The Welfare Cost of Inflation in the Presence of Inside Money

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October 24, 2003

In this paper, we ask what role an endogenous money multiplier plays for the estimated welfare cost of inflation.[1] The model is a variant of that used by Freeman and Kydland (2000), with inside and outside money in the spirit of Freeman and Huffman (1991). Unlike models in which the money-output link comes from either sticky prices or fixed money holdings, here prices and output are assumed to be fully flexible. Consumption goods are purchased using either currency or bank deposits. Two transaction costs affect these decisions. One is a cost of acquiring money balances, necessary to determine the demand for money and to make endogenous the velocity of money. The other is a fixed cost associated with using deposits. This cost is instrumental in determining the division of money balances into currency and interest-bearing deposits. Faced with these two costs and factors that may vary over time in equilibrium, such as over the business cycle, households make decisions that in the aggregate determine velocity of money and the money multiplier.

The model is consistent with several features of U.S. data: (i) $M1$ is positively correlated with real output; (ii) the money multiplier and deposit-to-currency ratio are positively correlated with output; (iii) the price level is negatively correlated with output [in spite of (i) and (ii)]; (iv) the correlation of M1 with contemporaneous prices is substantially weaker than the correlation of $M1$ with real output; (v) correlations among real variables are essentially unchanged under different monetary-policy regimes; and (vi) real money balances are smoother than money-demand equations would predict.

A key feature of the model is that households purchase a continuum of types of goods indexed by their sizes. It comes from assuming a Leontief-type utility function over these types. One could argue that the distinction

[1] The current version of this paper is available at:
`http://wpweb2k.gsia.cmu.edu/espen/inprogress/WelfareInsideMoney.pdf`.

between nondurable and (usually larger) durable consumption goods also should be taken into account. We shall not take that step here. Instead, compared with Freeman and Kydland (2000), we consider a more flexible utility function than before, which in equilibrium permits the implication that households wish to consume large goods in relatively greater quantities.

With the model economy calibrated to the usual long-run relations in the data, including selecting values of the two transaction-cost parameters so as to make the model consistent with the empirical average deposit-to-currency ratio and the fraction of capital that is intermediated, the estimated welfare cost of inflation turns out to be rather small. An interesting finding is that the welfare cost as a function of the steady-state inflation rate is very steep for low inflation rates (well under 10 percent), but quite flat for higher inflation rates. Moreover, we find that the welfare cost is sensitive to the values of the transaction-cost parameters.

1 Previous research

Beginning with the contributions of Bailey (1956) and Friedman (1969), a long line of research addresses the question of the cost of inflation. Among more recent general equilibrium models are: Cooley and Hansen (1989), İmrohoroğlu and Prescott (1991), Bullard and Russell (2000), and Lucas (2000). Cooley and Hansen (1989) address the question within a cash-in-advance model and their estimates for welfare costs range from 0.11% to 0.94% for reducing annual steady state inflation rate from 10% to 0% and 100% to 0%, respectively. In the model economy constructed by İmrohoroğlu and Prescott (1991) the agents face idiosyncratic labor income risk. The estimated welfare cost of reducing inflation from 10% to 0% is 0.9%. Bullard and Russell (2000) analyze the welfare costs of inflation within a life cycle economy with financial intermediation and gets an estimate of welfare costs as high as 12%. Lucas (2000) analyzes a representative agent model with shopping time and finds that the welfare gain from reducing inflation from 10% to 0% is less than 1%.[2].

[2] Among other recent contributions are: Gomme (1993), Jones and Manuelli (1995), Lacker and Schreft (1996), Dotsey and Ireland (1996).

