ESCAPIST POLICY RULES

JAMES BULLARD AND **IN-KOO CHO**

ABSTRACT. We study a simple, forward-looking macroeconomic system in which the monetary authority employs a Taylor-type policy rule. We analyze situations in which the self-confirming equilibrium is unique and learnable according to Bullard and Mitra (2000). The private sector harbors doubts about the policymaker, and recursively estimates a misspecified version of the policy rule using real time data. The policy authorities attempt to accommodate private sector beliefs by altering their inflation target in response to perceived "hawkishness." The private sector agents learn in a way that respects their uncertainty concerning the situation they face. We show that this system can sometimes depart from the unique equilibrium towards a point characterized by low nominal interest rates and low inflation rates. Thus we generate events that have some properties of "liquidity traps," even though there is no liquidity trap equilibrium.

1. Introduction

1.1. **The specter of Japan.** During the middle-to-late 1980s, the Japanese economy was widely admired in the business press and among academics. It had grown rapidly for many years, and seemed to threaten U.S. world economic leadership. But Japanese success faded in the 1990s as the economy became mired in a cycle of poor performance. The causes of this downfall have been widely debated, and we can only address one relatively narrow aspect of the debate in this paper, namely, the role of monetary policy.

Policymaking at the Bank of Japan is sometimes suspected of causing the change of fortunes. To critics, if the Bank of Japan had somehow behaved differently than it did, the 1990s Japanese experience might have been avoided. A difficult aspect of the critics' view is that the Bank of Japan did not appear to behave very differently during the 1990s than it had during the earlier, more successful periods for the economy. If the Bank of Japan's policy rule was the right one during the successful periods, why was essentially the same policy rule the wrong one during the 1990s?

Date: This version: January 11, 2001. PRELIMINARY AND INCOMPLETE.

Key words and phrases. Learning, monetary policy rules, escape dynamics.

Any views expressed are those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of St. Louis or the Federal Reserve System.

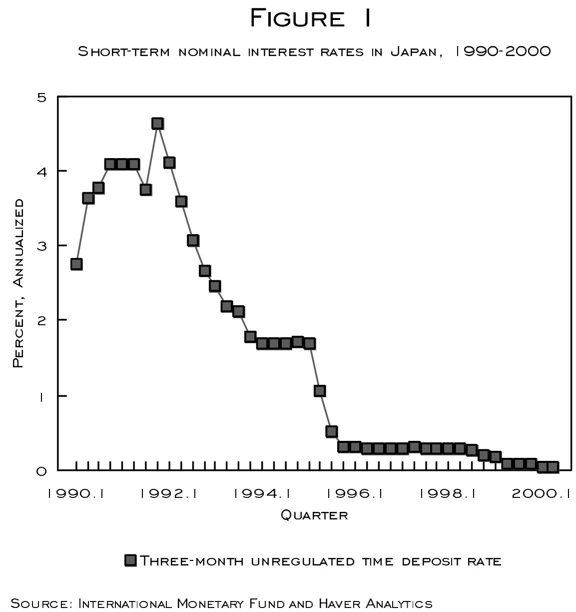

FIGURE 1. Low nominal interest rates in Japan.

This paper has a lot to say about this type of question. We study a stylized economy with a monetary policymaker following a seemingly fixed policy rule for the adjustment of a nominal interest rate target, like the one suggested by Taylor (1993). While such a policy can be quite successful at times in our model, we provide conditions under which the system may escape from the targeted equilibrium towards a non-equilibrium outcome. This escape outcome can be persistent, and has some of the characteristics present in the Japanese data.

1.2. **Data.** One of the features of the 1990s Japanese experience was a sharp decline in short-term nominal interest rates. Figure 1 shows annualized three-month unregulated time deposit rates in Japan from 1990 through 2000. These rates have remained below one percent per annum for the past five years, after beginning the decade near four percent. The low nominal interest rates have been associated with low inflation rates. Consumer prices were rising at a rate of 3 to 4 percent per year in Japan at the beginning of the 1990s, but the inflation rate has fallen to between ± 1 percent since 1995, when measured as a percent increase from the previous year (the exception is 1997, when it rose to about two percent). Real performance has been

