Optimal Taylor Rules in an Estimated Model of a Small Open Economy*

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June 2004

Abstract

We compute welfare-maximizing Taylor rules in a dynamic general equilibrium model of a small open economy. The model includes three types of nominal rigidities (domestic goods prices, imported goods prices and wages) and eight different structural shocks. We estimate its structural parameters using a maximum likelihood procedure with Canadian and U.S. data, and use a second-order approximation of the model's equilibrium conditions to measure the welfare effects of different Taylor rules. Estimating the model allows us to compare welfare levels with that attainable under the Taylor rule estimated for our sample period. Welfare gains from moving to the optimal Taylor rule are larger than those obtained in previous papers.

JEL classification: F2, F31, F33
Key words: Economic models; Open economy; Optimal monetary policy; Taylor rules

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*We thank Tiff Macklem for encouraging us to undertake this project, and Césaire Meh, Kevin Moran, Eva Ortega, Thomas Sargent, Christopher Sims, Gregor Smith, Raphael Solomon, Pedro Teles, Carolyn Wilkins, Alex Wolman, Michael Woodford, and participants at seminars at the American Economics Association, the Bank of Canada, the Canadian Economic Association, the North American Summer Meetings of the Econometric Society, the Society for Economic Dynamics and the Society for Computational Economics and the Université du Québec à Montréal for comments on an earlier draft. The views expressed in this paper are of the authors. No responsibility for them should be attributed to the Bank of Canada.

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1. Introduction

A large literature analyzes optimal monetary policy in the context of the New Open-Economy Macroeconomics (*NOEM*), a class of open-economy dynamic general equilibrium (*DGE*) models with explicit microfoundations, nominal rigidities, and imperfect competition.[1] Galí and Monacelli (1999) showed in a model with instantaneous pass-through of exchange rate changes to domestic prices that optimal monetary policy is identical in open and closed economies and involves stabilizing the overall price level, without regard to exchange rate fluctuations. Corsetti and Pesenti (2001) showed that with slow pass-through this is no longer the case: it is optimal for the central bank to minimize a CPI-weighted average of markups charged in the domestic market by domestic and foreign producers. Much of this literature uses highly stylized models with analytical solutions. Recently, more fully developed models have appeared. Smets and Wouters (2002) showed that optimal monetary policy with sticky domestic-goods prices and imported-goods prices involves minimizing a weighted average of domestic and import price inflation.

In this paper, we analyze optimal monetary policy (within a class of simple monetary rules) in a *NOEM* model of a small open economy with three types of nominal rigidities: wages and both domestic and imported goods prices are set in advance by monopolistically competitive agents. The model also incorporates eight different types of structural shocks. We estimate the model's structural parameters with Canadian and U.S. data using maximum likelihood via the Kalman filter. We then use the model to compute welfare-maximizing Taylor rules for setting domestic short-term interest rates. For these computations, we use a second-order approximation around the model's deterministic steady state. This methodology captures the effect of the Taylor rule coefficients on the stochastic means of consumption, leisure, and real balances as well as on their variances. Recent studies such as Kim and Kim (2003) have shown that solving models using first-order approximations can lead to misleading welfare comparisons.

Our main results can be summarized as follows. Our estimates for most of the model's parameters are precise. They are compatible with other small open economy models in the *NOEM* literature, for example Bergin (2003) and Dib (2003). The optimal Taylor rule involves responding more strongly to fluctuations in GDP than the Bank of Canada has done historically, and less strongly to fluctuations in money supply growth. The gains from optimal monetary policy are quite substantial: the gain in welfare is equivalent to a permanent increase of 1.40% in the level of consumption compared to the level of welfare under the historical (estimated) values of the Taylor rule coefficients. The optimal Taylor rule places the economy very close to a region in parameter space that implies local indeterminacy.

[1] The *NOEM* literature, spawned by the pioneering work of Obstfeld and Rogoff (1995), has been successful in explaining phenomena such as high real exchange rate volatility and the strong impact of monetary policy shocks on real exchange rates. See Sarno (2001), Lane (2001), and Bowman and Doyle (2003) for recent surveys.

Placing restrictions on the Taylor rule coefficients to move the economy further away from the region of local indeterminacy results in a smaller welfare gain.

Our results differ from those in the existing literature in three main respects. First, our estimate of the welfare gain from optimal monetary policy is larger than in other recent papers that analyze optimal monetary policy in small open economies (for example Kollmann, 2002 and Smets and Wouters, 2002).

Second, since we estimate the model's structural parameters we can compare welfare under the optimal Taylor rule to welfare under the historical (estimated) values of the Taylor rule coefficients. Previous studies compared welfare under optimal Taylor rules with welfare in the deterministic steady state. The deterministic steady state has the advantage of being invariant to the monetary policy rule and to the variance-covariance matrix of shocks. On the other hand, the deterministic steady state levels of consumption, leisure, and real balances can be quite different from the average values around which these variables fluctuate, because shocks can affect the stochastic means of the economy's endogenous variables.[2] Measuring welfare gains against the deterministic steady state means using a state in which the economy rarely if ever finds itself as a benchmark.

Third, we show that most of the welfare gains from optimized monetary policy come from its effects on the average levels of the arguments of the utility function rather than on their second moments. This underscores the importance of using higher-order approximations to solve the model. If the model is solved using a first-order approximation, measured welfare gains by construction can come only from reducing the size of the fluctuations of variables around their steady-state means. This explains why many previous studies found very small potential benefits from optimal monetary policy.

The rest of the paper is organized as follows. In section 2, we present the model. In section 3, we discuss the estimation strategy used to attribute values to the model's structural parameters and the parameter estimates themselves. We discuss the calculation of the optimal Taylor rule and present our results concerning the benefits of optimal monetary policy in section 4. Section 5 offers some conclusions. Our data sources are summarized in Appendix A. Appendix B summarizes the model's equilibrium conditions.

2. The Model

The economy faces fixed prices on world markets for imported goods. However, its domestic output is an imperfect substitute for foreign goods so that it faces a downward-sloping demand curve for its output on world markets. It also faces an upward-sloping supply curve for funds on international capital markets.

[2] This phenomenon is obscured when using a first-order approximation to solve the model. With a first-order approximation, the model's endogenous variables are on average equal to their deterministic steady-state values.

Different labor types are associated with particular households that act as monopolistic competitors in the labor market. Differentiated intermediate goods are produced by monopolistically competitive domestic firms using labor and a final composite good as inputs. Differentiated intermediate goods are also imported by monopolistically competitive importers. Domestic and imported intermediate goods are aggregated by competitive firms to form a composite domestic and a composite imported good. Some of the composite domestic good is exported. The remainder is combined with the composite imported good to form the final good. As in McCallum and Nelson (1999, 2001), imports enter the production process rather than being consumed directly.[3] The final good is used for consumption, government consumption, and as an input into the production of domestic intermediate goods.

There are therefore three sources of monopoly distortion and nominal rigidities. Households set wages in advance, and both importers and producers of domestic intermediate goods set prices in advance. Following Calvo (1983), price and wage setters maintain constant prices and wages unless they receive a signal to revise them, which arrives at the beginning of each period with a constant probability. This makes aggregation simple, allows us easily to vary the average duration of the nominal rigidities, and allows us to estimate the length of the nominal rigidities along with other structural parameters of the model.

2.1 Households

There is a continuum of different households on the unit interval, indexed by j. The j^{th} household's preferences are given by:

$$U_0(j) = E_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t(j), \frac{M_t(j)}{P_t}, h_t(j)\right), \tag{1}$$

where β is the discount factor, E_0 is the conditional expectations operator, $C_t(j)$ is consumption, $M_t(j)$ denotes nominal money balances held at the end of the period, P_t is the price level, and $h_t(j)$ denotes hours worked by the household. The single-period utility function is:

$$u(\cdot) = \frac{\gamma}{\gamma - 1} \log\left(C_t(j)^{\frac{\gamma-1}{\gamma}} + b_t^{\frac{1}{\gamma}}\left(\frac{M_t(j)}{P_t}\right)^{\frac{\gamma-1}{\gamma}}\right) + \eta \log\left(1 - h_t(j)\right), \tag{2}$$

where γ and η are positive parameters. Total time available to the household in the period is normalized to one. This functional form of the period utility function leads to a conventional money demand equation in which the short-term nominal interest rate is the opportunity cost of holding money, $-\gamma$ is the interest elasticity of money demand, and consumption is the scale variable. The b_t term is a shock to money demand. It follows the first-order

[3] Bergin (2003) and Kollmann (2002) develop models that are similar in this respect.

