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Export Decisions and International Business Cycles *

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ABSTRACT

It has been argued that the export decisions of firms are important for the dynamics of the trade balance (Baldwin and Krugman, 1989). Recent evidence from firm level data, find that exporters are bigger and more productive than non-exporters. These studies also find that the identity of exporting firms changes over time and that fixed entry and participation costs influence firm's decision to enter and exit export markets. This paper develops a model with firm level heterogeneity and exporter dynamics to study the propagation of international business cycles. With this external margin to trade, we find that the properties of international business cycles depend crucially on how consumers value an additional foreign variety. When consumers dislike additional varieties, the model is able to match the high comovement of economic activity across countries and the low comovement of consumption.

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1. Introduction

In an influential paper, Baldwin and Krugman (1989) suggest that the export decisions of firms strongly influence the transmission of business cycles across countries. They argue that in the presence of fixed costs of starting to export, firms will continue to export even when their goods become relatively expensive following an appreciation of their currency. This channel, named *exporter hysteresis*, is conjectured to contribute to the trade deficits apparent slow response to changes in the exchange rate. This idea received considerable attention following the large depreciation of the dollar in the late 80's. More recently, the large depreciation of the dollar and widening trade deficits of the last few years have revived interest in the idea. Despite its appeal, it is not known whether exporter hysteresis is a quantitatively important channel. In this paper, we develop a dynamic stochastic general equilibrium model with fixed costs of international trading to examine the role of export decisions for the international propagation of business cycles.

A number of features of the comovement of business cycles across countries are difficult to explain in theory.[1] In this paper we focus on two of these features. The first is the observation that consumption across countries is less correlated than output across countries. This is contrary to what models with risk sharing predict and is often described as the *consumption correlations puzzle*. The second feature of the data is that both employment and investment are positively correlated across countries. Standard single or multiple good models generate low or negative international comovements of investment and employment as in these models production is shifted across countries to take advantage of better productive opportunities. This observation is referred to as the *international comovements puzzle*. To address these puzzles, and the international propagation of business cycles, we focus on fixed costs of international trading which give rise to firm level dynamics in the export sector.

Our emphasis on the export decisions of firms is motivated in part by recent empirical studies of the characteristic of exporting and non-exporting firms. Two aspects of these studies suggest that the dynamics of the export sector may provide an important channel for the international transmission of business cycles. First, there is evidence that exporting firms are both more productive and much bigger, in terms of employment and capital, than non-exporters. Second, the identity of exporting firms changes substantially over time as firms enter and exit foreign markets. This evidence has been interpreted to imply that firms face fixed costs to export to foreign markets

[1] Baxter (1995) and Obstfeld and Rogoff (2001) provide comprehensive summaries of the puzzles in international macroeconomics.

and that the value of participating in foreign markets varies over time. In this paper, we explore the implications of this churning of the most productive and largest firms in the economy for the international transmission of business cycles.

We construct a dynamic general equilibrium model with firm level export dynamics. Three departures from the standard international real business cycle model are necessary to generate firm level export dynamics. First, we assume that in each country there is a continuum of monopolistically competitive firms producing differentiated intermediate goods. Second, we assume that firms are heterogenous in their technology and are subject to idiosyncratic technology shocks. Third, we assume that there are fixed costs of entering and participating in export markets. These fixed costs are of two types. There are *export penetration* costs which are the large, up-front fees that firms incur to establish a presence in foreign markets. Next, there are *continuation costs* which firms incur to maintain their position in foreign markets. We think of these fees as corresponding to the costs of establishing and maintaining foreign marketing campaigns and distribution channels. These features of the model give rise to characteristics of exporters and non-exporters which match the data. Moreover, by endogenizing the division between traded and non traded goods in the economy, these features introduce an extensive margin into the pattern of trade between countries.

Our main finding are mixed in that they depend on the taste for variety of consumers. With an export decision, the number of import varieties available in a market changes over time. If there are increasing returns to scale in variety so that consumers gain by spreading the same expenditure across as many goods as possible, then we find the dynamics of the export sector do not quantitatively alter the propagation of international business cycles. The model essentially reduces to a version of Backus, Kehoe and Kydland (1995).

However, if consumers would prefer to concentrate consumption in fewer varieties, then we find that the export decisions of firms substantially alter the propagation of business cycles. In this case, we find that the export decisions of firms lead to both lower comovements in consumption and higher comovements in economic activity.

The logic of this result can be best explained by considering the effect of a positive productivity shock in the home country. This shock lowers the cost of all home firms and leads the export sector to expand as its firms seek to new markets for their goods. This leads existing exporters from home to delay exiting and more non-exporters to enter. If consumers dislike additional variety, this expansion in the number of import varieties in foreign acts as a taste shock which shifts foreign demand away from home goods and leads to an increase in demand for home goods. Because the entry costs are sunk and productivity shocks are persistent, the home export sector expansion is persistent so that foreign demand for its own goods is persistently high, creating an incentive to increase foreign investment. This channel increases the comovement of output and inputs across countries and lowers the comovement of consumption across countries.

Many researchers have studied the international transmission of business cycles. Among the papers that focus on real factors as the source of fluctuations, our approach is most closely related to Backus, Kehoe and Kydland (1992), Stockman and Tesar (1995), Obstfeld and Rogoff (2000) and Betts and Kehoe (2002). To different degrees, these papers investigate the role of frictions in the goods market for the propagation of business cycles[2]. Our approach here differs in that we concentrate on the role of fixed international trading costs on the export decisions of heterogenous firms. Previous work by Baldwin (1988), Baldwin and Krugman (1989) and Dixit (1989a,b) studies the role of sunk entry costs on the export behavior of firms in partial equilibrium environments. These papers were developed to study export hysteresis following the swing in the value of the dollar in the 80's. Recent work by Bernard, Eaton, Jensen and Kortum (2000) and Melitz (2003) also consider the role of firm heterogeneity in an international context. These papers focus on the pattern of trade and welfare gains from trade liberalization and do not consider aggregate fluctuations. Finally, recent work by Evenett and Venables (2002), Hummels and Klenow (2002) and Kehoe and Ruhl (2002) study the growth in trade through the intensive and extensive margins.

The paper is organized as follows. The next section discusses the evidence of exporter characteristics in the data. Section 3 develops a two country dynamic general equilibrium model with export penetration and continuation costs. Section 4 discusses the quantitative implications of the model. Section 5 explores the sensitivity of the model to the structure of asset trade, aggregate shocks, firm-level shocks, and preferences. Section 6 concludes. All tables and figures are at the end of the paper.

2. Exporter characteristics and dynamics

We begin by summarizing some recent evidence about the characteristics of exporters. This evidence is drawn from plant and firm-level data in several countries and documents a number of margins along which exporters differ from non-exporters. Among other dimensions, exporters tend to have higher productivity, higher levels of output, and use more capital and labor inputs. Using the

[2] Another approach has focused on frictions in international asset markets (see Baxter and Crucini (1995), Heathcoate and Perri (2001) and Kehoe and Perri (2002)).

Longitudinal Research Database (LRD) of the Bureau of Census from 1984 to 1992 data, Bernard and Jensen (1999) find that U.S. exporters are 12 percent to 18 percent more productive, employ 77 percent to 95 percent more workers, use 13 percent to 20 percent more capital per worker and produce 104 percent to 115 percent more output than non-exporters.[3]

Second, there is substantial entry and exit into foreign markets as many firms that export in one period stop exporting in the next period and vice versa. Using a balanced panel of firms continuously present in the U.S. Census Bureau's Annual Survey of Manufacturers (ASM) from 1984 to 1992, Bernard and Jensen (2001) show that the fraction of last period's non-exporters that transition to exporting, which we term the *starter* ratio, is about 13.9 percent per year. The fraction of last period's exporters that transition to non-exporter status, which we term the *stopper* ratio, is about 12.6 percent on average. The starter and stopper ratios change over time leading to fluctuations in the ratio of exporters among all firms, which we term the *exporter* ratio. Since the starter ratio is slightly higher than stopper ratio, the exporter ratio exceeds 50 percent. During this period, the exporter ratio in the LRD increases from 47.6 percent of the firms in 1984 to 54.04 percent in 1992.[4] Bernard and Jensen (2001) attribute much of the growth in exporter ratio to a decline in the stopper ratio.

German and Colombian manufacturing data also show significant turnover in export status. Using annual manufacturing plant-level data from 1978 to 1992 in Lower Saxony, Germany, Bernard and Wagner (1998) find that on average, starter and stopper ratios are about 4.14% and 5.51%, respectively. In this region, the exporter ratio is only about 41.2% on average. Roberts and Tybout (1997) use annual data of Colombian manufacturing Census data from 1981 to 1989 to find that on average, the starter and stopper ratios are about 3.3% and 11.5%, respectively. The exporter ratio is about 11.8% on average. These findings are reproduced in Table 1.

The evidence about exporter characteristics and dynamics has been interpreted to imply that there are significant export penetration and continuations costs to international trade which influence the exporting decisions of firms.[5] These fixed costs are attributed to higher foreign marketing

[3]The LRD is not a representative sample of manufacturing firms but is biased towards larger firms. Consequently, this database tends to understate the differences between exporters and non-exporters compared to the Census of Manufactures.

[4]Since the starter and stopper ratios can be considered transition probabilities in export status, a higher starter ratio than stopper ratio does not necessarily mean that the exporter ratio is increasing all the time. If the starter and stopper ratios are n_0 and n_1 all the time, the exporter ratio converges to $\frac{n_0}{n_0+n_1}$.

[5]Roberts and Tybout (1997), Bernard and Wagner (1998), Aw, Chung an Roberts (1998), Bernard and Jensen (1999) and Das, Roberts and Tybout (2001) use Colombian, German, Taiwan, and Korean plant or firm-level manufacturing data, respectively. They commonly find evidence of large fixed export costs.

and distribution costs, additional bureaucratic procedures, and required changes in product characteristics to tailor products to the tastes of foreign consumers and to satisfy government regulations. Using annual firm level data on Colombian chemical producers from 1982 to 1991, Das, Roberts and Tybout (2001) estimate that export penetration costs account for between 18.4 percent and 41.2 percent of the annual value of a firms exports. In 1999 U.S. dollars, these costs are estimated to be between \$730,000 and \$1.6 million, depending on plant size. The continuation costs are considerably smaller and are estimated to be about 1 percent of the annual value of exports. These findings suggest that fixed costs of entry and continuation matter for the export decision of firms. This is a feature which is often overlooked in the previous literature on international business cycle.

3. Model

We develop a two-country model with infinitely lived consumers and heterogeneous firms to study the international transmission of business cycles. The production side of the model is developed to be consistent with the characteristics and dynamics of exporters described in the previous section. This requires taking a stand on what determines a firm. Our view is that a firm is associated with a unique variety of differentiated good with a production process that is subject to idiosyncratic technology shocks.

There are two countries, *home* and *foreign*. Each country is populated by a large number of identically, infinitely lived consumers. In each period of time, the economy experiences an event s_t. Let $s^t = (s_0, \cdots, s_t)$ denote the history of events from period 0 up to and including period t. The probability of a history s^t, conditional on the information available at period 0, is defined as $\pi(s^t|s^0)$. The initial realization of an event at period 0, s_0, is given.[6]

In each country there is a large number of monopolistically competitive firms each producing a differentiated intermediate good. The many intermediate good producers are normalized to a continuum with unit mass and are indexed $i \in [0, 1]$. An intermediate good producer uses capital and labor inputs to produce its variety of intermediate input. Firms differ in terms of total factor productivity, capital and the markets they serve. All firms sell their product in their own country but only some firms export their good abroad. When an intermediate good producer exports goods abroad, the producer incurs some international trading cost. The size of the cost depends on the producer's export status in the previous period. There is a (relatively) high initial entry cost τ_0 that must be borne to gain entry into the export market. In subsequent periods, to continue exporting

[6] The history of events, s^t, maps onto the state of the world.

firms incur a a lower but nonzero period-by-period fixed continuation cost τ_1. If a firm does not pay this continuation cost, then it ceases to export. In future periods, the firm can only begin exporting by incurring the entry cost τ_0 again.

In each country, competitive final goods producers purchase intermediate inputs from those firms actively selling in that country.[7] The cost of exporting implies that the set of goods available to competitive final goods producers differs across countries. The entry and exit of exporting firms implies that the set of intermediate goods available in a country is changing over time. The final goods are used for both domestic consumption or investment.

We assume that there are no economies of scale to exporting. In particular, it is not possible for a single firm to incur the fixed cost of exporting and then export multiple different varieties of intermediate goods. We take the view that τ_0 represents the cost of entering the foreign market per variety. In practice, these fixed costs represent those costs associated with tailoring a product to the standards and taste of foreign agents, establishing marketing and distribution networks, and learning about bureaucratic and administrative details in foreign markets. For diverse goods, it is unlikely exporting in one good reduces the fixed costs of exporting a second good.

In this economy, there exists a complete set of one period state contingent nominal bonds denominated in the home currency. Let $B\left(s^{t+1}, s^t\right)$ denote the home consumers holdings of a bond purchased in state s^t with payoff in state s^{t+1}. Let $B^*\left(s^{t+1}, s^t\right)$ denote the foreign consumers holding of this bond. The state contingent bond $B(s^t)$ pays 1 unit of home currency if s^t occurs, and 0 otherwise. Let $Q(s^{t+1}|s^t)$ denote the nominal price of the state contingent bond $B(s^{t+1})$ given s^t. All the intermediate and final good producers are owned by domestic consumers. It is assumed that these ownership claims can not be traded.

A. Consumers Problem

Home consumers choose consumption, labor and bond holdings to maximize their utility:

$$\max \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi\left(s^t|s_0\right) U\left[C\left(s^t\right), L\left(s^t\right)\right],$$

[7] Final good production technology does not require capital or labor inputs. The final good production technology regulates a country's preferences over local and imported varieties.

subject to the sequence of budget constraints,

$$P(s^t)C(s^t) + \sum_{s^{t+1}} Q(s^{t+1}|s^t)B(s^{t+1}) \leq P(s^t)W(s^t)L(s^t) + B(s^t) + \Pi(s^t) + P(s^t)T(s^t),$$

where $C(s^t)$ and $L\left(s^t\right)$ are the final good consumption and labor, respectively; $P(s^t)$ and $W\left(s^t\right)$ denote the price level and wage rate; $\Pi(s^t)$ is the sum of profits of the home country's intermediate good producers; and $T(s^t)$ is a lump sum transfer from the government. The discount factor is β.