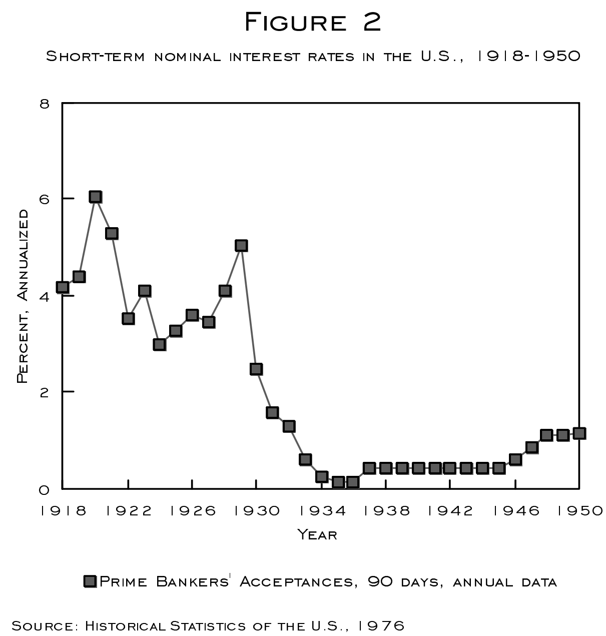

FIGURE 2. Low nominal interest rates in the U.S.

poor during the 1990s, especially when compared to the earlier Japanese experience.[1]

Of course, the current Japanese episode is not the only experience of a major industrialized economy with low nominal interest rates and low inflation. Figure 2 displays a famous example, namely short-term nominal interest rates in the U.S. from 1918 to 1950. The data are annual nominal interest rates on 90-day prime bankers' acceptances. The U.S. experience was associated with the upheaval of the Great Depression and the departure from the gold standard, and so interpretation is accordingly difficult. But short-term nominal interest rates averaged around four percent during the 1920s, and then fell to below one percent and remained there until 1946. And U.S. monetary policymakers can plausibly be viewed as following essentially the same monetary policy rule or operating procedure during the 1920s as they did during the 1930s.[2]

[1]Summers (1991) has argued that low nominal interest rates leave the economy more vulnerable to negative shocks, since monetary policymakers targeting nominal interest rates can do little when a shock hits.

[2]See Wheelock (1991) for a development of this thesis.

The data presented in these figures, along with the other economic events occurring during these periods, have been deeply influential in macroeconomics. They have spawned theories of "liquidity traps," and they have led many policymakers and financial market participants to spurn calls for policies targeting zero inflation, not to mention moderate rates of deflation. This is our motivation for studying a theory of what might be happening during these episodes.

1.3. **Policy rules and low nominal interest rates.** There has been a rapid expansion in the literature on monetary policy rules since the publication of Taylor (1993), which suggested a simple policy rule to describe the U.S. experience during the late 1980s and early 1990s.[3] Taylor's rule is expressed in terms of a short-term nominal interest rate as the policymakers' instrument. The rule is to adjust the nominal interest rate relative to a target value, based on a linear function of the deviation of inflation from target and the deviation of output from potential. Since Taylor's empirical description held over a period when U.S. macroeconomic performance was relatively satisfactory, the Taylor rule has also been taken as a prescription for good monetary policy, as described, for example, in Woodford (2001). Taylor (1999) has further argued that the policy rule will be more successful if it is *active*, meaning that nominal interest rates are adjusted more than one-for-one with inflation rates. While there have been many approaches to understanding low nominal interest rate phenomena, we wish to think in terms of the possible consequences of the use of a rule like the one Taylor suggested.

In an influential paper, Benhabib, Schmitt-Grohe and Uribe (1999) argued that the interaction between an active Taylor rule, a Fisher relation, and a zero bound on nominal interest rates creates a presumption of a second steady state equilibrium in most macroeconomic models. They constructed example economies in which two steady state equilibria exist, one characterized by inflation at target and a relatively high nominal interest rate, and a second characterized by unintentionally low inflation and relatively low nominal interest rates. For purposes of easy reference, we will label the former as a "Taylor" steady state and the latter as a "liquidity trap" steady state. Benhabib, *et al.,* (1999) also provide conditions under which an equilibrium sequence exists which begins arbitrarily close to the Taylor steady state and then follows an oscillatory path leading to the liquidity

[3]For a sample of the recent work, see the volumes edited by Taylor (1999) and King and Plosser (1999), and the survey by Clarida, Gali and Gertler (1999).

trap steady state. There were no equilibrium sequences moving in the opposite direction. One might interpret the Benhabib, *et al.,* (1999) work as providing a microfounded model which helps explain low nominal interest rate phenomena. The Benhabib, *et al.,* (1999) analysis has the interesting feature that the policymaker pursues a fixed rule, and one that otherwise has desirable features, but nevertheless could ultimately generate an undesirable outcome.[4]

Inflation is unintentionally low in the Benhabib, *et al.,* (1999) liquidity trap steady state because the government misses its inflation target on the low side every period. The low nominal interest rate outcomes in this paper will not have that property, as inflation will remain close to target at all times.