autoregressive process given by:

$$\log(b_t) = (1-\rho_b)\log(b) + \rho_b \log(b_{t-1}) + \varepsilon_{bt}, \tag{3}$$

with $0 < \rho_b < 1$ and where the serially uncorrelated shock, ε_{bt}, is normally distributed with zero mean and standard deviation σ_b. The household's budget constraint is given by:

$$P_t C_t(j) + M_t(j) + \frac{D_t^g(j)}{R_t} + \frac{e_t B_t^*(j)}{\kappa_t R_t^*} =$$

$$(1-\tau_t) W_t(j) h_t(j,\cdot) + M_{t-1}(j) + D_{t-1}^g(j) + e_t B_{t-1}^*(j) + T_t + D_t, \tag{4}$$

where $W_t(j)$ is the nominal wage rate set by the household. Labor income is taxed at an average marginal tax rate, τ_t. B_t^* and D_t^g are foreign-currency and domestic-currency bonds purchased in t, and e_t is the nominal exchange rate. Domestic-currency bonds are used by the government to finance its deficit. R_t and R_t^* denote, respectively, the gross nominal domestic and foreign interest rates between t and $t+1$; κ_t is a risk premium that reflects departures from uncovered interest parity. The household also receives nominal profits $D_t = D_t^d + D_t^m$ from domestic producers and importers of intermediate goods, and T_t is nominal lump-sum transfers from the government. The risk premium depends on the ratio of net foreign assets to domestic output:

$$\log(\kappa_t) = \varphi \left[\exp\left(\frac{e_t B_t^*}{P_t^d Y_t} \right) - 1 \right], \tag{5}$$

where P_t^d is the GDP deflator or domestic output price index. The risk premium ensures that the model has a unique steady state. If domestic and foreign interest rates are equal, the time paths of domestic consumption and wealth follow random walks.[4]

The foreign nominal interest rate, R_t^*, evolves according to the following stochastic process:

$$\log(R_t^*) = (1-\rho_{R^*})\log(R^*) + \rho_{R^*}\log(R_{t-1}^*) + \varepsilon_{R^*t}, \tag{6}$$

with $0 < \rho_{R^*} < 1$ and where the serially uncorrelated shock, ε_{R^*t}, is normally distributed with zero mean and standard deviation σ_{R^*}.

Household j chooses $C_t(j)$, $M_t(j)$, $D_t^g(j)$, and $B_t^*(j)$ (and $W_t(j)$ if it is allowed to change its wage) to maximize the expected discounted sum of its utility flows subject to three relationships: the budget constraint, equation (4), intermediate firms' demand for their labor type, and a transversality condition on their holdings of assets. Aggregate labor is given by:

$$h_t = \left(\int_0^1 h_t(j)^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}}, \tag{7}$$

[4]For an early discussion of this problem, see Giavazzi and Wyplosz (1984). Our risk premium equation is similar to the one used by Senhadji (1997). For alternative ways of ensuring that stationary paths exist for consumption in small open-economy models, see Schmitt-Grohé and Uribe (2003).

where σ is the elasticity of substitution between different labor skills. This implies the following conditional demand for labor of type j:

$$h_t(j) = \left(\frac{W_t(j)}{W_t}\right)^{-\sigma} h_t,$$

where h_t is aggregate employment. W_t is an exact average wage index given by:

$$W_t = \left(\int_0^1 W_t(j)^{1-\sigma} dj\right)^{\frac{1}{1-\sigma}}.$$

The household's first-order conditions are:

$$\frac{C_t(j)^{\frac{-1}{\gamma}}}{C_t(j)^{\frac{\gamma-1}{\gamma}} + b_t^{\frac{1}{\gamma}}\left(\frac{M_t(j)}{P_t}\right)^{\frac{\gamma-1}{\gamma}}} = \Lambda_t(j)\frac{P_t}{P_t^d}; \tag{8}$$

$$\frac{b_t^{\frac{1}{\gamma}}\left(\frac{M_t(j)}{P_t}\right)^{\frac{-1}{\gamma}}\left(\frac{P_t^d}{P_t}\right)}{C_t(j)^{\frac{\gamma-1}{\gamma}} + b_t^{\frac{1}{\gamma}}\left(\frac{M_t(j)}{P_t}\right)^{\frac{\gamma-1}{\gamma}}} = \Lambda_t(j) - \beta E_t\left[\frac{P_t^d}{P_{t+1}^d}\Lambda_{t+1}(j)\right]; \tag{9}$$

$$\frac{\Lambda_t(j)}{R_t} = \beta E_t\left[\frac{P_t^d}{P_{t+1}^d}\Lambda_{t+1}(j)\right]; \tag{10}$$

$$\frac{\Lambda_t(j)}{\kappa_t R_t^*} = \beta E_t\left[\frac{P_t^d}{P_{t+1}^d}\frac{e_{t+1}}{e_t}\Lambda_{t+1}(j)\right], \tag{11}$$

where $\Lambda_t(j)$ is the Lagrange multiplier associated with the time t budget constraint. With probability $(1-d_w)$ the household is allowed to set its wage. The first order condition is:

$$\tilde{W}_t(j) = \left(\frac{\sigma}{\sigma-1}\right)\frac{E_t\sum_{l=0}^{\infty}(\beta d_w)^l \frac{\eta h_{t+l}(j)}{1-h_{t+l}(j)}}{E_t\sum_{l=0}^{\infty}(\beta d_w)^l(1-\tau_{t+l})h_{t+l}(j)\Lambda_{t+l}(j)/P_{t+l}^d} \tag{12}$$

This first-order condition gives a New Keynesian Phillips curve for wage inflation (see section (2.6)). The wage index evolves over time according to:

$$W_t = \left[d_w(W_{t-1})^{1-\sigma} + (1-d_w)(\tilde{W}_t)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, \tag{13}$$

where $\tilde{W}_t$ is the average wage of those workers who revise their wage at time t.

2.2 Goods production

2.2.1 Domestic intermediate goods

Firms have identical production functions given by:

$$Y_t(i) = X_t(i)^{\phi} \left(A_t h_t(\cdot, i)\right)^{1-\phi}, \quad \phi \in (0,1), \tag{14}$$

where $h_t(\cdot, i)$ is the quantity of the aggregate labor input employed by firm i and $X_t(i)$ is the quantity of the final composite good used by firm i.[5] A_t is an aggregate technology shock that follows the stochastic process given by:

$$\log(A_t) = (1-\rho_A)\log(A) + \rho_A \log(A_{t-1}) + \varepsilon_{At}, \tag{15}$$

where ε_{At} is a normally distributed, serially uncorrelated shock with zero mean and standard deviation σ_A. The firm chooses $X_t(i)$ and $h_t(\cdot, i)$ to maximize its stock market value. When allowed to do so (with probability $(1-d_p)$ each period), it also chooses the price of its output, $\tilde{P}_t^d(i)$. It solves:

$$\max_{\{X_t(i), h_t(\cdot,i), \tilde{P}_t^d(i)\}} E_t \left[\sum_{l=0}^{\infty} (\beta d_p)^l \left(\frac{\Lambda_{t+l}}{\Lambda_t}\right) \frac{D_{t+l}^d(i)}{P_{t+l}^d} \right], \tag{16}$$

where Λ_t is the marginal utility of wealth for a representative household, and

$$D_{t+l}^d(i) \equiv \tilde{P}_t^d(i) Y_{t+l}(i) - W_{t+l} h_{t+l}(\cdot, i) - P_{t+l} X_{t+l}(i),$$

where P_t is the price of the final output good, Z_t. The maximization is subject to the firm's production function and to the derived demand for the firm's output (discussed in section (2.2.3)) given by:

$$Y_{t+l}(i) = \left(\frac{\tilde{P}_t^d(i)}{P_{t+l}^d}\right)^{-\theta} Y_{t+l}, \tag{17}$$

where P_t^d is the exact price index of the composite domestic good. The elasticity of the derived demand for the firm's output is $-\theta$. The first-order conditions are:

$$\frac{W_t}{P_t^d} = \xi_t(i)(1-\phi)\frac{Y_t(i)}{h_t(\cdot, i)}; \tag{18}$$

$$\frac{P_t}{P_t^d} = \xi_t(i)\phi\frac{Y_t(i)}{X_t(i)}; \tag{19}$$

[5] We include $X_t(i)$ in the production of domestic intermediates for two reasons. First, without $X_t(i)$, the response of the real wage to demand shocks is too highly countercyclical. Second, as shown in similar models by McCallum and Nelson (1999, 2001), the presence of intermediates in the production function for domestic goods affects the correlation between the nominal exchange rate and domestic inflation.

$$\tilde{P}_t^d(i) = \left(\frac{\theta}{\theta - 1}\right) \frac{E_t \sum_{l=0}^{\infty} (\beta d_p)^l \left(\frac{\Lambda_{t+l}}{\Lambda_t}\right) \xi_{t+l}(i) Y_{t+l}(i)}{E_t \sum_{l=0}^{\infty} (\beta d_p)^l \left(\frac{\Lambda_{t+l}}{\Lambda_t}\right) Y_{t+l}(i)/P_{t+l}^d}, \tag{20}$$

where $\xi_t(i)$ is the Lagrange multiplier associated with the production function constraint. It measures the firm's real marginal cost. The first-order condition with respect to the firm's price relates the price to the expected future price of final output and to expected future real marginal costs. It can be used to derive a New Keynesian Phillips curve relationship for the rate of change of domestic output prices (see section (2.6)).

