Structural changes in the US economy: Bad Luck or Bad Policy?

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Abstract

This paper investigates the relationship between changes in the output and inflation processes and monetary policy in the US over the last 25 years. It estimates a structural Bayesian TVC-VAR, where sign restrictions are used to identify monetary policy shocks, and analyzes the transmission of disturbances to the non-systematic and to the systematic component of monetary policy. Impulse responses are calculated as the difference between two conditional expectations, differing for a shock in the conditioning sets. We find structural variations the coefficients of the model but the changes are localized in time and unsyncronized with the changes in the monetary policy equation. Changes in the variance of structural shocks are qualitatively more relevant. The transmission of policy disturbances has hardly changed over the last 25 years and variations in the systematic component of policy have minor effects on the dynamics of the system. Inflation persistence varies over time but its variations are disconnected with the dynamics of monetary policy. Results are robust to a number of alterations in the auxiliary assumptions.

Key words: Monetary policy, Inflation persistence, Transmission of shocks, Time varying coefficients structural VARs

JEL Classification numbers:

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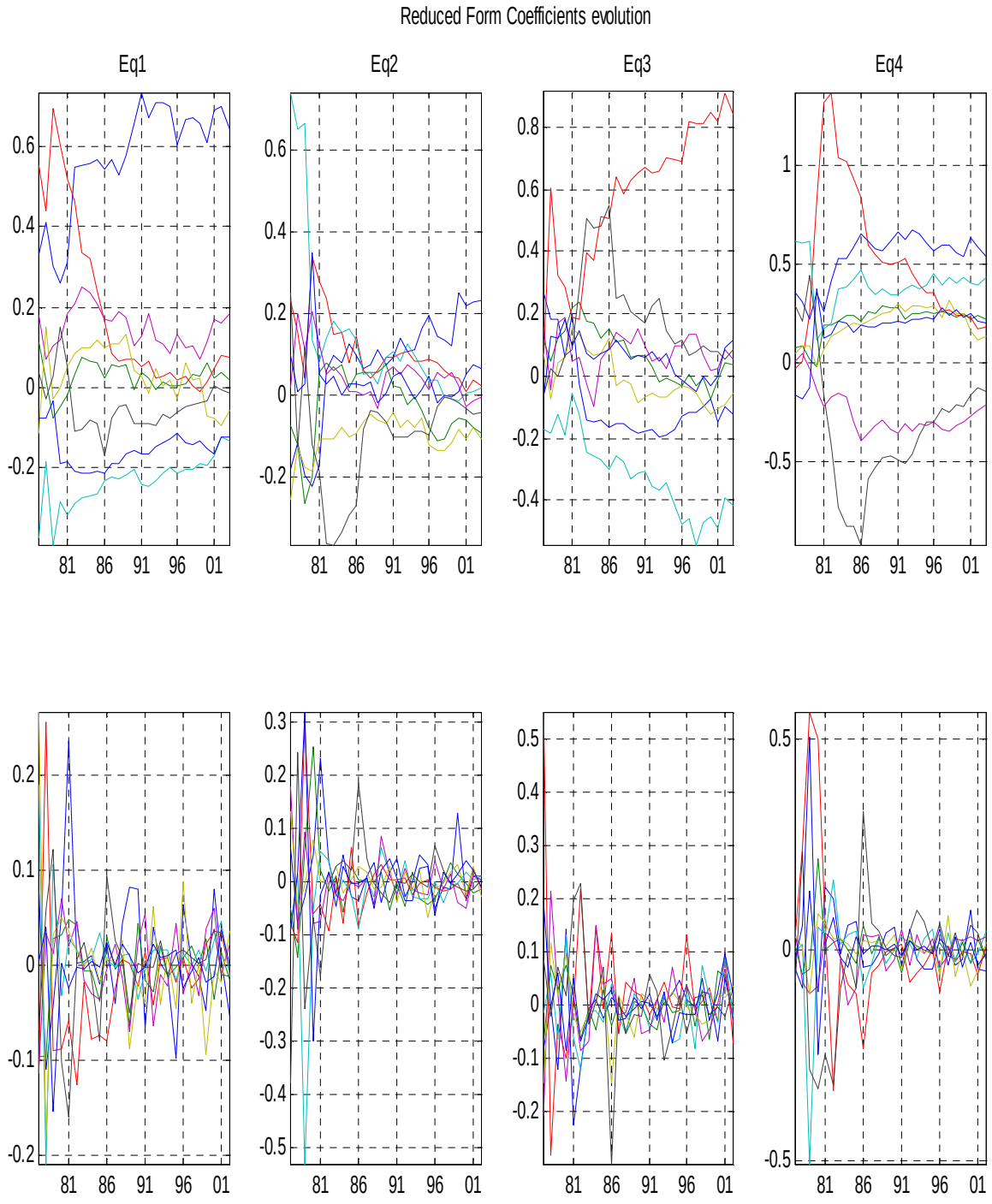


Figure 1: Reduced form coefficients

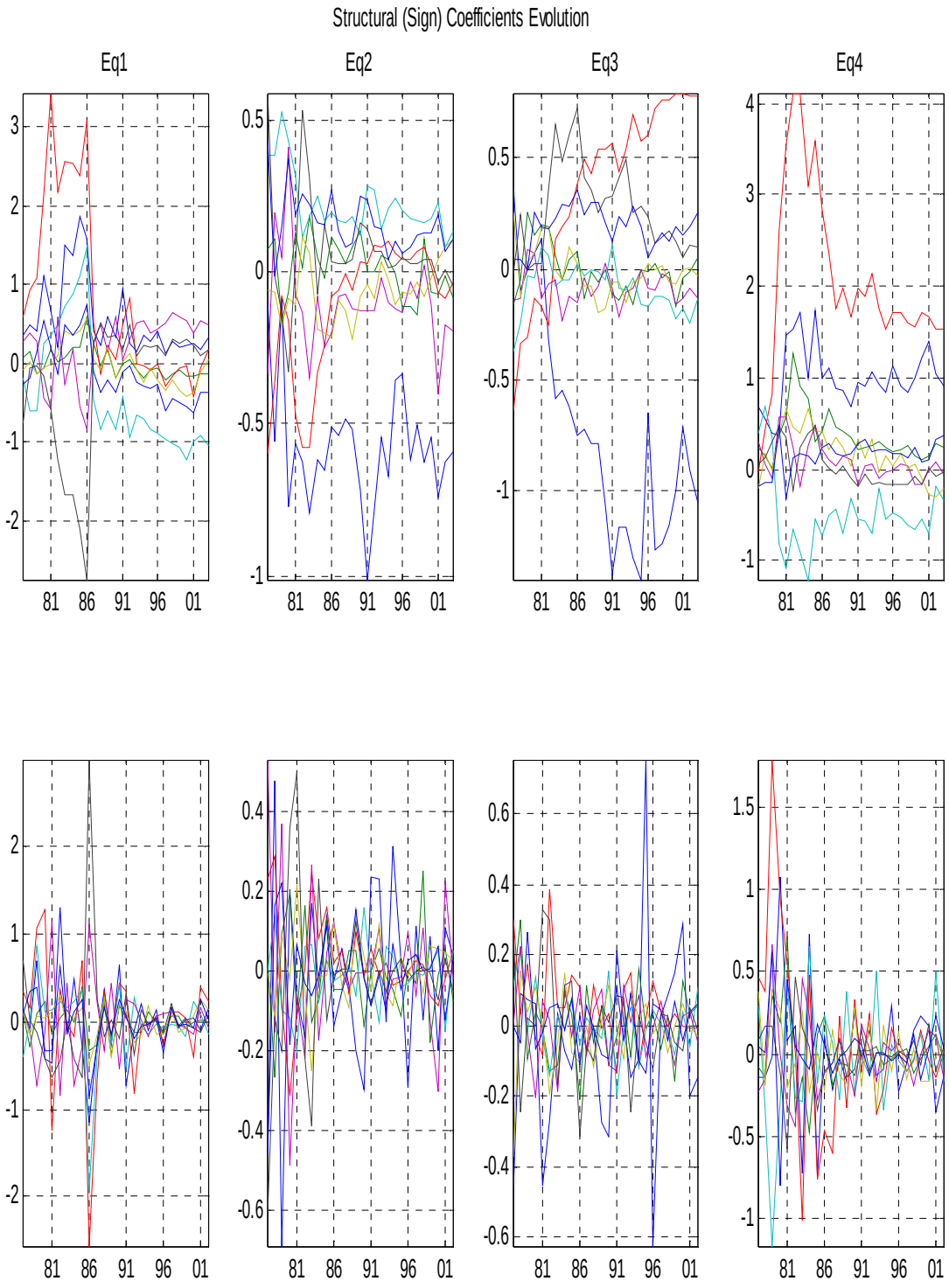


Figure 2: Structural coefficients

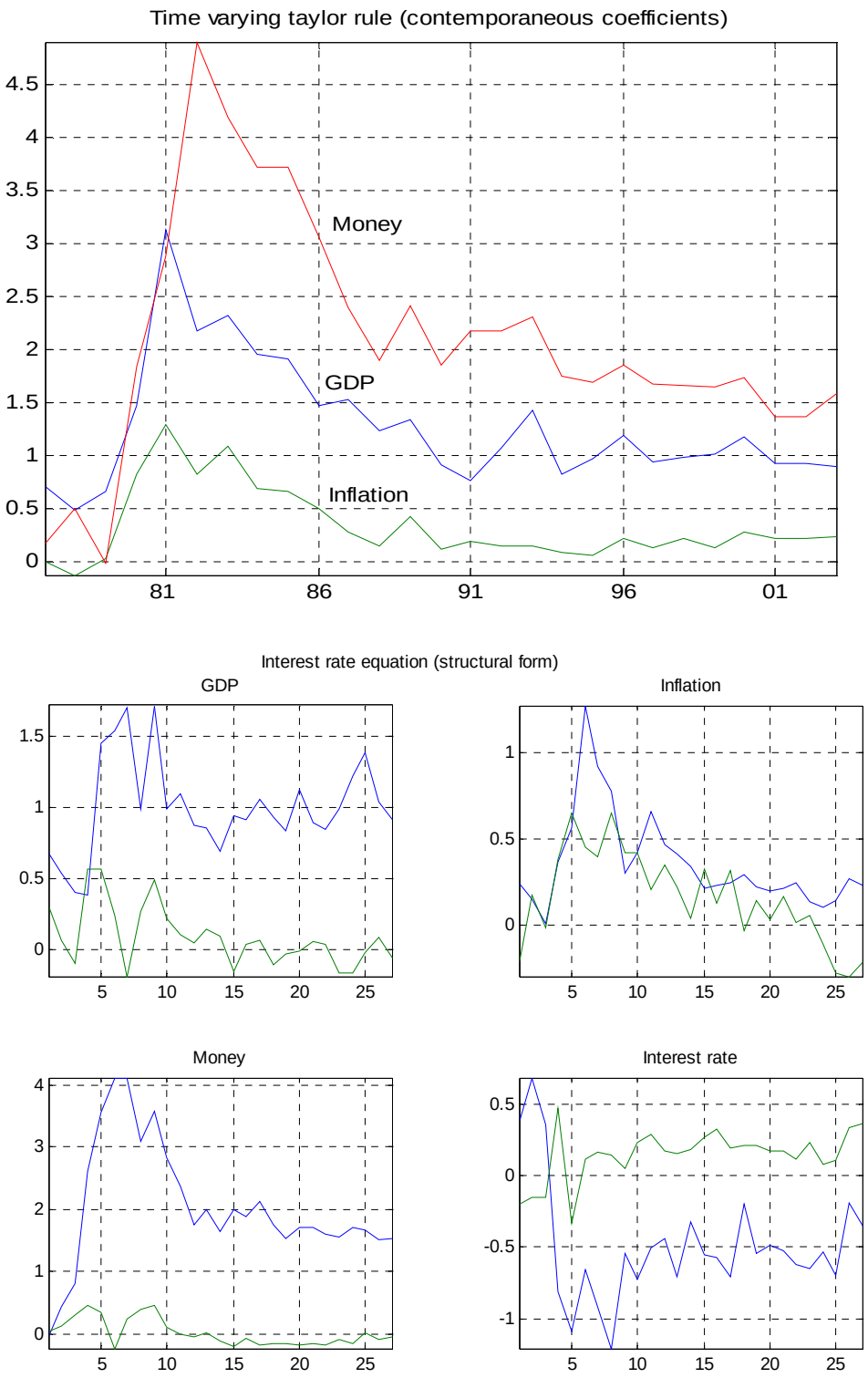
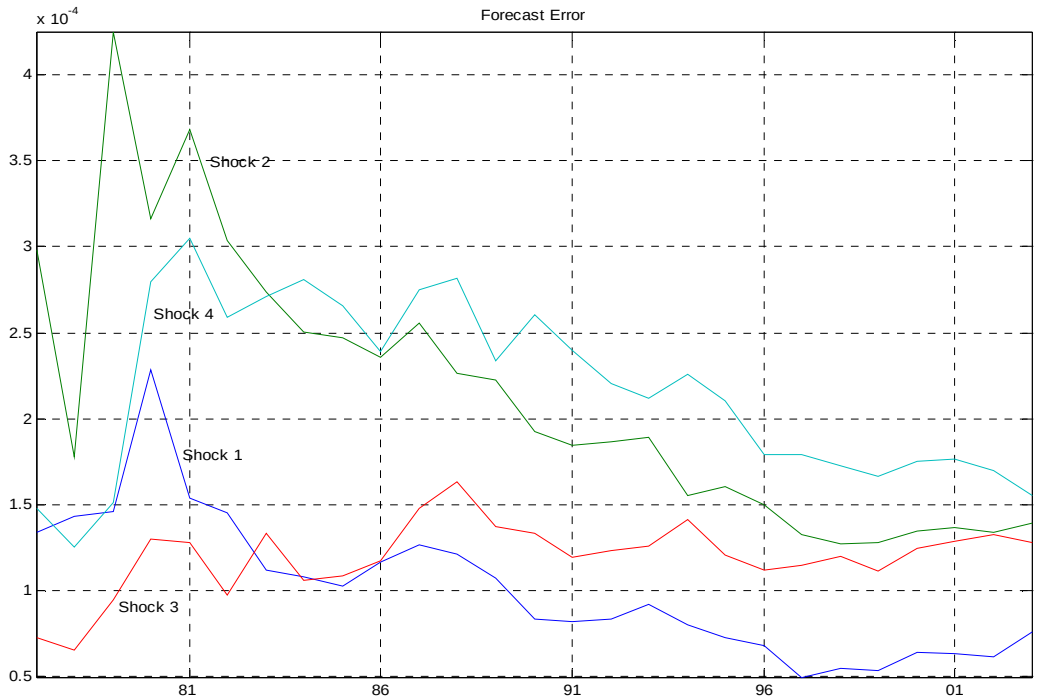


Figure 3: Policy Coefficients



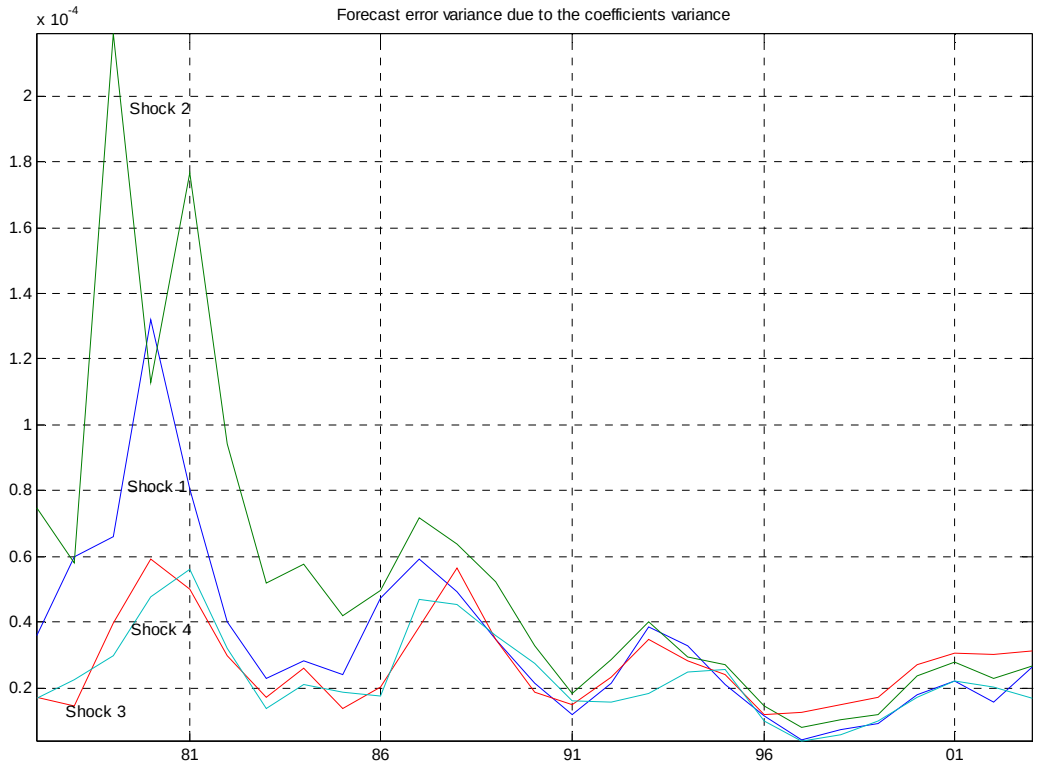
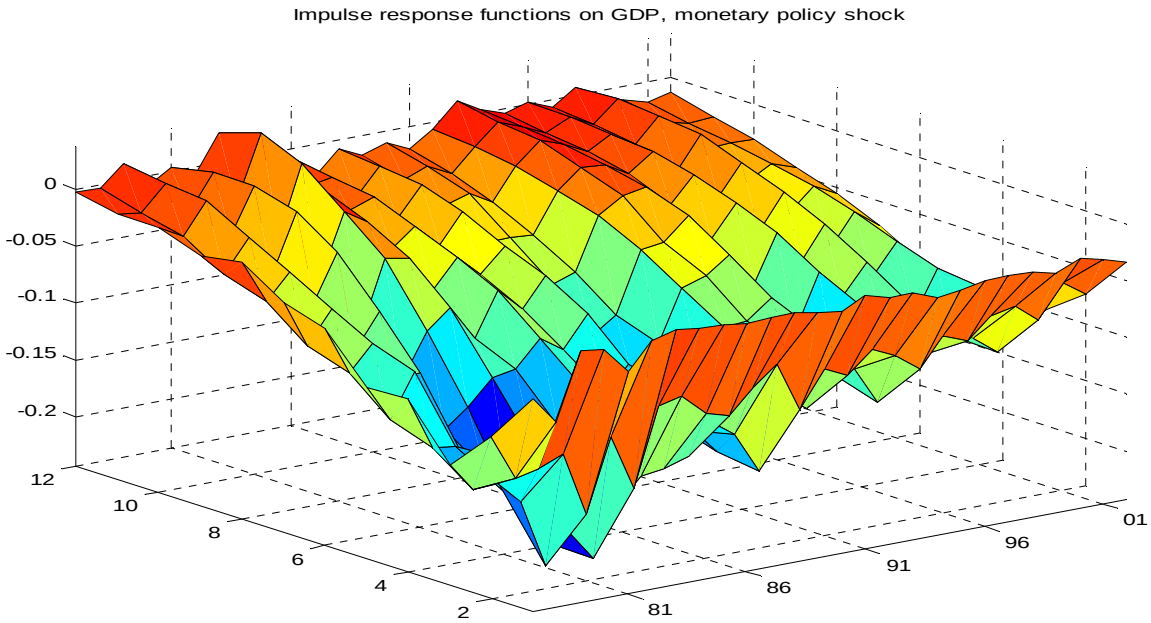