The problem of foreign consumers is analogous to this problem. Prices and allocations in the foreign country are represented with an asterisk. To be clear, money has no role in this economy. However, we do use the local currency as a unit of account so that the foreign budget constraint is expressed as

$$P^*(s^t)C^*(s^t) + \sum_{s^{t+1}} \frac{Q(s^{t+1}|s^t)}{e(s^t)} B^*(s^{t+1}) \leq P^*(s^t)W^*(s^t)L^*(s^t) + \frac{B^*(s^t)}{e(s^t)} + \Pi^*(s^t) + P^*(s^t)T^*(s^t),$$

where * denotes the foreign variables and $e(s^t)$ is the nominal exchange rate.

The first order conditions for home consumers' utility maximization problems are

$$-\frac{U_L(s^t)}{U_C(s^t)} = W(s^t),$$

$$Q(s^{t+1}|s^t) = \beta\pi(s^{t+1}|s^t)\frac{U_C(s^{t+1})}{U_C(s^t)}\frac{P(s^t)}{P(s^{t+1})}, \tag{1}$$

where $U_c(s^t)$, and $U_l(s^t)$ denote the derivatives of the utility function with respect to its arguments. The price of the state contingent bond is standard. With arbitrage, the complete asset markets assumption implies that the real exchange rate, $q(s^t)$, is proportional to the ratio of marginal utility of consumption at foreign to that at home or

$$q(s^t) = \frac{e(s^t)P^*(s^t)}{P(s^t)} = \kappa\frac{U_C^*(s^t)}{U_C(s^t)}, \tag{2}$$

where $\kappa = q(s^0)U_C(s^0)/U_C^*(s^0)$.[8]

B. Final Good Producers

In the home country, final goods are produced using only home and foreign intermediate goods. A final good producer can purchase from any of the home intermediate good producers, but

[8] In the simulation exercises, κ is normalized to be 1.

can only purchase from those foreign intermediate good producers that are actively selling in the home market. The set of foreign firms actively selling in the home country is denoted by $\mathcal{E}^*(s^t)$, where $i \in \mathcal{E}^*(s^t)$ if the i_{th} firm is a foreign exporter in s^t.

The production technology of the firm is given by a constant elasticity of substitution (henceforth CES) function

$$(3) \quad D(s^t) = \left\{ a_1 \left[\int_0^1 y_h^d(i, s^t)^\theta di \right]^{\frac{\rho}{\theta}} + (1 - a_1) \left[N^{*-\lambda} \left(s^t \right) \int_0^1 y_f^d(i, s^t)^\theta di \right]^{\frac{\rho}{\theta}} \right\}^{\frac{1}{\rho}},$$

where $D(s^t)$ is the output of final goods and $y_h^d(i, s^t)$ and $y_f^d(i, s^t)$ are inputs of intermediate goods purchased from home firm i and foreign firm i, respectively. The parameter a_1 determines the weight of home goods in final good consumption. The elasticity of substitution between intermediate goods that are produced in the same country is $1/(1 - \theta)$, and the elasticity of substitution between home and foreign aggregate inputs is $1/(1 - \rho)$.

With the export margin of the model, the measure of foreign varieties used in production of the composite foreign good changes over time. With a typical Dixit-Stiglitz aggregator there is a benefit to using smaller amounts of a greater number of varieties. To counteract this love of variety effect, we modify the aggregator of the foreign composite by introducing the additional term $N^{*-\lambda}$. Introducing this terms permits us to separate the love of variety effect from the degree of market power, which is related to elasticity of substitution between individual varieties (Benassy, 1996).

The final goods market is competitive. In each period t, given the final good price at home $P(s^t)$, the i_{th} home intermediate good price at home $P_h(i, s^t)$ for $i \in [0, 1]$, and the i_{th} foreign intermediate good price at home $P_f(i, s^t)$ for $i \in \mathcal{E}^*(s^t)$, a home final good producer chooses inputs $y_h^d(i, s^t)$ for $i \in [0, 1]$, and $y_f^d(i, s^t)$ for $i \in \mathcal{E}^*(s^t)$ to maximize profits,

$$(4) \quad \max P\left(s^t\right) D\left(s^t\right) - \int_0^1 P_h\left(i, s^t\right) y_h^d\left(i, s^t\right) di - \int_0^1 P_f\left(i, s^t\right) y_f^d\left(i, s^t\right) di,$$

subject to the production technology (3) and the constraint that $y_f^d\left(i, s^t\right) = 0$ for $i \notin \mathcal{E}^*(s^t)$. Solving the problem in (4) gives the input demand functions,

$$(5) \quad y_h^d(i, s^t) = a_1^{\frac{1}{1-\rho}} \left[\frac{P_h(i, s^t)}{P_h(s^t)} \right]^{\frac{1}{\theta-1}} \left[\frac{P_h(s^t)}{P(s^t)} \right]^{\frac{1}{\rho-1}} D(s^t),$$

$$(6) \quad y_f^d(i, s^t) = (1 - a_1)^{\frac{1}{1-\rho}} N^*\left(s^t\right) \left[\frac{P_f(i, s^t)}{P_f(s^t)} \right]^{\frac{1}{\theta-1}} \left[\frac{P_f(s^t)}{P(s^t)} \right]^{\frac{1}{\rho-1}} D(s^t), i \in \mathcal{E}^*(s^t)$$

where $P_h(s^t) = \left[\int_0^1 P_h(i,s^t)^{\frac{\theta}{\theta-1}} di\right]^{\frac{\theta-1}{\theta}}$, and $P_f(s^t) = \left[N^*\left(s^t\right)^{\frac{-\lambda}{1-\theta}} \int_{i\in\mathcal{E}^*(s^t)} P_f(i,s^t)^{\frac{\theta}{\theta-1}} di\right]^{\frac{\theta-1}{\theta}}$. The zero-profit condition in the perfectly competitive market determines the price level of the final good as

$$P(s^t) = \left[a_1^{\frac{1}{1-\rho}} P_h(s^t)^{\frac{\rho}{\rho-1}} + (1-a_1)^{\frac{1}{1-\rho}} P_f(s^t)^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-1}{\rho}}.$$

C. International Trading Costs

An intermediate good producer can sell its product without frictions in the domestic market. It is costly, however, to sell its product abroad. Producers that export to the foreign country face two sets of international trading costs. To enter the foreign market, an intermediate good producer has to pay a (relatively) high initial entry costs τ_0. From the following period on, to continue exporting the producer has to pay a lower but nonzero continuation costs τ_1 ($< \tau_0$). The export penetration costs (τ_0) and continuation costs (τ_1) are collected from foreign exporting firms by the domestic government, and distributed lump-sum to the domestic consumers. The government's budget constraint is given by

$$(7) \quad T(s^t) = \int_{i\in\mathcal{E}^*(s^t)} \left\{[1-m^*(i,s^{t-1})]\tau_0 + m^*(i,s^{t-1})\tau_1\right\} di,$$

where $m^*(i,s^t)$ is an indicator function denoting the export status of the i_{th} intermediate good producer in s^t. Let $m^*(i,s^t) = 1$ if the i_{th} foreign intermediate good producer is an exporter in s^t, 0 otherwise.[9] These trade costs imply that only a fraction

$$N^*(s^t) = \int_0^1 m^*(i,s^t) di$$

of foreign intermediate goods are available to home final good producers in state s^t.

D. Intermediate Goods Producers

In each country there is a large number of intermediate good producers normalized to a continuum with unit mass indexed $i \in [0,1]$ who behave as monopolistic competitors. An intermediate

[9] In reality, most of the costs are paid to agents that help exporting firms, not to the foreign government. However, no matter to whom the costs are paid initially, the payment goes to consumers ultimately. Additionally, in practice some of these costs are also paid to domestic agents. The results of the simulation exercises are not sensitive to the division of these costs across countries. To make matters simple, it is assumed that the costs are paid to the foreign government.

good firm produces its differentiated good with a Cobb-Douglas production technology,

$$(8) \quad F(i, s^t) = A(i, s^t) k(i, s^{t-1})^{\alpha} l(i, s^t)^{1-\alpha} = y_h^d(i, s^t) + y_h^{*d}(i, s^t),$$

where $y_h(i, s^t)$ and $y_h^*(i, s^t)$ are the amounts of good i sold in the home and foreign intermediate goods markets, respectively and $k(i, s^{t-1})$ and $l(i, s^t)$ are the capital and labor inputs of the firm i. Capital used in production is augmented by investment of final goods, $x(i, s^t)$. The law of motion for capital is given by

$$(9) \quad k(i, s^t) = (1 - \delta) k(i, s^{t-1}) + x(i, s^t),$$

where δ is the depreciation rate.

The term $A(i, s^t)$ denotes the productivity of the i_{th} firm and is composed of a country-wide component $z(s^t)$, and a firm-specific component $\eta(i, s^t)$ such that

$$\ln A(i, s^t) = z(s^t) + \eta(i, s^t).$$

The country-wide component $z(s^t)$ may be correlated across countries and evolves according to a vector autoregressive process (VAR) with the foreign country-wide productivity, $z^*(s^t)$,

$$Z(s^t) = M Z(s^{t-1}) + \nu(s^t), \nu(s^t) \overset{iid}{\sim} N(0, \Omega),$$

where M is a coefficient matrix; $Z(s^t) = [z(s^t), z^*(s^t)]'$ and $\nu(s^t) = [\epsilon(s^t), \epsilon^*(s^t)]'$. The firm-specific productivity is independently, identically distributed across countries, firms, and time, $\eta(i, s^t) \overset{iid}{\sim} N(0, \sigma_\eta^2)$.

Consider the problem of an intermediate good producer from the home country in state s^t. The state of an individual firm is summarized by the triple (η, k, m), where we temporarily drop the firm index and aggregate state. The intermediate good producer chooses current prices P, P^*, inputs of labor l, investment x and the export decision m' to solve

$$V\left(\eta,k,m,s^{t}\right)=\max\Pi(h,i;s^{t})+m_{t}\Pi^{*}(h,i;s^{t})+\sum_{s^{t+1}}\sum_{\eta'}Q\left(s^{t+1}|s^{t}\right)\Pr\left(\eta'\right)V\left(\eta',k',m',s^{t+1}\right),$$
$$\Pi(h,i;s^{t})=P_{h}(i,s^{t})y_{h}(i,s^{t})-P(s^{t})W(s^{t})l(i,s^{t})-P(s^{t})x(i,s^{t})$$
$$\Pi^{*}(h,i,s^{t})=e(s^{t})\left[P_{h}^{*}(i,s^{t})y_{h}^{*}(i,s^{t})-P^{*}(s^{t})[m\tau_{1}+(1-m)\tau_{0}]\right],$$

subject to the production technology (8), the law of motion for capital (9), and the constraints that supplies to home and foreign intermediate goods market $y_h(i,s^t)$ and $y_h^*(i,s^t)$ are equal to demands by home and foreign final good producers $y^d(h,i,s^t)$ and $y^{d*}(h,i,s^t)$ from (5) and its foreign analogue. Here, $\Pr(\eta')$ denotes the probability of an idiosyncratic shock η'. $Q(s^t)$ is the price of one unit of home currency in s^t in an abstract unit of account. Since firms are owned by domestic consumers, $Q(s^{t+1})/Q(s^t)$ should be the same as the intertemporal marginal rate of substitution. From the bond price condition (1), $Q(s^{t+1})/Q(s^t)=Q(s^{t+1}|s^t)$.

Let the value of the i_{th} producer if it exports in s^t be

$$V^{1}\left(\eta,k,m,s^{t}\right)=\max\Pi_{h}(i;s^{t})+\Pi_{h}^{*}(i;s^{t})+\sum_{s^{t+1}}\sum_{\eta'}Q\left(s^{t+1}|s^{t}\right)\Pr\left(\eta'\right)V\left(\eta',k',1,s^{t+1}\right),$$
$$\Pi_{h}(i;s^{t})=P_{h}(i,s^{t})y_{h}(i,s^{t})-P(s^{t})w(s^{t})l(i,s^{t})-P(s^{t})x(i,s^{t})$$
$$\Pi_{h}^{*}(i,s^{t})=e(s^{t})P_{h}^{*}(i,s^{t})y_{h}^{*}(i,s^{t})-e(s^{t})P^{*}(s^{t})[m\tau_{1}+(1-m)\tau_{0}],$$

and the value of the i_{th} producer if it does not export in s^t be

$$V^{0}\left(\eta,k,m,s^{t}\right)=\max\Pi_{h}(i;s^{t})+\sum_{s^{t+1}}\sum_{\eta'}Q\left(s^{t+1}|s^{t}\right)\Pr\left(\eta'\right)V\left(\eta',k',0,s^{t+1}\right),$$
$$\Pi_{h}(i;s^{t})=P_{h}(i,s^{t})y_{h}(i,s^{t})-P(s^{t})w(s^{t})l(i,s^{t})-P(s^{t})x(i,s^{t})$$

Then, the actual value of i_{th} producer can be defined as

$$V\left(\eta,k,m,s^{t}\right)=\max\left\{V^{1}\left(\eta,k,m,s^{t}\right),V^{0}\left(\eta,k,m,s^{t}\right)\right\}.$$

Clearly the value of a producer depends on its export status and are monotonically increasing and continuous in η, and V^1 intersects V^0 from below only once.[10] Hence, it is possible to solve for the

[10] If the difference between τ_0 and τ_1 is very large, $V^1 > V^0$ for all $\eta \in (-\infty,\infty)$ for some s^t. Since the data show

firm-specific productivity at which a firm is indifferent between exporting or not exporting. This level of technology differs by the firms current export status. The critical level of technology for exporters and non-exporters, η_1 and η_0, satisfy

$$(10)\quad V^1\left(\eta_1, k, 1, s^t\right) = V^0\left(\eta_1, k, 1, s^t\right),$$

$$(11)\quad V^1\left(\eta_0, k, 0, s^t\right) = V^0\left(\eta_0, k, 0, s^t\right),$$

In general these critical technology levels will differ across firms based on their capital level. However, the assumption that firm specific technology shocks are iid implies that each firm expects to draw the same level of technology tomorrow. Consequently, a firm's current capital stock is entirely determined by its export status in the previous period. As export status is a zero-one choice, the distribution of capital over firms is characterized by two mass points. This then implies that the critical technology level of an exporting firm also determines the technology of the marginal exporting firm, which we denote by $\eta_1\left(s^t\right)$. Among last period exporters, only those with a firm-specific productivity greater than $\eta_1(s^t)$ will continue to export in state s^t. Likewise, the critical technology of a non-exporter is denoted by $\eta_0\left(s^t\right)$.

From (10), (11), and the independence of the firm-specific productivity, the percentage of exporters in s^t among exporters and non-exporters in s^{t-1}, $n_1(s^t)$ and $n_0(s^t)$, respectively, can be defined as

$$n_1(s^t) = \Pr[\eta > \eta_1(s^t)],$$

$$n_0(s^t) = \Pr[\eta > \eta_0(s^t)].$$

Then, the law of motion for the export ratio among intermediate good producers, $N(s^t)$, is

$$(12)\quad N(s^t) = n_1(s^t)N(s^{t-1}) + n_0(s^t)[1 - N(s^{t-1})].$$

Figure 1 illustrates the values of firms across firm-specific productivity depending on export status. In the absence of trade costs, the value of a firm that exports always exceeds the value of not exporting for all firm-specific productivity. This is true because by exporting the firm has a larger market for its goods so that without the fixed costs all firms export their good abroad.

that some of the previous exporters exit from foreign markets each period, it is assumed through out that the shocks are small enough that this does not occur.