2.2.2 Imported intermediate goods

The economy imports a continuum of foreign intermediate goods on the unit interval. There is monopolistic competition in the market for imported intermediates, which are imperfect substitutes for each other in the production of the composite imported good, Y_t^m, produced by a representative competitive firm. When allowed to do so (with probability $(1 - d_m)$ each period), the importer of good i sets the price, $\tilde{P}_t^m(i)$, to maximize its weighted expected profits. It solves:

$$\max_{\{\tilde{P}_t^m(i)\}} E_t \left[\sum_{l=0}^{\infty} (\beta d_m)^l \left(\frac{\Lambda_{t+l}}{\Lambda_t}\right) \frac{D_{t+l}^m(i)}{P_{t+l}^d}\right], \tag{21}$$

where:

$$D_{t+l}^m(i) = \left(\tilde{P}_t^m(i) - e_{t+l} P_{t+l}^*\right) \left(\frac{\tilde{P}_t^m(i)}{P_{t+l}^m}\right)^{-\vartheta} Y_{t+l}^m. \tag{22}$$

For convenience, we assume that the price in foreign currency of all imported intermediates is P_t^*, which is also equal to the foreign price level. The elasticity of the derived demand for the imported good, i, is $-\vartheta$. The first-order condition is:

$$\tilde{P}_t^m(i) = \left(\frac{\vartheta}{\vartheta - 1}\right) \frac{E_t \sum_{l=0}^{\infty} (\beta d_m)^l \left(\frac{\Lambda_{t+l}}{\Lambda_t}\right) Y_{t+l}^m(i) e_{t+l} P_{t+l}^* / P_{t+l}^d}{E_t \sum_{l=0}^{\infty} (\beta d_m)^l \left(\frac{\Lambda_{t+l}}{\Lambda_t}\right) Y_{t+l}^m(i)/P_{t+l}^d}. \tag{23}$$

This equation can be used to derive a New Keynesian Phillips curve relationship for the rate of change of intermediate input prices (see section (2.6)).

2.2.3 Composite goods

The composite domestic good, Y_t, is produced using a constant elasticity of substitution (CES) technology with a continuum of domestic intermediate goods, $Y_t(i)$, as inputs:

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}}. \tag{24}$$

It is produced by a representative competitive firm that solves:

$$\max_{\{Y_t(i)\}} P_t^d Y_t - \int_0^1 P_t^d(i) Y_t(i) di, \tag{25}$$

subject to the production function (24). The first-order conditions yield the derived demand functions for the domestic intermediate goods given by (17). The exact price index for the composite domestic good is:

$$P_t^d = \left(\int_0^1 P_t^d(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}}. \tag{26}$$

This price index corresponds to a producer price index (PPI) for the economy. The price level obeys the following law of motion:

$$P_t^d = \left[d_p (P_{t-1}^d)^{1-\theta} + (1 - d_p)(\tilde{P}_t^d)^{1-\theta} \right]^{\frac{1}{1-\theta}}, \tag{27}$$

where $\tilde{P}_t^d$ is the price index derived by aggregating over all firms that change their price at time t.

Composite domestic output, Y_t, is divided between domestic use, Y_t^d, and exports, Y_t^x. Foreign demand for domestic exports is:[6]

$$Y_t^x = \alpha_x \left(\frac{P_t^d}{e_t P_t^*} \right)^{-\varsigma} Y_t^*, \tag{28}$$

where Y_t^* is foreign output.[7] The elasticity of demand for domestic output is $-\varsigma$, and $\alpha_x > 0$ is a parameter determining the fraction of domestic exports in foreign spending. Domestic exports form an insignificant fraction of foreign expenditures, and have a negligible weight in the foreign price index.

The foreign variables P_t^* and Y_t^* are both exogenous and, when stationarized, evolve according to

$$\log(P_t^*/P_{t-1}^*) = (1 - \rho_{\pi^*}) \log(\pi^*) + \rho_{\pi^*} \log(P_{t-1}^*/P_{t-2}^*) + \varepsilon_{\pi^* t}, \tag{29}$$

and

$$\log Y_t^* = (1 - \rho_{y^*}) \log(Y^*) + \rho_{y^*} \log(Y_{t-1}^*) + \varepsilon_{y^* t}, \tag{30}$$

[6]This condition can be derived from a foreign importing firm that combines non-perfectly substitutable imported goods.

[7]To ensure the existence of a balanced growth path for the economy, we assume that foreign output grows at the same trend rate as domestic output.

where π^* is steady-state foreign inflation, and ε_{π^*t} and ε_{y^*t} are zero-mean, serially uncorrelated shocks with standard errors σ_{π^*} and σ_{y^*}, respectively.

The composite imported good, Y_t^m, is produced using a CES technology with a continuum of imported-intermediate goods, $Y_t^m(i)$, as inputs:

$$Y_t^m \leq \left(\int_0^1 (Y_t^m(i))^{\frac{\vartheta-1}{\vartheta}} di \right)^{\frac{\vartheta}{\vartheta-1}}. \tag{31}$$

It is produced by a representative competitive firm. Its profit maximization gives the derived demand function for intermediate imported good j given by:

$$Y_t^m(i) = \left(\frac{P_t^m(i)}{P_t^m} \right)^{-\vartheta} Y_t^m. \tag{32}$$

The exact price index for the composite imported goods is given by:

$$P_t^m = \left(\int_0^1 P_t^m(i)^{1-\vartheta} di \right)^{\frac{1}{1-\vartheta}}. \tag{33}$$

The price index obeys the following law of motion:

$$P_t^m = \left[d_m (P_{t-1}^m)^{1-\vartheta} + (1-d_m)(\tilde{P}_t^m)^{1-\vartheta} \right]^{\frac{1}{1-\vartheta}}, \tag{34}$$

where $\tilde{P}_t^m$ is a price index derived by aggregating over all importers that change their price in time t.

2.2.4 Final goods

The final good, Z_t, is produced by a competitive firm that uses Y_t^d and Y_t^m as inputs subject to the following CES technology:

$$Z_t = \left[\alpha_d^{\frac{1}{\nu}} (Y_t^d)^{\frac{\nu-1}{\nu}} + \alpha_m^{\frac{1}{\nu}} (Y_t^m)^{\frac{\nu-1}{\nu}} \right]^{\frac{\nu}{\nu-1}}, \tag{35}$$

where $\alpha_d > 0$, $\alpha_m > 0$, $\nu > 0$, and $\alpha_d + \alpha_m = 1$. The final good, Z_t, is used for domestic consumption, C_t, as inputs to produce domestic intermediate goods, X_t, and government purchases, G_t. The final good is produced by a competitive firm that solves:

$$\max_{\{Y_t^d, Y_t^m\}} P_t Z_t - P_t^d Y_t^d - P_t^m Y_t^m, \tag{36}$$

subject to the production function (35). Profit maximization entails:

$$Y_t^d = \alpha_d \left(\frac{P_t^d}{P_t} \right)^{-\nu} Z_t, \tag{37}$$

and

$$Y_t^m = \alpha_m \left(\frac{P_t^m}{P_t}\right)^{-\nu} Z_t. \tag{38}$$

Furthermore, the final-good price, P_t, which corresponds to the consumer price index or CPI, is given by:

$$P_t = \left[\alpha_d (P_t^d)^{1-\nu} + \alpha_m (P_t^m)^{1-\nu}\right]^{1/(1-\nu)}. \tag{39}$$

2.3 Monetary authority

Following Taylor (1993), Dib (2003) and Ireland (2003) among others, the central bank manages the short-term nominal interest rate, R_t, in response to fluctuations in CPI inflation ($\pi_t = P_t/P_{t-1}$), money growth ($\mu_t = M_t/M_{t-1}$), and output (Y_t). Its interest rate reaction function is given by:

$$\log(R_t/R) = \varrho_\pi \log(\pi_t/\pi) + \varrho_\mu \log(\mu_t/\mu) + \varrho_y \log(Y_t/Y) + \varepsilon_{Rt}, \tag{40}$$

where π, μ and Y are the steady-state values of π_t, μ_t and Y_t, where R is the steady-state value of the gross nominal interest rate, and where ε_{Rt} is a zero-mean, serially uncorrelated monetary policy shock with standard deviation σ_R. The error term arises from the fact that the central bank can control short term interest rates only indirectly by setting the Bank rate. The error term thus reflects developments in money and financial markets that are not explicitly captured by our model.

Money growth is included as an argument in the Taylor rule because of the inclusion of money demand shocks in our model. They turn out to be important empirically and account for a significant fraction of fluctuations in output and inflation. If the central bank reacted only to inflation, money demand shocks could be exacerbated by the bank's behavior since a positive money demand shock would lead to a decrease in inflation, a reduction in short term interest rates, and thereby to an endogenous increase in money demand. The inclusion of CPI inflation rather than PPI inflation is motivated by the fact that the Bank of Canada does in fact target CPI inflation; also, reacting to CPI inflation allows for an indirect channel for reacting to exchange rate movements, since exchange rate fluctuations may be passed through much more quickly to the CPI than to the PPI.[8]

2.4 The government

The government budget constraint is given by:

$$P_t G_t + T_t + D_{t-1}^g = \tau_t W_t h_t + M_t - M_{t-1} + \frac{D_t^g}{R_t}. \tag{41}$$

[8]Ambler, Dib and Rebei (2003) present evidence that this is indeed the case for Canada.