Figure 4: Forecast error variance



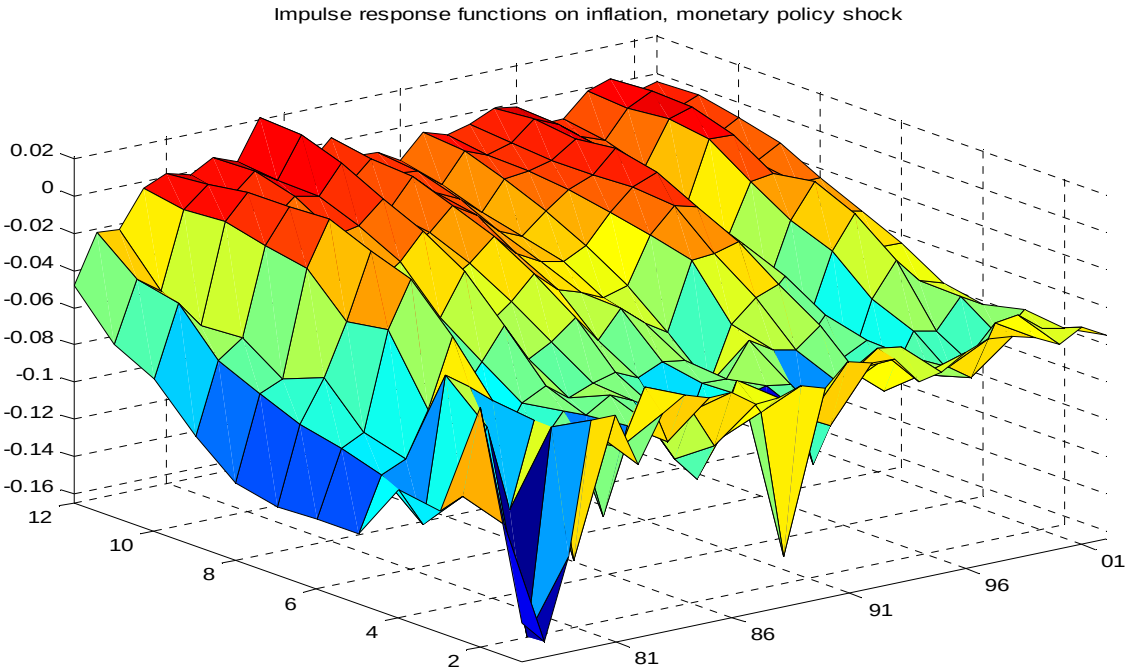
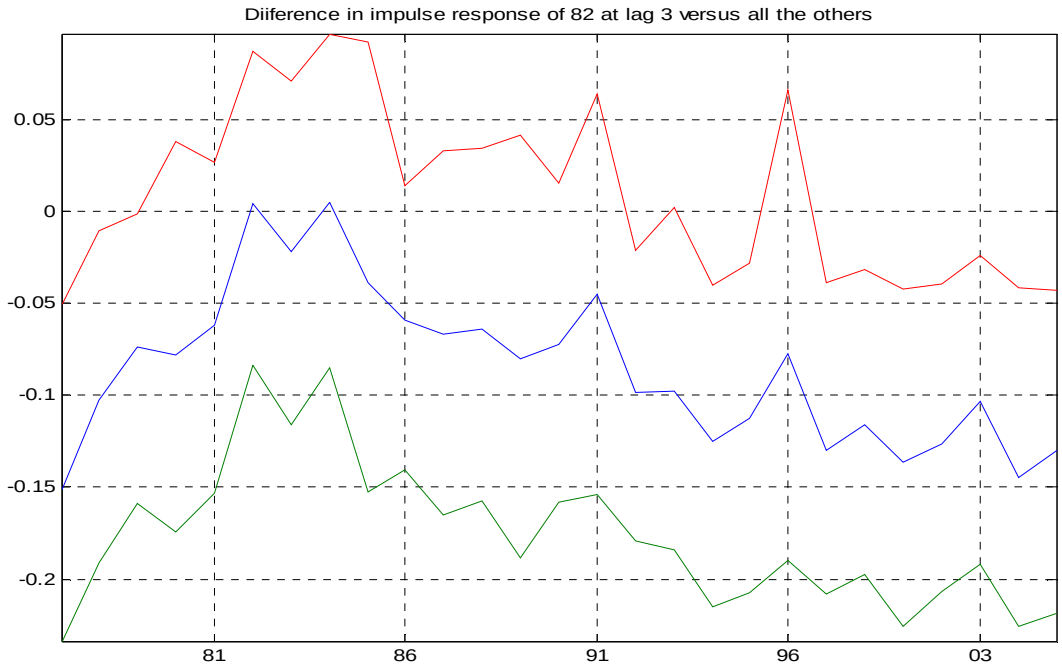


Figure 5: Structural Impulse responses



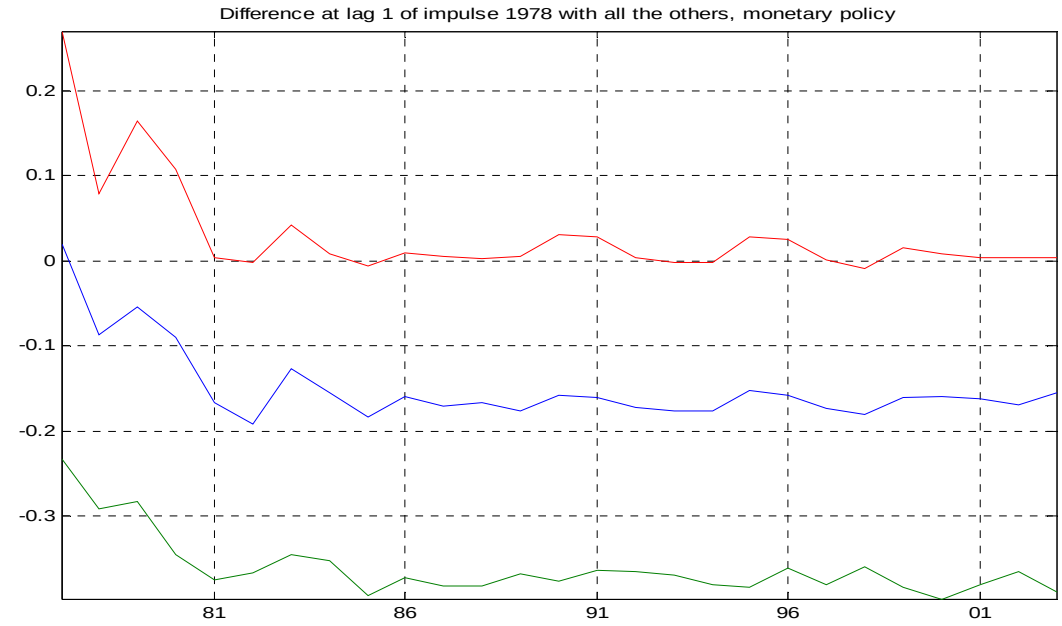
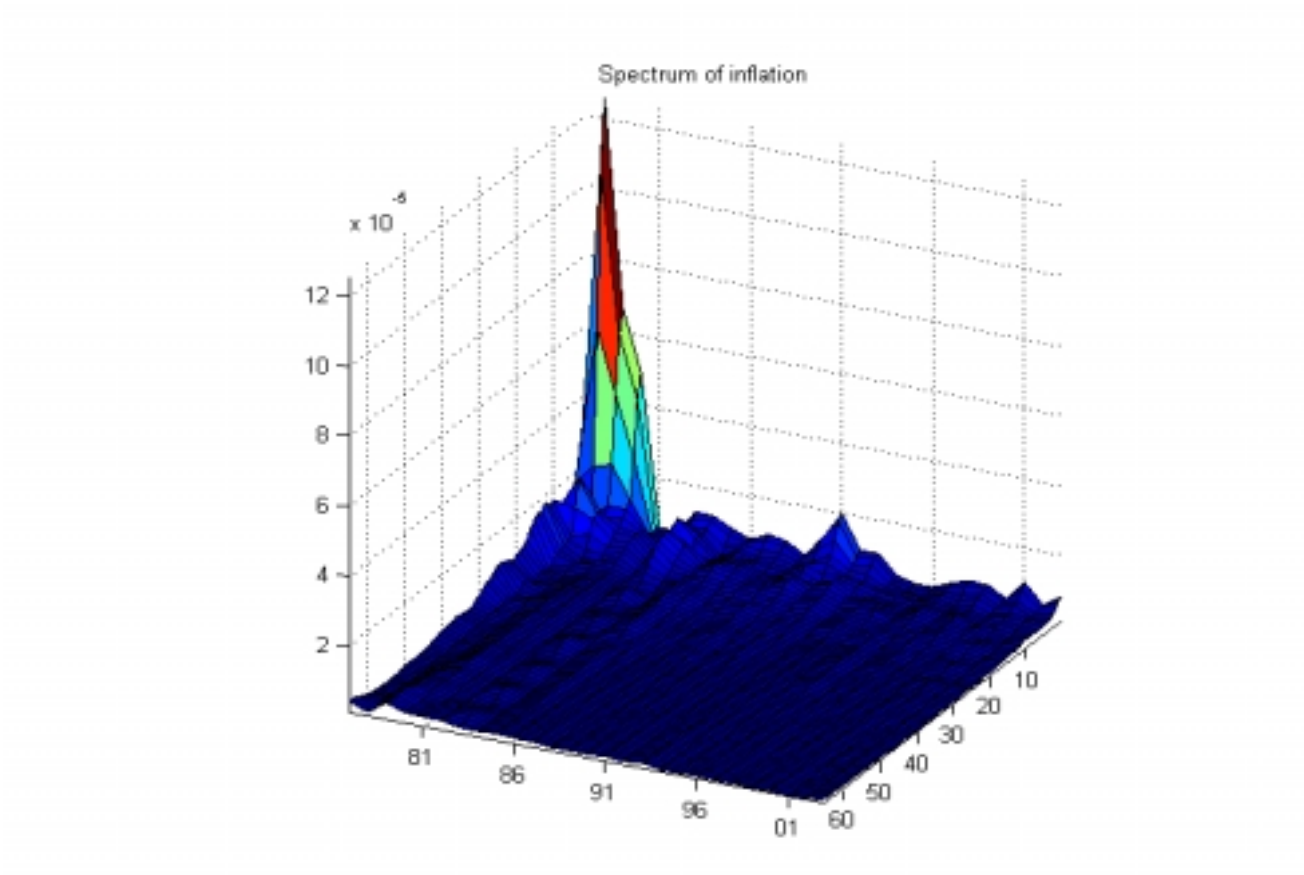