2 Model economy

2.1 Household's problem

There is a continuum of good types of measure c_t^*, ordered by size and indexed by j over $[0,1]$. The representative household has a Leontief-type instantaneous utility function over the continuum of good types,

$$\min\left[\frac{c_t(j)}{(1-\omega)\,j^{-\omega}}\right]$$

which gives us the parameterized distribution function for $c_t\,(j)$ over $[0,1]$

$$c_t(j) = (1-\omega)\,j^{-\omega}c_t^*. \tag{1}$$

The representative household has time separable preferences over total consumption (c_t^*) and leisure (d_t)

$$\max\sum_{t=0}^{\infty}\beta^t u\left(c_t^*, d_t\right), \tag{2}$$

where the instantaneous utility is given by

$$u\left(c_t^*, d_t\right) = \frac{1}{1-\nu}\left[\left(c_t^*\right)^{\zeta}\left(d_t\right)^{1-\zeta}\right]^{1-\nu}. \tag{3}$$

There are three vehicles of savings available to the household: nonintermediated capital (a_t), nominal bank deposits (h_t), and currency (m_t). Both bank deposits (h_t) and currency (m_t) can be used to purchase consumption goods, but use of deposits incurs an extra fixed cost, denoted γ. Because of this fixed cost of using deposits for purchases, the deposit rate of return net of transaction costs goes to negative infinity as the purchase size j goes to zero. Therefore, some j^* exists below which currency is a preferred means of payment and above which deposits are preferred.

The households' good budget constraint is given by

$$c_t^* + a_t + \frac{h_t}{p_t} + \frac{m_t}{p_t} + \gamma\left(1-j_t^*\right) = w_t l_t + r_t a_{t-1} + \tilde{r}_t\frac{h_{t-1}}{p_{t-1}} + \frac{m_{t-1}}{p_t} + \frac{x_t}{p_t}, \tag{4}$$

where p_t is the nominal price level, w_t is wage rate, r_t is real rate of return on capital, $\tilde{r}_t$ is the real rate of return on deposits, and x_t is government lump-sum transfers.

Available time for the households is normalized to 1 and the time available is spent on leisure (d_t), labor (l_t), and number of times money balances has to be replenished each period (n_t) multiplied by the time each replenishment takes (φ). The time constraint is

$$1 = d_t + l_t + n_t \varphi. \tag{5}$$

2.2 Production

Output is given by a constant-returns-to-scale production function with two inputs: capital (k_t) and labor (l_t);

$$y_t = z_t f\left(k_t, l_t\right)$$

The law of motion for the technology level z_t is given by

$$z_t = \rho z_{t-1} + \varepsilon_t, \qquad z_t \sim \mathcal{N}\left(\mu, \sigma^2\right),\ \mu > 0.$$

The depreciation rate is denoted δ, so the law of motion for the capital stock is

$$k_{t+1} = (1-\delta)\, k_t + i_t,$$

where i_t is gross investment.

2.3 Government

The government controls the supply of intrinsically worthless fiat money. The law of motion for the money stock is

$$M_t = \xi M_{t-1}.$$

Net revenues from printing money are transferred to the household in a lump sum fashion

$$x_t = (\xi - 1)\, M_{t-1}.$$

2.4 Financial intermediation

Banks accept deposits, hold required reserves fraction (θ) as cash and invest the proceeds in capital. Free entry ensures zero profit and the rate of return on deposits ($\tilde{r}$) is therefore a linear combination of the real return on capital (r_{t+1}) and return on holding currency (p_t/p_{t+1})

$$\tilde{r}_{t+1} = (1-\theta)\, r_{t+1} + \theta \frac{p_t}{p_{t+1}}.$$

The total stock of fiat money (the monetary base) is by definition equal to the combined stocks of currency and reserves

$$M_t = m_t + \theta h_t,$$

whereas the total money stock ($M1$) is the sum of nominal deposits and currency, which can be rewritten as the product of the monetary base and the money multiplier

$$M1_t = m_t + h_t = M_t \left[1 + \frac{h_t (1-\theta)}{m_t + \theta h_t} \right].$$