The left side of (41) represents uses of government revenue: goods purchases, transfers, and debt repayments. The right side includes tax revenues, money creation, and newly issued debt. The government also faces a no-Ponzi constraint that implies that the present value of government expenditures equals the present value of tax revenue plus the initial stock of public debt, D_0^g.

Because households have infinite horizons, there is Ricardian equivalence in the following sense: given the tax rate on labor income, a change in the mix between lump-sum taxes and borrowing does not affect the economy's equilibrium. We can simplify the budget constraint without loss of generality to:

$$P_t G_t + T_t = \tau_t W_t h_t + M_t - M_{t-1}. \tag{42}$$

This implies that D_t^g is zero in each period. Government spending and the tax rate are determined by:

$$\log(G_t) = (1-\rho_g)\log(G) + \rho_g \log(G_{t-1}) + \varepsilon_{gt}, \tag{43}$$

and

$$\log(\tau_t) = (1-\rho_\tau)\log(\tau) + \rho_\tau \log(\tau_{t-1}) + \varepsilon_{\tau t}. \tag{44}$$

Given these stochastic processes and that the nominal money stock is determined by money demand once the nominal interest rate is set, lump-sum taxes are determined residually to balance the government's budget.

2.5 Equilibrium

There are two different stochastic trends in the model. The first is in the foreign price level, and arises from the specification of the stochastic process for P_t^* in terms of rates of change in equation (29). The second is in the price of domestic output and all other domestic nominal variables, and arises from the fact that the monetary authority adjusts the domestic nominal interest rate as a function of inflation rather than the price level, according to equation (40).

Solving the model involves using stationary transformations of variables with unit roots. We use the following transformations: $p_t \equiv P_t/P_t^d$, $m_t \equiv M_t/P_t$, $p_t^m \equiv P_t^m/P_t^d$, $\tilde{p}_t^d \equiv \tilde{P}_t^d/P_t^d$, $\pi_t \equiv P_t/P_{t-1}$, $\pi_t^d \equiv P_t^d/P_{t-1}^d$, $w_t \equiv W_t/P_t^d$, $\pi_t^* \equiv P_t^*/P_{t-1}^*$, $b_t^* \equiv B_t^*/P_t^*$ and $s_t \equiv e_t P_t^*/P_t^d$. The complete system of equations in stationary variables that characterize the model's equilibrium is given in Appendix B.

2.6 New Keynesian Phillips curves

The price- and wage-setting equations cannot be used directly to simulate the model since they involve infinite summations. By linearizing these equations around the steady-state values of the variables, and assuming zero inflation in the steady state, we obtain three New Keynesian Phillips curves relationships that determine the rates of inflation of locally produced intermediate goods, imported intermediates, and the nominal-wage index. Defining $\pi_t^m \equiv P_t^m / P_{t-1}^m$, and $\pi_t^w \equiv W_t / W_{t-1}$, we get:

$$\hat{\pi}_t^d = \beta \hat{\pi}_{t+1}^d + \frac{(1-\beta d_p)(1-d_p)}{d_p} \hat{\xi}_t; \tag{45}$$

$$\hat{\pi}_t^m = \beta \hat{\pi}_{t+1}^m + \frac{(1-\beta d_m)(1-d_m)}{d_m} \hat{s}_t; \tag{46}$$

and

$$\begin{aligned} \hat{\pi}_t^w &= \beta \hat{\pi}_{t+1}^w \\ &+ \frac{(1-\beta d_w)(1-d_w)}{d_w} \cdot \left[\left(\frac{h}{1-h} \right) \hat{h}_t - \hat{\Lambda}_t + \left(\frac{\tau}{1-\tau} \right) \hat{\tau}_t - \hat{w}_t \right], \end{aligned} \tag{47}$$

where hats over variables denote deviations from steady-state values. The New Keynesian Phillips curve for domestic output inflation is the same as in Galí and Gertler (1999). It relates inflation to expected future inflation and to the real marginal cost of output. The equation for import price inflation is analogous, with real marginal cost captured by the real exchange rate. The wage inflation equation is also analogous. The term in square brackets measures the marginal rate of substitution (the real marginal cost to workers of their work effort) minus the real wage. The household's first-order condition for the nominal wage can be interpreted as a markup over the average marginal cost of work effort over the life of the wage contract.

3. Model Solution and Parameter Estimation

In order to estimate the model's parameters we use a linear approximation around its steady state, but for welfare analysis we use a second-order approximation using the *Dynare* program (Juillard, 2002). The Blanchard and Kahn (1980) algorithm is used to solve the linearized model. It leads to a state space representation with transition equations for the model's predetermined endogenous state variables and observation equations relating those states to observable macroeconomic aggregates. The model's forward-looking or jump state variables are eliminated from the state transition equations by the Blanchard and Kahn solution procedure. In the notation of Ireland (2004), we have:

$$\mathbf{s_t} = A \mathbf{s_{t-1}} + B \varepsilon_t, \tag{48}$$

The model is completed by the following set of observation equations relating the model's state variables to observable endogenous variables:

$$\mathbf{f_t} = C \mathbf{s_t}. \tag{49}$$

The column vector $\mathbf{s_{t-1}}$ contains the predetermined endogenous state variables of the model:

$$\mathbf{s_{t-1}}' \equiv [b_{t-1}\, , \, A_{t-1}\, , \, G_{t-1}\, , \, \tau_{t-1}\, , \, R^*_{t-1}\, , \, \pi^*_{t-1}\, , \, Y^*_{t-1}\, , \, w_{t-1}\, , \, p^m_{t-1}\, , \, m_{t-1}\, , \, b^*_{t-1}]$$

with all variables stationarized and measured in proportional deviations from their steady state values. With eight structural shocks in the model, we include a vector of eight observation equations in order to avoid the stochastic singularity problem discussed by Ingram, Kocherlakota and Savin (1994). This problem stems from the fact that, with more than eight observation equations, there would be exact or deterministic relationships among certain combinations of the model's endogenous variables. If these relationships did not hold exactly in the data, estimation by the maximum likelihood procedure would break down. We include the five state variables that are directly observable as well as consumption, CPI inflation, and the domestic interest rate:

$$\mathbf{f_t}' \equiv [C_t\, , \, \pi_t\, , \, R_t\, , \, G_t\, , \, \tau_t\, , \, R^*_t\, , \, \pi^*_t\, , \, Y^*_t],$$

once again with all variables measured in proportional deviations from their steady state values.

The Kalman filter is used to write down the model's log-likelihood function given its state space representation.[9] The same estimation method is used by Dib (2003) and Ireland (2003). The parameters are then estimated by maximizing the log-likelihood function over the sample period from 1981:3 to 2002:4.

3.1 Parameter estimates

Table 1 summarizes our parameter estimates. Not counting constants in the stochastic processes for the model's forcing variables, the model has 36 structural parameters. Of these, we were unable to estimate six because they were poorly identified. These parameters were assigned calibrated values, as outlined in the following paragraph.

The subjective discount rate, β, is given a standard value, which implies an annual real interest rate of 4 per cent in the steady state. The weight on leisure in the utility function, η, is calibrated so that the representative household spends about one third of its total time working in the steady state. The α_x parameter is a normalization that ensures that the current account is balanced in the long run. The demand elasticities, σ, θ, and ϑ, influence the stochastic properties of the model in a very indirect way. After linearization, they no longer appear in the three New Keynesian Phillips curve equations. By influencing the size of the markups over marginal cost, they do influence the steady-state levels of the domestic production of intermediate goods, imported intermediate goods, and employment. Because certain coefficients in the linearized model depend on the steady-state levels of

[9] See Hamilton (1994, ch.13) or Ireland (2004) for detailed descriptions.

endogenous variables, the moments predicted by the model are related to these parameters. Unfortunately, the influence is so weak that it is impossible to estimate them precisely. The θ and ϑ parameters give the elasticity of substitution across different types of intermediate goods in the production of the composite domestic good and the composite imported good. Setting $\theta = \vartheta = 8$ gives a steady-state markup of 14 per cent, which agrees well with estimates in the empirical literature of between 10 per cent and 20 per cent (see, for example, Basu 1995). The σ parameter gives the elasticity of substitution across different labor types in the production of individual domestic intermediate goods. The value $\sigma = 6$ corresponds to estimates from microdata in Griffin (1992).[10]

Of the estimated parameters in Table 1, most have small standard errors and are highly significant. In particular, the nominal rigidity parameters are highly significant. They are of plausible magnitude and within the range of values in previous empirical studies and in calibrated general-equilibrium models. The estimate of d_p implies that the prices of domestic intermediate goods remain fixed for, on average, 1.78 quarters. The other prices are revised less often on average, but still well within the range of plausibility. Import prices remain fixed for, slightly more than two quarters on average. Nominal wages remain fixed for 5.37 quarters on average.

