Payments Systems and Monetary Policy

Stephen D. Williamson

Department of Economics

University of Iowa

Iowa City, IA 52242

stephen-williamson@uiowa.edu

www.biz.uiowa.edu/faculty/swilliamson

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Abstract

A dynamic spatial model is constructed where there is a role for money and for centralized payments arrangements, and where there are aggregate fluctuations driven by fluctuations in aggregate productivity. With decentralized monetary exchange and no centralized payments arrangements, there is price level indeterminacy, and the equilibrium allocation is inefficient. A private clearinghouse arrangement is equivalent to payments system operated by the central bank. This arrangement improves efficiency but produces a real indeterminacy. The pricing of daylight overdrafts is irrelevant for the equilibrium allocation. Efficiency is achieved with a zero nominal interest rate on overnight central bank lending, or through private overnight interbank lending.

1.INTRODUCTION

The objective of this paper is to study the role of the central bank in a model which permits alternative types of payment arrangements. In the model there is a role for monetary exchange, and for centralized credit, and there are deterministic fluctuations in aggregate output, consumption, and employment, driven by fluctuations in aggregate productivity. The interaction between private payments arrangements and central bank credit is shown to have important implications for the variability of relative prices, the price level, consumption, output, and employment.

Economists have long been interested in the implications of private intermediation arrangements for monetary policy. Friedman (1960) argued that there should be a separation of money from credit, imposed through a 100% reserve requirement on intermediary liabilities which serve as a medium of exchange. He viewed fluctuations in the aggregate quantity of media of exchange and the price level as detrimental, and the 100% reserve requirement would improve monetary control by insulating the money stock from shocks to the quantity of credit. However, Sargent and Wallace (1982) countered that endogenous variability in the stock of media of exchange and the price level could be consistent with Pareto optimality, and inhibiting the creation of private media of exchange through legal restrictions on private intermediation would in general result in inefficiency. In the Sargent-Wallace model, Pareto optimality is achieved either through unrestricted private intermediation, or with restricted intermediation and unrestricted central bank credit. Champ, Smith and Williamson (1996) extended the Sargent and Wallace model to an environment with Diamond and Dybvig (1983) banks and circulating media of exchange. They showed that legal restrictions on private intermediation, in addition to implying inefficiency, could create banking panics, in a manner consistent with empirical evidence from the U.S. National Banking era and the same period in Canadian banking history.

A significant quantity of recent research has focussed on the design and functioning of payments systems. The payments system is the network of private and public intermediation arrangements through which transactions take place. It can be taken to include payments intermediated by government-supplied currency, payments by check cleared through private or central bank clearing facilities, electronic interbank payments cleared through the central bank or private intermediaries, and payments using electronic cash. Some research on payments examines issues related to the pricing of credit in payments systems, incentives, and risk sharing (Fujiki, Green and Yamazaki 1997, Rochet and Tirole, 1996a, 1996b), while other work looks at the general equilibrium implications of payments system arrangements (Freeman 1996a, 1996b, Lacker 1997, Kahn and Roberds 1998, Williamson 1998, Temzelides and Williamson 2000).

Payments system research is novel in monetary economics, in that it deals with the interaction between decentralized media of exchange (fiat money) and centralized payments arrangements. However, there are many issues associated with payments arrangements that are closely related to those addressed in the literature discussed above on the interaction between money and private intermediation. First, it is clear that intermediation by the central bank can be a close substitute for private intermediation in payments arrangements. For example, in the United States the Federal Reserve system is a major participant in check clearing and in interbank electronic payments through its Fedwire system. Second, some have argued that central bank credit related to payments system activity should be restricted, either through pricing or credit constraints (Greenspan 1996). This issue is closely-related to Friedman's notion that money and credit should be separated.

In this paper, I construct a model with spatial separation, which is related to the turnpike model of Townsend (1980). Monetary exchange is motivated by an absence of double coincidence problem, and there is a role for a centralized payments system with net settlement. The model is closely-related to the one constructed in Temzelides

and Williamson (2000), with the novelties here being that I include deterministic fluctuations in aggregate productivity and explore different issues. A period in the model is interpreted as being a day, so that issues associated with daytime payments system credit, the pricing of daylight overdrafts, and the effects of overnight central banking lending can be explored.

I study two alternative payments arrangements. In the first (the cash-in-advance, or CIA arrangement), there is no centralized payments system, and all transactions are carried out using fiat money. This leads to endogenous cash-in-advance constraints, and in equilibrium households accumulate cash two periods in advance of purchases. In an equilibrium with a fixed money supply there is an indeterminacy in the price level and in relative prices, and there is an equilibrium which has nice properties in terms of Friedman's (1960) goals for sound monetary control. That is, in this equilibrium the price level is constant and the money supply is constant. Further, in this equilibrium productivity fluctuations are accommodated as in the Pareto optimal allocation, but the equilibrium allocation is not Pareto optimal. This inefficiency arises for reasons that Friedman would understand, in that the rate of return on money is too low, and households economize excessively on real cash balances.

In the second payments arrangement, the clearinghouse arrangement, there is a centralized clearinghouse through which private credit is cleared on net, and settlement occurs at the end of each period in outside money. The clearinghouse relaxes some, but not all, cash-in-advance constraints and in general improves efficiency. However, this potentially produces real indeterminacy. Even though there is endogenous private intermediation which might potentially accommodate fluctuations in aggregate productivity, it is possible that an increase in the variability in aggregate productivity can increase the variability in relative prices (which in some sense reflects an increase in inefficiency) and in consumption across households within the period.

Central bank credit can be a perfect substitute for a private clearinghouse, in that

the same equilibrium allocation can be supported with the clearinghouse arrangement as with a central bank arrangement where the central bank operates a payments system with zero nominal-interest daylight overdrafts and settlement in outside money at the end of the period. So long as the central bank charges the same interest rate on daylight overdrafts as it pays on positive balances against the central bank-operated payments system, the pricing of daylight overdrafts is irrelevant. That is, transactions prices will change in such a way that the pricing of daylight overdrafts is neutral.

If the central bank makes overnight loans at a zero nominal interest rate, financed by the issue of outside money, then this supports a Pareto efficient allocation of resources. The price level on average decreases at the rate of time preference, and is above (below) trend when aggregate productivity is below (above) trend. This optimal arrangement can also be supported with a private intermediation arrangement where there is unrestricted interbank overnight lending, and outside money is not necessary.