Benhabib, *et al.,* (1999) analyzed perfect foresight, or rational expectations, equilibria. A fundamental question in economic theory is whether agents not initially possessing rational expectations might be able to learn an equilibrium using the data produced by the economy in which they operate. Bullard and Mitra (2000) asked this question, not in the context of the Benhabib, *et al.,* (1999) model, but in the context of a workhorse model from the policy rules literature put forward by Woodford (1999). The Woodford (1999) model describes a linearization about a Taylor steady state. The Bullard and Mitra (2000) answer was that *active* Taylor rules tended to be associated with the local stability of the Taylor equilibrium in the learning dynamics. Although there are differences between models, this suggests that the Taylor steady state of the Benhabib, *et al.,* (1999) model would also be locally stable in the learning dynamics. One would then expect that agents would be able to coordinate on the Taylor equilibrium according to this analysis, and that the liquidity trap outcome would not be observed.

1.4. **Instability under learning.** In this paper we take the results on learning from Bullard and Mitra (2000) seriously as suggesting that the economy could coordinate on the Taylor equilibrium if the central bank attempted to target it. We look for circumstances under which the stability under learning might break down, and cause the system to visit a low nominal interest rate, low inflation outcome, like the ones displayed in Figures 1 and 2.

To explore this possibility, we eliminate the liquidity trap steady state of Benhabib, *et al.,* (1999) altogether, and focus instead on the Woodford

[4]By undesirable here we mean unintentionally low nominal interest rates. We discuss welfare briefly in the conclusion.

linearization about a Taylor steady state. Thus there is no liquidity trap equilibrium in our model. We do not impose a lower bound on nominal interest rates, but we do ensure that such a bound is never violated in our simulations. We use 'large deviation' theory as employed by Sargent (1999) and Cho, Sargent, and Williams (2000) to generate departures from the Taylor equilibrium. These departures depend on a certain misspecification on the part of the private sector regarding the actions of the policy authorities, feedback from the beliefs of the private sector to the actions of the policy authority, and a learning rule that reflects the private sector's doubt about the accuracy of their specification. We now turn to developing this model.

2. Environment

2.1. **Rational expectations.** Our model is based on Woodford (1999). We use this model because it has been derived from microfoundations in Woodford and Rotemberg (1998), and because it is a workhorse model in the literature on monetary policy rules. The equations represent a simplified linearization about a Taylor steady state:

$$z_t - \bar{z}_t = E_t z_{t+1} - \bar{z}_t - \sigma^{-1}\left[r_t - \bar{r}_t - E_t \pi_{t+1} + \bar{\pi}_t\right] + w_t \tag{2.1}$$

$$\pi_{t+1} - \bar{\pi}_t = \kappa\left[z_t - \bar{z}_t\right] + \beta\left[E_t \pi_{t+1} - \bar{\pi}_t\right] \tag{2.2}$$

$$r_t - \bar{r}_t = \phi_\pi\left[\pi_t - \bar{\pi}_t\right] + \phi_z\left[z_t - \bar{z}_t\right] + \eta_t \tag{2.3}$$

where

$$w_t = \alpha w_{t-1} + \epsilon_t. \tag{2.4}$$

Here z_t is the level of real output at time t, r_t is the nominal interest rate at time t, and π_t is the inflation rate at time t. The parameters σ, relating to the elasticity of intertemporal substitution of the representative household, κ, relating to the degree of price stickiness in the economy, and β, the household's discount factor, are all fixed and positive. We view the Taylor rule coefficients ϕ_π and ϕ_z also as positive and fixed. We think of equations (2.1) and (2.2) as describing the optimizing behavior of the private sector in Woodford's (1999) framework, and the third equation as describing the behavior of the policy authorities who are committed to using a Taylor-type policy rule. While the first two equations are derived from a maximizing model of private sector behavior, the third equation is simply a description of policymaker behavior. We assume that ϵ_t and η_t are Gaussian white noise terms. The main difference from the original formulation of Bullard and Mitra (2000) and Woodford (1999) is that we write the variables in gross terms rather than as the deviation from the target. The target or long-run values are denoted by $\bar{z}_t$, $\bar{\pi}_t$ and $\bar{r}_t$.

We substitute equation (2.3) into (2.1) and endow the private sector with rational expectations concerning this system. The minimum state variable (MSV) solution is given by

$$z_t - \bar{z}_t = c_z w_t \tag{2.5}$$

and

$$\pi_t - \bar{\pi}_t = c_\pi w_t, \tag{2.6}$$

where

$$\begin{bmatrix} c_z \\ c_\pi \end{bmatrix} = \frac{1}{(\beta\alpha - 1)(1 + \sigma^{-1}\phi_z - \alpha) - \kappa\sigma^{-1}(\phi_\pi - \alpha)} \begin{bmatrix} \beta\alpha - 1 \\ -\kappa \end{bmatrix}. \tag{2.7}$$

