

Money, Credit, and Banking*

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Abstract

We construct a monetary model where banks reallocate idle cash by taking deposits and making loans. For any positive equilibrium nominal interest rate, the existence of banks leads to a higher level of steady state output and welfare. Existence of the banking equilibrium requires inflation to be sufficiently far from the Friedman rule. We then show that banks can amplify aggregate shocks by making individual consumption more volatile and derive the optimal stabilization response by the central bank. The optimal policy involves stabilizing nominal interest rates and therefore smoothing individual consumption.

Keywords : Money, Banking, Stabilization.

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1 Introduction

In many monetary models, some agents find themselves with large money balances but no particular urgency to spend them while there are others with low balances and a high marginal value of consumption.¹ This raises four questions. First, can financial intermediation improve the allocation by channeling money from those with idle balances to those who have an immediate need for it? Second, can the government achieve the same outcome by injecting additional money into the economy or by redistributing the existing money stock through lump-sum taxes and transfers? Third, if there are shocks to the economy, does financial intermediation dampen or amplify the shocks? Finally, can the central bank improve welfare by stabilizing the shocks? The objective of this paper is to answer these questions.

We model financial intermediation as being done by banks who accept deposits and make loans. All deposits and loans are nominal as is the interest rate paid to depositors and charged to borrowers. Banks have a record-keeping technology that allows them to keep track of financial histories but not trading histories in the good market amongst the agents themselves. Consequently, the existence of financial record keeping does not eliminate the need for money as a medium of exchange. Thus, money remains essential.² With regard to the first question, we show that under the Friedman rule, banks are not essential – the allocation without banks is the same as with banks. However, for any positive nominal interest rate such that loan repayment occurs, banks expand the set of allocations and thus are essential. By offering liquidity when needed, banks effectively provide consumption insurance for agents. As for the second question, we find that the provision of consumption insurance via the banking system cannot be replicated by a government policy of printing extra money or redistributing the existing money stock via lump-sum taxes and transfers. Thus, our model clearly shows that there is a fundamental difference between reallocating cash via a pricing mechanism and doing so through lump-sum redistribution.

To address the last two questions, we introduce shocks to the stock of idle balances. We demonstrate that even though banks increase average consumption they amplify these shocks such that individual consumption is more variable than would be the case without banks. Finally, we find that it is optimal for the central bank to stabilize nominal interest rates and therefore consumption.

¹See for example Bewley (1980), Lucas (1980), Levine (1991), Molico (1997), Camera and Corbae (1999), Berentsen (2002), Green and Zhou (2003), and Berentsen, Camera and Waller (2004a,b).

²By essential we mean that the use of money expands the set of allocations (Kocherlakota (1998), Kocherlakota and Wallace (1998), and Wallace (2001)).

This is equivalent to providing an elastic supply of currency - it injects money into the economy when demand for credit is high and withdraws it when demand is low.

The paper proceeds as follows. Section 2 contains a brief literature review and Section 3 describes the environment. In Section 4 we describe the agents' decision problems and derive the equilibrium. In Section 5 we introduce aggregate shocks to the economy and study the optimal stabilization policy by the central bank. The last section concludes.

2 Literature

There is a huge literature on banking which we cannot possibly review here. However, banking in models where money is essential is a rather small literature with notable papers being Cavalcanti and Wallace (1999a,b), Cavalcanti, Erosa, and Temzelides (1999), He, Huang and Wright (2003), and Wallace and Zhou (2004). Our model differs from these models in a number of ways. First, these models have indivisible money while we use the divisible money model of Lagos-Wright (2004).³ Second, we have competitive markets as opposed to bilateral matching and bargaining.⁴ Third, we do not have bank claims circulating as medium of exchange nor do we assume that agents' trading histories in the good markets are observable as is the case in these other models. Also, in contrast to He, Huang and Wright (2003), there is no security motive for depositing cash in the bank. Finally, since there is no collateral in our model bilateral trade credit cannot be supported as in Shi (1996). So a key contribution of our model is that we separate financial record-keeping from note issue and from goods market record-keeping.

3 The Environment

Time is discrete and in each period there are two markets that open sequentially and are perfectly competitive. There is a $[0, 1]$ continuum of infinitely-lived agents and one perishable good produced and consumed by all agents.

At the beginning of the first market agents get a preference shock such that they can either consume or produce. With probability $1 - n$ an agent can consume but cannot produce while with

³Recently, Faig (2004) has also developed a model of banking in the Shi (1997) framework in which money and goods are divisible.

⁴Competitive markets in the Lagos-Wright framework have been studied by Berentsen, Camera and Waller (2004), Aruoba, Waller and Wright (2003), Rocheteau and Wright (2004).

probability n the agent can produce but cannot consume. We refer to consumers as buyers and producers as sellers. Agents get utility $u(q)$ from $q > 0$ consumption in the first market, where $u'(q) > 0$, $u''(q) < 0$, $u'(0) = +\infty$, and $u'(\infty) = 0$. Furthermore, we impose that the elasticity of utility $e(q) = \frac{qu'(q)}{u(q)}$ is bounded. Producers incur utility cost $c(q) = q$ from producing q units of output. To motivate a role for fiat money, we assume that all goods trades are anonymous. In particular, trading histories of agents are private information. Consequently, sellers require immediate compensation so buyers pay with money.

In the second market all agents consume and produce, getting utility $U(x)$ from x consumption, with $U'(x) > 0$, $U'(0) = \infty$, $U'(+\infty) = 0$ and $U''(x) \leq 0$.⁵ Agents can produce one unit of x with one unit of labor h . Production of x units of output generates disutility h . The discount factor across dates is $\beta \in (0, 1)$.

We assume a central bank exists that controls the supply of fiat currency. The growth rate of the money stock is given by $M_t = \gamma M_{t-1}$ where $\gamma > 0$ and M_t denotes the per capita money stock in t . Agents receive lump sum transfers $\tau M_{t-1} = (\gamma - 1)M_{t-1}$ over the period. Some of the transfer is received at the beginning of market 1 and some during market 2. Let τ_1 and τ_2 denote the transfers in market 1 and 2 respectively with $(\tau_1 + \tau_2) M_{t-1} = \tau M_{t-1}$. Moreover, $\tau_1 = (1 - n)\tau_b + n\tau_s$ since the government might wish to treat buyers and sellers differently. If $\tau < 0$, we assume the central bank has the authority to levy taxes in the form of currency to extract cash from the economy.⁶ For notational ease variables corresponding to the next period are indexed by $+1$, and variables corresponding to the previous period are indexed by -1 .

The key contribution of this paper is the introduction of a banking sector. Its purpose is to reallocate cash from those with excess money balances to those with a need for cash. In the first market the banking sector opens before trading and agents can borrow and deposit after observing their production and consumption shocks. Then, buyers and sellers trade. In the second market the goods market and the banking sector opens simultaneously. We assume net settlements, i.e. all financial claims are settled at the end of the period. This essentially means that loans and deposits cannot be rolled over. Consequently, all financial contracts are one-period contracts.

In this paper we think of banking as a record keeping technology for financial transactions but

⁵The difference in preferences over the good sold in the last market allows us to impose technical conditions such that the distribution of money holdings is degenerate at the beginning of a period.

⁶Here, we assume that the central bank has the authority to control entry to the market. The lump-sum tax then is an entry fee. Note that it does not mean that the central bank can force agents to produce or consume certain quantities once they have entered the market. Nor does it mean the central bank knows the identity of the agents.

not good market transactions. This ensures that money is still essential for goods trade. We assume that banks do not issue tangible objects as deposit claims. Consequently, there are no bank notes in circulation. So the only role for banks in our paper is to reallocate fiat money within a period.⁷

Figure 1a: Cash only trades

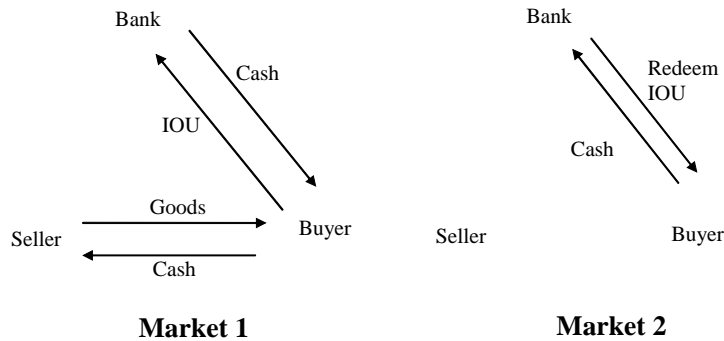


Figure 1b: Credit card with cash settlement

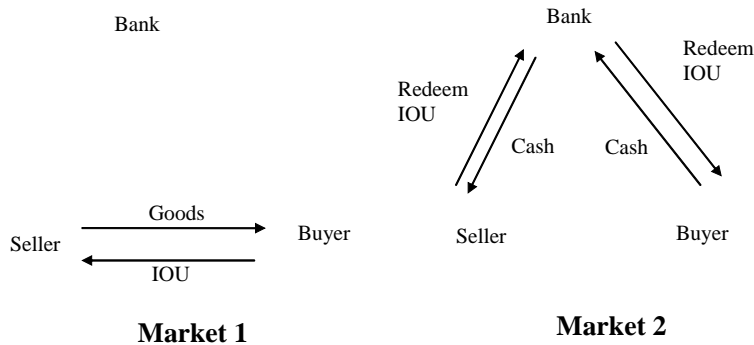


Figure 1. Cash trades vs credit cards.