The paper is organized as follows. In Section 2, the model is constructed, and a Pareto optimal allocation is characterized, while in Section 3 the CIA arrangement with no central bank intervention is analyzed. Section 4 contains a study of the clearinghouse arrangement, with an example and a discussion of the relationship between the velocity of money in the Section 3 and Section 4 payments regimes. In Sections 5 and 6, clearing and settlement through the central bank and overnight discount window lending are examined, respectively. Section 7 is a conclusion.

2. THE MODEL

This is a model of spatial frictions set up to motivate monetary exchange in a tractable way. There is a countable infinity of locations, indexed by $i = -\infty, ..., -1, 0, 1, ..., \infty$. In each location i there is a representative household, consisting of a *producer* and a

shopper. Each household has preferences given by

$$\sum_{t=0}^{\infty} \beta^t [u(c_t) - z(n_t)],$$

where $0 < \beta < 1$, c_t is consumption, and n_t is labor supply. Assume that $u(\cdot)$ is strictly increasing, strictly concave, twice continuously differentiable, with $u(0) = 0$ and $u'(0) = \infty$. Also assume that $z(\cdot)$ is strictly increasing, convex, and twice continuously differentiable with $z(0) = z'(0) = 0$.

There is an absence of double coincidence of wants, in that household i consumes only the goods produced by the household at location $i+1$. One unit of labor produces θ_t units of the consumption good if household i is productive, and zero units otherwise, where $\theta_t = \theta^e$ in even periods, and $\theta_t = \theta^o$ in odd periods, with $\theta^e > \theta^o$ Thus, productivity follows a two-cycle. At the beginning of each period, shopper i leaves location i and purchases goods at location $i + 1$, provided that the household at location $i+1$ is productive. Goods are then produced, there is exchange, and shoppers return to their initial locations with goods at the end of the period. Goods cannot be transported between locations, except by returning shoppers.

There are three types of households, denoted by $j = 1, 2, 3$. Type j households reside at locations ..., $j - 3$, j, $j + 3$, ... , and are unproductive in periods $3 - j$, $6 - j$, $9 - j$, ... Thus, every household follows a 3-period production cycle, and given that a household of type j buys goods from a household of type $j + 1$ (modulo 3), as in Figure 1, there will also be a 3-period consumption cycle. The production cycles are described in the following table, where P (NP) denotes a period when the household is productive (unproductive).

Table 1: Production Cycles

Type	$t=0$	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	...
1	P	P	NP	P	P	NP	...
2	P	NP	P	P	NP	P	...
3	NP	P	P	NP	P	P	...

Each household follows a three-cycle where it consumes and does not produce in a given period, then consumes and produces in the following period, and produce but does not consume in the next period hence. During any period, two thirds of the population is producing and two thirds is consuming.

The absence of double coincidence of wants problem implies that barter exchange is impossible, while the spatial structure implies that IOUs issued by households will not circulate in equilibrium. For trade to take place, we must introduce outside assets, or centralized credit arrangements.

The empirical length of time that corresponds to one period in the model is open to interpretation, but in terms of many of the issues addressed in this paper, a period is should be interpreted as one day. We thus have daily deterministic fluctuations in aggregate productivity, and much of our interest here will be in how the payments system arrangements and central bank policy accommodate these fluctuations. Indeed, though key monetary policy decisions typically relate to fluctuations at business cycle frequencies, much of the work of central banking concerns monetary control at daily, weekly, or monthly frequencies.

Optimality

If the social planner weights the utility of each household equally, Pareto optimal allocations are straightforward to determine. If household $i+1$ is productive in an even (odd) period, then it is Pareto optimal for household $i+1$ to produce $\frac{c^{e*}}{\theta^e}$ ($\frac{c^{o*}}{\theta^o}$) units of consumption goods, and for household i to consume c^{e*} (c^{o*}) goods, where

c^{e*} and c^{o*} satisfy

$$u'(c^{j*}) - \frac{1}{\theta^j} z'\left(\frac{c^{j*}}{\theta^j}\right), \text{ for } j = e, o. \tag{1}$$

3. NO CENTRALIZED PAYMENTS SYSTEM, NO CENTRAL BANK INTERVENTION

We will first study how this economy behaves when there is a fixed per capita quantity of fiat money, M, which enters as an endowment in period 0. For now, we assume that there are no centralized credit arrangements, i.e. no payments arrangements and no central bank. This economy has the feature that there are endogenous cash-in-advance constraints, and we will refer to the institutional arrangement in this section as *cash-in-advance (CIA),* since the cash that is required to purchase goods must be acquired one period in advance, given the spatial restrictions on exchange.

We will restrict attention here to steady state symmetric equilibria where the prices faced by a given household depend only on the phase of the household's production cycle, and whether the period is even or odd. In what follows, subscripts will denote the phase of the production cycle (1 denotes a period of consumption and no production, 2 a period of consumption and production, and 3 a period of production and no consumption), and superscripts denote even and odd periods (e for even, o for odd). Given exchange patterns among households (see Figure 1), in period 1 of the production cycle, households purchase goods at the price p_1^j, $j = e, o$, in period 2 of the production cycle, households purchase goods at the price p_2^j, $j = e, o$, and sell goods at the price p_1^j, $j = e, o$, while in period 3 of the production cycle, households sell goods at the price p_2^j, $j = e, o$. Thus, in period 1 of the production cycle, when the household consumes but does not produce, the Bellman equation is given by

$$v_1^j(m_1^j) = \max[u(c_1^j) + \beta v_2^k(m_2^k)] \tag{2}$$

subject to

$$c_1^j \leq m_1^j, \tag{3}$$

$$c_1^j + \frac{p_2^k}{p_1^j} m_2^k \leq m_1^j, \tag{4}$$

for $(j,k) = (e,o)$, (o,e). Here, m_i^j denotes real cash balances in period i of the production cycle, with $j = e, o$ denoting even and odd periods respectively, and $v_i^j(\cdot)$ is the value function, with subscripts and superscripts defined similarly. The inequality (3) is the cash-in-advance constraint, while (4) is the budget constraint. In period 2 of the production cycle, when the household consumes and produces, the Bellman equation is

$$v_2^j(m_2^j) = \max[u(c_2^j) - z(n_2^j) + \beta v_3^k(m_3^k)] \tag{5}$$

subject to

$$c_2^j \leq m_2^j, \tag{6}$$

$$c_2^j + \frac{p_2^k}{p_2^j} m_3^k \leq m_2^j + \frac{p_1^j}{p_2^j} n_2^j \theta^j, \tag{7}$$

for $(j,k) = (e,o)$, (o,e). Here, inequality (6) is the cash-in-advance constraint, and (7) is the budget constraint. Finally, in period 3 of the production cycle, when the household does not consume but produces, the Bellman equation is

$$v_3^j(m_3^j) = \max[-z(n_3^j) + \beta v_1^k(m_1^k)] \tag{8}$$

subject to

$$\frac{p_1^k}{p_2^j} m_1^k \leq m_3^j + n_3^j \theta^j, \tag{9}$$

for $(j,k) = (e,o)$, (o,e), where (9) is the budget constraint. We omit nonnegativity constraints on money balances, consumption, and labor supply in the above three problems, as our restrictions on preferences and the cash-in-advance constraints (3) and (6) imply that these nonnegativity constraints will not bind.

