-\alpha}, \quad \alpha \in (0, 1), \quad (16)$$

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<sup>5</sup>Note that when the riskiness of loans increases the agency costs rise and the lender's expected losses increase. A higher external finance premium paid by successful entrepreneurs offsets these higher losses and ensures that there is no change to the return on deposits for households.

where  $A_t$  is a technology shock that is common to all entrepreneurs. The technology shock  $A_t$  is assumed to follow the autoregressive process

$$\log A_t = (1 - \rho_A) \log(A) + \rho_A \log(A_{t-1}) + \varepsilon_{At}, \quad (17)$$

where  $\rho_a \in (-1,1)$ ,  $A > 0$ , and  $\varepsilon_{At}$  is normally distributed with zero mean and standard deviation  $\sigma_A$ .

The first-order conditions for this optimization problem are

$$r_{kt} = \alpha \frac{y_t \xi_t}{k_t \lambda_t}; \quad (18)$$

$$w_t = (1 - \alpha) \frac{y_t \xi_t}{h_t \lambda_t}; \quad (19)$$

$$y_t = k_t^\alpha (A_t h_t)^{1-\alpha}. \quad (20)$$

$$(21)$$

where  $\xi_t > 0$  is the Lagrangian multiplier associated with the technology function, and  $\xi_t/\lambda_t$  is the real marginal cost,  $MC_t/p_t$ .<sup>6</sup>

### 2.2.2 Capital producers

Capital producers use a linear technology to produce capital goods,  $i_t$ , sold at the end of period  $t$ . They also use a fraction of final goods purchased from retailers. The produced capital goods replace depreciated capital and add to the capital stock. We assume that capital producers are subject to quadratic capital adjustment costs. Their optimization problem, in real terms, is:

$$\max_{i_t} = E_t \left[ q_t i_t - i_t - \frac{\chi}{2} \left( \frac{i_t}{k_t} - \delta \right)^2 k_t \right]. \quad (22)$$

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<sup>6</sup>As in Bernanke, Gertler and Gilchrist (1999), we assume that entrepreneurial consumption is small and it drops out of the model

Thus, the optimal condition is

$$E_t \left[ q_t - 1 - \chi \left( \frac{i_t}{k_t} - \delta \right) \right] = 0; \quad (23)$$

which is the standard Tobin's  $Q$  equation that relates the price of capital to the marginal adjustment costs.

The quantity and price of capital are determined in the market for capital. The entrepreneurial demand curve for capital is determined by equations (11) and (18) and the supply of capital is given by equation (23). The intersection of these curves gives the quantity and price of capital. Capital adjustment costs slow down the response of investment to different shocks, which directly affects the price of capital.

Furthermore, the aggregate capital evolves according to

$$k_{t+1} = i_t + (1 - \delta)k_t. \quad (24)$$

### 2.2.3 Retailers

The retailers purchase the wholesale goods at a price equal to nominal marginal costs  $MC_t$  and differentiate them at no cost.<sup>7</sup> They then sell these differentiated retail goods on a monopolistically competitive market. Following Calvo (1983), we assume that retailers cannot change their selling prices unless they receive a random signal. The constant probability of receiving such a signal is  $(1 - \phi)$ . Thus, each retailer  $j$  sets the price  $\bar{p}_t(j)$  that maximizes the expected profit for  $l$  periods.<sup>8</sup> The retailer's optimization problem is

$$\max_{\{\bar{p}_t(j)\}} E_0 \left[ \sum_{t=0}^{\infty} (\beta\phi)^l \lambda_{t+l} D_{t+l}(j) / p_{t+l} \right], \quad (25)$$

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<sup>7</sup>The retail sector is used only to introduce nominal rigidity into this economy.

<sup>8</sup> $l$  is the average length of time a price remains unchanged,  $l = 1/(1 - \phi)$ .

subject to<sup>9</sup>

$$y_{t+l}(j) = \left( \frac{\bar{p}_t(j)}{p_{t+l}} \right)^{-\theta} y_{t+l}, \quad (26)$$

where the retailer's profit function is

$$D_{t+l}(j) = (\bar{p}_t(j) - MC_{t+l}) y_{t+l}(j). \quad (27)$$

The first-order condition is:

$$\bar{p}_t(j) = \frac{\theta}{\theta - 1} \frac{E_t \sum_{l=0}^{\infty} (\beta\phi)^l \lambda_{t+l} MC_{t+l} y_{t+l}(j) / p_{t+l}}{E_t \sum_{l=0}^{\infty} (\beta\phi)^l \lambda_{t+l} y_{t+l}(j) / p_{t+l}}. \quad (28)$$

The aggregate price is

$$p_t^{1-\theta} = \phi p_{t-1}^{1-\theta} + (1 - \phi) \bar{p}_t^{1-\theta}. \quad (29)$$

These equations lead to the following New Keynesian Phillips curve

$$E_t \hat{\pi}_{t+1} = \hat{\pi}_t - \frac{(1 - \beta\phi)(1 - \phi)}{\phi} \hat{m}c_t \quad (30)$$

where  $m c_t$  is real marginal cost and  $\hat{x}_t = \log(x_t/x)$ .

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<sup>9</sup>This demand function is derived from the definition of aggregate demand as the composite of individual final output (retail) goods and the corresponding price index in the monopolistic competition framework of Dixit and Stiglitz (1997) as follows

$$y_{t+l} = \left( \int_0^1 y_{t+l}(j)^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}$$

$$p_{t+l} = \left( \int_0^1 p_{t+l}(j)^{1-\theta} dj \right)^{\frac{1}{1-\theta}}$$

where  $y_{t+l}(j)$  and  $p_{t+l}(j)$  are the demand and price faced by each individual retailer  $j \in (0, 1)$

### 2.3 Monetary authority

Following Ireland (2004), the central bank adjusts the nominal interest rate,  $R_t^n$ , in response to deviations of inflation,  $\pi_t = p_t/p_{t-1}$ , output,  $y_t$ , and money growth rate  $\mu_t = M_t/M_{t-1}$  from their steady-state values. The policy rule also contains an interest rate smoothing term,  $R_{t-1}^n$ . Thus, the monetary policy rule evolves according to:

$$\log(R_t^n/R^n) = \varrho_R \log(R_{t-1}^n/R^n) + \varrho_\pi \log(\pi_t/\pi) + \varrho_y \log(y_t/y) + \varrho_\mu \log(\mu_t/\mu) + \varepsilon_{Rt} \quad (31)$$

where  $R^n$ ,  $\pi$ ,  $y$ , and  $\mu$  are the steady-state values of  $R_t^n$ ,  $\pi_t$ ,  $y_t$ , and  $\mu_t$ , respectively;  $\varepsilon_{Rt}$  is a monetary policy shock normally distributed with zero mean and standard deviation  $\sigma_R$ . The newly created money is transferred to the household, so  $T_t = M_t - M_{t-1}$ . By reacting to money growth deviations, the central bank tries to insulate the economy from the effects of money demand shocks.

We choose this policy rule to provide more flexibility in the characterization of monetary policy than the rule in BGG, which contains only the interest rate smoothing term and the lagged deviation of inflation from its steady state.<sup>10</sup> Allowing for a stronger output stabilizing response of monetary policy may have an impact on the conclusions regarding the importance of the financial accelerator. Also, the parameters of the policy rule will be estimated over two subsamples, so this flexibility will help to better characterize any change in Federal Reserve behaviour. For example, if  $\varrho_\mu$  is non-zero, monetary policy can be considered to influence a linear combination of the interest rate and money growth to achieve a target for inflation. In addition, Andrés, López-Salido and Nelson (2004) argue that including money growth in an interest-rate rule may be considered as an optimal reaction function

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<sup>10</sup>In addition, they set the weight on inflation deviations equal to 0.11.

when the central bank's loss equation contains money growth variability.