Buyers do not face a standard cash constraint in our model. They can borrow cash to supplement their money holdings at the cost of the nominal interest rate. This is equivalent to buying on credit. The only difference is that the seller is not involved with the credit transaction as occurs with a credit card transaction. Another difference is that unlike a credit card transaction the bank knows nothing about the goods transactions so that money is still essential. These differences are illustrated in Figure 1. Figure 1a describes the flow of goods, credit and money in our model in markets 1 and 2. Note the absence of links between the seller and the bank. Figure 1b describes

⁷This is fundamentally different from Cavalcanti-Wallace (1999a,b) who assume that some agents' goods market trades are public information, which allows these agents to issue tangible objects that serve as inside money. In fact, in our model banks are nothing more than cash machines that post interest rates for injections and withdrawals. In equilibrium these posted interest rates clear out the excess cash.

the same flow for a credit card transaction with cash settlement which has clear links between the seller and the bank.

Since goods are perishable across markets, loans are unsecured and thus repayment is a serious issue. We consider two cases. In the first case, banks are assumed to be able to force repayment. This allows us to focus on how the provision of liquidity via borrowing and lending affects the allocation. In the second case, we assume that a borrower who fails to repay his loan will be shut out of the banking sector in all future periods. That is, although banks compete they share information about agents' repayment histories. Given this punishment, we need to derive conditions to ensure voluntary repayment.⁸

At the beginning of the first market sellers will hold idle balances since they are not consuming. In contrast, buyers may have insufficient cash and so are willing to borrow to finance consumption in the first market. Banks accept nominal deposits and pay the nominal interest rate i_d and make nominal loans at nominal rate i . The banking sector is perfectly competitive, so banks take these rates as given. There are no operating costs but they may face reserve requirements on deposits. It is straightforward to show that in a symmetric equilibrium all borrowers take out the same size loan, l , and depositors deposit the same amount d .

The zero profit condition implies that

$$i(1-n)l = i_d n d \tag{1}$$

where the total loan size is

$$(1-n)l = (1-\mu)nd \tag{2}$$

and μ is the reserve requirement ratio. Consequently (1) and (2) imply

$$i_d = (1-\mu)i \tag{3}$$

To ensure that $i > i_d$ we impose throughout the paper that $\mu > 0$ where $i \rightarrow i_d$ when $\mu \rightarrow 0$. If $\mu = 1$, there is no financial intermediation so that the allocation is equivalent to one without banks.

⁸Aiyagari and Williamson (2000) proceed in a similar fashion in a dynamic contracting model with private information and money.

4 Symmetric equilibrium

In period t , let ϕ be the real price of money in the second market. We study equilibria where end-of-period real money balances are time-invariant

$$\phi M = \phi_{-1} M_{-1} = \Omega. \quad (4)$$

We refer to it as a stationary equilibrium.

Consider a stationary equilibrium. Let $V(m_1)$ denote the expected value from trading in market 1 with m_1 money balances conditional on the aggregate shock. Let $W(m_2, l, d)$ denote the expected value from entering the second market with m_2 units of money, l loans, and d deposits. In what follows, we look at a representative period t and work backwards from the second to the first market.

4.1 The second market

In the second market agents produce and trade good x , repay loans, redeem deposits and adjust their money balances. If an agent has borrowed l units of money, then he pays $(1+i)l$ units of money. If he has deposited d units of money, he receives $(1+i_d)d$. The representative agent's program is

$$W(m_2, l, d) = \max_{x, h, m_{1,+1}} [U(x) - h + \beta V(m_{1,+1})] \quad (5)$$

$$\text{s.t. } x + \phi m_{1,+1} = h + \phi(m_2 + \tau_2 M_{-1}) + \phi(1+i_d)d - \phi(1+i)l$$

where $m_{1,+1}$ is the money taken into period $t+1$ and ϕ is the real price of money. Rewriting the budget constraint in terms of h and substituting into (5) yields

$$\begin{aligned} W(m_2, l, d) &= \phi [m_2 + \tau_2 M_{-1} - (1+i)l + (1+i_d)d] \\ &\quad + \max_{x, m_{1,+1}} [U(x) - x - \phi m_{1,+1} + \beta V(m_{1,+1})]. \end{aligned}$$

The first-order conditions are

$$\begin{aligned} U'(x) &= 1 \\ -\phi + \beta V'(m_{1,+1}) &\leq 0 \quad (= 0 \text{ for } m_{1,+1} > 0) \end{aligned} \quad (6)$$

where $V'(m_{1,+1})$ is the marginal value of an additional unit of money taken into period $t+1$.

Notice that the optimal choice of x is the same across time for all agents. Furthermore, the m_1 chosen in t is independent of m_2 in $t-1$, since $W(m_2, l, d)$ is linear in m_2 so that the marginal

value of money is the same for all agents:

$$W_m = \phi. \tag{7}$$

The implication is that regardless of how much money the agent brings into the second market, all agents enter the following period with the same amount of money. Those who bring too much money into the second market, spend the excess cash on goods, while those with too little money produce extra output to sell for money. As a result, the distribution of money holdings is degenerate at the beginning of the following period. Also note that

$$W_l = -\phi(1+i) \tag{8}$$

$$W_d = \phi(1+i_d). \tag{9}$$

Finally, to ensure that all loans can be feasibly repaid in market two, let v be the ratio of aggregate nominal loan repayments, $(1-n)(1+i)l$, to the money stock, M . If $v \leq 1$, then borrowers can repay their loans with one trip to the bank only since the nominal demand for cash by borrowers for repayment of loans is less than M . If $v > 1$, borrowers cannot acquire sufficient balances in the aggregate to repay loans at once. This implies that they repay part of their loans which is then used to settle deposit claims and the cash reenters the goods market as depositors use the cash to acquire more goods. This recycling of cash occurs until all claims are settled.

4.2 The first market

Let q_b and q_s respectively denote the quantities consumed by a buyer and produced by a seller trading in market 1. Let p be the nominal price of goods in market 1. For any value of $\mu > 0$, it is straightforward to show that agents who are buyers will never deposit funds in the bank and sellers will never take out loans. Thus, $l_s = d_b = 0$. In what follows we let l denote loans taken out by buyers and d deposits of sellers. We also drop these arguments in $W(m, l, d)$ where relevant for notational simplicity.

An agent who has m_1 money at the opening of the first market has expected lifetime utility

$$\begin{aligned} V(m_1) = & (1-n)[u(q_b) + W(m_1 + \tau_b M_{-1} + l - pq_b, l)] \\ & + n[-c(q_s) + W(m_1 + \tau_s M_{-1} - d + pq_s, d)] \end{aligned} \tag{10}$$

where pq_b is the amount of money spent as a buyer, and pq_s the money received as a seller. Once the preference shock occurs, agents become either a buyer or a seller.

Sellers' decisions. If an agent is a seller in the first market, his problem is

$$\begin{aligned} \max_{q_s, d} & [-c(q_s) + W(m_1 + \tau_s M_{-1} - d + pq_s, d)] \\ \text{s.t.} & \quad d \leq m_1 + \tau_s M_{-1} \end{aligned}$$

The first-order conditions are

$$\begin{aligned} -c'(q_s) + pW_m &= 0 \\ -W_m + W_d - \lambda_d &= 0. \end{aligned}$$

where λ_d is the Lagrangian multiplier on the deposit constraint. Using (7), the first condition reduces to

$$-c'(q_s) + p\phi = 0. \quad (11)$$

Note that q_s is independent of m_1 and d . Consequently, sellers produce the same amount no matter what financial decisions they make. Using (7) and (9), the first-order condition for d reduces to

$$\phi i_d = \lambda_d.$$

Note that if the deposit constraint is non-binding, then $i_d = 0$. If the constraint is binding, then $i_d = \lambda_d/\phi > 0$.

Buyers' decisions. If an agent is a buyer in the first market, his problem is:

$$\begin{aligned} \max_{q_b, l} & [u(q_b) + W(m_1 + \tau_b M_{-1} + l - pq_b, l)] \\ \text{s.t.} & \quad pq_b \leq m_1 + \tau_b M_{-1} + l \end{aligned}$$

Notice that buyers cannot spend more cash than what they bring into the first market, m_1 , plus their borrowing, l , and the transfer $\tau_b M_{-1}$.

The buyers first-order conditions are

$$u'(q_b) - pW_m - p\lambda = 0 \quad (12)$$

$$W_m + W_l + \lambda = 0 \quad (13)$$

where λ is the multiplier on the buyer's cash constraint. Using (7) and (8), (13) reduces to

$$\phi i = \lambda \quad (14)$$

If the constraint is not binding, then $\lambda = i = 0$ and, using (7) and (11), (12) reduces to

$$u'(q_b) = c'(q_s).$$

Hence trades are efficient.⁹

When $\lambda > 0$ the buyer spends all of his money and consumes $q_b = (m_1 + \tau_b M_{-1} + l)/p$ and $i = \lambda/\phi > 0$. Then, equations (11), (12), and (14) imply

$$\frac{u'(q_b)}{c'(q_s)} = 1 + i \quad (15)$$

For $i > 0$, $u'(q_b) > c'(q_s)$ which means trades are inefficient. Note that (15) is a common condition found in money in the utility function or cash in advance models. It simply means that a positive nominal interest rate acts as tax on consumption. The nominal interest rate depends on real production and real consumption of the current period only.

Marginal value of money. The marginal value of money satisfies

$$V'(m_1) = (1 - n) \frac{u'(q_b)}{p} + n\phi(1 + i_d)$$

In the appendix we show that the value function is concave in m .

Using (11) with $c'(q_s) = 1$, $V'(m_1)$ reduces to

$$V'(m_1) = \phi [(1 - n) u'(q_b) + n(1 + i_d)]. \quad (16)$$

Note that banks increase the marginal value of money because sellers can deposit idle cash and earn interest. This is captured by the second term on the right-hand side. If there are no banks this term is just n .

4.3 Equilibrium allocation

We now derive the symmetric equilibrium. Market clearing implies $q_b = \frac{n}{1-n} q_s$. Note that by symmetry we impose that all sellers produce the same quantity q_s even though they are indifferent as to how much they produce.

