Payments Systems Design in Deterministic and Private Information Environments*

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Abstract

We study two dynamic models of payments systems. The first is a deterministic model with spatial separation where we consider alternative payments systems institutions and their implications for allocation and welfare. A payments system with period-by-period settlement in outside money improves on decentralized exchange using currency by reducing average cash balances and the average holding period of money. However, an efficient allocation is supported by a payments system with interbank credit where settlement is not

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imposed. In the second model, there is random matching and unobserved productivity shocks and preference shocks. We explore how risk should be shared optimally in this economy and interpret the results in terms of how actual payments systems should operate.

1. INTRODUCTION

Given advances in information technology, the volume of payments system transactions has recently increased greatly in the United States. For example, in terms of electronic payments by consumers, nominal payments by credit card increased by 72.6% from 1993 to 1997 in the United States, and nominal payments by debit card increased by 688.8%. The value of electronic interbank payments has also increased markedly over the same period, with nominal payments over the CHIPS system increasing 38.4% and nominal payments by Fedwire increasing 38.9% (see Bank for International Settlements 1998).

The increased flow of transactions through the payments system has raised questions about the optimal design of payments systems and the appropriate role for central banks in these systems (see Greenspan 1996 and Rochet and Tirole 1996a, 1996b). In particular, primary concerns are over whether settlement should take place on gross or net terms, how frequently settlement should take place, what constraints should be placed on within-settlement-period credit balances, and how payment services and credit should be priced.

To date there are few explicit models of economies where payments systems play an important role. Important contributions are by Freeman (1996a, 1996b, 1998) and related work by Kahn and Roberds (1998), Fujiki, Green, and Yamazaki (1997), Lacker (1997), and Rochet and Tirole (1996a). Our goal here is to construct dynamic models in which we can study different payments institutions and their efficiency properties.

We consider two related models. In the first model, we consider an environment with spatial separation and an absence-of-double coincidence problem, which bears some relation to Townsend (1980), though spatial itineraries and assumptions about communication are quite different, and we include production. We first suppose that there is no centralized payments system and that exchange takes place only using fiat money, which implies that the households in this economy essentially face endogenous cash-in-advance constraints. In equilibrium, there is price dispersion across markets (goods cannot be transported between locations), and the allocation is inefficient.

An arrangement that improves welfare for all agents, but is still not Pareto optimal, is to have a centralized payments system with net settlement in outside money at the end of each period. Here, though goods cannot be moved across locations, centralized payments improves on decentralized monetary exchange due to the fact that there is within-period credit. As a result, households can reduce their average cash balances and the average holding period of money.

An efficient arrangement in this environment involves having centralized payments and inter-period credit with no settlement imposed other than that households meet their lifetime budget constraints. In this arrangement, households (we can interpret representative households as banks) hold an account balance with a central clearinghouse, earning interest on positive balances and paying interest on negative balances at the same competitively-determined interest rate. Another interpretation of this arrangement is that the clearinghouse is a central bank which pays interest on reserves and charges interest on central bank loans at the same rate.

In the second model, we study an environment related to the first, but with random matching and idiosyncratic shocks which are private information. Here, we wish to study how idiosyncratic risk is allocated optimally, and to provide an interpretation of this optimal risk-sharing arrangement in terms of how real-world payments systems should work. The allocation problem for a social planner in the random

matching environment can be converted into a dynamic contracting problem under private information, following Aiyagari and Williamson (1996). This dynamic contracting problem can then be approached using methods developed in Atkeson and Lucas (1992, 1995), Aiyagari and Alvarez (1995), and Aiyagari and Williamson (1998). A key difference in this environment is that there is private information about productivity shocks as well as about preference shocks.

In this model, private information limits insurance, in that consumption and labor supply are not smoothed perfectly across states of the world as would be the case under full information. Also, as is the case in other dynamic contracting environments, the expected utility of an individual household will fluctuate over time. Here, this will affect the sequence of households that is randomly matched with this agent in transactions. That is, if an individual household receives a string of good luck, which is a string of positive productivity shocks here, then this is bad lack for the households that happen to be randomly matched with the good-luck household.

The random-matching private-information model implies that, to solve payments system incentive problems, credit and insurance should be an integral part of the payments system mechanism. An efficient payments system imposes endogenous credit limits, and insures each participant against the event that they cannot settle a transaction.

The remainder of the paper is organized as follows. In Section 2 we study the deterministic model, and then turn to the random matching model with private information in Section 3. Section 4 is a conclusion.

2. A DETERMINISTIC MODEL

This is a spatial model which contains frictions which will motivate transactions intermediated by currency and "interbank" payments mechanisms. There is a countable infinity of locations, indexed by $i = -\infty$, ..., $-1, 0, 1$, ..., ∞. Location i is inhabited

by a representative household, consisting of two members, a *producer* and a *shopper*. Each household has preferences given by

$$\sum_{t=0}^{\infty} \beta^t [u(c_t) - z(n_t)],$$

where $0 < \beta < 1$, c_t is consumption, and n_t is labor supply. Assume that $u(\cdot)$ is strictly increasing, strictly concave, twice continuously differentiable, and satisfies decreasing absolute risk aversion, with $u(0) = 0$ and $u'(0) = \infty$. Also assume that $z(\cdot)$ is strictly increasing, convex, and twice continuously differentiable with $z(0) = z'(0) = 0$.

The representative household at location i consumes only the goods produced by the household at location $i + 1$, so that there is an absence of double coincidence of wants. One unit of labor produces one unit of the non-storable consumption good when a household is productive, and produces zero units when the household is unproductive. Consumption goods cannot be transported between locations except by shoppers returning with goods that have been purchased. During a period, the timing is such that shoppers first leave their home locations and travel to the next location (i.e. shopper i travels to location $i + 1$), following which production takes place, there is exchange, and shoppers then return to their initial locations.

There are three types of households, denoted $j = 1, 2, 3$. Type j households reside at locations $..., j-3, j, j+3, ...$. The producer in a type 1 household is unproductive in periods $t = 2, 5, 8, 11, ...$, and productive in all other periods, the producer in a type 2 household is unproductive in periods $t = 1, 4, 7, 10, ...$, and the producer in a type 3 household is unproductive in periods $t = 0, 3, 6, 9, ...$. This then implies that type 1 households consume only in periods $t = 0, 1, 3, 4, 6, 7, ...$, type 2 households consume only in periods $t = 0, 2, 3, 5, 6, 8, 9, ...$, and type 3 households consume only in periods $t = 1, 2, 4, 5, 7, 8, ...$.

In each period, $\frac{2}{3}$ of households will be producing and $\frac{2}{3}$ of households will be consuming, however $\frac{1}{3}$ of households will be consuming and not producing, $\frac{1}{3}$ will be

consuming and producing, and $\frac{1}{3}$ will be producing and not consuming. Thus, each household follows a three-period *payment cycle* whereby in the first period of the cycle it consumes and does not produce, in the second period it consumes and produces, and in the third period it does not consume and produces. To help the reader visualize the patterns of trade among households, Figure 1 shows how exchange takes place among the three types of agents. If we take three consecutive periods where type 1 households consume in the first and second periods, then type 2 households consume in the first and third periods, and type 3 households consume in the second and third periods. In the model, in contrast to Figure 1, there is not a circular pattern of exchange, since the chain is broken between type 3 and type 1 agents, and the pattern is repeated over the double infinity of locations.

This environment has the following two key elements. First, households cannot engage in barter, since spatial separation and the pattern of meetings implies an absence-of-double-coincidence-of-wants problem. Second, households cannot trade personal IOUs, as there is an intertemporal absence of double coincidence problem for households who meet, and an IOU could not circulate as it would never find its way back to the issuer for redemption. Trade can take place only through trade in fiat money, or through a centralized "payments system."

Pareto Optimal Allocations

Here, we will suppose that the social planner weights the utility of each household type equally, and confine attention to allocations where households of a given type produce the same quantity and consume the same quantity in a given period. Let c_t^i (n_t^i) denote the consumption (production) of a type i household in period t. Then, the social planner solves

$$\sum_{t=0}^{\infty} \beta^t \left[\sum_{i=1}^{3} u(c_t^i) - \sum_{i=1}^{3} z(n_t^i) \right] \tag{1}$$

subject to

$$c_t^i = n_t^{(i+1)(\mathrm{mod}\,3)}$$

$$n_t^i = 0,\ i = 1 \text{ and } t = 0, 3, 6, ...,\ i = 2 \text{ and } t = 1, 4, 7, ...,\ i = 3 \text{ and } t = 2, 5, 8, ...$$

The solution is then

$$c_t^i = n_t^{(i+1)(\mathrm{mod}\,3)} = c^*,$$

for $i = 1$ and $t = 0, 1, 3, 4, 6, 7, ...,$ $i = 2$ and $t = 0, 2, 3, 5, 6, ...,$ $i = 3$ and $t = 1, 2, 4, 5, 7, 8, ...,$ where c^* solves

$$u'(c^*) = z'(c^*), \tag{2}$$

and $c_t^i = n_t^j = 0$ otherwise.