## 2.4 Symmetric equilibrium

In the symmetric equilibrium, all entrepreneurs are identical, so they make the same decision. In this economy, the symmetric equilibrium consists of an allocation  $\{y_t, c_t, m_t, i_t, h_t, k_t, n_t\}$  and a sequence of prices and co-state variables  $\{w_t, r_{kt}, R_t^n, R_t, f_t, q_t, \lambda_t, mc_t\}$  that satisfy the optimality conditions of households, capital producers, entrepreneurs, and retailers; the money-supply rule; and the stochastic processes for preferences, money demand, technology, and policy shocks (see Appendix A).

Taking a log-linear approximation of the equilibrium system around steady-state values and using Blanchard and Khan's (1980) procedure yields a state-space solution of the form:

$$\hat{\mathbf{s}}_{t+1} = \Phi_1 \hat{\mathbf{s}}_t + \Phi_2 \varepsilon_{t+1}, \quad (32)$$

$$\hat{\mathbf{d}}_t = \Phi_3 \hat{\mathbf{s}}_t. \quad (33)$$

The state variable vector,  $\hat{\mathbf{s}}_t$ , includes predetermined and exogenous variables;  $\hat{\mathbf{d}}_t$  is the vector of control variables; and the vector  $\varepsilon_t$  contains the random innovations. The coefficient matrices,  $\Phi_1$ ,  $\Phi_2$ , and  $\Phi_3$ , have elements that depend on the structural parameters of the model. Therefore, the state-space solution, (34)–(35), is used to estimate and simulate the model.

## 3 Calibration and Data

As in previous studies that estimate DSGE models by a maximum-likelihood procedure, some parameters have to be set prior to estimation because the data used contain little information about them. Thus, the parameter  $\eta$ , denoting the weight

on leisure in the utility function, is set equal to 1.315, so that the household spends around 33 per cent of its time in market activities. The degree of retailers' monopoly power,  $\theta$ , is set equal to 6, which implies a gross steady-state price markup of 1.20. The share of capital in production,  $\alpha$ , and the depreciation rate,  $\delta$ , are assigned commonly used values of 0.33 and 0.025, respectively. Finally, the probability that an entrepreneur will survive for the next period,  $\nu$ , is set equal to 0.9728, as in BGG (1999).<sup>11</sup>

The remaining non-calibrated parameters are estimated using a maximum-likelihood procedure with a Kalman filter. This method applies a Kalman filter to a model's state-space form to generate series of innovations used to evaluate the likelihood function for the sample.<sup>12</sup> Using quarterly US data from 1959Q1 through 2003Q3, we estimate two versions of the model. The first is a model with agency costs, i.e., a financial accelerator (hereafter referred to as the FA model). The second is the same model with the financial accelerator turned off, i.e.  $\psi = 0$  (the No-FA Model).

The behaviour of the monetary authorities has an impact on the quantitative importance of the financial accelerator. For example, calibrated studies have shown that policy rules that stabilize output will also counteract, and may eliminate, the impact of the financial accelerator on output or investment (BGG, Fukunaga (2002)). To best assess the quantitative importance of the financial accelerator we need to allow for what is widely believed to be a fundamental change in Federal Reserve policy that appeared in mid-1979, ex. Clarida, Galí, and Gertler (2000). As in Ireland (2003), we allow that the monetary policy parameters may change, so we split the full sample period into two separate subsamples. The first subsample runs from 1959Q1 to 1979Q2 and the second runs from 1979Q3 to 2003Q3. Both versions

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<sup>11</sup>Therefore, on the average, an entrepreneur may live 36 years.

<sup>12</sup>This method is described in Hamilton (1994, Chap. 13)

of the model are estimated for each subsample.

Output is measured by real GDP. Real balances are measured by dividing the M1 money stock by the GDP deflator. These two series are expressed in per capita terms using the civilian population aged 16 and over. The inflation rate is measured by changes in the GDP implicit price deflator, while the short-term nominal interest rate is measured by the rate on three-month treasury bills. All the series are HP-filtered before the estimation.<sup>13</sup>

## 4 Empirical Results

### 4.1 Parameter estimates

Table 1 reports the maximum-likelihood estimates and standard errors of the FA model's structural parameters for both pre- and post-1979 periods. Table 2 reports the same for the model estimated without the financial accelerator. The estimates of the discount factor  $\beta$  exceed 0.9925 for both subsamples. The estimates of  $\gamma$ , the constant elasticity of substitution between consumption and real balances, are around 0.0225 for the pre-1979 subsample and 0.040 for the post-1979 subsample. The estimates of  $b$ , the constant associated with money demand, slightly exceed 0.19 for the first period and 0.12 for the second one.

The capital adjustment cost parameter,  $\chi$ , is estimated precisely. In the pre-1979 subsample it has estimated values of 2.72 and 8.93 in the FA and No-FA models, respectively, as compared to 12.40 and 14.85 for the post-1979 period. These estimates are much higher than the 0.25 value for the adjustment cost parameter used by BGG. However, using a similar econometric methodology, Ireland (2001,2003)

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<sup>13</sup>Inflation and interest rates exhibit a small upward (downward) trend over the pre-1979 (post-1979) sample.

finds estimates of the adjustment cost parameter that are even larger.<sup>14</sup>

The estimates of  $\phi$ , the probability that prices remain unchanged for the next period, exceed 0.66 for the pre-1979 period, while they are relatively smaller in the second period with estimated values between 0.41 in the No-FA model and 0.52 in the model with the financial accelerator. These values indicate that prices remain unchanged for about 3 and 2 quarters in the pre- and post-1979 periods, respectively.<sup>15</sup>

The estimates of all the monetary policy parameters except  $\varrho_y$  are statistically different from zero. The estimates of the smoothing terms,  $\varrho_R$ , are relatively small in the pre-1979 period. They are around 0.70 in the pre-1979 period, but they exceed 0.82 in the post-1979 period. This parameter is always given a value between 0.8 and 0.9 in the calibration. In contrast, the estimates of  $\varrho_\pi$ , the coefficient that measures the response of monetary policy to inflation deviations are, at least, 0.32 in the pre-1979 sample while they exceed 0.82 in the post-1979 period. The estimates of  $\varrho_y$  are very small, even negative in the FA model, and statistically insignificant in all estimations. Ireland (2003) shows a similar result. The estimated values of  $\varrho_\mu$  exceed 0.37 in the pre-1979 period, but are much smaller in the post-1979 period, with estimated values around 0.05.

The estimates of the policy rule parameters over the two subsamples indicate that, during the pre-1979 period, the monetary authority responded to inflation and money deviations to a similar degree. In contrast, since 1979 the monetary authority has responded much more strongly to inflation deviations than to money growth fluctuations. This finding is similar to other empirical studies arguing that the

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<sup>14</sup>The estimated value for  $\chi$  in Ireland (2003) is 12.4 in the pre-1979 sample and 32.1 in the post-1979 sample.

<sup>15</sup>Prices are somewhat stickier in BGG with  $\phi = 0.75$  implying an average period between price adjustments of 4 quarters.

monetary policy rule followed by the Federal Reserve changed with the appointment of Paul Volcker as chairman in 1979. (for example Clarida, Gali and Gertler (2000)).