In a steady state equilibrium the real value of money is constant so $\phi M = \phi_{-1} M_{-1}$. Thus $\gamma = \frac{\phi_{-1}}{\phi}$ implying that the gross inflation rate equals the gross growth rate of the money supply in steady state.

⁹With $1 - n$ buyers and n sellers, the planner maximizes $(1 - n)u(q_b) - nc(q_s)$ s.t. $(1 - n)q_b = nq_s$. Use the constraint to replace q_s in the maximand. The first-order condition for q_b is $u'(q_b) = c'(q_s)$.

Use (6) lagged one period to eliminate $V'(m_1)$ and (15) to eliminate $u'(q_b)$ from (16) to get¹⁰

$$\frac{\gamma - \beta}{\beta} = (1 - n)i + ni_d \quad (17)$$

The right-hand side measures the value of bringing one extra unit of money into the first market. The first term reflects the interest saving from borrowing one less unit of money when a buyer and the second term is the value of depositing an extra unit of money when a seller.

Substituting for i_d yields

$$\frac{\gamma - \beta}{\beta} = (1 - n\mu)i \quad (18)$$

We can rewrite this in terms of q_b using (15) to get

$$\frac{\gamma - \beta}{\beta} = (1 - n\mu)[u'(q_b) - 1] \quad (19)$$

We now derive the conditions under which borrowers repay their loans. For buyers entering the second market with no money, who repay their loans, the expected discounted utility in a steady state is

$$\mathcal{U} = U(x^*) - h_b + \sum_{t=1}^{\infty} \beta^t [(1 - n)u(q_b) - nc(q_s) + U(x^*) - Eh]$$

where h_b is a buyer's production in the second market and Eh is expected production.

Consider the case of a buyer who reneges on his loan. The benefit of reneging is that he has more leisure in the second market because he does not work to repay the loan. The cost is that he is out of the banking system, meaning that he cannot borrow or deposit funds for the rest of his life. He cannot lend because the bank would confiscate his deposits to settle his loan arrears. Thus, a deviating buyer's expected discounted utility is

$$\widehat{\mathcal{U}} = U(\widehat{x}) - \widehat{h}_b + \sum_{t=1}^{\infty} \beta^t [(1 - n)u(\widehat{q}_b) - nc(\widehat{q}_s) + U(\widehat{x}) - E\widehat{h}]$$

where the hat indicates the optimal choice by a deviator.

A buyer has no incentive to deviate if

$$\mathcal{U} \geq \widehat{\mathcal{U}} \quad (20)$$

Both the value of being in the banking system \mathcal{U} as well as the expected discounted utility of defection $\widehat{\mathcal{U}}$ depend on growth rate of the money supply γ . This puts constraints on γ and the reserve ratio μ that the monetary authority can impose without destroying the banking system.

We can now state the following

¹⁰From this equation it follows that if nominal bonds could be traded in market 2, then their nominal rate of return i_b would be $(1 - n)i + ni_d$. For $\mu = 0$ the return would be $i_b = i_d$. Thus agents would be indifferent between holding a nominal bond or holding a bank deposit.

Definition 1 *A monetary equilibrium with banking is a quantity q_b satisfying (19) and (20).*

Equilibrium When Defection Is Not Possible. We first consider the role of banking and the optimal monetary policy when defection is not feasible. This allows us to focus on the provision of liquidity via borrowing and lending affects the allocation. In order to do so we drop the defection constraint (20) from Definition 1.

We can now state

Proposition 1 *Assume that defection is not possible. Then if $\gamma \geq \beta$, a monetary equilibrium with banking exists and is unique for $\gamma > \beta$. Equilibrium consumption is decreasing in μ and γ , and satisfies $q_b < q^*$ with $q_b \rightarrow q^*$ as $\gamma \rightarrow \beta$.*

It is clear from (19) that money is neutral, but not super-neutral. Increasing its stock has no effect on q_b , while changing the growth rate γ does. Moreover, when defection is impossible, the Friedman rule generates the first-best allocation. Away from the Friedman rule, by holding required reserves, banks restrict lending which makes consumption more costly. As a result, the required reserve ratio μ acts like a tax on consumption.

From Proposition 1 it follows that

Corollary 1 *Banks improve the allocation and welfare when $\mu < 1$ and $\gamma > \beta$.*

The key result of this section is that banks improve the allocation in the economy away from the Friedman rule.¹¹ Most interestingly, banks have their greatest impact on welfare for moderate values of inflation. The reason is that near the Friedman rule there is little gain from redistributing idle cash balances while for high inflation rates money is of little value anyway. This is shown in Figure 2 where the difference in expected lifetime utility with banks ($\mu = 0$) and without ($\mu = 1$) is drawn as a function of the inflation rate γ .

¹¹In a sense, banks in our model have a similar function as Kocherlakota's (2003) 'illiquid' bonds - they transfer money from those with low marginal value of money to those with a high one.

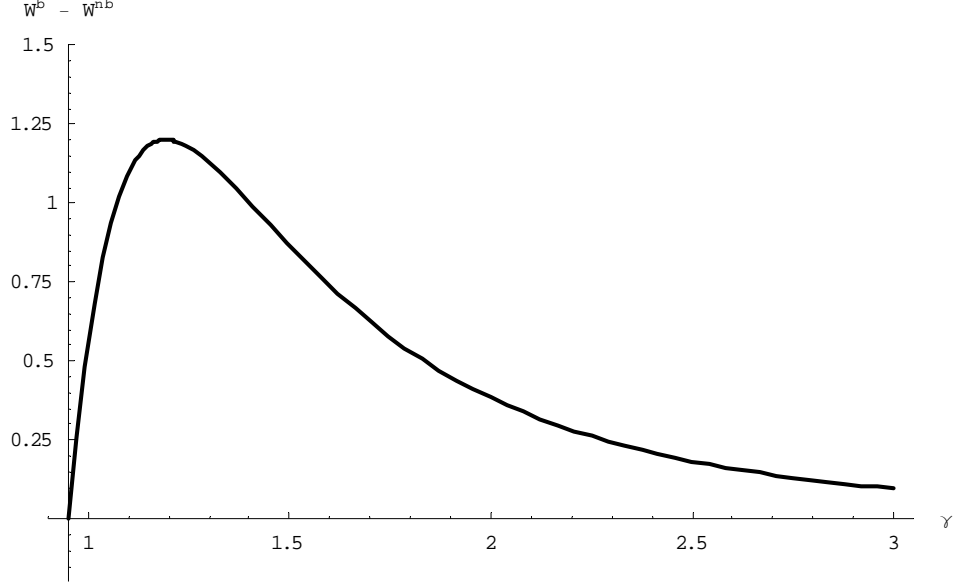


Figure 2: Difference in welfare

Given that banks improve the allocation by shifting idle cash from sellers to buyers, one wonders if such an improvement can be achieved by having the government redistribute the cash by imposing lump-sum taxes on sellers and giving the cash as lump-sum transfers to buyers. From Proposition 1 we can state:

Corollary 2 *The allocation with banks cannot be replicated by government policies using lump-sum transfers or taxes.*

The proof follows immediately from inspection of (19); neither τ_1 nor τ_b appear in this equation. Hence, varying the transfer across the two markets or by transferring cash from sellers to buyers via lump-sum taxes and transfers has no effect on q_b . It only affects the equilibrium price of money in the last market. To see this, note that in equilibrium the buyers' money holdings satisfy

$$(1 + \tau_b) M_{-1} + \frac{(1 - \mu)n}{1 - n} (1 + \tau_s) M_{-1} = \theta z M$$

where

$$\theta = \frac{1 - \mu n}{1 - n} \geq 1 \quad \text{and} \quad z = \frac{1 - n\mu + \mu(1 - n)\tau_b + (1 - \mu)\tau_1}{(1 - n\mu)\gamma}.$$

In equilibrium, $pq = \theta z M$ and $p = 1/\phi$ so

$$\phi = \frac{q}{\theta z M}.$$

For a given value of γ , if τ_b or τ_1 increases agents know that they will receive more cash when a buyer in market 1. Consequently, they simply reduce the amount of money they bring into market 1 and so the demand for money declines in market 2 which reduces the price of money.

This shows that the allocation with banks cannot simply be replicated by a policy of lump-sum taxes and transfers. Thus, reallocation of idle balances through a credit market is very different from a redistribution from sellers to buyers using lump-sum taxes and transfers. Redistributing money through the credit market affects marginal decisions; sellers are compensated on the margin by i_d and buyers pay i for a marginal unit of money. In contrast, redistribution through lump-sum transfers does not affect marginal decisions. This is why the banking system has real effects when moving money from sellers to buyers.

Equilibrium When Defection Is Possible. Since production is costly, those who borrow in market 1 have an incentive to default in market 2. To offset this short-run benefit we assume that if an agent defaults on his loan then the only punishment is permanent exclusion from the banking system. For a banking equilibrium to exist, it must be the case that borrowers prefer repaying loans to being banished from the banking system.

Proposition 2 *If $\beta = \gamma$, no monetary equilibrium with banking exists.*

The reason why banks cannot exist under the Friedman rule is quite intuitive. Suppose the Friedman rule holds. If a borrower works to repay his loan in market 2, he is strictly worse off than when he defaults since the outside option (trading with money only) yields the efficient consumption q^* in all future periods. In short, at the Friedman rule, agents are able to smooth consumption optimally without having access to financial markets. As a result, borrowers will not repay their loans and so financial intermediation is impossible. This result is related to Aiyagari and Williamson (2000) who also report a break-down of financial intermediation in a dynamic contracting model with private information at the Friedman rule. This is also similar to the arguments of Bulow and Rogoff (1989) who showed that a sovereign government would not repay its foreign debt if it has the ability to use the loan proceeds to buy consumption insurance on world financial markets.