The estimated values of the Taylor rule imply, since the sum of ϱ_π and ϱ_μ is greater than unity, that the long-run level of the inflation rate is determinate and the model is saddlepoint stable, with a unique dynamic solution in response to shocks. The value of ϱ_y suggests that the Bank of Canada intervened only weakly if at all during the sample period to fluctuations in real output.[11]

The stochastic processes for the model's forcing variables highly persistent. Except for $\rho_{\pi^*} = 0.2054$ and $\rho_\tau = 0.4320$, the estimated AR(1) parameters are greater than 0.64. The standard deviations of the innovations to the processes vary widely in magnitude, ranging from 0.0016 in the case of foreign interest rate shocks to 0.0718 in the case of money demand shocks. The volatility of foreign shocks is smaller than that of domestic shocks, which suggests the relative importance of domestic shocks for business cycle fluctuations in the Canadian economy.

Figure 1 shows the fitted and (within sample) predicted values of several of the model's time series (with error terms in the observation equations set equal to zero). For the series that are used to construct the model's likelihood function, the fit is quite good. The model does have difficulty tracking the nominal and real exchange rate series, even within sample. This is not surprising, given the generally poor performance of structural exchange rate

[10]It also agrees with the value estimated in Ambler, Guay, and Phaneuf (2003) using aggregate time series data. They succeeded in estimating the value of the equivalent parameter in their model by calibrating the equivalent of the d_w parameter.

[11]We also allowed monetary policy to respond to real exchange rate fluctuations in some of our estimations. The coefficient was very small in magnitude and insignificant. We did not allow for regime shifts when estimating the Taylor rule coefficients.

models and the fact the the real and nominal exchange rates were not included among the observable variables used to construct the likelihood function.

4. Optimal Monetary Policy

Given the estimated and calibrated values of the model's structural parameters, we optimized over the three coefficients of the Taylor rule to find the values that maximize unconditional welfare. The maximization problem can be written as follows:

$$\max_{\varrho_\pi,\varrho_\mu,\varrho_y} E\left\{u\left(C_t,m_t,h_t\right)\right\}. \tag{50}$$

The solution amounts to maximizing welfare in the steady state.[12] It ignores any costs involved in the transition between the initial stochastic steady state with the estimated values of the Taylor rule coefficients and the new stochastic steady state with optimized Taylor rule coefficients. We address this issue below.

It has been known for some time that for the purposes of welfare evaluation in *DGE* models, first-order approximations of their equilibrium conditions may not be adequate.[13] Kim and Kim (2003) provide a simple example of a model in which welfare appears higher under autarky than under complete markets because of the inaccuracy of the linearization method.[14] To avoid this problem, we computed the welfare-maximizing Taylor rules, using the *Dynare* program. *Dynare* calculates a second-order approximation of the model around its deterministic steady state. We used the program to calculate the theoretical first and second moments of the model's endogenous variables, including period utility. Our main results are presented in Table 2. The second column of the table reproduces the historical (estimated) values of the Taylor rule coefficients from Table 1 in order to facilitate comparison with their optimized values. The third column of the table shows the optimized Taylor rule coefficients. The last column shows the Taylor rule coefficients for an inflation stabilization scenario, in which the central bank pays attention only to inflation, and strongly resists fluctuations in the inflation rate around its long-run average value.

We measured the welfare gain associated with a particular monetary policy by means of the compensating variation. This measures the percentage change in consumption given the

[12]It has become standard practice in the literature to abstract from welfare gains and losses due to changes in real money balances. Because we find empirically that money demand shocks explain a substantial fraction of output fluctuations, we decided not to shut down the effects of money demand shocks on the model.

[13]Woodford (2001, chapter six) gives a series of conditions under which first-order approximations of the policy functions in DGE models are adequate for evaluating social welfare. The conditions do not hold here: the main condition that fails is that, because of the presence of distortions due to monopolistic competition, the deterministic steady state of the model has an allocation of resources that is not Pareto optimal.

[14]See Kim, Kim, Schaumburg and Sims (2003) for a more general discussion.

equilibrium with the historical values of the Taylor rule coefficients that would give households the same unconditional expected utility as in the indicated scenario. The compensating variation is defined as follows:

$$E\left\{u\left(C_t(1+\zeta), m_t, h_t\right)\right\} = E\left\{u\left(C_t^*, m_t^*, h_t^*\right)\right\}, \tag{51}$$

where variables without asterisks refer to variables under the historical values of the Taylor rule coefficients, and variables with asterisks refer to variables under the proposed Taylor rule coefficients. The compensating variations associated with the optimal Taylor rule and with the anti-inflation scenario are presented in the last row of Table 2.

The results are striking. The compensating variation for the optimal Taylor rule is quite large. Consumption in each period would have to increase by 1.40% in the model with the historical values of the Taylor rule coefficients in order for agents to be as well off as with the optimal coefficients. This is larger than the welfare gain calculated by Kollmann (2002). In order to compare our welfare gain with Kollmann's, it is important to note that he measures his welfare gains with respect to the level of welfare in the deterministic steady state, in which the variance of each shock is set equal to zero, rather than the stochastic steady state with historical values of the Taylor rule coefficients as we do.[15] His compensating variation is 0.39%. Note from Table 3 that in our model the deterministic steady state gives a welfare improvement over the stochastic steady state with the historical Taylor rule coefficients. The size of the compensating variation is 0.37%. The welfare gain in our model compared to the deterministic steady state is therefore equal to 1.02% in terms of compensating variation, just over two and one half times the increase in Kollmann's model. Even when the monetary policy has inflation stabilization as its exclusive focus, there is a substantial welfare gain compared to the stochastic steady state with the historical values of the Taylor rule coefficients: the compensating variation is equal to 0.73%.

Compared to the historical values of the Taylor rule coefficients, monetary policy with the optimal Taylor rule responds more strongly to fluctuations in inflation and output, and less strongly to fluctuations in the growth of nominal balances. Despite these differences, the coefficients of the optimized Taylor rule are quite close to the corresponding historical values. This suggests that the measured welfare gains may be quite sensitive to small variations in the Taylor rule coefficients. This is confirmed by a detailed analysis of the shape of the welfare function in the space of the Taylor rule coefficients. Figure 2 shows the shape of the welfare function in the ϱ_π/ϱ_y plane, holding constant the value of ϱ_μ at its optimal level of zero. The level of period utility in the neighborhood of the optimum takes the form of a tall, narrow peak. In addition, the peak is quite close to the region of parameter values in which the model is locally indeterminate. Local indeterminacy occurs when the number of stable roots of the linearized model is greater than the number of predetermined variables. In this

[15]He calibrates rather than estimates his model. Therefore, he has no estimated historical values of the Taylor rule coefficients with which to measure welfare.

case, there are infinitely many dynamic paths that converge to the model's deterministic steady state, starting in the immediate neighborhood of that steady state, and sunspot equilibria are a possibility. Roughly speaking, the model is locally indeterminate when

$$\varrho_\pi + \varrho_\mu - \varrho_y \leq 1.0.$$

Schmitt-Grohe and Uribe (2004) restrict the policy rules that they consider in order to yield a locally unique equilibrium within a radius of 0.15 around the optimized coefficients. They note (p.20) that, "welfare computations near a bifurcation point may be inaccurate." Restricting the coefficients of the Taylor rule so that

$$\varrho_\pi + \varrho_\mu - \varrho_y \geq 1.15$$

leads to the welfare contours illustrated in Figure 3. For small values of ϱ_μ (less than two), welfare is increasing in the value of ϱ_π, but is less than the optimal level of welfare attainable without imposing this additional restriction.

4.1 Level effect versus stabilization effect

Because the model is solved using a second-order approximation of its equilibrium conditions around the deterministic steady-state levels of its variables, both the variances of shocks and the monetary policy rule (which influences how the shocks are transmitted to the economy) can affect the **means** of the endogenous variables of the economy. Table 3 shows the average levels of various endogenous variables, and the standard deviations of the same variables, for the deterministic steady state and for the same monetary policy scenarios as reported in Table 2.