2.2. **Learning.** The private sector in our model harbors doubts about the nature of the government's monetary policy. These doubts do not involve the long-run level of potential output in the economy, and we simply normalize

$$\bar{z}_t = 0 \quad \forall t \geq 1. \tag{2.8}$$

But the private sector is not sure about the policy rule that the government is adopting. They believe that 'actions speak louder than words,' and accordingly they estimate the actual Taylor rule in use by the monetary authority. They employ a misspecified model for this purpose:

$$r_t = \hat{\phi}_0 + \hat{\phi}_\pi \pi_t. \tag{2.9}$$

We will discuss the nature of the misspecification in this equation in detail in the remainder of the paper, but we note here that the specification is actually quite good for many purposes. The private sector updates the coefficients according to the recursive least squares estimation:

$$\begin{bmatrix} \hat{\phi}_{0,t+1} \\ \hat{\phi}_{\pi,t+1} \end{bmatrix} = \begin{bmatrix} \hat{\phi}_{0,t} \\ \hat{\phi}_{\pi,t} \end{bmatrix} + a\Sigma_t^{-1} \begin{bmatrix} 1 \\ \pi_t \end{bmatrix} \left(r_t - \hat{\phi}_{0,t} + \hat{\phi}_{\pi,t}\pi_t \right) \tag{2.10}$$

$$\Sigma_{t+1} = \Sigma_t + a \left(\begin{bmatrix} 1 \\ \pi_t \end{bmatrix} \begin{bmatrix} 1 & \pi_t \end{bmatrix} - \Sigma_t \right). \tag{2.11}$$

The slope $\hat{\phi}_{\pi,t}$ represents the private sector's perception of how aggressive the government is in responding to inflation movements—that is, the perception of how 'active' the Taylor rule is. One could think of this as a measure of the "hawkishness" of the monetary authority. We stress that, in reality, the monetary authority is never changing the degree of aggressiveness toward inflation in the policy rule, because the actual value ϕ_π is fixed. It is in this sense that the policy authorities are committed to the use of an active Taylor rule.

The monetary authority is making some adjustments, however, of which the private sector is unaware. These adjustments would normally be small in size. In particular, policymakers allow their inflation target to drift in response to perceived aggressiveness, according to:

$$\bar{\pi}_t = \gamma_0 + \gamma_1 \hat{\phi}_{\pi,t}, \tag{2.12}$$

where γ_0 and γ_1 are fixed parameters. We think of γ_1 as 'small,' and in particular if γ_1 is zero, then the government's inflation target is fixed at γ_0. Since the Fisher equation must hold at the steady state, the nominal interest rate target is given by

$$\bar{r}_t = \rho + \bar{\pi}_t \tag{2.13}$$

where ρ is the long run real interest rate.

We note that (2.9) is misspecified mainly in the sense that the private sector presumes a fixed inflation target for the government, when in fact the inflation target is time-varying. It is also misspecified in the sense that the output gap is not included, but this turns out to be only a minor concern in this setting.

Our model consists of (2.5), (2.6), (2.4), (2.10), (2.11), (2.12) and (2.13). Note that the private sector has rational expectations regarding the dynamics induced by (2.1), (2.2) and (2.3). The only source of bounded rationality of the private sector is its perception of the government's Taylor rule policy. Our task is to investigate the asymptotic properties of the stochastic dynamic system as well as its large deviation properties. As a benchmark, we first analyze the self-confirming equilibrium.

2.3. **Self-confirming equilibrium.** When the gain function $a > 0$ is small, the stochastic process induced by (2.10) and (2.11) can be approximated by an ordinary differential equation (ODE):

$$\begin{bmatrix} \dot{\hat{\phi}}_0 \\ \dot{\hat{\phi}}_\pi \end{bmatrix} = \Sigma^{-1} \begin{bmatrix} \rho - \hat{\phi}_0 + (1 - \hat{\phi}_\pi)(\gamma_0 + \gamma_1 \hat{\phi}_\pi) \\ (\phi_\pi - \hat{\phi}_\pi)\sigma_\pi^2 + (\gamma_0 + \gamma_1 \phi_\pi)\left(\rho - \hat{\phi}_0 + (1 - \hat{\phi}_\pi)(\gamma_0 + \gamma_1 \hat{\phi}_\pi)\right) \end{bmatrix} \tag{2.14}$$

$$\dot{\Sigma} = \begin{bmatrix} 1 & \gamma_0 + \gamma_1 \hat{\phi}_\pi \\ \gamma_0 + \gamma_1 \hat{\phi}_\pi & (\gamma_0 + \gamma_1 \hat{\phi}_\pi)^2 + \sigma_\pi^2 \end{bmatrix} - \Sigma \tag{2.15}$$

where

$$\sigma_\pi^2 = \frac{\sigma_\epsilon^2}{1 - \alpha^2}. \tag{2.16}$$

The self-confirming equilibrium is the outcome that causes the right hand side of the ODE to vanish:

$$\hat{\phi}_\pi = \phi_\pi \tag{2.17}$$

$$\hat{\phi}_0 = \rho + (1 - \phi_\pi)(\gamma_0 + \gamma_1 \phi_\pi) \tag{2.18}$$

$$\Sigma = \begin{bmatrix} 1 & \gamma_0 + \gamma_1 \hat{\phi}_\pi \\ \gamma_0 + \gamma_1 \hat{\phi}_\pi & (\gamma_0 + \gamma_1 \hat{\phi}_\pi)^2 + \sigma_\pi^2 \end{bmatrix}. \tag{2.19}$$