In order to analyze the economy away from the Friedman rule assume that preferences are such that $q[u'(q) - 1]$ is monotonically decreasing in q . This condition simply ensures that a deviator takes more money into market 1 than non-deviators. This condition is reasonable since it implies that the deviator holds more money to compensate for the loss of consumption insurance provided by the banking system.¹²

¹²This condition is satisfied for example by the utility function $u(q) = \frac{q^{1-\sigma}-1}{1-\sigma}$ when $\sigma \geq 1$.

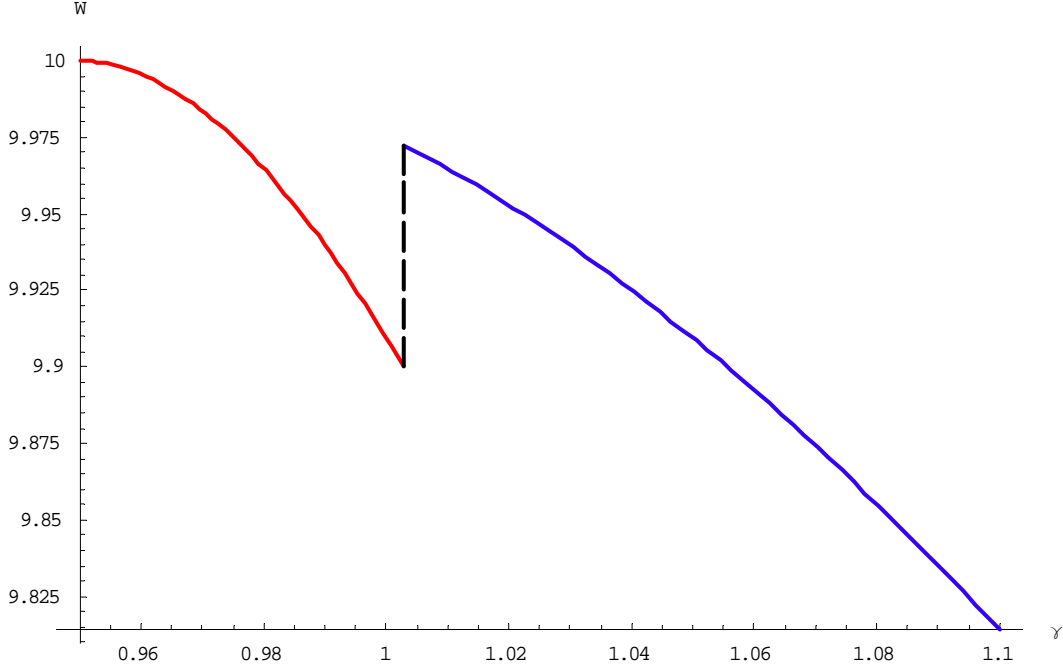


Figure 1: Figure 3: Welfare when Defection is Possible

Proposition 3 *For any $\gamma > 1$ there exists a critical value $1 > \tilde{\beta} > 0$ such that if $1 > \beta \geq \tilde{\beta}$, a monetary equilibrium with banking exists and is unique.*

The proof of existence of a monetary equilibrium with banking requires that the gross growth rate of the money supply is not too low. The intuition behind the lower bound on γ is related to Proposition 2. If γ is too close to β , agents are not willing to repay their loans because they can self-insure against consumption risk using only money since the cost of doing so is low. It should be noted that the condition $\gamma > 1$ is a sufficient condition. Equilibria with banking might also exist if this condition is violated as some numerical simulations have shown.

When defection is a serious issue, then we obtain an interesting result regarding welfare in our economy. For γ sufficiently close to 1 (price stability) the banking equilibrium exists for sufficiently high values of β . But if γ falls even slightly below one, such that there is a small deflation, then financial intermediation is not possible since agents would default on loans. Thus, there is a large drop in consumption, production and welfare. This is shown in Figure 3 which is a simulation of our economy.

While deflation at the Friedman rule maximizes welfare, small rate of deflation is associated with a significant drop in welfare relative to what occurs from a small rate of inflation.

We find this to be an interesting result because it seems to capture why central banks seem to fear deflation. Even though most models of money show that the Friedman rule is optimal there is a belief that deflation raises the real value of debt and creates incentives for debtors to default. Default in turn may have serious macroeconomic effects (Fisher (1933)). Our model captures this idea by pointing out that if default is the likely outcome, then financial intermediation collapses and the benefits of intermediation disappear.

5 Aggregate shocks

In this section we will consider aggregate shocks to the composition of buyers and sellers. When the measures of buyers and sellers are random the total demand for goods and the amount of idle cash balances in the economy is also random. Although the measure of sellers is random, with linear production costs the supply of output is horizontal, so equilibrium output is entirely demand determined. With banks, deposits and loans will be random also. This suggests that banks may amplify shocks to the economy. In fact, we show below that although banks increase consumption they also create more consumption volatility for individual agents. To keep the analysis simple, we assume repayment of loans.

Rather than being constant we now assume n is determined as follows:

$$n_t = \begin{cases} n^H & \text{with probability } \pi \\ n^L & \text{with probability } 1 - \pi \end{cases}$$

where H denotes a high seller state and L denotes a low seller state. This means when state H occurs, there are many sellers and few buyers so aggregate deposits are high and aggregate loan demand is low. Shocks to n can be thought of as shocks to idle balances. We also assume that $n = \pi n^H + (1 - \pi) n^L$ where n is the number of sellers in the deterministic model. This allows us to compare across models.

Because there is randomness in deposits, the central bank may choose to make the market 1 injections contingent on the aggregate state of the economy. Thus, let $\tau_1^H M_{-1}$ be the transfer in state H and $\tau_1^L M_{-1}$ be the transfer in state L where $\tau_1^j + \tau_2^j = \tau = \gamma - 1$ for $j = H, L$ with $\tau_b = \tau_s = \tau_1$.

The agents' optimization choices for how much to buy and sell are unchanged with the only difference being that the choices are state contingent. The value function in market 1 is

$$V(m_1) = \pi V^H(m_1) + (1 - \pi) V^L(m_1) \tag{21}$$

where for $j = H, L$

$$V^j(m_1) = \pi(1-n^j) \left[u(q_b^j) + W(m_1 + \tau_1^j M_{-1} + l^j - p^j q_b^j, l^j) \right] \\ + \pi n^j \left[-c(q_s^j) + W(m_1 + \tau_1^j M_{-1} - d^j + p^j q_s^j, d^j) \right]$$

The sellers choose to produce

$$\phi^j p^j = c'(q_s^j) = 1$$

while buyers set

$$u'(q_b^j) = 1 \quad \text{if } \lambda^j = 0 \\ q_b^j = \phi^j \theta^j z^j M \quad \text{if } \lambda^j > 0 \quad (22)$$

for $j = H, L$, where $z^j = \frac{1+\tau_1^j}{1+\tau}$, $\theta^j = \frac{1-\mu n^j}{1-n^j}$ and $\theta^H > \theta^L$. This last inequality follows from the fact that in the high state deposits are also high thus each borrower can borrow more. Finally, let $q_b^j = \left(\frac{n^j}{1-n^j}\right) q_s^j = q^j$ for $j = H, L$.

We can now state the following

Proposition 4 *Assume $z^H \theta^H \geq z^L \theta^L$. For $\gamma \geq \beta$, a monetary equilibrium exists with $q^H = q^L = q^*$ at $\gamma = \beta$. For $\gamma \in (\beta, \tilde{\gamma}]$ there is a unique monetary equilibrium with $q^L < q^H = q^*$. For $\gamma > \tilde{\gamma}$, there is a unique monetary equilibrium with $q^L < q^H < q^*$.*

Once again the Friedman rule replicates the first-best allocation. At the Friedman rule there is no value of allocating excess cash. Obviously there is no role for banks and no role for stabilization.

Near the Friedman rule, Proposition 4 states that buyers are constrained in the low deposit state but not in the high state. Thus, the nominal interest rate is zero in the high state. In the low state it is positive and satisfies

$$i^L = \frac{\gamma - \beta}{\beta(1-\pi)(1-n^L\mu)}$$

From these equations it is clear that the nominal interest rate is increasing in γ and the reserve ratio μ . Interestingly, since the nominal interest rate is independent of the injection z^L consumption q^L is also independent. Consequently, the central bank cannot affect consumption in market 1 in the low inflation economy given γ .

In contrast, when inflation is sufficiently high the central bank can affect consumption across states since

$$\frac{q^H}{q^L} = \frac{\theta^H z^H}{\theta^L z^L}$$

in the high inflation economy. From this condition it is clear that the central bank can affect the relative magnitudes of consumption in the two states by changing the values of z^H and z^L . It follows that if $z^H\theta^H \geq z^L\theta^L$, then $q^H > q^L$.

Finally, it should be noted that if $z^H\theta^H < z^L\theta^L$ then Proposition 4 must be modified such that when inflation is low agents are constrained in the high state and not in the low state and $q^L > q^H$ for all γ .

Role of banks We want to compare the equilibrium outcome in the banking economy to the no-banking case when these shocks occur. For this purpose we initially eliminate any policy responses to these shocks. Consequently, the central bank acts ‘passively’ and sets τ_1 such that $z^H = z^L = z$.

We can replicate the no-banking equilibrium simply by setting $\mu = 1$ so that $\theta^H = \theta^L = 1$. In this case the quantities consumed in the no-banking equilibrium are

$$q^H = q^L = q = z\Omega$$

where Ω solves

$$\frac{\gamma - \beta}{\beta} = (1 - n) [u'(z\Omega) - 1].$$

Since we have assumed $n = \pi n^H + (1 - \pi) n^L$, Ω has the same value as in the deterministic case. Hence, in the no banking equilibrium, buyers consume the same quantity across states since they can only spend the cash they bring into market 1 which is independent of the state that is realized. This occurs for all inflation rates. The problem is that while there are many sellers in state H , the idle cash cannot be reallocated to buyers. Aggregate consumption in the economy in state H is lower however since there are few buyers in this state.