It is also possible to summarize the extent to which the gains in welfare are coming from the effects of the change in policy on the levels of consumption, leisure and real balances versus changes in the volatility of these variables. We can approximate the difference between welfare under optimal policy and the estimated values of the Taylor rule coefficients as follows:

$$E\left(u\left(z_t^*\right)\right) - E\left(u\left(z_t\right)\right)$$
$$\approx u(z) + u_z E\left(\hat{z}_t^*\right) + \frac{1}{2}E(\hat{z}_t^*)' u_{zz}(\hat{z}_t^*) - u(z) - u_z E\left(\hat{z}_t\right) - \frac{1}{2}E(\hat{z}_t)' u_{zz}(\hat{z}_t),$$

where $z_t \equiv (C_t, m_t, h_t)$ is the vector of arguments of the utility function, z is the value of these arguments in the deterministic steady state, and variables with hats measure deviations from their levels in the deterministic steady state. This implies:

$$E\left(u\left(z_t^*\right)\right) = E\left(u\left(z_t\right)\right) + u_z E\left(\hat{z}_t^* - \hat{z}_t\right) + \frac{1}{2}E\left(\hat{z}_t^* - \hat{z}_t\right)' u_{zz}\left(\hat{z}_t^* - \hat{z}_t\right).$$

This allows us to decompose the gains in welfare from optimal monetary policy into a level effect and a stabilization effect. We define the level effect as:

$$E\left\{u\left(C_t(1+\zeta_L), m_t, h_t\right)\right\} = Eu\left(z_t\right) + u_z E\left(\hat{z}_t^* - \hat{z}_t\right). \tag{52}$$

We define the stabilization effect as follows:

$$E\left\{u\left(C_t(1+\zeta_S), m_t, h_t\right)\right\} = Eu\left(z_t\right) + \frac{1}{2}E\left(\hat{z}_t^* - \hat{z}_t\right)' u_{zz}\left(\hat{z}_t^* - \hat{z}_t\right). \tag{53}$$

The results are shown in the last two rows of Table 3. The overall effect in all cases is such that approximately:

$$(1+\zeta) \approx (1+\zeta_L)(1+\zeta_S). \tag{54}$$

The most important result is that the welfare gain from optimizing the Taylor rule coefficients comes from the level effect. In fact, period utility becomes more volatile with the optimal Taylor rule coefficients than with their historical values. Hours worked are slightly more volatile, consumption is slightly less volatile, and real balances are much more volatile under the optimal Taylor rule. This implies that the welfare gain due to the stabilization effect is actually negative. From the top three rows of Table 3, it is clear that the level effect comes mainly from an increase in the level of real money balances. As discussed in the next subsection, this suggests that neglecting the effects on welfare of the transition from the initial steady state to the final steady state of the model may give misleading results.

4.2 Transition costs

By maximizing unconditional welfare, we are implicitly maximizing welfare in the stochastic steady state. The welfare comparison ignores the possibility of losses in welfare on the transition path from one steady state to another. The possibility is potentially acute for open economies. Welfare in the new steady state with optimal policy may be higher because a higher level of net foreign assets allows individuals to enjoy a higher level of consumption. However, acquiring the additional foreign assets implies a lower level of consumption in the short run. The short term loss may even swamp the long term gain if individuals are sufficiently impatient and if this impatience is reflected in the social welfare function.

In our model, much of the increase in welfare comes from an increase in the level of real money balances. Table 3 shows that average consumption increases slightly, average hours worked increase slightly (which implies a small loss in welfare), and that there is a substantial increase in real money balances. For a small open economy, the costs of acquiring these additional real assets are of course different in nature from the costs of increasing the level of net foreign assets. An increase in real balances can arise from a fall in the overall price level, without a direct sacrifice of consumption in the short run. In order to analyze the impact of the transition to the final steadys state on welfare, we conducted the following Monte Carlo experiment. For 1000 different draws of the model's shocks from a multivariate normal distribution and of the model's structural parameters, we simulated the model's response to the shocks, with starting values for its predetermined state variables equal to their means in the stochastic steady state with the historical values of the Taylor rule coefficients.

The results are shown in Figure 4. Period utility is measured on the vertical axis. The blue dotted lines indicate the average level of period utility under the historical Taylor rule and under the optimized Taylor rule. The solid line shows the average of period utility across the 1000 different replications, with 90% confidence bands around this mean response given by the dashed lines. The graph indicates clearly that there are no short term costs associated with moving to the optimal monetary policy. Welfare increases even in the period immediately after the implementation of the new monetary policy.

5. Conclusions

This paper has shown that it is feasible to construct a fully-developed *NOEM* model of a small open economy such as Canada, to estimate almost all of its parameters using maximum likelihood techniques, and to use the model to analyze optimal monetary policy by calculating the values of the Taylor rule coefficients that maximize unconditional welfare. The time is perhaps not far off when central banks themselves will integrate the use of such models into the formulation of their monetary policy.

Our results show that it is possible to improve welfare substantially by getting the coefficients of a modified Taylor rule right. The welfare increase is equivalent to a permanent 1.40% increase in the level of consumption between the stochastic steady states with the estimated values of the Taylor rule coefficients and the optimal values. However, the optimal Taylor rule coefficients put the model quite close to the region of local indeterminacy. If the Taylor rule coefficients are restricted to place the economy further away from the region of local indeterminacy, the welfare gains from optimal policy decrease substantially.

Much work remains to be done. We need to incorporate capital into the model so that it can better reproduce the persistence of some of the main macroeconomic aggregates. We need to do more work on the difference between policies that maximize conditional versus unconditional welfare. We need to work on deriving the truly optimal feedback rule and to evaluate the welfare loss from using a Taylor rule that is necessarily an **approximation** to the fully optimal rule. We need to analyze the problem of time consistency. Finally, we need to examine whether the result that welfare gains are extremely sensitive to the coefficients of the optimal policy rule and to the structural parameters of the model is robust to different types of models.

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Appendix A: Data and Data Sources

Our data set is available on request. The data are from Canada and the United States and are quarterly from 1981Q3 to 2002Q4. The Canadian data are from *Bank of Canada Banking and Financial Statistics*, a monthly publication by the Bank of Canada. Series numbers are indicated in brackets and correspond to Cansim databank numbers.

- Consumption, C_t, is measured by real personal spending on non-durable goods and services in 1997 dollars (non-durables [v1992047] + services [v1992119]).
- The CPI inflation rate, π_t, is measured by changes in the consumer price index, P_t [v18702611].
- The short-term nominal interest rate, R_t, is measured by the yield on Canadian three-month treasury bills [v122531].
- Government spending, G_t, is measured by government expenditures on goods and services (total domestic demand [v1992068] − total personal expenditures [v1992115] − construction [v1992053 + v1992055] − machinery and equipment investment [v1992056]).
- The labor tax rate, τ_t, is measured by the effective labor tax rate (calculated following the methodology of Jones 2002; and Mendoza, Razin, and Tezar 1994).
- The series in per-capita terms are obtained by dividing each series by the Canadian civilian population aged 15 and over (civilian labor force [v2062810] / labor force participation [v2062816]).

The U.S. data are from the Federal Reserve Bank of St. Louis, with the series numbers in brackets. The world series are approximated by some of the U.S. series.

- World output, Y_t^*, is real U.S. GDP per capita in 1996 dollars [GDPC96] divided by the U.S. civilian non-institutional population [CNP16OV].
- The world nominal interest rate, R_t^*, is measured by the rate on U.S. three-month Treasury Bills [TB3MS].
- The world inflation rate, π_t^*, is measured by changes in the U.S. GDP implicit price deflator, P_t^* [GDPDEF].

Appendix B: Equilibrium Conditions

The following system of equations defines the economy's equilibrium:

$$\frac{C_t^{\frac{-1}{\gamma}}}{C_t^{\frac{\gamma-1}{\gamma}} + b_t^{\frac{1}{\gamma}} m_t^{\frac{\gamma-1}{\gamma}}} = \Lambda_t p_t; \tag{B.1}$$

$$\frac{b_t^{\frac{1}{\gamma}} m_t^{\frac{-1}{\gamma}}}{C_t^{\frac{\gamma-1}{\gamma}} + b_t^{\frac{1}{\gamma}} m_t^{\frac{\gamma-1}{\gamma}}} = \Lambda_t p_t \left(1 - \frac{1}{R_t}\right); \tag{B.2}$$

$$\frac{R_t}{\kappa_t R_t^*} = E_t\left[\frac{s_{t+1}\pi_{t+1}^d}{s_t\pi_{t+1}^*}\right]; \tag{B.3}$$

$$\frac{\Lambda_t}{R_t} = \beta E_t\left[\frac{\Lambda_{t+1}}{\pi_{t+1}^d}\right]; \tag{B.4}$$

$$\tilde{w}_t = \left(\frac{\sigma}{\sigma-1}\right)\frac{E_t\sum_{l=0}^{\infty}(\beta d_w)^l\ \eta h_{t+l}/(1-h_{t+l})}{E_t\sum_{l=0}^{\infty}(\beta d_w)^l(1-\tau_{t+l})\Lambda_{t+l}h_{t+l}\prod_{k=1}^{l}(\pi_{t+k}^d)^{-1}}; \tag{B.5}$$

$$w_t^{1-\sigma} = d_w\left(\frac{w_{t-1}}{\pi_t^d}\right)^{1-\sigma} + (1-d_w)\tilde{w}_t^{1-\sigma}; \tag{B.6}$$