The estimated values of the parameter  $\psi$ , the elasticity of the external finance premium with respect to the leverage ratio, are statistically significant and equal to 0.0754 and 0.0377 in the pre- and post-1979 periods, respectively. The parameter  $\kappa$ , the steady-state leverage ratio defined as the ratio of net worth to total assets, is estimated to be around 0.50 and 0.45 for the pre and post-1979 periods, respectively. These estimated values are close to those usually used to calibrate this parameter in models with a financial accelerator, for example BGG (1999), Hall (2001), and Fukunaga (2002). In contrast, this literature often sets  $\psi$  at a value of 0.05, between the the two values estimated here. This may be related to the fact that the parameters associated with the financial accelerator have been calibrated with an eye to matching long-run historical averages and thus rely on data that span both of our samples. The estimates of  $\psi$  indicate that the sensitivity of the cost of external funds to the leverage ratio is almost twice as high in the pre-1979 subsample as in the post-1979 subsample. In the BGG model  $\psi$  is a function of the monitoring costs in default (liquidation costs) and the riskiness (variability of returns) of investment projects. One potential explanation for a change in  $\psi$  over the sample is that low and stable inflation and more credible monetary policy have reduced the variance in project returns making the cost of funds less sensitive to leverage.

We use the likelihood-ratio test to test the restrictions imposed by the No-FA model ( $\psi = 0$  and  $\kappa = 1$ ) against the model with the financial accelerator (FA model). Let  $L^u$  and  $L^c$  denote the maximum values of the log-likelihood function for the unconstrained (FA) and constrained (No-FA) models, respectively. The likelihood ratio statistic  $-2(L^c - L^u)$  has a chi-square distribution with two degrees

of freedom under the null hypothesis that the No-FA is valid. The values of  $L^u$  equal to 1372.6 and 1541.6 for the pre- and post-1979 periods, respectively; while those of  $L^c$  equal to 1361.5 and 1534.4 for both subsamples. The 2 per cent critical value for a  $\chi^2(2)$  is 9.21. Therefore, the likelihood ratio test easily rejects the restrictions of the No-FA model in favour of the model that includes a financial accelerator.<sup>16</sup>

## 4.2 Impulse responses

Next we compare the responses of various macroeconomic variables to four different shocks when the financial accelerator is present and when it is not. Figures 1 to 5 display the impulse responses of a set of macroeconomic variables to a 1 per cent shock to the short-term nominal interest rate (tightening of monetary policy), technology (increase in  $A_t$ ), money demand (increase in  $b_t$ ), and preferences (increase in  $z_t$ ). Each variable's response is expressed as the percentage deviation from its steady-state level.

Figures 1 to 4 show the impulse responses generated in the estimated FA model in the post-1979 sample with two other impulse responses in which there is no financial accelerator effect. The green line shows the impulse responses generated by setting  $\psi$  equal to 0, but keeping all of the other parameter estimates from the FA model. This is the same approach as taken in the existing calibrated literature. The red (dash-dot) lines are the impulse responses from the model if we set  $\psi$  equal to 0 and re-estimate all of the other parameters. We refer to the former as the No-FA model and the latter as the Estimated No-FA model. The No-FA model is a useful benchmark because it keeps the policy reaction function and other behaviour constant. Since the likelihood-ratio test rejects the estimated model in which  $\psi$

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<sup>16</sup>For the pre-1979 period,  $-2(L^c - L^u) = 22.2$ , while, for the post-1979 period,  $-2(L^c - L^u) = 14.4$ .

is constrained to equal zero its impulse responses are a less relevant benchmark. Nonetheless, we show them for completeness.

Figure 1 shows that the presence of a financial accelerator does not amplify or propagate the impact of a monetary policy shock on real variables apart from investment. However, the impulse responses of the financial accelerator model show more amplification than those for the estimated No-FA model. This is due to the difference in the estimated values of the structural parameters that are not associated with the financial accelerator. Despite the small impact of the accelerator on real variables, the basic mechanism evident in the impulse responses. Net worth falls because of the declining return to capital and the higher real interest costs associated with existing debt (debt-deflation effect). Since the external finance premium depends negatively on the leverage ratio, the external funds rate goes up and the funding cost of purchasing new capital increases. The demand for new capital decreases further and the expected price of capital falls.

As in previous studies, the FA amplifies the impact of the monetary policy shock on investment, but this amplification is very small. The reason for this result is the high estimate of the capital adjustment cost parameter in this model, which slows the adjustment of the capital stock. As an experiment, we set the capital adjustment cost parameter equal to 0.25, the value considered by BGG, and generated another set of impulse responses. The peak response of investment to the monetary policy shock was much stronger (4.5 per cent) and similar in magnitude to that shown in BGG.

Figure 2 shows that following a positive technology shock the amplification of the output response by the financial accelerator is present but more muted than for the monetary policy shock. Here the technology shock increases the return to

capital pushing up net worth. The decline in inflation that results from the shock increases the real cost of repaying existing debt with a negative effect on net worth. The positive impact on net worth from the higher return to capital dominates, in part due to the endogenous policy response that reduces the disinflationary impact, and net worth rises. Higher net worth decreases the external finance premium and increases the demand for capital. Again the response of investment to the shock is larger when the FA is present. The price of capital rises reinforcing the rise in net worth. As is often found in sticky-price models, hours worked declines after the technology shock as the wealth effect from higher marginal product of labour outweighs the substitution effect. However, the decline in hours worked is less pronounced in the model with the FA. The model estimated with no financial accelerator shows a more persistent response of output, but this is due to a higher estimated persistence coefficient of technology shocks.

Figure 3 shows the impulse responses to a money demand shock. In this case the presence of the FA makes very little difference to the response of most macro variables. The dynamics of net worth, the price of capital, the external finance premium and investment are all consistent with a shock to the demand for nominal balances which reduces the funds available for lending and raises the deposit rate.

Figure 4 shows the impulse responses to a preference (demand) shock. Here also the presence of a financial accelerator has little effect. The preference shock increases the return to capital since output and hours worked rise. This and the reduced real cost of debt repayment due to the increase in inflation contribute to a rise in net worth and a decline in the external finance premium. As a result, the financial accelerator model shows a less severe decline in investment. Again the impact on investment would likely be larger if adjustment costs were not so large.

Figure 5 shows the impulse responses to a monetary policy shock using the parameter estimates from the pre-1979 sample. In general these impulse responses show some interesting differences from those of the post-1979 sample (Figure 1). They demonstrate a aggressive monetary policy regime in the earlier sample. Also of interest is the size of the decline in investment in the FA model. In this sample the estimated adjustment cost parameter is 2.7, well below the 12.4 in the post-79 sample. As a consequence the decline in investment is larger and the FA effect on output is increased slightly.

### 4.3 Volatility and autocorrelation

Table 3 reports the volatilities of output, real balances, interest rates and inflation for both subsamples and for the three simulated versions of the model. The standard deviations are expressed in percentage terms, as computed from the data and generated by the simulated versions of each model.<sup>17</sup> In the data, output has a standard deviation of 1.63 and 1.37 per cent for pre- and post-1979 periods. Real balances, measured by real M1 per capita, has a standard deviation of 1.45 percent in the pre-1979 sample, while volatility is much higher in the post-1979 period with a standard deviation of 3.51 per cent. The short-term nominal interest rate and inflation are less volatile; their standard deviations are less than 0.33 per cent in both subsamples.

The simulation results show that all three versions of the model (FA on, FA off and re-estimated with FA off) overpredict the volatility of output in both subsamples. In the post-1979 sample the FA model comes closest to replicating the volatility of output. All versions of the model also underpredict the volatility of

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<sup>17</sup>In the data, all series are HP-filtered before calculating their standard deviations.

real balances in both subsamples. In the pre-1979 subsample, the No-FA models successfully reproduce the volatility observed in the data for inflation and interest rates, but in the FA model these volatilities are too high. Post-1979 all three models generate inflation volatility consistent with that seen in the data. However, all models also generate too little volatility in interest rates.