Let λ_i^j and μ_i^j denote the multipliers associated with the cash-in-advance and budget constraints, respectively, for $i = 1, 2, 3$, and $j = e, o$, and assume that the value functions are concave and differentiable. Then, the first-order conditions from the optimization problems on the right-hand sides of Bellman equations (2), (5), and (8) are

$$u'(c_1^j) - \lambda_1^j - \mu_1^j = 0, \tag{10}$$

$$\beta v_2^{k\prime}(m_2^k) - \frac{p_2^k}{p_1^j}\mu_1^j = 0, \tag{11}$$

$$u'(c_2^j) - \lambda_2^j - \mu_2^j = 0, \tag{12}$$

$$-z'(n_2^j) + \frac{p_1^j}{p_2^j}\theta^j \mu_2^j = 0, \tag{13}$$

$$\beta v_3^{k\prime}(m_3^k) - \frac{p_2^k}{p_2^j}\mu_2^j = 0, \tag{14}$$

$$-z'(n_3^j) + \mu_3^j \theta^j = 0, \tag{15}$$

$$\beta v_1^{k\prime}(m_1^k) - \frac{p_1^k}{p_2^j}\mu_3^j = 0, \tag{16}$$

for $(j, k) = (e, o)$, (o, e). Next, the envelope conditions are

$$v_i^j(m_i^j) = \lambda_i^j + \mu_i^j, \tag{17}$$

$$v_3^j = \mu_3^j, \tag{18}$$

for $i = 1, 2$, $j = e, o$. Finally, given that households in period 1 (2) of the production cycle buy from households in period 2 (3) of the production cycle, the following equilibrium conditions must hold.

$$c_1^j = n_2^j, \text{ for } j = e, o, \tag{19}$$

$$c_2^j = n_3^j, \text{ for } j = e, o. \tag{20}$$

From Temzelides and Williamson (2000), we know that when $\theta^e = \theta^o$, so that even periods are identical to odd periods, the cash-in-advance constraint does not bind in

the first period of the production cycle for each household, but binds in the second period. Thus, by continuity, if θ^e is close to θ^o, then the cash-in-advance constraint will bind in even and odd periods only for households in the second period of their production cycle. We then consider the symmetric steady state equilibrium where $\lambda_1^j = 0$ and $\lambda_1^j > 0$ for $j = e, o$. Then, from (10)-(20), the following equation solves for c_i^j,

$$\beta^2 u'(c_i^j) - z'\left(\frac{c_i^j}{\theta^j}\right)\frac{1}{\theta^j} = 0, \text{ for } i = 1, 2, \ j = e, o, \tag{21}$$

so that $c_1^j = c_2^j = c^j$ for $j = e, o$, so that consumption and labor supply depend only on aggregate productivity and not on an individual household's period in the production cycle. Also, it is clear from (1) and (21) that the competitive equilibrium allocation is not Pareto optimal, and that consumption is smaller at even and odd dates and at each period in the production cycle than at the optimum. Further, note from (21) that consumption is higher in even periods (when aggregate productivity is higher) than in odd periods.

From (10)-(20), we can determine relative prices to be

$$\frac{p_2^o}{p_1^e} = \frac{\beta u'(c^o)}{u'(c^e)}, \tag{22}$$

$$\frac{p_2^e}{p_1^o} = \frac{\beta u'(c^e)}{u'(c^o)}, \tag{23}$$

but the relative price $\frac{p_1^o}{p_1^e}$ is indeterminate (and therefore by implication $\frac{p_2^o}{p_2^e}$ is as well), though there is no indeterminacy in equilibrium quantities. From (12) and (21), $\lambda_2^j \geq 0$ holds for $j = e, o$ (the cash-in-advance constraint always binds in period 2 of the production cycle) if and only if

$$\frac{p_2^j}{p_1^j} \leq \frac{1}{\beta^2}, \tag{24}$$

for $j = e, o$.

Given the relative price indeterminacy, it is possible to have equilibria with or without price dispersion within a period. That is, (24) permits an equilibrium where

$\frac{p_2^j}{p_1^j} \neq 1$ for all j, so that there is always price dispersion or where $\frac{p_2^j}{p_1^j} = 1$, in which case there is no price dispersion across locations. Also, it is possible to have an equilibrium where there are no fluctuations in money prices across periods, or where

$$\frac{p_i^e}{p_i^o} = 1, \ i = 1, 2. \tag{25}$$

Necessary and sufficient conditions for such an equilibrium to exist, from (22)-(24), are that

$$\frac{\beta^3 u'(c^o)}{u'(c^e)} \leq 1, \tag{26}$$

$$\frac{\beta^3 u'(c^e)}{u'(c^o)} \leq 1. \tag{27}$$

Now, since $c^e > c^o$, (27) holds, but we need to construct an example where (26) holds. Suppose that $u(c) = \log c$ and $z(n) = \gamma n$ with $\gamma > 0$. Then (26) becomes $\beta^3 \frac{\theta^e}{\theta^o} \leq 1$, which clearly holds so long as β is sufficiently small. Therefore, there can be an equilibrium which has very nice properties in terms of monetarist doctrine, in that the money supply and the price level are constant. Further, the allocation has the property that productivity shocks are accommodated, in the sense that, at the optimum and in the equilibrium allocation, consumption and output during odd (even) periods depends only on (odd) even period productivity. In spite of these properties of the equilibrium allocation, this allocation is inefficient. As well, given the relative price indeterminacy, price level variability is essentially unrelated to the equilibrium allocation of resources.