For the representative household the per period demand for real deposits (h_t/p_t) is

$$n_t \frac{h_t}{p_t} = \int_{j^*}^{1} c_t(j) \mathrm{d}j = \int_{j^*}^{1} (1-\omega)\, j^{-\omega} c_t^* \mathrm{d}j = \left[j^{1-\omega} c_t^* \right]_{j^*}^{1} = \left(1 - (j^*)^{1-\omega} \right) c_t^* \tag{6}$$

and the per period demand for real fiat-money balances (m_t/p_t) is:

$$n_t \frac{m_t}{p_t} = \int_{0}^{j^*} c_t(j) \mathrm{d}j = \int_{0}^{j^*} (1-\omega)\, j^{-\omega} c_t^* \mathrm{d}j = \left[j^{1-\omega} c_t^* \right]_{0}^{j^*} = (j^*)^{1-\omega}\, c_t^*. \tag{7}$$

2.5 Equilibrium

At any point, the economy is characterized by the state of the technology (z), by the growth of money stock (ξ), the price level in the previous period (p_{-1}), and by the holdings of non-intermediated capital (a_{-1}), deposits (h_{-1}) and currency (m_{-1}) in the previous period. [**to be completed...**]

3 Mapping data to the model

In steady state investment is one-quarter of output and the annual capital-output ratio 2.5. The depreciation rate is then calibrated to 0.025. The parameter α in the production function is calibrated such that labor share of national income is 0.64. The autocorrelation coefficient ρ in the technology process is set equal to 0.95 with a standard deviation of 0.0076.

Setting the average allocation of households' time (net of sleep and personal care) to market activity equal to 0.30 restricts the value of the utility parameter ζ. The risk aversion parameter $\nu = 2$.

The reserve requirement ratio θ is equal to 0.10.

3.1 Utility function

Let the continuum of good types $c_t(j)$ be of measure $c_t^* = 1$. Eq. (1) can then be simplified as

$$c_t(j) = (1-\omega)\, j^{-\omega} c_t^*.$$

In Figure 1, $c_t(j)$ is plotted for three different values of ω.

As is apparent from the expression and visualized in the figure, for $\omega > -1$ (e.g. $\omega = -0.5$), the amount consumed of a good is a concave function of the size of the good, whereas for $\omega < -1$, the amount consumed of a good is a convex function of the size of the good.

Combining equations (6) and (7) gives us the cut-off size for purchase above which deposits are preferred to currency

$$j^* = \left(1 + \frac{h_t}{m_t}\right)^{\frac{1}{\omega-1}}, \tag{8}$$

[**To be continued and completed: properties of j^*, pinning down ω, business cycle findings.**]

4 Quantitative findings

We begin by describing the steady state properties of our economy under different inflation regimes.

4.1 Steady state

Figures 2 and 3 (Figure 3 is just a subset of Figure 2) plot the benchmark welfare cost function λ, defined such that

$$u\left(\lambda c\left(\pi\right), d\left(\pi\right)\right) = u\left(c\left(\tilde{\pi}\right), d\left(\tilde{\pi}\right)\right),$$

where $\tilde{\pi}$ is set equal to the average inflation rate the last 15 years, about 1.03.

The economy is calibrated such that for gross annual inflation rate equal to 1.03 the currency-to-deposit ratio is equal to 9 and non-reserve portion of $M1$ divided by the capital stock is 0.05. This gives us calibrated values for $\gamma = 0.00529$ and $\phi = 0.00060$, which implies that at this inflation rate the fixed cost $\gamma\left(1 - j^*\right)$ is 0.36 percent of GDP and ϕ corresponds to approximately 55 minutes per quarter.

We see from Figures 2 and 3 that as steady state inflation approaches an annual rate of 50%, the welfare cost is slightly less than 0.4% of consumption

compared with the steady state associated with 3% inflation. As the steady state inflation rate increases further, the associated welfare flattens out. At an annual inflation rate of 400% the cost inflation in terms of consumption compensation is still less than 0.8%.

The most striking feature of the graph is the predicted welfare gain from reducing inflation below 3% annually. As we see from the graph, the effect of reducing inflation towards its lower bound of -0.01644% gives a welfare improvement that is almost of the same magnitude as the welfare cost of increasing inflation from 3% to 50%.[3]

The main underlying variables underlying these results are presented in Table 1. As inflation increases, individuals become less and less willing to hold non-interest bearing assets such as currency. The cut-off value of j^* below which currency is preferred to deposits is decreasing and will eventually converge to zero as inflation increases towards infinity. Hence, the deposit-to-currency ratio will increase and consequently more resources in the economy will be spent on facilitating transactions, both through the fixed cost $\gamma(1-j^*)$ for purchasing goods with deposits and by time spent $n^*\phi$ to withdraw currency.