However, in the presence of international trade costs, it is not optimal for some firms to export goods abroad. The value of an exporting firms is reduced by the amount of the trade costs, τ_0 or τ_1 depending on the export status last period. Since the cost of being a new exporter exceeds the cost of continuing to export, $\tau_0 > \tau_1$, the value of being a new exporter is always lower than the value of being a continuing exporter. This implies that $\eta_1(s^t) < \eta_0(s^t)$ for all s^t. Hence, the probability of being an exporter in s^t is always higher for last period exporters than last period non-exporters $(n_1(s^t) > n_0(s^t))$.

E. Equilibrium Definition

In an equilibrium, variables satisfy several resource constraints. The final goods market clearing conditions are given by $c(s^t) + \int x(i, s^t)di = D(s^t)$, and $c^*(s^t) + \int x^*(i, s^t)di = D^*(s^t)$. The intermediate goods market clearing conditions are $y_h^d(i, s^t) = y_h(i, s^t)$ for $i \in [0, 1]$, $y_f^d(i, s^t) = y_f(i, s^t)$ for $i \in \mathcal{E}^*(s^t)$, $y_f^{d*}(i, s^t) = y_f^*(i, s^t)$ for $i \in [0, 1]$, and $y_h^{d*}(i, s^t) = y_h^*(i, s^t)$ for $i \in \mathcal{E}(s^t)$. The labor market clearing conditions are $L(s^t) = \int_0^1 l(i, s^t)di$, and $L^*(s^t) = \int_0^1 l^*(i, s^t)di$. The profits of firms are distributed to the shareholders, $\Pi(s^t) = \int_0^1 \Pi_h(i, s^t) + \Pi_h^*(i, s^t)di$, and $\Pi^*(s^t) = \int_0^1 \Pi_f(i, s^t) + \Pi_f^*(i, s^t)di$. The government budget constraint is given by (7) and the foreign analogue. The international bond market clearing condition is given by $B(s^t) + B^*(s^t) = 0$. Finally, our decision to write the budget constraints in each country in units of the local currency permits us to normalize the price of consumption in each country as $P(s^t) = P^*(s^t) = 1$.

An equilibrium of the economy is a collection of allocations for home consumers $C(s^t)$, $L(s^t)$, $B(s^{t+1})$; allocations for foreign consumers $C^*(s^t)$, $L^*(s^t)$, $B^*(s^{t+1})$; allocations for home final goods producers $D(s^t)$, $y^d(h, i, s^t)$ for $i \in [0, 1]$, and $y^d(f, i, s^t)$ for $i \in \mathcal{E}^*(s^t)$; allocations for foreign final good producers $D^*(s^t)$, $y_f^{d*}(i, s^t)$ for $i \in [0, 1]$, and $y_h^{d*}(i, s^t)$ for $i \in \mathcal{E}(s^t)$; allocations and prices for home intermediate good producers $l(i, s^t)$, $x(i, s^t)$, $y_h(i, s^t)$, and $P_h(i, s^t)$ for $i \in [0, 1]$, $y_h^*(i, s^t)$ and $P_h^*(i, s^t)$ for $i \in \mathcal{E}(s^t)$; allocations and prices for foreign intermediate good producers $l^*(i, s^t)$, $x^*(i, s^t)$, $y_f(i, s^t)$ and $P_f(i, s^t)$ for $i \in \mathcal{E}^*(s^t)$, $y_f^*(i, s^t)$ and $P_f^*(i, s^t)$ for $i \in [0, 1]$; the export statuses of home and foreign intermediate good producers $m(i, s^t)$ and $m^*(i, s^t)$ for $i \in [0, 1]$; transfers $T\left(s^t\right), T^*\left(s^t\right)$ by home and foreign governments; real wages $W(s^t), W^*(s^t)$, real and nominal exchange rates $q\left(s^t\right)$ and $e\left(s^t\right)$; and bond prices $Q(s^{t+1}|s^t)$ that satisfy the following conditions: (i) the consumer allocations solve the consumer's problem; (ii) the final good producers' allocations solve their profit maximization problems; (iii) the intermediate good producers' allocations, prices, and export statuses solve their profit maximization problems; (iv) the market clearing conditions hold; and (v) the transfers satisfy the government budget constraint.

In what follows we concentrate on a stationary equilibrium. A stationary equilibrium consists of stationary decision rules and pricing rules that are functions of the state of the economy. The state of the economy is completely described by the distribution of the state variables (η, k, m) for all individual firms in both countries and the aggregate technology shocks. The state of the economy when firms make their export decisions must record the joint distribution of the capital stock, technology and export status of firms in both countries. In general, keeping track of this distribution over time is computationally difficult. However, the assumption that firm specific technology shocks are iid greatly simplifies the analysis as it implies that last period's export status is sufficient to determine a firm's current capital stock. As firms are either exporters or non-exporters, at any point in time firms will have either a relatively low capital stock if they did not export yesterday or a relatively high capital stock if they did export yesterday. Consequently, the distribution of the capital stock in the economy is completely summarized by the aggregate shocks, Z and Z^*,the capital stock of exporters, K_1 and K_1^*, the capital stock of non-exporters, K_0 and K_0^*, and the share of exporters in each country, N and N^*.

F. Calibration

We now describe the functional forms and parameters values considered for our benchmark economy. The parameter values used in the simulation exercises are reported in Table 2. The instantaneous utility function is given as

$$U(C, L) = \frac{[C^\gamma (1-L)^{1-\gamma}]^{1-\sigma}}{1-\sigma},$$

where $1/\sigma$ is the inter-temporal elasticity of substitution, and γ is the share parameter for consumption in the composite commodity.

In the steady state, the real interest rate is equal to $(1-\beta)/\beta$. The annual real return to capital is around 4%. This gives $\beta = 0.99$. The steady state constraint gives $Y = C + \delta K$. Dividing both sides by K, $\delta = \frac{Y}{K}\left(1 - \frac{C}{Y}\right)$. With the annual capital output ratio of 2.5 and consumption to output ratio of 0.75 as the average of the post-war U.S. data, $\delta = 0.025$. The curvature parameter, σ, determines the inter-temporal elasticity of substitution and the relative risk aversion of consumers. Empirical studies using the U.S. time series, such as that of Eichenbaum et al. (1988), suggest that σ lies between 0.5 and 3. We consider a value of $\sigma = 2$ as this is widely used in the international business cycle literature, e.g., Backus et al. (1992), Stockman and Tesar (1995), Kollman (1997), and Kehoe and Perri (2002).

The parameter θ determines an intermediate good producer's markup. Schmitt-Grohe (1997) summarizes the results of empirical studies estimating this markup. These estimates vary widely from 3% to 70%. Based on Basu and Fernald (1994), θ is set to be equal to 0.9 and yields an intermediate good producer's markup of about 11%. The parameter ρ determines the elasticity of substitution between home and foreign aggregates, $1/(1-\rho)$. There is considerable disagreement over an appropriate value. Using the U.S. quarterly data of 163 industries at the 3-digit SIC level from 1980:1 to 1988:4, Gallaway, et al. (2000) estimate that the elasticities range from 0.14 to 3.49. Reinert and Roland-Holst (1992) estimate the elasticities using the U.S. monthly data of 309 industries at the 4-digit SIC level from 1989:1-1995:12. Their estimates range from 0.52 to 4.83. In the simulation exercises, ρ is set to 1/3 so that the elasticity equals to 1.5 as in Backus et al. (1992) and Chari et al. (2001).

The parameter λ determines the love of variety. To our knowledge, there are no empirical estimates of this parameter. Consequently, we follow the literature which implicitly assumes that the love of variety is tied to the elasticity of substitution across varieties and set $\lambda = 0$. In this case, consumers have a preference for spreading consumption across more varieties. We examine two other cases for $\lambda \in \{1-\theta, 1\}$. When $\lambda = 1-\theta$, then there are constant returns to scale as consumers are indifferent between consuming n units of a single good or 1 unit of n identical goods. When $\lambda = 1$, consumers dislike variety and would rather concentrate all of their consumption in a single variety.

In the model, we assume that profit income is attributed proportionally to labor and capital. We choose capital's share of income from post-war U.S. data to be $\alpha = 0.36$. The share parameter for consumption in the composite commodity, γ, is set to be equal to 0.294. This value is obtained from the observation that the average time devoted to work is 1/4 of the total available time, and the consumption-output ratio is about 0.75 in the post-war period.

The parameters τ_0, τ_1, a_1 and σ_η jointly determine the amount of trade, characteristics of exporters and non exporters, and the dynamics of export status.[11] To pin these parameters down,

[11] An alternate approach to calibrate the firm shocks is to use previous estimate for the firm-specific productivity process. However, these studies tend to rely heavily on the sample of firms. With very small firms in the sample, the variance becomes very large. With only large firms, such as firms that can be found in S&P 500, the size of the variance becomes very small. Bernard, et al. (2003) estimate the distribution across plants of value added per workers using ASM 1992. They find that the sample standard deviation of the productivity across firms is about 0.76. However, their estimate differs due to the differences in production functions and the processes of technology shocks. For the robustness of the simulation results, various values of the standard deviation for the firm-specific productivity are considered.

we consider the following evidence. First, as in Table 1, using annual data Bernard and Jensen (1999) find that about 87.8 percent of exporters continue exporting in the next period, and among those that did not export last period, about 85.6 percent of firms remain in the non-exporter status. Consequently, we set $n_1 = n_0 = 3.5\%$ to match an average of the quarterly starter and stopper ratios from Bernard and Jensen (1999). Second, Bernard and Jensen (1999) find that exporters are 12 percent to 18 percent more productive than non-exporters. Finally, we note that for the U.S., the import to output ratio is approximately 15 percent. Choosing these parameters jointly, to match these statistics yields values of $\tau_0 = 0.24897, \tau_1 = 0.05043, a_2 = 0.321$ and $\sigma_\eta = 0.5$.[12] The choice of $\sigma_\eta = 0.5$ is made as it leads exporters to be 15.5 percent more productive and to ship 90.2 percent more output (and hire 90.2 percent more workers). The characteristics of exporters in terms of employment, and output matches up well with the data as exporters produce 104 to 115 percent more output than non-exporters and hire 77 percent to 95 percent more workers. With these parameter values, on average, a non-exporter expects to pays about 16.5 percent of sales as entry costs, while an exporter expects to pay about 1.7 percent of sales to remain in the foreign market in the steady state. In total, these international trading costs represent 1.3 percent of gdp, or about 8 percent of exports.

There are several sets of parameters values for the country specific productivity process that are used in the literature. The benchmark model follows the parameter values in Kehoe and Perri (2002)

$$\mathbf{M} = \begin{bmatrix} .95 & 0 \\ 0 & .95 \end{bmatrix}$$

with $Var[\epsilon(s^t)] = Var[\epsilon^*(s^t)] = \sigma_\epsilon^2 = 0.007^2$, and $Corr[\epsilon(s^t), \epsilon^*(s^t)] = 0.25$. The model is simulated for 1000 times with 120 periods using the linearization methods suggested by King, et al. (1988a,b), and Klein (2000).

4. Findings

We report the H-P filtered statistics for the data, the benchmark economy, and some variations on that economy in Table 3. As a baseline, to evaluate the role of export characteristics and dynamics for the propagation of international business cycles, we compare the results from the

[12] Under the zero export penetration costs, $\tau_0 = \tau_1 = 0$, a_2 set to be equal to 0.315 to match the exports to output ratio of 0.15.

model with the export penetration and continuation costs to a model without the costs, denoted no costs. The no cost model is essentially a version of the Backus et al. (1994) with heterogenous monopolistically competitive firms.

A. Benchmark Model

We find that the benchmark model is nearly identical to the no cost model. In fact, exporter dynamics seem to worsen slightly both the consumption correlations and international comovement puzzles. In Table 3, we see that in the benchmark model output is less positively correlated across countries than consumption (0.22 vs. 056). Both the international correlation of investment (-0.01) and employment (0.18) are also quite low. Compared to a model with no costs, these export costs generate slightly less comovement in economic activity across countries and more comovement in consumption. The only dimension in which the export model does better than the no cost model is in terms of persistence, as the persistence of all macro aggregates rises slightly. The largest increase in persistence is for net exports which rises from 0.68 in the no cost model to 0.73 in our baseline calibration. The model also generates a larger negative correlation between the terms of trade and net exports (0.38 vs. 0.34).

B. Impulse Response

To compare how the mechanisms for international transmissions of business cycles differ between models with and without export penetration costs, the responses of variables to a positive one standard deviation home country-wide productivity shock are plotted in Figure 2.

The aggregate dynamics of output, consumption, labor and investment are almost identical to what one would find in the no cost model. There is strong production shifting and risk sharing motives leading to an increase in output, employment, investment and consumption at home. This is a good time for foreign agents to cut back on investment and hours worked, and increase consumption. Foreign agents finance investment in home and this leads home to run a trade deficit initially. As the productivity shock dies out, this trade deficit reverses.

The dynamics of the export sector differ across countries. The number of exporters from each country expands initially, but only the Home exporter expansion is sustained, building slowly and peaking almost 5 years after the shock. The Foreign exporter expansion is more immediate and peaks about 1 year after shock. In time, the Foreign export sector begins to contract and bottoms out almost 6 years after the initial shock. These dynamics are driven by the sustained productivity gain of the home exporters, which as low cost producers seek to expand market access. The foreign exporters initially experience a spike in demand as consumption and investment boom in Home, but as demand weakens their relatively high costs lead them to receive a smaller share of the market and only the most productive firms remain.

The source of changes in the size of the export sector is similar across countries. Both countries experience a relatively large drop in the stopper ratio and a smaller increase in the starter ratio. These changes are much larger for Foreign exporters and they are reversed relatively quickly, hence the export ratio peaks after only 4 quarters. The stopper and starter ratio of Home exporters gradually declines with time leading to the later peak. The relatively large increase in the stopper ratio implies that relatively low productivity, current exporters continue to export. The increase in the starter ratio implies that newest exporters are less productive than marginal exporters in the past.

The model generates slightly lower comovements across countries than in the no cost model because of the extensive margin to trade. When there is a home productivity shock the increase in the number of foreign exporters, most of whom are relatively unproductive, makes it easier for the home final goods producers to increase production by purchasing smaller quantities of a broader range of foreign inputs. Because in our baseline case there is a love of variety effect, the home final good producer can produce more of its composite foreign intermediate using lower foreign quantities. This weakens the demand for foreign intermediates. It also makes investment at home cheaper than in the no cost model, and this leads to more investment at home and less in foreign.