Figure 6 plots the autocorrelation functions for output, money, nominal interest rates and inflation generated by our models and in the data. The Estimated No-FA model does the poorest job of matching the autocorrelations seen in the data for output, the nominal interest rate and inflation. Differences between the estimated FA model with the financial accelerator turned on and turned off are small, with the FA-on model matching the autocorrelations marginally better for output and inflation.

## 5 Conclusion

There is a growing literature focusing on the importance of financial frictions in the amplification and propagation of transitory shocks in the context of DSGE models. Almost all of these studies use calibrated, rather than estimated, models. In this paper, we introduce the financial accelerator à la Bernanke, Gertler, and Gilchrist (1999) into a standard sticky-price model to econometrically assess the role of the financial accelerator in post-war US business cycles.

Using quarterly data and a maximum-likelihood procedure with a Kalman filter, we estimate two versions of the model, one with and one without the financial accelerator. Both versions are estimated for two subsamples: pre-1979 and post-1979. The estimation results show that the estimated values of the elasticity of the external finance premium with respect to the leverage ratio are statistically

significant, but different in both subsamples. In the post-1979 period, the external finance premium is less sensitive to the firm leverage and monetary policy responds more aggressively to inflation and more modestly to money growth. In both samples, the likelihood ratio test rejects the model without a financial accelerator in favour of the one with it. However, the impulse response functions show that the financial accelerator does little to amplify or propagate the impact of monetary policy shocks on output. The impact of the accelerator is more apparent for investment.

Future work could extend this model to include further real frictions, more sources of persistence, and some exogenous financial shocks. This might allow the model to better match the responses of macroeconomic variables to different shocks. We may also extend this work to analyze the role of the financial accelerator in a small open economy model.

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**Table 1:** Maximum-likelihood estimates: Model with the Financial Accelerator

Parameters	Pre-1979		Post-1979	
	Estimates	Std. errors	Estimates	Std. errors
$\beta$	0.9975	0.0001	0.9925	0.0006
$\gamma$	0.0225	0.0060	0.0428	0.0084
$\chi$	2.7182	0.2911	12.3985	1.6753
$\phi$	0.6933	0.0458	0.5237	0.0836
$\psi$	0.0754	0.0279	0.0377	0.0143
$\kappa$	0.4937	0.2687	0.4468	0.1802
$\varrho_R$	0.6909	0.1099	0.9067	0.1524
$\varrho_\pi$	0.4540	0.0893	0.8254	0.1389
$\varrho_y$	-0.0026	0.0089	-0.0101	0.0265
$\varrho_\mu$	0.4235	0.1060	0.0453	0.0195
$\sigma_R$	0.0026	0.0005	0.0026	0.0002
$A$	4.3989	1.3566	5.4058	0.8407
$\rho_A$	0.7499	0.0575	0.9253	0.0103
$\sigma_A$	0.0134	0.0029	0.0093	0.0004
$b$	0.1947	0.0074	0.1254	0.0040
$\rho_b$	0.6854	0.0592	0.8122	0.0532
$\sigma_b$	0.0148	0.0031	0.0142	0.0014
$\rho_z$	0.9570	0.0166	0.9204	0.0267
$\sigma_z$	0.0193	0.0020	0.0137	0.0018
$LL$	1372.6		1541.6	

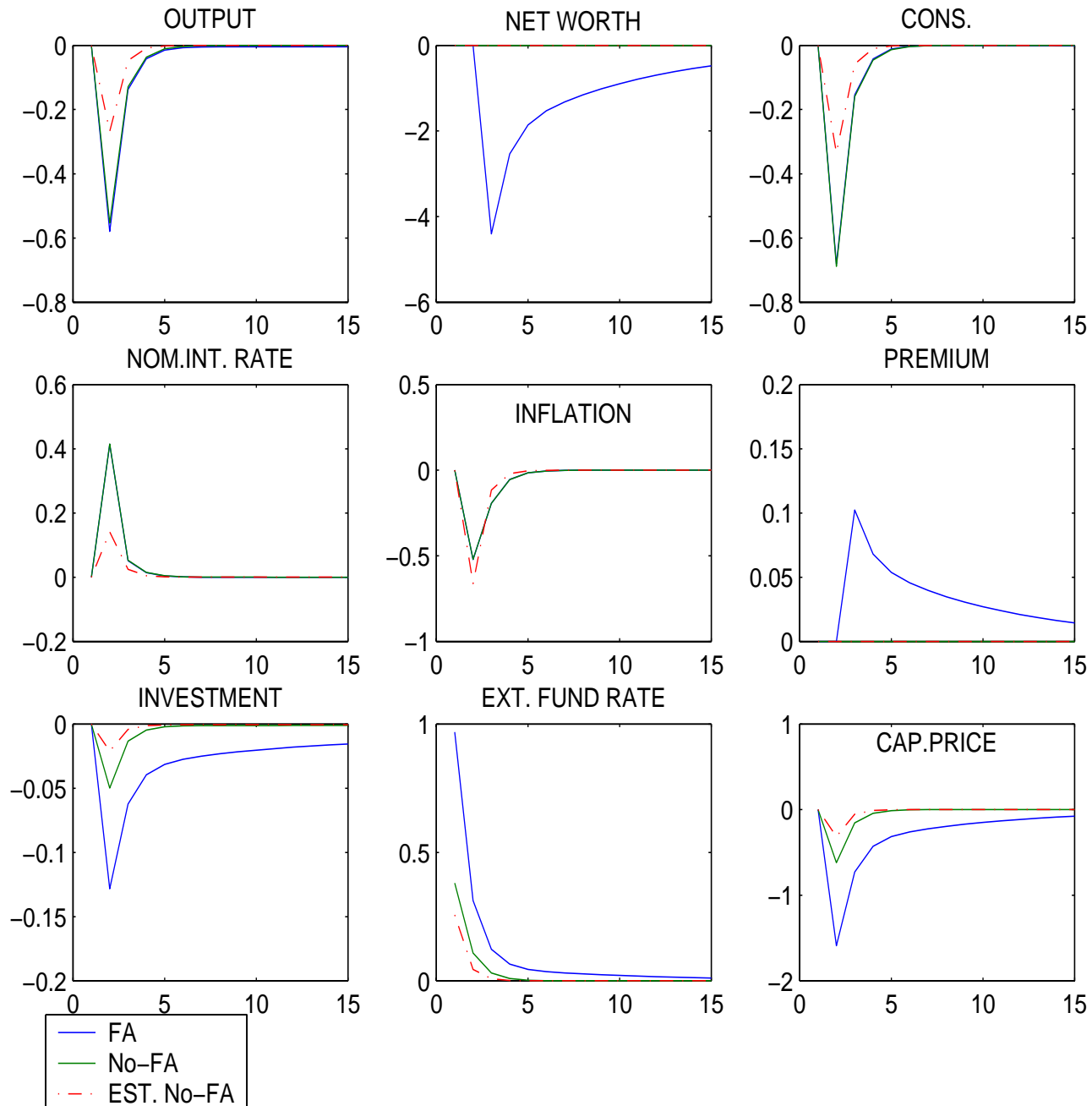
**Table 2:** Maximum-likelihood estimates: Model with No Financial Accelerator