Monetary Exchange Without a Centralized Payments System

To show how the equilibrium allocation changes with alternative payments technologies, we first consider a case where there is no means for communication among households other than through the movements of shoppers between adjacent locations. Fiat money is distributed in some arbitrary fashion among households in period $t = 0$, and we will suppose for now that the aggregate stock of money then remains fixed for all time at M. This environment is then essentially a cash-in-advance economy where households accumulate cash during production periods in order to consume during consumption periods. We will refer to this institutional arrangement as *cash-in-advance* (CIA).

Given a fixed money supply, consider a steady state symmetric equilibrium where each household faces the same prices in each period of the three-period payments cycle. That is, the shopper purchases consumption goods at the price p_1 (in terms of money) when the household does not produce, consumption goods are purchased at price p_2 and sold at price p_1 in a period when the household produces and consumes,

and consumption goods are sold at price p_2 in a period when the household produces but does not consume. Therefore, when the household consumes and does not produce, the Bellman equation associated with the household's optimization problem is

$$v_1(m_1) = \max_{c_1, m_2 \geq 0} [u(c_1) + \beta v_2(m_2)] \tag{3}$$

subject to

$$c_1 \leq m_1 \tag{4}$$

$$c_1 + \frac{p_2 m_2}{p_1} \leq m_1, \tag{5}$$

Here, c_1 denotes consumption during period 1 of the payments cycle, m_i denotes real cash balances at the beginning of period i of the payments cycle, for $i = 1, 2$, $v_i(\cdot)$ values real cash balances at the beginning of period i of the payments cycle, (4) is the cash-in-advance constraint, and (5) is the budget constraint. Similarly, in period 2 of the payments cycle, when the household produces and consumes, the Bellman equation associated with the household's problem is

$$v_2(m_2) = \max_{c_2, n_2, m_3 \geq 0} [u(c_2) - z(n_2) + \beta v_3(m_3)] \tag{6}$$

subject to

$$c_2 \leq m_2 \tag{7}$$

$$c_2 + m_3 \leq m_2 + \frac{p_1 n_2}{p_2}, \tag{8}$$

and in period 3 of the payments cycle we have

$$v_3(m_3) = \max_{n_3, m_1 \geq 0} [-z(n_3) + \beta v_1(m_1)] \tag{9}$$

subject to

$$\frac{p_1 m_1}{p_2} \leq m_3 + n_3, \tag{10}$$

In equilibrium we will have

$$c_1 = n_2, \ c_2 = n_3. \tag{11}$$

Assuming that the value functions are strictly concave and differentiable, and letting λ_i denote the Lagrange multiplier associated with the cash-in-advance constraint for periods $i = 1, 2$ of the payment cycle, and μ_i the multiplier associated with the budget constraint for $i = 1, 2, 3$, we have the following first-order conditions from problems (3), (6), and (9):

$$u'(c_1) - \lambda_1 - \mu_1 = 0, \tag{12}$$

$$\beta v_2'(m_2) - \frac{p_2}{p_1}\mu_1 = 0, \tag{13}$$

$$u'(c_2) - \lambda_2 - \mu_2 = 0, \tag{14}$$

$$-z'(n_2) + \frac{p_1}{p_2}\mu_2 = 0, \tag{15}$$

$$\beta v_3'(m_3) - \mu_2 = 0, \tag{16}$$

$$-z'(n_3) + \mu_3 = 0, \tag{17}$$

$$\beta v_1'(m_1) - \frac{p_1}{p_2}\mu_3 = 0. \tag{18}$$

We also have the following envelope conditions:

$$v_i'(m_i) = \lambda_i + \mu_i, \text{ for } i = 1, 2, \tag{19}$$

$$v_3'(m_3) = \mu_3. \tag{20}$$

Lemma 1 *Some cash-in-advance constraint must bind in equilibrium, i.e. we cannot have* $\lambda_1 = \lambda_2 = 0$.

Proof. Suppose that $\lambda_1 = \lambda_2 = 0$. Then, given (11)-(20), we obtain the following four equations which c_1 and c_2 must satisfy in equilibrium:

$$\beta u'(c_2) - \frac{p_2}{p_1}u'(c_1) = 0, \tag{21}$$

$$-z'(c_1) + \frac{p_1}{p_2}u'(c_2) = 0, \tag{22}$$

$$\beta z'(c_2) - u'(c_2) = 0, \tag{23}$$

$$\beta u'(c_1) - \frac{p_1}{p_2} z'(c_2) = 0. \tag{24}$$

But then, (21), (23), and (24) imply that

$$(\beta^3 - 1)u'(c_1) = 0,$$

which holds if and only if $\beta = 1$, a contradiction.□

Lemma 2 $\lambda_1 = 0$.

Proof. Suppose not. Then, (4) holds with equality, and (5) then implies that $m_2 = 0$. Then, from (7), $c_2 = 0$ which cannot hold in equilibrium as $u'(0) = \infty$. We therefore have a contradiction.□

>From Lemmas 1 and 2, we can conclude that $\lambda_1 = 0$ and $\lambda_2 > 0$, that is the cash-in-advance constraint binds in equilibrium only in the second period of the payments cycle. Thus, cash accumulated from production in the second and third periods of the payments cycle is spent on consumption goods in the first and second periods of the payments cycle, and the entire stock of cash is exhausted at the end of period 2 of the payments cycle.

Now, let $p \equiv \frac{p_2}{p_1}$. We can use (11)-(20) to derive the following four equations, which solve for c_1, c_2, p, and λ_2:

$$\beta u'(c_2) - p u'(c_1) = 0, \tag{25}$$

$$u'(c_2) - \lambda_2 - p z'(c_1) = 0, \tag{26}$$

$$\beta z'(c_2) - p z'(c_1) = 0, \tag{27}$$

$$\beta u'(c_1) - \frac{1}{p} z'(c_2) = 0. \tag{28}$$

Then, it is straightforward to show that the solution to (25)-(28) is

$$c_1 = c_2 = \hat{c}, \tag{29}$$

$$p = \beta, \tag{30}$$

$$\lambda_2 = (1 - \beta^3)u'(\hat{c}) > 0, \tag{31}$$

where $\hat{c}$ solves

$$\beta^2 u'(\hat{c}) - z'(\hat{c}) = 0. \tag{32}$$

Thus, from (2) and (32), we have $\hat{c} < c^*$, and the equilibrium allocation is not Pareto optimal. Note that the equilibrium exhibits price dispersion, that is $p_1 > p_2$, so that prices are higher in exchanges between households in period 1 of the payments cycle and households in period 2 of the payments cycle, versus. exchanges between households in period 2 of the payments cycle and households in period 3 of the payments cycle. There are two effects at work here. First, shoppers in the first type of exchange are not cash-constrained while shoppers in the second are, which tends to lower p_2 relative to p_1. Second, producers in the first type of exchange will not be able to spend the money they receive for two periods, while producers in the second type of exchange need only wait one period. This acts to reduce production in the first type of exchange and to increase p_1 relative to p_2.

We can solve for m_i, $i = 1, 2, 3,$ from (5), (7), (8), (29), and (30), to get

$$m_1 = (1 + \beta)\hat{c}, \tag{33}$$

$$m_2 = \hat{c}, \tag{34}$$

$$m_3 = \frac{\hat{c}}{\beta}. \tag{35}$$

We can then solve for prices by using (30) and the equilibrium condition for the money market,

$$p_1(1 + \beta)\hat{c} + p_2\hat{c} + p_2\frac{\hat{c}}{\beta} = M,$$

to obtain

$$p_1 = \frac{M}{2(1 + \beta)\hat{c}},$$

$$p_2 = \frac{\beta M}{2(1+\beta)\hat{c}}.$$

The velocity of money is then

$$V_c = \frac{2\hat{c}}{\sum_{i=1}^{3} m_i} = \frac{2\beta}{2\beta + \beta^2 + 1}.$$

Note that the symmetric equilibrium that we have constructed is a steady state equilibrium, but it is a stationary equilibrium given the correct distribution of money balances among types at $t = 0$.

Banking with Settlement at the End of Each Period

Now, suppose that there is a centralized clearing arrangement among the households, which can be interpreted as the clearing of transactions among banks, where there is one bank for each location. As before, consumption goods cannot be transported between locations, except when shoppers go to adjacent locations. However, outside money can now be transferred among locations at the end of each period through the clearing mechanism, which we will denote the *clearinghouse*.

At the beginning of each period, shoppers go to the next location and exchange IOUs (interpreted as checks or electronic messages) for consumption goods. At the end of the period, IOUs are received by the clearinghouse, and these debts must be settled on net by each household with the clearinghouse, in outside money. The key difference between this arrangement and the previous one is that the household does not face a cash-in-advance constraint in the second period of the payments cycle, when it consumes and produces. However, note that there is still a cash-in-advance constraint in the first period of the payments cycle. Thus, what changes from the previous section is that we can drop (7), and set $\lambda_2 = 0$ in (14). We will refer to the institutional arrangement here as *payments system with settlement (*PSS). It is important that this be a net settlement system. If households are required to engage in gross settlement, then the arrangement is exactly the same as with CIA.