It is straightforward to verify that the Hessian of the right hand side of the ODE has negative real eigenvalues, which proves that the self-confirming equilibrium is the stable solution of the ODE. The standard result of the learning literature proves that the learning dynamics converges to the self-confirming equilibrium in a probabilistic sense.

3. Learning Dynamics

3.1. **Decreasing gain sequences.** At the self-confirming equilibrium, the agent's misspecified model is indistinguishable from the true model because the estimator of the slope of the government's policy is precisely what the government actually implements, namely, $\hat{\phi}_\pi = \phi_\pi$. Not surprisingly, if the private sector uses the least squares learning algorithm by setting the gain sequence

$$a \sim \frac{1}{t}, \tag{3.1}$$

then the learning algorithm converges to the self-confirming equilibrium with probability 1.

However, if we fix the gain sequence to a small positive number, then the dynamics of the stochastic system change dramatically. A fixed gain learning algorithm is one way of allowing the private sector agents to guard against the possibility that they do not fully understand the economy in which they operate. Thus we are allowing the agents to acknowledge their uncertainty concerning their specification of the government's monetary policy. Should the system begin to behave differently than they had expected, the fixed gain algorithm will allow the private sector agents to track the time-variation in the system effectively.

Constant gain learning provides the third ingredient necessary to generate escape dynamics in this model (the other two being private sector misspecification of the policy rule, and feedback from private sector beliefs to government actions). We begin with a simulation example of the escape dynamics, and then we proceed to a discussion.

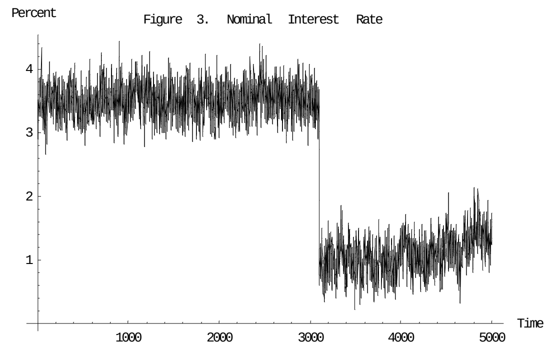

FIGURE 3. A large deviation from the equilibrium nominal interest rate. The system maintains interest rates in a neighborhood of 3.5 percent for many periods, but eventually the system departs to a low nominal interest rate outcome.

3.2. **A quantitative illustration.** We simulate our system under constant gain learning to generate an escape from the Taylor equilibrium. We note that the main qualitative feature of our simulation—that the system eventually displays a large deviation from the self-confirming equilibrium—is quite robust across parameter choices. But for purposes of illustration, we used the following parameter values. For the structural parameters, we took the calibrated values from Woodford (1999), $\sigma = 0.157$, $\kappa = 0.024$, and $\beta = .99$. We set ρ, which puts a lower bound on the nominal interest rate in this model, equal to zero. In the stochastic processes, we set $\alpha = .9$, $\sigma_\epsilon = .00372$, and $\sigma_\eta = .002$. This represents a high degree of serial correlation and a low level of noise in the system relative to Woodford (1999). This is mainly so that the noise does not interfere with our observation of the escape dynamics. We set the feedback parameters equal to $\gamma_0 = 0$ and $\gamma_1 = .01$. We keep the constant gain factor small by setting $a = .005$. This leaves only the coefficients in the government's Taylor rule to be set. We want to choose values that are consistent with both determinacy and learnability in the Bullard and Mitra analysis. This requires roughly that $\phi_\pi > 1$ in this model. Of course, we want to analyze an active Taylor rule as well, which also means $\phi_\pi > 1$. Accordingly, we set $\phi_\pi = 3.5$ and $\phi_z = 1$. These parameter choices mean that the government's target inflation rate at the self-confirming equilibrium is 3.5 percent. Since we have set $\rho = 0$, this is also the target nominal interest rate for this example.

The escape outcome is not a self-confirming equilibrium because the private sector perceptions are $\hat{\phi}_\pi = 1$, whereas the reality is that $\phi_\pi > 1$.