Figure 6: Posterior 68\% band for the difference in impulse responses



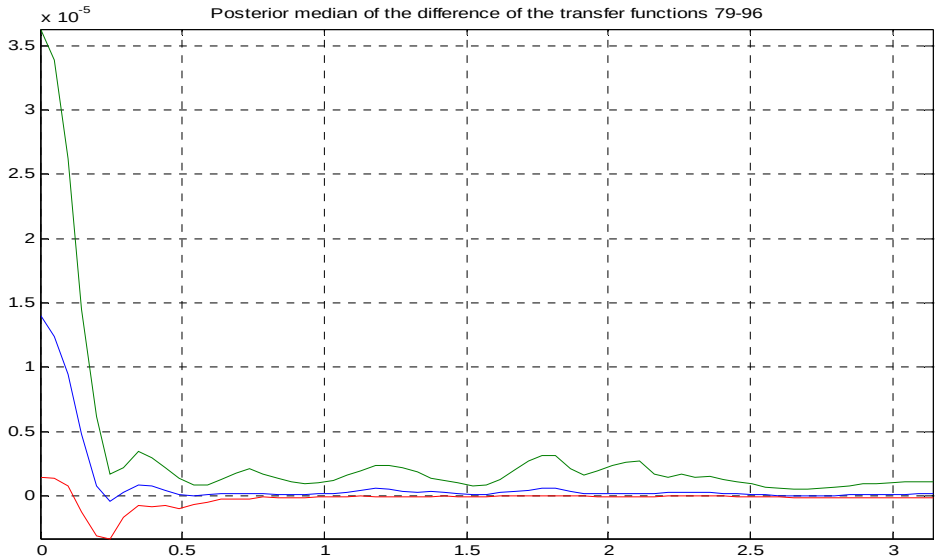


Figure 7: Spectrum of Inflation

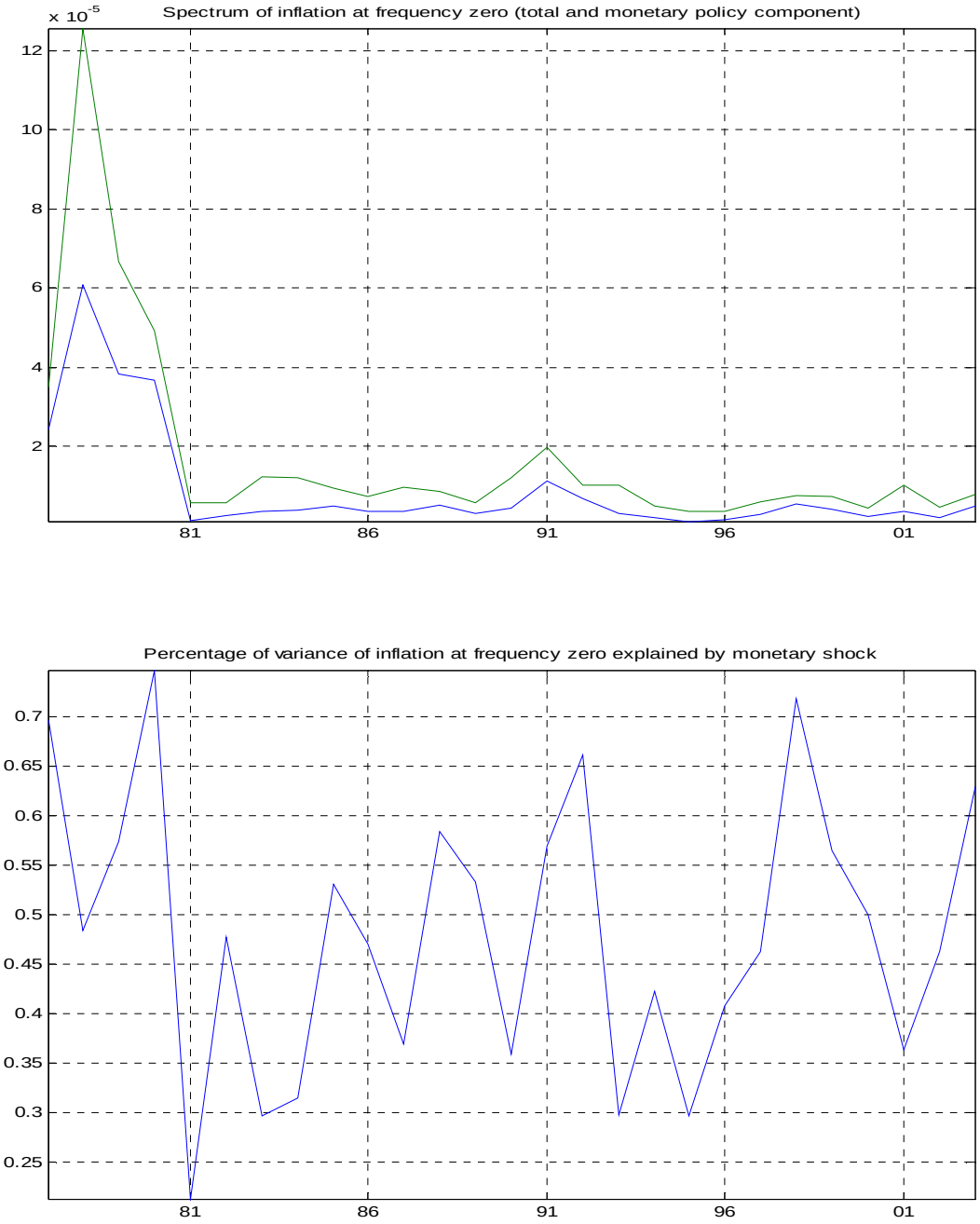


Figure 8: Contribution of Monetary policy shocks to the spectrum of inflation

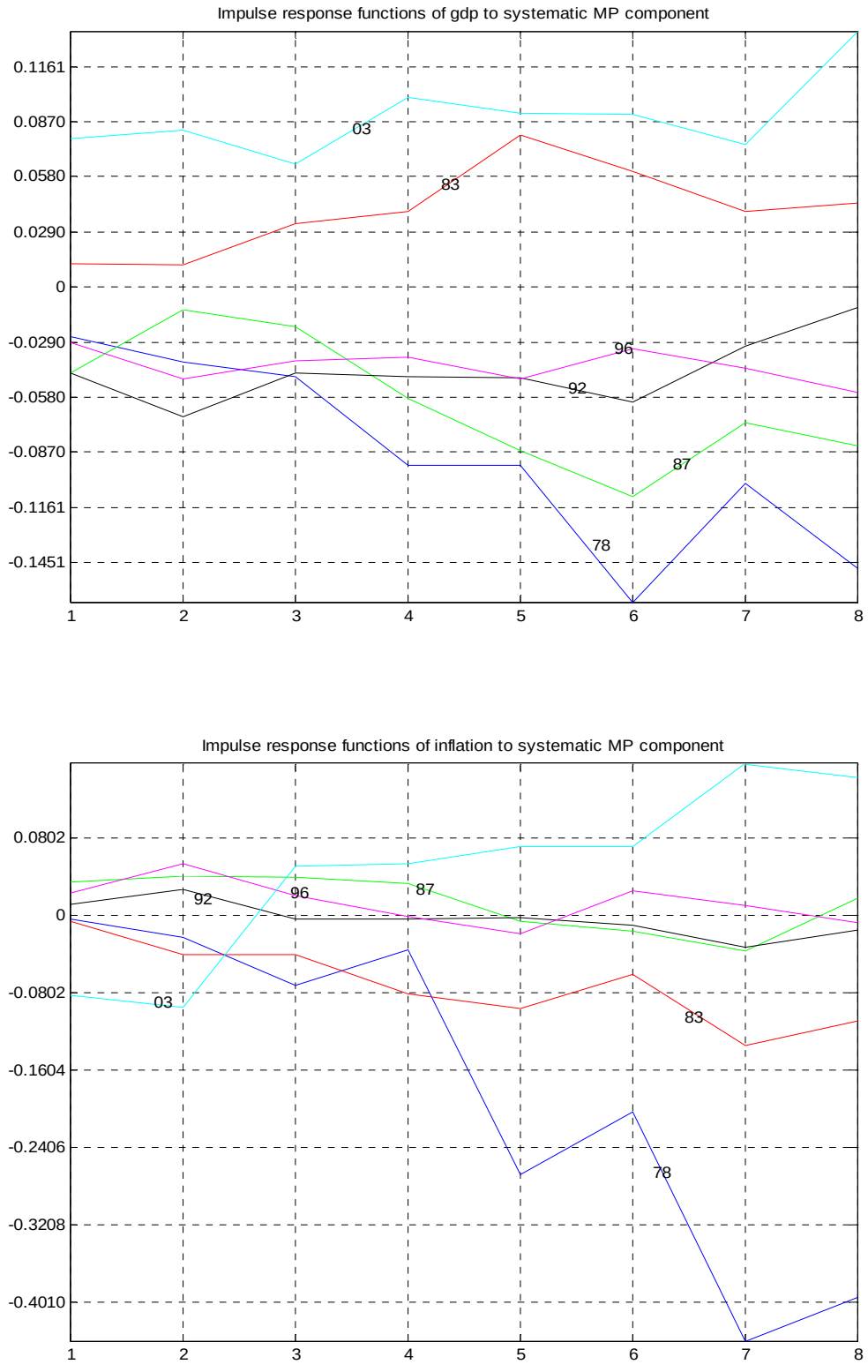


Figure 9: Responses to a more aggrestive stance on inflation

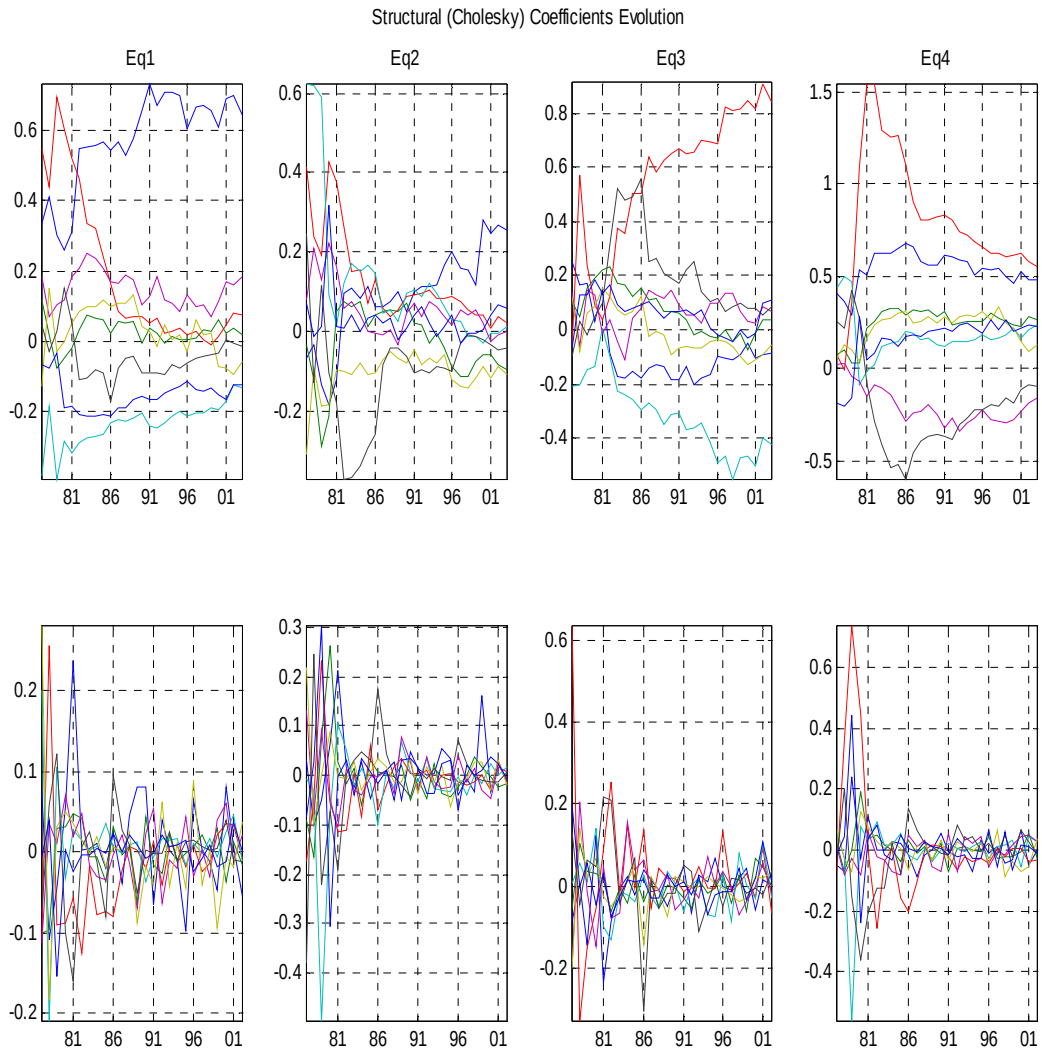


Figure 10a) Structural Coefficients, Choleski identification

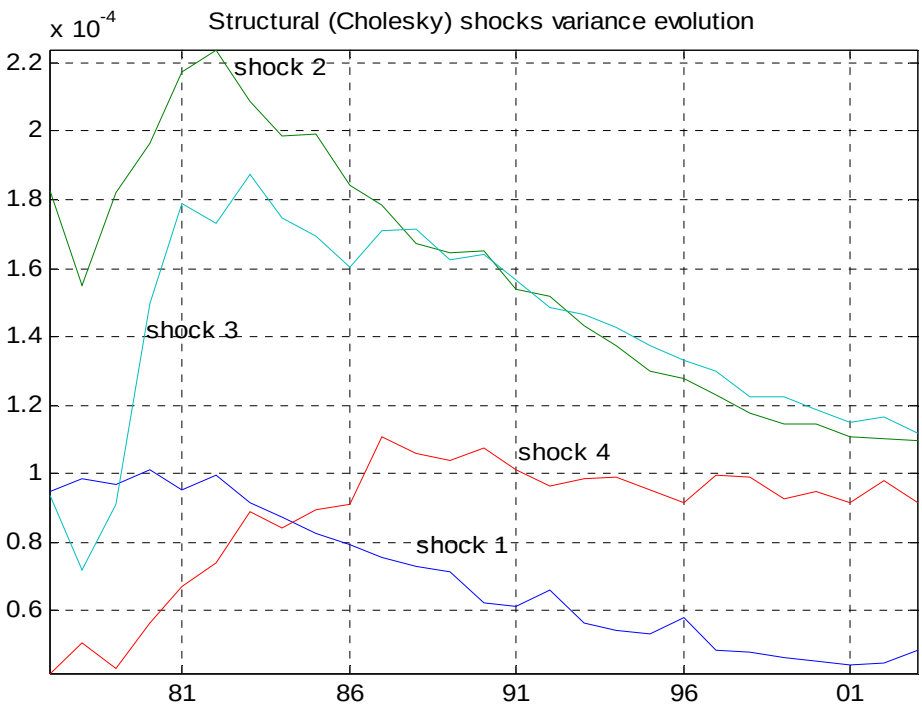
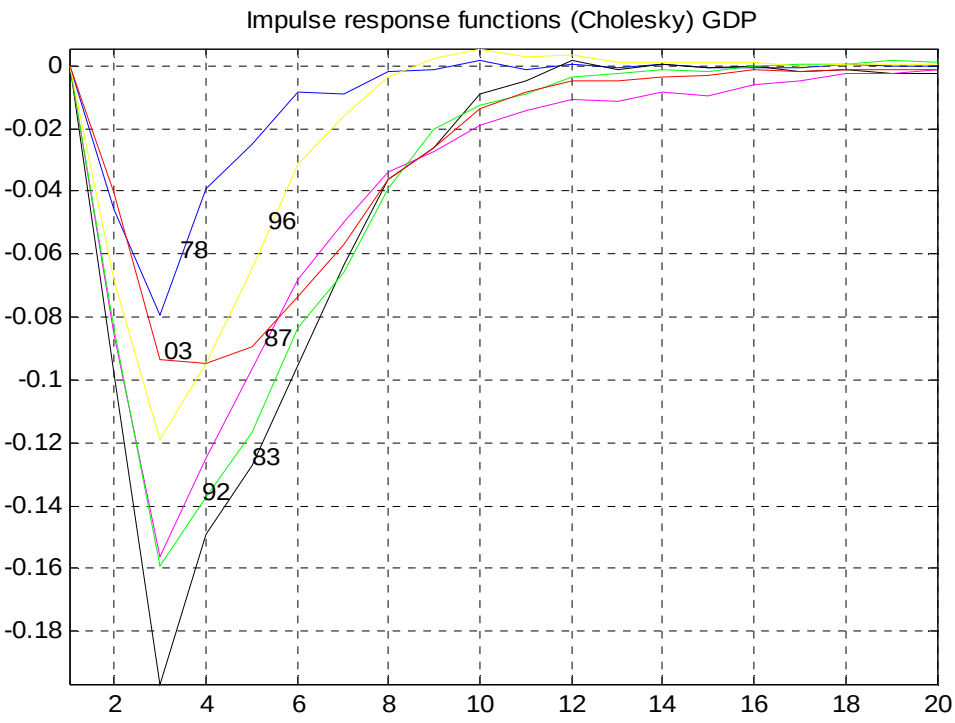