$$Y_t = A_t X_t^{\phi} h_t^{1-\phi}; \tag{B.7}$$

$$w_t = (1-\phi)\xi_t\frac{Y_t}{h_t}; \tag{B.8}$$

$$p_t = \phi\xi_t\frac{Y_t}{X_t}; \tag{B.9}$$

$$\tilde{p}_t^d = \left(\frac{\theta}{\theta-1}\right)\frac{E_t\sum_{l=0}^{\infty}(\beta d_p)^l\Lambda_{t+l}Y_{t+l}\xi_{t+l}}{E_t\sum_{l=0}^{\infty}(\beta d_p)^l\Lambda_{t+l}Y_{t+l}\prod_{k=1}^{l}(\pi_{t+k}^d)^{-1}}; \tag{B.10}$$

$$1 = d_p\left(\frac{1}{\pi_t^d}\right)^{(1-\theta)} + (1-d_p)(\tilde{p}_t^d)^{(1-\theta)}; \tag{B.11}$$

$$\tilde{p}_t^m = \left(\frac{\vartheta}{\vartheta-1}\right)\frac{E_t\sum_{l=0}^{\infty}(\beta d_m)^l\Lambda_{t+l}Y_{t+l}^m s_{t+l}}{E_t\sum_{l=0}^{\infty}(\beta d_m)^l\Lambda_{t+l}Y_{t+l}^m\prod_{k=1}^{l}(\pi_{t+k}^d)^{-1}}; \tag{B.12}$$

$$(p_t^m)^{(1-\vartheta)} = d_m\left(\frac{p_{t-1}^m}{\pi_t^d}\right)^{(1-\vartheta)} + (1-d_m)\left(\tilde{p}_{mt}\right)^{(1-\vartheta)}; \tag{B.13}$$

$$(p_t)^{(1-\nu)} = \alpha_d + \alpha_m\left(p_t^m\right)^{(1-\nu)}; \tag{B.14}$$

$$Z_t = C_t + X_t + G_t; \tag{B.15}$$

$$Y_t = Y_t^x + Y_t^d; \tag{B.16}$$

$$Y_t^x = \alpha_x s_t^{\varsigma} Y_t^*; \tag{B.17}$$

$$Y_t^d = \alpha_d \left(\frac{1}{p_t}\right)^{-\nu} Z_t; \tag{B.18}$$

$$Y_t^m = \alpha_m \left(\frac{p_{mt}}{p_t}\right)^{-\nu} Z_t; \tag{B.19}$$

$$\frac{b_t^*}{\kappa_t R_t^*} - \frac{b_{t-1}^*}{\pi_t^*} = \frac{Y_t^x}{s_t} - Y_t^m; \tag{B.20}$$

$$\log(\kappa_t) = \varphi \left[\exp\left(\frac{s_t b_t^*}{Y_t}\right) - 1\right]; \tag{B.21}$$

$$\log(R_t/R) = \varrho_\pi \log(\pi_t/\pi) + \varrho_\mu \log(\mu_t/\mu) + \varrho_y \log(Y_t/Y) + \varepsilon_{Rt}; \tag{B.22}$$

$$\pi_t = \frac{m_{t-1}}{m_t}\mu_t; \tag{B.23}$$

$$\log(A_t) = (1-\rho_A)\log(A) + \rho_A \log(A_{t-1}) + \varepsilon_{At}; \tag{B.24}$$

$$\log(b_t) = (1-\rho_b)\log(b) + \rho_b \log(b_{t-1}) + \varepsilon_{bt}; \tag{B.25}$$

$$\log(G_t) = (1-\rho_g)\log(G) + \rho_g \log(G_{t-1}) + \varepsilon_{gt}; \tag{B.26}$$

$$\log(\tau_t) = (1-\rho_\tau)\log(\tau) + \rho_\tau \log(\tau_{t-1}) + \varepsilon_{\tau t}; \tag{B.27}$$

$$\log(R_t^*) = (1-\rho_{R^*})\log(R^*) + \rho_{R^*} \log(R_{t-1}^*) + \varepsilon_{R^* t}; \tag{B.28}$$

$$\log(\pi_t^*) = (1-\rho_{\pi^*})\log(\pi^*) + \rho_{\pi^*} \log(\pi_{t-1}^*) + \varepsilon_{\pi^* t}; \tag{B.29}$$

$$\log Y_t^* = (1-\rho_{y^*})\log(Y^*) + \rho_{y^*} \log(Y_{t-1}^*) + \varepsilon_{y^* t}. \tag{B.30}$$

Equation (B.20) gives the trade balance of the economy.

Table 1: Parameter Estimates

Parameter	Value	Standard deviation	t-stat	p-value
Stochastic processes				
ρ_a	0.8797	0.0258	34.10	0.00
ρ_b	0.6450	0.0453	14.24	0.00
ρ_g	0.7919	0.0663	11.94	0.00
ρ_τ	0.4320	0.0961	4.50	0.00
ρ_{r^*}	0.8973	0.0200	44.87	0.00
ρ_{y^*}	0.8280	0.0412	20.10	0.00
ρ_{π^*}	0.2054	0.0951	2.16	0.03
σ_a	0.0204	0.0017	12.00	0.00
σ_b	0.0718	0.0046	15.61	0.00
σ_g	0.0072	0.0006	12.00	0.00
σ_τ	0.0251	0.0019	13.21	0.00
σ_{r^*}	0.0016	0.0001	16.00	0.00
σ_{y^*}	0.0066	0.0005	13.20	0.00
σ_{π^*}	0.0018	0.0002	9.00	0.00
σ_r	0.0109	0.0009	12.11	0.00
b	0.3532	0.0402	8.79	0.00
Nominal rigidity				
d_w	0.8257	0.0491	16.82	0.00
d_p	0.4398	0.0479	9.18	0.00
d_m	0.5508	0.0275	20.03	0.00
Interest rate rule				
ϱ_π	1.0223	0.0863	11.85	0.00
ϱ_μ	0.6567	0.0745	8.81	0.00
ϱ_y	-0.0147	0.0498	-0.30	0.76
Foreign supply/demand				
α_x	0.074	calibrated		
φ	-0.0204	0.0311	-0.66	0.51
ς	0.5962	0.0288	20.70	0.00
Production				
ν †	0.5962	0.0288	20.70	0.00
α_d	0.6406	0.0616	10.40	0.00
ϕ	0.3788	0.0333	11.38	0.00
σ	6.00	calibrated		
θ	8.00	calibrated		
ϑ	8.00	calibrated		
Preferences				
γ	0.3561	0.0354	10.06	0.00
β	0.99	calibrated		
η	1.35	calibrated		

† — ν was constrained to equal ς

Table 2: Optimized Taylor Rule Coefficients

	Historical	Base Case	Inflation Stabilization
ϱ_π	1.0223	1.2000	30.0000
ϱ_μ	0.6567	0.0000	—
ϱ_y	-0.0147	0.2000	—
CV*	—	1.3976	0.7322

*: compensating variation in percent

Table 3: Average Values and Standard Deviations

	Deterministic Steady State	Initial Stochastic Steady State	Optimal Stochastic Steady State	Inflation Stabilization
		Averages		
Consumption	0.0805	0.0804	0.0811	0.0812
Hours Worked	0.3230	0.3238	0.3254	0.3265
Real Balances	0.1374	0.1392	0.2478	0.2335
Period Utility	-3.0572	-3.0608	-3.0472	-3.0537
		Standard Deviations		
Consumption	—	0.0025	0.0023	0.0059
Hours Worked	—	0.0092	0.0098	0.0147
Real Balances	—	0.0080	0.0764	0.0856
Period Utility	—	0.0367	0.0495	0.0614
		Compensating Variations (%)		
CV†	0.3716	—	1.3976	0.7322
Level Effect	0.2876	—	2.1865	1.8932
Stabilizaton Effect	0.0843	—	-0.7718	-1.1404