C. Love of Variety

In this section we show that the taste for variety crucially alters the international propagation of business cycles. We find that reducing the love of variety effect leads to an increase in the comovement of economic activity across countries. Figure 4 reports the relationship between international commovements and the love of variety. Table 4 reports some properties of the model for two particular cases. In the first case, $\lambda = 1 - \theta$, so that we eliminate the variety effect. In the second case, we set $\lambda = 1$ so that production of the foreign composite is a geometric average of consumption of each individual variety. We think this second case is of interest as many models with a continuum of goods implicitly make this assumption.

In the case where there is no variety effect, either positive or negative, we find slightly greater comovements in economic activity and a slightly lower correlation of consumption. Compared to our baseline case, following a positive home productivity shock, demand stays stronger for foreign goods initially and in the future. This additional demand encourages more investment in the export sector. The combination of the increase in the measure of exporters and the smaller decrease in capital per firm, implies that investment in foreign declines less following a positive home productivity shock. Consequently, the model generates greater comovements in investment and output than both our baseline model and the standard, no-cost model. However, this effect is quantitatively minor so that both major puzzles persist.

In the case where $\lambda = 1$, the model no longer generates either the comovement or consumption correlation puzzles. In fact, the model now closely matches the international correlation of output, consumption, investment and employment in the data. In this case, the model also differs from the previous cases in that both output and net exports are less volatile. Moreover, net exports have become slightly more countercyclical than before. Finally, we find that the terms of trade and net exports are now almost perfectly negatively correlated.

To understand the behavior of the model when $\lambda = 1$, consider the impact of introducing an additional, imported variety. If the price of this additional variety is equal to the average price of existing varieties, then the increase in foreign varieties makes it harder to produce a unit of the composite import using the same amount of inputs. Thus, holding all else equal, an increase in foreign varieties raises the price of imports relative to the price of domestic goods. This shifts demand towards domestically produced intermediates. It also implies that the price of a final unit of consumption will increase.

Now, with a positive home productivity shock, as before there is a large and persistent increase in home exporters. Compared to the baseline case, the starter margin is relatively more important than the stopper margin so that more of the most productive firms enter the foreign market. This increase in home exporters shifts foreign taste towards its own goods. Because the rise in exporters is persistent, this leads to a persistent increase in demand for foreign goods, which leads to an increase in production and investment. At the same time, because the cost of producing a final good at homes has increased, consumption at home only increases very slightly. It is this effect that generates greater comovement in economic activity and lower consumption correlations.

5. Sensitivity Analysis

In this section we modify the benchmark model to include changes in the persistence of export status and incomplete asset markets. We study the impact of the changes in the persistence of export status in two ways. First, in a case we call *endogenous persistence of export status*, we examine how the model's properties vary with the stopper probability. Second, in a case we call *exogenous persistence of export status* we change our model so that firms no longer pay a continuation cost each period with certainty. Instead, we assume that exporting firms receive a shock which forces them to repay the entry cost again in order to continue in the export market. We allow firms to enter as before. When $\lambda = 0$, we find that these changes generally do not have quantitatively important effects on the properties of the baseline model. However, when $\lambda = 1$, there are some significant changes in the model occur. In particular, we find there is a gap between the endogenous and exogenous exit probability, with the endogenous exit model performing substantially better than the exogenous exit status model.

A. Endogenous Persistence of Export Status

Table 3 shows how the propagation of business cycles varies with changes in the persistence of export status. We compare the baseline model to one with low persistence in which nearly 50 percent of the firms exporting today continue to export in the next quarter to a model in which export status is quite persistent, so that there is a 1/2 percent chance of a current exporter exiting in the next quarter. In both cases, we maintain an exporter ratio of 50 percent. To increase the persistence of exporter status requires raising the ratio of entry cost to continuation cost, τ_0/τ_1. This increases the cutoff for the idiosyncratic shock to enter the export market, η_0, and decreases the cutoff for the idiosyncratic shock to exit the export market, η_1. As we increase τ_0/τ_1 we lower the productivity difference between exporters and non-exporters, but we raise the gap in capital (k_0/k_1).

In our baseline case, these changes have very small effects on international comovements. Reducing the persistence moves us closer to the no-cost model. Lowering the persistence of export status also lowers the persistence of net exports and exporters. In general, the effect on the persistence of net exports is minor, while the effect on export status is much greater.

Figure 4 reports how the model behaves for different persistence of export decisions when $\lambda = 1$. We see that international comovement is highly non-linear in export status. For very persistent and very transitory export status, we find that there is less comovement in economic activity in the model than in the data. For intermediate levels we find that there is relatively more comovement in economic activity in the model. We find that consumption correlations are minimized, and employment/investment correlations maximized, when about 13 percent of exporters exit each period.

B. Exogenous Persistence of Export Status

We now modify the model slightly by assuming that existing exporters draw their continuation cost from a two point distribution. With probability n_1, exporters draw a continuation cost of τ_0 and with probability $(1 - n_1)$, existing exporters can continue to export for free. The rest of the model is the same as before so that those firms with relatively poor productivity shocks that receive the high continuation cost will exit. This modification allows us to explore the effect of the exit margin on the propagation of shocks.

Table 3 shows that having a fixed exit rate does little to change the aggregate properties of the model when $\lambda = 0$. The only noticeable changes in the model come from the dynamics of the export sector. Now, the number of exporters is much more persistent and there is less comovement in exporting across countries. When $\lambda = 1$, the model generates nearly the same amount of comovement in economic activity across countries as in the endogenous exit model, but substantially more comovement in consumption. This difference can be explained by looking at the dynamics in the export sector. With endogenous exit it is much cheaper to expand the export sector by reducing the exit margin, consequently we get a larger increase in exporters by high productivity countries. This export expansion is driven by the least productive firms in the export sector. This leads to large negative taste shocks in the destination market and lower comovements. With exogenous exit, the export sector expansion is driven by more of the most productive firms entering the export market. This tends to shift production towards more productive firms in the economy. As more of the more productive firms account for a larger share of production, this raises aggregate productivity in both countries and encourages greater consumption.

C. Incomplete Asset Markets

We now examine how our results change when there are incomplete asset markets. In the benchmark model, it is assumed that home and foreign consumers can buy or sell a complete menu of state contingent bonds. In this section, we modify the model so that only one period non-state contingent risk-free bonds issued by home consumers are available. The budget constraint for home consumers with the bond becomes

$$P(s^t)C(s^t) + \overline{Q}(s^t)\overline{B}(s^t) \leq P(s^t)W(s^t)L(s^t) + \overline{B}(s^{t-1}) + \Pi(s^t) + P(s^t)T(s^t),$$

where $\overline{Q}(s^t)$ is the nominal price of the risk-free bond $\overline{B}(s^t)$ at period t. $\overline{B}(s^t)$ is the amount of the bonds that the consumer buys at period t. The bond $\overline{B}(s^t)$ pays 1 unit of home currency at period

$t+1$ regardless of the state at period $t+1$. Under this constraint, the equation for the real exchange rate (2) changes to

$$\sum_{s^{t+1}} \pi(s^{t+1}|s^t) \frac{U_C(s^{t+1})P(s^t)}{U_C(s^t)P(s^{t+1})} = \sum_{s^{t+1}} \pi(s^{t+1}|s^t) \frac{U_C^*(s^{t+1})q(s^t)P(s^t)}{U_C^*(s^t)q(s^{t+1})P(s^{t+1})}.$$

The results with the modified model are summarized in Table 4.[13] Without the trade costs, introducing incomplete asset markets lowers the consumption correlation across countries, and increases the employment correlation compared to the complete asset market assumption. Quantitatively, the main effect of market incompleteness is to lower international consumption correlations. This effect appears to be independent of the love of variety effect.

6. Conclusions

The idea that sunk costs of exporting are important for the dynamics of net exports has received considerable attention. However, previous work on this exporter hysteresis has studied this issue outside of a business cycle framework. In this paper we develop a dynamic stochastic general equilibrium model of trade driven by firms facing fixed costs of entering and participating in foreign markets to study the propagation of business cycles across countries. We find that a model calibrated to match certain characteristic of the export sector generates comovements in economic activity across countries that match the data. In particular, the model developed here offers a potential resolution of the consumption correlation and international comovement puzzles.

Our results depend crucially on how consumers value additional varieties. When there are economies to scale from additional varieties, we find that exporter hysteresis leads to international business cycles that differ substantially from the data. When economies to scale to variety are small, exporter hysteresis has almost no role for aggregate fluctuations. Only when there are diseconomies to variety do international business cycles closely resemble the data. Unfortunately, there is very little empirical evidence to quantify the taste for variety. This is not a unique problem to international macroeconomics as Benassy (1996) shows that the implications of much of the endogenous growth literature depend importantly on the assumptions about this effect.

The mechanism described in the model suggests a particular pattern of export participation across countries and over the business cycle. To determine whether the channel explored here is important in practice requires more empirical work into how these export decisions vary over the

[13] In the simulations a small quadratic cost is imposed so that the law of motion for bonds is stationary.

business cycle. This type of empirical work will also shed light on how consumers value additional varieties. We are in the process of conducting this analysis.

The current model has a number of shortcomings. First, we have concentrated on a limited set of facts about exporters. In particular, we have focused on the differences between the average exporters and average non-exporters with little concern about the difference among firms within these sectors. Clearly, there are large differences between major exporters like GM, Ford, and Boeing and the rest of the export sector which may matter. Second, we have focused on a number of puzzles in the movements of quantities across countries over time. There are a number of important features of fluctuations in prices that this model is not designed to address, but may matter for the export decisions of firms.

Appendix

Consumer's Problem: The first order conditions for the home consumer are:

$$
\begin{aligned}
(13)\quad -\frac{U_L(s^t)}{U_C(s^t)} &= W(s^t), \\
(14)\quad Q(s^{t+1}|s^t) &= \beta Pr(s^{t+1}|s^t)\frac{U_C(s^{t+1})P(s^t)}{U_C(s^t)P(s^{t+1})}.
\end{aligned}
$$

Similarly, the first order conditions for the foreign consumer are:

$$
\begin{aligned}
(15)\quad -\frac{U_L^*(s^t)}{U_C^*(s^t)} &= W^*(s^t), \\
(16)\quad Q^*(s^{t+1}|s^t) &= \beta Pr(s^{t+1}|s^t)\frac{U_C^*(s^{t+1})P^*(s^t)}{U_C^*(s^t)P^*(s^{t+1})}, \\
(17)\quad Q(s^{t+1}|s^t) &= \beta Pr(s^{t+1}|s^t)\frac{U_C^*(s^{t+1})e(s^t)P^*(s^t)}{U_C^*(s^t)e(s^{t+1})P^*(s^{t+1})}.
\end{aligned}
$$

From the state contingent bond equations (14) and (17), we get

$$(18)\quad \frac{U_C(s^{t+1})P(s^t)}{U_C(s^t)P(s^{t+1})} = \frac{U_C^*(s^{t+1})e(s^t)P^*(s^t)}{U_C^*(s^t)e(s^{t+1})P^*(s^{t+1})}.$$

The real exchange rate is defined as $q(s^t) = \frac{e(s^t)P^*(s^t)}{P(s^t)}$. Iterating on (18) yields

$$(19)\quad q(s^t) = \kappa\frac{U_C^*(s^t)}{U_C(s^t)},$$

where $\kappa = q(s^0)U_C(s^0)/U_C^*(s^0)$. For the simulations κ is normalized to be 1.

Final Good Producer's Problem: The focs for the home final good producer give the input demand functions

$$
\begin{aligned}
(20)\quad y^d(h,i,s^t) &= a_1^{\frac{1}{1-\rho}}\left[\frac{P(h,i,s^t)}{P(h,s^t)}\right]^{\frac{1}{\theta-1}}\left[\frac{P(h,s^t)}{P(s^t)}\right]^{\frac{1}{\rho-1}} D(s^t), \\
(21)\quad y^d(f,i,s^t) &= a_2^{\frac{1}{1-\rho}}\left[\frac{P(f,i,s^t)}{P(f,s^t)}\right]^{\frac{1}{\theta-1}}\left[\frac{P(f,s^t)}{P(s^t)}\right]^{\frac{1}{\rho-1}} D(s^t),
\end{aligned}
$$

where $P(h,s^t) = \left[\int_0^1 P(h,i,s^t)^{\frac{\theta}{\theta-1}}di\right]^{\frac{\theta-1}{\theta}}$, and $P(f,s^t) = \left[\int_{i\in\mathcal{E}^*(s^t)} P(f,i,s^t)^{\frac{\theta}{\theta-1}}di\right]^{\frac{\theta-1}{\theta}}$. The zero-profit condition in final goods implies that

$$(22)\quad P(s^t) = \left[a_1^{\frac{1}{1-\rho}}P(h,s^t)^{\frac{\rho}{\rho-1}} + a_2^{\frac{1}{1-\rho}}P(f,s^t)^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-1}{\rho}}.$$

The resource constraint for the final goods gives

$$(23)\quad D(s^t) = C(s^t) + I(s^t).$$

Intermediate Good Producer's Problem: The first order conditions for the i_{th} home intermediate good producer give

$$(24)\quad \frac{P(h,i,s^t)}{P(s^t)} = q(s^t)\frac{P^*(h,i,s^t)}{P^*(s^t)} = \frac{W(s^t)}{\theta F_L(i,s^t)},$$

$$P(s^t) = \sum_{s^{t+1}}\sum_{\eta(i,s_i^{t+1})}\frac{Q(s^{t+1})}{Q(s^t)}Pr[\eta(i,s^{t+1})]\left\{\left(\frac{\alpha}{1-\alpha}\right)\right.$$

$$(25)\quad \left.\cdot\frac{P(s^{t+1})W(s^{t+1})L(i,s^{t+1})}{K(i,s^t)} + P(s^{t+1})(1-\delta)\right\}.$$

The marginal cost of production is equal to $W(s^t)/F_L(i,s^t)$ and prices are a constant mark-up over marginal cost. Since firms in a country are owned by domestic consumers, clearly $Q(s^{t+1})/Q(s^t) = Q(s^{t+1}|s^t)$.