Figure 10b) Forecast error variance, Choleski system



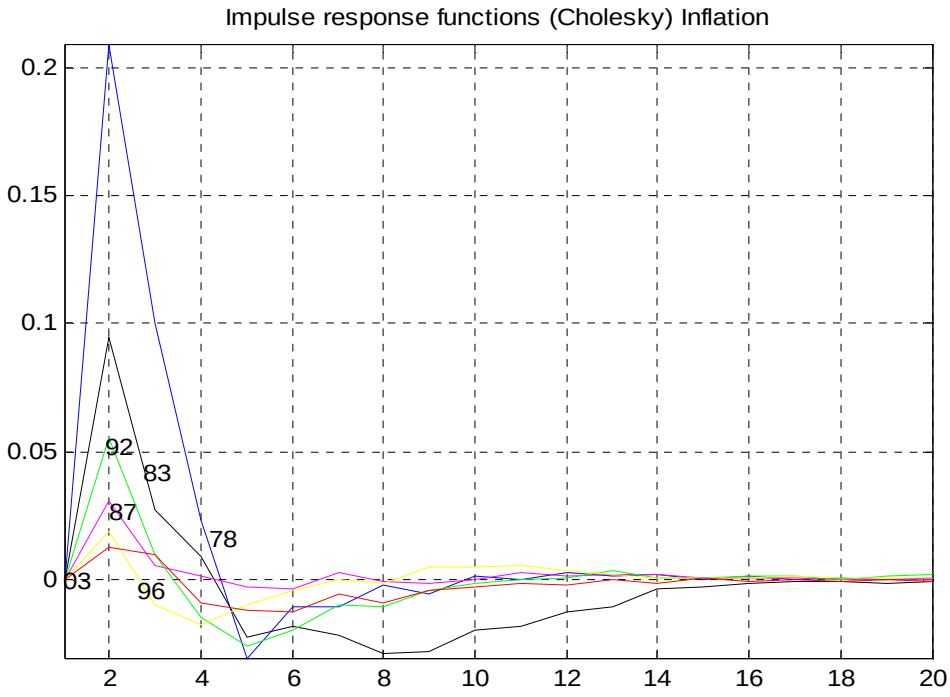


Figure 10c) Output and inflation responses to policy shocks, Choleski System

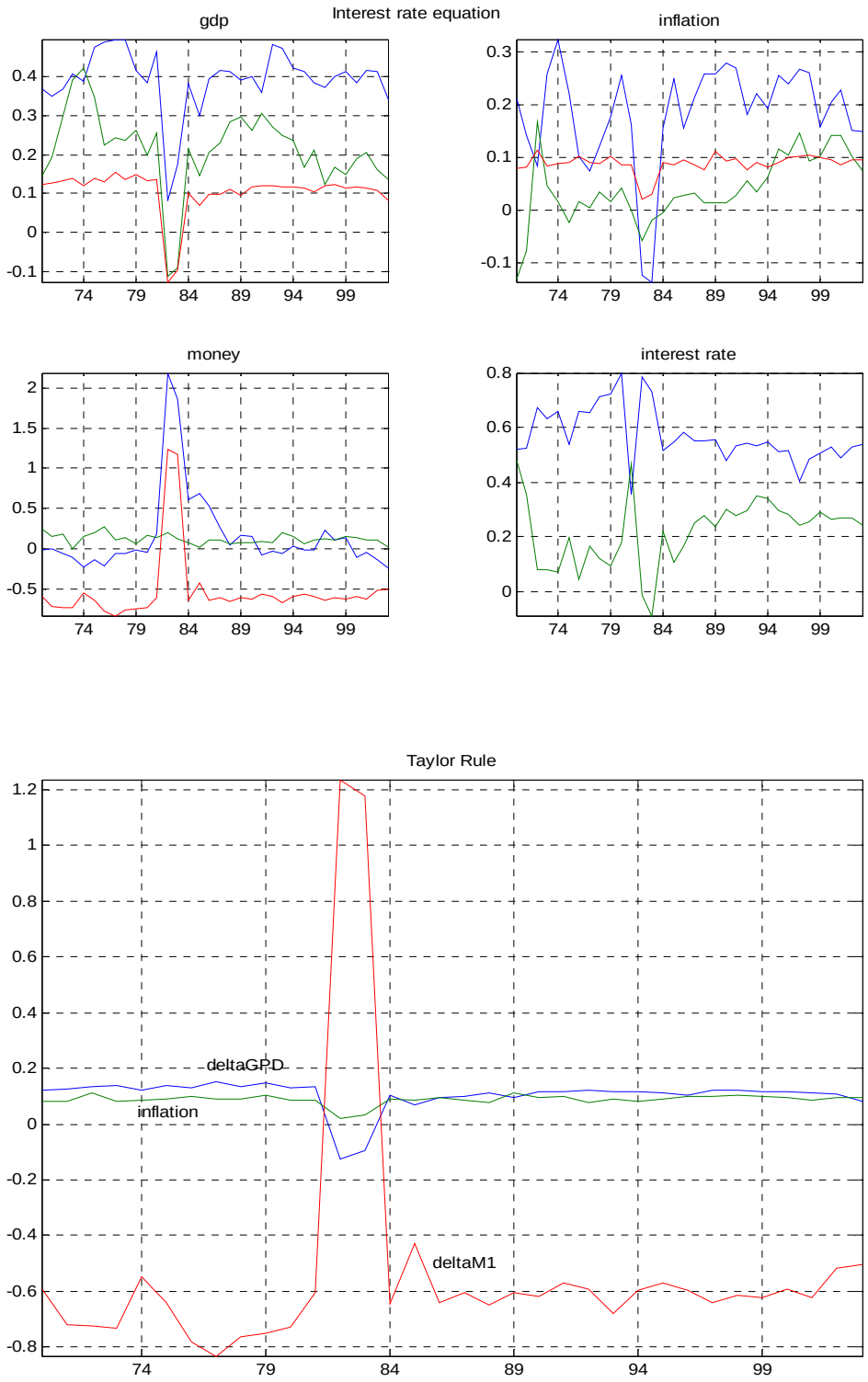


Figure 11a): Structural coefficients, VAR with output and M1 in growth rates

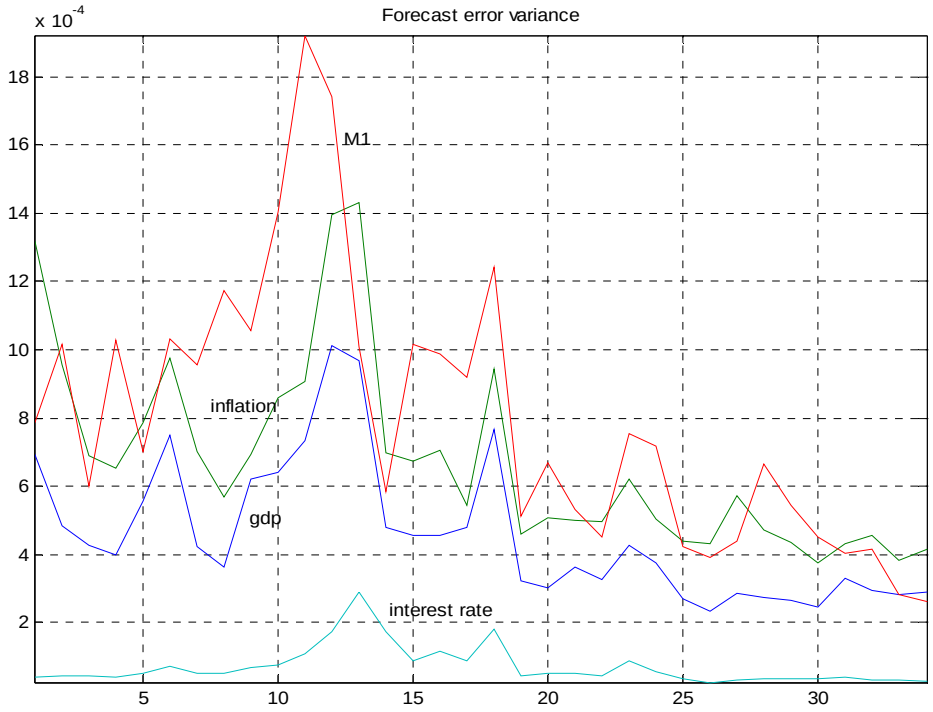
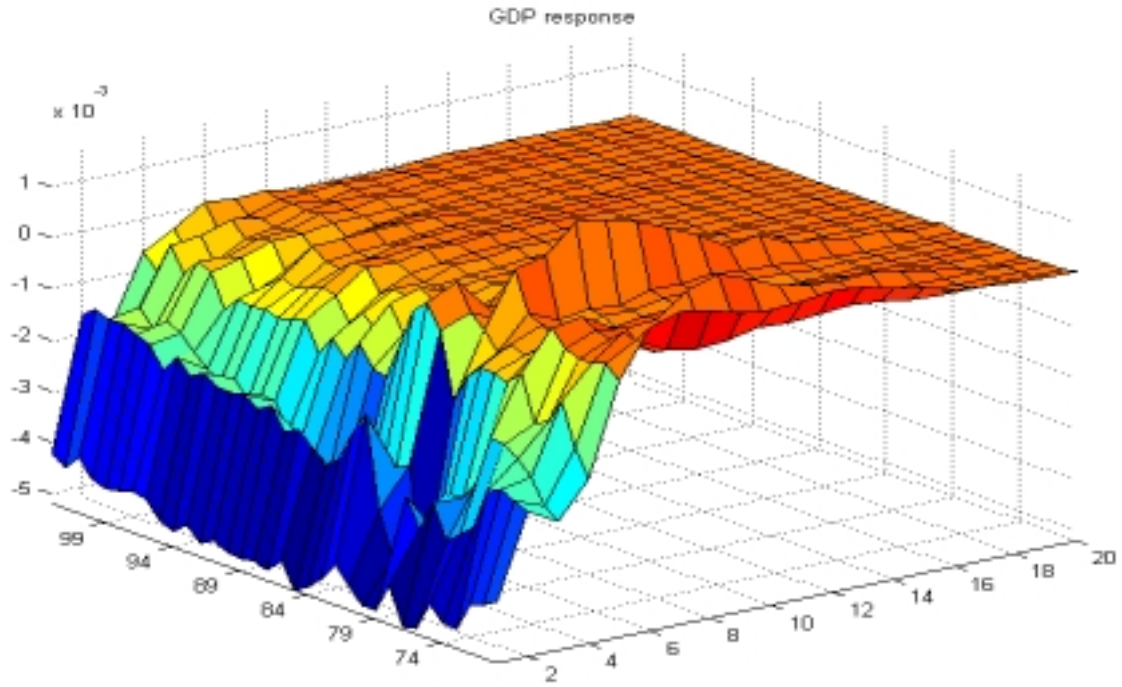


Figure 11b): Variances, VAR with output and M1 in growth rates



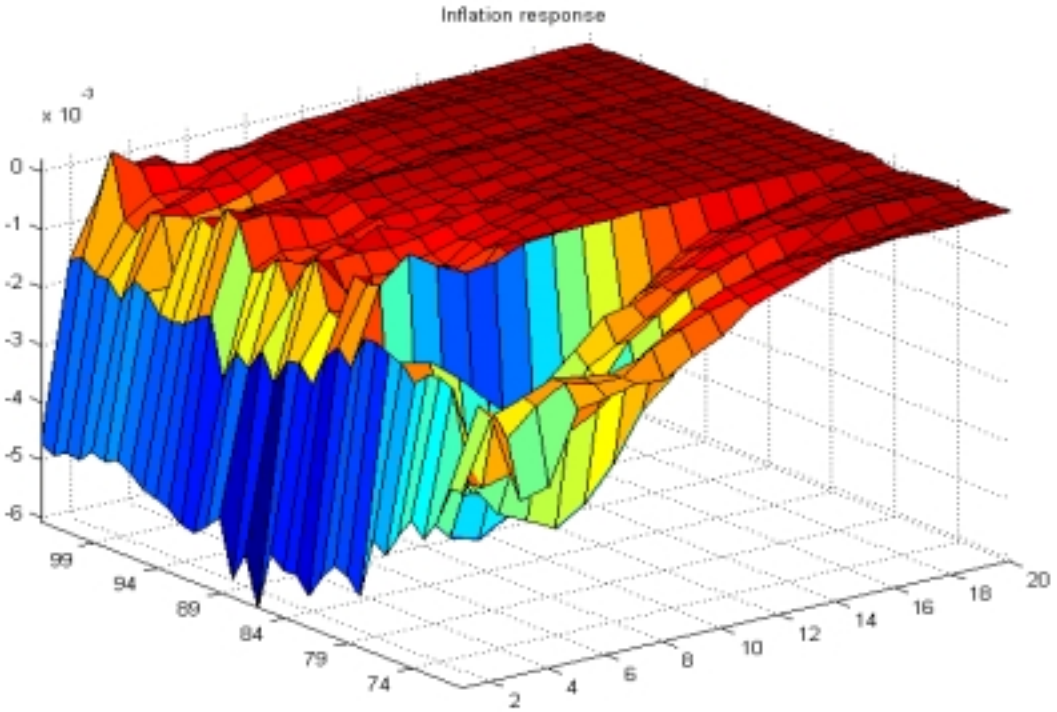


Figure 11c): Responses of output and inflation to policy shock (VAR in growth rates)

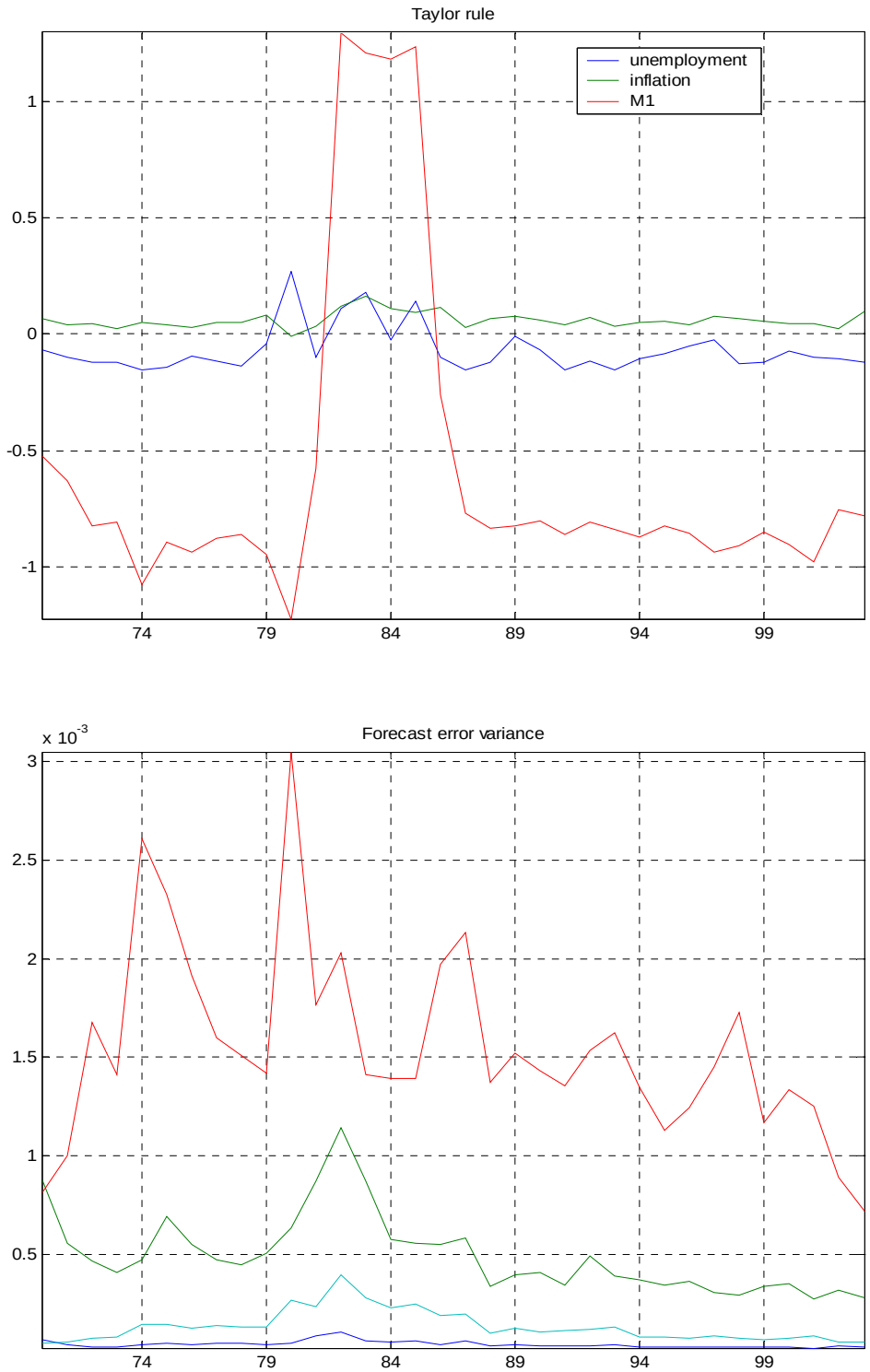
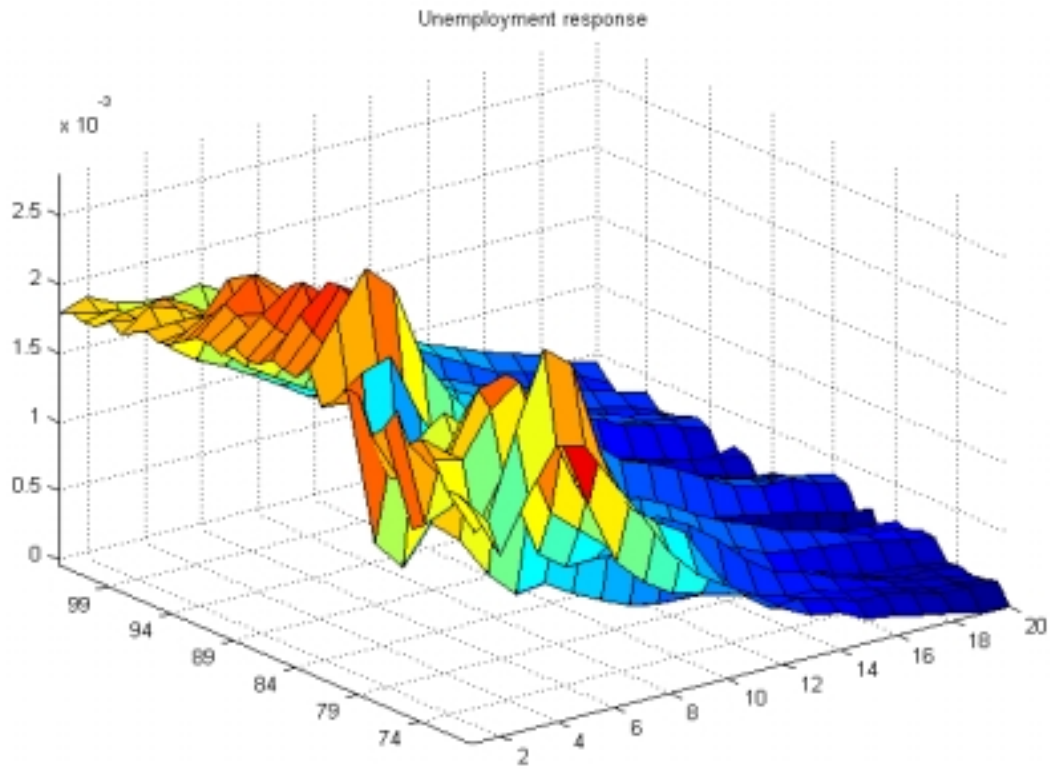


Figure 12 a) Structural coefficients, VAR with unemployement, inflation, M1 and FFR



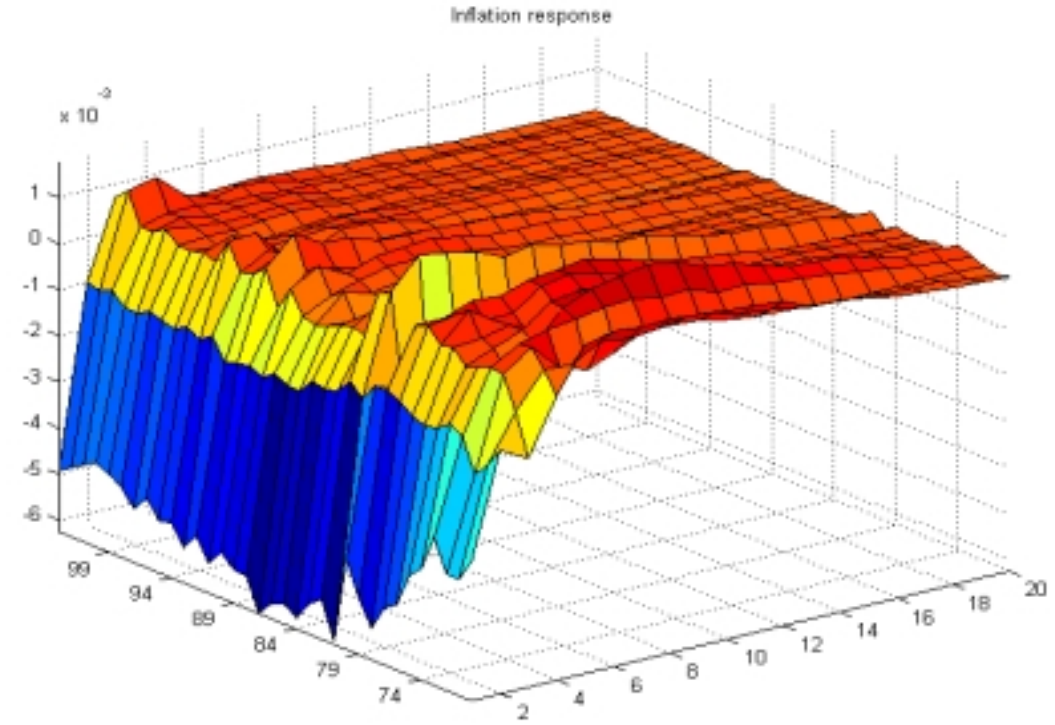


Figure 12b): Structural Response, VAR with unemployment, M1, inflation, and FFR

1 Introduction

There is considerable evidence suggesting that the US economy has fundamentally changed over the last couple of decades. In particular, several authors have noted a marked decline in the variance of real activity and inflation since the early 1980s (see e.g. Blanchard and Simon (2000), McConnell and Perez Quiroz (2001) and Stock and Watson (2003)). What are the reasons behind such a decline? A stream of literature attributes these changes to alterations in the mechanisms through which exogenous shocks spread across sectors and propagate over time. Since the transmission mechanism depends on the structure of the economy, the main implication of this viewpoint is that the underlying characteristics of the economy have changed. Several factors could be responsible for this evolution, for instance, changes in the behavior of consumers and firms or changes in preferences of policymakers. The recent literature has paid particular attention to monetary policy. Several studies, including Clarida, Gali and Gertler (2000), Cogley and Sargent (2001) (2003), Boivin and Giannoni, (2002a), (2002b), have argued that monetary policy was "loose" in fighting inflation in the 1970s but become more aggressive since the early 1980s and see in this change of attitude the reason for the observed reduction of inflation and output volatility. This view, however, is far from unanimous. For example, Bernanke and Mihov (1998), Orphanides (2001), Leeper and Zha (2003) find little evidence of significant changes in the policy rule used over the last 25 years and in the propagation of monetary shocks to the economy while Hanson (2001) claims that the dynamics of output and inflation change because of alterations taking place in the rest of the economy. In addition, Sims (2001) and Sims and Zha (2002) claim that changes in the variance of exogenous shocks (and, in particular, of monetary policy innovations) are probably responsible for the observed changes in output and inflation variance.

This controversy is not new. In the past rational expectations econometricians (e.g. Sargent (1984)) have argued that policy changes involving regime switches, dramatically alter private behavior decisions and, as a consequence, the dynamics of the macroeconomic variables and searched for historical episodes supporting this view (see e.g. Sargent (1999)). VAR econometricians, on the other hand, often denied the empirical relevance of this argument suggesting that the systematic portion of monetary policy has rarely been altered and that policy changes are better characterized as random draws for the non-systematic part (Sims (1982)). This long standing debate now has been cast into the dual framework of "bad policy" (failure to respond to inflationary pressure) vs. " bad luck" (shocks are drawn from a distribution whose moments vary over time) and new ground has been broken thanks to the development of tools which explicitly allow the examination of time variations in the structure of the economy and in the variance of the exogenous processes and the investigation of the timing of the changes.

This paper provides novel evidence on the contribution of monetary policy to the structural changes occurred in the US economy over the last three decades. Our basic framework of analysis is a time varying coefficients VAR model (TVC-VAR), similar to the one employed by Cogley and Sargent (2001), where the coefficients evolve according to a nonlinear

transition equation which puts zero probability on paths associated with explosive VAR roots. We use Markov Chain Monte Carlo (MCMC) methods to estimate posterior distributions of the quantities of interest. Contrary to the existing literature, we explicitly conduct a structural analysis and consider the relationship between monetary policy and the rest of the economy both in short and long run. Moreover, in addition to studying the timing of the changes in various equations, we also measure the magnitude of the variations in the propagation of monetary policy disturbances and the potential effect that changes in preference of the Fed would have had in particular historical episodes.

The structural setup we employ is particularly suited to study the main issues in the debate. In fact, we are able to separate structural variations produced by i) changes in the systematic component of policy, ii) changes in the propagation of policy shocks, iii) changes in the variance of the monetary policy and other shocks and iv) changes in the rest of the economy. The distinction between i) and ii) is important and both reduced form time varying approaches and structural but constant coefficient approaches are unable to separate the relative contribution of these two types of changes to the alterations in the output and inflation processes observed over the last 20 years.