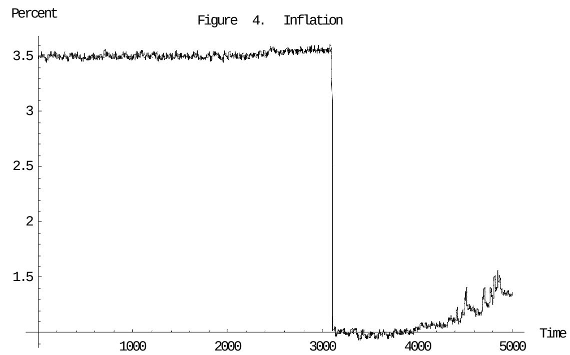

FIGURE 4. Inflation also falls as the system departs from the Taylor equilibrium.

Figure 3 shows the nominal interest rate dynamics for this example, in a simulation of 5,000 periods initialized at the self-confirming equilibrium. The system remains in a neighborhood of the self-confirming equilibrium for about 3,000 periods before an abrupt escape to the low nominal interest rate, low inflation outcome occurs. Since the low nominal interest rate outcome is not a self-confirming equilibrium, the system does not remain there indefinitely. Instead, the private sector begins to revise its estimate of government "hawkishness" on inflation upward, away from one and toward the actual value ϕ_π. This attempt to climb back toward the self-confirming equilibrium is more apparent in Figure 4, which shows the inflation rate gradually rising following the escape.

Figure 5 shows the private sector's estimate of the slope of the Taylor rule, $\hat{\phi}_\pi$, as the simulation unfolds. The initial estimated slope is exactly correct during the early portion of the simulation, but then rises gradually before the escape occurs. In simulations with alternative parameter values, we found the tendency of the estimated slope to rise, sometimes much more substantially, to be a generic feature of the dynamics.

3.3. **The escape outcome.** The escape outcome has agents becoming pessimistic concerning the aggressiveness of the government's response to inflation. In the neighborhood of the self-confirming equilibrium, the estimated "hawkishness" $\hat{\phi}_\pi$ fluctuates slightly because of the random perturbations. Points such as A, B, C, and D in Figure 6 will be part of the normal fluctuation about the self-confirming equilibrium. As the perceived "hawkishness" changes, so does the target inflation rate set by the government. In some circumstances, the observed data (π_t, r_t) can be generated around $(\bar{\pi}_t, \bar{r}_t)$, which is moving along the Fisher relation. These circumstances are akin to

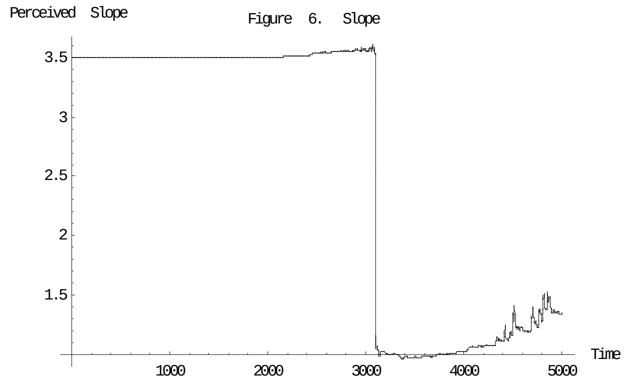

FIGURE 5. The perceived slope is 3.5, consistent with the Taylor equilibrium for many periods. But eventually, the agents estimate a slope of one, consistent with a Fisher relation. This latter perception is inconsistent with reality, and the system begins to move back toward the Taylor equilibrium.

observations along BC in Figure 6. The estimated slope will move close to unity, as illustrated by the regression line in the Figure. In turn, the government moves its target $(\bar{\pi}_t, \bar{r}_t)$ away from the self-confirming equilibrium and toward the origin. As the target moves away from the self-confirming equilibrium, even more data is generated in the neighborhood of the Fisher relation, because both π_t and r_t are falling. This reinforces the private sector's belief that the government's attitude toward inflation has softened, and leads the government to lower its inflation target still further. This process continues until the private sector's recursive estimate of $\hat{\phi}_\pi$ falls to unity, and the $(\bar{\pi}_t, \bar{r}_t)$ pair falls to a point consistent with the belief $\hat{\phi}_\pi = 1$, labelled $(\bar{\pi}_{escape}, \bar{r}_{escape})$ in Figure 6. This is the escape outcome of the model.

The escape outcome will occur with probability 1 so long as $\gamma_1 > 0$.