We now have to take account of the nonnegativity constraint on money balances at the beginning of period 3 of the payments cycle. That is, problem (6) is solved subject to

$$m_3 \geq 0, \tag{36}$$

in addition to (8). Note, that we did not need to impose (36) for the CIA setup, as it was satisfied given the cash-in-advance constraint (7). Along the lines of Lemma 1, it is straightforward to show that at least one of constraints (4) and (36) must bind in a steady state symmetric equilibrium. Let δ denote the Lagrange multiplier associated with constraint (36). Then there are three possibilities. First, we could have $\lambda_1 \geq 0$ and $\delta = 0$, so that the first period cash-in-advance constraint binds and the nonnegativity constraint on end-of-period money balances in the second period does not bind; second, we could have $\lambda_1 \geq 0$ and $\delta \geq 0$, so that both constraints bind; third, we could have $\lambda_1 = 0$ and $\delta \geq 0$, so that the cash-in-advance constraint does not bind but the nonnegativity constraint does. In the first case, period 1 consumption is financed by cash accumulated from sales in periods 2 and 3, and period 2 consumption is financed from period 2 sales. In the second case, period 1 consumption is financed from cash accumulated from sales in period 3, and period 2 consumption is financed from period 2 sales. Finally, in the third case, period 1 consumption is financed by cash accumulated from period 3 sales, and period 2 consumption is financed by cash accumulated from period 3 sales and from period 2 sales. We need to consider each case in turn.

Case 1: $\lambda_1 \geq 0$, $\delta = 0$.—

Here, we use (11)-(20) to obtain the following three equations, which solve for c_1, c_2, and p :

$$-z'(c_1) + \frac{1}{p}u'(c_2) = 0, \tag{37}$$

$$\beta z'(c_2) - pz'(c_1) = 0, \tag{38}$$

$$\beta u'(c_1) - \frac{1}{p} z'(c_2) = 0. \tag{39}$$

In contrast to the CIA case, a simple solution is not available here, and we will need to resort to local analysis. Note first that if $\beta = 1$, then (37)-(39) (for the PSS case) and (25), (27), and (28) (for the CIA case) solve to give $c_1 = c_2 = c^*$ and $p = 1$, where $u'(c^*) = z'(c^*)$. Now, if we totally differentiate (37)-(39), we can solve to get

$$\frac{dc_1}{d\beta} = \frac{2u'}{z'' - u''} > 0, \tag{40}$$

$$\frac{dc_2}{d\beta} = \frac{-u'}{z'' - u''} < 0, \tag{41}$$

$$\frac{dp}{d\beta} = \frac{(z'' + u'')}{(u'' - z'')},$$

where all derivatives are evaluated at $\beta = 1$. Thus, for β close to 1, we will have $c_1 < c^*$ and $c_2 > c^*$, but p may be greater or less than 1 (depending on the sign of $z'' + u''$). As with the CIA case, the equilibrium allocation is not Pareto optimal, and prices are different across locations. Here, however, we may have $p_1 < p_2$ or $p_1 > p_2$. From (4) and (8), we have $m_3 = \frac{c_1}{p} - c_2$, so for (36) to be satisfied, we must have $\frac{c_1}{p} - c_2 \geq 0$. Similar to the above, for β close to 1 this constraint will be satisfied if and only if

$$3u' + c^*(z'' + u'') \leq 0. \tag{42}$$

We can solve for λ_1 to get $\lambda_1 = u'(c_1) - \frac{\beta}{p} u'(c_2)$, and it is straightforward to show that, given the solution above, $\lambda_1 \geq 0$ as required, if β is close to 1.

Using (4), (5), (8), and (10) with equality, velocity is given by

$$V_{p1} = \frac{c_1 + c_2}{m_1 + m_2 + m_3} = \frac{c_1 + c_2}{(\frac{1}{p} + 1)c_1 - c_2}$$

When $\beta = 1$, since (37)-(39) and (25), (27) each solve to give $c_1 = c_2 = c^*$ and $p = 1$, we have $V_c = \frac{1}{2}$ and $V_{p1} = 2$. Thus, by continuity, for β close to 1 velocity is approximately 4 times higher with PSS than with CIA in this first case. A payments

system with net settlement allows households to reduce their average cash holdings and to reduce the average holding period of money, which then increases velocity.

Case 2: $\lambda_1 \geq 0$, $\delta \geq 0$.—

We take a similar approach here to case 1, obtaining the following three equations which solve for c_1, c_2, and p in the symmetric steady state:

$$-z'(c_1) + \frac{1}{p}u'(c_2) = 0, \tag{43}$$

$$\beta u'(c_1) - \frac{1}{p}z'(c_2) = 0, \tag{44}$$

$$c_2 - \frac{1}{p}c_1 = 0. \tag{45}$$

Then, totally differentiating (43)-(45) and solving, we get

$$\frac{dc_1}{d\beta} = \frac{u'(-u' - c^*u'')}{(-z'' + u'')[2u' + c^*(z'' + u'')]}, \tag{46}$$

$$\frac{dc_2}{d\beta} = \frac{u'(-u' - c^*z'')}{(-z'' + u'')[2u' + c^*(z'' + u'')]}, \tag{47}$$

$$\frac{dp}{d\beta} = \frac{-u'}{2u' + c^*(z'' + u'')}.$$

For this to be a symmetric equilibrium for β close to 1, we need $\lambda_1 \geq 0$ and $\delta \geq 0$, or

$$\frac{3u' + c^*(2z'' + u'')}{2u' + c^*(z'' + u'')} \geq 0, \tag{48}$$

$$\frac{3u' + c^*(z'' + 2u'')}{2u' + c^*(z'' + u'')} \geq 0, \tag{49}$$

respectively. Further, velocity in case 2 is given by

$$V_{p2} = \frac{c_1 + c_2}{c_1},$$

so that, as in case 1, $V_{p2} = 2$ when $\beta = 1$.

Case 3: $\lambda_1 = 0$, $\delta > 0$.—

Here, taking a similar approach to the first two cases, we obtain the following three equations which solve for c_1, c_2, and p:

$$\beta u'(c_2) - pu'(c_1) = 0,$$

$$-z'(c_1) + \frac{1}{p}u'(c_2) = 0,$$

$$\beta u'(c_1) - \frac{1}{p}z'(c_2) = 0,$$

and totally differentiating and solving we get

$$\frac{dc_1}{d\beta} = \frac{-u'}{z''}, \tag{50}$$

$$\frac{dc_2}{d\beta} = \frac{2u'}{z''}, \tag{51}$$

$$\frac{dp}{d\beta} = \frac{2u'' + z''}{z''}.$$

In contrast to case 1, where $c_1 < c^*$ and $c_2 > c^*$ for β close to 1, here we will have $c_1 > c^*$ and $c_2 < c^*$ for β close to 1. This is due to the fact that there is a binding nonnegativity constraint on money balances at the end of period 2 of the payments cycle. That is, in period 2 of the payments cycle, households would like to borrow to finance current consumption, but they are constrained from doing so.

The condition $\delta \geq 0$ is always satisfied here, but to guarantee that the cash-in-advance constraint holds in period 1 of the payments cycle, i.e. $c_1 \leq m_1$, we must have

$$3u' + c^*(2u'' + z'') \leq 0. \tag{52}$$

Velocity is given by

$$V_{p3} = \frac{c_1 + c_2}{(1+p)c_2 - \frac{1}{p}c_1},$$

and, as in the other two cases, we have $V_{p3} = 2$ when $\beta = 1$.

An Example.—

Inspection of inequalities (42), (48), (49), and (52) indicates that there is a possibility of multiple symmetric steady states with a payments system with settlement. An example will show what can happen. Suppose that $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$ and $z(n) = n$, where $\gamma > 0$. Then, the Pareto optimum has $c_1 = c_2 = c^* = 1$. In the CIA case, we have

$$\begin{aligned} p &= \beta, \\ c_1 &= c_2 = \beta^{\frac{2}{\gamma}}. \end{aligned}$$

With PSS, case 1, we get

$$\begin{aligned} p &= \beta, \\ c_1 &= \beta^{\frac{2}{\gamma}}, \\ c_2 &= \beta^{-\frac{1}{\gamma}}, \end{aligned}$$

and this equilibrium exists if and only if $\gamma \geq 3$. With PSS, case 2, the solution is

$$\begin{aligned} p &= \beta^{\frac{1}{\gamma-2}}, \\ c_1 &= \beta^{\frac{\gamma-1}{\gamma(\gamma-2)}}, \\ c_2 &= \beta^{-\frac{1}{\gamma(\gamma-2)}}, \end{aligned}$$

and this equilibrium exists if and only if $\gamma \leq \frac{3}{2}$ or $\gamma \geq 3$. For PSS, case 3, we get

$$\begin{aligned} p &= \beta^{-2}, \\ c_1 &= \beta^{-\frac{1}{\gamma}}, \\ c_2 &= \beta^{\frac{2}{\gamma}}, \end{aligned}$$

and this equilibrium exists if and only if $\gamma \geq \frac{3}{2}$.

Therefore, with PSS, for $\gamma < 3$ there exists a unique symmetric steady state equilibrium, but for $\gamma \geq 3$ there are three such equilibria.