Parameters	Pre-1979		Post-1979	
	Estimates	Std. errors	Estimates	Std. errors
$\beta$	0.9971	0.0008	0.9928	0.0002
$\gamma$	0.0287	0.0138	0.0387	0.0011
$\chi$	8.9349	1.5786	14.8560	2.0715
$\phi$	0.6621	0.0520	0.4142	0.0561
$\varrho_R$	0.7964	0.0472	0.8280	0.1914
$\varrho_\pi$	0.3154	0.0737	1.1547	0.1794
$\varrho_y$	-0.0140	0.0293	0.0031	0.0164
$\varrho_\mu$	0.3675	0.0942	0.0528	0.0205
$\sigma_R$	0.0025	0.0006	0.0029	0.0005
$A$	2.2687	0.0624	3.5679	0.0790
$\rho_A$	0.7572	0.0656	0.9610	0.0735
$\sigma_A$	0.0117	0.0009	0.0121	0.0021
$b$	0.2287	0.0201	0.1453	0.0012
$\rho_b$	0.6273	0.0633	0.8587	0.0258
$\sigma_b$	0.0160	0.0033	0.0141	0.0007
$\rho_z$	0.9548	0.0290	0.9519	0.1192
$\sigma_z$	0.0214	0.0017	0.0179	0.0017
$LL$	1361.5		1534.4	

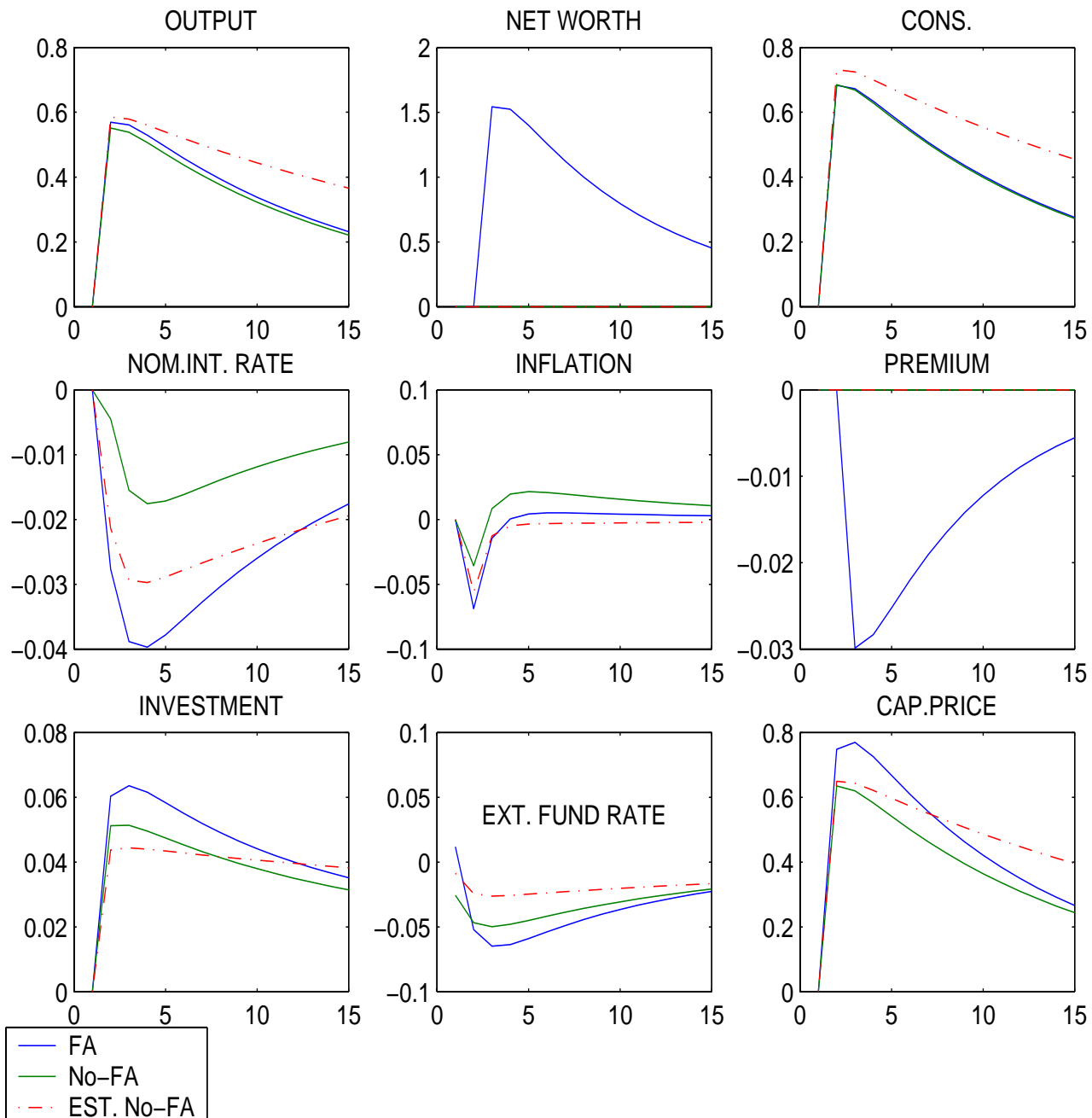
**Table 3:** Standard deviations: data and models

Variables	Data	FA Model	FA Model, $\psi = 0$	Est. No-FA Model
Pre-1979				
$y$	1.63	2.65	2.62	2.48
$m_t$	1.45	0.51	0.35	0.34
$R_t^n$	0.25	0.43	0.25	0.288
$\pi_t$	0.33	0.59	0.31	0.34
Post-1979				
$y$	1.37	1.93	2.69	4.84
$m_t$	3.51	0.33	0.43	0.77
$R_t^n$	0.31	0.18	0.15	0.18
$\pi_t$	0.21	0.24	0.24	0.21

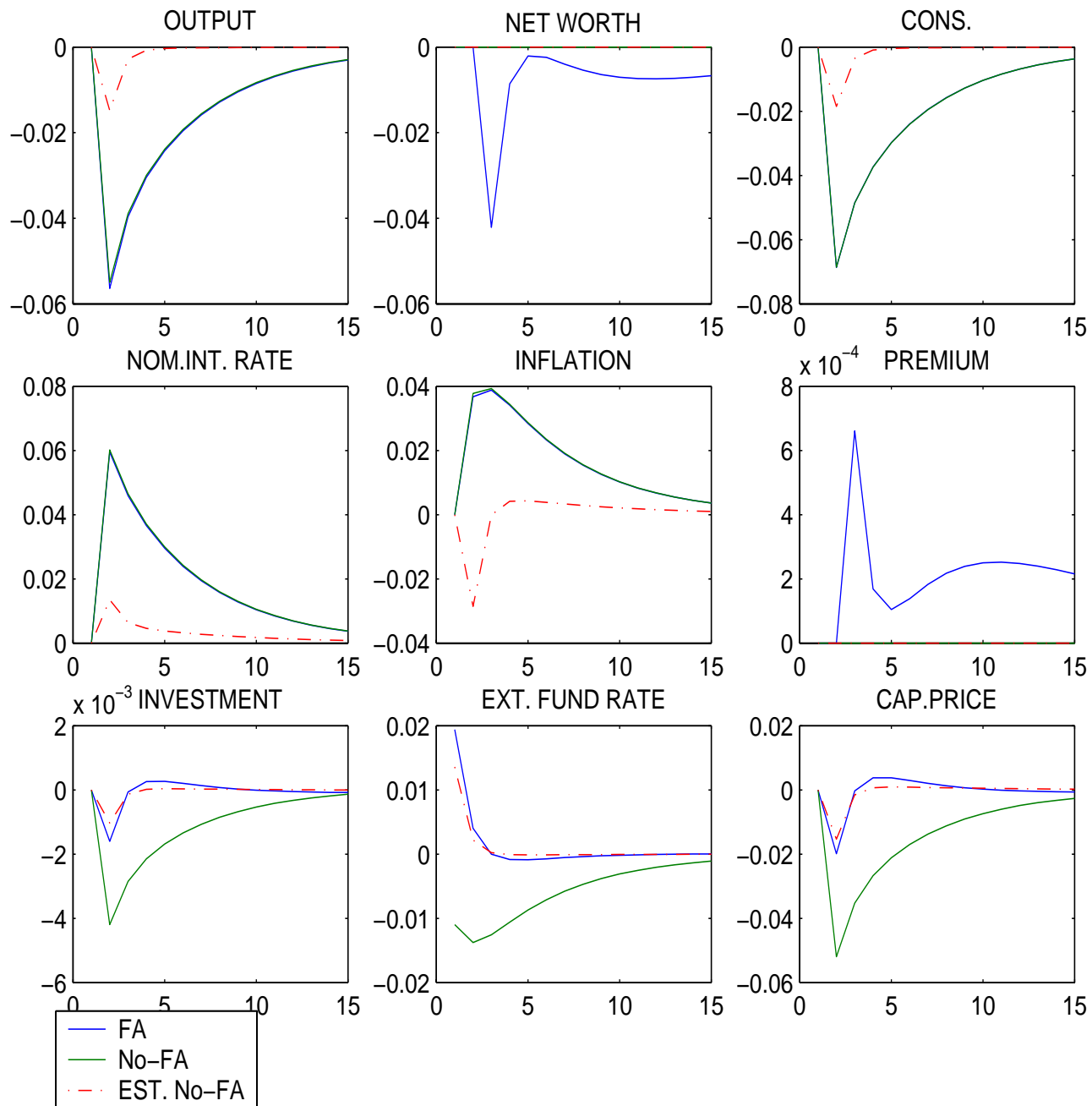
**Figure 1:** Monetary policy shock, Post-1979