The resource constraint is defined as for good $i \in [0,1]$

$$(26)\quad y(h,i,s^t) + m(i,s^t)y^*(h,i,s^t) = A(i,s^t)K(i,s^{t-1})^{\alpha}L(i,s^t)^{1-\alpha}.$$

From the demand functions for intermediate goods (20) and (21), and the price decisions (24), the labor demand function can be obtained from (26).

$$L(i,s^t) = \left[\frac{W(s^t)}{\theta(1-\alpha)}\right]^{\frac{\nu}{\theta-1}} A(i,s^t)^{\frac{1-\nu}{\alpha}}K(i,s^{t-1})^{1-\nu}\left\{\left[\frac{P(h,s^t)}{P(s^t)}\right]^{\mu} D_t\right.$$

$$(27)\quad \left.+m(i,s^t)a_2^{\frac{1}{1-\rho}}q(s^t)^{\frac{1}{1-\theta}}\left[\frac{P^*(h,s^t)}{P^*(s^t)}\right]^{\mu} D_t^*\right\}^{\nu},$$

where $\nu = \frac{\theta-1}{1-\theta(1-\alpha)}$, and $\mu = \frac{1}{1-\theta} - \frac{1}{1-\rho}$. Since $\eta(i,s^t)$ follows an *iid.* normal distribution, equation (25) implies that $K(i,s^t)$ is independent of $\eta(i,s^t)$ but depends on the firm's export status, $m(i,s^t)$, and the state of the world, s^t.

$$(28)\quad K(i,s^t) = \begin{cases} K_0(s^t) & \text{if } m(i,s^t) = 0, \\ K_1(s^t) & \text{if } m(i,s^t) = 1. \end{cases}$$

Hence, the sufficient statistics for the distribution of the capital among home intermediate good producers are $K_0(s^t)$, $K_1(s^t)$, and $N(s^t)$.

Marginal Exporters: Let $L_{m,m'}(i,s^t)$ and $I_{m,m'}(i,s^t)$ be the potentially sub-optimal levels of labor inputs and investment for the i_{th} firm when $m(i,s^{t-1}) = m$ and $m(i,s^t) = m'$, respectively. Clearly $I_{m,m'}(i,s^t) = I_{m,m'}(s^t) = K_{m'}(s^t) - (1-\delta)K_m(s^t)$. The problem of firm i with state (η,k,m) in aggregate state s^t is to solve the following problem

$$V\left(i,\eta,k,m;s^t\right) = \max\left\{V^0\left(i,\eta,k,m;s^t\right), V^1\left(i,\eta,k,m;s^t\right)\right\}$$

where V^0 is the maximal value of not exporting in the current period and V^1 is equal to the maximal value of exporting this period. From the mark-up pricing (24), the value of the i_{th} firm can be rewritten as

$$V^{m'}\left(\eta,k,m,s^t\right) = \left[\frac{1-\theta(1-\alpha)}{\theta(1-\alpha)}\right]P(s^t)W(s^t)L_{m,m'}(i,s^t) - P(s^t)I_{m,m'}(s^t) - m'e(s^t)P^*(s^t)\tau_m$$

$$(29)\quad +\sum_{s^{t+1}}\sum_{\eta(i,s^{t+1})}Q(s^{t+1}|s^t)V\left(\eta',k',m',s^{t+1}\right)$$

where m, $m' \in \{0,1\}$. The firm-specific productivity of marginal exporters among last period exporters and non-exporters, $\eta_1(s^t)$ and $\eta_0(s^t)$, satisfy

$$V^1\left(\eta_j(s^t), K_j(s^{t-1}), m; s^t\right) = V^0\left(\eta_j(s^t), K_j(s^{t-1}), m; s^t\right), \tag{30}$$

where $m, j = \{0,1\}$. Let $\zeta_j \in [0,1]$ denote the identity of the firm with a shock such that $\eta\left(\zeta_j, s^t\right) = \eta_j\left(s^t\right)$, then the marginal exporter conditions (30) can be rewritten as

$$\begin{aligned} 0 \quad &= \left[\frac{1-\theta(1-\alpha)}{\theta(1-\alpha)}\right] P(s^t)W(s^t)\left[L_{m,1}(\zeta_j, s^t) - L_{m,0}(\zeta_j, s^t)\right] - P(s^t)[K_1(s^t) - K_0(s^t)] \\ &\quad -e(s^t)P^*(s^t)\tau_m + + \sum_{s^{t+1}} \sum_{\eta(\zeta_j, s^{t+1})} Q(s^{t+1}|s^t)\left(V\left[\eta\prime, K_1(s^t), 1; s^{t+1}\right] - V\left[\eta\prime, K_0(s^t), 0;, s^{t+1}\right]\right) \end{aligned} \tag{31}$$

$$\begin{aligned} L_{m,m'}(i,s^t) \quad &= \quad \left[\frac{W(s^t)}{\theta(1-\alpha)}\right]^{\frac{\nu}{\theta-1}} e^{\frac{1-\nu}{\alpha}[z(s^t)+\eta(i,s^t)]} K_m(s^{t-1})^{1-\nu} \\ &\cdot \left[\left(\frac{P(h,s^t)}{P(s^t)}\right)^{\mu} D_t + m' a_2^{\frac{1}{1-\rho}} q(s^t)^{\frac{1}{1-\theta}} \left(\frac{P^*(h,s^t)}{P^*(s^t)}\right)^{\mu} D_t^*\right]^{\nu}. \end{aligned} \tag{32}$$

Among last period exporters, if the firm-specific productivity $\eta(i,s^t)$ is greater (less) than $\eta_1(s^t)$, the producer will (will not) export goods abroad in s^t. Among last period non-exporters, if the firm-specific productivity $\eta(j,s^t)$ is greater (less) than $\eta_0(s^t)$, the producer will (not) export goods abroad in s^t. Thus, the percentage of exporters in s^t among non-exporters and exporters in s^{t-1}, $n_0(s^t)$ and $n_1(s^t)$, respectively, can be defined as

$$n_m(s^t) = 1 - \Phi[\eta_m(s^t)], \tag{33}$$

where $m = \{0,1\}$. $\Phi(\eta)$ is the *cdf*. of $\eta(i,s^t)$. $N(s^t)$ is the percentage of exporters in s^t among all intermediate good producers. $N(s^t)$, evolves as

$$N(s^t) = n_1(s^t)N(s^{t-1}) + n_0(s^t)[1 - N(s^{t-1})]. \tag{34}$$

Aggregate Variables

Capital and Investment: The aggregate capital at home in s^t is defined as

$$\begin{aligned} K(s^t) \quad &= \quad \int_{i\in\mathcal{E}(s^t)} K_1(s^t)di + \int_{i\notin\mathcal{E}(s^t)} K_0(s^t)di \\ &= \quad [1 - N(s^t)]K_0(s^t) + N(s^t)K_1(s^t). \end{aligned} \tag{35}$$

The aggregate investment at home in s^t is defined as

$$I(s^t) = K(s^t) - (1-\delta)K(s^{t-1}). \tag{36}$$

Labor Demand: The average labor demands in s^t from last period non-exporters and exporters, $L_0(s^t)$ and $L_1(s^t)$, can be defined as

$$L_0(s^t) = \frac{\int_{i\notin\mathcal{E}(s^{t-1})} L(i,s^t)di}{1 - N(s^{t-1})}, L_1(s^t) = \frac{\int_{i\in\mathcal{E}(s^{t-1})} L(i,s^t)di}{N(s^{t-1})}.$$

As η is iid from (32)

$$L_m(s^t) = \left[\frac{W(s^t)}{\theta(1-\alpha)}\right]^{\frac{\nu}{\theta-1}} e^{\frac{1-\nu}{\alpha}z(s^t)} K_m(s^{t-1})^{1-\nu} \left\{\left[\left(\frac{P(h,s^t)}{P(s^t)}\right)^{\mu} D_t\right]^{\nu} \cdot \int_{-\infty}^{\eta_m(s^t)} e^{\frac{1-\nu}{\alpha}\eta}\phi(\eta)d\eta + \left[\left(\frac{P(h,s^t)}{P(s^t)}\right)^{\mu} D_t + a_2^{\frac{1}{1-\rho}} q(s^t)^{\frac{1}{1-\theta}} \cdot \left(\frac{P^*(h,s^t)}{P^*(s^t)}\right)^{\mu} D_t^*\right]^{\nu} \int_{\eta_m(s^t)}^{\infty} e^{\left(\frac{1-\nu}{\alpha}\right)\eta}\phi(\eta)d\eta\right\}, \tag{37}$$

where m ={0,1}. $\phi(\eta)$ is the *pdf*. of η. The home aggregate labor demand is defined as

$$L(s^t) = [1 - N(s^{t-1})]L_0(s^t) + N(s^{t-1})L_1(s^t). \tag{38}$$

Capital Decision Rules: The capital decision rules (25) as

$$1 = \sum_{s^{t+1}} Q(s^{t+1}|s^t)\frac{P(s^{t+1})}{P(s^t)}\left[\left(\frac{\alpha}{1-\alpha}\right)\frac{W(s^{t+1})L_m(s^{t+1})}{K_m(s^t)} + (1-\delta)\right]. \tag{39}$$

Price Indices: From the mark-up pricing (24), and the labor demand function for the i_{th} firm, the price of the i_{th} firm can be rewritten as

$$\left[\frac{P(h,i,s^t)}{P(s^t)}\right]^{\frac{\theta}{\theta-1}} = \left[\frac{W(s^t)}{\theta(1-\alpha)}\right]^{\frac{\nu+\theta-1}{\theta-1}} K_{m(i,s^{t-1})}^{1-\nu} e^{\left(\frac{1-\nu}{\alpha}\right)[z(s^t)+\eta(i,s^t)]} \left\{\left[\frac{P(h,s^t)}{P(s^t)}\right]^{\mu} D(s^t) + m(i,s^t)a_2^{\frac{1}{1-\rho}} q(s^t)^{\frac{1}{1-\theta}} \left[\frac{P^*(h,s^t)}{P^*(s^t)}\right]^{\mu} D(s^t)^*\right\}^{\mu-1}, \tag{40}$$

$$\left[\frac{P^*(h,i,s^t)}{P^*(s^t)}\right]^{\frac{\theta}{\theta-1}} = q(s^t)^{\frac{\theta}{1-\theta}}\left[\frac{P(h,i,s^t)}{P(s^t)}\right]^{\frac{\theta}{\theta-1}}. \tag{41}$$

Then, the aggregate export price $P^*(h,s^t)$ can be expressed as

$$\left[\frac{P^*(h,s^t)}{P^*(s^t)}\right]^{\frac{\theta}{\theta-1}} = \left[\frac{W(s^t)}{\theta(1-\alpha)}\right]^{\frac{\nu+\theta-1}{\theta-1}} q(s^t)^{\frac{\theta}{1-\theta}} e^{\left(\frac{1-\nu}{\alpha}\right)z(s^t)} \left\{\left[\frac{P(h,s^t)}{P(s^t)}\right]^{\mu} D(s^t) + a_2^{\frac{1}{1-\rho}} q(s^t)^{\frac{1}{1-\theta}} \left[\frac{P^*(h,s^t)}{P^*(s^t)}\right]^{\mu} D^*(s^t)\right\}^{\nu-1} \cdot \left\{[1-N(s^t)]K_0(s^{t-1})^{1-\nu}\int_{\eta_0(s^t)}^{\infty} e^{\left(\frac{1-\nu}{\alpha}\right)\eta}\phi(\eta)d\eta + N(s^t)K_1(s^{t-1})^{1-\nu}\int_{\eta_1(s^t)}^{\infty} e^{\left(\frac{1-\nu}{\alpha}\right)\eta}\phi(\eta)d\eta\right\}. \tag{42}$$

Similarly, the aggregate home price $P(h,s^t)$ can be expressed as

$$\begin{aligned}\left[\frac{P(h,s^t)}{P(s^t)}\right]^{\frac{\theta}{\theta-1}} &= q(s^t)^{\frac{\theta}{\theta-1}}\left[\frac{P^*(h,s^t)}{P^*(s^t)}\right]^{\frac{\theta}{\theta-1}}+\left[\frac{W(s^t)}{\theta(1-\alpha)}\right]^{\frac{\nu+\theta-1}{\theta-1}} \\ &\cdot e^{\left(\frac{1-\nu}{\alpha}\right)z(s^t)}\left\{\left[\frac{P(h,s^t)}{P(s^t)}\right]^{\mu}D(s^t)\right\}^{\nu-1} \\ &\cdot\left\{[1-N(s^t)]K_0(s^{t-1})^{1-\nu}\int_{-\infty}^{\eta_0(s^t)}e^{\left(\frac{1-\nu}{\alpha}\right)\eta}\phi(\eta)d\eta\right. \\ &\left.+N(s^t)K_1(s^{t-1})^{1-\nu}\int_{-\infty}^{\eta_1(s^t)}e^{\left(\frac{1-\nu}{\alpha}\right)\eta}\phi(\eta)d\eta\right\}. \qquad (43)\end{aligned}$$

From the aggregate price index (22),

$$1=a_1^{\frac{1}{1-\rho}}\left[\frac{P(h,s^t)}{P(s^t)}\right]^{\frac{\rho}{\rho-1}}+a_2^{\frac{1}{1-\rho}}\left[\frac{P(f,s^t)}{P(s^t)}\right]^{\frac{\rho}{\rho-1}}. \qquad (44)$$

Values of Firms: Let $V_m(s^t), m=\{0,1\}$, be the average values of firms among the firms that have the same export status, m, in s^{t-1}. Clearly

$$\begin{aligned}V_0(s^t) &= \frac{1}{1-N(s^{t-1})}\int V\left[\eta,K_0(s^{t-1}),0,s^t\right]\phi(\eta)\,\partial\eta, \\ V_1(s^t) &= \frac{1}{N(s^{t-1})}\int V\left[\eta,K_1(s^{t-1}),1,s^t\right]\phi(\eta)\,\partial\eta.\end{aligned}$$

These average values of firms can be rewritten as

$$\begin{aligned}V_m(s^t) &= \left[\frac{1-\theta(1-\alpha)}{\theta(1-\alpha)}\right]P(s^t)W(s^t)L_m(s^t)-P(s^t)\left\{[1-n_m(s^t)]K_0(s^t)\right. \\ &\left.+n_m(s^t)K_1(s^t)\right\}+(1-\delta)P(s^t)K_m(s^{t-1})-n_m(s^t)e(s^t)P^*(s^t)\tau_m \\ &+\sum_{s^{t+1}}Q(s^{t+1}|s^t)\left\{[1-n_m(s^t)]V_0(s^{t+1})+n_m(s^t)V_1(s^{t+1})\right\}, \qquad (45)\end{aligned}$$

and the difference between $V_1(s^t)$ and $V_0(s^t)$ gives

$$\begin{aligned}V_1(s^t)-V_0(s^t) &= \left[\frac{1-\theta(1-\alpha)}{\theta(1-\alpha)}\right]P(s^t)W(s^t)[L_1(s^t)-L_0(s^t)] \\ &-P(s^t)\left\{[n_0(s^t)-n_1(s^t)][K_0(s^t)-K_1(s^t)]\right\} \\ &+(1-\delta)P(s^t)[K_1(s^{t-1})-K_0(s^{t-1})] \\ &-e(s^t)P^*(s^t)[n_1(s^t)\tau_1-n_0(s^t)\tau_0] \\ &+[n_1(s^t)-n_0(s^t)]\sum_{s^{t+1}}Q(s^{t+1}|s^t)[V_1(s^{t+1})-V_0(s^{t+1})]. \qquad (46)\end{aligned}$$

The conditions for marginal exporters (**??**) can be rewritten as

$$
\begin{aligned}
0 \quad = \quad & \left[\frac{1-\theta(1-\alpha)}{\theta(1-\alpha)}\right] P(s^t) \left[\frac{W(s^t)}{\theta(1-\alpha)}\right]^{\frac{\nu+\theta-1}{\theta-1}} e^{\left(\frac{1-\nu}{\alpha}\right)[z(s^t)+\eta_m(s^t)]} K_m(s^{t-1})^{1-\nu} \\
& \cdot \left\{ \left[\left(\frac{P(h,s^t)}{P(s^t)}\right)^{\mu} D_t + a_2^{\frac{1}{1-\rho}} q(s^t)^{\frac{1}{1-\theta}} \left(\frac{P^*(h,s^t)}{P^*(s^t)}\right)^{\mu} D_t^* \right]^{\nu} \right. \\
& \left. - \left[\left(\frac{P(h,s^t)}{P(s^t)}\right)^{\mu} D_t \right]^{\nu} \right\} - P(s^t)[K_1(s^t) - K_0(s^t)] - e(s^t)P^*(s^t)\tau_m \\
& + \sum_{s^{t+1}} Q(s^{t+1}|s^t)[V_1(s^{t+1}) - V_0(s^{t+1})]. \qquad (47)
\end{aligned}
$$

Notice that by substituting $[V_1(s^{t+1}) - V_0(s^{t+1})]$ with (46), (47) becomes a static equation.