Welfare.—

We would like to compare the welfare properties of PSS versus. CIA equilibria, and also see if we can arrive at a ranking of PSS equilibria for the cases where there is more than one. The welfare measure we will use is the welfare function (1) which the social planner maximizes in determining the Pareto optimal allocation. Thus, for a symmetric steady state allocation, welfare is given by

$$W = u(c_1) + u(c_2) - z(c_1) - z(c_2).$$

As in much of the above, we do a local analysis for β close to 1, given that the equations characterizing equilibria yield the same solution of $c_1 = c_2 = c^*$ and $p = 1$ for $\beta = 1$. If we differentiate the above welfare function with respect to β, and evaluate derivatives at $\beta = 1$, we obtain

$$\frac{dW}{d\beta} = (u' - z')(\frac{dc_1}{d\beta} - \frac{dc_2}{d\beta}) = 0,$$

so that the first-order effect on welfare is zero. However, the second derivative of the welfare function, evaluated at $\beta = 1$, is

$$\frac{d^2W}{d\beta^2} = (u'' - z'')\left[\left(\frac{dc_1}{d\beta}\right)^2 + \left(\frac{dc_2}{d\beta}\right)^2\right] < 0, \tag{53}$$

so knowing the derivatives $\frac{dc_1}{d\beta}$ and $\frac{dc_2}{d\beta}$ allows us to compare welfare across equilibria for β close to 1.

Now, let

$$\phi_C = \left(\frac{dc_1}{d\beta}\right)^2 + \left(\frac{dc_2}{d\beta}\right)^2,$$

evaluated at $\beta = 1$, for the CIA case, and similarly let ϕ_{Pi} apply to PSS case i for $i = 1, 2, 3$. Then, from (29), (32), (40), (41), (46), (47), (50), and (51), we get

$$\phi_C = \frac{8(u')^2}{(u'' - z'')^2}, \tag{54}$$

$$\phi_{P1} = \frac{5(u')^2}{(u'' - z'')^2}, \tag{55}$$

$$\phi_{P2} = \frac{(u')^2\left[(u'+c^*u'')^2+(u'+c^*z'')^2\right]}{(u''-z'')^2\left[2u'+c^*(z''+u'')\right]^2}, \tag{56}$$

$$\phi_{P3} = \frac{5(u')^2}{(z'')^2}. \tag{57}$$

It is then clear from (54), (55), and (57) that

$$\phi_{P1} < \phi_C, \tag{58}$$

and

$$\phi_{P1} < \phi_{P3}. \tag{59}$$

Also, we have

$$\phi_C - \phi_{P3} = \frac{5(u'')^2 - 10z''u'' - 3(z'')^2}{(z'')^2(u''-z'')^2} > 0 \tag{60}$$

if the case 3 PSS equilibrium exists, i.e. if (52) holds. Further, we have

$$\phi_{P1} - \phi_{P2} = \frac{(3u'+2c^*z''+c^*u'')\,(6u'+4c^*u''+2c^*z'')}{(u''-z'')^2\left[2u'+c^*(z''+u'')\right]^2} > 0 \tag{61}$$

if the case 2 PSS equilibrium exists, i.e. if (48) and (49) hold. Therefore, from (58)-(61) we have

$$\phi_{P2} < \phi_{P1} < \phi_{P3} < \phi_C,$$

and from (53) the welfare ranking of equilibria is then the reverse of the above inequalities, i.e. the welfare ranking is (from low to high): CIA, PSS case 3, PSS case 1, PSS case 2.

Here, the payments system with settlement yields an efficiency gain, due to the fact that households reduce average money holdings and the average holding period of money. However, the payments system also introduces the possibility of multiple steady state equilibria. Though all equilibria dominate the CIA arrangement in welfare terms, they can be ranked in terms of welfare, so there is a clear multiplicity problem.

Banking with Interbank Lending

Suppose now that payments are arranged so that, instead of settling at the end of each period with outside money, as above, households (i.e. banks) settle by borrowing and lending on an interbank market. This can also be interpreted as an arrangement where each household carries an account balance with the clearinghouse, with the account balance increasing when goods are sold to other households and decreasing when the household makes purchases. Further, we could think of the clearinghouse as a central bank, with banks at each location holding interest-earning reserve balances at the central bank and taking out central bank loans.

We wish to show that this arrangement can support the Pareto optimal allocation as a steady state symmetric equilibrium. If the interest rate on clearinghouse account balances is $\frac{1}{\beta} - 1$, then it is optimal for each household to produce c^* in productive periods and to consume c^* when consumption is possible. Letting b_i denote the household's net position against the clearinghouse at the beginning of period i of the payments cycle in the steady state symmetric equilibrium, from the household's budget constraints we have, when the interest rate is $\frac{1}{\beta} - 1$,

$$b_1 = \frac{c^*(1-\beta^2)}{1-\beta^3},$$

$$b_2 = \frac{-c^*\beta(1-\beta)}{1-\beta^3},$$

$$b_3 = \frac{-c^*(1-\beta)}{1-\beta^3},$$

and so $\sum_{i=1}^{3} b_i = 0$, and this is a steady state equilibrium. Here, the household borrows in the first period of the payments cycle to finance consumption, sells just enough output in period 2 of the payments cycle to finance period 2 consumption, and accumulates credits from sales in period 3 of the cycle so as to come into period 1 of the cycle with a positive account balance.

This payments system works efficiently solely through the exchange of inside money, and it is unnecessary for payments to be settled in outside money. Indeed, note that there is no settlement in the steady state symmetric equilibrium, since there will be no period in which all households have a zero account balance with the clearinghouse. We could impose some kind of settlement rule, but this would only imply inefficiency. For example, suppose that there is a rule that households must settle with the clearinghouse once in each payment cycle. Without loss of generality, let settlement occur at the ends of periods 2, 5, 8, 11, ... That is, at the end of each settlement period, each household is required to hold a zero account balance with the clearinghouse. This constraint must bind on some households in a steady state symmetric equilibrium, and therefore leads to Pareto inefficiency.

A Pareto optimum could certainly be supported in other ways than the arrangement with a clearinghouse and interbank lending. For example, there are versions of the Friedman rule which would yield efficiency under CIA or PSS. This would in general require that the appropriate money transfers be made in a type-specific fashion. The Friedman rule solution here suffers from the usual problems. First, equilibrium can be indeterminate when the monetary authority is behaving optimally. Second, permitting the monetary authority to make individual-specific transfers appears to violate the spirit of the spatially-separated environment.

3. A MODEL WITH IDIOSYNCRATIC RISK AND PRIVATE INFORMATION

In the model above, it is efficient for the payments system to operate without outside money and to have settlement postponed to the indefinite future. However, in many discussions of payments system design and policy, the allocation of risk plays a central role (e.g. Rochet and Tirole 1996a, 1996b, Fujiki, Green and Yamazaki 1997), and Federal Reserve policy towards payments by Fedwire is to move towards

real time gross settlement of payments with outside money, in the belief that this implies optimal risk-bearing (see Greenspan 1996).

In the model we study next, agents make transactions at dispersed locations subject to unobserved idiosyncratic risk. We wish to study the nature of optimal risk sharing in this setting, with the view that this will be useful in telling us how risk should be shared optimally within a payments system. The model bears some relation to the deterministic setup in the previous section in that exchange is not possible without some kind of monetary or payments arrangement. The key differences will be that households are randomly matched over time with would-be trading partners, and there are idiosyncratic shocks which are private information.

There is a continuum of households with unit mass, half of whom are *private-information households* (type 1), with the remaining half being *full-information households* (type 2). A household of type i has preferences given by

$$E_0(1-\beta_i)\sum_{t=0}^{\infty}\beta_i^t[\theta_{1t}u(c_t)-z(n_t)],$$

where the functions $u(\cdot)$ and $z(\cdot)$ have the same properties assumed previously. For private information households, θ_{1t} is a preference shock. Private-information households also receive a productivity shock, θ_{2t}, where output is produced by the household according to $y_t = \theta_{2t}n_t$. Assume that $\theta_t = (\theta_{1t}, \theta_{2t})$ is an i.i.d. draw from the distribution $F(\theta_t)$. There is an upper bound on labor supply, that is $n_t \leq h$, where $h > 0$, and the consumption good is perishable. Further, we assume that $\beta_1 < \beta_2$.[1] Full-information households always have $\theta_{1t} = \theta_{2t} = 1$.

As in the previous setup, households consist of producer/shopper pairs. Each shopper from a private (full) information household is randomly matched with a

[1] There will be a limiting distribution of expected utilities with mobility if and only if the risk neutral agent is more patient than the risk averse agent (see Aiyagari and Williamson 1996 and Williamson 1998).

producer from a full (private) information household each period. Households cannot produce for themselves.

Efficient Allocations

As in the deterministic model, we could consider allocations here as equilibrium allocations with different kinds of monetary/payments institutions. However, to concentrate on optimal risk-sharing implications, we will suppose that the payments system operates as if there were a central planner who can communicate with all households in the economy, but cannot transfer goods across locations (i.e. between pairs of matched agents). The planner takes the initial distribution of expected utilities across private information households, $\psi_0(w)$, as given and maximizes the initial expected utilities of full information households, w^* (restricted to be the same for each full information household). The planner thus wishes to write efficient contracts with private information households and, as in Aiyagari and Williamson (1996), the fact that in each meeting there is only a one-sided asymmetry of information, and that idiosyncratic shocks are i.i.d., implies that the planner's contracting problem can be solved as if a private information household were paired with the same full information household forever.