A structural model is obtained from the reduced form TVC-VAR, identifying structural disturbances by means of sign restrictions. While our focus is on monetary policy disturbances, and therefore arbitrarily orthogonalize the other shocks, the methodology can be employed to jointly identify multiple sources of structural disturbances if needed (see e.g. Canova and De Nicolo' (2002)). While we focus attention primarily on the system identified with sign restrictions, we show in the last section of the paper that our conclusions are robust to the identification procedure and to a number of other auxiliary assumptions we have made.

Because time variations in the coefficients induce important non-linearities in the structure, standard statistics summarizing the dynamics in response to impulses in the structural shocks are inappropriate. For example, since at each point in time the coefficient vector is perturbed by a shock, assuming that between $T+1$ and $T+\tau$ no shocks other than the monetary policy disturbance hit the system is unappealing and can give misleading conclusions. To study the evolution the economy in response to structural shocks when coefficients vary over time, we therefore employ a different concept sharing similarities with those used in Koop, Pesaran and Potter (1996), Koop (1996), and Gallant, Rossi and Tauchen (1996). In particular, impulse response functions are defined as the difference between two conditional expectations, which differ in the arguments of their conditioning sets.

Several important results emerge from our investigation. First, we find changes in the estimated structural relationships, but these changes are localized in time and only involve particular coefficients in certain equations. In particular, we show that excluding the Volker experiment of the beginning of the 1980s, the monetary policy rule has been quite stable over time. Interestingly, the posterior mean of the (sum of) inflation coefficients fails to satisfy the so-called Taylor principle, not only in the 1970s but also in the 1990s. Hence, both in terms of timing of the changes and of aggressiveness in response to inflation movements, variations in the systematic component of monetary policy are unlikely to have driven

the changes in the observed output and inflation processes. Second, we find evidence of a decrease in the uncertainty surrounding the structural disturbances of the system, including those of monetary policy shocks, and that the evolution of the changes roughly coincides with those observed in the output and inflation equations. Taken together, these results suggest that the "bad luck" hypothesis has considerable more posterior support than the "bad policy" hypothesis in accounting for the observed dynamics of the US economy. Third, we show that also the transmission of monetary policy shocks has been very stable over time. In fact, both the shape and the persistence of output and inflation responses are very similar over time and the quantitative differences in the posterior mean response are statistically negligible at almost all horizons for all years from 1979 to 2002. Fourth, we find that inflation persistence (measured by the height of the zero frequency of the spectrum of inflation in the structural model) has statistically changed over time. We show that both monetary and non-monetary factors account for the magnitude of inflation persistence and that, although the relative contribution of monetary policy fluctuates over time, it is acually increasing since 1981.

We investigate whether a more aggressive response to inflation would have made a difference in the dynamics of output and inflation. We show that such a stance would have reduced inflationary pressures in short and medium run in 1979 but not in any other date after that. Furthermore, it would have induced negative output effects at several dates in the sample. Hence, while the Fed could had some room to improve economic performance at the end of the 1970s, it is not that clear that the economy would have benefitted from a tougher inflation stance except in 1983. However, since the magnitude of output and inflation effects tends to decline over time, it appears that the stance of the Fed regarding inflation has become less and less important for the dynamics of both real and monetary variables. Finally, we show that our conclusions are robust to a number of changes in the auxiliary assumptions used. In particular, we show that our results do not depend on the identification procedure, on the treatment of trends and the variables included in the VAR.

The rest of the paper is organized as follows. Section 2 presents the reduced form model, describes our identification scheme and the computational approach used to obtain posterior distributions of the structural coefficients of the model. Section 3 defines impulse response functions which are valid in our TVC-VAR model and describes how to compute dynamics to shocks in the non-systematic and the systematic component of the model. Section 4 presents the results and Section 5 concludes. A series of appendices describes the technical details involved in the computation of posterior distributions and of impulse responses.

2 The Reduced form Model

Let y_t be a $n \times 1$ vector of time series with the representation

$$y_t = A_{0,t} + A_{1,t} y_{t-1} + A_{2,t} y_{t-2} + ... + A_{p,t} y_{t-p} + \varepsilon_t \tag{1}$$

where $A_{0,t}$ is a $n \times 1$ vector, $A_{i,t}$, for $i = 1, ..., p$ are $n \times n$ matrices of coefficients and ε_t is a $n \times 1$ Gaussian white noise process with zero mean and covariance Σ_t. Let $A_t =$

$[A_{0,t}, A_{1,t}...A_{p,t}]$, $x_t' = [1_n, y_{t-1}'...y_{t-p}']$, where 1_n is a row vector of ones of length n, let $vec(\cdot)$ denote the stacking column operator and let $\theta_t = vec(A_t')$. Then (1) can be more compactly written as

$$y_t = X_t'\theta_t + \varepsilon_t \tag{2}$$

where $X_t' = (I_n \bigotimes x_t')$ is a $n \times (np+1)n$ matrix, I_n is a $n \times n$ identity matrix, and θ_t is a $(np+1)n \times 1$ vector. If we treat θ_t as a hidden state vector, equation (2) represents the observation equation of a state space model. We assume that θ_t evolves according to the following nonlinear transition equation

$$p(\theta_{t+1}|\theta_t, \Omega_t) \propto \mathcal{I}(\theta_{t+1}) f(\theta_{t+1}|\theta_t, \Omega_t) \tag{3}$$

where $\mathcal{I}(\theta_{t+1})$ is an indicator function discarding explosive paths of y_t. Such an indicator is necessary to make dynamic analysis sensible and, as we will see below, it is easy to implement numerically. We assume that $f(\theta_{t+1}|\theta_t, \Omega_t)$ can be represented as

$$\theta_{t+1} = \theta_t + u_{t+1} \tag{4}$$

where u_t is a $(np+1)n \times 1$ Gaussian white noise process with zero mean and covariance Ω_t. We select this simple specification because more general AR and/or mean reverting structures were always discarded in out-of-sample model selection exercises. We assume that $\Sigma_t = \Sigma \ \ \forall t$; that $corr(u_t, \varepsilon_t) = 0$, and that Ω_t is diagonal. At first sight, these assumptions may appear to be restrictive but they are not. For example, the first assumption does not imply that the forecast errors of the model are homoschedastic. In fact, as shown in Canova (1993) substituting (4) into (2) we have that $y_t = X_t'\theta_{t-1} + e_t$ where $e_t = \varepsilon_t + X_t'u_t$. Hence, one-step ahead forecast errors have a time varying non-normal heteroschedastic structure and important non-linearities in higher moments even without assuming that Σ_t or Ω_t vary over time. The assumed structure is appealing also because the structure that time variations imposed on the variance of the forecast errors allows us to sort out variations due to changes the coefficients from variations due to changes the variances of the shocks. (see Sims and Zha (2002) or Cogley and Sargent (2003) for specifications where Σ_t is allowed to vary over time). The second assumption is standard but somewhat stronger and implies that the dynamics of the model are conditionally linear [1] and this facilitates the derivation of posterior estimates and substantially reduces the computational costs. Sargent and Hansen (1998) showed how to relax this assumption by equivalently letting the innovations of the measurement equation to be serially correlated. Since in our setup ε_t is, by construction, a white noise process, the loss of information caused by imposing uncorrelation between the shocks is likely to be small. The third assumption implies that each element of θ_t evolves independently but it is irrelevant for the outcomes since structural coefficients will be allowed to evolve in a correlated manner.

Let S be the square root of Σ, i.e., $\Sigma = SS'$. Since Σ is time invariant also S is time invariant. Let H_t be an orthonormal matrix, independent of ε_t, such that $H_tH_t' = I$ and let

[1] This means, for instance, that we can not study whether shocks of different sign or of different magnitude have different dyanmic effects on the system.

$K_t^{-1} = H_t' S^{-1}$. K_t is a particular decomposition of Σ which transforms (2) in two ways: it produced uncorrelated innovations via the matrix S; it gives a structural interpretation to the equations of the system via the matrix H_t. Premultiplying y_t by K_t^{-1} we obtain

$$K_t^{-1} y_t = K_t^{-1} A_{0,t} + \sum_j K_t^{-1} A_{j,t} y_{t-j} + e_t \tag{5}$$

where $e_t = K_t^{-1} \varepsilon_t$ satisfies $E(e_t) = 0$, $E(e_t e_t') = I_n$. Equation (5) represents the class of "structural" representations of y_t we are interested in. For example, a standard Choleski representation can be obtained setting S to be lower triangular and $H_t = I_n$ and more general patterns by choosing S to be non-triangular and $H_t = I_n$. In this paper S is arbitrarily chosen and H_t implements interesting economic restrictions.

Letting $C_t = [K_t^{-1} A_{1t} ... K_t^{-1} A_{pt}]$, and $\gamma_t = vec(C_t')$, (5) can be written as

$$K_t^{-1} y_t = X_t' \gamma_t + e_t \tag{6}$$

As in standard fixed coefficient VARs there is a mapping between γ_t and θ_t since $\gamma_t = (K_t^{-1} \bigotimes I_{np}) \theta_t$ where I_{np} is a $(np+1) \times (np+1)$ identity matrix. Whenever $\mathcal{I}(\theta_{t+1}) = 1$, we also have

$$\gamma_{t+1} = \gamma_t + \eta_{t+1} \tag{7}$$

where $\eta_t = (K_t^{-1} \bigotimes I_{np}) u_t$ is the vector of shocks to structural parameters and satisfies $E(\eta_t) = 0$, $E(\eta_t \eta_t') = E((K_t^{-1} \bigotimes I_{np}) u_t u_t' (K_t^{-1} \bigotimes I_{np})')$.

Hence, the vector of structural shocks $\xi_t' = [e_t', \eta_t']'$ is a white noise process with zero mean and covariance matrix $E \xi_t \xi_t' = \begin{bmatrix} I_n & 0 \\ 0 & E((K_t^{-1} \bigotimes I_{np}) u_t u_t' (K_t^{-1} \bigotimes I_{np})') \end{bmatrix}$. Note that since each element of γ_t depends on several u_{it} via K_t, shocks to structural parameters are no longer independent

The structural model (6)-(7) contains two types of shocks: disturbances to the observations equations, e_t, and disturbances to structural parameters, η_t. While the former have the same interpretation as those in a fixed coefficients VARs, the second are new. To understand their meaning, suppose that the $n-th$ equation of (6) is the monetary policy equation and suppose we split it into a systematic component, summarized by $\tilde{\gamma}_t = [\gamma_{(n-1)(np+1),t}, ..., \gamma_{n(np+1),t}]'$, and describing say, how interest rates respond to the developments in the economy, and the non-systematic component, summarized by the policy shock $e_{n,t}$. Then, innovations $\tilde{\gamma}_t$ represent changes in the preferences of the monetary authorities with respect to developments in the rest of the economy.

In our setup, identifying structural shocks is equivalent to choosing a matrix H_t. Here as in Faust (1998), Uhlig (2001), and Canova and De Nicoló (2002), we select H_t so that the sign of the impulse response functions at $t+j, j = 1, 2, \ldots, J$ matches some theoretical restriction. In particular, we assume that a contractionary monetary policy shock must generate a non-positive effects on output, inflation and nominal balances and a non-negative effect on the interest rate for two quarters after the shock.

We choose sign restrictions to identify shocks to the observation equation for two reasons. First, the contemporaneous zero restrictions conventionally used to identify VARs

are often absent in those theoretical (DSGE) models economists like to use to guide the interpretation of the results. Second, in our framework, standard decompositions impose strong restrictions on the structure of time variations of the model, restrictions which have no a-priori justification. To illustrate this point consider, for example, a Choleski decomposition. Since Σ is time invariant, its Choleski factor S is time invariant. Hence, since $H_t = I$, the contemporaneous effects of a monetary policy shock are time-invariant. That is, contemporaneous impulse responses will be constant no matter what point in time we choose to compute them and time varying responses will be obtained only if there are variations in the lagged coefficients of the reduced form model. Our identification approach, on the contrary, allows for time variations in both contemporaneous and lagged effects. In fact, by restricting the sign of the impulse responses for at least two periods we make H_t depend on the conditional distribution of the states. Since this is time varying, H_t must vary over time therefore making the contemporaneous effects K_t vary over time as well. For sensitivity analysis we report, in the last section of the paper, responses obtained identifying policy shocks as the third element of a Choleski system, i.e. we let monetary policy reacts to output and inflation movements but assume that it has no effects within a quarter on these variables. Since this paper is interested in recovering the systematic and non-systematic part of monetary policy and in analyzing how the economy respond their changes over time, we arbitrarily diagonalize the remaining disturbances without giving them any structural interpretation We describe the algorithm implementing sign restrictions in the model in appendix B.

3 Impulse Responses

One important question we would like to address in our study is whether the transmission of monetary policy shocks has changed over time. In a fixed coefficient model, impulse response functions provide information on how the variables of the system react to policy shocks. Impulse responses are typically computed as the difference between two realizations of $y_{i,t+\tau}, \tau = 1, 2, \ldots$: these realizations are identical up to time t; one assumes that between $t+1$ and $t+\tau$ a shock in e_j occurs only at time $t+1$ and the other that no shocks will take place at all dates between $t+1$ and $t+\tau$.

In a TVC model, responses computed this way are inappropriate since they disregard the fact that between $t+1$ and $t+\tau$ the coefficients of the system may also change. Hence, meaningful impulse response functions ought to measure the effects of a shock in $e_{j,t+1}$ on $y_{it+\tau}$, without restricting future coefficients shocks to be non-zero. For this reason, our impulse responses are obtained as the difference of two conditional expectations of $y_{t+\tau}$. In both cases we condition on the history of the data $y_1, \ldots, y_t$, of the states $\theta_1, \ldots \theta_t$, of the structural parameters of the transition equation (which are function of K_t) and all future shocks. However, in the first case we condition on a draw for the current shock, while in the second we condition on current shock being zero.

Formally speaking, let $y^t = [y_1', ...y_t']'$ be a history for y_{t+1}; $\theta^t = [\theta_1', ...\theta_t']'$ be a trajectory for the states up to t. Also, let $y_{t+1}^{t+\tau} = [y_{t+1}', ...y_{t+\tau}']'$ denote a collection of future

observations and $\theta_{t+1}^{t+\tau} = [\theta'_{t+1}, ...\theta'_{t+\tau}]'$ a future trajectory of states from $t+1$ up to time $t+\tau$. Let $V = (\Sigma, \Omega)$; recall that $\xi'_t = [e'_t, \eta'_t]'$ and let $\zeta_t = [\varepsilon'_t, u'_t]'$. Let $\xi^{\delta}_{i,t+1}$ be a particular realization of size δ of $\xi_{i,t+1}$ and let $I_1 = \{y^t, \theta^t, V, K_t, \xi^{\delta}_{i,t+1}, \xi_{-i,t+1}, \xi^{t+\tau}_{t+2}\}$ and $I_2 = \{y^t, \theta^t, V, K_t, \xi_{-i,t+1}, \xi^{t+\tau}_{t+2}\}$ be two conditioning sets. Then an impulse response function to a shock $\xi^{\delta}_{i,t+1}$, $i = 1, \ldots, n$ for each $\tau = 1, \ldots, T$ is defined as: [2]

$$IR_y(t,\tau) = E(y_{t+\tau}|I_1) - E(y_{t+\tau}|I_2) \tag{8}$$

(8) resembles the impulse response function suggested by Gallant et al. (1996), Koop et al. (1996) and Koop (1996). Three important differences however need to be noted. First, rather than treating histories as random variables, we condition on a particular realization of y^t. Since we analyze how responses vary over time, we want impulse responses to be history dependent. Second, our impulse responses are independent of the sign and the size of the shocks (as it is in a fixed coefficient case). This is the result of the assumption that shocks to the observation and the transition equations are uncorrelated. Third, since the parameters of the transition equation appear in the conditioning set, we do not condition on a particular realization of θ_t (for example, its conditional mean) but instead treat θ_t as a random variable and integrate it out when calculating impulse responses. This implies that $IR_y(t,\tau)$ are random variables. Integrating θ_t out of impulse responses allows us to concentrate attention on time difference which depend on the history of y_t but not on the size of the sample. Finally, note that $IR_y(t,\tau)$ can be made state dependent if we condition on a particular stretch of a history (a boom or a recession) and that when coefficients are constant, (8) coincides with the standard impulse response definition.

Since there are two types of shocks in the system, we describe how to trace out the dynamics effects of each of then separately. Let $\xi_{i,t+1} = e_{i,t+1}$. Then

$$\begin{aligned} IR_y(t,1) &= K_t^{-1,i} e_{i,t+1} \\ IR_y(t,\tau) &= \Psi^i_{t+\tau,\tau} e_{i,t+1} \text{for} \quad \tau = 2, ...T. \end{aligned} \tag{9}$$

where $\Psi_{t+\tau,\tau} = \mathcal{S}_{n,n}[(\prod_{h=1}^{\tau-1} \mathbf{A}_{t+\tau-h}) \times S \times H_t]$, $\mathbf{A}$ is the companion matrix of the VAR; $\mathcal{S}_{n,n}$ is a selection matrix which extracts the first $n \times n$ block of $[(\prod_{i=1}^{\tau-1} \mathbf{A}_{t+\tau-i}) \times S \times H_t]$ and $\Psi^i_{t+\tau,\tau}$ is the column of Ψ corresponding to the ith shock.

When the coefficients are constant, $\prod_h \mathbf{A}_{t+\tau-h} = \mathbf{A}^{\tau}$ and $\Psi^i_{t+\tau,\tau} = \mathbf{A}^k \times S \times H$ for all τ. Hence (18) collapses to traditional impulse response function to unitary structural shocks. Clearly, IR_y depends on the identifying matrix K_t and is not explosive, since $\Psi_{t+\tau,\tau}$ is the product of matrices whose eigenvalues are non-explosive.