Some of the properties of the equilibrium with the CIA arrangement result from the fact that the holding period of cash is effectively two periods. There is a standard monetary inefficiency in that, with a fixed money supply, the rate of return on money is too low, and households hold less than the optimal quantity of real balances and consume too little. However, because households acquire cash two periods in advance of when it is spent, the rate of return on money will depend only on aggregate

productivity in the current period and aggregate productivity two period's hence, and given the two-cycle in aggregate productivity, these two are the same. Thus, in (21) the solution has the property that consumption in odd (even) periods is independent of aggregate productivity in even (odd) periods, which is also a feature of the Pareto optimal allocation described by (1). The indeterminacy in relative prices, which is essentially an indeterminacy in the variability in the price level across periods, also results from the match between the holding period of money and the frequency of aggregate productivity fluctuations. Fundamentally, in a symmetric steady state equilibrium, trade in even periods is completely unrelated to trade in odd periods, and the fraction of the money stock held in odd (even) periods is indeterminate. This property of the model is obviously special, but it indicates the importance of the relationship between the transactions velocity of money and the frequency of fluctuations for equilibrium outcomes.

4. A PAYMENTS SYSTEM WITH NET SETTLEMENT AND NO CENTRAL BANK INTERVENTION

Now, suppose a private centralized clearinghouse arrangement, whereby households exchange IOUs for goods, with the IOUs settled at the end of the period with the clearinghouse in fiat money. It is important here that settlement is made on net with the clearinghouse, since gross settlement implies that transactions arrangements will be identical to those in the CIA regime, with households facing cash-in-advance constraints in periods 1 and 2 of the production cycle. We will refer to this institution as the *clearinghouse arrangement.*

The clearinghouse receives IOUs from households at the beginning of the period, in exchange for promises to pay fiat money at the end of the period. IOUs held by the clearinghouse are then redeemed at the end of the period for fiat money, and the clearinghouse makes good on its promises. Given free entry into this intermediation

activity, where we permit the flow of IOUs and fiat money across locations, but not the flow of goods, the clearinghouse will earn zero profits, and IOUs will be priced at their redemption value.

The clearinghouse arrangement implies that, in contrast to the CIA arrangement, households are not subject to a cash-in-advance constraint period 2 of the production cycle, when a household consumes and produces. Since households do not produce in period 1 of the production cycle, they have no opportunity to acquire IOUs, and so they continue to face a cash-in-advance constraint in that period. With the clearinghouse arrangement, we will have to take account of the nonnegativity constraint on money balances at the beginning of period 3 of the production cycle. In the CIA arrangement considered in the previous section, the cash-in-advance constraints in periods 1 and 2 of the production cycle implied that nonnegativity constraints on money balances would never bind, so these constraints were ignored. Thus, the optimization problems in (2)-(9) remain the same, except that we drop the cash-in-advance constraints (6), and add the nonnegativity constraints on money balances

$$m_3^j \geq 0, \text{ for } j = e, o. \tag{28}$$

We will let δ_2^j, $j = e, o$, denote the multipliers associated with these nonnegativity constraints. Then, in a steady state symmetric equilibrium, the first order conditions (10)-(13), (15), and (16) will be satisfied, with (14) replaced by

$$\beta v_3^{k\prime}(m_3^k) - \frac{p_2^k}{p_2^j}\mu_2^j + \delta_2^j = 0, \tag{29}$$

for $j = e, o$, with $\lambda_2^j = 0$, for $j = e, o$, since the cash-in-advance constraint in period 2 of the production cycle is irrelevant. The envelope conditions and equilibrium conditions (17)-(20) remain the same.

In Temzelides and Williamson (2000) we show that, in the case where $\theta^e = \theta^o$, there are potentially three steady state symmetric equilibria. In the first, the cash-in-advance constraint (3) binds in period 1 of the production cycle for each household

and the nonnegativity constraint (28) does not bind. In the second equilibrium both constraints bind, and in the third equilibrium the cash-in-advance constraint does not bind, and the nonnegativity constraint binds. Similar to the previous section, we consider only these three types of equilibria. For example, in the first equilibrium the cash-in-advance constraint binds for households in period 1 of the production cycle whether t is odd or even. By continuity, these three equilibria exhaust the possibilities for θ^e close to θ^o.

Case 1: $\lambda_1^j \geq 0,\ \delta_2^j = 0$

In this case, households in the first period of their production cycle spend all of their cash holdings to purchase consumption goods, and then the sales of goods in periods 2 and 3 of the production cycle finance consumption in periods 2 and 1 of the production cycle. From (10)-(13), (15)-(20) and (29) we can solve for equilibrium prices and quantities as follows. The following four equations solve for c_i^j, $i = 1, 2$, $j = e, o$.

$$\beta z'\left(\frac{c_2^e}{\theta^e}\right) c_2^e - \theta^e c_2^o u'(c_2^o) = 0, \tag{30}$$

$$\beta z'\left(\frac{c_2^o}{\theta^o}\right) c_2^o - \theta^o c_2^e u'(c_2^e) = 0, \tag{31}$$

$$\beta^2 u'(c_1^j) - \frac{1}{\theta^j} z'\left(\frac{c_1^j}{\theta^j}\right) = 0, \text{ for } j = e, o. \tag{32}$$

Then, relative prices are given by

$$\frac{p_2^e}{p_2^o} = \frac{c_2^o}{c_2^e}, \tag{33}$$

$$\frac{p_1^e}{p_2^e} = \frac{u'(c_1^e)}{u'(c_2^e)}, \tag{34}$$

$$\frac{p_1^o}{p_2^o} = \frac{u'(c_1^o)}{u'(c_2^o)}. \tag{35}$$

From (32), note that consumption in period 1 of the production cycle depends only on current productivity, while from (30) and (31) consumption in period 2 of the

production cycle for t even is jointly determined with consumption in period 2 of the production cycle for t odd. Therefore, in this equilibrium output in even periods is not independent of productivity in odd periods, and vice versa, as is the case at the optimum. Further, note that, with regard to price level fluctuations, from (33)-(35), that the relative price of goods traded across markets fluctuates, and that the price level fluctuates as well in equilibrium.