We denote consumption and production of the private information agent at t by $c_t(w_0, \theta^t)$ and $n_t(w_0, \theta^t)$ respectively, where $\theta^t = (\theta_0, \theta_1, \ldots, \theta_t)$ denotes an agent's history of reports to date t and w_0 is the private information agent's date 0 expected utility entitlement.

Definition 1 *For each given w_0, and all t, θ', an allocation that is feasible, incentive compatible, and satisfies promise-keeping is given by a pair (c, n) is $\{c_t(w_0, \theta^t), n_t(w_0, \theta^t\}_{t=0}^{\infty}$ such that:*

$$w_0 = E_0(1 - \beta_1) \sum_{t=0}^{\infty} \beta_1^t \left\{ \theta_{1t} u[c_t(w_0, \theta^t)] - z[n_t(w_0, \theta^t)] \right\}$$

$$
\begin{aligned}
& E_t \sum\nolimits_{s=t}^{\infty} \beta_1^{s-t} \left\{ \theta_{1s} u[c_s(w_0, \theta^s)] - z[n_s(w_0, \theta^s)] \right\} \\
\geq \; & \theta_{1t} u[c_t(w_0, \theta_0, \ldots, \theta_{t-1}, \theta')] - z[n_t(w_0, \theta_0, \ldots, \theta_{t-1}, \theta')] \\
& + E_t \sum\nolimits_{s=t+1}^{\infty} \beta_1^{s-t} \left\{ \begin{array}{c} \theta_{1s} u[c_s(w_0, \theta_0, \ldots, \theta_{t-1}, \theta', \theta_{t+1}, \ldots, \theta_s)] \\ -z[n_s(w_0, \theta_0, \ldots, \theta_{t-1}, \theta', \theta_{t+1}, \ldots, \theta_s)] \end{array} \right\}, \\
& \text{for all } t, \ \theta^t, \ \text{and } \theta',
\end{aligned}
$$

$$
0 \leq c_t(w_0, \theta^t), n_t(w_0, \theta^t) \leq h
$$

In the above definition, the first equation is the promise-keeping constraint, the second inequality describes the temporary incentive compatibility constraints, and the last inequality captures the feasibility constraint.

An allocation (c, n) attains $\psi_0(w)$ for the private information agents with expected utility for the full information agents $\phi(\psi_0)$ if

$$
\phi(\psi_0) = (1 - \beta_2) \sum_{t=0}^{\infty} \beta_2^t \int \int \left\{ u[\theta_{2t} n_t(w_0, \theta^t)] - z[c_t(w_0, \theta^t)] \right\} dF(\theta^t) d\psi_0(w).
$$

Next, define maximized expected utility ϕ^* by

$$
\phi^* = \sup_{c,n} \phi(\psi_0).
$$

Definition 2 *An allocation (c, n) is efficient if it attains $\psi_0(w)$ at a maximum expected utility for the full information agents.*

Bellman equation

We consider a special case of the above model which will allow us to relate this environment to the deterministic model studied in the previous section. Consider the case where there are 3 states for a private information household. In state 1, $\theta_t = (1, 0)$

(the household consumes and does not produce), in state 2, $\theta_t = (1,1)$ (the household consumes and produces), and in state 3, $\theta_t = (0,1)$ (the household produces and does not consume). Following Aiyagari and Williamson (1996) and Atkeson and Lucas (1994), the social planner's problem can be solved as a set of component planning problems, where it is as if a given private information household and full information household met forever. The private information household comes into the period with an expected utility promise w from the social planner, and receives consumption $c_i(w)$ in state i, produces $n_i(w)$, and receives continuation expected utility $w_i(w)$.

Clearly, it is efficient to have $c_3(w) = 0$ as the private information household receives no utility from consumption in state 3 and this cannot hurt incentives. We must have $n_1(w) = 0$ as zero labor supply is all that is feasible in state 1. Thus, the promise-keeping constraint for the planner is

$$w = \frac{1}{3}\left\{(1-\beta_1)\left[u(c_1(w)) + u(c_2(w)) - z(n_2(w)) - z(n_3(w))\right] + \beta_1 \sum_{i=1}^{3} w_i(w)\right\}. \tag{62}$$

To induce truth-telling, the contract must satisfy a set of incentive compatibility conditions. Assuming that $n_i(w) > 0$ for $i = 2,3$, it is impossible for a private information household in state 1 to report states 2 or 3, as it cannot produce. For it to be in the interest of a household in state 2 to report the truth we must have

$$(1-\beta_1)[u(c_2(w)) - z(n_2(w))] + \beta_1 w_2(w) \geq (1-\beta_1)u(c_1(w)) + \beta_1 w_1(w), \tag{63}$$

$$(1-\beta_1)[u(c_2(w)) - z(n_2(w))] + \beta_1 w_2(w) \geq -(1-\beta_1)z(n_3(w)) + \beta_1 w_3(w), \tag{64}$$

and similarly for an agent in state 3,

$$-(1-\beta_1)z(n_3(w)) + \beta_1 w_3(w) \geq -(1-\beta_1)z(n_2(w)) + \beta_1 w_2(w), \tag{65}$$

$$-(1-\beta_1)z(n_3(w)) + \beta_1 w_3(w) \geq \beta_1 w_1(w). \tag{66}$$

Consumption and labor supply must satisfy

$$0 \leq c_i(w), n_j(w) \leq h, \text{ for } i = 1, 2, \ j = 2, 3. \tag{67}$$

The lowest possible expected utility that can be imposed on a private information agent is $w = 0$, since consumption can be reduced to zero, but households cannot be forced to work as they can always report that the state is 1. The highest possible level of expected utility is what the household receives if consumption is always h and labor supply is zero, that is $w = \frac{2}{3}u(h) = \bar{w}$. The continuation expected utilities in the contract must then satisfy

$$w_i(w) \in [0, \bar{w}], \text{ for } i = 1, 2, 3. \tag{68}$$

An efficient contract is then the solution to the following Bellman equation

$$v(w) = \max \frac{1}{3} \left\{ \begin{array}{c} (1-\beta_2)[-z(c_1(w)) - z(c_2(w)) + u(n_2(w)) + u(n_3(w))] \\ +\beta_2 \sum_{i=1}^{3} v(w_i(w)) \end{array} \right\} \tag{69}$$

subject to (62)-(68).

Next, we wish to establish some properties of the value function $v(\cdot)$, and the policy functions $c_i(w)$, $i = 1, 2$, $n_i(w)$, $i = 2, 3$, and $w_i(w)$, $i = 1, 2, 3$. That a unique $v(\cdot)$ exists that is bounded, continuous, decreasing, and concave follows from applying Theorems 9.6, 9.7, and 9.8 in Stokey, Lucas, and Prescott (1989).

Proposition 1 $w_1(w) \leq w_2(w)$.

Proof. Suppose $w_1(w) > w_2(w)$ at the optimum. Then, increase $w_2(w)$ and decrease $w_1(w)$ by the same small amount. This relaxes constraints (63), (64), and (68), and so (63)-(68) continue to hold. Further, (62) holds. Since $v(w)$ is a concave function,

the value of the objective function on the right-hand side of (69) increases, which is a contradiction.□

Proposition 2 $w_1(w) \leq w_3(w)$.

Proof. This follows immediately from the incentive constraint (66).□

Proposition 3 $c_1(w) \leq c_2(w)$.

Proof. Suppose that $c_1(w) > c_2(w)$ at the optimum. Then, reduce $c_1(w)$ and increase $c_2(w)$ such that (62) continues to hold. This relaxes constraints (63) and (64), so that (62)-(68) hold. Further, given the convexity of $z(\cdot)$ and the strict concavity of $u(\cdot)$, the value of the objective function on the right-hand side of (69) increases, a contradiction.□

Proposition 4 *If β_1 is sufficiently large, then $n_2(w) \geq n_3(w)$ and $w_2(w) \leq w_3(w)$.*

Proof. First, if $n_2(w) = n_3(w)$ and $w_2(w) > w_3(w)$, or if $n_2(w) < n_3(w)$ and $w_2(w) = w_3(w)$, or if $n_2(w) < n_3(w)$ and $w_2(w) > w_3(w)$ then the incentive constraint (65) does not hold. Next, suppose that $n_2(w) > n_3(w)$ and $w_2(w) > w_3(w)$. If $\beta_1 = 0$, then (65) holds, but if $\beta_1 = 1$, then (65) does not hold. Now, $(1-\beta_1)[-z(n_3(w)) + z(n_2(w))] + \beta_1[w_3(w) - w_2(w)]$ is continuous in β_1, and it is decreasing in β_1 given that $n_2(w) > n_3(w)$ and $w_2(w) > w_3(w)$. Therefore, for β_1 sufficiently large, (65) does not hold if $n_2(w) > n_3(w)$ and $w_2(w) > w_3(w)$. Finally, if $n_2(w) < n_3(w)$ and $w_2(w) < w_3(w)$, then for $\beta_1 = 1$ the incentive constraint (64) does not hold. Then, by continuity (64) does not hold for β_1 sufficiently large if $n_2(w) < n_3(w)$ and $w_2(w) < w_3(w)$. We therefore conclude that, if β_1 is sufficiently large, then $n_2(w) \geq n_3(w)$ and $w_2(w) \leq w_3(w)$ are necessary in order for the incentive constraints (64) and (65) to hold.□

Propositions 1-4 tell us that, if β_1 is sufficiently large, then the optimal contract between the social planner and an individual private information household satisfies several intuitively-appealing properties. We can think of state 1 as corresponding to a state of the world where the private information household makes a transaction (consumes), but cannot settle the transaction in the current period (cannot produce). It is optimal for the payments system to insure against this state by permitting the household to make a transaction in this state even though the household cannot settle, but there is an incentive problem in that the planner does not want households in states 2 and 3 to claim that they are state 1 households so as to avoid producing. The incentive problems are solved by allowing the state 2 household to make a larger transaction than the state 1 household ($c_2(w) \geq c_1(w)$), and permitting the state 2 household and the state 3 household to make larger transactions and produce less in the future than does the state 1 household ($w_2(w) \geq w_1(w)$ and $w_3(w) \geq w_1(w)$). There are further incentive problems in that the planner does not want a state 3 household to misreport that it is a state 2 household, and vice-versa. Provided that the discount factor of a private information household is sufficiently high, these incentive problems are solved by having the state 3 household produce less and receive a higher continuation utility than the state 2 household. Thus, the ability to produce during the current period always increases a household's current consumption and its claims to future consumption, and if the household does not consume during the current period this reduces its current production and increases future claims on consumption.