When $\xi_{i,t+1} = \eta_{j,t+1}$ for $j = (n-1)(np+1), ..., n(np+1)$, appendix A shows that

$$IR_y(t,1) = [E(\mathbf{A}_{t+1,1}|I_1) - E(\mathbf{A}_{t+1,1}|I_2)]\, y_t \tag{10}$$

[2] An alternative definition of impulse responses is obtained averaging out future shocks. Our definition is preferrable for two reasons: it is easier to compute and produces numerically more stable distributions; it produces impulses responses which are more similar to those produced by constant coefficient impulse responses when shocks to the measurement equations are considered. However, since future shocks are not averaged out, our impulse responses display larger variability.

and for $\tau = 2, ..., T$

$$\begin{aligned} IR_y(t,\tau) &= E(\tilde{A}_{0,T+\tau}|I_1) - E(\tilde{A}_{0,T+\tau}|I_2) \\ &+ E\left(\tilde{\mathbf{\Phi}}_{T+\tau,\tau}|I_1\right)\mathbf{y}_T - E\left(\tilde{\mathbf{\Phi}}_{T+\tau,\tau}|I_2\right)\mathbf{y}_T \\ &+ E\left(\sum_{j=0}^{\tau-1}\Psi_{T+\tau,j}|I_1\right)e_{T+\tau-j} - E\left(\sum_{j=0}^{\tau-1}\Psi_{T+\tau,j}|I_2\right)e_{T+\tau-j} \end{aligned} \tag{11}$$

where$\tilde{\mathbf{\Phi}}_{t+\tau,\tau} = \mathcal{S}_{(n,n^2p)}\prod_{h=0}^{j-1}\mathbf{A}_{t+\tau-h}$. There are three components in the responses to shocks in the structural coefficients: the first line in equation (11) shows how the shocks spread through the system via lags of y_t; the second and the third lines how it spreads through the constant term and the last two lines how they spread through future shocks to the structural equations. Note that when a shock hits the systematic component of policy, IR_y depends on K_t only because $\eta_t = (K_t^{-1} \bigotimes I_{np})u_t)$. Note that, also in this case, IR_y are non-explosive.

4 Estimation

The model (6)-(7) is estimated using Bayesian methods. That is, we specify prior distributions for θ^t, Σ, Ω, and H^t and use data up to t to compute posterior estimates of the structural parameters and of continuous functions of them. Since our sample covers quarterly data from 1960:1 to 2002:4, we initially estimate the model for the sample 1960:1-1978:3 and then reestimate it 97 times moving the terminal date by one quarter from 1978:4 to 2002:4.

Posterior distributions for the structural parameters are not available in a closed form. MCMC methods are used to simulate sequences from the posterior distributions obtained with the information up to time t. Estimation of reduced form TVC-VAR models with or without time variations in the variance of the shocks to the transition equation is standard (see e.g. Cogley and Sargent (2001)): it requires treating parameters which are time varying parameters as a block in a Gibbs sampling algorithm. Therefore, at each t and in each cycle of the Gibbs sampler, one runs the Kalman filter and the Kalman smoother, conditional on the draw of the other time invariant parameters. In our setup the calculations are complicated by the fact that at each cycle, we need to obtain structural estimates of the time varying features of the model. This means that we need to apply at each step the identification scheme, discarding paths which are explosive and paths which do not satisfy the restrictions we impose. While this complicates the Gibbs sampler loop, and dramatically reduces the number of draws available for inference, it is straightforward to implement and relatively cheap to compute.

Because of the heavy notation involved in the construction of posterior distributions and the technicalities needed to produce draws from these posteriors, we defer the presentation of the details of the estimation to appendix B.

5 The Results

The data we use is taken from the FREDII data base of the Federal Reserve Bank of San Louis. In our basic exercise we use the log of (linearly) detrended (linear) real GDP, the log of first difference of GDP deflator, the log of (linearly) detrendend M1 and the federal funds rate in that order. Systems containing the growth rate of GDP and M1 or the (linearly) detrendend unemployment rate and nonborrowed reserves are analyzed in the next section. Distribution of structural parameters and of interesting functions of them are obtained with the amount of information available up to that point in time. That is, to produce posterior estimates as, say, 1981:1 we use (revised) data from 1960:1 up to 1981:1 and to produced posterior estimates for 1981:2 we use (revised data) from 1960:1 up to 1981:2.

We organize the presentation of the results around four general themes: (i) Do reduced form coefficients display significant changes? (ii) Are there synchronized changes in the structural coefficients of different equations and/or in the structural variances of the model? (iii) Are there changes in the propagation of monetary policy disturbances in the short and the long run? (iv) Would it have made a difference in terms of macroeconomic performance if monetary policy would had been more aggressive in fighting inflation at the end of the 1970's or at any other date in the sample?

5.1 The evolution of reduced form coefficients

Figure 1 plots the evolution of the mean of the posterior distribution of the reduced form coefficients in each of the four equations (top panel) and their change (bottom panel). The first date corresponds to estimates obtained with the information available up to time 1978:3, the last one to estimates obtained with the information up to time 2002:4.

Several interesting aspects of the figure deserve some comments. First, consistent with the evidence of Sargent and Cogley (2001) and (2003) all equations display some coefficient variation. In terms of size, the money and interest rate equations are those with the largest changes, while in the other two equations time variations are smaller. Interestingly, variations in the coefficients of the inflation equation are the smallest of all. Second, while variations appear to be stationary in nature, there are few coefficients which display a clear trend over time. For example, in output equation the coefficient on the first lag of money is drifting downward from 0.6 in 1979 to essentially zero at the end of the sample; while in the interest rate equation, the first lagged money coefficient is drifting upward from roughly zero in 1978 to about 0.9 in 2002. In general, and excluding for the 1979-1986 period, coefficients drift is smooth and relatively slow. Perhaps more importantly, we find little evidence that the coefficients of the output and inflation equation present a once-and-for-all structural break (i.e. coefficients do not jump at some date and stays there afterward). Third, the majority of the changes appear to be concentrated at the beginning of the sample. The period 1979-1982 is the one which displays the most radical changes; there is some coefficient drift up to 1986, and after that date variations appear to be random in nature and small. Furthermore, these changes seem to involve primarily the coefficients on the first lag of money (this is the case in three equations) or of interest rates (one equation).

Finally, centered 68% posterior bands for the coefficients at the beginning (1978:3) and at the end of the sample (2002:4) overlap in many cases. Therefore, barring few relevant exceptions, instabilities appear to be associated with the Volker (1979-1982) experiment and the adjustments following it. Furthermore, they are temporary and mean reverting in nature.

To go beyond the simple documentation of patterns of time variations in reduced form coefficients and study whether there are structural alterations in the economy and whether monetary policy is responsible for the changes, we next examine the dynamics of structural coefficients.

5.2 Structural time variations

The upper panel of figure 2 presents the evolution of the posterior mean of lagged structural coefficients of each equation , obtained via the Gibbs sampler and our identification scheme, and the bottom panel their changes at each date in the sample. The first date corresponds again to estimates obtained with the information up to time 1978:3, the last one to estimates obtained with the information up to time 2002:4. Figure 3 presents the evolutions of the coefficients of the monetary policy equation (which is normalized to be the last one of the system). Contemporaneous coefficients appear in the top panel and lagged coefficients in the bottom panel.

It is immediate to notice that changes in the structural coefficients are typically larger and more generalized than those in the reduced form coefficients. The output and the monetary policy equations are those displaying the largest absolute coefficient changes - these are up to 4 times as large as the largest absolute changes present in the other two equations - while the coefficients of the structural inflation equation are the most stable ones. Furthermore, except for the money (demand) equation, most the variations are concentrated in the first part of the sample, are large in size, statistically and often economically significant. Consistent with the conventional wisdom the money (demand) equation displays trending coefficients (its own first lagged one goes from -0.6 in 1979 to 0.7 in 2002) and large swings in the output and interest rate coefficients from 1991 on. More interestingly from our point of view, there is a pattern in the structure of time variations. The output equation displays two regimes of coefficient variations (one with high variations up to 1986 and one with low variations thereafter) and, within the high volatility regime, the largest coefficient variations occur in 1986. The inflation equation shows the largest coefficient changes up to 1982 and, barring few exceptions, a more stable pattern has resulted since then. Finally, our identified monetary policy equation displays large and erratic lagged coefficient changes up to 1986 and coefficients variation is considerably reduced after that. Since the timing of the variations in the structural coefficients of the output and inflation equations are somewhat asynchronous with those of the monetary policy equation, figure 2 casts some doubts on a causal interpretation of the observed changes running from changes in the policy equation to changes in the dynamics of output and inflation.

Figure 3 present in more details the pattern of time variations present in the monetary policy rule Three facts stand out. First, posterior mean estimate of all contemporaneous

coefficients are humped shaped: they significantly increase from 1979 to 1982 and smoothly decline afterwards. Second, although all contemporaneous coefficients are higher at the end than at the beginning of the sample, they are typically lower than the conventional wisdom would suggest. In particular, the contemporaneous inflation coefficient peaks at about 1.2 in 1982 and then declines to a low 0.3, on average, in the 1990s. This pattern is shared by the two lagged inflation coefficients: they both peak in 1982 and smoothly decline afterward. In other words, except for the beginning of 1980s, the sum of coefficients on current and lagged inflation fails to satisfy the so-called Taylor principle. In this sense, the Greenspan's regime was only marginally more effective than Burns's in insuring inflation stability: interest rate responses to inflation movements were barely more aggressive in the 1990s than they were in the 1970s. Note also that, again excluding the beginning of the 1980's, the estimated monetary policy rule displayed considerable stability, in line with the evidence presented, e.g. by Bernanke and Mihov (1998). Hence, the fact that the macroeconomic performance was considerably different in the two time periods seems to suggest that the size and characteristics of the shocks hitting the US economy in the two periods were different. We will elaborate on this issue later on.

Our estimated policy rule displays a six fold-increase in all contemporaneous and first lagged coefficients from 1979 to 1982. Interestingly, this increase is not limited to the inflation coefficients, but also involve output and the money coefficients. The high responsiveness of interest rates to economic conditions is consistent with the idea that by targeting monetary aggregates the Fed forced interest rates to jump to equilibrate a "fixed" money supply with a largely varying money demand (the period was characterized by a number of important financial innovations). The pervasive instability characterizing this period and the subsequent three years adjustments contrasts with the substantial stability of the coefficients of the monetary policy rule in the rest of the sample. Hence, excluding the "Volker experiment", the systematic component of monetary policy has hardly changed over time and if, any change must be noted, it is more toward a decline in the responsiveness of interest rates to economic conditions. Therefore this outcome is consistent with the "business as usual" characterization over the last 30 years put forward by Leeper and Zha (2003). Furthermore, it replicates very closely the time profile of the policy rule obtained recursively estimating a small scale DSGE model with Bayesian methods (see e.g. Canova (2004)).

The evidence we have so far collected seems to give little credence to the "bad policy" hypothesis. If policy mistakes were made in the late 1970's, they seemed to have been repeated in the 1990's. Still, the dynamics and the volatility of output and inflation were quite different. The "back luck" hypothesis suggests that policy has little to do with the observed changes and that instead it is alterations in the distribution of the shocks hitting the economy that is responsible for the improved macroeconomic outcome.

Figure 4 presents some evidence on this issue. In the top panel we report the evolution of the posterior mean estimate of the variance of the forecast errors of the structural model and, in the bottom panel, the variations produced by its heteroschedastic component, i.e the variations induced by product of the estimated innovations in the coefficient times the regressors of the model. Four features of the figure 4 are of interest. First, forecast

error variances in three of the four equations are humped shaped: they show a significant increase from 1979 to 1982 followed by a smooth decline. As it happened with structural coefficients, the posterior mean estimate of the variance of the shocks in 2002 is roughly similar in magnitude to the posterior mean estimate obtained in 1978. Second, the time profile of the changes in the forecast error variances of the output and the inflation equations are sufficiently well synchronized with the variations in the forecast error variance of our estimated monetary policy equation. In particular, since 1981, the point estimate of the variance of the forecast error of these three equations displays a common and significantly declining trend. Third, time variations in the posterior mean of the forecast error variance of the output and inflation equations due to changes in the coefficients are much larger than in the other two equations up to 1982. Since that date, the heteroschedastic component of the forecast error variance of all equations has common pattern and similar size. Finally, the proportion of the variance of the forecast error attributable to the variance of the shocks of the model has generally declined after 1982. The decline is stronger in the inflation equation, but somewhat cyclical in the output equation. Note that shocks to the model contribute most to the variability of the forecast error between 1979 and 1981: in fact, in the output and inflation equation they account for about 50% of the variance.

In sum, it appears that the variance of the forecast errors in several equations have declined and this decline seems to be the equally due to a decline in the contribution of the heteroschedastic component of the forecast error and in the size of the shocks to the model. Since the decline is, to a large extent, simultaneous the probability distribution from which shocks to the model and to the coefficients are drawn has become less dispersed over time.

Given the systematic component of monetary policy appears to have been remarkably stable over time and that the contribution of the non-systematic component of policy to the variance of forecast errors is roughly constant, we next examine whether there are changes in the propagation of monetary policy disturbances over time and whether can be related in any meaningful way to the changes in structural coefficients in the output and inflation equations.

5.3 Changes in the propagation of monetary policy disturbances?

Figure 5 reports the posterior mean responses of output and inflation to identified monetary policy shocks for each date in the sample for horizons from 1 to 12 quarters. We do not report interest rate responses because they are similar across time, quite standard in shape and magnitude: after the initial impulse, the initial increase dissipates quite quickly and responses become insignificantly different from zero after the 3th quarter for each date in the sample. Figure 6 presents 68% posterior confidence bands at selected horizons for the difference between the 1982 output response or the 1978 inflation response and all the other dates in the sample.

The shape of both output and inflation responses is quite similar over time. Output responses are U-shaped; there is a though response occurring after about 3 quarters and a smooth convergence to zero after that. Inflation responses are also slightly U-shaped; the effect at horizon one is typically the largest, and responses smoothly converge toward

zero afterwards. Note also that, consistent with a-priori expectations, output responses are persistent and significant for about 5-6 quarters after the shock in every date of the sample.

Some quantitative difference over time in the mean posterior responses of output and inflation is present but small. For output, the posterior mean of the instantaneous response is always centered around -0.15 and only the size of the through responses at lag 3 varies (the range is from -0.20 to -0.05). For inflation, minor magnitude difference occur at lag one (posterior mean varies between -0.07 to -0.16) while 1978 responses are slightly more persistent than all the others at horizons ranging from 3 to 8.

Differences in inflation responses at all horizons are both statistically and economically small. In fact, the posterior 68% confidence band for the largest discrepancy (the one at lag 1) includes zero at all horizons and if we exclude the initial three years, the time path of inflation responses appears to be unchanged over time. The posterior 68% confidence band for the largest discrepancy in output responses (the one at lag 3) does at times exclude zero. In particular, the trough response in 1982 appear to be significantly deeper than the though response in 1978 and 1979 and at most dates after 1992. However, differences between the 1990s and the end of 1970s responses are quite small. Note that the differences are also economically insignificant: over the 12 quarters horizon, the maximum difference in the cumulative output multiplier is only 0.5%. In other words, a one percent increase in interest rates produced output responses which differ over time on average by 0.04% points at each horizon.

Overall, the dynamics induced by monetary policy shocks have not significantly changed over time: responses in the end of the 1990's look very similar in shape and size to those of the end of the 1970's.

5.4 Inflation Dynamics and Monetary Policy

Sargent and Cogley (2001) and (2003) have examined measures of core inflation to establish their claim that monetary policy is responsible for the observed changes in inflation dynamics. They define core inflation as the persistent component of inflation, statistically measured by the zero frequency of the spectrum (that is, by the sum of all autocovariances of estimated inflation process), and show i) persistence has substantially declined over time and ii) there is synchronicity between the changes in persistence and a narrative account of monetary policy changes. Pivetta and Reis (2004), using univariate conventional classical methods, dispute the first claim showing that differences over time in two measures of inflation persistence are statistically insignificant. Since, our examination of the relationship between monetary policy and inflation has so far concentrated on short/medium run frequencies, we next investigate their longer run relationship. In particular, we are curious as to whether different frequencies of the spectrum carry different information and whether our basic conclusions on the role of monetary policy for the observed changes in inflation dynamics is robust.

Our analysis differs from existing ones in two important respects: first, we use output in place of unemployment in the estimated system; second, we measure persistence using the estimated structural model. While the first difference is minor, the second is not. In fact,

thanks the orthogonality of the structural shocks and the orthogonality of the ordinates of the spectrum, our setup allows not only to describe the evolution of the spectrum of inflation over time, but also measure of proportion of the spectral power at frequency zero due to monetary policy shocks and describe its evolution over time. In fact, from the structural MA representation of the system we have that $\pi_t = \sum_{i=1}^{n} \phi_{it}(\ell) e_{it}$ where e_{it} is orthogonal to e_{jt}. Hence the spectrum at Fourier frequencies ω is $S_\pi(\omega) = \frac{1}{2\pi} \sum_{i=1}^{n} |\phi_{it}(\omega)|^2 \sigma_i^2$ and the component at frequency zero due to monetary policy shocks is $S_\pi^*(\omega = 0) = \frac{1}{2\pi} |\phi_{nt}(\omega = 0)|^2 \sigma_n^2$.

The top panel of figure 7 shows the evolution of the posterior mean of the spectrum of inflation. On the vertical axis we report the size of the spectral density, and on the two horizontal axis the frequency (on the side) and the date of the sample(in front). The estimate of the zero frequency displays an initial increase in 1979-1980 followed by a sharp decline the year after; since 1981 the estimated posterior mean of the zero frequency of the spectrum has been very stable (with the exclusion of 1991). The initial four fold jump and the following ten fold decrease are visually large and statistically significant. In fact, the bottom panel of figure 7 indicates that the 68% posterior band for difference between the log spectrum at frequency zero in 1979 and 1996 (the date with the lowest estimate) does not include zero, contrary to Pivetta and Reis's conclusion. At all other frequencies there are very few measurable differences over time both in terms of size and shape. Hence, except for the zero frequency, the posterior distribution of the spectrum of inflation has also been relatively stable over time.

Figure 8 provides visual evidence on the role that monetary policy had in shaping the inflation persistence. The top panel plots the evolution of $S_\pi(\omega = 0)$ and $S_\pi^*(\omega = 0)$ over time and the bottom panel reports, at each date, the percentage of the inflation persistence at each date due to monetary policy shocks. Three important conclusions can be derived. First, the two graphs in the top panel track each other very well, in particular in the 1978-1982 period and in 1991, suggesting that, in at least in terms of timing, monetary policy shocks are important in determining inflation dynamics. Second, the contribution of monetary policy to inflation persistence varies over time: fluctuations are large and over the 1978-2002 sample the percentage explained ranges from about 20 to about 75 percent. Interestingly, there is a significant trend increase since 1981 and the percentage found in 2002 is roughly the same as it was in 1978. Third, there is a substantial portion of inflation persistence (roughly 50 percent on average over the 1981-2002 period) which has nothing to do with monetary policy shocks. Determining what are the forces which account for this large percentage is beyond the scope of this paper. Nevertheless, one can conjecture that the pattern obtained between 1980 and 1982 occurs because of changes in the structure of the economy (possibly due to financial innovations and changes in the real side of the economy) and because the Volker experiment made interest rates very volatile in response to these changes. The subsequent decline is consistent with the reduction of interest rate volatility produced by a substantially more stable macroeconomic environment and the return to the pre-1979 orthodoxy.