3.4. **Misspecification.** It is instructive to understand how the belief of the private sector can be self-confirmed despite the fact that its model is misspecified. For the sake of discussion, let us fix the market outcome at the self-confirming equilibrium. Because of the random perturbations w_t and η_t, (π_t, r_t) deviates from the self-confirming equilibrium level. In the neighborhood of the self-confirming equilibrium, the slope of the perceived Taylor rule must be precisely ϕ_π in order to minimize the forecasting error. Once $\hat{\phi}_\pi = \phi_\pi$ is achieved, the target outcome induced by the perception of the private sector becomes the self-confirming equilibrium outcome.

FIGURE 6

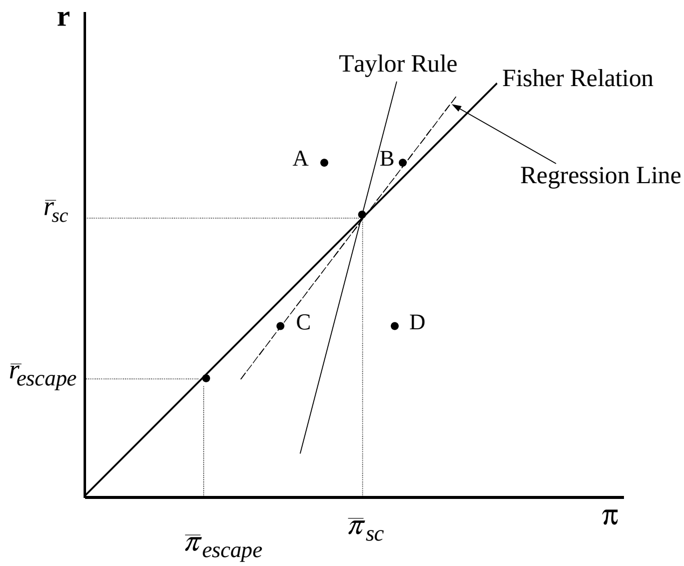

FIGURE 6. Schematic escape dynamics. Data generated along BC induces a slope estimate near unity. The values of $(\bar{\pi}, \bar{r})$ are then lowered, reinforcing the perceived passiveness of monetary policy.

If the private sector maintains a correctly specified model of the economy and estimates its parameters according to the least squares estimation method, the asymptotic properties of the decreasing gain algorithm should not be very different from that of the constant gain as long as the constant gain is sufficiently small. The dramatic contrast of the asymptotic dynamics of the two closely related learning algorithms thus suggests that the model of the private sector is misspecified in a certain sense.

One obvious flaw is that the agent does not include the output target in its regression equation. However, this is *not* the fundamental flaw. This is because we fixed the target output level as $\bar{z}_t = \bar{z} = 0$ for all $t \geq 1$. As a result, z_t is moving around a small neighborhood of $\bar{z}$. Therefore, the missing variable z_t in the private sector's perceived Taylor rule is nothing more than a constant plus small noise, which is properly captured by the intercept term $\hat{\phi}_{0,t}$ in the regression equation.

The fundamental flaw of the private sector's model is the implicit assumption that the government's inflation target does not respond to the private sector's perception. Within the confine of the self-confirming equilibrium, this assumption is viable, because the government's target price level is pinned down by the equilibrium belief of the private sector. However, if the private sector's belief about the government's attitude toward inflation changes, then the sustainable level of the inflation target, $\bar{\pi}_t$, changes accordingly. The government's policy is fixed: whatever the inflation target may be, it is committed to maintain that inflation target through an aggressive response of the nominal interest rate to changes in the inflation rate. Yet the government's target moves in response to the perception of the private sector. This change is not captured in the regression equation of the private sector, because the intercept term $\hat{\phi}_0$ is implicitly assumed to be constant (or close to constant) under the recursive least squares estimation process.

3.5. **Caveats.** In this preliminary version of the paper, we are only displaying the results from a single simulation. We do stress, however, that all other simulations, both with these parameter values and with others, displayed the same qualitative patterns. We can report on a number of caveats based on the additional simulations we have run so far.

First, the simulation above shows a system initialized at the self-confirming equilibrium, and then escaping from that equilibrium. One might wonder if the system eventually returns to the self-confirming equilibrium. The answer is that it may or it may not. For the parameters used to produce Figures 3, 4, and 5, when the system is simulated for a much longer period of time, one observes the nominal interest rate rising from the escape outcome up to about 2.5 percent. However, at that point, nominal interest rates again collapse to one before beginning the process again. Essentially, the system "escapes" from the self-confirming equilibrium before it actually gets all the way *to* the self-confirming equilibrium. However, for other parameter values the system can display a pattern of arriving at the self-confirming equilibrium, then escaping rapidly, then again achieving a neighborhood of the self-confirming equilibrium. This pattern is the same one observed in a different model by Cho, Sargent, and Williams (2000) and Sargent (1999).

Second, one might think that, from a quantitative perspective, the expected waiting time to an escape is quite long. But escape can occur much more frequently if we increase the value of the fixed gain parameter.