Examples

To illustrate the features of an optimal contracts, we compute two examples, using methods similar to those in Aiyagari and Williamson (1999) and Williamson (1998). We choose a constant absolute risk aversion utility function, $u(c) = 1 - e^{-\alpha c}$, where

$\alpha > 0$, and a quadratic disutility of labor function, $z(n) = \gamma n^2$, where $\gamma > 0$, and set $\alpha = \gamma = 1$.

In the first example, we choose $\beta_1 = .5$ and $\beta_2 = .6$. Also, we choose $h = 1$, but the constraint on hours worked did not bind in the solution. Here, since the discount factor of the private information household is small, intertemporal incentives will not work well. The binding incentive constraints are (except at the lower boundary on expected utilities) (63) and (65), that is the key incentive problem in the example involves keeping producers from reporting that they are non-producers. In Figure 2, note that consumption is higher in state 2 than in state 3, which induces the household in state 2 to report the truth. Also, labor supply is higher in state 2 than in state 3. The incentive constraint (65) does not bind, so this cannot be the reason why $n_2 > n_3$. However, since (63) and (65) bind, a household in state 2 can then be induced to report the truth by giving her more consumption in state 2 than in state 1 and requiring her to work less in state 2 than might otherwise be the case. For an agent in state 3, within-period incentives can be generated only by lowering n_3, so we have $n_2 > n_3$.

In Figure 3, we have $w_1 < w_2 < w_3$, which again is the result of binding incentive constraints. The continuation utility of a household in state 3 is higher than for a household in state 1 to provide the state 3 household intertemporal incentives to report the truth. The same applies to state 2 versus. state 1. We have $w_2 < w_3$ as the household in state 2 receives an intratemporal incentive which the household in state 3 does not, i.e. $c_1 < c_2$. Thus, intertemporal incentives are used more for the state 3 household.

Figure 4 shows the limiting distribution of expected utilities for private information households. Note that a significant fraction of households are "credit constrained," i.e. they are at the lower bound on expected utilities, and they consume zero in state 1 when they cannot produce.

In the second example, we use the same period utility function and have higher discount factors, i.e. $\beta_1 = .99$ and $\beta_2 = .991$. In Figure 5 we show consumption and labor supply for private information households. Here, note that risk-sharing is much better than in the previous example, due to the fact that, with a higher discount factor for private information households, intertemporal incentives are more effective. Figure 6 shows the resulting limiting distribution of expected utilities for private information households.

Discussion

Households in this economy are tied together through randomly constructed chains of transactions. The structure of the optimal allocation mechanism implies imperfect risk-sharing among households, so that one household's bad or good luck will affect the households that are engaged with that household in transactions, indefinitely into the future. For example, if a household has a long string of periods with an urgent need to consume, this will imply that the household's claims on consumption through the payments system will be lower than otherwise in the future, and that the household must supply more output than otherwise to the payments system. That is, the household's future trading partners will receive more consumption and have to produce less than would otherwise be the case. Thus, payments system risk is shared through the chain of transactions.

In some policy discussions of payments system design (e.g. Greenspan 1996) it seems to be taken as given that it is optimal to minimize payments system credit balances and the risk borne by the payments system as a whole. A message that should be taken from this model is that the optimal mechanism for solving payments system incentive problems involves credit and risk-bearing in an important way. Payments system participants should not be encouraged to seek credit and insurance contracts outside the payments system; in fact, the payments system will work more efficiently

if credit contracts and insurance contracts are an integral part of the relationship among the participants.

4. CONCLUSION

Here, we have looked at two models which capture different aspects of payments system design. In a deterministic model with spatial separation and an absence-of-double-coincidence problem, we have shown how a payments system can improve efficiency by substituting for currency transactions. The model illustrates the role of net settlement systems and shows why, in an environment without risk, imposing settlement is inefficient. In a random matching model with private information, we modeled an efficient payments system as a solution to a social planner's dynamic contracting problem. Credit arrangements and insurance within the payments system were key to mitigating the incentive problems implied by private information. Key elements of an efficient payments system were credit limits and insurance against the inability to settle transactions.

We think that these models are potentially useful for addressing other problems in monetary economics and in the economics of payments systems. For example, the deterministic model with spatial separation might be used to analyze the role of central bank intervention through the payments system. As well, the random matching model with private information might be extended to address issues associated with payments system risk. For example, suppose there is idiosyncratic risk and transactions relationships take a form similar to those in the first model, with repeated transactions between the same households rather than random matching. Then it is possible that idiosyncratic shocks could be propagated from household to household and over time, potentially with amplification of these shocks. The question then is whether such "systemic risk" phenomena represent a policy problem or not.

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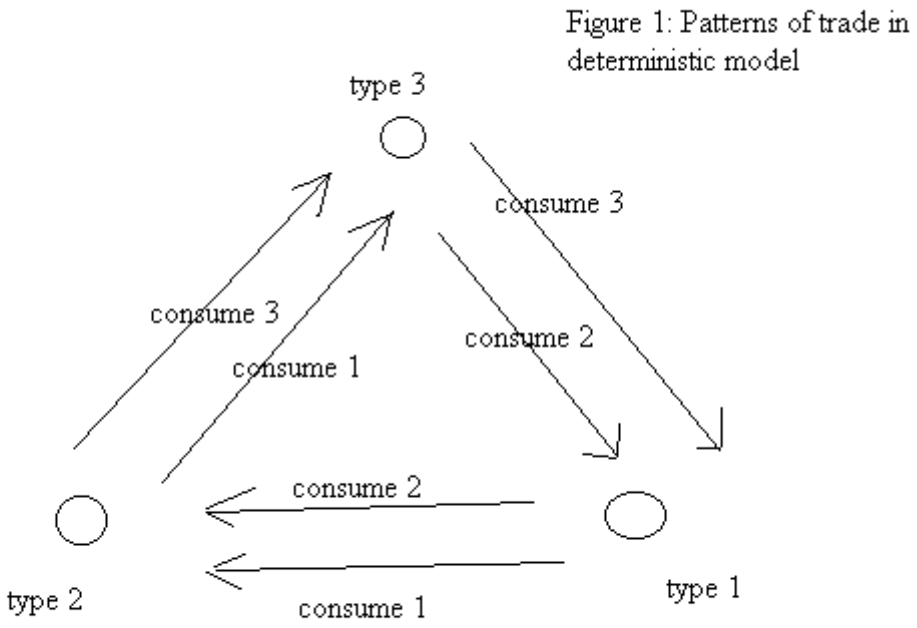
Figure 1: Patterns of trade in deterministic model
type 3
consume 3
consume 3
consume 1
consume 2
consume 2
type 2
consume 1
type 1

FIG. 1.