In conclusions, as in Sargent and Cogley, we find visual evidence instabilities in the

posterior mean of our measure of inflation persistence. We also find that a-posteriori these changes are not negligible. Changes in the point estimates of the posterior of inflation persistence go hand in hand with changes in the contribution of monetary policy shocks but their quantitative contribution is varying significantly over time. Perhaps more importantly, we find that the contribution of monetary policy shocks to the variations in the posterior means of inflation persistence is smaller than expected.

5.5 What if monetary policy would have been more aggressive?

It is common in the literature to argue by mean of counterfactuals that monetary policy failed to perform an inflation stabilization role in the 1970s (see e.g Clarida, Gali and Gertler (2000) or Boivin and Giannoni (2002b)) and that, had it followed a more aggressive stance against inflation, dramatic changes in the economic performance would have resulted. While exercises of this type are meaningful only in dynamic models with clearly stated microfundations, our structural setup allows us to approximate the ideal type of exercise without falling into standard Lucas-critique type traps. In fact, to the extent that the monetary policy equation we have identified is structural we can examine what would have happened, e.g., if a shock to the preferences of the Fed would have made the inflation coefficients in the policy rule significantly higher. Given the estimated distribution of coefficients, we interpret "significantly higher" as a (temporary) two standard deviations increase in the inflation coefficients above the estimate posterior mean.

Figure 9 plots output and inflation dynamics which would have been obtained if such a change would have occurred at selected dates in the sample. It is clear that inflation would have considerably decreased if such a shock therapy would have been applied in 1978. However, at all the other dates, the effect would have been small and a-posteriori not different from zero. In addition, a more aggressive stance on inflation would have induced negative output effects. In fact, in four of the reported dates output falls, and the largest fall is in 1978. Note also that a more aggressive stance on inflation in 2002:4 would have temporarily decreased inflation but then increases it. At the same time it would have produced the largest output effects in the sample. Hence, there appears to be a significant Phillips curve trade-off, but the slope of the trade-off changes over time. In general, stabilizing inflation is costly and only in 1983 that inflation reductions would have been accompanied beneficial output effects.

In sum, while there was room for improvement in the conduct of monetary policy at the end of the 1970, it is not that clear that the economy would have benefitted from a tougher inflation stance except in 1983. However, since the size of negative output effects and the positive inflation effects tends to decline over time, it appears that the inflation stance of the Fed has become less and less important for the dynamics of output and inflation.

6 Robustness analysis

There are a number of choices we have made which may affect the results we obtain. In the section we analyze the sensitivity of our conclusions to three particular choices: the identifying restrictions, the treatment of trends, the variables included in the VAR.

All figures we have presented so far have been produced identifying monetary policy shocks using sign restrictions on the dynamics of money, inflation and output. Would the pattern of time variations, the estimated policy rule and the time profile of impulse responses be altered if another identification scheme is used? Figure 10 shows the estimated structural monetary policy coefficients, the estimated variances of the forecast error and the responses of output and inflation at selected dates when monetary shocks are identified via a Choleski decomposition where interest rates contemporaneously respond to output and inflation. Since contemporaneous coefficients are constant over time, the evolution of structural coefficients reproduces the pattern of time variations present in the reduced form coefficients (they are simply multiplied by a constant over time). Therefore, all the discussion subsection 5.1 apply here without a change. In particular, the sum of the inflation coefficients in the interest rate equation is less than one at every date in the sample and by a substantial amount (maximum value is 0.6). Note also that the evolution of the estimated forecast error variance matches to a large extent the one present in figure 4. Finally, while the pattern of impulse responses changes with identification scheme - output falls in response to interest rate increases but inflation increases for at least a year after the shocks - the main conclusion that the propagation of monetary shocks has not changed over time is still valid. In fact the maximum discrepancy in the output responses occurs at the third lag for responses computed in 1982 and in 1978, the mean gap is 0.10 and a posterior 68% band for the difference includes zero. Similarly, the maximum discrepancy in the inflation responses occur at lag 2 when comparing 1978 and 1996 responses; the mean gap is 0.19 and again the 68% posterior band includes zero.

Some may feel uncomfortable with dynamic exercises conducted in a system where linearly detrended output and linearly detrended money are used. The typical argument is that after these transformations these two variables are still close to be integrated and are not necessarily cointegrated. Hence, the dynamics we trace out may be spurious. While we have serious doubts about this argument since the residuals of our structural VAR are well behaved, we have redone our exercises using the growth rate of output and of M1 in place of the detrended values of output and M1. We report our results in figure 11: we plot the contemporaneous coefficients of the monetary policy rule and the variances of the forecasts errors of the model and the time profile of output and inflation responses to a monetary policy shock, identified using sign restrictions as in our benchmark case.

Figure 11 indicates that there is even less evidence of structural instability in the policy coefficients: if we exclude the spike occurred between 1981 and 1984 the coefficients appear to be very stable over the entire the sample. This pattern is also present in the lagged coefficients of the policy rule and in the coefficients of the other equations. However, we still find that the variance of all four forecast errors decline significantly over time: for

example, the variability of GDP and inflation forecast errors in the 1990's is about half what it was in the 1980's and 1970's. Third, it is still the case that the sum of coefficients on inflation in the policy equation is less than one, and now this occurs for the entire sample. Finally, the transmission of monetary policy shocks has been largely unchanged over time. The only important variation concerns the response of inflation in the medium run, which was stronger at the beginning of the sample than at the end. Overall, our basic results are robust to the treatment of trends in the variables in the system.

Finally, we have examined the sensitivity of our conclusions to changes in the variables of the VAR. It is well known that small scale VAR models are appropriate only to the extent that omitted variables exert no influence on the dynamics of the included ones. However, a-priori it is hard to know what variables are more important and to check if our system effectively marginalized the influence of all relevant variables. We have therefore repeated our exercise substituting unemployment rate to detrended output and non-borrowed reserves to money. Figure 12 reports the evolution of the posterior mean of the coefficients of the monetary policy rule, of the variances of the forecast error and the responses of unemployment and inflation to monetary shock identified via sign restrictions. Also in this case, our conclusions appear to be robust: excluding the Volker period, variations in the policy coefficients are small; there is a significant decline in the uncertainty surrounding the economy; the transmission of monetary policy shocks is similar over the entire time period.

7 Conclusions

This paper provides novel evidence on the contribution of monetary policy to the structural changes in the US economy over the last 25 years. We use time varying structural representation of the US economy which allows to evaluate the magnitude of the changes in the coefficients in different equations and in the variance of the forecast errors at each point in time, and to analyze the syncronicity in the timing of the changes. Our framework of analysis also allow us to assess how much time variation there is in the propagation of policy shocks both in the short and in the long run and to run some counterfactuals, to understand whether changes in the systematic component of policy would have significantly altered macroeconomic performance.

Structural disturbances are identified using sign restrictions, but the main trust of the results is independent of the identification scheme employed. Because time variations in the coefficients induce important non-linearities in the dynamics of the model, we provide and implement a new definition of impulse responses, based on the difference of two conditional expectations with different arguments in their conditioning set.

Our results indicate that while there are changes in the estimated structural relationships, they tend to be localized in time and involve particular coefficients in certain equations. We show that, if we exclude the beginning of the 1980s, the monetary policy rule has been quite stable over time and that the posterior mean of the (sum of) inflation coefficients fails to satisfy the so-called Taylor principle, not only in the 1970s but also in the 1990s. Hence, both in terms of timing of the changes and in terms of aggressiveness to inflation

movements, changes in the systematic component of monetary policy are unlikely to drive the changes in the output and inflation process. We also find evidence of a generalized decrease in the uncertainty surrounding the US economy over time and that the timing of the changes roughly coincides with those in the output and inflation equations. Taken together, these facts suggest that the "bad luck" hypothesis has considerable more posterior support than the "bad policy" hypothesis in accounting for the observed structural changes in the US economy.

We show that the transmission of monetary policy shocks to output and inflation has also been very stable over time. We also show there is posterior evidence that inflation persistence (as measured by the height of the zero frequency of the spectrum) has changed over time, that both monetary and non-monetary factors account for the magnitude of inflation persistence and that the relative contribution of monetary policy fluctuates over time but is is increasing since 1981.

Finally, we investigate the claim that a more aggressive stance on inflation would have made a difference on output and inflation dynamics. We find that such a policy it would have reduced inflationary pressures in short and medium run in 1979 but not afterwards but also that the output costs of such a policy would not have been negligible. Hence, while there was room for improvement in the conduct of monetary policy at the end of the 1970, it is not that clear that the economy would have benefitted from a tougher inflation stance. Also, since the size of negative output effects and the positive inflation effects decline over time, the inflation stance of the Fed has become less and less important for the dynamics of output and inflation.

Since our conclusions differ somewhat from those presented in the literature, it is important to highlight what may have produced the differences. As repeatedly emphasized, our analysis uses a structural model. Previous studies which used the same level of econometric sophistication (such as Cogley and Sargent (2001) (2003)) have concentrated on reduced form analysis and were forced to use the timing of the observed changes to infer the contribution of monetary policy to changes in output and inflation variances and dynamics. Our approach allows not only informal tests based on timing of the changes but also to quantify the contribution of monetary policy changes to observed changes in output and inflation dynamics. Relative to earlier studies such as Bernanke and Mihov (1998), Hanson (2001) or Leeper and Zha (2003), which use subsample analyses to characterize the changes over time in structural VAR coefficients and in monetary policy equations, we are able to precisely track the evolution of the coefficients over time and produce a more complete and reliable picture of the insignificance of the observed monetary policy changes.

Our results broadly agree with those obtained recursively estimating a small scale DSGE model with Bayesian methods (see Canova (2004)) and contrast with those of Boivin and Giannoni (2002b) who use an indirect inference principle to estimate the parameters of a DSGE model over two subsamples. We conjecture that differences in the estimation method could be responsible for the difference in the conclusions. Finally, our results agree with those of Sims and Zha (2002), despite the different methodologies employed. Relative to their work, our analysis emphasizes that factors other than monetary policy could be more

important in explaining the structural changes that the US economy has witnessed over the last 25 years.

While our conclusion that the decline in the variance of the shocks hitting both the economy and the coefficients of its structural representation seems to point to exogenous reasons for the changes in the US economy, it is important to emphasize that our conclusions are consistent with the analysis of McConnel and Perez Quiroz (2001) and with the idea that a more transparent policy process has reduced the volatility of agent's expectations over time. It is therefore important to extend the current study, by enlarging the number of variables included in the structural model, by trying to identify other shocks and by disentangling the factors which may be behind the decline in the volatility of structural shocks. Also, we have repeatedly mentioned that the monetary policy rule failed to satisfy the Taylor principle in the 1990s. Why is it that inflation did not follow the same pattern as in the 1970? What is the contribution of technological changes to this improved macroeconomic framework? We plan to conduct an investigation of these issues in future work.

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Appendix

A. Impulse Responses

The structural model is

$$y_t = A_{0,t} + A_{1,t}y_{t-1} + A_{2,t}y_{t-2} + ... + A_{p,t}y_{t-p} + S \times H_t \times e_t \tag{12}$$

and its companion form is

$$\mathbf{y}_t = \mathbf{A}_{0,t} + \mathbf{A}_t\mathbf{y}_{t-1} + {}^2{}_t \tag{13}$$

where $\mathbf{A}_{0,t} = [A_{0,t}, 0, ..., 0]$, $\mathbf{y}_t = [y_t', y_{t-1}', ..., y_{t-p}']'$, ${}^2{}_t = [(S \times H_t \times e_t)', 0, ..., 0]'$ are $n(np+1) \times 1$ vectors and

$$\mathbf{A}_t = \begin{pmatrix} A_{1t} & A_{2t} & \cdots & A_{p-1t} & A_{pt} \\ I_k & 0 & \cdots & 0 & 0 \\ 0 & I_k & \ddots & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & I_k & 0 \end{pmatrix}$$

is an $n(np+1) \times n(np+1)$ matrix. Substituting into the companion form we obtain

$$\mathbf{y}_{t+k} \quad = \quad \mathbf{A}_{0,t+k} + \sum_{j=1}^{k-1} \mathbf{\Phi}_{t+k,j}\mathbf{A}_{0,t+k-j} + \mathbf{\Phi}_{t+k,k}\mathbf{y}_t + \sum_{j=0}^{k-1} \mathbf{\Phi}_{t+k,j}\,{}^2{}_{t+k-j} \tag{14}$$

where $\mathbf{\Phi}_{t+k,j} = \prod_{i=0}^{j-1} \mathbf{A}_{t+k-j}$ for $j = 1, 2, ...$, $\mathbf{\Phi}_{t+k,0} = I$. Let $\mathcal{S}_{(h,k)}(X)$ be a selection matrix extracting h-rows and k-columns of the matrix X. Since $y_t = \mathcal{S}_{(n,1)}(\mathbf{y}_t)$, setting $\tilde{\mathbf{\Phi}}_{t+k,k-1} = \mathcal{S}_{(n,n^2p)}(\mathbf{\Phi}_{t+k,k-1})$ and $\Phi_{t+k,j} = \mathcal{S}_{(n,n)}(\mathbf{\Phi}_{t+k,j})$ we have

$$y_{t+k} = \tilde{A}_{0,t+k} + \tilde{\mathbf{\Phi}}_{t+k,k}\mathbf{y}_t + \sum_{j=0}^{k-1} \Psi_{t+k,j}e_{t+k-j} \tag{15}$$

where $\tilde{A}_{0,t+1} = A_{0,t+1}$; $\tilde{A}_{0,t+k} = A_{0,t+k} + \sum_{j=1}^{k-1} \Phi_{t+k,j}A_{0,t+k-j}$ for $k > 1$; $\Psi_{t+k,j} = \Phi_{t+k,j} \times S \times H_{t+k-j}$ and $\Psi_{t+k,0} = \Phi_{t+k,0} \times S \times H_t = S \times H_t$.

Partition $e_t = (e_{i,t}, e_{-i,t})$, where $e_{i,t}$ is an element of e_t and $e_{-i,t}$ is the vector containing the other $n-1$ elements of e_t, and $H_t = (h_{it}, h_{-it})$, where h_{it} is a column of H_t corresponding to $e_{i,t}$ and h_{-it} is the matrix formed by remaining $n-1$ columns. Then equation (15) is

$$y_{t+k} = \tilde{A}_{0,t+k} + \tilde{\mathbf{\Phi}}_{t+k,k}\mathbf{y}_t + \sum_{j=0}^{k-1} \Psi^{i}_{t+k,j}e_{i,t+k-j} + \sum_{j=0}^{k-1} \Psi^{-i}_{t+k,j}e_{-i,t+k-j} \tag{16}$$

where $\Psi^{i}_{t+k,j} = \Phi_{t+k,j} \times S \times h_{it}$ and $\Psi^{-i}_{t+k,j} = \Phi_{t+k,j} \times S \times h_{-it}$.

Shocks to the Non-Systematic Component

Suppose $e_{i,T+1}$ is a shock to the non-systematic component (of monetary policy) occurring at $T+1$. Define the conditioning sets $I_1 = \{y^t, \theta^t, \Sigma, \Omega, K_t, \xi^{\delta}_{i,t+1}, \xi_{-i,t+1}, \xi^{t+\tau}_{t+2}\}$ and $I_2 = \{y^t, \theta^t, \Sigma, \Omega, K_t, \xi_{-i,t+1}, \xi^{t+\tau}_{t+2}\}$. Taking conditional expectations we obtain

$$\begin{aligned} E\left(y_{T+\tau}|I_j\right) &= E(\tilde{A}_{0,T+\tau}|I_1) + E\left(\tilde{\mathbf{\Phi}}_{T+\tau,\tau}\mathbf{y}_T|I_j\right) + \\ &+ E\left(\sum_{j=0}^{\tau-1}\Psi^i_{T+\tau,j}e_{i,T+\tau-j}|I_j\right) + E\left(\sum_{j=0}^{\tau-1}\Psi^{-i}_{T+\tau,j}e_{-i,T+\tau-j}|I_j\right) \end{aligned} \tag{17}$$

$j = 1, 2$. Using the fact that e_t and η_t are orthogonal and taking the difference between the two conditional expectations we have

$$\begin{aligned} E\left(y_{T+\tau}|I_1\right) - E\left(y_{T+\tau}|I_2\right) &= E(\Psi^i_{T+\tau,\tau}e_{i,T+\tau-\tau}|I_1) - E\left(\Psi^i_{T+\tau,\tau}e_{i,T+\tau-\tau}|I_2\right) \\ &= \Psi^i_{T+\tau,\tau}e_{i,T+\tau-\tau} - \Psi^i_{T+k,k}E(e_{i,T+k-k}|I_2) \\ &= \Psi^i_{T+\tau,\tau}e_{i,T+\tau-\tau} \end{aligned} \tag{18}$$

Shocks to the Systematic Component

Let $\eta_{i,T+1}$ be a shock to the systematic component of the monetary policy and set $\eta_{-i,t+1} = 0$. Taking expectations with respect to I_j, $j = 1, 2$ we have

$$E\left(y_{T+\tau}|I_j\right) = E(\tilde{A}_{0,T+\tau}|I_j) + E\left(\tilde{\mathbf{\Phi}}_{T+\tau,\tau}|I_j\right)\mathbf{y}_T + E\left(\sum_{j=0}^{\tau-1}\Psi_{T+\tau,j}|I_j\right)e_{T+\tau-j} \tag{19}$$

And taking the difference between conditional expectations we obtain equation (11).