In this equilibrium, a portion of the money stock is held for two periods, while the remaining portion is held for one period. In particular, (32) reflects the fact that, in exchanges between shoppers in period 1 of the production cycle and producers in period 2 of the production cycle, some of the cash brought to the transaction was acquired two periods previously. However, in exchanges between shoppers in period 2 of their production cycle and producers in period 3 of the production cycle, all purchases are made with cash acquired in the previous period (see (30) and (31)). Therefore, in contrast to the CIA arrangement, consumption in even (odd) periods will in general depend on aggregate productivity in odd (even) periods, and price level variability across periods is determinate.

Case 2: $\lambda_1^j \geq 0,\ \delta_2^j \geq 0.$

In this equilibrium the cash-in-advance constraint binds in period 1 of the payment cycle, as does the nonnegativity constraint on money balances at the end of period 2 of the payments cycle. This implies that production in period 2 of the payments cycle finances consumption in period 2, and that production in period 3 finances consumption in period 1. From (10)-(13), (15)-(20) and (29), we obtain

$$c_2^e u'(c_2^e) - z'\left(\frac{c_1^e}{\theta^e}\right)\frac{c_1^e}{\theta^e} = 0, \tag{36}$$

$$c_2^o u'(c_2^o) - z'\left(\frac{c_1^o}{\theta^o}\right)\frac{c_1^o}{\theta^o} = 0, \tag{37}$$

$$\beta c_1^o u'(c_1^o) - z'\left(\frac{c_2^e}{\theta^e}\right)\frac{c_2^e}{\theta^e} = 0, \tag{38}$$

$$\beta c_1^e u'(c_1^e) - z'\left(\frac{c_2^o}{\theta^o}\right)\frac{c_2^o}{\theta^o} = 0, \tag{39}$$

$$\frac{p_2^e}{p_2^o} = \frac{c_2^o}{c_2^e}, \tag{40}$$

$$\frac{p_1^e}{p_2^e} = \frac{c_2^e}{c_1^e}, \tag{41}$$

$$\frac{p_1^o}{p_2^o} = \frac{c_2^o}{c_1^o}. \tag{42}$$

Here, (36)-(39) solve for consumptions, and then (40)-(42) solve for relative prices. Here, it is clear that consumption in periods 1 and 2 of the payments cycle and in even and odd periods are interdependent, and consumption in odd periods will depend on even period productivity and vice versa. This equilibrium therefore has properties similar to the case 1 equilibrium, but in the case 2 equilibrium all cash is held for only one period.

Case 3: $\lambda_1^j = 0$, $\delta_2^j \geq 0$.

In this equilibrium the cash-in-advance constraint does not bind in period 1 of the payment cycle, while the nonnegativity constraint on money balances binds at the end of period 2. Here, period 3 production finances consumption in period 1 and period 2 of the production cycle, while period 2 production finances consumption in period 2 of the production cycle. From (10)-(13), (15)-(20) and (29), we get

$$c_1^e u'(c_1^e) - \beta z'\left(\frac{c_1^o}{\theta^o}\right)\frac{c_1^o}{\theta^o} = 0, \tag{43}$$

$$c_1^o u'(c_1^o) - \beta z'\left(\frac{c_1^e}{\theta^e}\right)\frac{c_1^e}{\theta^e} = 0, \tag{44}$$

$$\beta^2 u'(c_2^j) - z'\left(\frac{c_2^j}{\theta^j}\right)\frac{1}{\theta^j} = 0, \text{ for } j = e, o, \tag{45}$$

$$\frac{p_2^e}{p_2^o} = \frac{u'(c_2^e)c_1^e u'(c_1^e)}{u'(c_2^o)c_1^o u'(c_1^o)}, \tag{46}$$

$$\frac{p_1^e}{p_2^e} = \frac{c_1^o u'(c_1^o)}{\beta c_1^e u'(c_2^e)}, \tag{47}$$

$$\frac{p_1^o}{p_2^o} = \frac{c_1^e u'(c_1^e)}{\beta c_1^o u'(c_2^o)}. \tag{48}$$

This equilibrium again has the property that consumption in odd (even) periods depends on aggregate productivity in even (odd) periods, and there is determinacy in price level variability across periods. Here, in transactions between shoppers in period 1 of the production cycle and producers in period 2 of the production cycle, all cash used in the exchange was acquired in the previous period. However, in transactions between shoppers in period 2 of the production cycle and producers in period 3 of the production cycle, some cash used in the transaction was acquired two periods previously.

An Example

Suppose that $u(c) = \frac{c^{1-\alpha}-1}{1-\alpha}$ and $z(n) = n$. From Temzelides and Williamson (2000), we then know that, if $\theta^e = \theta^o$, then the case 1 equilibrium exists for $\alpha \geq 3$, the case 2 equilibrium exists for $\alpha \leq \frac{3}{2}$ or $\alpha \geq 3$, and the case 3 equilibrium exists for $\alpha \geq \frac{3}{2}$. Thus, in this case a steady state symmetric equilibrium always exists, and there exist multiple equilibria when $\alpha \geq 3$. There is therefore an interesting tension here between efficiency and determinacy.[1] In the CIA arrangement, there was a determinate steady state symmetric equilibrium, with a price level indeterminacy that was irrelevant for the real allocation. Here, in the clearinghouse arrangement, there can be real indeterminacy. However, we show in Temzelides and Williamson that with $\theta^e = \theta^o$ and β close to 1, that any steady state symmetric equilibrium with the clearinghouse

[1] See Smith (1991) and Woodford (1994) which also feature a tension between efficiency and multiplicity of equilibrium.

arrangement dominates the steady state equilibrium with the CIA arrangement in terms of welfare.