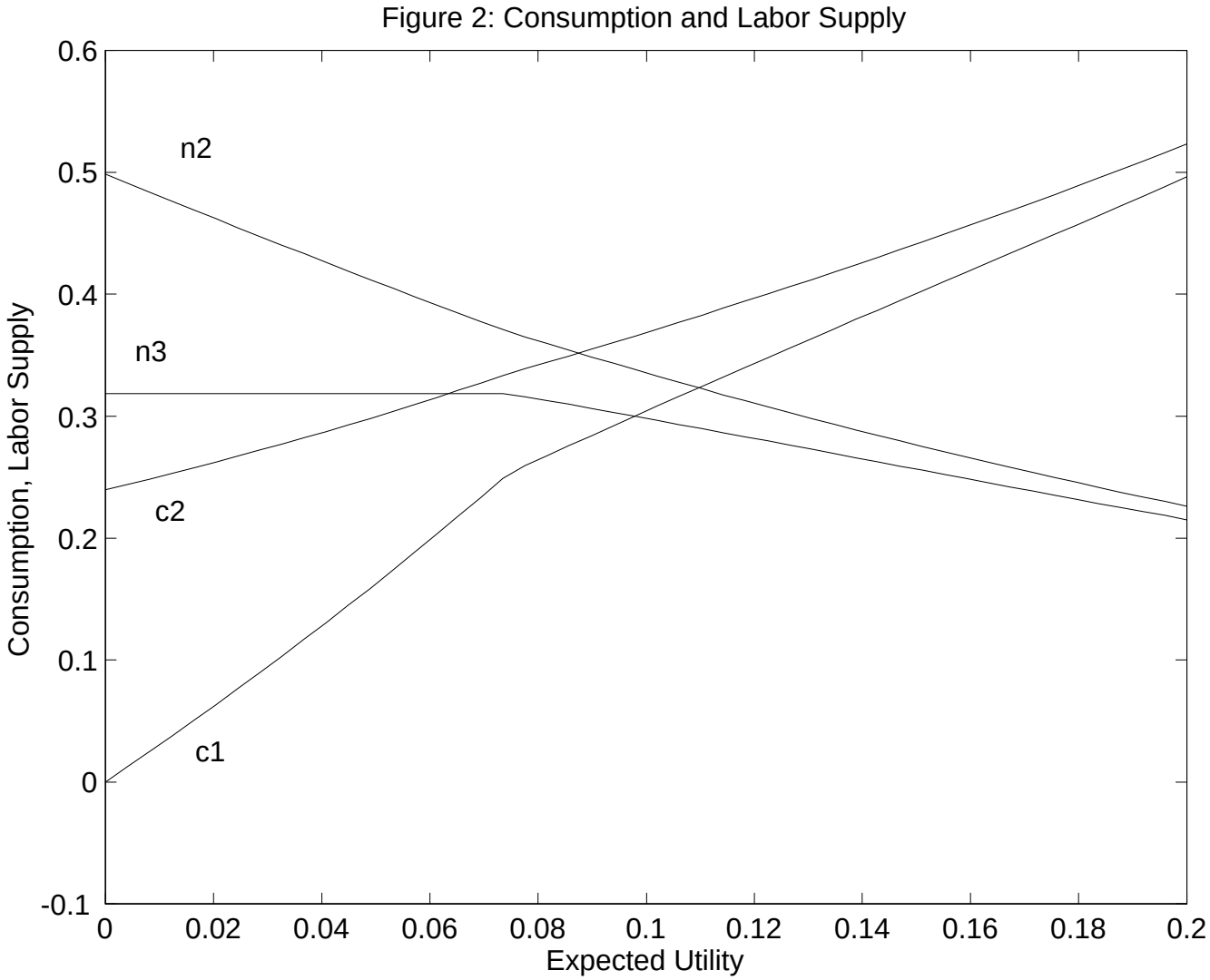
Figure 2: Consumption and Labor Supply
Consumption, Labor Supply
Expected Utility
n2
n3
c2
c1
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2

FIG. 2.

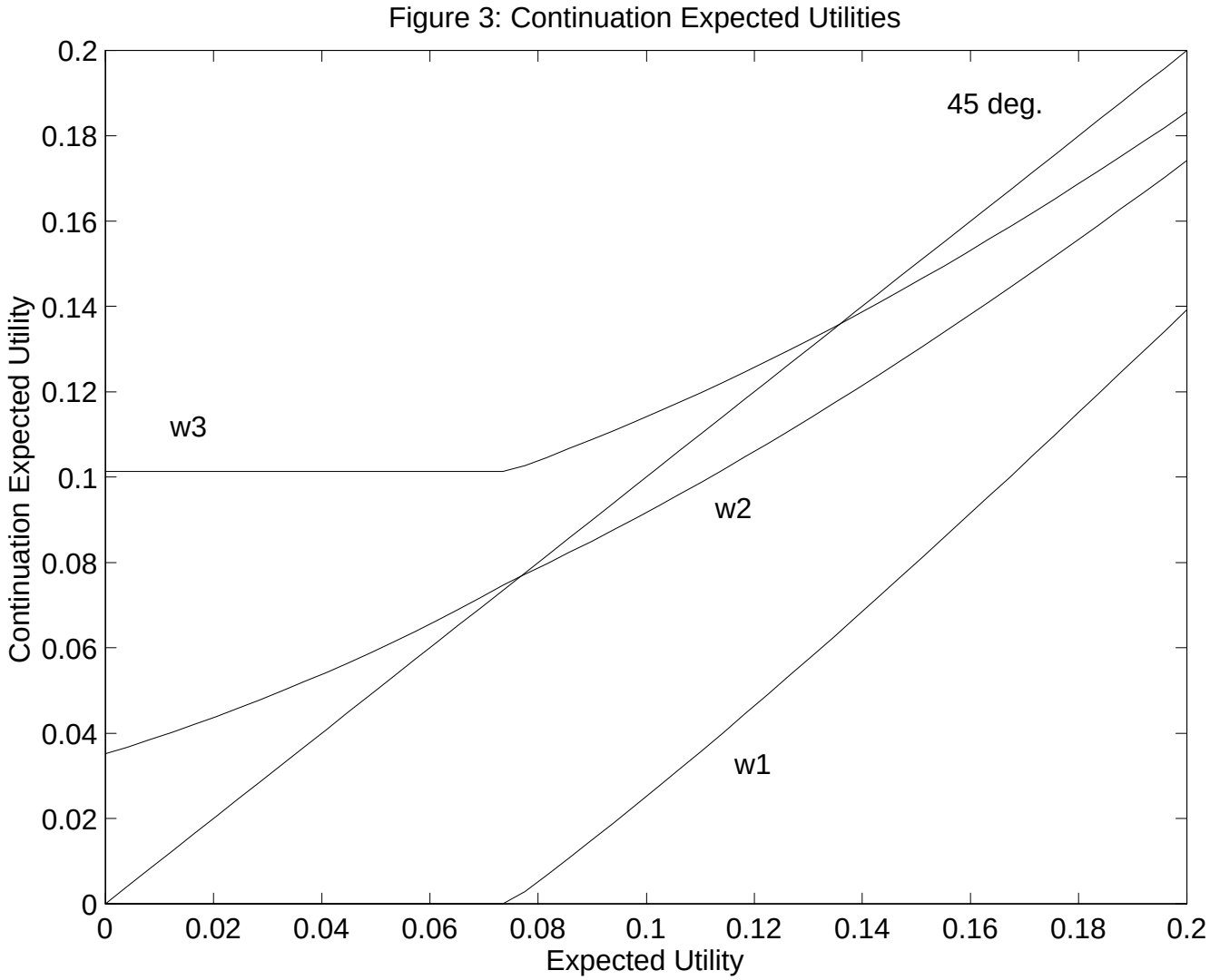
Figure 3: Continuation Expected Utilities
Continuation Expected Utility
Expected Utility
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
45 deg.
w3
w2
w1

FIG. 3.

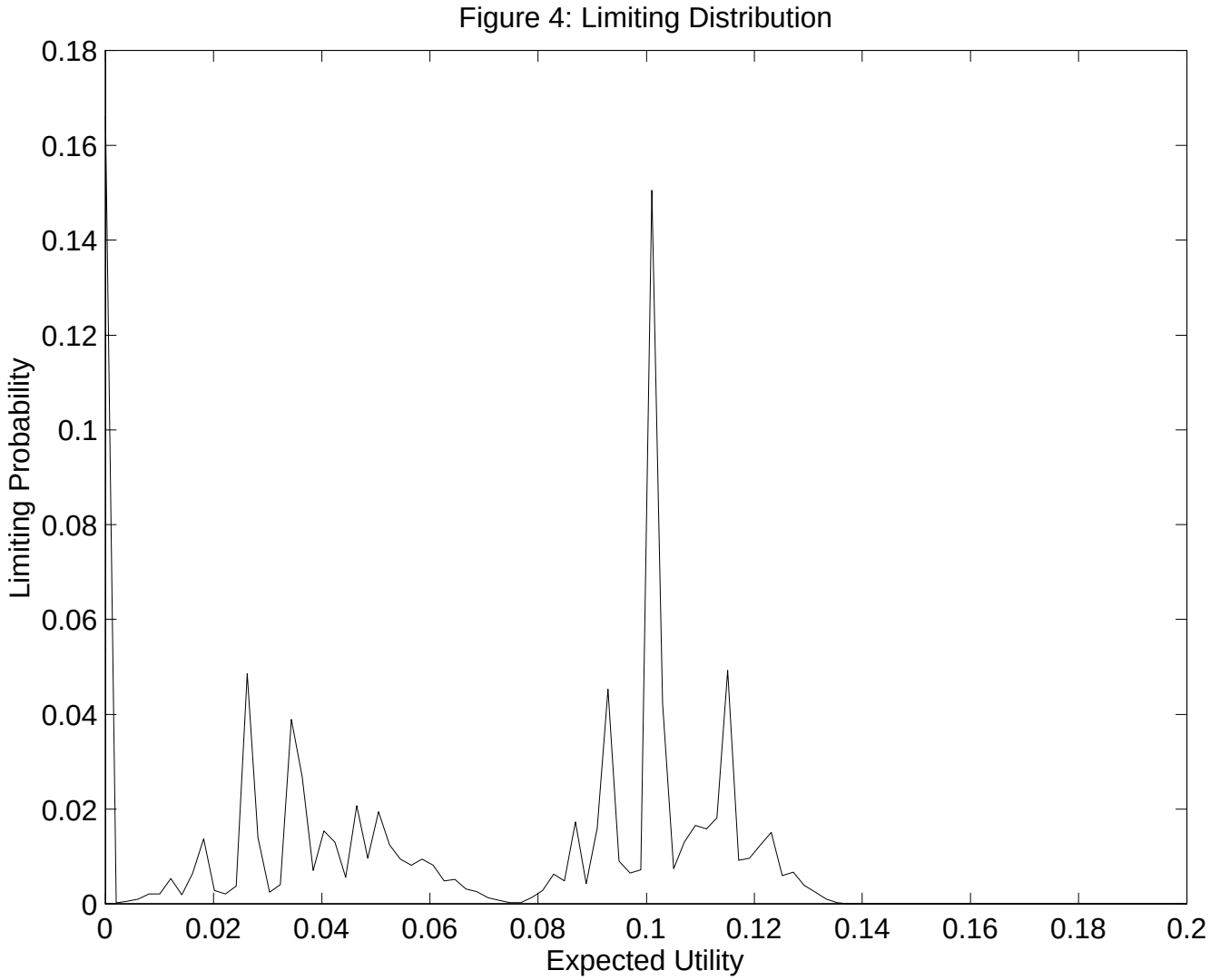
Figure 4: Limiting Distribution
Limiting Probability
Expected Utility
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2

FIG. 4.