Now consider what happens when banks exist, i.e. $\theta^H > \theta^L > 1$. Now the idle cash from sellers is deposited into the banking system and lent back out to buyers. In state H , there is a large number of depositors and relatively few borrowers. Hence, the banks must lower the nominal interest rate to get rid of the excess cash. The lower nominal interest rate expands borrowing and leads to higher quantities of goods consumed by buyers. In state L , the opposite occurs - there are relatively few depositors and a large number of borrowers. This forces the nominal interest rate to rise and reduces the quantity of goods per buyer that can be purchased. Thus, $q^H > q^L$ in equilibrium.

Corollary 3 *Assume $\gamma > \beta$. Banks make individual consumption more volatile but stabilize aggregate consumption.*

The interesting aspect of this result is that while banks raise average consumption across states, they transmit deposit shocks to buyers making individual consumption more volatile. Nevertheless, it is straightforward to show that aggregate consumption is more stable when measured by the coefficient of variation (See appendix). The next question to address is whether or not a passive monetary policy is optimal.

Optimal stabilization policy What is the optimal response of the central bank to these shocks? We assume the central bank's stabilization policy maximizes the welfare of the representative agent for a given a steady-state inflation rate. In short, suppose the central bank could pick the quantities consumed and produced in each state subject to the constraint that the chosen quantities satisfy the conditions of a stochastic steady state competitive equilibrium. This is similar to a Ramsey problem.¹³ We now assume that the growth rate of the money supply is determined exogenously to finance a fixed amount of government consumption in market 2. This implies $\tau > 0$. However, the central bank has the power to set τ_1^j and τ_2^j such that the growth rate is maintained at the exogenously given value. By choosing τ_1^j and τ_2^j the central bank determines the quantities consumed in each state.

As shown in the appendix, the steady state welfare of the representative agent at the beginning of period t is given by

$$(1 - \beta) V(M_{-1}) = U(x) - x - g + \pi(1 - n^H)[u(q^H) - q^H] + (1 - \pi)(1 - n^L)[u(q^L) - q^L]$$

where g is the real amount of government spending that is financed in market 2. It is obvious that $x = x^*$ so all that remains is to choose q^H and q^L .

The problem facing the central bank is

$$\begin{aligned} & \underset{q^H, q^L}{Max} \quad \pi(1 - n^H)[u(q^H) - q^H] + (1 - \pi)(1 - n^L)[u(q^L) - q^L] \\ \text{s.t.} \quad & \frac{\gamma - \beta}{\beta} = \pi(1 - n^H)\theta^H[u'(q^H) - 1] + (1 - \pi)(1 - n^L)\theta^L[u'(q^L) - 1] \end{aligned} \quad (23)$$

where the constraint is derived in the appendix. From now on we consider high enough levels of g and γ such that buyers are liquidity constrained in both states.

¹³Although our model is a dynamic, by studying optimal policy in a stochastic steady state, our approach looks very much like the static Ramsey problem as opposed to the dynamic problem studied by Lucas and Stokey (1984). In particular, we are ignoring the time inconsistency problem that arises in a dynamic analysis.

The first-order conditions are

$$\begin{aligned} u'(q^H) - 1 + \lambda \theta^H u''(q^H) &= 0 \\ u'(q^L) - 1 + \lambda \theta^L u''(q^L) &= 0. \end{aligned}$$

To ensure that this corresponds to a maximum we assume $u''(q)u''(q) - [u'(q) - 1]u'''(q) > 0$ which can be shown to be a sufficient condition so that the first-order conditions describe a maximum. Since the constraint binds, $\lambda > 0$ and so q^H and q^L are inefficiently low. The first-order conditions can be rewritten as

$$\frac{u'(q^L) - 1}{u''(q^L)} \frac{u''(q^H)}{u'(q^H) - 1} = \frac{\theta^L}{\theta^H} \quad (24)$$

We can now state the following:

Proposition 5 *The constrained planner's choice of quantities yields $q^L = q^H$ when there are no banks and $q^L > q^H$ with banks.*

Without banks $\theta^H = \theta^L = 1$ and it is clear from (24) that the planner chooses $q^H = q^L$ which implies $z^H = z^L = z$. The reason is that although the central bank can alter q^H and q^L via its choice of τ_1^H and τ_1^L , doing so would only create consumption variability which is welfare reducing. Consequently, the best stabilization policy is to keep quantities constant across states. Note that this implies aggregate consumption still varies across states.

Now suppose banks exist implying $\theta^H > \theta^L > 1$. With banks the ordering on the quantities is exactly reverse from what happens when the central bank is passive. With a passive policy, buyers consume more in state H since loans are plentiful and cheap in that state. However, this is just the opposite of what is optimal. From the viewpoint of the representative agent looking into the future, he wants to consume more when he is more likely to desire consumption, which is when he is a buyer. This corresponds to state L . However this is the state when there is little liquidity in the banking system since the number of depositors is low. Thus, what the planner would like to do is put more liquidity in the system when loan demand is high and deposits are low then take it out when loan demand is low and deposits are high. To accomplish this, the central bank injects more cash into the system in state L and less cash in state H . Hence, $z^L > z^H$ and these values are chosen to generate the quantities solving (23) and (24).

An interesting implication of this policy is that the central bank is essentially providing an elastic supply of currency to the economy – when demand for liquidity is high, the central bank provides additional currency and withdraws it when the demand for liquidity is low. Note what this does

NOT imply – a constant nominal interest rate across states, which would imply that consumption is the same in both states, i.e. $q^H = q^L$. It is easy to show that stabilizing interest rates would be welfare improving compared to the passive monetary policy but having them fluctuate in the correct fashion is optimal.

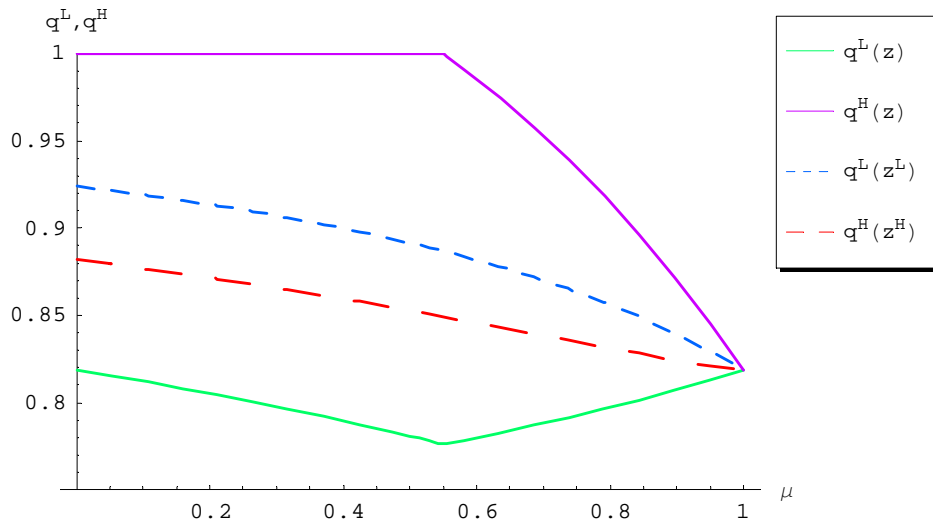


Figure 4. Equilibrium vs optimal consumption

Figure 4 shows equilibrium and optimal consumption as functions of the reserve ratio μ for a given inflation rate γ . Note that for low reserve ratios equilibrium consumption is efficient in state H and inefficiently low in state L . Interestingly, increasing the reserve ratio decreases consumption in the low state as long as the economy is in the first equilibrium. The opposite is true in the second equilibrium. If we compare the central banks solution with the equilibrium quantities we see two things. First, the ordering of the quantities are reversed across states. Second, the central bank smooths consumption across states.

6 Conclusion

In this paper we showed that reallocating idle cash via a banking system can expand output and improve welfare. Such an improvement cannot be achieved through a government policy of lump-sum taxes and transfers. Interestingly, banks have there greatest impact on welfare for moderate rates of inflation. However, banks may amplify shocks and create variability in consumption and interest rates. We then show that there is a role for central bank policy to smooth consumption and interest rates by providing an elastic supply of currency. However, the optimal policy does not call for complete elimination of these fluctuations.

Our framework is open to many extensions such as private bank note issue, financing of investment instead of consumption, and longer term financial contracts. We could also extend the model to investigate the role of banks in transmitting other type of shocks such as productivity shocks or preference shocks. Finally, the interaction of government regulation and stabilization policies would allow analysis of different monetary arrangements such as expressed by the real-bill doctrine or the quantity theory as studied in Sargent and Wallace (1982).