Now suppose that we let $\alpha = \frac{1}{2}$. Allowing for fluctuations in productivity, if θ^e is sufficiently close to θ^o then by continuity there is a unique steady state symmetric equilibrium given the clearinghouse arrangement, which is the case 2 equilibrium. Solving for quantities and relative prices using (36)-(42), we obtain

$$c_1^e = (\theta^e)^{\frac{8}{5}}(\theta^o)^{\frac{2}{5}}\beta^{\frac{2}{3}}, \tag{49}$$

$$c_1^o = (\theta^e)^{\frac{2}{5}}(\theta^o)^{\frac{8}{5}}\beta^{\frac{2}{3}}, \tag{50}$$

$$c_2^e = (\theta^e)^{\frac{6}{5}}(\theta^o)^{\frac{4}{5}}\beta^{\frac{4}{3}}, \tag{51}$$

$$c_2^o = (\theta^e)^{\frac{4}{5}}(\theta^o)^{\frac{6}{5}}\beta^{\frac{4}{3}}, \tag{52}$$

$$\frac{p_2^e}{p_2^o} = \frac{c_2^o}{c_2^e} = \left(\frac{\theta^o}{\theta^e}\right)^{\frac{2}{5}}, \tag{53}$$

$$\frac{p_1^e}{p_2^e} = \frac{c_2^e}{c_1^e} = \left(\frac{\theta^o}{\theta^e}\right)^{\frac{2}{5}}\beta^{\frac{2}{3}}, \tag{54}$$

$$\frac{p_1^o}{p_2^o} = \frac{c_2^o}{c_1^o} = \left(\frac{\theta^e}{\theta^o}\right)^{\frac{2}{5}}\beta^{\frac{2}{3}}. \tag{55}$$

Further, from (49)-(55), we obtain

$$\frac{p_1^e}{p_1^o} = \frac{c_1^o}{c_1^e} = \left(\frac{\theta^o}{\theta^e}\right)^{\frac{6}{5}}. \tag{56}$$

This equilibrium has some of the properties we would anticipate. In particular, from (53) and (56), consumption is high and the price level is low when aggregate productivity is high, and consumption is low and the price level is high when aggregate productivity is low, i.e. variability in consumption and the price level increases with variability in aggregate productivity. Further, note from (49)-(51) that consumption in even (odd) periods depends on aggregate productivity in odd (even) periods, so that the clearinghouse arrangement does not accommodate productivity fluctuations.

A feature of the equilibrium we might not anticipate is that relative price variability across markets depends on the variability in aggregate productivity over time from (53) and (56). The relative price variability across markets reflects inefficiency, and since the clearinghouse arrangement does not sufficiently accommodate productivity fluctuations, the inefficiency becomes more pronounced as variability in productivity increases.

The Velocity of Money

In Temzelides and Williamson (2000), we show that, for the case where $\theta^e = \theta^o$ and β close to 1, the velocity of money approximately quadruples with the clearinghouse arrangement relative to the CIA arrangement. In the CIA arrangement, approximately half of the money stock is used in transactions each period, as money is acquired two periods in advance of when a transaction is made. However, with the clearinghouse arrangement, the money stock turns over approximately twice during the period. This is easiest to see in the case 2 equilibrium, where at the beginning of the period the money stock is held by shoppers in period 1 of their production cycle, and is exchanged for goods with producers in period 2 of their production cycle. Then, money is used at the end of the period by the households in period 2 of their production cycle to make settlement with the clearinghouse, and the entire stock of money is received from the clearinghouse at settlement time by households in period 3 of the production cycle. This flow of transactions is illustrated in Figure 2.

5. CLEARING AND SETTLEMENT THROUGH THE CENTRAL BANK

We will now consider an alternative financial arrangement, where the payments system is run through the central bank. We could think of this mechanism working

in a way similar to the private clearinghouse arrangement in Section 4, except that IOUs are accepted and cleared through the central bank rather than through the clearinghouse. Alternatively, we could have the central bank make zero-nominal-interest loans to households financed by the issue of outside money, with the loans repaid at the end of the period.

So long as the central bank charges zero nominal interest on what we can interpret as daylight overdrafts (the loans made by the central bank during the period), the equilibrium allocation will be identical to what it was with the private clearinghouse arrangement. Thus, given the equivalence between private clearing and settlement arrangements and clearing and settlement through the central bank, the model appears to be silent on the appropriate role for a central bank in clearing and settlement. However, though it may be the case in a frictionless world that the appropriate nominal interest rate on central bank loans is zero, it is not readily apparent that such a policy is appropriate here. This is because the equilibrium allocation is inefficient, and therefore some alternative pricing rule for daylight overdrafts might increase welfare. A private clearinghouse subject to free entry necessarily earns zero profits and its pricing is constrained by the market, but a monopoly central bank might be able to bring about an improvement, as it is not market-constrained.

Consider the case where the central bank charges a gross nominal interest rate on daylight overdrafts of R^e in even periods and R^o in odd periods. So that we separate the equilibrium effects of pricing from the effects of changes in the money growth rate (which would clearly be important here), we will assume that the central bank pays the same interest rate on positive daylight balances with the central bank as it charges on daylight overdrafts. Thus, the central bank will earn zero profits and the pricing of daylight overdrafts will not change the stock of money from period to period.

With the overdraft pricing policy in place, the effective price in exchanges between

a shopper in period 2 of his or her production cycle and a producer in period 3 of his or her production cycle is then $p_2^j R^j$, for $j = e, o$. The only effect of the overdraft pricing policy, then, would be a change in p_2^j leaving the effective price, $p_2^j R^j$, unchanged. Thus, this overdraft pricing policy is neutral.

It is possible however, that if the central bank charged a higher rate of interest on daylight overdrafts than on positive balances, that this could lead to a welfare improvement. The presumption is that deflation is welfare-enhancing, following the standard logic behind the Friedman rule. Thus, if there is zero nominal interest on positive daily balances with the central bank, and positive nominal interest on daylight overdrafts, with the profits on transactions intermediation used to reduce the stock of money over time, there may be a welfare improvement. This is in spite of the fact that this type of pricing rule implies a wedge between buying and selling prices in the market where households in periods 2 and 3 of their production cycles trade. However, as a result of the distortion that is introduced, it would likely be more efficient to implement negative growth in the money stock through a tax-transfer policy.

6. OVERNIGHT DISCOUNT WINDOW LENDING

Now, suppose that, given either the CIA arrangement or the clearinghouse arrangement, the central bank makes one-period zero-nominal-interest loans at the end of each period, with repayment due at the end of the following period. These discount window loans are financed by the issue of outside money, and this regime will be one where all outside money is backed by discount window loans.

We will look for a steady state symmetric equilibrium, given this policy, where quantities depend only on the period in the payments cycle and whether t is odd or even. We will want to take account of the fact that there might be long run growth in the price level in equilibrium. Households will solve the same optimization

problems as in (2)-(9), except that we drop all cash-in-advance constraints, as well as the nonnegativity constraints on money balances which needed to be imposed in the clearinghouse arrangement in Section 4. Cash-in-advance constraints do not bind, as the household can effectively borrow against the income it will receive at the end of the period in order to make purchases during the period. Nonnegativity constraints on money balances do not bind since, given this policy, a discount window loan is effectively a negative money balance.