Looking at steady states with lower inflation rates, we see the mirror image of the high inflation regimes: since the alternative value of holding deposits over holding currency diminishes, the cut-off value j^* increases and the deposit-to-currency ratio decreases. Henceforth, the welfare costs associated with individual liquidity management will decrease.[4]

[3] From the first order conditions of the representative household's optimization problem, we get the following expression for the lower bound for $1/\xi$ for a given calibration of the model economy

$$\frac{1}{\xi} = r\left\{1 - \frac{1}{\varphi w c^* \left[(1-\theta)\frac{(1-\omega)}{\gamma}\right]^2}\right\}.$$

[4] *Intuition:* In steady state the two variables that enter into the utility function are given by

$$d^* = 1 - l^* - n^*\phi$$

and

$$c^* = \underbrace{w^* l^* + (r^*-1)\,a^* + (\tilde{r}^*-1)\,\frac{h^*}{p^*} + \frac{x^*}{p^*}}_{=f(k^*)-\delta k^* \text{ in steady state}} - \gamma(1-j^*)$$

So the only two variables that directly change steady state leisure (d^*) and consumption (c^*) are n^* and j^*.

For the benchmark calibration with $\omega = -1$, at 3% annual inflation the value of j^* below which currency is preferred to deposits is 0.316. (This implies currency is preferred to deposits for 0.316 of the *number* and 0.10 of the *value* of the total purchases).

4.1.1 Sensitivity/alternative calibration

The results presented above are sensitive to the calibration of the parameters γ and ϕ. Our empirical deposit-currency ratio range from 12 early in the sample to 7 late in the sample. The highest estimate for non-reserve portion $M1$ divided by the capital stock we have encountered is 0.20. So here we choose the extreme values and calibrate the model economy such that at gross annual inflation rate 1.03 the deposit-to-currency ratio is 7 and non-reserve portion of $M1$ dividend by the capital stock is 0.20, the calibrated values for γ and ϕ are equal to 0.02414 and 0.01088, respectively. This corresponds to that fixed cost of making purchases with deposits amounts to 1.8% of output.

As we see from Figures 4 and 5 and Table 2 the shape of the welfare cost functions are very similar to those resulting from the benchmark calibration. The measure, however, differs significantly. Under this alternative calibration the steady state cost of inflation at 50% net annual inflation compared with at .03 is almost 2%, which is about five times as high as under the benchmark calibration. Likewise, if we look at the steady states with inflation rates below .03 the qualitative picture is the same, but the quantitative difference is of about the same magnitude.

As inflation increases, the cost of holding currency increases and the cut-off value therefore decreases and in the limit as inflation goes to infinity j^* converges to 0. Steady state consumption (c^*) will then converge towards $f(k^*) - \delta k^* - \gamma$.

Analogously, as the rate of inflation decreases and in the limit is equal to the inverse of the rate of return on non-intermediated assets, j^* will equal 1. Steady state consumption will then be equal to $f(k^*) - \delta k^*$.

The intuition for the concavity of welfare cost as a function of inflation is then straightforward: If for gross inflation rate equal to 1.03, $j^* = 0.32$, the gain to steady state consumption from decreasing the rate of inflation to r will be 0.68γ, whereas the loss to steady state consumption from increasing the rate of inflation from 1.03 to ∞ will be 0.32γ.