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Appendix

Marginal value of money Differentiating (10) with respect to m_1

$$V'(m_1) = (1-n) \left[u'(q_b) \frac{\partial q_b}{\partial m_1} + W_m \left(1 - p \frac{\partial q_b}{\partial m_1} + \frac{\partial l}{\partial m_1} \right) + W_l \frac{\partial l}{\partial m_1} \right] \\ + n \left[-c'(q_s) \frac{\partial q_s}{\partial m_1} + W_m \left(1 + p \frac{\partial q_s}{\partial m_1} - \frac{\partial d}{\partial m_1} \right) + W_d \frac{\partial d}{\partial m_1} \right]$$

Recall from (7), (8), and (9) that $W_m = \phi$, $W_l = -\phi(1+i)$ and $W_d = \phi(1+i_d) \forall m_2$. Furthermore, $\frac{\partial q_s}{\partial m_1} = 0$ because the quantity a seller produces is independent of his money holdings. We also know that $\frac{\partial d}{\partial m_1} = 1$ since a seller deposits all his cash when $i > 0$. Hence,

$$V'(m_1) = (1-n) \left[u'(q_b) \frac{\partial q_b}{\partial m_1} + \phi \left(1 - p \frac{\partial q_b}{\partial m_1} + \frac{\partial l}{\partial m_1} \right) - \phi(1+i) \frac{\partial l}{\partial m_1} \right] + n\phi(1+i_d)$$

Since $i > 0$ implies $pq_b = m_1 + \tau_b M_{-1} + l$ we have $1 - p \frac{\partial q_b}{\partial m_1} + \frac{\partial l}{\partial m_1} = 0$. Hence¹⁴

$$V'(m_1) = (1-n) \frac{u'(q_b)}{p} + n\phi(1+i_d)$$

In a symmetric equilibrium $q_b = \frac{1-n}{n} q_s$. Define $m^* = pq^*$ where q^* solves $u'(q) = 1$. Then if $m_1 < m^*$, $0 < q_b < q^*$, implying $\frac{\partial q_b}{\partial m_1} > 0$ so that $V''(m_1) < 0$. If $m_1 \geq m^*$, $q_b = q^*$ implying $\frac{\partial q_b}{\partial m_1} = 0$, so that $V''(m_1) = 0$. Thus, $V(m_1)$ is concave $\forall m$. ■

Proof of Proposition 1: Because $u(q)$ is strictly concave there is a unique value q that solves (15), and for $\gamma > \beta$, $q < q^*$ where q^* is the efficient quantity solving $u'(q^*) = 1$. As $\gamma \rightarrow \beta$, $u'(q) \rightarrow 1$, $q \rightarrow q^*$, and from (18) $i \rightarrow 0$. In this equilibrium, the Friedman rule sustains efficient trades in the first market. Furthermore, from (15) q is decreasing in μ . Since there is no lending when $\mu = 1$ the allocation corresponds to the model without banking. Since $V(m_1)$ is concave, then for $\gamma > \beta$, the choice m_1 is maximal. ■

Proof of Proposition 2 and 3: We now determine the conditions under which (20) holds. Recall that, due to idiosyncratic trade shocks and financial transactions, money holdings are heterogeneous after the first market closes. Therefore, if we set $m_1 = M_{-1}$, the money holdings of agents at the opening of the second market are $m_2 = 0$ for buyers and $m_2 = \left(\frac{1-n}{n}\right) \theta (1 + \tau_1) M_{-1}$ for sellers.

¹⁴Note that $u'(q_b) \frac{\partial q_b}{\partial m_1} - \phi(1+i) \frac{\partial l}{\partial m_1} = u'(q_b) \frac{\partial q_b}{\partial m_1} - \phi(1+i) \left[p \frac{\partial q_b}{\partial m_1} - 1 \right] \\ = \frac{\partial q_b}{\partial m_1} (u'(q_b) - \phi(1+i)p) + \phi(1+i) = \phi(1+i) = \frac{u'(q_b)}{p}$.

We can now solve for equilibrium consumption and production in the second market. Equation (6) gives us $x^* = U'^{-1}(1)$. The buyer's production in the second market can be derived as follows

$$\begin{aligned}
h_b &= x^* + \phi \{m_{1,+1} + (1+i)l - \tau_2 M_{-1}\} \\
&= x^* + \phi \{M - \tau_2 M_{-1} + (1+i)l\} \\
&= x^* + \phi \{(1+\tau_1)M_{-1} + (1+i)l\} \\
&= x^* + q_b + \phi il
\end{aligned}$$

since in equilibrium $m_{1,+1} = M = M_{-1} + \tau_1 M_{-1} + \tau_2 M_{-1}$ and $pq_b = (1+\tau_1)M_{-1} + l$. Thus, an agent who was a buyer in market 1 has to work to recover the production cost of his consumption and the interest on his loan.

The seller's production is

$$\begin{aligned}
h_s &= x^* + \phi \{m_{1,+1} - [pq_s + (1+i_d)d + \tau_2 M_{-1}]\} \\
&= x^* + \phi \{(1+\tau_1)M_{-1} - pq_s - (1+i_d)d\} \\
&= x^* - q_s - \phi i_d d
\end{aligned}$$

since in equilibrium $d = (1+\tau_1)M_{-1}$.

Note that the expected hours worked Eh satisfies

$$Eh = (1-n)h_b + nh_s = x^*$$

since in equilibrium $q_b = \frac{n}{1-n}q_s$ and $(1-n)li = ndi_d$.

Production can be also expressed in terms of q as in the following table

Trading history:	Production in the last market:
Buy	$h_b = x^* + \frac{q_b}{\theta} + \left(\frac{\theta-1}{\theta}\right) e(q_b) u(q_b)$
Sell	$h_s = x^* - \left(\frac{1-n}{n}\right) \left[\frac{q_b}{\theta} + \left(\frac{\theta-1}{\theta}\right) e(q_b) u(q_b)\right]$

Since we assumed that the elasticity of utility $e(q_b)$ is bounded, we can scale $U(x)$ such that there is a value $x^* = U'^{-1}(1)$ greater than the last term for all $q_b \in [0, q^*]$. Hence, h_s is positive for all $q_b \in [0, q^*]$ ensuring that the equilibrium exists.

We now derive the conditions under which borrowers repay their loans to ensure that the equilibrium exists. For buyers entering the second market with no money, who repay their loans, the expected discounted utility is

$$\mathcal{U} = U(x^*) - h_b + \sum_{t=1}^{\infty} \beta^t [(1-n)u(q_b) - nc(q_s) + U(x^*) - Eh]$$

where all variables are evaluated at their equilibrium values.

Consider the case of a buyer who reneges on his loan. The benefit of renegeing is that he has more leisure in the second market because he does not work to repay the loan. The cost is that he is out of the banking system, meaning that he cannot borrow or lend for the rest of his life. He cannot lend because the bank would confiscate his deposits to settle his loan arrears. Thus, a deviating buyer's expected discounted utility is

$$\widehat{U} = U(\widehat{x}) - \widehat{h}_b + \sum_{t=1}^{\infty} \beta^t \left[(1-n)u(\widehat{q}_b) - nc(\widehat{q}_s) + U(\widehat{x}) - E\widehat{h} \right]$$

where the hat indicates the optimal choice by a deviator. We now derive \widehat{x} , \widehat{h} , \widehat{q}_b , and \widehat{q}_s . In the last market the deviating buyer's program is

$$\begin{aligned} W(\widehat{m}_2) &= \max_{\widehat{x}, \widehat{h}, \widehat{m}_{1,+1}} \left[U(\widehat{x}) - \widehat{h} + \beta V(\widehat{m}_{1,+1}) \right] \\ \text{s.t. } &\widehat{x} + \phi \widehat{m}_{1,+1} = \widehat{h} + \phi(\widehat{m}_2 + \tau_2 M_{-1}) \end{aligned}$$

As before, the first-order conditions are

$$\begin{aligned} U'(\widehat{x}) &= 1 \\ -\phi + \beta V'(\widehat{m}_{1,+1}) &\leq 0 \quad (= 0 \text{ for } \widehat{m}_{1,+1} > 0) \end{aligned} \tag{25}$$

Thus, $\widehat{x} = x^*$. Note that $W_m = \phi$, which is the same marginal value for non-deviators. The production and consumption decisions are affected as follows. The first-order condition if the deviator becomes a seller in the first market is

$$-c'(\widehat{q}_s) + pW_m = 0.$$

Since $W_m = \phi$ and agents take prices as given the deviator is indifferent as of how much to produce. In what follows we assume that $\widehat{q}_s = \frac{1-n}{n}q_s$. Any other quantity would render the same result.

The first-order condition if the deviator becomes a buyer is

$$u'(\widehat{q}_b) - pW_m - p\lambda = 0$$

Finally, the marginal value of the money satisfies

$$V'(\widehat{m}_1) = \phi \left[\frac{(1-n)u'(\widehat{q}_b)}{c'(\widehat{q}_s)} + n \right]$$

which means that (25) can be written as

$$\frac{\gamma - \beta}{\beta} = (1-n) \left[\frac{u'(\widehat{q}_b)}{c'(\widehat{q}_s)} - 1 \right] \tag{26}$$

Now if we compare (26) with (19) we find that

$$u'(\hat{q}_b) - c'(\hat{q}_s) = \theta [u'(q_b) - c'(q_s)]$$

which implies that $\hat{q}_b < q_b$ since $\theta > 1$. Therefore $u(q_b) - c(q_s) > u(\hat{q}_b) - c(\hat{q}_s)$.

We first show that the money holdings of the deviator at the beginning of the period grow at the same rate as the one of the representative agent. To see this note that from (26) \hat{q}_b is constant across time. Consequently, we have

$$\frac{p_{+1}}{p} = \frac{\hat{m}_{1,+1} + \tau_1 M}{\hat{m}_1 + \tau_1 M_{-1}}$$

which implies that $\gamma(\hat{m}_1 + \tau_1 M_{-1}) = \hat{m}_{1,+1} + \tau_1 M$. Rearranging yields

$$\hat{m}_{1,+1} - \gamma \hat{m}_1 = \tau_1 (\gamma M_{-1} - M) = 0$$

implying

$$\frac{\hat{m}_{1,+1}}{\hat{m}_1} = \gamma.$$

The money holdings of the deviator at the opening of the second market are $\hat{m}_2 = 0$ for buyers and $\hat{m}_2 = \hat{m}_1 + \tau_1 M_{-1} + pq_s$ for sellers. The buyer's production in the second market can be derived as follows

$$\begin{aligned} \hat{h}_b &= x^* + \phi [\hat{m}_{1,+1} - \tau_2 M_{-1}] \\ &= x^* + \phi [(1 + \tau_1 + \tau_2) \hat{m}_1 + \tau_1 M_{-1} - \tau_1 M_{-1} - \tau_2 M_{-1}] \\ &= x^* + \phi [p \hat{q}_b + (\tau_1 + \tau_2) (\hat{m}_1 - M_{-1})] \\ &= x^* + \hat{q}_b + \phi (\tau_1 + \tau_2) (\hat{m}_1 - M_{-1}) \end{aligned}$$

since for a deviator $p \hat{q}_b = \hat{m}_1 + \tau_1 M_{-1}$. The last term reflects the fact that the deviator is subject to the inflation tax in a different way than the representative agent.