From (10)-(20), we then obtain

$$\beta u'(c_2^k) - \frac{p_2^k(t+1)}{p_1^j(t)} u'(c_1^j) = 0, \tag{57}$$

$$-z'\left(\frac{c_1^j}{\theta^j}\right)\frac{1}{\theta^j} + \frac{p_1^j(t)}{p_2^j(t)} u'(c_2^j) = 0, \tag{58}$$

$$\beta z'\left(\frac{c_2^k}{\theta^k}\right)\frac{1}{\theta^k} - \frac{p_2^k(t+1)}{p_2^j(t)} u'(c_2^j) = 0, \tag{59}$$

$$\beta u'(c_1^k) z'\left(\frac{c_2^k}{\theta^k}\right)\frac{1}{\theta^k} - \frac{p_1^k(t+1)}{p_2^j(t)} z'\left(\frac{c_2^j}{\theta^j}\right)\frac{1}{\theta^j} = 0, \tag{60}$$

where we use the obvious notation to allow for the fact that there can be long-run changes in prices in equilibrium. It is straightforward to show that there is no equilibrium where prices depend only on the period in the payment cycle and whether the period is odd or even. A solution to (57)-(60) is $c_1^j = c_2^j = c^j$ for $j = e, o$, with c^j satisfying

$$u'(c^j) - \frac{1}{\theta^j} z'\left(\frac{c^j}{\theta^j}\right), \text{ for } j = e, o, \tag{61}$$

and

$$p_i^j(t) = p_i^j \beta^t,$$

where

$$\frac{p_1^j}{p_2^j} = 1, \text{ for } j = e, o,$$

$$\frac{p_i^e}{p_i^o} = \frac{u'(c^e)}{u'(c^o)}, \text{ for } i = 1, 2.$$

Clearly, from (1) and (61), the equilibrium allocation is Pareto optimal. From the budget constraints (4), (7), and (9), we can solve for the assets held by agents in each period of the production cycle and in each period. Letting b_i^j denote the net assets held by an agent at the beginning of period i of the production cycle, $j = e, o$, we obtain

$$b_1^e = \frac{(1-\beta^2)\left(\beta^3 \frac{u'(c^o)}{u'(c^e)} c^o + c^e\right)}{1-\beta^6},$$

$$b_1^o = \frac{(1-\beta^2)\left(\beta^3 \frac{u'(c^e)}{u'(c^o)} c^e + c^o\right)}{1-\beta^6},$$

$$b_2^e = \frac{(1-\beta^2)\beta^2 c^e - \frac{u'(c^o)}{u'(c^e)}(1-\beta^4)c^o}{1-\beta^6},$$

$$b_2^o = \frac{(1-\beta^2)\beta^2 c^o - \frac{u'(c^e)}{u'(c^o)}(1-\beta^4)c^e}{1-\beta^6},$$

$$b_3^e = \frac{(1-\beta^2)\frac{u'(c^e)}{u'(c^o)} c^e - (1-\beta^4)c^o}{1-\beta^6},$$

$$b_2^o = \frac{(1-\beta^2)\frac{u'(c^o)}{u'(c^e)} c^o - (1-\beta^4)c^e}{1-\beta^6}.$$

From the above, we have $b_1^j + b_2^j + b_3^j = 0$ for $j = e, o$, so that outside money is backed one-for-one with discount window loans in equilibrium. It is straightforward to show that the central bank's budget constraint holds in even and odd periods given the above, so that this is an equilibrium. However, there may be other equilibria under a policy of unrestricted lending by the central bank at a zero nominal interest rate, and we do not study these equilibria here.

Thus, a Pareto optimal allocation can be achieved through a policy of offering discount window loans at a zero nominal interest rate, with transactions carried out using outside money. Note that, at the optimum, there is no price dispersion within the period, but the price level varies over time relative to trend, being high (low) relative to trend when consumption is low (high) relative to trend. It is important

to note that this Pareto optimal allocation could also be supported without central banking or outside money, with private "inter-bank" lending.

7. CONCLUSION

In this paper, I constructed a model with spatial separation in which there is a role for money and centralized payments arrangements, with aggregate fluctuations driven by fluctuations in aggregate productivity. When transactions are conducted only using fiat currency in the absence of a payments system, there is an indeterminacy in the price level and in relative prices. However, there exists an equilibrium where the price level and the money stock are constant, and where productivity fluctuations are accommodated. However, this equilibrium allocation is not Pareto optimal. With a private clearinghouse which permits payments activity with net settlement at the end of each period, there is an improvement in efficiency, but a real indeterminacy can result. An example shows that productivity shocks may not be accommodated, and the variability in relative prices (which reflects an inefficiency) increases with the variability in productivity. The pricing of intraday overdrafts proves to be irrelevant for the equilibrium allocation. A Pareto optimal allocation is achieved with overnight lending by the central bank at a zero nominal interest rate, and this equilibrium allocation can also be supported with unregulated interbank lending and without outside money.

This paper does not address any of the issues associated with risk-bearing and incentives which seem important for some aspects of payments system design. Given what is known about the nature of private insurance and credit contracts under private information, there are good reasons to believe that some restrictions on credit arrangements in payments systems would be appropriate. Temzelides and Williamson (2000) and Williamson (1998) consider dynamic environments with private information which show how credit constraints and imperfect insurance can arise endoge-

nously in payments arrangements. Whether or not there are "systemic risk" issues that need to be addressed in payments system design is a question which needs to be answered by future research.