B. Estimation

Priors

We choose prior densities for the unknowns which gives us analytic expressions for the conditional posteriors of subvectors of the unknowns. Let T be the end of the estimation sample and let $\bar{K}$ be the number of periods for which the identifying restrictions must be satisfied. Let $H_t = \rho(\omega_t)$ be a rotation matrix whose columns represents orthogonal points in the hypershere and let ω_t is a vector in R^6 whose elements are $U[0,1]$ random variables. Let $\mathcal{M}_t$ the set of impulse response functions at time t satisfying the restrictions and let $F(\mathcal{M}_t)$ be an indicator function which takes the value one if the identifying restrictions are satisfied, that is it is one if $(\Psi^i_{t+1,1}, ..., \Psi^i_{t+K,K}) \in \mathcal{M}_t$, and zero otherwise. Let the joint prior for $\theta^{T+\bar{k}}$, Σ, Ω and ω_T be

$$p(\theta^{T+\bar{K}}, \Sigma, \Omega, \omega_T) = p(\theta^{T+K}|\Sigma, \Omega)p(\Sigma, \Omega)F(\mathcal{M}_T)p(\omega_T) \tag{20}$$

Here $p(\theta^{T+K}|\Sigma, \Omega) \propto I(\theta^{T+K})f(\theta^{T+K}|\Sigma, \Omega)$ where $f(\theta^{T+K}|\Sigma, \Omega) = f(\theta_0)\prod_{t=0}^{T+K-1} f(\theta_{t+1}|\theta_t, \Sigma, \Omega)$ and $I(\theta^{T+K}) = \prod_{t=0}^{T+K} I(\theta_t)$. Hence, $p(\theta^{T+K}|\Sigma, \Omega)$ is truncated normal.

We assume that Σ and Ω have independent inverse Wishart distributions with scale matrices Σ_0^{-1}, Ω_0^{-1} and degrees of freedom T_{01} and T_{02}. We also assume that the prior for the initial state is a truncated Gaussian random variable distributed independent of Σ and Ω, i.e. $f(\theta_0) \propto I(\theta_0)N(\bar{\theta}, \bar{P})$. Note that the uniform prior $p(\omega_T)$ is justified by the fact that all the trajectories satisfying the restrictions are a-priori equally likely.

Collecting the pieces the joint prior is:

$$p(\theta^{T+\bar{K}}, \Sigma, \Omega, \omega_T) \propto I(\theta^{T+K})F(\mathcal{M}_T)f(\theta_0) \prod_{t=0}^{T+K-1} f(\theta_{t+1}|\theta_t, \Sigma, \Omega)p(\Sigma)p(\Omega) \tag{21}$$

Note that when $H_t = I_n$,the prior reduces to

$$p(\theta^{T+\bar{K}}, \Sigma, \Omega, H_T) = I(\theta^{T+K})f(\theta_0) \prod_{t=0}^{T+K-1} f(\theta_{t+1}|\theta_t, \Sigma, \Omega)p(\Sigma)p(\Omega) \tag{22}$$

We "calibrate" the prior by estimating a fixed coefficients VAR using data from 1960:I up to 1969:I. We set $\bar{\theta}$ equal to the point estimates of the coefficients and $\bar{P}$ to the estimated covariance matrix. Σ_0 is equal to the estimated covariance matrix of VAR innovations and $\Omega_0 = \varrho\bar{P}$. The parameter ϱ measures how much the time variation is allowed in coefficients. Although as t grows the likelihood dominates, the choice of ϱ matters in finite samples. We choose ϱ on the basis of the sample size i.e. for the sample 1969:I-1981:II $\varrho = 0.0025$, 1969:I-1983:II $\varrho = 0.003$, 1969:I-1987:II $\varrho = 0.0035$, for 1969:I-1989:II $\varrho = 0.004$, 1969:I-1995:IV $\varrho = 0.007$, 1969:I-1999:I $\varrho = 0.008$, 1969:I-2002:IV $\varrho = 0.01$. This range of values implies a quiet conservative prior coefficient variations: in fact, time variation accounts from a 0.35 and a 1 percent of the total coefficients standard deviation.

The primary goal of this paper is to compute impulse response functions, which depend on $\Phi_{t+k,k}$, S and H_t. Therefore, we characterize first the posterior of $\theta^{t+K}, \Sigma, \Omega, H$ which are used to construct $\Phi_{t+k,k}$'s, S and H_t and then describe an approach to sample from them. Note that when identification is achieved via sign restrictions H_t is a matrix of random variable while under standard schemes H_t is a matrix of constants. In this latter case we only need to characterize the posterior distribution of $(\theta^{T+K}, \Sigma, \Omega)$.

Posteriors

To draw posterior sequences we need to $p(h_{T+1}^{T+K}, \omega_T, \theta_{T+1}^{T+K}, \theta^T, \Sigma, \Omega|y^T)$, which is analytically intractable. Luckily we can decompose it into simpler components. First, note that

$$\begin{aligned} p(\omega_T, \theta_{T+1}^{T+K}, \theta^T, \Sigma, \Omega|y^T) &\equiv p(\omega_T, \theta^{T+K}, \Sigma, \Omega|y^T) \\ &\propto p(y^T|\omega_T, \theta^{T+K}, \Sigma, \Omega)p(\omega_T, \theta^{T+K}, \Sigma, \Omega) \end{aligned} \tag{23}$$

Second, since the likelihood is invariant to any orthogonal rotation $p(y^T|\omega^T, \theta^{T+K}, \Sigma, \Omega) = p(y^T|\theta^{T+K}, \Sigma\Omega)$. Third, $p(\omega_T, \theta^{T+K}, \Sigma, \Omega) = p(\theta^{T+K}, \Sigma, \Omega)F(\mathcal{M}_T)p(\omega_T)$. Thus we have

$$p(\omega_T, \theta^{T+K}, \Sigma, \Omega|y^T) \propto p(\theta^{T+K}, \Sigma, \Omega|y^T)F(\mathcal{M}_T)p(\omega_T) \tag{24}$$

where $p(\theta^{T+K}, \Sigma, \Omega | y^T)$ is the posterior distribution for $\theta^{T+K}, \Sigma, \Omega$ (i.e. the posterior for reduced form parameters). Such a posterior can be factored as

$$p(\theta^{T+K}, \Sigma, \Omega | y^T) = p(\theta_{T+1}^{T+K} | y^T, \theta^T, \Sigma, \Omega) p(\theta^T, \Sigma, \Omega | y^T) \tag{25}$$

where the first term of the right hand side represents beliefs about the future and the second term represents the posterior density for states and hyperparameters. Note that $p(\theta_{T+1}^{T+K} | y^T, \theta^T, \Sigma, \Omega) = p(\theta_{T+1}^{T+K} | \theta^T, \Sigma, \Omega) = \prod_{k=1}^{K} p(\theta_{T+k} | \theta_{T+k-1}, \Sigma, \Omega)$ because the states are Markov. Finally, since θ_{T+k} is conditionally truncated normal with mean θ_{T+k-1} and variance Ω, we can write

$$\begin{aligned} p(\theta_{T+1}^{T+K} | \theta^T, \Sigma, \Omega) &= I(\theta_{T+1}^{T+K}) \prod_{k=1}^{K} f(\theta_{T+k} | \theta_{T+k-1}, \Sigma, \Omega) \\ &= I(\theta_{T+1}^{T+K}) f(\theta_{T+1}^{T+K} | \theta_T, \Sigma, \Omega) \end{aligned} \tag{26}$$

The second term in (25) can be factored as

$$p(\theta^T, \Sigma\Omega | y^T) \propto p(y^T | \theta^T, \Sigma, \Omega) p(\theta^T, \Sigma, \Omega) \tag{27}$$

The first term is the likelihood function which, given the states, has a Gaussian shape so that $p(y^T | \theta^T, \Sigma, \Omega) = f(y^T | \theta^T, \Sigma, \Omega)$. The second term is the joint posterior for states and hyperparameters and can be written as:

$$p(\theta^T, \Sigma, \Omega | y^T) \propto f(y^T | \theta^T, \Sigma, \Omega) p(\theta^T | \Sigma, \Omega) p(\Sigma, \Omega) \tag{28}$$

The conditional density for the states can be written as $p(\theta^T | \Sigma, \Omega) \propto I(\theta^T) f(\theta^T | \Sigma, \Omega)$ where $f(\theta^T | \Sigma, \Omega) = f(\theta_0 | \Sigma, \Omega) \prod_{t=1}^{T} f(\theta_t | \theta_{t-1}, \Sigma, \Omega)$ and $I(\theta^T) = \prod_{t=0}^{T} I(\theta_t)$, thus

$$p(\theta^T, \Sigma, \Omega | y^T) \propto I(\theta^T) f(y^T | \theta^T, \Sigma, \Omega) f(\theta^T | \Sigma, \Omega) p(\Sigma, \Omega) = I(\theta^T) p_u(\theta^T, \Sigma, \Omega | y^T) \tag{29}$$

where $p_u(\theta_T, \Sigma, \Omega | y^T) = f(y^T | \theta^T, \Sigma, \Omega) f(\theta^T | \Sigma, \Omega) p(\Sigma, \Omega)$ is the posterior density obtained if no restrictions are imposed. Collecting pieces we finally have

$$p(\omega_T, \theta_{T+1}^{T+K}, \theta^T, \Sigma, \Omega | y^T) \propto F(\mathcal{M}_T) \left[I(\theta^{T+K}) f(\theta_{T+1}^{T+K} | \theta^T, \Sigma, \Omega) p_u(\theta^T, \Sigma, \Omega | y^T) \right] p(\omega_T) \tag{30}$$

Note that for $H_t = I$ $p(\theta_{T+1}^{T+K}, \theta^T, \Sigma, \Omega | y^T) = I(\theta^{T+K}) f(\theta_{T+1}^{T+K} | \theta^T, \Sigma, \Omega) p_u(\theta^T, \Sigma, \Omega | y^T)$.

Drawing from the posterior of structural parameters

Given (30) draws for the structural parameters can be obtained as follows

1. Draw a vector of θ^T from the unrestricted posterior $p_u(\cdot)$ via the Gibbs sampler (see below). Apply the filter $I(\theta^T)$ and discard explosive paths.

2. Draw a sequence of future states θ_{T+1}^{T+K}, given θ^T i.e. obtain N draws of u_t from $N(0, \Omega)$ and iterate in $\theta_{T+i} = \theta_{T+i-1} + u_{T+i}$ K times. Apply the filter $I(\theta^{T+K})$ and discard explosive paths.

3. Draw $\omega_{i,T}$ for $i = 1, ..., 6$ from a $U[0,1]$. Construct $H_t = \rho(\omega_t)$.

4. Draw Σ from the posterior. Find the matrix S, such that $\Sigma = SS'$. Construct K_t^{-1}.

5. Compute $(\Psi^{i,\ell}_{T+1,1}, ..., \Psi^{i,\ell}_{T+K,K})$ and $F(\mathcal{M}_T)^\ell$ and apply the filter $F(\mathcal{M}_T)$. If the identification restrictions are satisfied keep the draw, otherwise discard it.

Computing Posteriors of reduced form parameters: the Gibbs Sampler

The Gibbs Sampler we use to compute the posterior for the reduced form parameters iterate on two steps. The implementation is identical to Cogley and Sargent (2001).

• Step 1: States given hyperparameters

Conditional y^T, Σ, Ω, the unrestricted posterior of the states is normal and $p_u(\theta^T|y^T, \Sigma, \Omega) = f(\theta_T|y^T, \Sigma, \Omega) \prod_{t=1}^{T-1} f(\theta_t|\theta_{t+1}, y^t, \Sigma, \Omega)$. All densities on the right end side are Gaussian they their conditional means and variances can be computed using the backward recursion of the Kalman filter. Let $\theta_{t|t} \equiv E(\theta_t|y^t, \Sigma, \Omega); P_{t|t-1} \equiv Var(\theta_t|y^{t-1}, \Sigma, \Omega); P_{t|t} \equiv Var(\theta_t|y^t, \Sigma, \Omega)$. Given $P_{0|0}$, $\theta_{0|0}$, Ω and Σ, we compute forward Kalman filter recursions

$$\begin{aligned} P_{t|t-1} &= P_{t-1|t-1} + \Sigma \\ K &= (P_{t|t-1}X_t)(X_t' P_{t|t-1} X_t + \Omega)^{-1} \\ \theta_{t|t} &= \theta_{t-1|t-1} + K_t(y_t - X_t'\theta_{t-1|t-1}) \\ P_{t|t} &= P_{t|t-1} - K_t(X_t' P_{t|t-1}) \end{aligned}$$

The last iteration gives $\theta_{T|T}$ and $P_{T|T}$ which are the conditional means and variance of $f(\theta_t|y^T, \Sigma, \Omega)$. Hence $f(\theta_T|y^T, \Sigma, \Omega) = N(\theta_{T|T}, P_{T|T})$. The other $T-1$ densities can be computed using the backward Kalman filter recursions, i.e

$$\theta_{t|t+1} = \theta_{t|t} + P_{t|t} P^{-1}_{t|t+1}(\theta_{t+1} - \theta_{t|t-1}) \tag{31}$$

$$P_{t|t+1} = P_{t|t} - P_{t|t} P^{-1}_{t+1|t} P_{t|t} \tag{32}$$

where $\theta_{t|t+1} \equiv E(\theta_t|\theta_{t+1}, y^t, \Sigma, \Omega)$ and $P_{t|t+1} \equiv Var(\theta_t|\theta_{t+1}, y^t, \Sigma, \Omega)$ are the conditional means and variances of the remaining terms in $p_u(\theta^T|y^T, \Sigma, \Omega)$. Thus $f(\theta_t|\theta_{t+1}, y^t, \Sigma, \Omega) = N(\theta_{t|t+1}, P_{t|t+1})$ Therefore, to sample θ^T from the conditional posterior we proceed backward, sampling θ^T from $N(\theta_{T|T}, P_{T|T})$ and θ^t from $N(\theta_{t|t+1}, P_{t|t+1})$ for all $t < T$.

• Step 2: Hyperparameters given states

Since Σ, Ω are independent, we can sample them separately. Conditional on the states and the data ε_t and u_t are observable and Gaussian. Combining a Gaussian likelihood with an inverse-Wishart prior results in an inverse-Wishart posterior, so that $p(\Sigma|\theta^T, y^T) = IW(\Sigma_1^{-1}, T_{11}); p(\Omega|\theta^T, y^T) = IW(\Omega_1^{-1}, T_{12})$ where $\Sigma_1 = \Sigma_0 + \Sigma_T$, $\Omega_1 = \Omega_0 + \Omega_T$ and $T_{11} = T_{01} + T$ $T_{12} = T_{02} + T$ and Σ_T and Ω_T are proportional to the covariance estimator $\frac{1}{T}\Sigma_T = \frac{1}{T}\sum_{t=1}^T \varepsilon_t \varepsilon_t'$; $\frac{1}{T}\Omega_T = \frac{1}{T}\sum_{t=1}^T u_t u_t'$.

Under the regularity conditions presented in Tierney (1994) and after a burn-in period, iterations on these two steps produce draw from $p_u(\theta^T, \Sigma, \Omega | y^T)$.

In our exercises T varies from 1979:1 to 2002:4 (these are the dates at which we compute impulse responses). For each of these T 10000 iteration of the Gibbs sampler are made. We have constructed CUMSUM graphs to check for convergence and found that the chain had converged roughly after 2000 draws for each date in the sample. The densities for the parameters obtained with the remaining draws are always well behaved and none is multimodal. We keeping one every four of the remaining 8000 draws and discard all the draws generating explosive paths. The autocorrelation function of the 2000 draws which are left is somewhat persistent We could reduce the persistence taking draws more largely spaced (but this come at the price of reducing the number of draws which satisfy the identification restrictions and therefore substantially reduce the precision of the exercise. In fact, only about 10% of the draws satisfy the identification restrictions. In the end, we have no more than 300 draws for each date to conduct structural inference.

Computation of impulse responses

Given the algorithm to draw from the posterior of the structural parameters, the calculation of impulse responses to shocks to the non-systematic component is straightforward. In fact, given the accepted draws for $(\theta^{t+K}, \Sigma, \Omega, H_t)$ we calculate $\Psi_{t+k,k}$ at each of the draw, compute the posterior median and the 68% central credible set at each horizon k.

To compute responses to shocks to the systematic component of monetary policy we proceed as follows.

1. Draw θ_T and Σ, Ω and H_t from the posterior distribution.
2. Compute S and draw sequences for ε_{t+1}^{T+K} and u_{t+2}^{T+K}.
3. Fix $\eta_{i,t+1}^{\delta}$ and draw $\eta_{-i,t+1}$ from the conditional distribution of $\eta_{-i,t+1} | \eta_{i,t+1} = \delta$ and form the vector η_{t+1}^{δ}.
4. Compute $u_{t+1}^{\delta} = (K_t^{-1} \bigotimes I_{np+1})^{-1} \eta_{t+1}^{\delta}$ and define $u_{t+1}^{\delta,t+k} = \{u_{t+1}^{\delta}, u_{t+2}^{T+K}\}$
5. Use $u_{t+1}^{\delta,t+k}$, ε_{t+1}^{T+K} and θ_t to compute $\tilde{\theta}_{T+1}^{T+K}$ compute $\tilde{\Phi}_{T+k,i}$ and $\sum_{i=1}^{k} \tilde{\Phi}_{T+k,j} \tilde{A}_{0,T+k-i}$ $\sum_{i=1}^{k} \tilde{\Phi}_{T+k,j} \varepsilon_{t+k,j}$ for $k = 1, ..., K$ and $i = 1, ...K-1$. Using these draws compute y_{t+k}^{δ}. This is a draw for $E(y_{t+k} | I_1)$..
6. Fix $\eta_{i,t+1}^{0}$ and draw $\eta_{-i,t+1}$ from the conditional distribution of $\eta_{-i,t+1} | \eta_{i,t+1} = 0$ and form the vector η_{t+1}^{0}.
7. Compute $u_{t+1}^{0} = (K_t^{-1} \bigotimes I_{np+1})^{-1} \eta_{t+1}^{0}$ and define $u_{t+1}^{0,t+k} = \{u_{t+1}^{0}, u_{t+2}^{T+K}\}$
8. Use $u_{t+1}^{0,t+k}$, ε_{t+1}^{T+K} and θ_t to compute $\tilde{\theta}_{T+1}^{T+K}$ compute $\tilde{\Phi}_{T+k,i}$ and $\sum_{i=1}^{k} \tilde{\Phi}_{T+k,j} \tilde{A}_{0,T+k-i}$ $\sum_{i=1}^{k} \tilde{\Phi}_{T+k,j} \varepsilon_{t+k,j}$ for $k = 1, ..., K$ and $i = 1, ...K-1$. Using these draws compute y_{t+k}^{0}. This is a draw for $E(y_{t+k} | I_2)$..
9. Take the difference of the two realizations in 5. and 8.