The seller's production in the second market can be derived as follows

$$\begin{aligned} \hat{h}_s &= x^* + \phi [\hat{m}_{1,+1} - (\hat{m}_1 + \tau_1 M_{-1}) - p \hat{q}_s - \tau_2 M_{-1}] \\ &= x^* + \phi [(\tau_1 + \tau_2) \hat{m}_1 - \tau_1 M_{-1} - p \hat{q}_s - \tau_2 M_{-1}] \\ &= x^* + \phi [-p \hat{q}_s + (\tau_1 + \tau_2) (\hat{m}_1 - M_{-1})] \\ &= x^* - q_s + \phi (\tau_1 + \tau_2) (\hat{m}_1 - M_{-1}) \end{aligned}$$

In summary, we get the following values for \hat{h}_b and \hat{h}_s :

Trading history for deviator Production in the last market:

$$\text{Buy} \quad \hat{h}_b = x^* + \hat{q}_b + \phi(\tau_1 + \tau_2)(\hat{m}_1 - M_{-1})$$

$$\text{Sell} \quad \hat{h}_s = x^* - \hat{q}_s + \phi(\tau_1 + \tau_2)(\hat{m}_1 - M_{-1})$$

From this it follows that the expected hours worked for a deviator are

$$\begin{aligned} \hat{h} &= (1-n)\hat{h}_b + n\hat{h}_s \\ &= x^* + (\gamma-1)\left(\hat{q}_b - \frac{q_b}{\theta}\right) \end{aligned}$$

since $\phi(\tau_1 + \tau_2)(\hat{m}_1 - M_{-1}) = (\gamma-1)\left(\hat{q}_b - \frac{q_b}{\theta}\right)$.

Thus, the expected difference in hours worked are

$$\hat{h} - h = (\gamma-1)\left(\hat{q}_b - \frac{q_b}{\theta}\right)$$

Then, $\hat{h}_b - h_b$ satisfies

$$\begin{aligned} \hat{h}_b - h_b &= x^* + \hat{q}_b + \phi(\tau_1 + \tau_2)(\hat{m}_1 - M_{-1}) - [x^* + q_b + \phi il] \\ &= \hat{q}_b - q_b + (\gamma-1)\left(\hat{q}_b - \frac{q_b}{\theta}\right) - \phi il \end{aligned}$$

Finally, define $\Psi = \{u(q_b) - q_b - [u(\hat{q}_b) - \hat{q}_b]\}$ and consider the difference $\mathcal{D} = \mathcal{U} - \hat{\mathcal{U}}$

$$\begin{aligned} \mathcal{D} &= \hat{h}_b - h_b + \frac{(1-n)\beta}{1-\beta} [u(q_b) - u(\hat{q}_b)] - \frac{n\beta}{1-\beta} (q_s - \hat{q}_s) + \frac{\beta}{1-\beta} E(\hat{h} - h) \\ &= \hat{q}_b - q_b + (\gamma-1)\left(\hat{q}_b - \frac{q_b}{\theta}\right) - \phi il + \frac{\beta(1-n)}{1-\beta} \Psi + \frac{\beta(\gamma-1)}{1-\beta} \left(\hat{q}_b - \frac{q_b}{\theta}\right) \\ &= -\phi il + \frac{\gamma-1}{1-\beta} \left(\hat{q}_b - \frac{q_b}{\theta}\right) + (\hat{q}_b - q_b) + \frac{\beta(1-n)}{1-\beta} \Psi \\ &= -i \frac{n(1-\mu)}{1-n\mu} q_b + \frac{\gamma-1}{1-\beta} \left(\hat{q}_b - \frac{q_b}{\theta}\right) + (\hat{q}_b - q_b) + \frac{\beta(1-n)}{1-\beta} \Psi \end{aligned}$$

since $\phi l = \frac{\phi n d(1-\mu)}{1-n} = \frac{\phi n(1-\mu)zM}{1-n} = \frac{\phi n(1-\mu)z\theta M}{(1-n)\theta} = \frac{n(1-\mu)}{1-n\mu} q_b$.

Then,

$$\mathcal{D} = -\frac{(\gamma-\beta)n(1-\mu)q_b}{\beta(1-n\mu)^2} + (\hat{q}_b - q_b) + \frac{\gamma-1}{(1-\beta)\theta} (\theta\hat{q}_b - q_b) + \frac{\beta(1-n)}{1-\beta} \Psi \quad (27)$$

since $i = \frac{\gamma-\beta}{\beta(1-n\mu)}$.

Assume that $\gamma = \beta$. Then $\hat{q}_b = q_b = q^*$ and

$$\mathcal{D}|_{\gamma=\beta} = -\left(\frac{\theta-1}{\theta}\right) q^* < 0$$

Thus, a monetary equilibrium with banking does not exist at the Friedman rule. This completes the proof of Proposition 2.

For the proof of Proposition 3, note that the first two terms in (27) are bounded for any finite value of γ . Also, the last term is positive and approaches infinity as $\beta \rightarrow 1$. Now consider the third term. The sign of this term is ambiguous in general. However using (19) and (26) we obtain

$$\theta = \frac{u'(\hat{q}_b) - 1}{u'(q_b) - 1}$$

which makes the third term equal to

$$\frac{(\gamma - 1) \{ \hat{q}_b [u'(\hat{q}_b) - 1] - q_b [u'(q_b) - 1] \}}{(1 - \beta) [u'(\hat{q}_b) - 1]} \quad (28)$$

If the function $q [u'(q) - 1]$ is decreasing in q then

$$\hat{q}_b [u'(\hat{q}_b) - 1] - q_b [u'(q_b) - 1] \geq 0$$

for all γ . Thus, (28) is non-negative for $\gamma \geq 1$. Consequently, the last two terms are positive when $\gamma > 1$ and approach infinity as $\beta \rightarrow 1$. Hence for a sufficiently high value of $1 > \beta > \tilde{\beta}$ we will have $\mathcal{D} \geq 0$. So, the banking equilibrium will exist and all loans will be repaid. ■

Proof of Proposition 4: Taking the expectation of (16) with respect to the shocks and using (6) lagged one period yields

$$\begin{aligned} \frac{\phi_{-1}}{\beta} &= \pi \phi^H [(1 - n^H) u'(q^H) + n^H (1 + i_d^H)] \\ &\quad + (1 - \pi) \phi^L [(1 - n^L) u'(q^L) + n^L (1 + i_d^L)]. \end{aligned}$$

where $q_b^H = \left(\frac{n^H}{1-n^H}\right) q_s^H = q^H$ and $q_b^L = \left(\frac{n^L}{1-n^L}\right) q_s^L = q^L$.

In a steady state equilibrium the real value of money is constant so $\phi^H M = \phi^L M = \Omega$ which implies $\phi^H = \phi^L = \phi = \frac{1}{\gamma} \phi_{-1}$. This implies $p^H = p^L = 1/\phi$. Thus, using (3) we can write the first-order condition as

$$\frac{\gamma}{\beta} = \pi [(1 - n^H) u'(q^H) + n^H (1 + i_d^H)] + (1 - \pi) [(1 - n^L) u'(q^L) + n^L (1 + i_d^L)] \quad (29)$$

In any equilibrium $p^j q^j \leq \theta^j (1 + \tau^j) M_{-1}$ where $\theta^H = \frac{1 - \mu n^H}{1 - n^H} > \theta^L = \frac{1 - \mu n^L}{1 - n^L}$. Then, the seller's first-order condition $p = 1/\phi$ implies

$$q^j \leq \theta^j z^j \Omega$$

where $z^j = \frac{1 + \tau^j}{1 + \tau}$ is greater in one state than the other state. The efficient quantity is defined to be $u'(q^*) = 1$. If trades are efficient, $q = q^* \leq \theta^j z^j \Omega$. Therefore, two types of equilibria are feasible.

Low Inflation Economy: For the conjectured equilibrium in the low inflation economy,

$$q^H = q^* < \theta^H z^H \Omega \text{ and } q^L = \theta^L z^L \Omega < q^*.$$

Using these expressions we obtain $\theta^L z^L \Omega < q^* < \theta^H z^H \Omega$ respectively

$$\frac{q^*}{\theta^H z^H} < \Omega < \frac{q^*}{\theta^L z^L} \quad (30)$$

which is a non-empty interval.