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Figure 1: Patterns of Trade

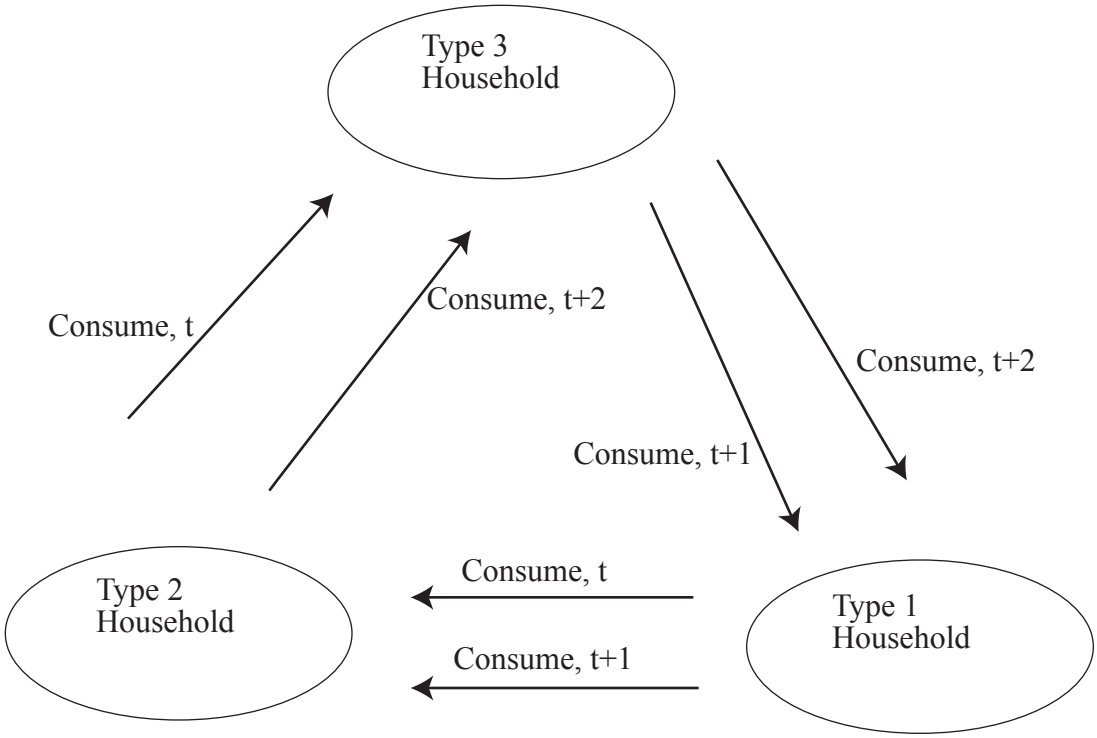
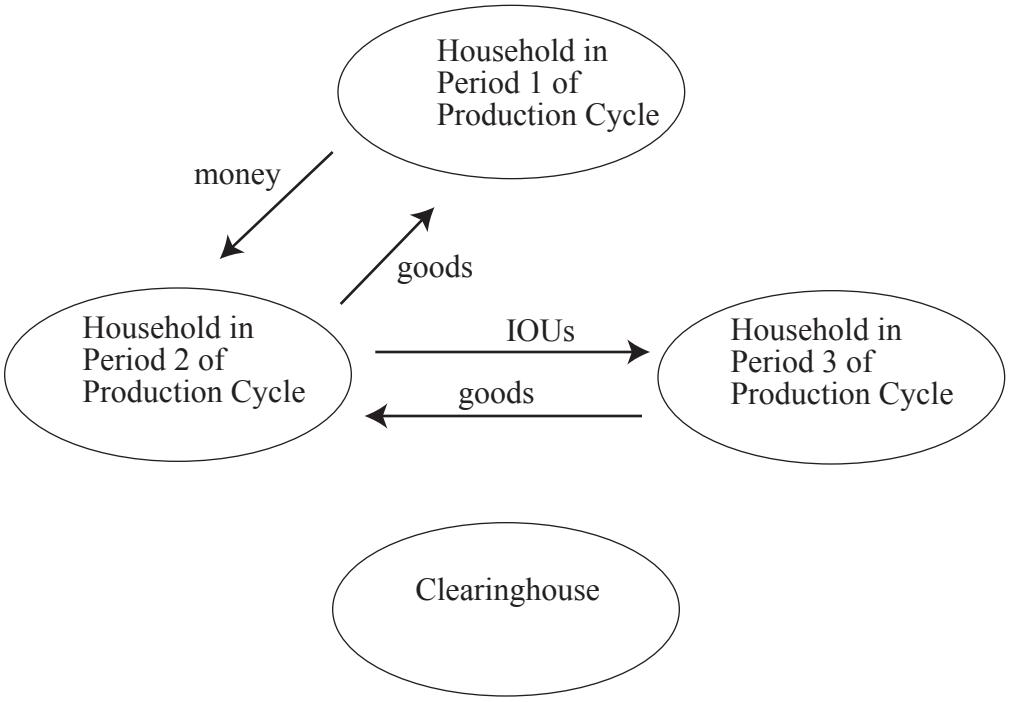
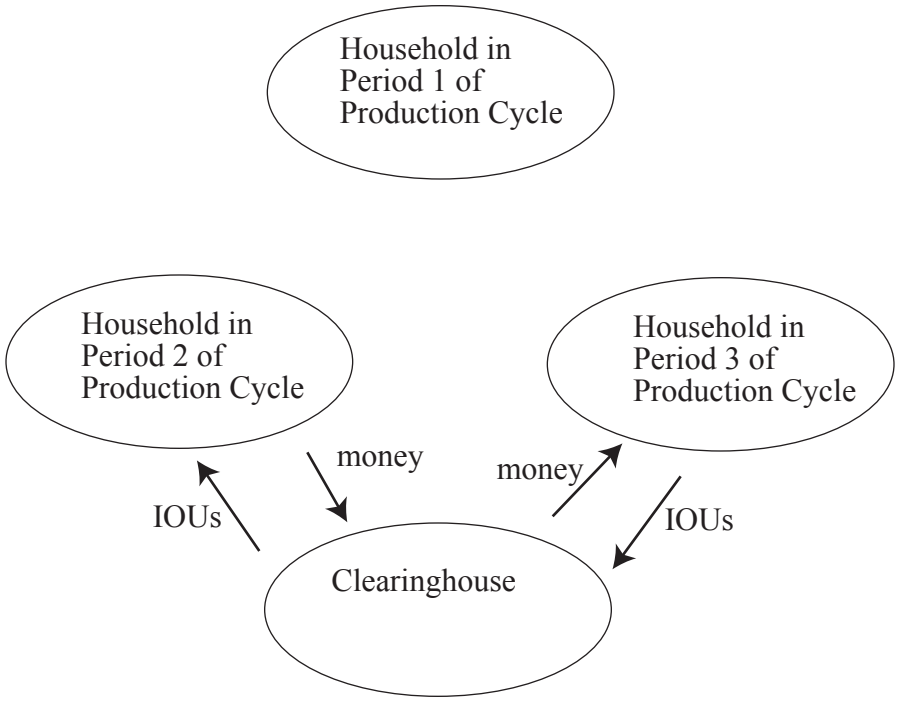


Figure 2: Transactions With the Clearinghouse Arrangement

Beginning of the Period



End of the Period