For given currency-to-deposit ratio the derivative of j^* with respect to ω is

$$\frac{\mathrm{d}j^*}{\mathrm{d}\omega} = (-1)(\omega - 1)^{-2} \ln\left\{1 + \frac{h_t}{m_t}\right\}\left(1 + \frac{h_t}{m_t}\right)^{\frac{1}{\omega - 1}} < 0.$$

So for a given currency-to-deposit ratio (h_t/m_t), the larger (closer to zero) ω is, the lower is j^* and welfare cost as a function of inflation rate be more concave than in the benchmark case. Similarly, the smaller (further from zero) ω is, the higher is j^* and welfare cost as a function of inflation rate be less concave than in the benchmark case.

4.2 Transition

Comparing steady states ignores, however, several important aspects that are relevant in order to answer our question. We therefore conduct a series of policy experiments where we reduce inflation from a moderate level (0.03, 0.06, 0.10 and 0.25) to zero. When conducting these policy experiments, we calibrate the economy to the benchmark case above where γ and ϕ are equal to 0.0276 and 0.0147, respectively. Already after two quarters is the model economy very close to its steady state. We can therefore ignore the transition.

Table 3 presents the results from these experiments. The welfare gains from reducing inflation are smaller than when comparing the steady states and range from 0.1% for initial inflation equal to 3% to 0.3% for initial inflation equal 15%.

When we for the steady states analyzed the sources of the welfare gains we found that they solely came from the reduction in resources spent on facilitating transactions. Here, we have in addition the effect through changes in expectations of monetary policy. If the rate of money growth (ξ) is decreased, anticipated inflation decreases, demand for real money balances increase, and price *level* must decrease in equilibrium. This is known as the "Friedman surge effect".

When the price level drops sharlpy, the real value of deposits increase. The non-reserve part of deposits are invested in real capital. The result is that the capital stock is increasing, reducing the marginal product of capital.

This is the opposite of the "Tobin effect". The Tobin effect predicts that as inflation decreases and hence the rate of return on holding currency increases, individuals will hold more currency relative to deposits and non-intermediated capital. The Tobin effect then predicts that the capital stock should decrease and the marginal product of capital should increase.

5 Conclusion

[**To be added**].