**Figure 2:** Technology shock, Post-1979



**Figure 3:** Money demand shock, Post-1979



**Figure 4:** Preference shock, Post-1979

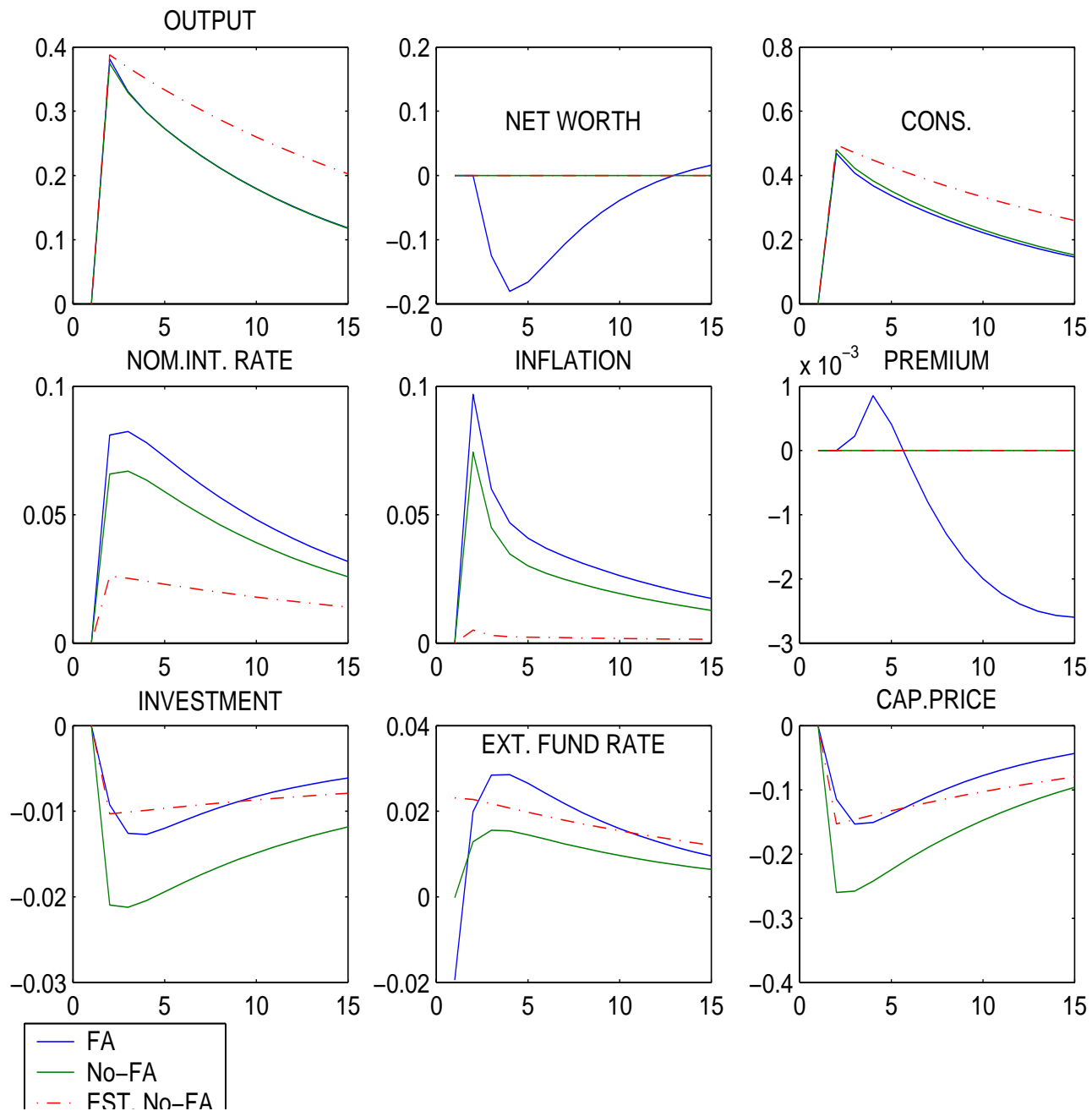


Figure 5: Monetary Policy shock, Pre-1979