In the low inflation economy in equilibrium, $q^H = q^*$. This implies from (15) and (3) that $i^H = i_d^H = 0$ and

$$u'(q^L) = 1 + i^L = 1 + \frac{1}{1-\mu} i_d^L. \quad (31)$$

Thus, (29) can be solved to obtain a single equation in Ω

$$\frac{\gamma - \beta}{\beta} = (1 - \pi) (1 - n^L) \theta^L [u'(\theta^L z^L \Omega) - 1]$$

or a single equation in i^L

$$i^L = \frac{\gamma - \beta}{\beta (1 - \pi) (1 - n^L \mu)} \quad (32)$$

It is clear that the value of i^L solving (32) is greater than zero for $\gamma > \beta$ and approaches zero as $\gamma \rightarrow \beta$. Furthermore, we have $q^L \rightarrow q^*$ as $\gamma \rightarrow \beta$. Given this solution for i^L we can solve for q^L , i_D^L and Ω . The right-hand side of (32) is monotonically increasing in γ . Thus, an increase in the money growth rate, raises the nominal borrowing rate, increasing the cost of consumption and reducing q . Since q decreases in γ , Ω decreases as well. Given this behavior on i^L it is straightforward to show that as $\gamma \rightarrow \beta$, $\Omega \rightarrow \frac{q^*}{\theta^L z^L}$ so that the right-hand inequality is binding in (30). Thus, as γ increases from the Friedman rule, Ω declines and satisfies (30). For a sufficiently high value of γ , the left-hand inequality is violated. Denote $\tilde{\gamma}$ the value of γ that solves $q^* = \theta^H z^H \Omega$. Then, a unique monetary equilibrium exists for $\gamma \in [\beta, \tilde{\gamma})$.

High inflation economy: For the conjectured equilibrium in the high inflation economy,

$$q^H = \theta^H z^H \Omega < q^* \text{ and } q^L = \theta^L z^L \Omega < q^*. \quad (33)$$

Note that $q^H = \frac{\theta^H z^H}{\theta^L z^L} q^L > q^L$ because $\theta^H z^H > \theta^L z^L$.

From (3) and (15) we have

$$u'(q^j) = 1 + i^j = 1 + \frac{1}{1-\mu} i_d^j \quad j = L, H \quad (34)$$

Thus, (29) can be solved to obtain a single equation in Ω

$$\frac{\gamma - \beta}{\beta} = \pi (1 - n^H) \theta^H [u'(\theta^H z^H \Omega) - 1] + (1 - \pi) (1 - n^L) \theta^L [u'(\theta^L z^L \Omega) - 1]. \quad (35)$$

The right-hand side expression is monotonically decreasing in Ω and is equal to $\frac{\tilde{\gamma} - \beta}{\beta}$ at $\Omega = \frac{q^*}{\theta^H z^H}$. Thus for $\gamma > \tilde{\gamma}$, a unique value of Ω exists such that (33) holds. ■

Proof of Corollary 2: The result that individual consumption is more volatile is a straightforward implication of Proposition 2. We now show that banks reduce aggregate volatility. Consider the no bank case. Aggregate consumption is given by:

$$\begin{aligned} Q^H &= (1 - n^H) q \quad \text{when there are few buyers} \\ Q^L &= (1 - n^L) q \quad \text{when there are many buyers} \end{aligned}$$

where q is the steady state value of individual consumption. Expected aggregate consumption is:

$$\begin{aligned} \bar{Q}_n &= \pi Q^H + (1 - \pi) Q^L \\ &= \pi (1 - n^H) q + (1 - \pi) (1 - n^L) q \\ &= [\pi (1 - n^H) + (1 - \pi) (1 - n^L)] q \end{aligned} \quad (36)$$

The variance of aggregate consumption is:

$$\begin{aligned} V_n(Q) &= \pi [Q^H - \bar{Q}_n]^2 + (1 - \pi) [Q^L - \bar{Q}_n]^2 \\ &= \pi [(1 - n^H) q - [\pi (1 - n^H) + (1 - \pi) (1 - n^L)] q]^2 \\ &\quad + (1 - \pi) [(1 - n^L) q - [\pi (1 - n^H) + (1 - \pi) (1 - n^L)] q]^2 \\ &= 2\pi (1 - \pi) [(1 - n^L) - (1 - n^H)]^2 q^2 \end{aligned}$$

Using (36) the coefficient of variation is

$$\rho_n \equiv \frac{\sqrt{V_n(Q)}}{\bar{Q}_n} = \sqrt{2\pi(1-\pi)} \left[\frac{(1 - n^L) - (1 - n^H)}{\pi(1 - n^H) + (1 - \pi)(1 - n^L)} \right] \quad (37)$$

Now consider the equilibrium with banks. Aggregate consumption is given by:

$$\begin{aligned} Q^H &= (1 - n^H) q^H \quad \text{when there are few buyers} \\ Q^L &= (1 - n^L) q^L \quad \text{when there are many buyers} \end{aligned}$$

Expected aggregate consumption is:

$$\begin{aligned}\bar{Q}_b &= \pi Q^H + (1 - \pi) Q^L \\ &= \pi (1 - n^H) q^H + (1 - \pi) (1 - n^L) q^L\end{aligned}$$

We know that in this equilibrium $q^H = \theta^H \Omega$ and $q^L = \theta^L \Omega$ with $\theta^H > \theta^L$ implying

$$q^H = \frac{\theta^H}{\theta^L} q^L$$

Substitute in expected aggregate consumption to get

$$\bar{Q}_b = \left[\pi (1 - n^H) \frac{\theta^H}{\theta^L} + (1 - \pi) (1 - n^L) \right] q^L \quad (38)$$

The variance of consumption is given by

$$\begin{aligned}V_b(Q) &= \pi [Q^H - \bar{Q}_n]^2 + (1 - \pi) [Q^L - \bar{Q}_n]^2 \\ &= \left[(1 - n^H) \frac{\theta^H}{\theta^L} q^L - \left[\pi (1 - n^H) \frac{\theta^H}{\theta^L} + (1 - \pi) (1 - n^L) \right] q^L \right]^2 \\ &\quad + (1 - \pi) \left[(1 - n^L) q^L - \left[\pi (1 - n^H) \frac{\theta^H}{\theta^L} + (1 - \pi) (1 - n^L) \right] q^L \right]^2 \\ &= 2\pi (1 - \pi) \left[(1 - n^L) - (1 - n^H) \frac{\theta^H}{\theta^L} \right] (q^L)^2\end{aligned}$$

Using (38) the coefficient of variation is

$$\rho_b \equiv \frac{\sqrt{V_b(Q)}}{\bar{Q}_b} = \sqrt{2\pi (1 - \pi)} \left[\frac{(1 - n^L) - (1 - n^H) \frac{\theta^H}{\theta^L}}{\pi (1 - n^H) \frac{\theta^H}{\theta^L} + (1 - \pi) (1 - n^L)} \right] \quad (39)$$

It is clear that the numerator of (39) is smaller than the numerator in (37) while the denominator of (39) is larger than the denominator in (37). Thus, $\rho_b < \rho_n$. In this sense, the bank make aggregate consumption less disperse than when there are no banks. ■

Stabilization Problem: Suppose that there is a government that must finance a given level of government spending per capita in market 2 via the use of money creation only. So the government gives the agents cash and requires them to buy goods and then turn them over. So $g = \phi (\gamma - 1) M > 0$ which requires $\gamma > 1$.

In market two the agents' problem is

$$\begin{aligned}W(m_2, l, d) &= \max_{x, h, m_{1,+1}} [U(x) - h + \beta EV_{+1}(m_{1,+1})] \\ \text{s.t. } &x + \phi m_{1,+1} + g = h + \phi m_2 + \phi (1 + i_d) d - \phi (1 + i) l + \phi (\gamma - 1) M\end{aligned} \quad (40)$$

Rewriting the budget constraint yields

$$W(m_2, l, d) = \phi m_2 + \phi(\gamma - 1)M + \phi(1 + i_d)d - \phi(1 + i)l - g \\ + \max_{x, m_{1,+1}} [U(x) - x - \phi m_{1,+1} + \beta EV_{+1}(m_{1,+1})]$$

The first-order conditions are the same as in (6). Furthermore, the agents problem in market 1 is unaffected by these actions. Consequently, (29) is the constraint for a competitive equilibrium. Also welfare is then given by

$$(1 - \beta)EV(M) = U(x) - x - g + \pi(1 - n^H)[u(q^H) - q^H] + (1 - \pi)(1 - n^L)[u(q^L) - q^L] \quad (41)$$

We consider high enough levels of g and thus γ such that buyers are always liquidity constrained in any state. This reduces (29) to

$$\frac{\gamma - \beta}{\beta} = \pi(1 - n^H)\theta^H[u'(q^H) - 1] + (1 - \pi)(1 - n^L)\theta^L[u'(q^L) - 1] \quad (42)$$

Thus, the central bank's problem is to choose x , q^H and q^L to maximize (41) subject to (42). It is obvious that $x = x^*$ so the remaining problem is

$$\max_{q^H, q^L} \pi(1 - n^H)[u(q^H) - q^H] + (1 - \pi)(1 - n^L)[u(q^L) - q^L] \\ s.t. \quad \frac{\gamma - \beta}{\beta} = \pi(1 - n^H)\theta^H[u'(q^H) - 1] + (1 - \pi)(1 - n^L)\theta^L[u'(q^L) - 1]$$

which is what we have in the text. ■

Proof of Proposition 5: Since this is a well-defined maximization problem over a compact set for $q \in [0, q^*]$, there is a unique pair (q^L, q^H) in the interval $[0, q^*]^2$ that solves it.

First consider the problem when there are no banks. In this case we can simply set $\mu = 1$ which makes $\theta^H = \theta^L = 1$. It is clear from (24) that the planner chooses $q^H = q^L$.

Now consider the problem with banks. The first-order conditions must satisfy (24). Let $q^L = q^H < q^*$. Then the left-hand side of (24) is one which is larger than $\frac{\theta^L}{\theta^H}$. Let $f(q) = \frac{u'(q) - 1}{u''(q)}$. We have $f(q) < 0$ and $f'(q) > 0$ because $u''(q)u''(q) - [u'(q) - 1]u'''(q) > 0$. Fix q^H then the left-hand side is monotonically decreasing to zero in q^L between $[0, q^*]$. Thus there is a unique pair $q^* > q^L > q^H$ such that (24) is satisfied. ■