†: compensating variation in percent

Figure 1: Fitted and Actual Values

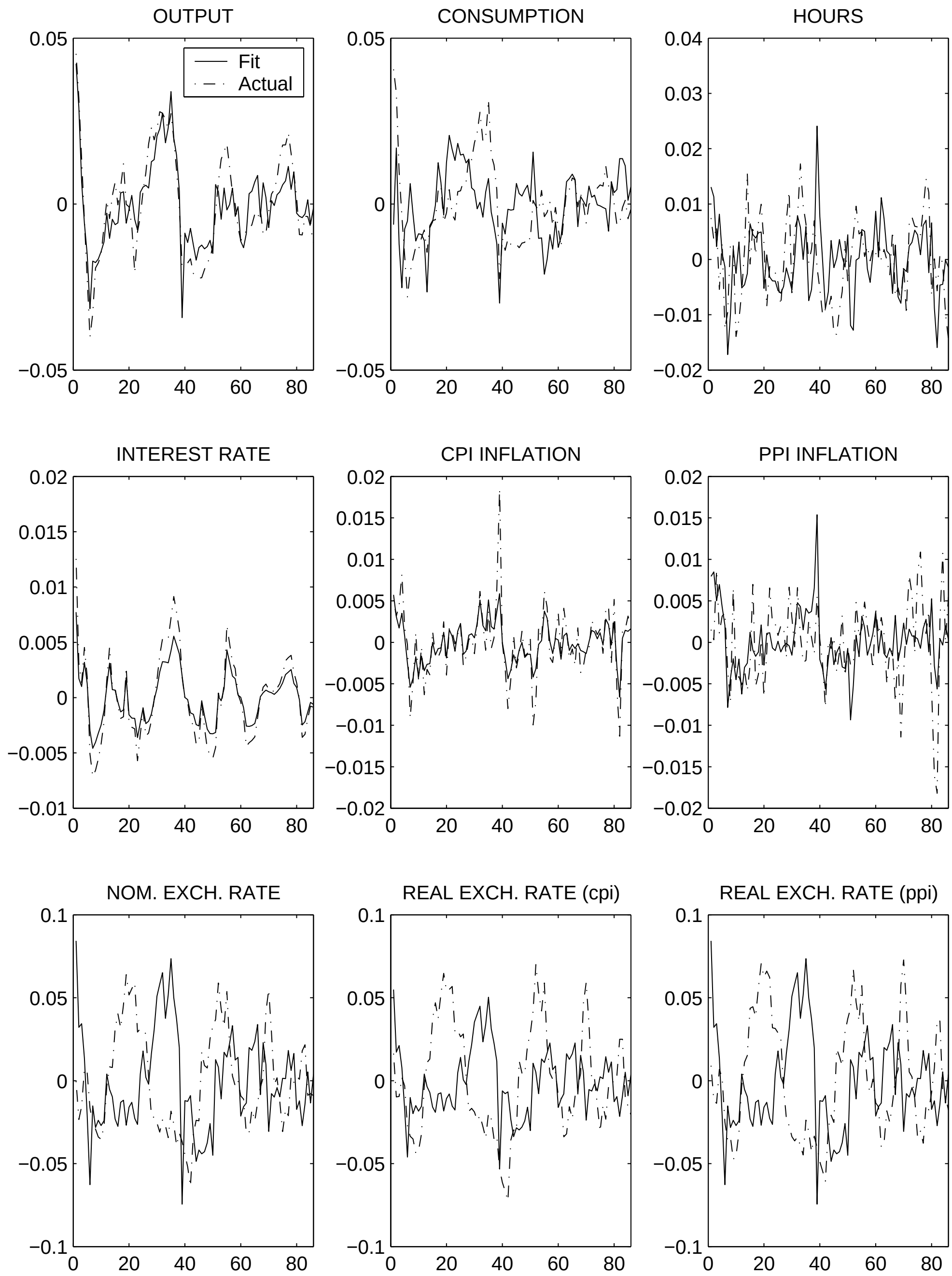

Figure 2: Objective Function with $\varrho_\mu = 0.0$

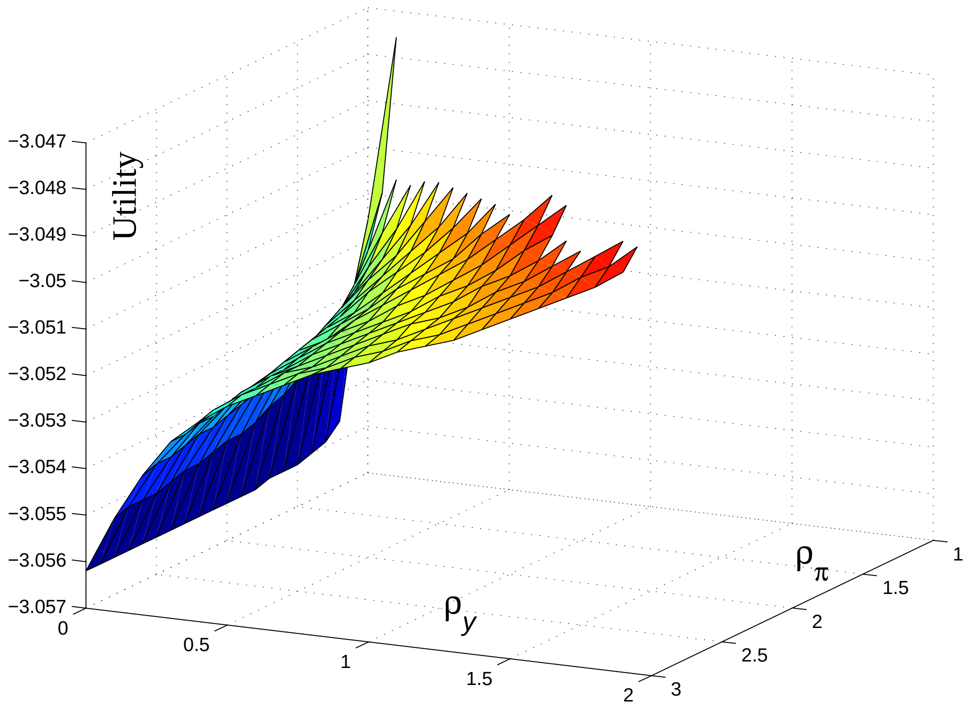

Figure 3: Objective Function with $\varrho_\pi + \varrho_\mu - \varrho_y \geq 1.15$

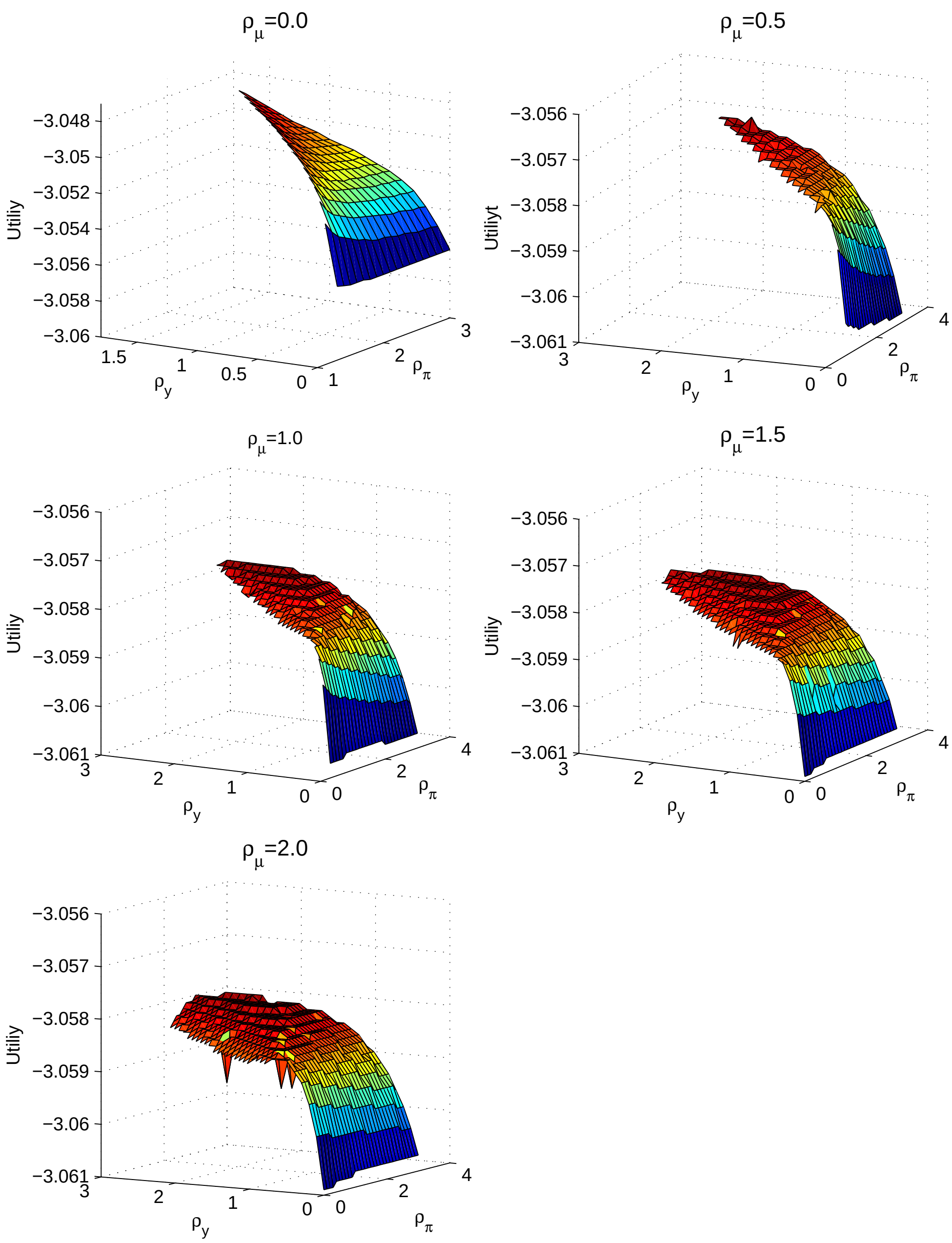

Figure 4: Fitted and Actual Values