Exports and Imports: The real imports are defined as

$$
(48) \quad IM(s^t) = \int_{i\in\mathcal{E}^*(s^t)} \frac{P(f,i,s_i^t)y(f,i,s^t)}{P(s^t)} di = a_2^{\frac{1}{1-\rho}} \left[\frac{P(f,s^t)}{P(s^t)}\right]^{\frac{\rho}{\rho-1}} D(s^t).
$$

Similarly, the real exports are defined as

$$
(49) \quad EX(s^t) = \int_{i\in\mathcal{E}(s^t)} \frac{e(s^t)P^*(h,i,s^t)y^*(h,i,s^t)}{P(s^t)} di = a_2^{\frac{1}{1-\rho}} q(s^t) \left[\frac{P^*(h,s^t)}{P^*(s^t)}\right]^{\frac{\rho}{\rho-1}} D^*(s^t).
$$

The gross domestic product, $Y(s^t)$ is defined as

$$
(50) \quad Y(s^t) = C(s^t) + I(s^t) + NX(s^t),
$$

where $NX(s^t) = EX(s^t) - IM(s^t)$, the real net exports.

Incomplete Asset Markets: We there is a single non-contingent asset, so the foreign consumer's budget constraint becomes

$$
P^*(s^t)C^*(s^t) + \frac{\overline{Q}(s^t)\overline{B}^*(s^t)}{e(s^t)} \le P^*(s^t)W^*(s^t)L^*(s^t) + \frac{\overline{B}^*(s^{t-1})}{e(s^t)} + \Pi^*(s^t) + P^*(s^t)T^*(s^t).
$$

The first order conditions give

$$
(51) \quad \overline{Q}(s^t) = \sum_{s^{t+1}} Pr(s^{t+1}|s^t) \frac{U_C(s^{t+1})P(s^t)}{U_C(s^t)P(s^{t+1})} = \sum_{s^{t+1}} Pr(s^{t+1}|s^t) \frac{U_C^*(s^{t+1})e(s^t)P^*(s^t)}{U_C^*(s^t)e(s^{t+1})P^*(s^{t+1})}.
$$

And the budget constraint is written as

$$
\begin{aligned}
P^*(s^t)C^*(s^t) + \frac{\overline{Q}(s^t)\overline{B}^*(s^t)}{e(s^t)} \quad = \quad & \left[\frac{1}{\theta(1-\alpha)}\right] P^*(s^t)W^*(s^t)L^*(s^t) - P^*(s^t)I^*(s^t) \\
& + \frac{\overline{B}^*(s^{t-1})}{e(s^t)} - \frac{P(s^t)}{e(s^t)} \left[n_0^*(s^t)\tau_0 + n_1^*(s^t)\tau_1\right] \\
& + P^*(s^t)\left[n_0(s^t)\tau_0 + n_1(s^t)\tau_1\right]. \qquad (52)
\end{aligned}
$$

Hence, The equation for the real exchange rate (19) is replaced by (51), and the budget constraint (52) is newly introduced with the additional variable $\overline{B}^*(s^t)$.

Equilibrium: Under the normalization of price indices, $P(s^t) = P^*(s^t) = 1$, we have 6 dynamic equations[14]:

(39) and foreign analogue for $K_0(s^t)$ and $K_0^*(s^t)$, and $K_1(s^t)$ and $K_1^*(s^t)$;
(46) and foreign analogue for $V_1(s^t) - V_0(s^t)$ and $V_1^*(s^t) - V_0^*(s^t)$; and, 37 static equations:
(13) and foreign analogue for $W(s^t)$ and $W^*(s^t)$;
(19) ((51) under the incomplete asset market condition) for $q(s^t)$;
(23) and foreign analogue for $D(s^t)$ and $D^*(s^t)$;
(33) and foreign analogue for $n_0(s^t)$ and $n_0^*(s^t)$, and $n_1(s^t)$ and $n_1^*(s^t)$;
(34) and foreign analogue for $N(s^t)$ and $N^*(s^t)$;
(35) and foreign analogue for $K(s^t)$ and $K^*(s^t)$;
(36) and foreign analogue for $I(s^t)$ and $I^*(s^t)$;
(37) and foreign analogue for $L_0(s^t)$ and $L_0^*(s^t)$, and $L_1(s^t)$ and $L_1^*(s^t)$;
(38) and foreign analogue for $L(s^t)$ and $L^*(s^t)$;
(42) and foreign analogue for $P^*(h, s^t)$ and $P(f, s^t)$;
(43) and foreign analogue for $P(h, s^t)$ and $P^*(f, s^t)$;
(44) and foreign analogue for $P(s^t)$ and $P^*(s^t)$;
(47) and foreign analogue for $\eta_0(s^t)$, $\eta_0^*(s^t)$, $\eta_1(s^t)$, and $\eta_1^*(s^t)$[15];
(48) and foreign analogue for $IM(s^t)$ and $IM^*(s^t)$;
(49) and foreign analogue for $EX(s^t)$ and $EX^*(s^t)$;
(50) and foreign analogue for $Y(s^t)$ and $Y^*(s^t)$.

With incomplete asset markets we have one more equation for bond holdings
(52) for $B^*(s^t)$.

[14] $Q(s^{t+1}|s^t)$ is substituted by other variables using (14) and (17).

[15] By substituting $[V_1(s^{t+1}) - V_0(s^{t+1})]$ with (46), (47) becomes a static equation.

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	Starter Ratios			Stopper Ratios			Exporter Ratios		
Year	US[a]	GR[b]	CO[c]	US[a]	GR[b]	CO[c]	US[a]	GR[b]	CO[c]
1979	-	3.34	-	-	6.45	-	-	37.12	-
1980	-	3.01	-	-	5.95	-	-	36.66	-
1981	-	3.26	-	-	5.93	-	-	37.19	-
1982	-	3.42	-	-	4.93	-	-	37.20	-
1983	-	4.31	2.6	-	6.22	16.8	-	38.10	11.3
1984	-	4.30	2.9	-	5.17	13.5	-	39.19	10.7
1985	-	5.58	4.3	-	4.64	13.1	-	41.59	11.7
1986	-	4.81	3.7	-	5.62	10.8	-	42.96	11.2
1987	13.61	3.79	2.6	14.79	6.39	15.8	48.66	42.91	11.9
1988	16.40	5.29	2.8	12.64	5.18	8.6	50.93	44.12	12.4
1989	12.59	4.74	4.2	12.02	4.86	1.7	50.99	45.14	13.5
1990	14.24	4.21	-	10.83	5.68	-	52.45	44.60	-
1991	13.40	4.21	-	9.48	4.74	-	53.85	45.27	-
1992	16.10	3.64	-	13.48	5.38	-	54.02	44.94	-
Average	14.39	4.14	3.3	12.20	5.51	11.5	51.82	41.21	11.8

Sources: a. Recalculation of Table 5 in Bernard and Jensen (2001); b. Recalculation of Table 2 in Bernard and Wagner (1998); c. Table 2 in Roberts and Tybout (1997).

Notes: Starter ratio – the ratio of non-exporters that transition to exporter status to the number of last period exporters; Stopper ratio – the ratio of exporters that transition to non-exporter status to the number of last period non-exporters; Exporter ratio – the ratio of exporters to the number of all firms.

Table 1: Starter, Stopper, and Exporter Ratios

Preferences	$\beta = 0.99, \quad \sigma = 2, \quad \theta = 0.9, \quad \rho = 1/3, \quad \gamma = 0.294$
Production	$\alpha = 0.36, \quad \delta = 0.025, \quad a_1 = 1, \quad a_2 = 0.321$
Productivity	$M = \begin{bmatrix} 0.95 & 0 \\ 0 & 0.95 \end{bmatrix}$ $Var(\epsilon) = Var(\epsilon^*) = \sigma_\epsilon^2 = 0.007^2$ $Corr(\epsilon, \epsilon^*) = 0.25$ $\sigma_\eta = 0.50$
Trade costs	$\tau_0 = 0.24897, \quad \tau_1 = 0.05043$

Note: Under the zero export penetration costs, $\tau_0 = \tau_1 = 0$, and $a_2 = 0.315$.

Table 2: Parameter Values

Table 3: Business Cycle Statistics

Statistics	Data	No cost	λ=0 Baseline	λ=0 High pers. (n=0.005)	λ=0 Low pers. (n=0.4995)	λ=0 Fixed Exit	λ=1-θ Baseline	λ=1 Baseline	λ=1 Fixed Exit
Std dev (%)									
Output	1.72	1.29	1.28	1.28	1.27	1.28	1.26	1.13	1.22
NX/Output	0.15	0.17	0.17	0.17	0.16	0.17	0.15	0.05	0.08
Std dev (relative to Y)									
Consumption	0.79	0.34	0.36	0.36	0.36	0.36	0.36	0.38	0.36
Investment	3.24	3.16	3.39	3.38	3.37	3.38	3.34	3.09	3.20
Employment	0.63	0.47	0.47	0.47	0.47	0.47	0.47	0.46	0.47
TOT	3.68	0.43	0.45	0.46	0.46	0.46	0.47	0.54	0.65
RER	4.34	0.28	0.31	0.31	0.32	0.32	0.33	0.49	0.38
Domestic correl w/Y									
Consumption	0.88	0.95	0.95	0.95	0.95	0.95	0.96	0.96	0.96
Investment	0.93	0.98	0.98	0.98	0.98	0.98	0.98	0.99	0.98
Employment	0.86	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.98
NX/Output	-0.36	-0.51	-0.50	-0.50	-0.49	-0.50	-0.50	-0.54	-0.36
Corr(NX,TOT)		-0.34	-0.38	-0.40	-0.46	-0.43	-0.57	-0.99	-0.58
Int'l correl									
Output	0.51	0.23	0.23	0.22	0.23	0.21	0.23	0.30	0.34
Consumption	0.32	0.52	0.56	0.56	0.55	0.55	0.54	0.29	0.46
Investment	0.29	0.01	-0.01	-0.01	0.02	-0.01	0.04	0.28	0.29
Employment	0.43	0.21	0.18	0.18	0.20	0.17	0.21	0.38	0.39
Exporters			0.62	0.66	0.57	0.11	0.66	0.66	0.31
Persistence									
Output	0.87	0.69	0.69	0.70	0.69	0.69	0.69	0.67	0.69
Consumption	0.85	0.73	0.74	0.74	0.73	0.73	0.73	0.72	0.73
Investment	0.91	0.67	0.68	0.68	0.67	0.68	0.68	0.66	0.66
Employment	0.95	0.67	0.68	0.68	0.68	0.68	0.68	0.65	0.67
NX/Output	0.61	0.68	0.71	0.72	0.69	0.70	0.69	0.70	0.62
Exporters			0.88	0.90	0.70	0.95	0.88	0.86	0.95

Notes: The statistics in the data column are from Kehoe and Perri (2002). The model statistics are computed from a simulation of 1000 iterations with 120 periods. All series but net exports have been logged and H-P filtered with a smoothing paramer of 1600.

Table 4: Business Cycle Statistics Incomplete Markets

Statistics	Data	No cost	λ=0 Baseline	λ=0 High pers. (n=0.005)	λ=0 Low pers. (n=0.4995)	λ=0 Fixed Exit	λ=1−θ Baseline	λ=1 Baseline	λ=1 Fixed Exit
Std dev (%)									
Output	1.72	1.31	1.27	1.27	1.26	1.27	1.25	1.13	1.21
NX/Output	0.15	0.18	0.19	0.19	0.18	0.18	0.16	0.06	0.11
Std dev (relative to Y)									
Consumption	0.79	0.36	0.37	0.37	0.37	0.37	0.37	0.38	0.37
Investment	3.24	3.19	3.41	3.39	3.39	3.39	3.35	3.11	3.26
Employment	0.63	0.49	0.46	0.46	0.46	0.46	0.46	0.45	0.47
TOT	3.68	0.45	0.39	0.39	0.40	0.39	0.42	0.58	0.63
RER	4.34	0.29	0.28	0.28	0.28	0.28	0.29	0.43	0.33
Domestic correl w/Y									
Consumption	0.88	0.95	0.97	0.96	0.97	0.96	0.97	0.96	0.97
Investment	0.93	0.98	0.97	0.98	0.98	0.98	0.98	0.99	0.98
Employment	0.86	0.99	0.98	0.99	0.98	0.98	0.99	0.99	0.98
NX/Output	-0.36	-0.51	-0.52	-0.53	-0.52	-0.53	-0.53	-0.54	-0.41
Corr(NX,TOT)		-0.34	-0.41	-0.45	-0.49	-0.46	-0.63	-0.99	-0.62
Int'l correl									
Output	0.51	0.25	0.24	0.23	0.24	0.23	0.24	0.30	0.35
Consumption	0.32	0.43	0.45	0.46	0.44	0.45	0.44	0.26	0.40
Investment	0.29	0.03	-0.01	-0.01	0.01	-0.01	0.04	0.26	0.26
Employment	0.43	0.23	0.24	0.24	0.26	0.24	0.27	0.40	0.42
Exporters			0.58	0.66	0.47	0.12	0.63	0.74	0.33
Persistence									
Output	0.87	0.69	0.69	0.69	0.69	0.69	0.69	0.67	0.69
Consumption	0.85	0.73	0.73	0.73	0.73	0.73	0.73	0.72	0.73
Investment	0.91	0.67	0.68	0.69	0.67	0.68	0.68	0.65	0.66
Employment	0.95	0.67	0.68	0.68	0.67	0.68	0.68	0.65	0.67
NX/Output	0.61	0.68	0.70	0.71	0.67	0.68	0.68	0.63	0.59
Exporters			0.88	0.90	0.69	0.95	0.87	0.86	0.94

Notes: The statistics in the data column are from Kehoe and Perri (2002). The model statistics are computed from a simulation of 1000 iterations with 120 periods. All series but net exports have been logged and H-P filtered with a smoothing paramer of 1600.