Third, because we have very little feedback to output in this model, there is very little going on with output here other than the serially correlated shock w_t.

4. Conclusion

In this paper we have sketched a theory of inadvertently low nominal interest rates. The theory is based on the existence of a self-confirming equilibrium in which inflation and nominal interest rates are relatively high. Our dynamic system can make sudden departures from that equilibrium towards a persistent low inflation, low nominal interest rate outcome which looks something like observed episodes in major industrialized countries. These escape dynamics are a consequence of the large deviation properties of our system. We have stressed that three key ingredients are required to generate the escape dynamics. The first of these is that the private sector's model of the government's policy is misspecified. The important element of this misspecification is subtle, however, in that the private sector does not realize that the monetary authority is adjusting the inflation target in response to private sector beliefs. The second element is that there is some feedback from beliefs to policy actions. And finally, the private sector needs to learn using a constant gain algorithm, which might be interpreted as allowing these agents to acknowledge their own uncertainty concerning the system in which they operate. With these elements in place, we showed that the long-run behavior of our small macroeconomic model includes recurrent visits to the "liquidity trap" outcome, even though that outcome is not a self-confirming equilibrium of the system.

From the government's point of view, perhaps little can be done to stop the private sector from continually using available data to update their estimates of the policy rule the government uses. And similarly, the nature of the econometric procedure the private sector employs may also be something the government cannot reliably influence. However, the third element needed to generate escape dynamics in this model is the feedback from private sector beliefs to the inflation target. If the government could credibly commit to a constant long-run inflation target, there could be no escape from the Taylor equilibrium in this model. A number of central banks have, in recent years, begun to state their inflation target more explicitly, although not the Bank of Japan or the Federal Reserve.

In this paper, we have treated the low nominal interest rate outcome as a point to be avoided. Low nominal interest rates have sometimes been associated with poor economic performance in actual economies. In many other contexts in monetary theory, however, low nominal interest rates are welfare-improving. We think of our problem as one where, for reasons exogenous to this model, the nominal interest rate and the inflation rate associated with the self-confirming equilibrium are socially optimal, and the goal of the government is to cause these values to come about. The large deviation from this equilibrium is then inadvertent and unwanted.

References

[1] Benhabib, Jess, Stephanie Schmitt-Grohe, and Martin Uribe. 1999. "The Perils of Taylor Rules." Working paper, Rutgers University, forthcoming in the *Journal of Economic Theory.*

[2] Bullard, James, and Kaushik Mitra. 2000. "Learning About Monetary Policy Rules." Working paper, Federal Reserve Bank of St. Louis.

[3] Cho, In-Koo, Thomas Sargent, and Noah Williams. 2000. "Escaping Nash Inflation." Manuscript, Stanford University.

[4] Clarida, Richard, Jordi Gali, and Mark Gertler. 1999. "The Science of Monetary Policy: A New Keynesian Perspective." *Journal of Economic Literature* XXXVII(4): 1661-1707.

[5] King, Robert, and Charles Plosser, editors. 1999. "Special Issue: Monetary Policy Rules." *Journal of Monetary Economics*, 43 (June).

[6] Rotemberg, Julio, and Michael Woodford. 1998. "Interest-Rate Rules in an Estimated Sticky-Price Model." In John Taylor, ed., *Monetary Policy Rules.* Chicago: University of Chicago Press.

[7] Sargent, Thomas. 1999. *The Conquest of American Inflation.* Princeton University Press.

[8] Summers, Lawrence. 1991. "How Should Long Term Monetary Policy Be Determined?" *Journal of Money, Credit and Banking*, 23: 625-631.

[9] Taylor, John. 1993. "Discretion Versus Policy Rules in Practice." *Carnegie-Rochester Conference Series on Public Policy*, 39: 195-214.

[10] Taylor, John. 1999. "A Historical Analysis of Monetary Policy Rules." In John Taylor, ed., *Monetary Policy Rules.* Chicago: University of Chicago Press.

[11] Wheelock, David. 1991. *The Strategy and Consistency of Federal Reserve Monetary Policy 1924-1933.* Cambridge University Press.

[12] Woodford, Michael. 1999. "Optimal Monetary Policy Inertia." NBER Working Paper #7261, July.

[13] Woodford, Michael. 2001. "The Taylor Rule and Optimal Monetary Policy." Forthcoming, *AEA Papers and Proceedings.*

Research Department, Federal Reserve Bank of St. Louis, 411 Locust Street, St. Louis, MO 63102

E-mail address: bullard@stls.frb.org

URL: www.stls.frb.org/research/econ/bullard

Department of Economics, 330 Wohlers Hall, University of Illinois at Urbana-Champaign, 1206 S. Sixth Street, Champaign, IL 61820

E-mail address: inkoocho@uiuc.edu