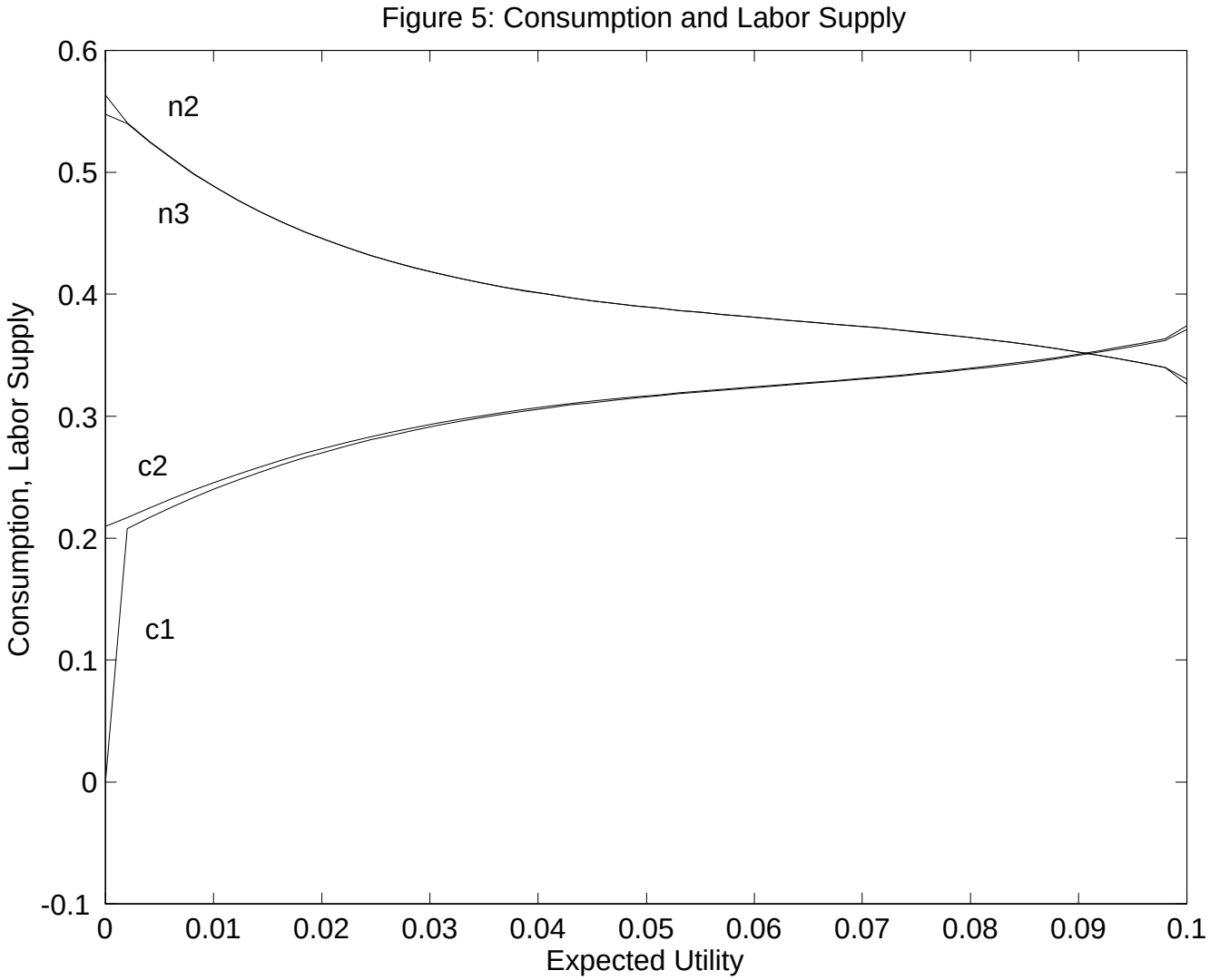
Figure 5: Consumption and Labor Supply
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
Consumption, Labor Supply
n2
n3
c2
c1
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
Expected Utility

FIG. 5.

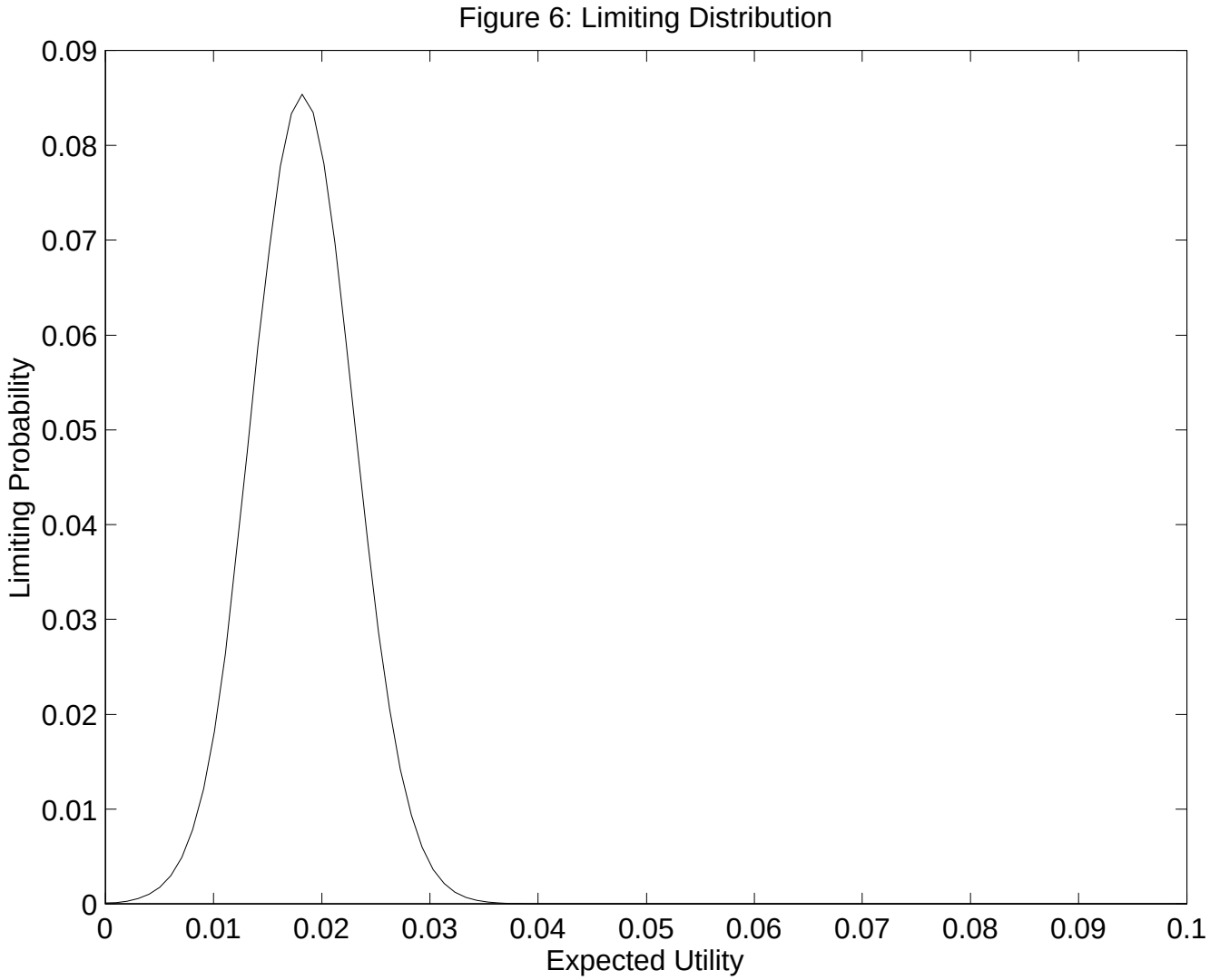
Figure 6: Limiting Distribution
Limiting Probability
Expected Utility
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1

FIG. 6.