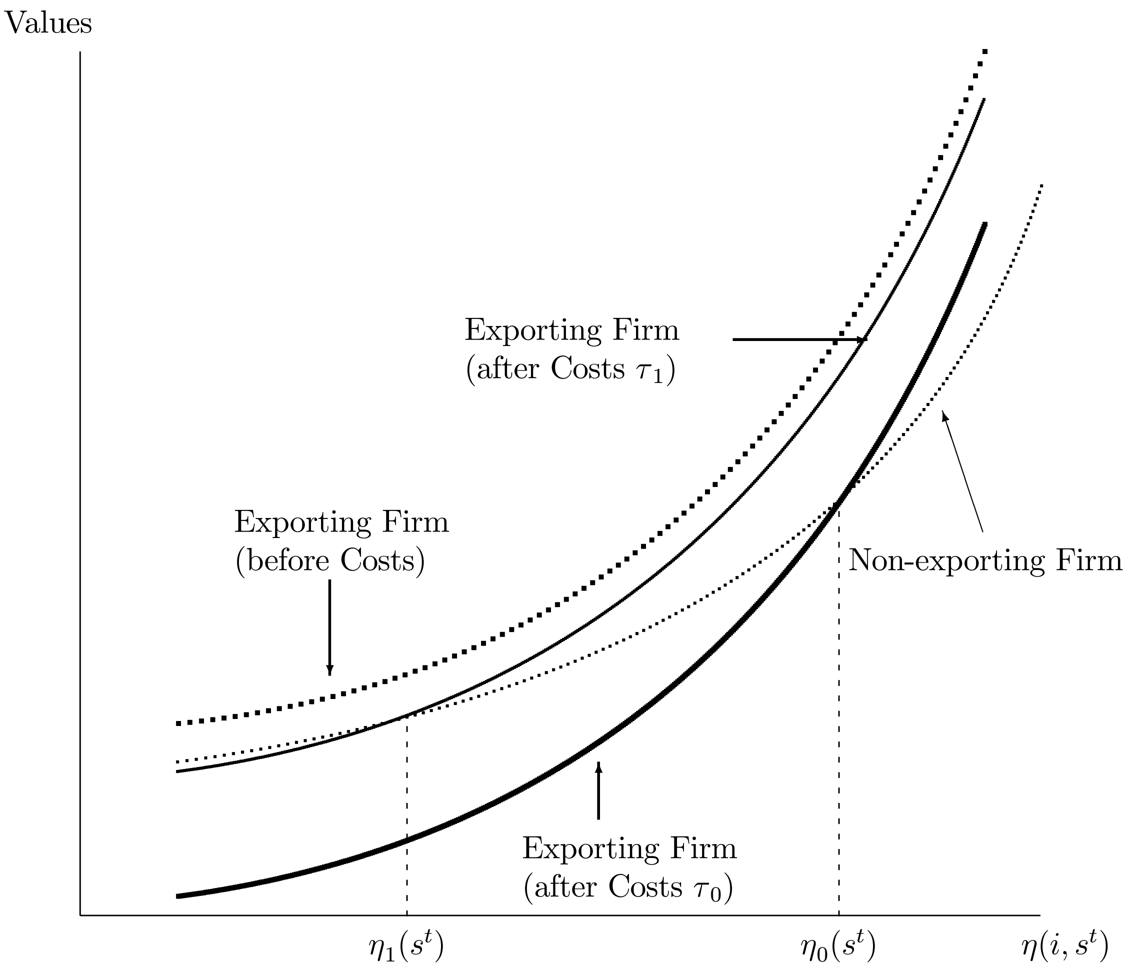
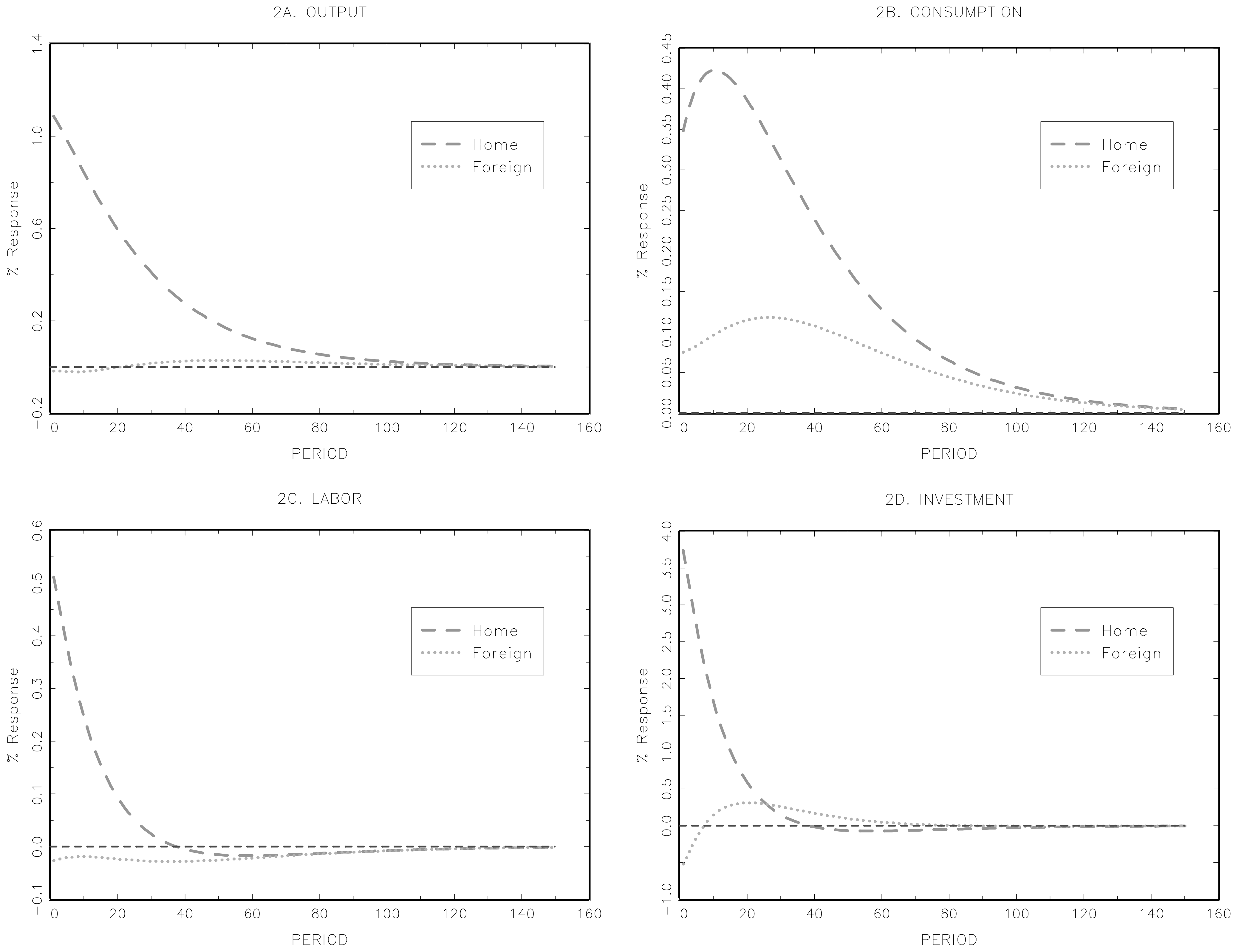
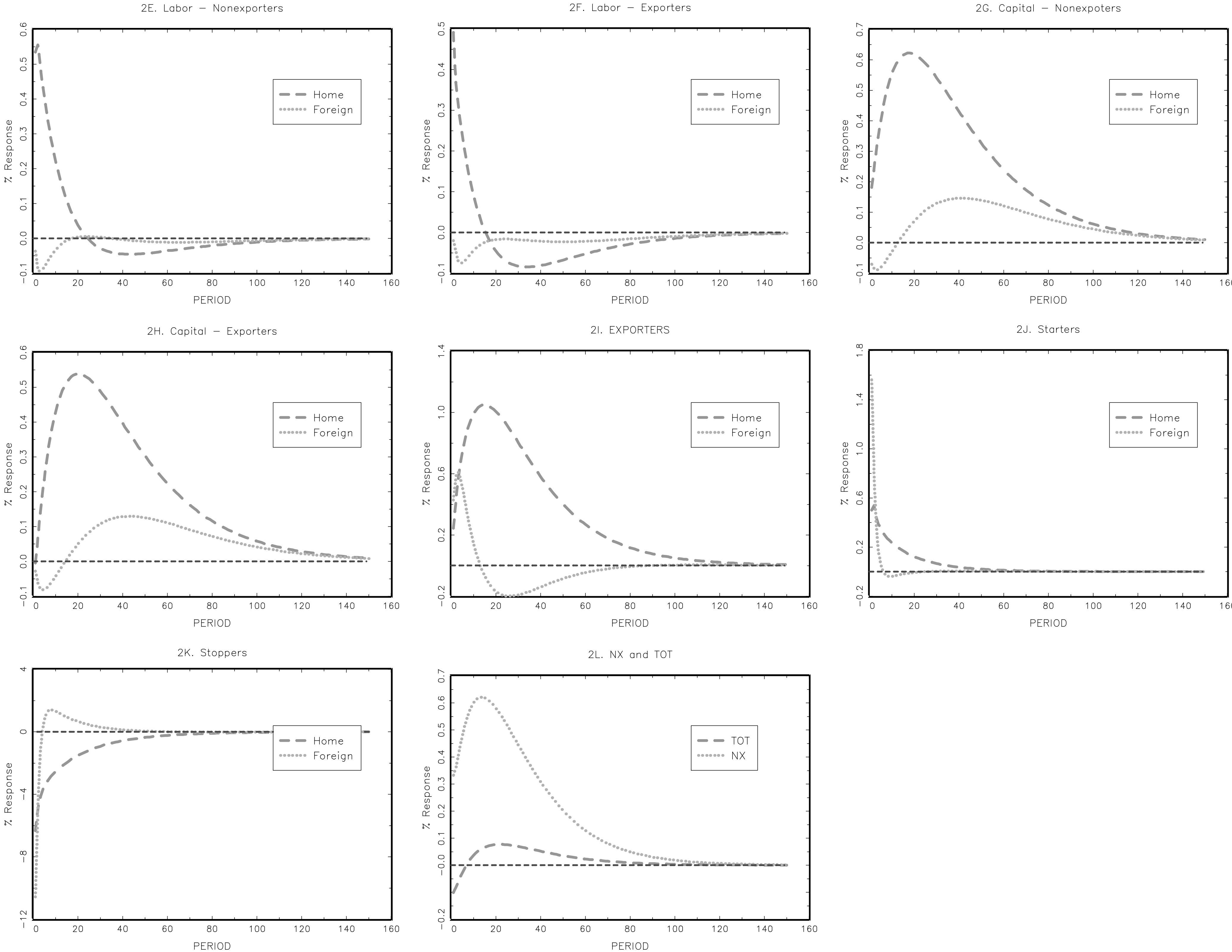
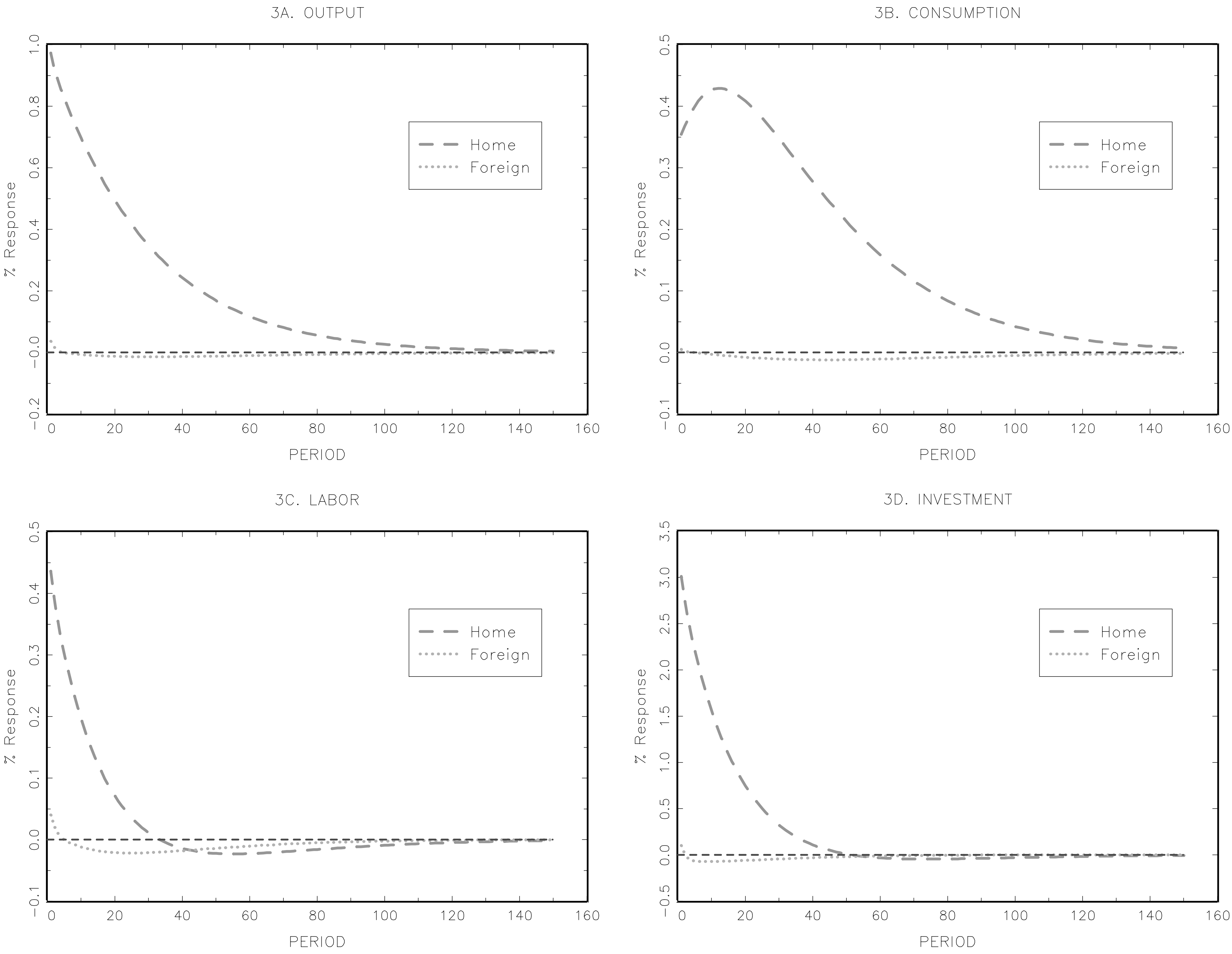