References

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A Figures and tables

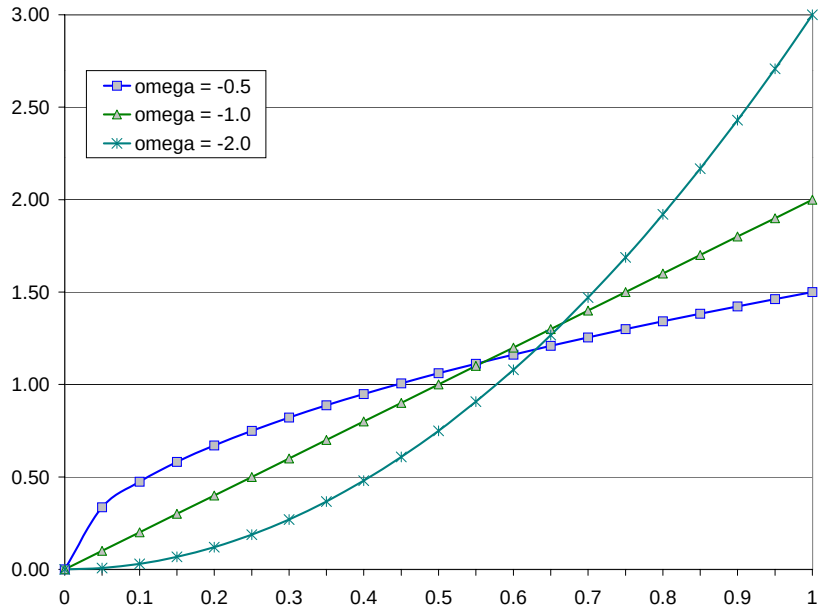

Figure 1: $c_t(j)$ for $0 \leq j \leq 1$

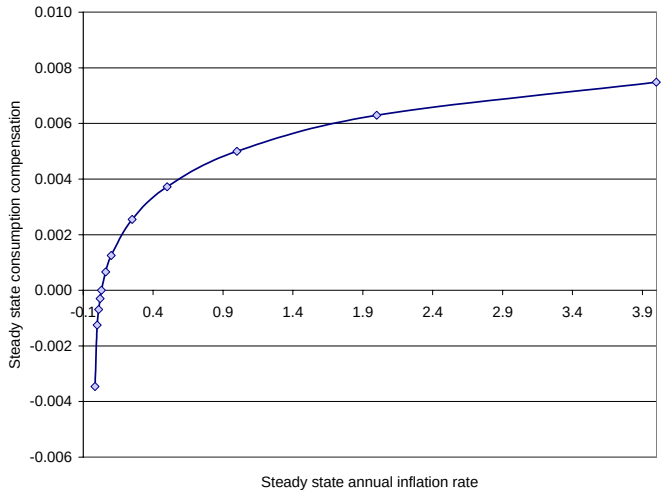

Figure 2: Welfare costs relative to 3% annual money growth

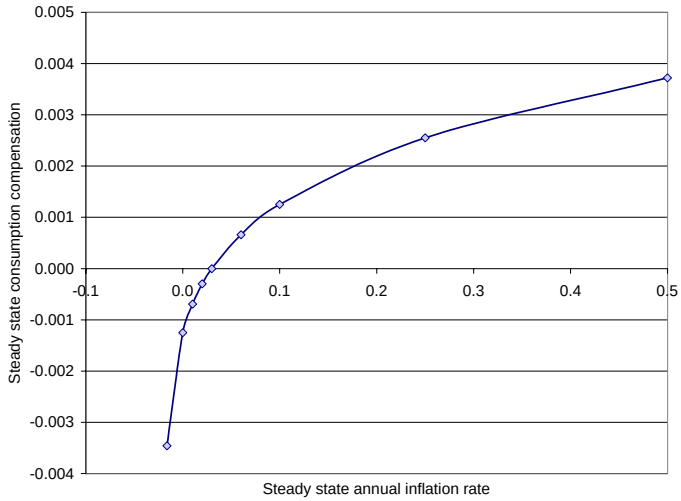

Figure 3: Welfare costs relative to 3% annual money growth

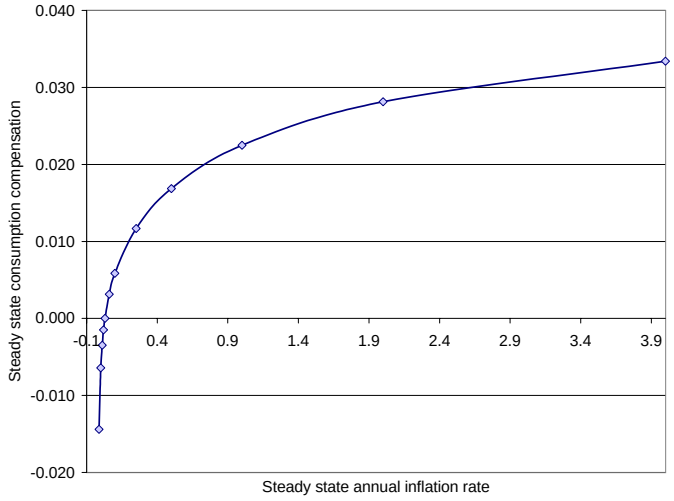

Figure 4: Welfare costs relative to 3% annual money growth, alt. calibration

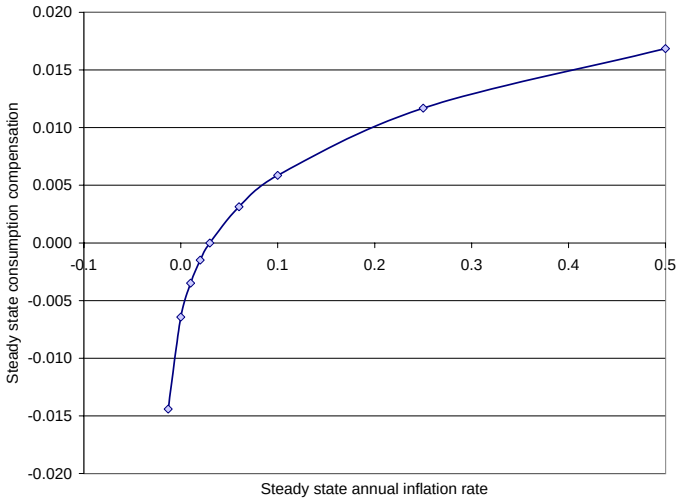

Figure 5: Welfare costs relative to 3% annual money growth, alt. calibration

Table 1: Steady state welfare costs

	-0.0164	0.00	0.01	**0.03**	0.06	0.10	0.25	0.50
Nom. interest (Q)	1.00569	1.00985	1.01237	1.01734	1.02467	1.03420	1.06779	1.11759
h/m	0.01048	3.04712	5.06039	8.99999	14.55445	21.35746	42.63267	69.27504
j^*	0.99584	0.57167	0.48641	0.39811	0.33363	0.28856	0.22084	0.18251
$(\xi - 1)\,\frac{M}{p}/Y$	-0.00171	0.00000	0.00035	0.00077	0.00117	0.00155	0.00247	0.00345
c	0.74998	0.74773	0.74728	0.74681	0.74647	0.74623	0.74588	0.74567
d	0.69892	0.69919	0.69921	0.69918	0.69911	0.69902	0.69874	0.69843
$c/c_{0.03}$	1.00424	1.00123	1.00063	1.00000	0.99954	0.99922	0.99874	0.99847
$d/d_{0.03}$	0.99963	1.00002	1.00004	1.00000	0.99990	0.99977	0.99937	0.99893
$\lambda - 1$	-0.00346	-0.00125	-0.00069	0.00000	0.00066	0.00125	0.00255	0.00372

Table 2: Steady state welfare costs, alt. calibration