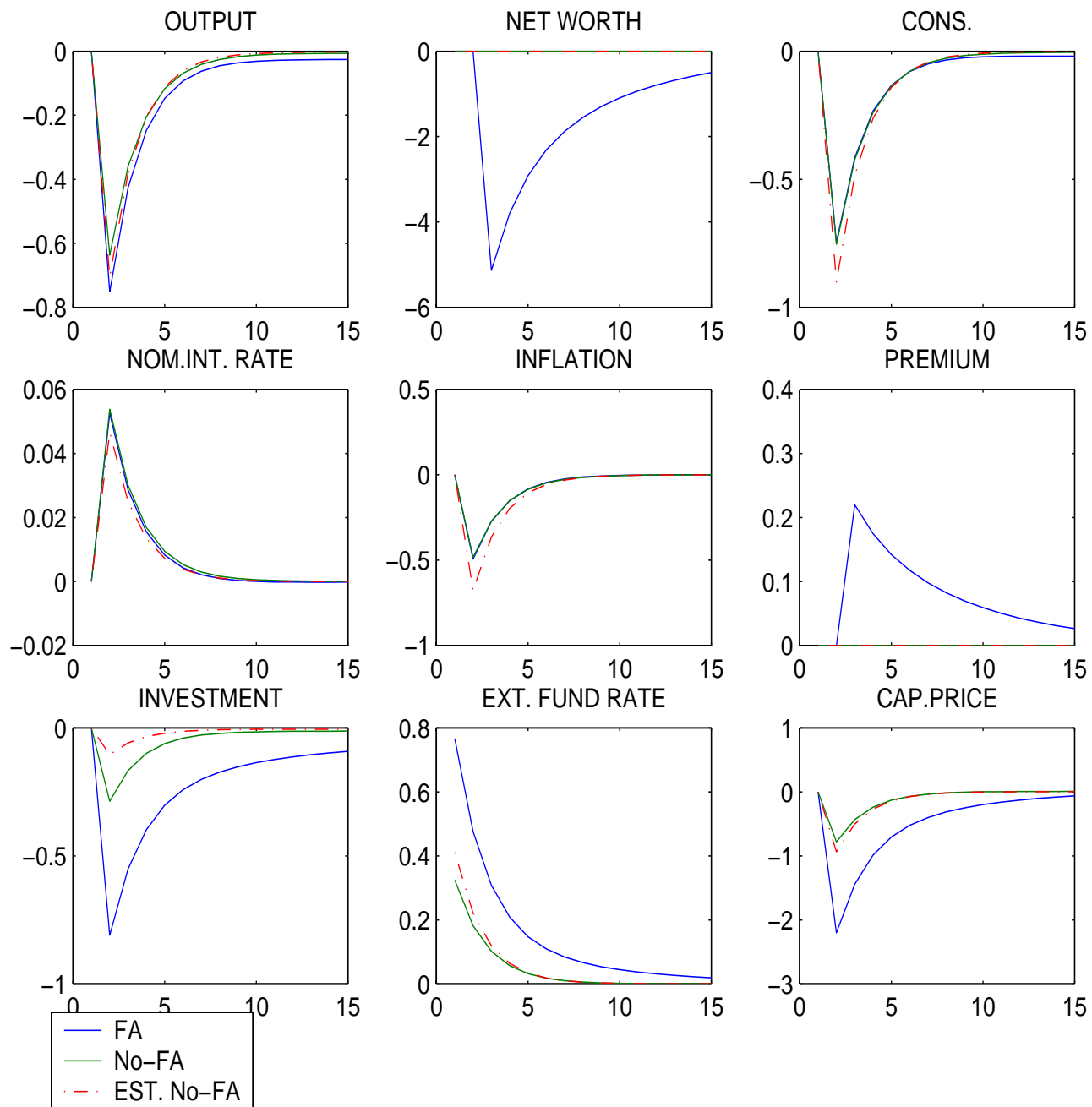
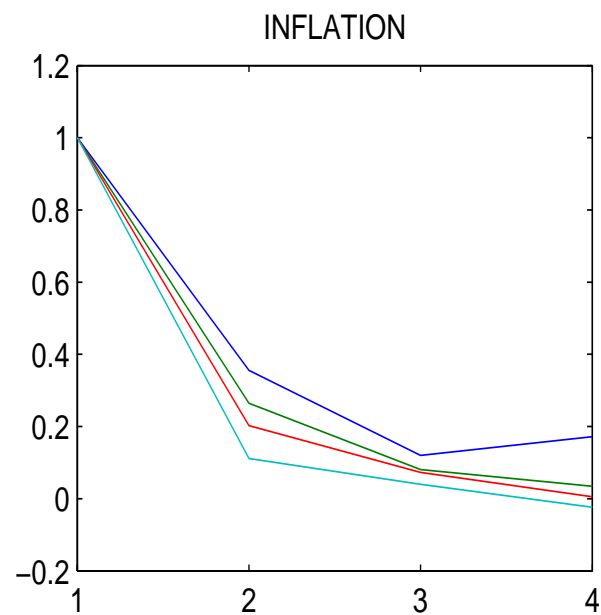
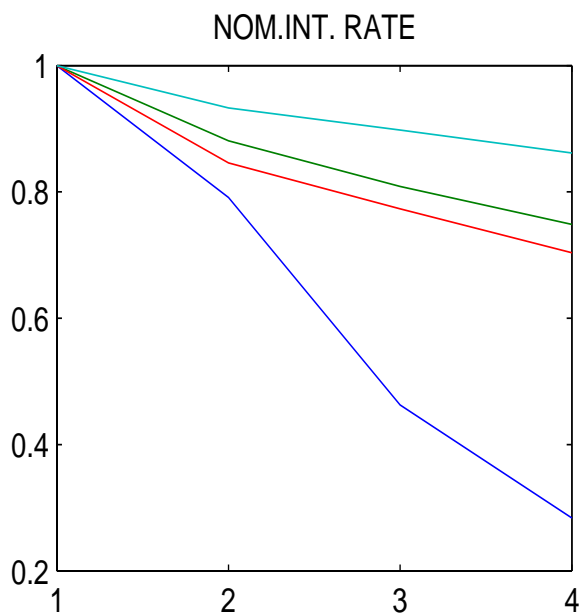
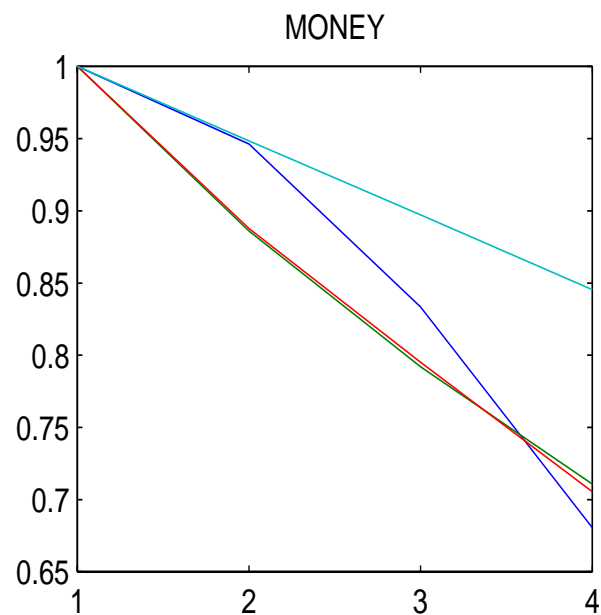
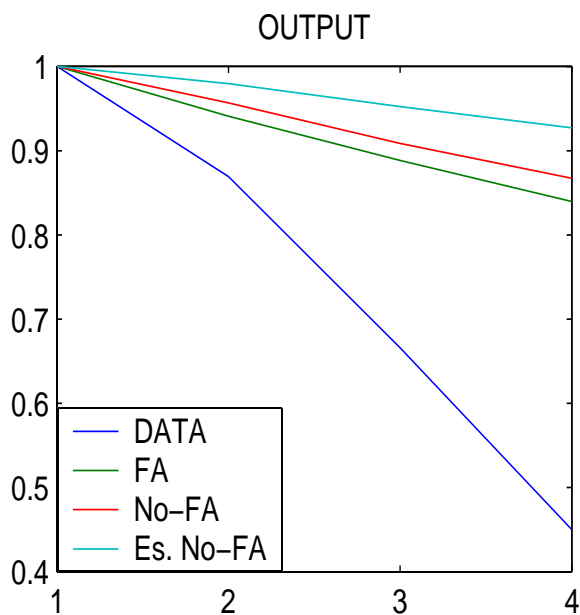