Figure 1: Value of Firms

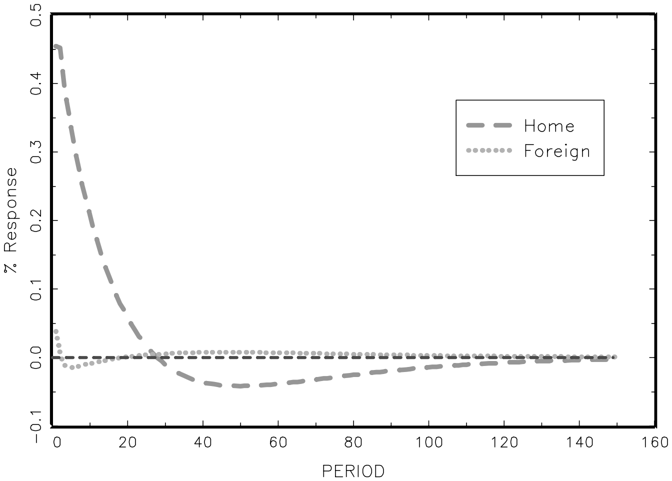

2A. OUTPUT
% Response
PERIOD
Home
Foreign
1.4
1.0
0.6
0.2
−0.2
0
20
40
60
80
100
120
140
160
2B. CONSUMPTION
% Response
PERIOD
Home
Foreign
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
2C. LABOR
% Response
PERIOD
Home
Foreign
0.6
0.5
0.4
0.3
0.2
0.1
0.0
−0.1
2D. INVESTMENT
% Response
PERIOD
Home
Foreign
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
−1.0



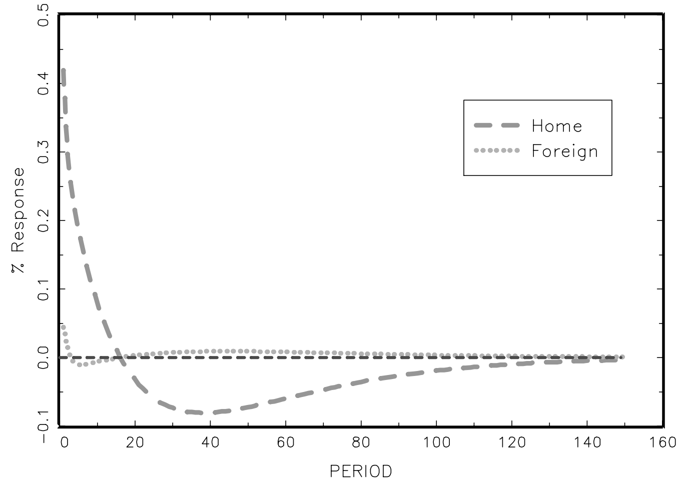
2E. Labor – Nonexporters
2F. Labor – Exporters
2G. Capital – Nonexpoters
2H. Capital – Exporters
2I. EXPORTERS
2J. Starters
2K. Stoppers
2L. NX and TOT
Home
Foreign
TOT
NX
% Response
PERIOD


3A. OUTPUT
% Response
1.0
0.8
0.6
0.4
0.2
-0.0
-0.2
0
20
40
60
80
100
120
140
160
PERIOD
Home
Foreign
3B. CONSUMPTION
% Response
0.5
0.4
0.3
0.2
0.1
0.0
-0.1
PERIOD
3C. LABOR
% Response
0.5
0.4
0.3
0.2
0.1
0.0
-0.1
PERIOD
3D. INVESTMENT
% Response
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
-0.5
PERIOD

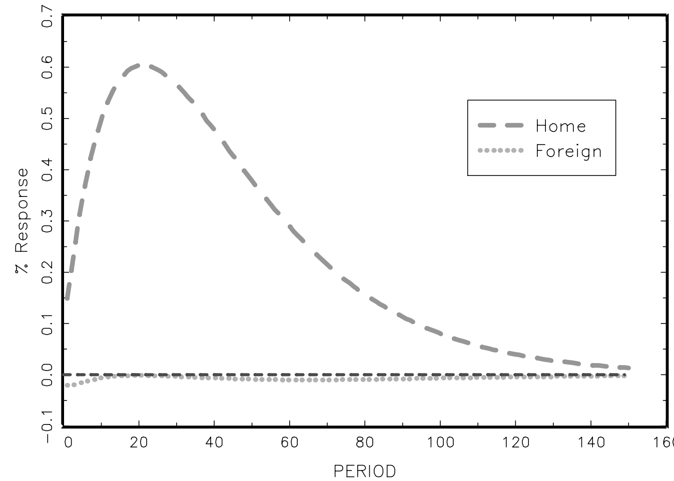
3E. Labor – Nonexporters



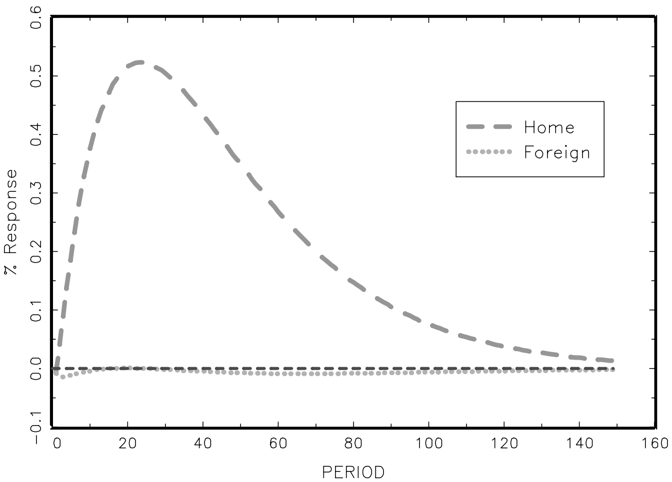
3F. Labor – Exporters



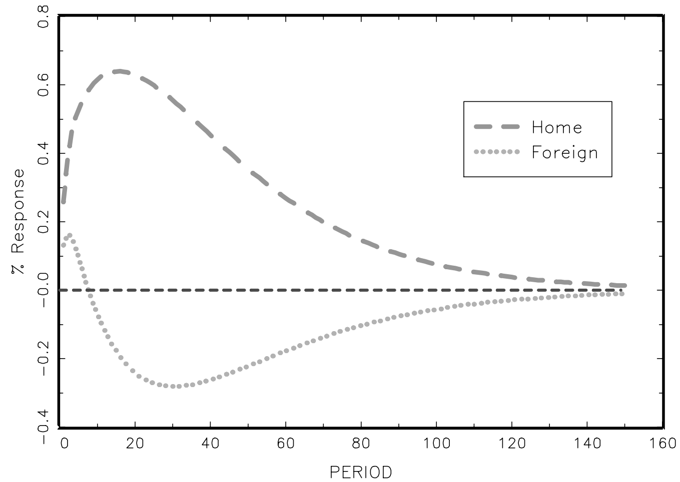
3G. Capital – Nonexpoters



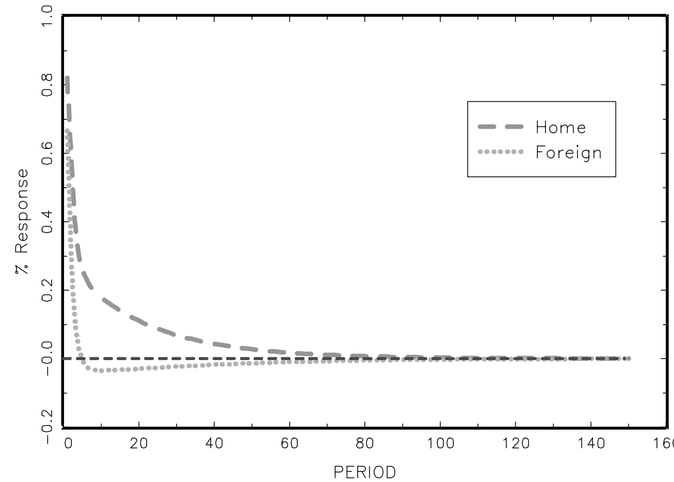
3H. Capital – Exporters



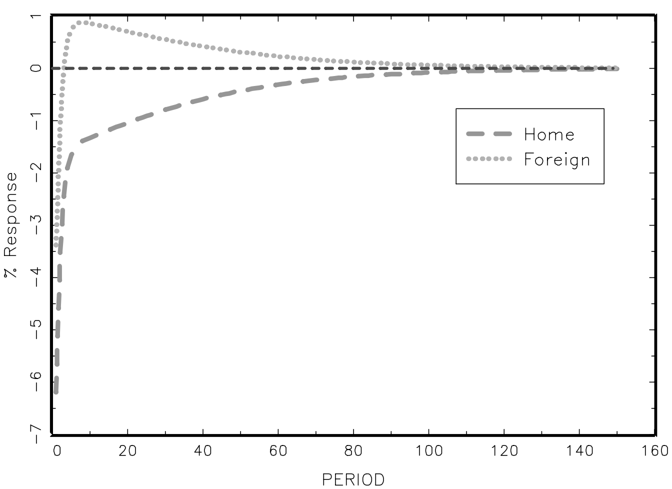
3I. EXPORTERS



3J. Starters



3K. Stoppers



3L. NX and TOT

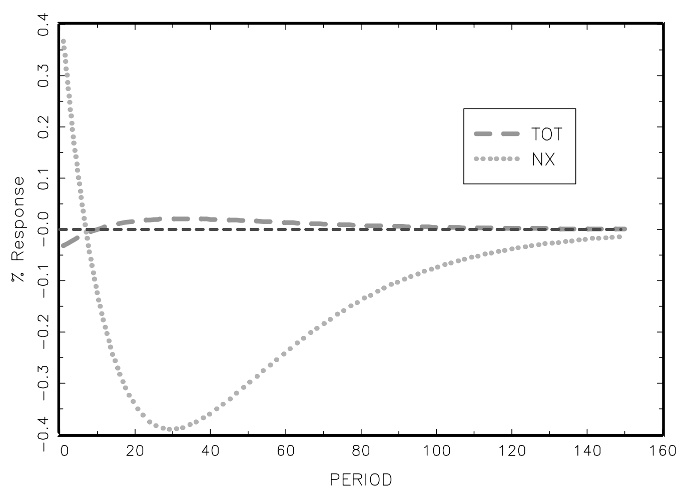


Figure 4: International Correlations & Love of Variety

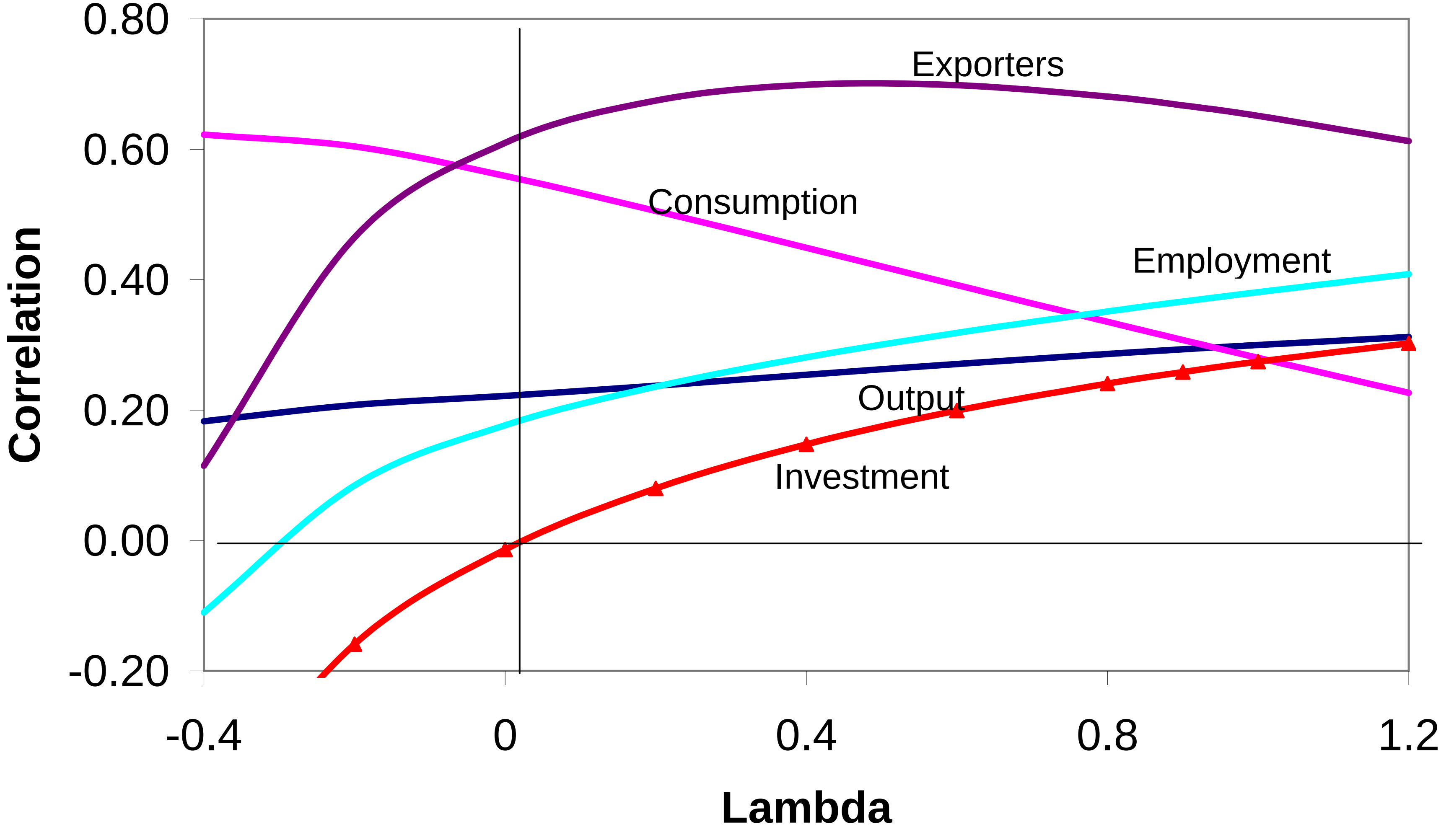


Fig. 5: International Correl & Exporter Status