	-0.013	0.00	0.01	**0.03**	0.06	0.10	0.25	0.50
Nom. interest (Q)	1.00656	1.00985	1.01237	1.01734	1.02467	1.03420	1.06779	1.11759
h/m	0.00822	1.95626	3.63543	7.00000	11.80190	17.71170	36.23990	59.45900
j^*	0.99673	0.64819	0.54146	0.43528	0.36065	0.30985	0.23529	0.19383
$(\xi - 1)\,\frac{M}{p}/Y$	-0.00541	0.00000	0.00549	0.01626	0.03018	0.04535	0.08496	0.12750
c	0.74992	0.74151	0.73893	0.73637	0.73456	0.73334	0.73154	0.73054
d	0.69507	0.69617	0.69635	0.69634	0.69612	0.69576	0.69462	0.69335
$c/c_{0.03}$	1.01841	1.00698	1.00348	1.00000	0.99755	0.99589	0.99344	0.99208
$d/d_{0.03}$	0.99817	0.99975	1.00001	1.00000	0.99967	0.99916	0.99753	0.99570
$\lambda - 1$	-0.01441	-0.00642	-0.00349	0.00000	0.00313	0.00586	0.01168	0.01686

Table 3: Steady state welfare costs

	$0.03 \to 0.00$	$0.06 \to 0.00$	$0.10 \to 0.00$	$0.25 \to 0.00$
h/m initial	9.0000	14.5545	21.3575	42.6326
h/m new s.s.	4.4197	6.7786	10.5585	19.5078
j^* initial	0.3981	0.3336	0.2886	0.2208
j^* new s.s.	0.5086	0.4402	0.3757	0.2987
c initial st. st.	0.7468	0.7465	0.7462	0.7459
c new s.s.	0.7471	0.7473	0.7474	0.7480
c change (net)	0.0003	0.0011	0.0016	0.0028
d initial s.s.	0.6991	0.6991	0.6990	0.6987
d new s.s.	0.6994	0.6993	0.6992	0.6990
d change (net)	0.0004	0.0003	0.0003	0.0003
output initial s.s.	1.0000	1.0000	1.0000	1.0000
output new s.s.	0.9995	1.0011	1.0023	1.0045
y change (net)	-0.0005	0.0011	0.0023	0.0045
welfare gain (c comp)	0.0012	0.0018	0.0023	0.0035