Figure 6: Autocorrelations, Post-1979



## A. The non-linear equilibrium system

$$\frac{z_t c_t^{-\frac{1}{\gamma}}}{c_t^{\frac{\gamma-1}{\gamma}} + b_t^{1/\gamma} m_t^{\frac{\gamma-1}{\gamma}}} = \lambda_t; \quad (\text{A.1})$$

$$\left(\frac{b_t c_t}{m_t}\right)^{1/\gamma} = \frac{R_t^n - 1}{R_t^n}; \quad (\text{A.2})$$

$$\frac{\eta}{1 - h_t} = \lambda_t w_t; \quad (\text{A.3})$$

$$\frac{1}{R_t^n} = \beta E_t \left[ \frac{\lambda_{t+1}}{\pi_{t+1} \lambda_t} \right]; \quad (\text{A.4})$$

$$R_t = E_t \left[ \frac{R_t^n}{\pi_{t+1}} \right]; \quad (\text{A.5})$$

$$r_{kt} = \alpha \frac{y_t}{k_t} m c_t; \quad (\text{A.6})$$

$$w_t = (1 - \alpha) \frac{y_t}{h_t} m c_t; \quad (\text{A.7})$$

$$y_t = k_t^\alpha (A_t h_t)^{1-\alpha}; \quad (\text{A.8})$$

$$y_t = c_t + i_t; \quad (\text{A.9})$$

$$\bar{p}_t = \frac{\theta}{\theta - 1} \frac{E_t \sum_{l=0}^{\infty} (\beta \phi)^l \lambda_{t+l} m c_{t+l} y_{t+l} / p_{t+l}}{E_t \sum_{l=0}^{\infty} (\beta \phi)^l \lambda_{t+l} y_{t+l} / p_{t+l}}; \quad (\text{A.10})$$

$$p_t^{1-\theta} = \phi p_{t-1}^{1-\theta} + (1 - \phi) \bar{p}_t^{1-\theta}; \quad (\text{A.11})$$

$$E_t f_{t+1} = E_t \left[ \left( \frac{n_{t+1}}{q_t k_{t+1}} \right)^{-\psi} R_t \right]; \quad (\text{A.12})$$

$$E_t f_{t+1} = E_t \left[ \frac{r_{kt+1} + (1 - \delta) q_{t+1}}{q_t} \right]; \quad (\text{A.13})$$

$$E_t n_{t+1} = \nu [f_t q_{t-1} k_t - E_{t-1} f_t (q_{t-1} k_t - n_t)]; \quad (\text{A.14})$$

$$k_{t+1} = i_t + (1 - \delta) k_t; \quad (\text{A.15})$$

$$q_t = 1 + \chi \left( \frac{i_t}{k_t} - \delta \right); \quad (\text{A.16})$$

$$\frac{R_t^n}{R^n} = \left( \frac{R_{t-1}^n}{R^n} \right)^{\varrho_R} \left( \frac{\pi_t}{\pi} \right)^{\varrho_\pi} \left( \frac{y_t}{y} \right)^{\varrho_y} \left( \frac{\mu_t}{\mu} \right)^{\varrho_\mu} \exp(\varepsilon_R); \quad (\text{A.17})$$

$$\mu_t = m_t \pi_t / m_{t-1}. \quad (\text{A.18})$$

## B. The steady-state equilibrium

$$\mu = \pi = 1; \quad (\text{B.1})$$

$$q = 1; \quad (\text{B.2})$$

$$mc = \frac{\theta - 1}{\theta}; \quad (\text{B.3})$$

$$R = R^n = 1/\beta; \quad (\text{B.4})$$

$$f = r_k + 1 - \delta; \quad (\text{B.5})$$

$$f = \left(\frac{n}{k}\right)^{-\psi} R; \quad (\text{B.6})$$

$$i = \delta k; \quad (\text{B.7})$$

$$\lambda c = \left[ 1 + b \left( \frac{\pi}{\pi - \beta} \right)^{\gamma-1} \right]^{-1}; \quad (\text{B.8})$$

$$\lambda m = \lambda c b \left( \frac{\pi}{\pi - \beta} \right)^{\gamma}; \quad (\text{B.9})$$

$$\frac{k}{y} = \frac{\alpha mc}{r_k}; \quad (\text{B.10})$$

$$\frac{c}{y} = 1 - \delta \frac{k}{y}; \quad (\text{B.11})$$

$$wh\lambda = \frac{(1 - \alpha)(\lambda c)mc}{c/y}; \quad (\text{B.12})$$

$$h = \frac{wh\lambda}{\eta + wh\lambda}; \quad (\text{B.13})$$

$$y = Ah \left( \frac{k}{y} \right)^{\alpha/(1-\alpha)}; \quad (\text{B.14})$$

$$(\text{B.15})$$

## C. The log-linearized equilibrium system

### Static equations

$$\left(\frac{1-\gamma}{\gamma} - \frac{\lambda}{\gamma c}\right) \hat{c}_t = \frac{(R^n - 1)m}{R^n c} \left(\frac{1}{\gamma} \hat{b}_t + \frac{\gamma - 1}{\gamma} \hat{m}_t\right) - \frac{\lambda}{c} \hat{\lambda}_t; \quad (\text{C.1})$$

$$\hat{b}_t + \hat{c}_t - \hat{m}_t = \hat{R}^n_t / (R^n - 1); \quad (\text{C.2})$$

$$h \hat{h}_t / (1 - h) - \hat{w}_t = \hat{\lambda}_t; \quad (\text{C.3})$$

$$\hat{y}_t = \hat{A}_t + \alpha \hat{k}_t + (1 - \alpha) \hat{h}_t; \quad (\text{C.4})$$

$$y \hat{y}_t = c \hat{c}_t + i \hat{i}_t; \quad (\text{C.5})$$

$$\hat{w}_t = \hat{y}_t + \hat{m} c_t - \hat{h}_t; \quad (\text{C.6})$$

$$\hat{r}_{kt} = \hat{y}_t + \hat{m} c_t - \hat{k}_t; \quad (\text{C.7})$$

$$\hat{\mu}_t = \hat{m}_t - \hat{m}_{t-1} + \hat{\pi}_t \quad (\text{C.8})$$

$$\hat{R}_t^n = \varrho_R \hat{R}_{t-1}^n + \varrho_\pi \hat{\pi}_t + \varrho_\mu \hat{\mu}_t + \varrho_y \hat{y}_t + \varepsilon_{Rt} \quad (\text{C.9})$$

$$\hat{f}_t + \hat{q}_{t-1} = \frac{r_k}{f} \hat{r}_{kt} + \frac{1 - \delta}{f} \hat{q}_t; \quad (\text{C.10})$$

$$\hat{q}_t = \chi(\hat{i}_t - \hat{k}_t). \quad (\text{C.11})$$

$$(\text{C.12})$$

## Dynamic equations

$$\beta \hat{\pi}_{t+1} = \hat{\pi}_t - \frac{(1 - \beta\phi)(1 - \phi)}{\phi} \hat{m}c_t; \quad (\text{C.13})$$

$$\hat{\lambda}_{t+1} = \hat{\lambda}_t - \hat{R}_t; \quad (\text{C.14})$$

$$\hat{\pi}_{t+1} = \hat{R}_t^n - \hat{R}_t; \quad (\text{C.15})$$

$$\hat{k}_{t+1} = \delta \hat{i}_t + (1 - \delta) \hat{k}_t; \quad (\text{C.16})$$

$$\hat{f}_{t+1} + \psi \hat{n}_{t+1} - \psi \hat{k}_{t+1} = \hat{R}_t + \psi \hat{q}_t; \quad (\text{C.17})$$

$$\frac{\hat{n}_{t+1}}{\nu f} = \frac{k}{n} \hat{f}_t - \left( \frac{k}{n} - 1 \right) \hat{R}_{t-1} - \psi \left( \frac{k}{n} - 1 \right) (\hat{k}_t + \hat{q}_{t-1}) + \left( \psi \left( \frac{k}{n} - 1 \right) + 1 \right) \hat{q}_t; \quad (\text{C.18})$$

$$\hat{m}_t = \hat{m}_t; \quad (\text{C.19})$$

$$\hat{q}_t = \hat{q}_t. \quad (\text{C.20})$$